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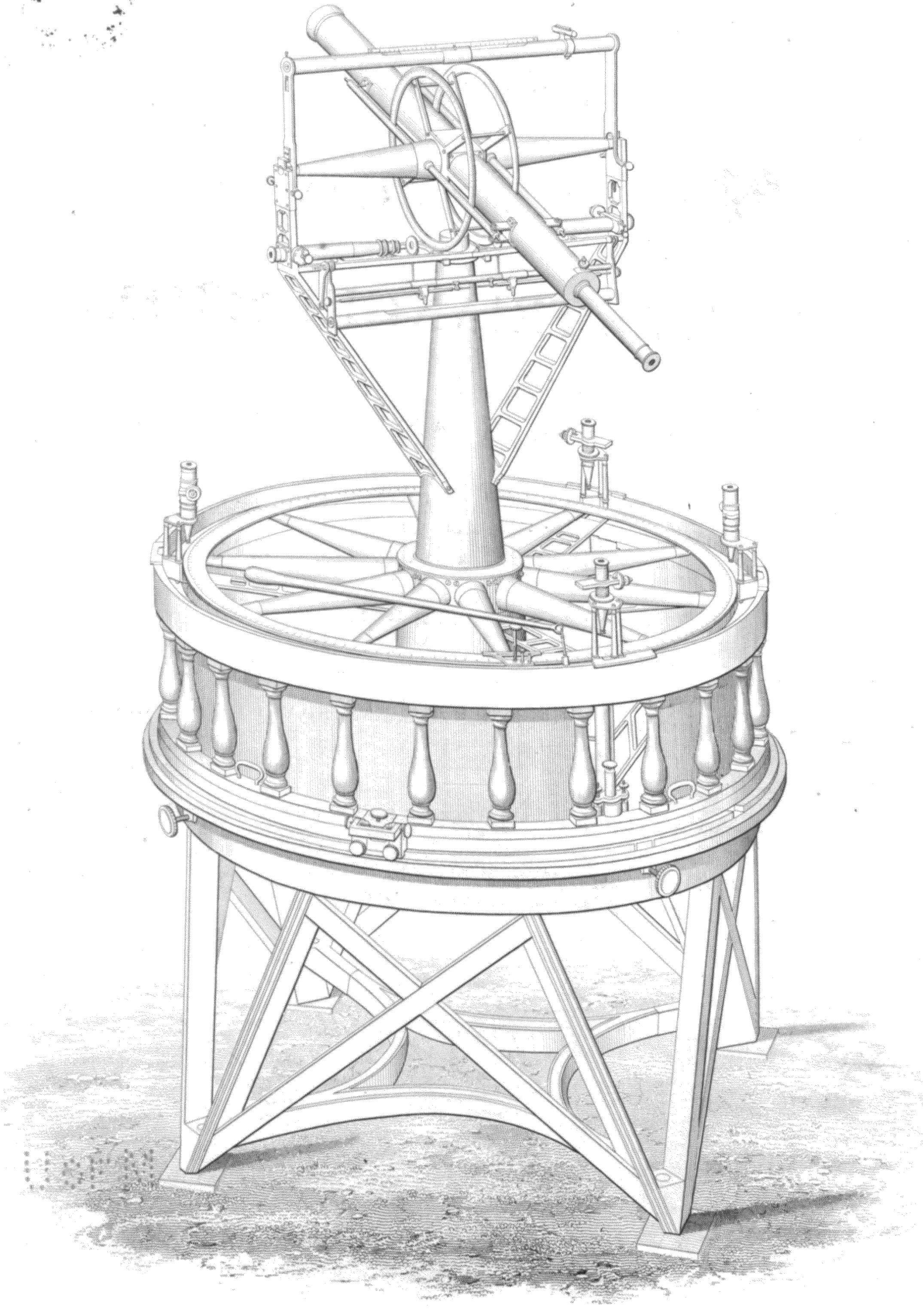


Gt. Brit. Ordnance survey

ORDNANCE TRIGONOMETRICAL SURVEY
OF
GREAT BRITAIN AND IRELAND

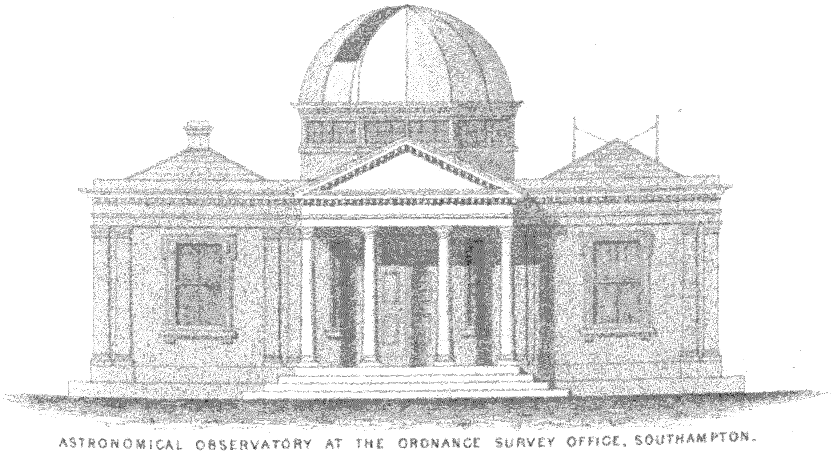
SOLD BY H.M. STATIONERY OFFICE

ACCOUNT
OF THE
OBSERVATIONS AND CALCULATIONS,
OF THE
PRINCIPAL TRIANGULATION;
AND OF THE
FIGURE, DIMENSIONS AND MEAN SPECIFIC GRAVITY,
OF THE
EARTH
AS DERIVED THEREFROM.

PUBLISHED BY ORDER OF THE MASTER-GENERAL AND BOARD OF ORDNANCE.

Drawn up by **CAPTAIN ALEXANDER ROSS CLARKE, R.E. F.R.A.S.** under the direction of

LT. COLONEL H. JAMES, R.E. F.R.S. M.R.I.A. &c.
SUPERINTENDENT OF THE ORDNANCE SURVEY.



ASTRONOMICAL OBSERVATORY AT THE ORDNANCE SURVEY OFFICE, SOUTHAMPTON.

LONDON
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PRINTERS TO THE QUEENS MOST EXCELLENT MAJESTY.
1858.

Rec'd 10-17-35-MEW

TO

THE RIGHT HONORABLE

LORD PANMURE, G.C.B, K.T. &c.

SECRETARY OF STATE FOR WAR.

THIS WORK IS WITH PERMISSION,

RESPECTFULLY DEDICATED

BY

HENRY JAMES,

LT COLONEL, R.E.

SUPERINTENDENT OF THE ORDNANCE SURVEY.

ORDNANCE SURVEY OFFICE, SOUTHAMPTON. 2nd FEBY 1858.

189353

PREFACE.

THE Principal Triangulation of the United Kingdom, which was commenced in 1783, under General Roy, for the more immediate purpose of connecting the Observatories of Greenwich and Paris, and determining the difference of longitude between them, has been gradually extended, under the successive direction of Colonel Williams, General Mudge, General Colby, Colonel Hall, and myself, over the entire United Kingdom, to form the basis of the National Survey; it is now finished, and an account of all the operations connected with the work is given in this volume.

But the details of the measurement of the bases, with a full description of the instruments employed for taking the angles between the Trigonometrical Stations and the Astronomical Observations, and a full account of the successive steps which have been taken in the progress of the work, will be found in the previously published works of the Survey, a list of which is subjoined.

I have also given a list of the names of all the Officers and Non-commissioned Officers who have been personally engaged either in the measurement of the Bases or in taking the Trigonometrical or Astronomical Observations, and I deeply regret that the late Major-General Colby, who for twenty-seven years held the office of Superintendent of the Survey, and under whose able direction the Triangulation was extended over the whole of Ireland and the far greater portion of Great Britain, did not survive to see the results of this great work given to the public.

The voluminous computations connected with the reduction of the observations have been made by Lieutenant-Colonel Yolland, Captain Cameron, and Captain Alexander Clarke, assisted by Quartermaster Young and Mr. O'Farrell, and a great number of computers; but the chief and most important portion of the work, including all the calculations connected with the determination of the Figure, Dimensions, and Mean Specific Gravity of the Earth, has been performed by Captain Clarke.

I gladly avail myself of this opportunity of expressing my best thanks to G. B. Airy, Esq., the Astronomer Royal, for the valuable advice and assistance which we have uniformly received from him during the progress of the work.

HENRY JAMES,
Lieut.-Colonel R.E., and Colonel.

ORDNANCE SURVEY OFFICE,
2nd February 1858.

List of Published Works of the Ordnance Survey.

An Account of the Operations carried on for Accomplishing a Trigonometrical Survey of England and Wales, &c. By Order of the Honourable Board of Ordnance. By Captain Mudge, R.A., Mr. Dalby, and Captain Colby, R.E. 3 vols.

Astronomical Observations made with Ramsden's Zenith Sector, &c. Published by Order of the Honourable Board of Ordnance. 1842.

An Account of the Measurement of the Lough Foyle Base in Ireland, &c. Published by Order of the Honourable Board of Ordnance. By Captain William Yolland, Royal Engineers, F.R.A.S. 1847.

Astronomical Observations made with Airy's Zenith Sector from 1842 to 1850, &c. Published by Order of the Master-General and Board of Ordnance. By Captain William Yolland, Royal Engineers, F.R.A.S. 1852.

Abstracts from the Meteorological Observations taken at the Stations of the Royal Engineers in the Year 1853-4. Edited by Lieut.-Colonel H. James, R.E., F.R.S., M.R.I.A., F.G.S., &c.

Abstracts of Principal Lines of Spirit Levelling in Ireland carried on during the Years 1839 to 1843, &c. Published by Order of the Master-General and Board of Ordnance. Edited by Captain Cameron, R.E. 1855.

Meteorological Observations taken during the Years 1829 to 1852 at the Ordnance Survey Office, Phœnix Park, Dublin. Published by Order of the Honourable Board of Ordnance. Edited by Captain Cameron, R.E. 1856.

Account of the Observations and Calculations of the Principal Triangulation, and of the Figure and Mean Density of the Earth, &c. Published by Order of the Master-General and Board of Ordnance. Edited by Captain Alexander Clarke, Royal Engineers, F.R.A.S. 1858.

Names of the Officers, Non-commissioned Officers, and others who were employed on the Measurement of the Bases.

Bases.	Names.
Hounslow Heath, 1784	Major-General Roy, R.E., Dr. Usher, and Mr. Dalby, assisted occasionally by Lieutenant-Colonel Calderwood, Horse Guards, and Lieutenant-Colonel Pringle, R.E.
Hounslow Heath, 1791—Steel Chain	Colonel E. Williams, R.A., Major-General Mudge, R.A., and Mr. Dalby.
Salisbury, 1794..........................	Major-General Mudge, R.A., and Mr. Dalby.
Misterton Carr, 1801	Major-General Mudge, R.A., and Mr. Woolcot, assisted occasionally by Mr. Berge.
Rhuddlan Marsh, 1806	Major-General Colby, R.E., and Mr. Woolcot.
Belhelvie, 1817	Major-General Colby, R.E., and Mr. Gardner.
Lough Foyle, 1827–8	Major-General Colby, R.E., Lieutenant-Colonel Pringle, R.E., Captains Henderson and Drummond, and Lieutenants Murphy and Mould, R.E.
Salisbury, 1848	Sergeant-Major Steel, R.E., Sergeant Jenkins, R.E., and Corporal Harkin, R.E. Captain Gosset, R.E., was present at the commencement, and Captain Hawkins, R.E., at the termination.

Names of the Officers, Non-commissioned Officers, and others who were employed in observing with the large Instruments.

Officers.	Stations.
Major-General Roy, R.E.	The Triangulation for connecting Greenwich and Dunkirk.
Major-General W. Mudge, R.A.	Hanger Hill; St. Ann's; Hampton Poorhouse; King's Arbour; Banstead; Leith Hill; Butser; Ditchling; Dunnose; Beachy Head; Fairlight; Crowborough; Dean Hill; Four Mile Stone; Old Sarum Gun; Beacon Hill; Wingreen; Blackdown; Pillesdon; Karnminnis; Maker; Hensbarrow; St. Agnes; Pertinny; Karnbonellis; Deadman; Trevose; Mendip; Inkpen.
Major-General T. F. Colby, R.E.	Plynlimmon; Cyrn-y-Brain; Llanelian; Garreg; Delamere; Moelfre Issa; Axedge; Whittle Hill; Ingleboro'; Balta; Kellie Law; Cowhythe; Knock; Mount Battock; Calton Hill; Hart Fell; Dunrich; East Lomond; Glashmeal; Ben Cleugh; Ben Lomond; Corryhabbie; Ben Wyvis; Ben Cheilt; Fair Isle; Foula; Ronas; Yell; Fetlar; Brassa; Fitty; Hanger Hill; Fairlight; Crowborough; Leith Hill; Wrotham; Severndroog Tower; Chingford; Divis; Slieve Donard; Sawel.
Captain J. Vetch, R.E.........	Fair Isle; Foula; Ronas; Yell; Feltar; Brassa; Fitty; Wart Hill Hoy; South Renaldsha; Stronsa; Deerness; Ben Tartevil; Jura; Ben More, Mull; Ben Heynish; Chingford; Berkhampstead; Dunstable.

Officers.	Stations.
Colonel W. R. Ord, R.E.	Divis.
Major J. W. Pringle, R.E.	Slieve Donard.
Lieutenant-Colonel A. W. Robe, R.E.	Ben Hutig; Fashven; Dunnet Head; Corryhabbie; Ben Wyvis; Ben Cheilt.
Captain A. Henderson, R.E.	Divis.
Major-General J. E. Portlock, R.E.	Bencorr; Vicar's Cairn; Trostan; Taur; Tara; Slieve Snaght; Slieve League; Sawel; Nephin; Lyons Hill; Knocknaskagh; Knocklayd; Knockanaffrin; Kippure; Hungry Hill; Howth; Forth; Feaghmaan; Dublin Observatory; Divis; Doolieve; Cuilcagh; Croghan; Carrigfadda; Caherbarnagh; Baurtregaum; Keeper.
Captain T. Drummond, R.E.	Fair Isle; Foula; Ronas; Yell; Fetlar; Brassa; Fitty; Wart Hill Hoy; Stronsa; South Ronaldsha; Deerness; Chingford; Berkhampstead; Dunstable; Divis.
Colonel R. K. Dawson, R.E.	Corryhabbie; Ben Wyvis; Ben Cheilt; Ben Tartevil; Jura; Ben More, Mull; Ben Heynish; Cundtham; Drung Point.
Lieutenant F. H. Murphy, R.E.	Cundtham; Divis; Drung Point; Mount Sandy; Knockalongy.
Lieutenant-Colonel T. A. Larcom, R.E.	Slieve Donard; Slieve League.
Colonel W. Robinson, R.E.	Ben Hutig; Cnoc-Ghiubhais; Fashven; Dunnet Head; Scarabin; Ben Clibrig; Monach; Cleisham; Ben More, S.U.; Clifton Beacon; Bardon Hill.
Captain J. H. Pipon, R.E.	Cnoc-Ghiubhais; Fashven; Dunnet Head; Scarabin; Ben Clibrig; Great Whernside; Botton Head; Whittle Hill; Sea Fell; Cross Fell; Axedge; Snowdon.
Captain P. J. Hornby, R.E.	Scarabin; Ben Clibrig; Monach; Cleisham; Ben More, S.U.; Criffel; Merrick; Ben Lawers; Garforth Cliff.
Lieutenant A. F. H. Da Costa, R.E.	Black Comb; Pendle Hill; Delamere; Lincoln Minster; Ely Minster.
Lieutenant J. B. Luyken, R.E.	Clifton Beacon; Bardon Hill; Precelly; Cradle; Long Mount.
Captain Kater, 12th Regiment	Hanger Hill; Fairlight; Crowborough; Leith Hill; Wrotham; Severndroog Tower; Chingford.
Mr. J. Gardner	Wordeslow; Collier Law; Water Crag; Criffel; Cross Fell; Wisp; Sayrs Law; Lumsden; Mordington; Blackheddon; Mormonth; Little Stirling; Dudwick; Blue Hill; Tarbathy; Layton; Over Hill; Brimmond; Balta; Kellie Law; Cowhythe; Knock; Mount Battock; Calton Hill; Hart Fell; Dunrich; East Lomond; Glashmeal; Ben Cleugh; Ben Lomond; Saxavord; Hanger Hill; Fairlight; Crowborough; Leith Hill; Wrotham; Chingford; Severndroog Tower.
Mr. S. Woolcot	Mendip; White Horse Hill; Broadway Tower; Arbury Hill; Naseby Tower; Bardon Hill; Tilton; Beacon Hill; Dunkery; Balsham; Gringley Beacon; Brandon; Malvern; Precelly; Paracombe; Gwaunysgaer; Burleigh Moor; Easington; Botton Head; Crowle Beacon; Clifton Beacon; Moelfre Issa; Pertinny.
Mons. Arago	Folkstone; Fairlight.

Non-commissioned Officers.	Stations.
Sergeant-Major J. Steel, R.E.	South Berule; Wingreen; Holme Moss; Acklam Wold; Crowle; Norwood; Boston Tower; Naseby Tower; Keysoe Spire; Hingham Tower; Easton Tower; Swaffham Tower; Norwich Spire; Gorleston Tower; St. Peter's Church (Thanet); Thaxted Spire; Danbury Spire; Otley Tower; Naughton Tower; Mickfield Tower; Beachy Head; Boniface Down; Boniface, S.E.; Cyrn-y-Brain; Dunnose; Gerth of Scaw; Great Stirling; High Port Cliff; Little Stirling; Littletown Down; Nodes Beacon; Peterhead, Old W.M.; Saxavord; Shanklin Down; Week Down; Wroxall Down; St. Paul's Cathedral.
Colour-Sergeant Donelan, R.E.	Garforth Cliff; Arbury Hill; Dunstable; Tharfield; Happisburgh; Orford Castle; Lawshall Tower; Leith Hill Tower; Paddlesworth; Frittenfield; Wrotham; Fairlight; Ditchling; Butser Hill; Coringdon; Swyre Barrow; Pillesdon; High Willhays; Ryders Hill; Barrow Hill; Karnminnis; Pertinny; Goonhilly; Deadman; Trevose Head; Maker Tower; Scournalapich; Cowhythe; Mormonth; Mount Battock; Jura; Hanger Hill; Chingford; Severndroog; Banstead; Blackdown; Old Sarum Gun; Old Lodge; Old Sarum Castle; Beacon Hill; Dean Hill; Four Mile Stone; Milk Hill; Dunrich; Goat Fell; Knocknadober.
Colour-Sergeant J. Winzer, R.E.	Blackheddon; Mordington; Lumsden; Sayrs Law; Ben Nevis; Kellie Law; Ben Macdui; Storr; Mamsuil; Ru Rea; Ben Cleugh; Wingreen; Beachy Head; Nodes Beacon; Slieve Donard.
Sergeant J. Mulligan, R.E.......	Tilton.
Sergeant A. Bay, R.E.	Lynn, Old Tower; Docking Tower; Baconsthorpe Tower; Walpole, St. Peter's; Walton Tower; Stoke Tower; Balsham Tower; Epping Cupola; Bunwell Tower; Southwold Tower; South Lopham Tower; Brandon (Suffolk); Tofts Tower; Ely Tower; Laxfield Tower; Gads Hill; York Minster; Easington; Wordeslow; Cheviot; Hart Fell; Burnswark.
Sergeant R. Forsyth, R.E.	Slieve Donard; South Berule.
Sergeant J. Beaton, R.E........	Danbury Spire; Otley Tower.
Sergeant W. Grose, R.E.........	Brandon Down; Collier Law; Goat Fell; Water Crag.
Sergeant W. Jenkins, R.E. ...	Old Lodge; Westbury Down; Upcot Down; Broadway Tower; Ben Lawers; Ballycreen; Corryhabbie; Balta; Gerth of Scaw; Merrick; Mowcopt; Nive Hill; Stoke Hill; Ben More, South Uist.
Corporal T. Cosgrove, R.E. ...	Long Mount; Malvern; White Horse Hill; Inkpen; Dunnose; Wingreen; Cradle.
2nd Corporal J. Stewart, R.E.	Long Mount; Malvern; White Horse Hill; Inkpen; Dunnose; Wingreen; Mendip; Dunkery; Paracombe; Lundy Island; Hensbarrow; Cradle; Slieve Donard.
2nd Corporal J. Wotherspoon, R.E.	Peninnis, W.M.; Telegraph Tower; Brown Willy; Beacon Hill, Trescow; Carn Galver; Cyrn-y-Brain; St. Paul's Cathedral.

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INTRODUCTION.

Standard of Length.

The National or Parliamentary Standard Yard having been destroyed in the fire which consumed the Houses of Parliament in 1834, a commission was issued to G. B. Airy, Esq., Astronomer Royal, Rev. R. Sheepshanks, and Francis Baily, Esq., to make a new National Standard of Length from the data derived from the previous comparisons which had been made between the old Standard Yard and several existing standards, such as the two belonging to the Ordnance Survey, the Royal Astronomical Society's Standard, and others.

The New Standard Yard (of which two copies—one of bronze, No. 29, and the other of wrought iron, No. 55—have been sent to me by the Astronomer Royal) is a bar 38 inches long and 1 inch square in section, with wells at either extremity drilled out to the depth of half the metal; at the bottom of these wells transverse lines are drawn on platina, the distance between which, at the temperature engraved on the bar, is one yard.

The Commissioners have very properly not given the length of the new Standard as compared with any other Standard; they have restored the Standard with the greatest possible accuracy, and the lengths of all others must be found from it by comparison.

The lengths of all the bases and the distances between the Trigonometrical Stations which are given in this volume are in terms of the 10-feet Standard O_1, which is the Standard of the Ordnance Survey; but it is obviously necessary that the length of O_1 and of the trigonometrical distances should be given in terms of the National Standard of Length.

I have therefore had a Standard I O (Intermediate Ordnance) made, on which $3\frac{1}{3}$ lengths of the Standard Yard have been set off, and the 10 feet thus obtained has been compared with the 10-feet Standard O_1. A detailed account of the comparisons will be given in a separate volume; but to obtain the lengths and distances in this volume in terms of the National Standard of Length, we must add to the log. distances the correction + ·0000005 (approximate). The amount of difference in 10 feet is ·00014 inches, and in a mile ·075 inches.

Base Measurement Apparatus.

The base-lines from which all the trigonometrical distances have been computed are those which were measured on Salisbury Plain, and on the shore of Lough Foyle in Ireland: they are respectively 6·93 and 7·89 miles long, and were measured with General Colby's Compensations Bars. The difference between the measured lengths and their lengths as computed through the Triangulation is ± 0·4178 feet, or about 5 inches.

Mean Base.

This difference has been divided in proportion to the square roots of the lengths of the measured bases, from which we have obtained the *Mean Base* which has been used in the Triangulation; there is, therefore, a difference of + or — 0·2 feet, or 2½ inches, between the measured and computed length of these bases.

Four other base-lines were measured with Ramsden's 100-feet Steel Chains,—one on Hounslow Heath, one at Misterton Carr near Doncaster, and one at Belhelvie in Aberdeenshire,—and the greatest difference between the measured lengths and the computed lengths of these bases from the mean base does not amount to 3 inches.

The fourth base was measured in an unfavourable position on Rhuddlan Marsh in North Wales, and the difference between the measured and computed length in this case is 1·596 feet.

Horizontal Angles:

The Horizontal Angles, as well as the Azimuthal Bearings of the Stations were principally obtained from observations with Ramsden's great 3-feet Theodolites; and the perfection of these instruments may be judged of from the fact, that the sum of the angles in the triangles rarely differed 3″·4 from the true sum.

Correction of.

To correct the discrepancies thence arising by the theory of probabilities, so as to render the Triangulation consistent in itself, and as perfect as possible, has been a work of immense labour; but we should not have done justice to a great national work of this kind if we had failed to render it as perfect as possible before giving it to the public.

Lengths of the Sides of the Triangles.

The sum of all the distances or sides in the Principal Triangulation is about 206,710,000 feet, or, in round numbers, exactly ten times the radius of the Earth (radius of the Earth = 20,890,000). The mean length of a side is 35·4 miles. There are 37 lines whose lengths are between 80 and 90 miles; 18 between 90 and 100 miles in length; and 11 exceeding 100 miles in length. The longest side in the Triangulation is 111 miles, viz. that from Slieve Donard to Sca Fell.

Latitudes. The latitudes of 32 of the Stations have been determined with Ramsden's Zenith Sector of 8 feet radius, which was destroyed in the fire at the Tower in 1841, and with Airy's Zenith Sector of 20·5 inches radius, by Troughton and Simms. From the very close approximation of the observed zenith distances of the stars with each instrument, and the perfection of the instruments themselves, we have every confidence in the results obtained.

Longitudes. The longitudes of the different Stations were in the first instance calculated on Airy's Figure of the Earth, by using the observed latitude of Greenwich and the observed azimuth of the meridian-mark, and transferring this azimuth with the distances to each Station in succession; the observed latitudes and the observed azimuths are then used to determine corrections to Airy's Figure, and the Greenwich latitude and azimuth; the longitudes and latitudes then received their necessary corrections.

The longitudes were also determined chronometrically by the transmission of chronometers, and by transit observations; the difference of longitude obtained in this way between the extreme points, viz. Greenwich and Feaghmaan in the Island of Valentia, including an arc of about ten degrees, was found to correspond to a difference of 461 feet in the position of the Station, as determined geodetically.

It will be observed that there is a difference of about 170 feet in the distance from Dunnose to Clifton as given in this volume and that previously published; this great error was detected when the detailed Triangulation was extended over the same district.

Altitudes. The altitudes of many of the Stations have been determined by actual levelling with a spirit level, and are all referred to the mean tidal level at Liverpool; but the vertical angles, or zenith distances of the Stations, were also reciprocally observed with the theodolites.

Since the printing of the computed height (at page 558) of Ben Macdui, the height of that hill has been determined by spirit levelling. By levelling up the western side the height was found to be 4295·70 feet, and by levelling down the eastern side the result was 4295·76 feet. The agreements of these results with one another and also with the computed height, 4295·6 feet, are very remarkable.

Refraction. The refraction was found to vary in a normal state of the atmosphere from $\frac{1}{15\cdot4}$ to $\frac{1}{9\cdot4}$ of the angles subtended by the Stations from the centre of the Earth; the mean refraction is $\frac{1}{12\cdot6}$, but we have cases in which the refraction reaches $\frac{1}{2}$, and others in which there is no refraction, and a few in which the refraction was actually negative.

The great amount of refraction in the morning, its diminution towards the middle of the day, and its increase again towards evening, is obviously caused by the greater amount of aqueous vapour in the lower portion of the atmosphere in the morning and evening, as compared with the amount in the middle of the day.

Figure and Dimensions of the Earth.

In Airy's Figure of the Earth, the equatorial radius is 20,923,713 feet, and the polar radius is 20,853,810 feet, giving an ellipticity of $\frac{1}{299 \cdot 33}$; but from the Ordnance and other more recent surveys abroad, the axes are found to be 20,926,500 feet and 20,855,400 feet, and the ellipticity about $\frac{1}{294}$; but the results are slightly dependent on the results to be obtained from the measurement of the Russian arc of meridian, which will probably modify them in some degree; the account of the measurement of the Russian arc is not yet published. The result obtained from pendulum observations gives an ellipticity of $\frac{1}{284}$.

Density of the Earth.

The mean density of the Earth, as determined from the observations at Arthur's Seat, is 5·316.

Local Attraction.

I cannot help regretting that a series of Stations in plains, and in localities where, from the geological structure of the country, we might infer that little or no deflection of the plumb-line could exist, had not been selected for the express purpose of measuring an arc of meridian. The summits of mountains were necessarily selected for the connecting points in the Triangulation; but, from the want of symmetry in the mountains themselves, the irregular manner in which other mountain masses surround the Stations, the diversity in their geological structure, and the unknown distance to which the igneous rocks especially extend below or around, must always leave a doubt upon the determination of the latitudes of such Stations.

The minor Triangulation for the detailed survey of the kingdom is now rapidly extending, and I think it not improbable that we shall be able at no distant period to select such a series of Stations as I have described, and thus obtain other arcs of meridians more free from the probability of error than those in the Principal Triangulation. When I directed the observations for the investigation of the amount of the local attraction at Arthur's Seat to be made, we had not only a perfectly contoured plan from which the distribution and quantity of matter in the hills and mountains round it was accurately known, but we knew also the exact geological structure of the country and the specific gravity of the rock, and yet there is a discrepancy of 2″·0 between the observed and the computed latitudes unaccounted for. I am not satisfied with this result, and believe we should obtain better results if the course I have indicated had been followed.

The late Dr. M'Culloch, when engaged on the Geological Survey of Scotland, was requested to keep in view the intentions of the late General Colby to have observations made for determining the mean density of the Earth, and Dr. M'Culloch was requested to point out such mountains as he thought well suited for the purpose; and I find in the correspondence connected with this subject, that he recommended Ben Stack and Seul Vein mountains, in the South West of Sutherlandshire, as well suited for the purpose.

Geological Structure.

I have myself examined the geological structure of the district in which these mountains are situated. It consists of a great series of quartzose micaceous stratified rocks, on which the old red sandstone of Caithness rests unconformably, and which has at the base of the series bands of limestone and pure white quartz strata. These white quartz strata rest unconformably upon a coarse red conglomerate, which again rests unconformably upon gneiss.

Seul Vein.

Seul Vein is the most remarkable mountain I have ever examined; it is composed of the red conglomerate in horizontal strata, and stands out by itself on a plain of gneiss, a monument of the great degradation of the surface of the Earth in former geological periods: it is about 1500 feet high, with nearly vertical sides, but the mass of the mountain is not large; and as I fear we should have great irregularity in the refraction if observations were taken near it, I do not, therefore, consider it well suited for the purpose of observations for determining the mean density of the Earth.

Ben Stack.

Nor do I think Ben Stack well suited for the purpose; this mountain is composed of the white quartz strata resting on the gneiss, and although its form is good, its mass is not large, and it has other mountains round it, and a great range at but a short distance on the northern side, all of which would affect the observations.

I still think we ought to have further observations for determining the mean specific gravity of the Earth, and I proposed last year to send a party for this purpose to the Peak of Teneriffe, and I now purpose, as soon as the detailed survey reaches Ben Nevis, to have observations taken at this the highest mountain in Great Britain.

H. JAMES.

SECTION I.

DESCRIPTION OF STATIONS.

Acklam Wold, 1842, is a large moor in Yorkshire, about 1 mile east of the village of Acklam, and 3 miles north of the village of Kirby Underdale, and on the left-hand side of the road leading from the latter place to Malton. The station is on the south-eastern rise of the most easterly of two barrows, distant from each other 293 feet, and is marked by a piece of white limestone with a hole in it, placed 1·8 feet below the surface.

Arbury Hill, 1843, is an insulated eminence in a field belonging to Righton Farm, in the parish of Badby, Northamptonshire, about 1·5 miles west of the village. The hill was visited in 1799 and 1800, and again in 1843, with the Ordnance 3-foot theodolite; but of the point from which the earlier observations were made, no trace is to be found. The station of 1843 is marked by a stone 18 inches square, with a hole to denote the centre, and the words "Arbury Hill, 1843," cut into it, and sunk 3·8 feet below the general surface. In the bottom of the hole in the stone, a fourpenny-piece of the present reign, dated 1841, was inserted, but when the stone was examined in 1850 it had been removed. A pole 19 feet high has been erected over the stone. By comparison of the old and new angles, the site of the earlier station is supposed to have been about 5 or 6 feet due east of the new station.

Axedge, 1842. This station is on the highest part of a hill, about 2·25 miles south-west from Buxton, and about a quarter of a mile to the west of the road from Buxton to Leek, in the county of Derby. Observations were taken from the hill in 1807 and in 1842 with the great instruments, but the old station is altogether lost. A pile 17 feet high, built of stone to the height of 4·5 feet, and the remainder of turf, denotes the station of 1842, the centre of which is marked by a stone with a hole in it, and inscribed with a broad arrow and the letters "B.O. 1842." The surface of the stone is about 1 inch below the tops of the pickets of the frame on which the instrument rested, which were left in the ground. The frame consisted of four upright posts or pickets, 5 feet long, mortised into the corners of a square frame, and strongly braced together.

Baconsthorpe Tower, 1843. This station is in the centre of the top of the tower of the parish church of Baconsthorpe, in the county of Norfolk, about 4 miles south-east of the town of Holt.

BALLYCREEN, 1852. This station is on a sharp-topped mountain, known in its neighbourhood by the name of Mottee, three quarters of a mile from the fifth milestone on the road from Rathdrum to Baltinglass, in the county of Wicklow. The station point is marked by a hole 1·5 inches deep and 1 inch in diameter, in a sandstone measuring 12 inches by 11, and 9 inches deep, which is jammed between two pieces of rock, and its upper surface level with the general surface of the ground. The frame which supported the instrument in 1852 was left in position, and a pile of turf, 60 feet in circumference at the base and 10·75 feet high, built over it.

BALSHAM TOWER, 1844. This tower is that of the parish church of Balsham in Cambridgeshire, situated about 6 miles east of Babraham and about 12 miles south-east of Cambridge.

BALTA, 1817, 1847. This station is situated in the small island of Balta, which lies to the east of the island of Unst, one of the Shetland Isles. The island is rather more than one mile in length from north to south, and about half a mile in breadth at the broadest part, forming an excellent natural breakwater to Balta Sound. The station is on the highest part of the south end of the island, and is marked by a hole bored in a stone sunk about 2 feet below the natural surface of the ground, and by a pile of earth and stones, 6 feet high, erected over it.

From this station the zenith-sector station on Balta is distant 215·5 feet, and bears 90° 11′ from the south. It is marked by a hole in a stone, sunk about 2 feet below the surface.

BANSTEAD, 1848. The stations at Banstead, in the county of Surrey, are about half a mile west of the parish church, in a grass field called Tumble Field, on the west side of the road leading from Sutton to Reigate, and opposite the Black Boy Beer-house. The new station of 1848 is 748 feet from the chimney of the house or cottage at the entrance of the field, and is in a line with the chimney and Banstead Church spire: it is marked by a piece of freestone about 12 inches square, with a hole 3 inches deep in the centre, placed 3·2 feet under the surface.

The old station of 1792 is 6·2 feet north of the former, and is well marked with a stone, about 6 inches diameter, shaped like a truncated cone, placed with its large end downwards, and about 24 inches below the surface.

BARDON HILL, 1842, is in Leicestershire, nearly equidistant from the towns of Leicester, Loughboro', and Ashby-de-la-Zouch; the turnpike road from Leicester to Ashby-de-la-Zouch passing about half a mile from and on the south side of the hill, at a distance of 9 miles from the former town. The station is situated on a rock near the western side of the summit, and 273·7 feet distant, in a west-by-north direction, from a summer-house on the top of the hill. There have been two sets of observations taken from this hill, both from the same site. A pile was erected, at the conclusion of the first series, above a centre mark bored in the solid rock. This centre was found and the instrument was placed above it for the observations of 1842, and the site was also marked by four holes excavated in the rock for the pickets of the frame of the instrument. These pickets were each about 4 feet long, and after being tightly wedged, were run in with lead and well braced together. The pile was restored.

BARROW HILL, 1845. This station is situated in a field called "Inner Mill Field," belonging to Boro' Farm, in the parish of Chivelstone, in the southern part of Devonshire, and about 8 miles south-east of Kingsbridge. The station is marked by a stone, with a hole drilled in it 2 inches deep, denoting the exact spot from which the observations were made, and sunk 16 inches below the surface of the ground.

BAURTREGAUM, 1831. The station is on the western head of a large hill or mountain, about 8 miles south-west of Tralee, and on the left side of the road leading from Tralee to Dingle through Blenerville. Curraheen Chapel is on the Dingle and Tralee road; and a person intending to visit Baurtregaum station from Tralee should leave the Dingle road at this chapel, and pursue the course of a stream upwards for about 2·5 or 3·0 miles. This stream leads to Caum Brack Lough, which is about 1·5 miles east of the station; the passage from Curraheen Chapel to Caum Brack Lough is of easy ascent, the remainder is difficult. The station is marked by a large pile of stones, within which will be found the centre stone, an irregular piece of sandstone, level with the general surface, and marked with a hole.

BEACHY HEAD, 1845. This headland in the English Channel is in the parish of Eastbourne, county of Sussex, about 2 miles south of the town. Observations were taken on the hill in 1793 with the Ordnance 3-foot theodolite, and in 1845 with the 18-inch theodolite; but the sites from whence the angles were taken are not identical, though both are adjacent to the houses occupied by the Coast Guard. The site of the earlier station was marked by an old gun with its breech imbedded in the soil, and its muzzle about 2 or 3 feet above the surface; but when the place was visited in 1842 it had been removed, and a signal-house to which the site was referred had also been pulled down; no vestige therefore of the site could be found, though careful search was made. In 1844 renewed attempts were made to find some indication of the site, but these being unsuccessful, a point about 105 feet north-east of the well in the rear of coast guard's buildings, was selected as a site for the new station, and marked with a centre stone, measuring 13 inches by 11, with a hole 2 inches deep, and a broad arrow cut on the top, over which a pile of turf was erected. The 18-inch theodolite was placed above this centre stone in 1845.

BEACON HILL, 1849. The station on Beacon Hill, in the parish of Bulford near Amesbury, in the county of Wilts, is about 3 miles east-south-east of Amesbury, and about a quarter of a mile north of the Amesbury and Andover road.

The station, being the north-east end of the base line measured in 1794 on Salisbury plain, was marked by an iron cannon with its muzzle 8 or 10 inches above the surface. The calibre of the gun is a little more than 4 inches, and its centre was in the observations of 1849 adopted as the observing station.

BEACON HILL, TRESCOW, 1850. The station so called is situated on the highest part of Beacon Hill, in the island of Trescow, one of the Scilly Isles, and is about 1·5 miles from Old Grimsby. The station is about 60 feet from an old watchhouse.

BEN CHEILT, 1819, is a large mountain in the county of Caithness. The station was on the north-western end of the highest part of the hill, which is about a foot higher than the station, and about 400 yards south-east of it. The peat moss, which was about 2 feet deep, was dug out in a circular pit of 8 feet in diameter, which being filled in with clay, the pickets for the instrument were driven into it and braced together. Large stones were laid upon and round the braces to steady them, so that the station is easily discovered, even if the pile which surrounds the staff were removed.

BEN CLEUGH, 1848. This station is situated on the most northerly and one of the most prominent points of the Ochill Hills, in the county of Clackmannan, about 2 miles north of the village of Tillicoultry, and 4 miles north-east of the small town or village of Alva. The station is distinguished by a stone pile, 13 feet high and 36 feet in circumference, erected around the frame on which the instrument stood. The centre of the station is marked by a large flat stone, with a hole in its centre about 4 inches deep, placed 3 feet below the general surface of the top of the mountain, and by a similar stone placed level with the surface and within the wooden frame or stand which supported the instrument. There is no permanent mark or object near the station from which measurements can be given, but there are several protruding pieces of rock only a few yards distant. The best and easiest way of ascending this hill is from Alva by the Blackforde Horse path, which passes about 1 mile west of the station.

BEN CLIBRIG, 1839. This mountain is situated in the exact centre of Sutherlandshire. With the exception of Ben More in Assynt, which exceeds it by about 70 feet, it is the highest in that county. It lies to the right of the high road from Golspie (through Lairg) to Tongue, and its summit is distant in a south-easterly direction about 4 miles from the inn of Ault-na-harrow. The top of the hill as seen from the north has a ridge falling gently to the east, but abruptly to the west. The station was not quite at the top of the hill, but at an angle of the ridge, where there was a knoll with a few large stones projecting from the earth. The highest of these stones is from 3 to 4 feet above the centre mark of the pile which was erected on the site of the station. On taking down the old pile, the centre mark was found. The pickets on which the frame of the theodolite rested, were placed in four holes, 6 or 8 inches deep and 4 inches square, drilled in the solid rock. These pickets were about 18 inches long, and after being tightly wedged were run in with lead. The pile was rebuilt 18 feet high and about 16 feet in diameter.

BEN HEYNISH, 1822, is an isolated hill in the island of Tiree, one of the Western Isles of Scotland. The hill is composed entirely of gneiss and is flat-topped, but furrowed by shallow ravines dividing it into several points, that on which the station is situate being rather the highest and, by calculation, 462 feet above the mean level of the sea. The centre of the station is marked by a hole bored in the solid rock to the depth of 4 or 5 inches, four other holes of a similar description were also made, into which wooden pickets were driven for the feet of the large theodolite to rest on; these holes, and a large angular block of gneiss 10 or 12 feet high, about 57·5 feet distant from the station in the direction of Canna, will serve to identify the spot. The trigonometrical point is identical with the zenith-sector point.

BEN HUTIG, 1838, is a large mountain situated near the centre of the north coast of Sutherlandshire. Its summit is about a mile from the edge of the coast on the north, and about 3 miles south-east from the Whiten Head. From the village and inn at Kirkiboll by the Moin House the distance to the top is about 11 miles, and from Hailaim Inn on Loch Eriboll by the Moin House it is 10 miles.

The route by the Moin House is the best way for carriage of heavy articles to the hill; but by crossing the moor from the Tongue or Hope ferries the distance will be less by a mile or two, but it is rougher walking. There is a good road also from Tongue Ferry by Talmin, round to a river and bay called the "Strath," which is at the foot of the hill on the east side within about 2 miles.

The top of the hill is rocky and rugged, and has two ridges running north-east and south-west; the station was on the highest part of the rock on the western ridge, and had a very large block of stone a few feet to the north of it. The old pile put up in 1819 was found undisturbed, but no centre mark was discovered. The centre was, however, taken accurately by a tape as soon as the pile was cleared away, and a hole 5 inches deep and 1 inch in diameter was jumped to mark the spot.

In 1844 observations for latitude were made with Airy's zenith sector, the position which it occupied being identical with the trigonometrical station above described.

BEN LAWERS, 1850. This mountain is in Perthshire, and lies 5 miles north-west of Loch Tay and about 8 miles north-east of Killin in that county. The station is on a mass of rock at the western extremity and highest point of the ridge, which runs east and west; it is marked by a hole bored in the rock, and the frame which supported the instrument, over which is a pile of stones 12·5 feet high and 15 feet in diameter. Two series of observations were made at this station, the first in 1841 and the second in 1850.

BEN LOMOND, 1818, is a large mountain in Stirlingshire. The station is 52·4 feet south-east by east of the cliff at the western end of the summit of the mountain. It is on the broadest part of the summit, and a little lower than the narrow flat above the western cliff, which was not sufficiently large to hold the observation tent. The site is marked by a wooden picket having a hole bored in it, and by the pickets on which the instrument rested in 1818. The mountain is composed almost entirely of micaceous schist and quartz rock; the ascent is by no means difficult, and the prospect from it on a clear day is not only very extensive, but extremely beautiful and magnificent. The point on which Airy's zenith sector was placed is identical with this station.

BEN MACDUI, 1847. Ben Macdui is the highest of the Grampian Hills, and is about 12 miles north-west of Castleton of Braemar, in the county of Aberdeen.

The station is on the southernmost summit above Glen Lui-beg, and near the united boundaries of the counties of Inverness, Banff, and Aberdeen.

The site is marked by a hole about 6 inches deep, in an enormous stone, over which a large pile, about 22 feet high, was erected. The pile was circular, but not of a shape similar to those generally erected to denote trigonometrical stations; it is perpendicular for a few feet, and then reduced in circumference.

BEN MORE, MULL, 1822. This mountain is in the island of Mull, one of the Western Islands of Scotland. The station is situated on the highest and most salient point, about 30 feet from the edge of the precipice.

The rock on the summit was found so splintery and frangible that the station could not be prepared in the usual manner, by securing pickets into the rock. A strong framework of wood was therefore constructed, and buried in the soil, resting upon the solid rock. This, when tightly rammed, was found perfectly free from vibration. The framework was left to mark the station, and a stone 18 inches in diameter, with a hole in it 2 inches deep, was placed in the centre; above these a pile of stones 13 feet in diameter was erected.

BEN MORE, S. UIST, 1851. The mountain of Ben More, in the island of South Uist, is on the eastern coast, and not far from the centre of the island; it is about 2 miles north of Loch Eynort, and about 5 miles east of Howmore in that island. The top of the mountain consists of two ridges meeting at an angle: the lower ridge is very rocky, sharp, and abrupt; the higher ridge has a rather flat top about 200 yards wide, with an immense precipice along the north side.

The station is on the highest part of the latter ridge, and at the junction of the ridges close to the precipice. The stations of 1840–1 and 1851 are identical, and the site was marked in 1851 by a pile of stone 49 feet in circumference at the base, and 14·5 feet high; the centre is denoted by a hole pierced in the rock 2·7 feet below the surface of the ground beneath the pile.

BEN TARTEVIL, 1822, is a large hill situated in the parish of Portnahaven, in the island of Islay. The station is on the summit, and is rendered permanent by the holes bored in the rock for the pickets of the theodolite. A wooden centre mark was also left, and the whole covered by a pile 14 feet in diameter, the lower part of stone, and the remainder of turf.

The station was examined in 1841, and a centre stone was placed above the wooden centre, without disturbing it or the pickets, and the pile raised to the height of 20 feet.

The road from Skiba to Portnahaven passes about 1 mile east of the station.

BEN NEVIS, 1848. This mountain, the roughest and highest in Scotland, is in the parish of Kilmanivaig, in the county of Inverness, and about 7 miles east of Fort William. The top of the mountain has a flat rugged surface, of considerable extent.

The station, which is upon the highest part of the mountain, and within a few yards of a precipice of about 1500 feet, is marked by a hole in a very large stone resting on a rock about level with the surface, and by a similar stone about 3 feet above the surface, and within and level with the frame which supported the instrument. The frame of the instrument and centre marks, are protected by a stone pile 60 feet in circumference and 25 feet high.

BEN WYVIS, 1819. The station on this mountain, in the county of Ross-shire, is marked by a pile enclosing a pole and centre stone about 2·5 feet square and 6 inches thick, weighing about 3·5 cwt., with a hole in the centre 4 inches deep.

BENCORR, 1830. This station is about 9 miles east of the town of Clifden, in the county of Galway, and is on the summit of one of the Twelve Pins of Connemara. The centre stone measures 32 inches by 20 inches, with a hole in its centre, not very deep, but sufficiently defined. The length of the stone is in an east-north-east and a west-south-west direction. The pile is 8 feet high.

BERKHAMPSTEAD, 1823. The station at Berkhampstead, in Hertfordshire, is on the gazebo or observatory tower belonging to Mr. W. Stratton, of Little Berkhampstead. The tower is circular, and about 100 feet high; the instrument was placed about 11 inches eastward of the centre of the tower.

BLACK COMB, 1841, is a large rocky hill in the parish of Whitbeck, in the county of Cumberland. The station is on the top of the hill, and is marked by a pile of stones 14·5 feet high and 50 feet in circumference, erected above a centre stone with a hole in it 4 inches deep and an inch in diameter. The station was restored in 1852.

BLACKDOWN, 1848–9. This is a large hill in the parish of Portisham, in Dorsetshire, about 6 miles west-south-west of Dorchester, and 1·75 miles south of the village of Winterbourne.

A column was erected here in 1845 in honour of the late Admiral Sir Thomas Hardy. It is an octagonal tower, 71·25 feet high, ascended by a winding stair of 120 stone steps, the newel of which in the centre of the tower corresponds with the centres of the stations observed from in 1797 and 1848–9, and is identical with the zenith-sector station in 1842–3. A hole was drilled in the centre of the topmost stone of the newel, and run in with lead, to mark the station.

BLACKHEDDON, 1846. This station is situated on the highest point of Blackheddon Hill, in the county of Northumberland, about 5 miles north-west of the town of Belford, 3 miles east of Lowick, and 10 miles south of Berwick-on-Tweed. Two piles, about 30 feet distant from each other, mark the sites on which the instruments were placed during the observations of 1809 and 1846, the smaller pile denoting the former or old station, and the larger pile, built of stone 18 feet high, distinguishing the latter or new station. The new station is marked by a hole in a large stone 3·5 feet below the surface. The hill commands views of the sea to the north-east and east, and the Cheviot and Lammermuir Mountains.

BLUE HILL, 1814. This station is about 4 miles from Aberdeen, and half a mile on the right of the road from Aberdeen to Stonehaven, by the stone bridge over the Dee; it is situated on a low barren place partially covered with heath, and planted with fir very stunted in its growth. The site is marked by a centre stone sunk below the general surface, and a well-built pile of stones 16 feet high, with a diameter of 13 feet, erected above it.

BONIFACE DOWN, 1846. The trigonometrical station on Boniface Down, in the Isle of Wight, is identical with the station from which the observations with Airy's zenith sector were made in 1846. The station is in the centre of an apparently artificial mound of decayed vegetable matter on the Down, about 100 yards south-west of, and about 10 feet lower than its highest part, which is due south of Dunnose station. A point 2237 feet eastwards from the second and principal junction of the roads from Ventnor and Wroxhall to Shanklin, measured along the road, is in line between this station and the station on Dunnose; the former lying about 60 yards to the south of this point.

The station is marked by a centre stone with a hole bored in it, sunk 2 feet below the surface.

BOSTON TOWER, 1842, is the tower of the parish church of Boston, in the county of Lincoln; it is of an octagonal form, with 16 pinnacles, viz., a large pinnacle at each angle, and a smaller pinnacle between the larger ones. In consequence of the obstruction which they presented to the field of the instrument, it was placed upon two different points, the first being 5·8 feet south of the centre of the tower, and the other 2·8 feet north of the centre: the first is the trigonometrical station.

BOTTON HEAD, 1840, is a large hill about 3 miles south-east of the village of Ingleby-green-hoe, in the northern part of Yorkshire. The station is situated on the south-west edge of a large barrow, and is 4 feet south and 12 feet west of the southern edge of a deep indentation or hollow, extending from the north edge of the barrow southwards. It is marked by a hole in a stone, sunk about 1·75 feet below the surface.

BRANDON DOWN, 1853. This Down separates the villages of East and West Brandon, in the county of Durham, from the former of which it is about a mile distant; and the lane between the two villages passes close to the hill. The station is upon the flattest part of the highest point of the Down at the western extremity of a sharp ridge, which appears to have been thrown up from a quarry near it many years ago.

BRANDON, 1845. This station is in the parish of Brandon, in the county of Suffolk; it is situated on the highest part of the ground, in a large rabbit warren, called Wangford Rabbit Warren. A small house, called Shaker's Lodge, inhabited by the keepers, is 22 chains eastward of the station, which is marked by a centre stone with a hole in it, placed 8 feet below the surface in consequence of the ground being very fine sand, and to afford proper foundation and support for the pole, 44 feet long, which was erected above it. The plantations of Brandon Park obstructing the view in the north and east direction, and another plantation obstructing the view on the south-east side of the station, it was necessary to erect a stage for the 2-foot theodolite, which was 39 feet high.

BRANDON. This station is on the summit of a well-known hill, about 8 miles due north of the town of Dingle, in the west of the county of Kerry. The station is marked by an excellent stone pile 14 feet high. The centre is marked by a hole in a piece of sandstone, level with the surface. Forty links north of the station there is a small grave-like excavation, having its sides and eastern end rudely lined with stones, and opposite the west end is a small upright flagstone, presenting the appearance of a grave or headstone; this is called by some the chapel, and by others St. Brandon's bed. About 60 links north-north-west of it there is a small well.

BRASSA, 1821. This station is so called from its being situated in Brassa, one of the Shetland Isles. On the top of the Wart Hill, where the instrument was fixed, are two mounds that appear to be the ruins of ancient watch-towers and furnaces. Similar remains are found on the top of every hill in Zetland and Orkney bearing the name of Wart or Ward; such hills, as the term implies, having in ancient times been selected by the inhabitants on account of their commanding situations to descry their enemies at a distance, and spread the intelligence of their approach. The western of the two mounds upon Wart Hill, Brassa, was selected as the station, and was marked by a stone sunk in the ground to denote the centre. A pole and a stone pile were erected over the station.

BRIMMOND or BREMEN STATION, 1817, is about 6 miles north-west of New Aberdeen and about 1 mile west of New Hills Kirk, and is situated on a low heathy mountain, cultivated to within three quarters of a mile of the top on its south side. The station is marked by a pile of turf and stone 23 feet in height, and 53 feet in circumference. The centre is denoted by a stone hammer weighing about three pounds, with a handle about a foot long, thrust through the old centre picket of the frame of the instrument; a large flagstone more than 2 feet wide was laid on the hammer, and on this flagstone a piece of fir about 4 feet long was erected as a guide to the centre.

BROADWAY TOWER, 1850. This is an hexagonal tower with 3 turrets, one at each alternate angle, situated on a prominent hill in the parish of Broadway, Worcestershire, about 1 mile to the east of Broadway New Church Tower, and 2·5 miles to the south of Campden Church Tower. The station is upon the top of the tower, and its site is marked by a copper nail driven in the lead near the centre of the tower.

BROWN WILLY, 1849, is a high and rugged topped hill, about 5 miles south-east of Camelford, in the county of Cornwall. The station was upon a barrow of loose stones on the summit of the hill, and was marked by a flat stone with a hole through it, placed in the earth level with the general surface of the hill. A pile of stones, 11 feet high and 10 feet in diameter at the base, was erected above the centre stone.

BUNWELL TOWER, 1844. The tower of the parish church of Bunwell in the county of Norfolk is the site of this station. The instrument was placed exactly over the centre of the tower, and the frame on which the instrument was fixed rested on the battlements.

BURLEIGH MOOR, 1806, is in the North Riding of Yorkshire, about a mile and a half south-east of Kirkleatham, and a mile north-west of Upleatham. The station is close to a bend in a stone wall and is marked by a pole 30 feet high.

BUTSER HILL, 1845, is an eminence in the parish of East Meon, in the county of Hants, about 2·5 miles west of Petersfield, and about three-quarters of a mile west of the road from Petersfield to Portsmouth. The station is marked by the pickets on which the instrument rested, cut off about 18 inches below the surface, and a stone measuring 2·7 feet by 1·4 feet, and about 6 inches thick, with a hole in it denoting the centre, placed between the pickets 2·4 feet below the surface. A pile 14 feet high and 11 feet in diameter was erected over the site.

CADER IDRIS, 1844. This station is situated on the highest point of Cader Idris mountain, 6 miles south of Dolgelly, and 8 miles east of Barmouth in North Wales. It is about 2914 feet above the level of the sea, and 92 feet south of a small hut built near the mountain top as a place of shelter to tourists.

CAHERBARNAGH STATION, 1832, is situated on a mountain about 6 miles west of the town of Mill Street, and about 1 mile south of an old road leading from Mill Street to

Killarney. The surface of the entire mountain is bog 2 or 3 feet deep, and particularly so about the station, which is marked by a turf pile 7 feet high, enclosing a smaller pile of stones. The centre stone is a piece of clay slate measuring 28 inches by 17 inches, and 3 feet below the general surface of the mountain; the length of the stone is in a north and south direction.

CARN GALVER, 1850. The station called Carn Galver is situated on the top of Morvah-look-out-Hill near Penzance, in the county of Cornwall, from which town it is about 7 miles north-west, and about 2 miles north-east of the village of Morvah, and 2·5 miles south-west of Zennor. The station is marked by a centre stone sunk about 2 feet below the top of the barrow. The point known as Carn Galver by the country people is about 0·75 of a mile eastward of the station.

CARRIGFADDA, 1832. This station, in the south of the county of Cork, is situated on the western summit of a small hill about 8 miles south-south-east of the town of Dunmanivay, and about 4·5 miles north-north-west of Roscarberry. From Dunmanivay there is a cart road, by the club house and Lough Thariv, skirting the foot of the hill on the north side; the hill is of easy ascent, and covered with heath on the top. The centre stone, which measures 27 inches by 17 inches, is level with the surface, and marked by a cross cut in its upper surface. The pile is about 7 feet high.

CHEVIOT, 1846. This station is on the summit of the Cheviot. The ground is flat and very boggy, but a gravelly soil was found at a depth of 9 feet. A triangular frame was sunk 2 feet into the gravel, and uprights mortised into each corner, and well braced together, for the feet of the instrument to rest upon. When the observatory was first set up, it sank into the ground at the rate of half an inch per diem, and at length rested upon the framework supporting the instrument. A number of piles 11 feet long were therefore procured and driven into the ground, and their tops connected by two cross beams, upon which the observatory was raised by means of levers.

CHINGFORD, 1848. This station is in a field near Epping in Essex. It is 13·54 feet from the centre of Greenwich north meridian mark, 9·67 feet from the south-west corner, and 10·12 feet from the south-east corner, of the base of the obelisk; and is marked by a round hole bored in a square stone, the upper surface of which is level with the surface of the ground. This station is identical with that used by Captain Kater.

CLEISHAM, 1840. This, the highest mountain in the Hebrides, is in that part of the Island of Lewis which is in the county of Inverness. It is near the south and west shore of Loch Seaforth, and, being the most conspicuous object in that part, cannot easily be mistaken.

The huts nearest to the mountain are at Marvig, from which it is distant about 3 or 4 miles. West Tarbert, a small village on the south side of the hill, is about 6 or 7 miles distant. On account of the accommodation which the observing party in

1840 required, the men and stores were landed at a farm house and shooting lodge at Mole-na-harig, about 6 miles south-west from the hill. The station is upon the highest point of the mountain. A pile 18 feet high occupied the site of the station previous to the preparation of the ground for the reception of the frame of the instrument in 1840, but on taking it down no centre mark was found. The centre of the pile was therefore ascertained by careful measurement, and the instrument placed over it.

On quitting the station the wooden pickets were left in the rock, a large stone with a centre hole in it was placed even with the surface, and a stone pile 21 feet on slope, and 18 or 19 feet in diameter, erected. A few feet from the pile is a large rock.

Clifton Beacon, 1842. This station is about a quarter of a mile south-by-west of the village of Clifton, which is 1·5 mile south by east of Conisboro', a small town half way between the towns of Rotherham and Doncaster in Yorkshire. The station is in an arable field close to and on the south side of a hedge, across one field from a lane leading out of the village of Clifton in a westerly direction. In 1842, when additional observations were made at the station, two centre stones were left, one at the depth of 4 feet 6 inches below the surface, and the other at the surface; the beacon pole was replaced, and a stone pile built round it, 11·5 feet high and 30 feet in circumference. From this point the zenith sector station is 3·5 feet distant, bearing due south.

Cnoc Ghubhais, 1838, is a low round hill about 17 miles west of Ben Hutig, and 2·75 miles south-east from Cape Wrath Lighthouse. The road from Durness to the Lighthouse passes near it, the nearest point of the road across the moor being about 1·25 miles.

The station is at the top of the hill, and is marked by a pile 15 feet in diameter and 15 feet high, erected above a centre stone and the four pickets on which the table of the instrument rested, which were wedged into holes bored in the rock and run in with lead.

Collier Law, 1851. On this hill, in the parish of Stanhope, in the county of Durham, there are two stations, about 247 feet distant from each other, and called respectively Collier Law Old, and Collier Law New, stations. They are situated on the top of the mountain, about 4·5 miles north-east of Stanhope, and about 5 miles north of Frosterley; and are marked by two piles, each 14 feet in circumference and 15 feet high. The western pile marking the old station has a pole of 13 feet in its centre, braced to the pickets left by the instrument; the other pile marking the new station has a centre stone and a short pole.

The easiest ascent of the mountain is from Frottesley.

Coringdon, 1845. This station is upon a hill about 2 miles north of the village of Swanage, and an equal distance south-west of Studland, in the county of Dorset, and is nearly 3·5 miles east of Corfe Castle, in the island of Purbeck.

The remains of an old fence, the reputed boundary of the parishes of Studland and Swanage, may be traced over the hill from Nine Barrow Down in a south-east direction, and passes the station 50 feet to the north-east. The station is marked by a turf pile, about 6 feet high, built above a centre stone inserted about a foot below the general surface, and the piles and bracings which supported the instrument.

CORRYHABBIE, 1850. This hill is situated in the parish of Mortlach, Banffshire. The mountain has a long flat or ridge-like top, extending north-east and south-west, consisting of loose fragments of quartz rock, very imperfectly covered with vegetable matter. The station is on the highest part of the hill, about 2 miles south of Glen Rinnes Kirk, and about 1·5 miles south-east of Cook's Cairn, and is indicated by a stone pile 13 feet high and 46 feet in circumference at the base. The centre is marked by a large flat block of quartz, of about 4 cwt., with a hole 4 inches deep and 15 inches in diameter bored in it, and by the frame which supported the instrument being left in position.

COWHYTHE, 1847. This station is situate on the north edge of an arable field, upon a hill in the parish of Fordyce, county of Banff; it is about one quarter or three-eighths of a mile from the edge of the cliff above the sea, and about one mile east of the village of Portsoy, where the hill is called "Cairn Rankee."

The original centre mark of the station was found, in 1846, about 30 inches below the surface; it consisted of a wooden picket, mounted on the upper part with a strong iron hoop, 4·25 inches in diameter; the wood was quite decayed, but the iron hoop was firmly set in the earth.

The wood was replaced without disturbing the iron ring, and a stone, 20 inches by 14 inches, with a hole 1·75 inches deep bored in it, was placed above the picket, its centre corresponding with the centre of the iron ring. The stone and the piles on which the instrument stood in 1846, with their bracings, &c., are enclosed in a pile of stones and turf, 9 feet high.

The site on which Airy's zenith sector was placed is 42 feet due north of the trigonometrical station, and is identical with that stated to have been used for Ramsden's zenith sector in 1813; but no centre mark or pickets were found that would indicate it had ever been used as a station.

The altitude of Cowhythe, as obtained by spirit levelling on the 25th August 1813, is 272·75 feet above low water.

CRADLE, 1843, or Pen Cader Fawr, or, as it is usually called by the country people in its neighbourhood, "the Cader," is situated about 8 miles north-east of the town of Crickhowell, in Brecknockshire, South Wales, in the range of the Black Mountains. A steep mound, which when viewed from a distance has the appearance of a truncated cone, forms the summit, on the north side of which at the top is the station. A stone, 2·5 feet long and 2 feet wide, with a hole in it to mark the centre of the station, was left 2·5 feet below the surface, and a stone pile, 19 feet high and 18 feet in diameter, was built over it.

Criffel, 1841. This station is on the top and close to the south-west edge of "Douglas Cairn," which is on the highest part of the well-known mountain named Criffel, 10 miles south of Dumfries, and 3 miles south of the village of New Abbey. It is marked by a hole in the solid rock, in the centre of a square formed by four pickets on which rested the instrument. A wall runs over the top of the mountain in a north-east and south-west direction, forming an angle near the station, which is 80 feet due east from the nearest point of the wall.

Croghan, 1828, is on the highest pinnacle of Croghan Mountain, about 8 miles west of Arklow, in the county of Wicklow. The station is either on or close to the line separating the county of Wexford from the county of Wicklow, and is well marked by a stone measuring 26 inches by 10 inches, having a good centre hole bored in it. The centre stone is level with the surface, and is covered by a small pile.

Cross Fell, 1841, is a remarkable mountain, about 9 miles south of Alston, in Cumberland. The station is on its summit, and may be discovered by means of the stones placed around the staff.

Crowle Beacon, 1806. This station was situated on a barrow where once stood a windmill, nearly 150 feet east of a mill. The station was 36·9 feet from the corner of the hedge next the road and 43·9 feet from the road; the barrow has, however, lately been removed, and with it every trace of the old station.

Cuilcagh, 1828. This mountain which is in the counties of Fermanagh and Cavan, is about 4 miles west of Swanlinbar, and 12 miles south-west of Enniskillen. It is ridge-formed, exhibiting two precipices, one to the north and the other to the east, the former in the direction of the dip; the station is on an ancient carn near the north-eastern angle formed by the junction of these precipices. A pile and staff were first placed on the mountain by Major-General Colby's direction in 1825.

As a station Cuilcagh is of the first order, commanding a vast number of churches and other points connected with the district triangulation, and thereby assisting to furnish bases for the surveys of about 20 counties.

Cyrn-y-Brain, 1852. This station is on the most westerly of two headlands nearly half a mile apart, forming a saddle-top to the broad moorish mountain of this name. It is about 1857 feet above the level of the sea; and the ascent to it from Llangollen, through a dreary journey of 5 or 6 miles, is neither steep nor difficult. There is an old round tower, about 25 feet high, within 50 feet of the station; and though it is on ground several feet lower than the base of the small pile built over it, this tower is very liable to be observed from any distant point instead of the station pile. It seems not improbable that this tower has been built with the stones described in the third volume of the "Account of the Trigonometrical Survey," as forming a large cairn about 30 yards north-west of the station. This cairn is not now in existence, but there is a large heap of stones on the eastern summit.

DANBURY CHURCH SPIRE, 1844. This church is in the parish of Danbury, in the county of Essex, about 5 miles distant from the towns of Chelmsford and Maldon, on the high road between them. The station is above the centre at the top of the spire.

The instrument was supported on a scaffold erected from the top of the tower, and the observatory was supported by another scaffold similar in plan and construction; both rested on the walls of the tower, and were similarly cross braced, that bearing the instrument had its four uprights resting on the middle of the walls, and the other bearing the observatory rested on the angles. The spire of Danbury Church is built of wood, and was considered a much less stable structure for the instrument than the scaffold here described.

DEADMAN, 1846. This bold headland is in the parish of Gorran, about 2·5 miles south of the parish church, in the county of Cornwall. The station is in the north-east corner of a small field, and is marked by a small stone pile, 8 feet high, erected over the pickets on which the instrument stood, which were left in the ground, and a centre stone, with a small hole in it, sunk 1 foot below the common surface.

DEAN HILL, 1850. This hill is in the parish of Dean, in the county of Wilts. The station is about 1·25 miles south-west of the village of Dean, and nearly the same distance north-north-east of the village of Whiteparish. It is in the north-west corner of a large field, bounded on the north by a plantation of mixed trees. A circular block of sandstone, with its upper surface smooth, and about 20 inches in diameter, having a hole in it 3·75 inches deep and about 1 inch in diameter, marks the centre of the station. The stone is imbedded in the ground about 20 inches below the surface.

DEERNESS, 1821, is a large hill in the Mainland, island of Orkney. The station was upon a small mound on the highest part of the mountain, and was marked by a centre stone and pile with a pole in it.

DELAMERE, 1842. This station is in Delamere Forest, near Tarporley.

The station from which the observations of 1805 were taken is described in the observation book to have been 22·7 feet west of a hedge running north and south over the top of the hill, and 129 feet south of another hedge, running at right angles from the former. In 1840 the station was visited for the purpose of restoration, but no picket, or other remains serving to point out its site, could be found. By means, however, of observations taken with a small theodolite, the observer was enabled to fix upon a point very near the old station. This was marked by a stone, 1·25 feet long by 1 foot wide, with a hole cut in it, placed about 18 inches below the surface of the ground. A pole 25 feet long was erected over the centre. From a combination of the 3-foot theodolite angles taken in 1805 with those taken in 1842 over the point selected in 1840, and marked as here described, the position of the old station appears to be 3·7 feet from the new station, and to bear 201° 1′ 16″ from the south, reckoning by the west and north.

DOOLIEVE STATION, 1832, is situated on a low dark-coloured hill, about 9 miles south of the city of Cork, and about 4 miles south-east of the small village of Ballinhassig, which is on the old road from Cork to Kinsale. The station occupies the summit of a very small mound on the western extremity of the hill, and is marked by an irregular piece of clay slate level with the surface, about 36 or 40 inches in length, and pointed at both ends, the greatest breadth not exceeding 14 inches, and having a small hole bored in it about three quarters of an inch deep. The length of the stone is in a north-east and south-west direction, and it is covered by a pile of stones about 8 feet high.

DITCHLING, 1844. This station is at Ditchling, in the county of Sussex, and about 6 miles from Brighton. It is in the middle of a small rising, which has the appearance of having once been a barrow, and is marked by a hole in a stone sunk 2·5 feet below the general surface.

DIVIS, 1825. This station is on the summit of a well-known mountain of the same name, about 3·5 miles west of the Exchange Buildings, in the town of Belfast. It may be approached by the Shanklin Road for rather more than a mile, then by a bye-road skirting the mountain on the east side. The station is marked by a pile of large coarse stones, having a diameter at base of 16 feet and raised to a height of about 5 feet; this truncated section of a pile has a small quantity of bog turf on its top.

The centre stone has a smooth upper surface, with a well-formed hole in it, 2 inches deep and 2 inches in diameter. It is level with the surface of the mountain.

Divis Station is about 140 links due south of a fence which crosses the mountain in an east and west direction.

DUDWICK, 1816. Dudwick Hill, in the parish of Ellon, Aberdeenshire, is the site of this station, which is distant about 6 miles north-north-east of Ellon, and about 1 mile to the left of the road from Ellon to Peterhead. The station is marked by a pile 20 feet high.

DUNKERY BEACON, 1844. This well-known hill, in the parish of Luckham, Somersetshire, is about 3·5 miles south of Porlock, 1·5 mile south-south-east of the village of Stoke Pero, and 2 miles south-south-west of Luckham. There are three barrows on the hill, composed of stone, touching each other at their bases. The station of 1844 is upon the south top of the central or most eastern barrow, and is marked by a centre stone, with a hole jumped in it, and sunk about 2 feet below the surface of the barrow. A pile 19 feet high and 53 feet in circumference was erected over it.

DUNNET HEAD, 1838. The station on this remarkable headland, on the north coast of the county of Caithness, is on the highest point of the rising ground in the rear of the Lighthouse, from which it is distant 1273 feet in a south-easterly direction. Four holes were jumped in the rock, at a depth of 7 feet below the surface, to receive the feet of the posts, which were wedged in and run with lead. The framework was left, and pile of stone 11 feet in diameter and 10 feet high built over it.

DUNNOSE, 1844. This station is situated about 30 yards north-west of the summit of Shanklin Down, 1·5 mile south-west of the village of Shanklin, in the Isle of Wight. The centre is marked by a large gun imbedded in the soil, the top being nearly level with the natural surface of the ground. In 1846 observations were made with Airy's zenith sector at this point.

DUNRICH, 1850. This station is about 7 miles south of the town of Peebles, and 7 miles west of the village of Traquair in Selkirkshire; it is on the summit of the hill called by the inhabitants Gumscleugh, and is named Dunrich, after the peak of the mountain immediately north of the observing station. The old centre-mark, and framework used for the instrument in 1816, were refound in 1841, and covered with a well-made pile.

In 1850, when additional observations were taken from this station, the pile was taken down, and the instrument placed on the original centre, as near as possible in the position indicated to have been occupied by the instrument at the previous observations. The framework which supported the instrument on this occasion was left in position, and a mound of turf and mud erected above it to the height of 15 feet, having a piece of deal, 4 inches by 3 inches and 12 feet long, standing on the centre stone. At a distance of 435·5 feet eastward from the station, will be found a stone weighing about 4 or 5 cwt., placed by Major-General Colby in the Meridian of Edinburgh Castle, having a centre mark and the letters "E.M. 1816."

DUNSTABLE, 1843. This station is upon Dunstable Down, in the county of Bedford, and is marked by a pile erected over a block of Portland stone weighing 17 cwt. Two sets of observations have been taken at this station, one in 1823 and the other in 1843. About 95 feet from the centre stone in a south-east direction will be found Kensworth station, which is marked by a centre stone.

EASINGTON, 1846. The station is in the south-west corner of a cultivated field, about a mile north of the village of Easington, in Yorkshire. It is on the highest ground, close to the cliff, about twenty yards from the ruins of an old house, and marked in the usual manner by a centre stone and a pile.

East Lomond Station, 1818, is situated on the mountain of that name in Fifeshire, and about 1 mile south-south-west from the town of Falkland. In 1818 the site of the station was marked by a stone 4 feet long and 13 inches square sunk in the ground, with the date "1818" and the letters "T. S." cut on its upper surface. This stone was refound in 1841, but the letters were obliterated. A hole was bored in it, and a pile of turf 12 feet high, with a diameter of 16 feet, was raised above it.

Ely Minster Station, 1842–3, is in the centre of the octagon top of the western tower.

Easton Tower, 1843. This station is on the tower of Easton church, about 2 miles south-west of Stamford. The instrument was supported on a scaffold erected over the centre of the tower, and just high enough to enable the observer to obtain a clear view over the parapet and staircase.

Epping Cupola, 1844. The cupola on which this station is situated, and from which it is named, is that of the Union workhouse at Epping, in the county of Essex. The cupola is about the centre of the middle building, and is surmounted by a ball; the centre of the instrument was made to coincide with the centre of this ball, but should the ball be removed, the station may be found by bisecting the ridge of the central building, which runs north and south.

Fair Isle, 1821. This station is upon the highest part of Wart Hill, in Fair Isle, on the north coast of Scotland, nearly equidistant between the Shetland and Orkney Isles. The hill is the highest in the island, and lies near its north-west extremity. A large stone 4 feet in length, with 3 holes in it, marks the station; the central hole denotes the point over which the instrument was fixed, and in the other two were inserted pickets for the feet of the instrument. A staff was erected over the stone, and a pile 14 feet in diameter and about 16 feet high built round the staff.

Fairlight, 1844. This station is on Fairlight Down, in the county of Sussex. The site on which the instrument was in position is about 81·75 feet from the centre of the octagonal tower of Fairlight Windmill, and is marked by a pole or beacon 30·6 feet high.

Fashven, 1838, is a mountain on the north coast of Sutherlandshire, about 1·25 miles south from the sixth milestone on the road between Durness and Cape Wrath Lighthouse. It rises very abruptly on three sides, and on the fourth in a long slope from the south to the north, terminating in a kind of saddle-top. The station was not on the highest point, but on a more convenient spot, 343 feet from the highest projecting piece of rock at the north extremity. Four holes will be found in the rock, covered by a pile, 10 feet in diameter and 12 feet high.

Feaghmaan, 1832, or Geokaun, is about 2·5 miles west of the ferryhouse opposite Reenard, and three quarters of a mile south of Reenardrolaun Point, in the island of Valencia, county of Kerry, Ireland. The station is on the highest part of the hill, immediately above the slate quarries.

The zenith sector station is identical with the trigonometrical station, and is marked by a centre stone, sunk about 1 foot under the surface, which by calculation is 884 feet above the mean level of the sea.

FETLAR, 1821. This station is on the top of Vord, the highest hill in the island of Fetlar, one of the Shetland Isles. The top of the hill is flat to a considerable extent, and on the north side there is a large mass of stones, apparently the ruins of some ancient building, from which the station lies about 150 yards south. On the west side of the top of the hill is another mass of stones, which appears to be the remains of one of the rude furnaces used in former times as signals of alarm; from this the station is between 70 and 80 yards south-east. The pickets on which the instrument stood were left in the ground, and a large flat stone, with a hole 3 inches deep bored in it, marks the centre of the station, over which a plank was erected, and a stone pile, 12 feet in diameter and 15 feet high.

FITTY HILL, 1821. The station on the top of this hill, in the island of Westray, one of the most northerly of the Orkney Islands, is on a mound of earth and stones, called by the inhabitants "the Watchhouses." The mound is consolidated by time, and forms one mass with the hill. The pickets on which the instrument stood were left in the ground; a large stone with a hole in it 3 inches deep, corresponding to the centre of the instrument, was sunk in the ground to mark the station, and a stone pile with a pole in it was erected above them.

FORTH MOUNTAIN, 1829, 1843. On the top of this mountain, which is about 4 miles south-west of the town of Wexford, there are several large projecting rocks, called the Carrickadee Rocks, on the summit of the highest of which is the station. The centre is marked by a hole, rather more than 4 inches deep and about an inch and a half in diameter, in the north-north-west slope of the rock. The holes made for the stand of the zenith sector will also be found in the rock. The station is covered by a large stone pile, 14·5 feet in diameter at the base and about 10 feet high.

FOULA, 1821. This station is on the top of Snood Hill, Foula, one of the Shetland Isles. It is marked by a large stone with a hole in the centre, which corresponded to the centre of the instrument. The pickets on which the instrument rested were left in position, and a pile of stones, 13 feet in diameter and 17 feet high, with a pole resting on the central stone, was erected above them.

FOUR MILE STONE, 1850. The station so called is on the summit of a small hill in the parish of South Newton, in the county of Wilts, and about 3·75 miles from Salisbury, and a little to the west of the road from Salisbury to Devizes. The centre of the station is marked by a rough piece of black flint stone, with a natural tapering hole in its upper surface of about 1·5 inches deep, and about the same diameter at the top. The stone is placed 20 inches below the general surface.

GADS HILL, 1845. This hill is near Chatham, in the county of Kent. The station is 250 feet north-west of the obelisk upon the hill.

GARFORTH CLIFF, 1842. The station on Garforth Cliff, in the parish of Garforth in Yorkshire, is about 8 miles east of Leeds, and about 315 feet north of the road from Leeds to Selby, through Whitkirk, in a line nearly with the churches of Kippax and Garforth. The frame on which the instrument rested was left in the ground, the top of the pickets being about 9 inches above the surface. The centre stone is 2·25 feet under the common surface.

GERTH OF SCAW, 1847. This station is situated a short distance to the west of the neck of land that connects Lambaness with the mainland of Unst, one of the Shetland Islands, which neck is occasionally in very stormy weather, with the wind from the north-west, washed over by the sea. The site of the station is marked by a stone, with a hole in its centre, placed about 2 feet below the surface, with a small pile of stones and turf above it.

GLASHMEAL, 1841. This station is on the highest and western summit of the mountain of this name, in the junction of the three counties, Perth, Forfar, and Aberdeen, about 8 miles south from Braemar, and 3 miles eastward of the road leading from Braemar to Glenshee. It is marked by a hole 2·5 inches deep in a centre stone, and built over by a pile of stones 20 feet high and 45 feet in circumference at base.

GOAT FELL, 1852. This station is marked by a hole 1·5 inches in diameter, and 3 inches deep, cut in the rock on the highest point of Goat Fell Mountain, 5 miles north-west of Brodick village in the Isle of Arran. It is the centre of a square formed by four pickets, on which the theodolite was supported. These pickets are wedged into the rock, and are covered by a pile of stone and turf 10·5 feet high, and 30 feet in circumference. The road to it by Brodick and the mill-dam is very rough, but not steep till within 2 chains of the top.

GOONHILLY, 1846. This station is situated on a bleak moor known by the name of Goonhilly Downs, about 6 miles south-east from the town of Helston, and 1·75 miles nearly due south of St. Martin's, in the county of Cornwall. The site of the station is a small mound, from which the undefined boundaries between the estates of different proprietors diverge. The point on the mound from which these boundaries radiate is marked by a beacon placed over the site of that which has obtained the local name of the "Dry Tree;" and it is now marked in addition by a stone having a hole bored in it, and placed 3 feet under the surface, and the beacon re-erected thereon is supported by three braces, each 12 feet long, and a small turf pile. This point is identical with that from which the zenith sector observations were made in 1850.

GORLESTON CHURCH TOWER, 1843. The station at Gorleston Church, in the county of Norfolk, is in the centre of the top of the tower.

GREAT STIRLING STATION, 1850, is about 1 mile south-west of the village of Boddam in the parish of Peterhead and county of Aberdeen. It is 25 feet north-west of the highest point of the hill, on which a large hole has been bored in the protruding rock to receive the base of a flagstaff. This rock is about 4·5 feet higher than the site of the sector station, and, being used as a sea mark in conjunction with the old Windmill near Peterhead, is not permitted to be removed. The quarrymen have, however, approached within 27 feet of it. The centre of the station is marked by a hole bored in the rock.

GREAT WHERNSIDE, 1840. This station is on the top of a conspicuous mountain, a few miles west from New Malton in the North Riding of Yorkshire.

GRINGLEY. Joining the east end of Gringley-on-the-Hill, a village near Gainsborough in Lincolnshire, is a field in which is a very large barrow. The centre of this barrow marks the station.

GALTYMORE. The station is about 8 miles south of the town of Tipperary, and situated on the summit of the Galtymore, which is the highest point of the Galty range of mountains. About the middle of the summit there is a small mound well covered with moss, and rising 4 or 5 feet above the ordinary surface of the mountain. On the summit of this mound is the station, marked by a piece of conglomerate rock, the upper surface of which measures about 24 inches by 17 inches. The centre hole is 2 inches deep and well formed. On the north of Galtymore there is a tremendous precipice.

HANGER HILL TOWER, 1848, is a slight square brick-built building on Hanger Hill, near Ealing, in the county of Middlesex. The station was upon the centre of the roof.

HAPPISBURGH, 1843. This station is on the tower of the parish church of Happisburgh, on the north-eastern coast of the county of Norfolk. The station is upon the top of the roof, the centre of the theodolite being made to correspond with that of the tower.

HART FELL, 1847, is a mountain in Dumfriesshire, about 6 miles north of the town of Moffatt; the station is on the highest part of the mountain, and is marked by a pile, 14 feet in diameter at base and 16 feet high, built of stones to the height of 3 feet, and the remainder of turf. The site on which the instrument was in position is denoted by a centre stone with a hole drilled in it, in which is inserted a halfpenny (date 1806).

It may be well to remark that there is the appearance of a station mark somewhat lower and to the westward of the true station, consisting of a circle of 8 small holes round a centre one.

HENSBARROW, 1845. The trigonometrical station on this large and well-known barrow near Roche, in the county of Cornwall, is 17·9 feet due south from a stone denoting the boundary of two estates, and is marked by a stone with a hole bored in its centre, placed 1 foot below the surface. The zenith sector station is 72 feet south of this point.

HIGH PORT CLIFF, 1846, is on the highest and most easterly of two headlands between Ventnor and Bonchurch in the Isle of Wight. The cliff consists of a chalky marl, and rises perpendicularly from the sea to the height of about 130 feet. The station was 9 feet from the edge of the cliff at an angle projecting southwards, and 6 and 13 feet respectively from two re-entering angles on the west and east sides; but, from the crumbling nature of the soil, the site will probably be found nearer the edge of the cliff than as above mentioned. The station on which Airy's zenith sector was placed in Port Valley is in the bottom of the valley, 263 feet on the north side of the High Port Cliff Station, and in a line with it and the south-east station on Boniface Down.

HIGH WILHAYS, 1845. This station is on the rock of the same name in the parish of Oakhampton, in the county of Devon, from which town it is distant about 5 miles south, and is about 1 mile south of a well known rock called "Yes Tor," which may be seen from the common called Oakhampton Park. The site is marked by a good stone pile, 20 feet high, erected above the centre mark of the station, which is a hole, 3·5 inches deep, drilled into the solid rock. The pickets on which the instrument rested were left in position, and being of hard wood, and let into the rock and fixed with lead, are likely to remain some time.

HILL OF HOWTH. This station is on the highest of a series of peaks and ridges of nearly equal altitude in the promontory of Howth, in the county of Dublin, and close to the eastern edge of a quarry. It was marked by a centre stone with a wooden plug driven into it, and the posts (about 1·5 feet long) which supported the instrument left in the ground. A pole, with pile surrounding it, was left over the station.

HINGHAM TOWER, 1843. This station is over the centre of the tower of Hingham parish church, in Norfolk. The tower is a plain stone building, about 120 feet high, without pinnacles, and the instrument was supported on three uprights let into two strong joists, which were let into the walls of the tower a few feet below the roof.

HOLME MOSS, 1841. This station is on the highest part of a large broad-topped moorish hill south-west of Holmfirth, and crossed by the road leading from Further to Huddersfield. About 9 feet of the soil at Holme Moss is soft black turf or moss, good for fuel.

The station is marked by a pile, which at an examination in 1841 was found to have been erected over the framework on which the instrument was placed.

HUNGRY HILL, 1832, 1843, is in the parish of Adrigole, about 5 miles north-east from Castletown, Bantry Bay, county of Cork. The hill has two summits, about three quarters of a mile apart. The station is on the higher and more northern one. The site is marked by a hole bored in the rock, which is within 1 foot of the surface, and the pickets which supported the instrument; the whole is covered by a pile. The zenith sector and trigonometrical stations are identical.

INGLEBOROUGH, 1807. This station is on the top of the mountain of this name, 5 miles east of Ingleton Village, in the West Riding of Yorkshire; it is about 60 yards from the highest part of an old building towards the north-east, which building is on the nearest brink of Ingleborough from the station south-westward.

INKPEN BEACON, 1844. This station is situated on or near the highest point of a cultivated field known by that name, about 2 miles south-east of the village of Inkpen and 1 mile north of the village of Coombe, in the county of Wilts.

The stone marking the centre of the station is imbedded in the soil at a depth of 4 feet.

There are no measurements for refinding this point, as there were no permanent objects in the vicinity to which it might be referred.

JURA, NORTH PAP, 1847. The station thus denominated is situated on the summit of the mountain called Beinn-an-oir, the highest and most northerly of three hills at the south-west extremity of Jura in Argyleshire. A pile, 16 feet high and 17 feet in diameter, is erected over the centre stone.

KARNBONELLIS. This hill is situated in the parish of Wendron, in the county of Cornwall. The station is on a flat, between two eminences on the top of the hill, and is marked by a round hole cut in a stone, with a pile of stone 15 feet high and 38 feet in circumference at base erected over it. A new stone fence has lately been built 51·5 feet south of the station.

KARNMINNIS, 1845. This station is in the parish of Towednack, in the county of Cornwall, about 3 miles south-west of St. Ives, and about a quarter of a mile south of the road leading from St. Ives to Zennor Churchtown. The site of the station is a bare rock, close on the south-west side of an old ill-defined cairn (the summit of the cairn is about 8 feet higher than the station), from which the station is separated by an old fence, which at this place is the boundary of the parishes of Towednack and Zennor. A turf pile indicates the site, and as the centre mark of the station a hole 2 inches deep was drilled in the rock beneath the pile. The piles which supported the instrument were let into the rock with lead and left in position enclosed in the pile.

KEEPER, 1830. This station is on the top of the Keeper Mountain, about 6 miles north-north-east of the village of Newport, in the county of Tipperary, and 3 miles east of the chapel and national school-house of Killascully. On the north-north-east summit of the hill is a large cairn of stones, on the top of which is the station. The centre stone, which is level with the surface, measures 31 inches by 20 inches, and has a hole 2 inches deep drilled in its upper surface. This, with the pickets of the great instrument frame, is covered by a pile 6 feet high, or 10 feet above the general surface. About one chain south-south-west of the station there is a collection of stones resembling that upon which the station is, but not so high or so well formed.

Kellie Law, 1847. This station is about 4 miles north-west of Anstruther, in Fifeshire, on a hill of the same name, and is 30 feet north-west from the centre of a large barrow. It is marked by a hole 4 inches deep in the centre of a stone measuring 2·5 feet by 1·3 feet, and placed 3·3 feet below the surface of the ground. There is a stout pole 30 feet long, with a cross on its top, resting on the centre stone, and a small pile built around it. From this point the zenith sector station is distant 210·1 feet, bearing 302° 53′ from the south.

Keysoe Spire, 1843. This station is coincident with the centre of Keysoe Church Spire, in a small village near Kimbolton, in Huntingdonshire.

The spire is tall and slender, and to obtain a bearing for the instrument, about 20 feet of the top was removed, and a simple frame of joisting was laid on the base of the soundholes below, on which the feet of the instrument stood, just high enough to give a clear view over the wall top, and with barely sufficient room for the observer, also inside of the spire, to move round the instrument.

Kippure, 1829. This station is on the summit of the mountain of this name in the county of Wicklow, 10 miles south of Dublin, and about the same distance west of Bray. It is marked by the frame on which the instrument rested, left in the ground, and by two centre stones; the original one, which is large, is sunk 4 feet below the surface, and marked by a deeply-cut cross; the other is smaller, and rests between the upper ends of the posts. Three shallow cup-like holes are in line north of the true centre hole. A pile 18 feet in diameter,—the lower part of stones, the upper of sods,—is erected over the station. A small pole is supported in the centre of the pile.

Knockalongy, 1828. This mountain is the highest of the Ox mountains in the county of Sligo, 14 miles west-south-west of the town of Sligo, and 4 miles south-south-west of Skreen. The station, which is on the summit, is marked by a cross cut on a large stone. The top of the mountain is covered with soft bog about 8 feet deep.

Knockanaffrin, 1829. This station is on the summit of the mountain of this name in the parish of Rathcormac, in the county of Waterford. It is marked by a hole 1·5 inches in depth and 1 inch in diameter, drilled in a stone, and covered with a stone pile 12 feet high, and 35·5 feet in circumference at base. There is a large piece of rock 8 feet north-west of the station.

KNOCKLAYD, 1827, is in the county of Antrim, and 2 miles south of Ballycastle. The top of the mountain is covered with bog or peat to the depth of nearly 10 feet. In many parts this turf has been abraded to considerable extent, and on the highest of the lumps thus formed is the station. A large hole was dug through the turf, and the frame for supporting the instrument made to rest on the firm ground; this frame was left in position, and a very weighty centre stone was placed at the bottom of the hole; another stone was placed under the pole marking the station.

KNOCKMEALDOWN. This station is on the confines of the counties of Waterford and Tipperary; it is about 3 miles south of Clogheen, and about 15 miles south-west of Clonmel. When visited in 1852 no trace of the station could be found, a boundary fence across the top of the mountain having been constructed over the station.

KNOCKNADOBER, 1840. This station is about 6 miles north-east of the town of Cahirciveen, in the south of the county of Kerry, and on the highest point of a high mountain, the north base of which forms the south coast of Dingle Bay. The station is marked by a stone pile 7 feet high, within which is a small centre stone slightly imbedded in the gravel or common surface.

KNOCKNASKAGH, 1831, in the north of the county of Cork, is about 3 miles south-west of Rathcormac, a village situated on the road from Cork to Fermoy. The station is on the eastern spur of the mountain, which slopes away from this point in a south-west direction, and extends to a distance of about 3 miles. In the immediate vicinity of the station the mountain has a thin covering of peat moss, but without any heath. The centre stone is a piece of sandstone, having a hole about three quarters of an inch deep in its smooth upper surface, and placed about 24 inches under the surface, over it is a pile about 8 feet high. The remains of the old cookhouse are about 100 yards north-east of the station; and from the circumstance of there being a little turf or heath where it was built, it is likely to remain a permanent object.

LAWSHALL CHURCH TOWER, 1843. The station on the church of this parish, in the county of Suffolk, is on the top of the tower, and at the intersection of two lines drawn from the corners of the parapet inside the tower.

LAXFIELD TOWER, 1845. The station so designated is upon the tower of the parish church of Laxfield in the county of Suffolk. The station is in the centre of the top of the tower.

LAYTON HILL, 1817, is situated in Aberdeenshire. It formed the northern terminus of the Belhelvie Base measured in 1817, but the exact station cannot be identified, as the ground about it has been reclaimed and cultivated.

LEITH HILL, 1844. This station is on the centre of Leith Hill Tower, near Dorking, in Sussex. The station of 1822 bears from this point 137° 58′, and is 78·17 feet distant from it. The station of 1795 bears 207° 55′, and is 42·83 feet distant.

LINCOLN MINSTER, 1842. The station at Lincoln Minster was on the great tower about 4·2 feet to the north and 6·25 feet eastward of the centre of the tower.

LITTLE STIRLING, 1814. This station is in the parish of Cruden, Aberdeenshire, about 28 miles north of Aberdeen, 5 miles south of Peterhead, and about 1·5 miles to the left of a new road leading from Aberdeen to Peterhead. The four pickets which supported the instrument frame, and a centre picket, with a hole in it 9 inches deep, were left at the station; a centre stone was also placed over the centre picket, and a pile of turf, 18 feet high, with a circumference at base of 45 feet, erected around them.

LOUGH FOYLE BASE, NORTH END. This station is situated in the townland of Ballymulholland, parish of Magilligan, in the county of Londonderry, about 2·75 miles south of the martello tower on Magilligan Point at the entrance of Lough Foyle. The ground between this station and Mount Sandy is composed of low sand hills, and is much broken and very rugged.

LOUGH FOYLE BASE, SOUTH END. This station is on a small rising ground called Sheep Hill, about a quarter of a mile south-east from Ballykelly Church, in the townland of Drummond, parish of Tamlaght, and county of Londonderry.

These stations are marked by dots made with the point of a needle in platina wires, $\frac{1}{20}$th of an inch in diameter, run with lead into holes 1·5 inch in diameter and 6 inches deep, bored into blocks of Dungiven sandstone, 4 feet square and 20 inches deep. These blocks are laid in cement above other and similar blocks roughly chiselled, and placed on beds of solid masonry. The whole at each station is enclosed in a chamber of masonry 6 feet square, with walls 2 feet thick and 3 feet deep, covered over with a lid of flagstone, with bolts and rings passing through them, by which they may be removed with safety to the dots. On the upper surface of the flags cross lines are drawn, with the crosses vertically over the dots. This masonry is covered over with a tumulus of earth; and a circular wall 2 feet thick, with eight internal buttresses, is built as a base for an iron railing 4·5 feet high, enclosing a space 30 feet in diameter. The zenith sector station is in the same field with the south end of the base, from which point it is distant 559 feet due east.

LONGMOUNT POLE, 1843. The station so denominated is situated on the range called the Long Mynd, and is about 2 miles in a westerly direction from the town of Church Stretton in Shropshire. On the completion of the observations in 1843, a stone 2 feet by 18 inches and 3 inches thick, with a hole jumped in it to mark the centre of the station, was placed two feet and a half below the surface of the ground. On the upper surface of the stone are cut the words "Longmount Pole Station, 1843." A pole, standing 42 feet above the surface, was erected over the centre of the station, and a pile 18·5 feet high and 17·5 feet in diameter built around it.

LUMSDEN, 1846, is a hill in Berwickshire, close to the southern side of the entrance to the Firth of Forth, about 4 miles west of Coldingham, and a short distance from the Edinburgh and Berwick Road. The station is about half a mile from Lumsden Farmhouse, and about 50 yards south-west of a large barrow; it is marked by the centre picket which supported the frame of the instrument during the observations in 1809. A stone, with a hole cut in it 3 inches deep, is placed over it, and a pile of turf 18 feet high and 54 feet in circumference, surmounted by a pole and cross in its centre, is erected above the centre stone.

LUNDY ISLAND, 1845. The station on Lundy Island, in the Bristol Channel, is in the centre of a ruin, said to have been formerly a chapel dedicated to St. Ann; the outline of the walls is yet visible. The lighthouse on Lundy Island is 141·25 feet due west from the station, which is marked by a centre stone, with a hole bored in it, and can be easily found.

LYNN OLD TOWER, 1843. This station is upon the top of the old Grey Friars tower at Lynn, in the county of Norfolk. The tower is hexagonal, with a flagstaff in its centre, and a staircase, turret, and vane at its most northerly angle. The instrument was placed nearly in the centre of the tower, in the spot where the flagstaff stands. The flagstaff stood originally 3 feet further south, but at the conclusion of the observations it was fixed above the site of the station.

MAKER CHURCH TOWER, 1846. This station is upon the tower of the parish church of Maker, near Plymouth, in the county of Devon. The instrument was placed in the centre of the tower.

MALVERN, 1844. This station is at Great Malvern, in the county of Worcester. Two sets of observations have been taken with the great instrument at this place, viz., in 1803 and 1844. The site upon which the earlier observations were made is not to be found. The present station is within a mile of the town, and the centre stone by which it was marked was 2 feet by 1·5 feet, and inscribed "Great Malvern station, 1843-4." It was imbedded in the soil, and the framework on which the instrument stood was also left beneath the pile.

MAMSUIL, 1848. The mountain of Mamsuil is on the borders of the counties of Ross and Inverness, about 7 miles north-west of Captain Inge's shooting lodge in Glen Affaric, and 20 miles west of Invercannich. The station is on the highest point of the mountain, and is marked by a stone pile 23 feet high and 60 feet in circumference at the base, erected above the frame upon which the instrument was fixed; the frame was 10 feet high.

The only permanent object near the station is a pile on Carnet, a mountain three quarters of a mile north, and about the height of and having a similar appearance to Mamsuil.

Mendip, 1844. This station is on the Mendip Hills, in the south-east corner of a field in Oak Hill, Somersetshire, near the junction of four roads leading respectively to Bristol, Frome, Shepton Mallet, and Wells. The site is about 161·5 feet north-west from the finger-post at the junction of the roads, and about 94 feet north of the mile-stone. A centre stone is placed 18 inches below the surface.

Merrick, 1852. This mountain is in Kirkcudbrightshire, about 4 miles east of Polgowan Farmhouse, 12 miles north of Newton Stewart, and about 5 miles east of the road leading from that place to Ayr. The whole of the northern side of the mountain is bounded by a steep cliff, from the highest point of which a sharp narrow ridge extends towards the north, connecting the mountain with an adjoining hill called Shalloch of Menach. The station is about 10 yards from the point where the ridge joins the top of the mountain, and in the angle formed by two fences. The frame of the instrument was left in position, and a substantial stone pile, 48 feet in circumference at the base and 16 feet high, was erected above it. The dimensions of the centre stone are 2 feet by 1·4 feet, and 5 inches thick; the hole marking the centre is 2 inches deep.

Mickfield Church Tower, 1845. This church is near Debenham, in the county of Suffolk. It has a square tower, and the station is nearly in the centre, being 8·1 feet distant from the north parapet wall, 7·1 feet from the south wall, 8·8 feet from the east wall, and 7·5 feet from the west wall. The instrument was raised by scaffolding about 30 feet above the parapet.

Milk Hill, 1850. This hill, also known as Alton Hill, is in the parish of Stanton, in Wiltshire, about 2 miles north of Honey-street wharf. The station on this hill is in a cultivated field, about 877 links west from an artificial pond or reservoir, surrounded with a thorn hedge, and about a mile and a quarter east of a house on the summit of St. Ann's Hill. A coarse stone, with a small hole in its centre, is placed 2 feet below the ordinary surface, to denote the precise spot from which the observations were made.

Monach, 1839, is a saddle-backed hill, in the parish of Stornoway, in the northern part of the island of Lewis. The surface of the ground is 813·8 feet above the mean level of the sea, as obtained by spirit levelling; but from the fact of the hill rising from an extensive flat moor, with no other high ground in its vicinity, it is a very conspicuous object from every direction, and cannot be mistaken. It has two summits, lying east and west of each other; the station, which has also been adopted as a sector station, is on the highest part of the eastern summit.

The ground surrounding the station being all soft and peaty, and the instrument when in position not being insulated, a slight tremor was detected on closely watching the bubble of the level when any jumping or concussion of the ground on the outside took place; otherwise the instrument was firm and steady. A pile, 14 feet high and 57 feet in circumference, marks the station.

MOUNT BATTOCK, 1847, is a high mountain at the eastern extremity of the Grampian Hills, and in the county of Kincardine, about 4 miles west of the "Muckle Stane" of Clachnabane, which nearly overhangs the high road from Kincardine O'Neil to Fettercairn, and about 9 miles west of the bridge of Dye on the road leading from Banchory to Fettercairn. The station on this mountain is marked by a pile of stone, 16 feet high with a diameter of 15 feet. The framework which supported the instrument during the observations in 1847 was left in the ground, and the centre from which these observations were made is marked by a stone, with a hole 1·5 inches deep, placed level with the surface in the centre of the four piles on which the instrument stood, and level with their tops in the true vertical line of the original centre-mark, which was removed.

MORMONTH, 1847. The hill on which this station is situate is called Mormond, and is about 3·5 miles north-east of the village of Mormonth, otherwise known as Strichen, in the parish of Rathen, in Aberdeenshire. The station is on the south-east summit of the hill, but there is no permanent object near it by which its position could be pointed out. It is, however, marked by a stone with a hole 1·5 inches deep, placed 5·75 feet below the surface of the ground, in the centre of the frame on which the instrument was fixed, which was left in position. A pile of turf, 16 feet high and 15 feet in diameter at the base, was erected above the station.

MOUNT LEINSTER station is on the summit of the mountain known by this name, about 5 miles north-north-west of the village of Kiltealy, and about an equal distance west-south-west of the town of Newtown Barry. The station is marked by a large and irregular pile of stones, having a circumference at base of about 70 feet. The centre stone, which is a piece of granite measuring 32 inches by 12 inches, having a good hole in its centre, with four other holes round it, each about a quarter of an inch deep, is in the centre of the pile, about 6 feet above the general surface of the ground.

MOWCOPT, 1851. Mowcopt is the highest of a range of hills, extending in a south-west and north-east direction in the county of Cheshire. The station on this hill is situated about a mile and three quarters north of Newchapel, and about 3 miles south-west of Biddulph Church tower. The site of the station is marked by a well-defined hole bored in a large stone, the upper surface of which is placed about 8 inches below the general surface of the ground, and is about 24 feet 10 inches south-east of a large cut picket or cross on the edge of a precipice. A pole 21·5 feet high was set up over the centre, and a stone pile 41 feet in circumference at base and 7 feet high erected around it.

NASEBY TOWER, 1842. This station is over the centre of Naseby Church tower in Northamptonshire. The tower is free from pinnacles, and the instrument was raised on a scaffold just sufficiently high to enable the observer to isolate the instrument from the roof of the tower.

NAUGHTON CHURCH TOWER, 1845. The station on this church tower, in the county of Suffolk, is in the centre of the roof of the tower. The instrument was placed over the centre of the tower on a platform about 14·5 feet high. The station was not free from motion.

NODES BEACON, 1845. This hill is in the parish of Freshwater, Isle of Wight, about 6 miles south-west of the town of Yarmouth. On the highest part of this well-known spot, and about 1·5 miles from the lighthouse on the Needles Point, there is a large barrow on which there is a beacon about 40 feet high.

The station commands an open view of the sea between St. Catherine's Point and St. Alban's Head.

The station is on the northern side of the barrow, about 32·75 feet from the centre of the beacon, therefore no object was erected over the site. A large stone, with a hole in its centre, was sunk 3 feet below the surface to denote the station.

NORTH RONA, 1850. This station is on the south-east and highest point of the island of this name, about 47 miles north-north-east of the Butt of Lewis. The hill is 355·2 feet above the level of the sea, and is very precipitous on the east side, but sloping gently towards the west. The zenith sector station of the same name is 24 feet north-north-west of the trigonometrical station, and both are marked by holes bored in the rock.

NORWICH SPIRE, 1844. This station is on the top of the spire of the cathedral church of the city of Norwich in the county of Norfolk. The vane and the upright stone were removed, and the instrument placed on the flat stone composing the leaves of the finial, the centre of the instrument coinciding with the centre of the vane. The observatory was supported by a scaffolding built round the spire, and connected at various heights with its inside frame-work to within 30 feet of the top, above which height the scaffolding supported itself quite independently. The height of the scaffolding was 81 feet, and the instrument was 299 feet above the ground.

NORWOOD, 1844. This station is near the north-west corner of a cultivated field on the east side of the road leading from Sheerness through East Church in the Isle of Sheppey. It is marked by a well cut square stone with a hole bored in its centre, and the letters B.O. and a broad arrow cut on it. The stone is about 3 feet under the surface. The instrument was raised on a stage 31 feet above the ground for the purpose of obtaining a clear view over various trees in its immediate vicinity on the west side.

OLD LODGE, 1849. This station is in Old Lodge Farm, in the parish of Lower Wallop, Hampshire, about 3 miles west of Lower Wallop Church. The site of the station is in a plantation about 40 chains north-east of Old Lodge farmhouse, and is marked by a square stone, with a hole in it 1·5 inches deep, placed 15 inches below the general surface.

OLD SARUM CASTLE, 1849. This station is on the highest part of the north-west side of the inner ring of Old Sarum Castle, in Wiltshire.

The station is marked by a piece of green sandstone sunk 2·8 feet below the surface. The stone has a hole drilled in it to the depth of about 1·5 inches, which was afterwards filled in with lead around a piece of copper wire, which projects about an eighth of inch above the surface of the lead; the piece of wire marks very accurately the point above which the instrument was placed in 1849.

OLD SARUM GUN, 1849. This station is the south-west end of the base line measured on Salisbury Plain during the summer of 1794, and was then marked by an iron cannon imbedded in the ground, with its muzzle projecting about 12 inches above the surface. It is situated on the right hand side of the road from Salisbury to Amesbury, and 617 links on the Salisbury side of the second milestone on that road, and about 25 feet east of the boundary of the road. The centre of the bore at the muzzle was adopted as the centre of the station of 1849. A piece of hard wood driven into the bore, and having a needle inserted in its centre, denotes the precise station, and the extremity of the base line.

ORFORD CASTLE, 1843, is in the town of Orford, on the east coast of Suffolk.

The instrument was placed exactly above the centre of the spot occupied by the flagstaff, which was cut down for the purpose, but afterwards replaced. Should the flagstaff at any time be removed, the station may readily be found from the circumstance of the heel of the staff having rested on the newel of the stone staircase.

PADDLESWORTH, 1844. This station is situated in a grass field near Paddlesworth Church, in the county of Kent, and not far from the highest ground. It is about 726 feet east-south-east of a circular fir plantation, and marked by a large stone, having the words "Station of 1844" cut on it, with a hole 3 inches deep to mark the centre, sunk 1·75 feet below the surface. A pole 34 feet high was also erected over the centre.

PARACOMBE, 1845. The station at Paracombe, on the summit of a high hill in the north-west coast of Devonshire, is about 2 miles south-east of the village of that name, and about 1·25 miles west of Longstone Bar. The hill forms part of the Exmoor range, and has several barrows on its top, and a stone wall running along it. The station was upon the third barrow from Paracombe, and about 23 feet from the wall.

PENDLE HILL, 1841. There is a round conical barrow about 10 feet diameter upon the hill. The centre of the barrow is the station, and is marked by a stone, measuring 30 inches by 18, with a hole jumped in it.

PENINNIS, 1850, is in the Isle of St. Mary's, one of the Scilly isles. The station is upon the top of the windmill, and in the centre.

PERTINNEY, 1845. This station is on a low round hill about 2·5 miles south-south-east of the village of St. Just in Cornwall, and about half a mile west of the road leading from St. Just to Sancreed Church. The station is within the ruins of an old fort and near the north-west side; it is marked by a stone having a hole drilled in it 2 inches deep and placed level with the surface. The pickets on which the instrument rested were left in the ground, and a pile of stones raised above them to the height of 12 feet, with a diameter at the base of 9 feet.

PILLESDON, 1845. This station is on the hill known as Pillesdon Pen, in the county of Dorset, about 2 miles north of the village of Pillesdon, 3 miles south-west of Broadwinsor, and 7 miles north-west of Bridport. Two series of angles have been taken from the hill, the first in 1795, and the latter in 1845, on both occasions with the Ordnance 3-foot theodolite. The present station is on the south-east point of the remains of an ancient fortification on the top of the hill, about 40 perches to the right of the road leading from Broadwinsor to Axminster. A stone with a small hole in it to denote the centre of the station, placed 18 inches below the general surface, marks the precise position over which the instrument was placed in 1845. In 1850 a pole was erected above the centre stone.

PLYNLIMMON, 1805. The station on this celebrated mountain in Wales is 9 feet north of the centre cairn on the top of the hill, and marked by a stone pile.

PRECELLY, 1842. This station is on Precelly mountain about 14 miles from Haverfordwest, in Pembrokeshire, South Wales, and in a north-easterly direction from it. It is about 2 miles north-east of the New Inn, and about half way on the high road between the towns of Haverfordwest and Cardigan. The mountain is of a conical shape, and has two small barrows on its summit; on the centre of the south-western barrow is the station.

On completing the observations in 1842, a centre stone 2 feet 6 inches long, 1 foot 9 inches broad, and 1 foot 4 inches thick, with a hole in it, was left to mark the site, the upper surface of the stone being about 18 inches below the surface of the ground. In attempting to place the stone in the hole, it fell with the side in which the hole had been jumped undermost, and as its great weight rendered it very difficult to move it, a fresh hole was jumped in the upper surface of the stone, and marks the centre of the station.

RONAS, 1821, is a large flat-topped hill in the north-west of the Mainland, one of the Shetland Isles. The hill is covered with blocks or boulders, and the débris of granite. The station is on the highest part of the hill, about 45 yards north-west-by-north of the ruins of an ancient watchhouse, and is marked by an oblong stone about 5 feet long, with a hole bored in it to mark the centre. The wooden centre, and the pickets on which the stand of the instrument was fixed, were also left in the ground, and a large pile of stone 13 feet in diameter and 16 feet high, was erected around a staff placed over the centre stone.

RUE REA, 1848. This station is on the highest ground about Rue Rea Head in Ross-shire. It is about 10 miles north-west of Poolewe, about 5 miles west of Inveransdale, about 1·5 miles north from Melvaig, and about 2 miles south-east from the extreme headland forming the entrance to Loch Ewe. The ground for some miles about the station is of a very soft and boggy nature, so much so that it is a difficult matter to approach the station in wet weather; but the station itself is on a bed of firm and sandy ground. A turf pile, 45 feet in circumference at the base and 15 feet high, indicates the position of the station, which is also marked by a hole in a large stone placed about 2·5 feet below the surface, and within the frame on which the instrument stood; the top of the frame is level with the general surface. There is not a mark of any description near the pile that could be taken for it from any direction.

RYDER'S HILL, 1845, is a flat hill situated about 6 miles west-by-north of Buckfastleigh Church and about 4 miles from the small village of Holne, and nearly an equal distance from the Two Bridges Inn in Devonshire. The station is on the summit of the hill, and is well known by the name of "Peter on the Mount," by the inhabitants of the South Dartmoor. About 12 feet south from the station stands a stone with the letter H rudely cut on it, to mark the limit of the parish of Holne in this direction; the parochial authorities in their annual perambulation describe the stone by the name Peter on the Mount. The entire framework used in the support of the great instrument was left in position, and a stone, with a small hole 2 inches deep to mark the centre, was also placed on the gravel within the frame; the interior of the frame is filled up with stones level with the surface, and a turf pile, 12 feet in diameter and 13 feet high, was erected above it.

ST. AGNES STATION, 1850, is situated 219 feet to the north-east of the lighthouse in the parish and island of St. Agnes, one of the Scilly Islands. It is close to the road which passes the lighthouse and the church, on a small common used as a playground by the children attending the parish school. The centre is marked by a hole in a stone sunk 2 feet below the surface.

ST. ANN'S HILL, 1792. This station is upon St. Ann's Hill, in the county of Surrey; it is about 20 feet below the top of the hill, and on being revisited and identified in 1844 was marked by a centre stone and a pole. In consequence of some obstructions to the view a new station was prepared on the top, about 285·5 feet from the old station, but no observations were made from it.

ST. MARTIN'S HEAD, 1850. This is a station on St. Martin's Island, one of the Scilly Isles, on the coast of Cornwall. The site of the station is about 50·6 feet from a large beacon known as St. Martin's Daymark.

ST. PAUL'S, 1848. This station is on the centre of the top of the cross on St. Paul's Cathedral. A cradle to fit the cross was constructed of scantling, on which the instrument stood, and a scaffold was erected from the golden gallery to the same level, and on the floor of this scaffold the observatory rested.

ST. PETER'S CHURCH TOWER, 1844. This church tower is that of the parish church of St. Peter's, in the Isle of Thanet, Kent.

The station is on the centre of the tower above a signal house on the top of the tower, formerly used for telegraphic despatches. The stage on which the instrument was fixed was erected around the original house upon the same base, but was otherwise perfectly isolated from it.

SAWEL, 1827, is situated 7 miles south-west from Dungiven and 2 miles from Learmont, in the county of Derry; its summit is round and covered with bog to the depth of 6 feet. The sub-stratum is mica slate, varying much in hardness. A centre stone, 3 feet long, 2·75 feet wide, and nearly 1 foot thick, and marked with a hole, rests between the tops of the frameposts, is wedged tightly with stones, and further secured by mortar; on this was erected a staff, and around it a pile of stones and turf 15 feet high and of rather greater diameter, was built.

SAYRS LAW, 1846, is the highest point of the Lammermuir Hills, in Haddingtonshire, about 10 miles south-east of the town of Haddington, and 7 miles east of the village of Gifford. There is no permanent object near the station, except a pile on West Hopes Hill, about 1 mile west. The site of the station is marked by a pile of turf, 20 feet high and about 58 feet in circumference at base, erected above a large stone about 2 feet long and 10 inches wide with a hole through it, under which is an oaken picket. The frame upon which the instrument was fixed was left in position over the centre stone and picket.

SAXAVORD, 1817. This station is on the hill of that name in the parish and island of Unst, the most northerly of the Shetland Isles; it is an isolated hill at the northern extremity of the island, and rises abruptly from the sea to the east of the north extremity of a range of hills, extending from the north-west point of the island at Hermaness to the south-east point at Mouness; the hill sloping off rather more suddenly to the north than towards the south. The station is marked by a stone with a hole in its centre placed about two feet below the surface, and a small pile of stones about 8 feet in height above the general surface, which is by calculation 937 feet above the mean level of the sea. Observations were taken from this station with Airy's zenith sector.

SCA FELL PIKE, 1841. This hill lies between Eskdale and Wastdale, in the county of Cumberland; its summit is about three quarters of a mile north-east of Sca Fell, from which it is separated by a deep chasm called the Mickle Door. The station is marked by a pile, 21 feet high and 18 feet in diameter, beneath which the frame on which the instrument stood, and a centre stone with a hole in it marking the exact centre of the station, will be found. The nearest houses are at Wastdale Head, from whence the station is best approached.

SCARABIN, 1839, in the county of Caithness, is situated about 4 miles left of the road between Berriedale and Dunleath, both on the mail-coach road from Inverness to Thurso. The nearest houses to the foot of Scarabin are at Braemar, about 2 miles north. The top of the hill is covered with loose stones. The central and highest summit, upon which is the station, has rather a flat surface just below the highest ridge, and the pile denoting the station is a little to the left of it.

The pile constructed at the conclusion of the observations in 1839 was of stone, 20 feet in diameter and 20 feet on slope.

SCOURNALAPICH, 1846, is a high mountain in the north-west of the county of Inverness, close to the borders of Ross-shire. The site of the station of 1846 is about 100 feet west-south-west of the highest peak of the mountain, and is covered by a pile of stones 22 feet high and about 16 feet in diameter, raised above the frame and platform on which the instrument rested.

The piles of the framework were not driven into the soil, but were mortised into a square frame, which was placed on the natural surface of the mountain, and built round with stones to the height of 3 feet, forming a foundation for the platform of the observatory. In the centre of the 4 piles, a stone with a hole drilled in it about 2 inches deep, was inserted to mark the centre of the station.

The route to the station is by Struy Bridge to Invercannich Bridge and the head of Loch Molardich.

SEVERNDROOG TOWER, 1848. The station so called is the centre of the north-west turret of Severndroog Castle on Shooter's Hill, in the parish of Eltham, county of Kent.

From this point which is the station of 1822 and 1848, General Roy's station of 1792 is 10·73 inches distant, and bears 157° 30′ from the south.

SLIEVE DONARD, 1846. This station is on the top of the highest of the Mourne mountains, in the county of Down, in Ireland It is marked by a hole bored in the centre of a square stone, measuring 3·5 by 3·5 feet, and by a pile of stones built over it.

SLIEVE LEAGUE, 1828, is a high mountain, about 12 miles west of Killybegs, in the county of Donegal, and forms the extreme south-west promontory of the county in which it is situated. The hill has three not very conspicuous summits; on the western of these is the station. On the eastern brow,—close to the cliff,—of the western summit, a pile of stones, 30 feet in diameter and 4 feet high, indicates the spot where the great instrument was once in position on the wildest headland probably in Great Britain or Ireland. The centre stone is a piece of mica slate, measuring 1·67 feet by 1·5 feet, imbedded in gravel and small stones, having a hole an inch and a half deep in its centre; the hole cannot be called a well formed one, being a little too wide at the opening, but still sufficiently defined for its use. About 1 mile east of the station, and probably 1,000 feet below its level, is a limestone quarry.

SLIEVE MORE, 1831, is an isolated hill in the north end of the island of Achill, in the county Mayo. Its length is in an east and west direction, and the pile denoting the station is about 2 chains west of the eastern extremity of its summit. It is marked by a hole in a piece of mica slate, measuring 24 inches by 14 inches, level with the surface, and having its length in a north and south direction. The centre hole in the stone is an ill-shaped one, being nearly 3 inches in diameter, and scarcely 1·5 inches deep.

Slieve More station is about 2·5 miles north-west of the chapel Dukenella, and 3 miles north-north-east of the village of Keel. It would be difficult to ascend the mountain from any side except the west.

SLIEVE SNAGHT, 1827. This mountain is in the parish of Carnadongh, in the county of Donegal. The station, which is on the highest part of the mountain at the east end, is marked by a stone about 2 feet square, having a hole 2 inches deep drilled in its centre, with a pile of stone, 14 feet high and 50 feet in circumference at base, erected over it.

SNOWDON, 1842. The station on this celebrated mountain in North Wales is upon the highest peak, and is marked by a pile 20 feet in diameter, beneath which a hole is bored in the rock to denote the centre.

SOUTHAMPTON STATION. This station was over the chimney of the west wing of the Ordnance Map Office, 4 inches south from a broad arrow cut on the south edge of the partition between the two central funnels.

From this station the zenith sector station is distant 108·0 feet, and bears 285° 53′ from the south.

SOUTH BERULE, 1840, forms one of a range of hills running north-east and south-west throughout the Isle of Man; it is about 7 miles north-east of Castletown, and 6 miles south-south-east from Peel.

The station is close to the south edge of the hill, and nearly in the centre of the diameter of a semicircular wall, distant about 200 yards from the station, which wall is supposed to have been the boundary of an ancient Danish camp, and is so called at the present time. A large slate stone with a hole bored in it, sunk 3 feet below the surface of the ground, marks the station. The sector station of 1842 is identical with this station, but that of 1845 is 74 feet due south, and about 16 feet lower.

SOUTH LOPHAM TOWER, 1844. This is the parish church of South Lopham, in the county of Norfolk. The instrument was placed above the centre of the top of the tower, upon a platform 19·25 feet above the battlements.

SOUTH RONALDSHAY, 1821. The station on the island of South Ronaldshay, one of the Orkney islands, is upon the top of the Wart Hill, where a mound of turf and stone marks the situation of an ancient watch-house. The instrument was placed on the top of the mound; a centre stone was sunk in the ground, and a pole and pile erected above it to mark the site.

SOUTHWOLD TOWER, 1844. This station is on the centre of the square tower of Southwold church, on the east coast of Suffolk. The instrument was raised 23 feet above the top of the tower by scaffoldings.

STOKE TOWER, 1844. This station is on the church tower of the parish of Stoke, in the county of Suffolk. The instrument was placed in the centre of the roof.

STORR, 1847. This station is on the top of Storr Head, a remarkably rocky and precipitous mountain, near the village of Rig, on the east coast of the Isle of Skye.

STRONSAY, 1821. The station on Stronsay, one of the Orkney Isles, is on the highest ground of the island, and near the Burgh Head. It is marked by a centre stone with a hole in it, and by a pile of stone and turf.

SWAFFHAM, 1843. Swaffham church spire, in the county of Norfolk, was selected for a station in the month of September 1843, to correct the triangles of Cambridgeshire with those of the north and east of Norfolk. The station was on the north-east corner of the tower, and the instrument was placed on a stage which raised it within 19 feet of the top of the spire, which was sufficient for the purpose of observing all the principal stations that had taken the spire.

SWYRE BARROW, 1845. This station is on the west side of an artificial mound raised about 70 or 80 years ago by the proprietor, Mr. Pitt, on Swyre Head, about 1 mile west of Encombe House, the seat of Lord Eldon, and about 2 miles north-west of St. Alban's Head, and nearly an equal distance south-east of the small village of Kimeridge, on the sea-coast of Dorsetshire. A block of stone weighing about 1·5 cwt., with a hole in it, placed 4 inches under the ordinary surface, marks the station. In the centre of the hole is a piece of rod iron (a quarter of an inch in diameter) run in with lead and projecting 1 inch above the surface of the stone; the centre of this piece of iron denotes the precise centre from which the observations were made. The four piles on which the instrument rested were left in the ground. The ground about the station is the property of the Earl of Eldon, and, as it is used as a preserve for game, the station may remain a long time undisturbed.

TARA HILL, 1829, is about 3 miles north-east of the town of Gorey, in the county of Wexford; it is a rough rocky hill, and in an ordinary sized carn of stones in its summit is the station; it is marked by a centre stone, having a good round hole in it 1·5 inches deep, level with the surface of the mountain, and a stone pile above it 12 feet high.

Tarbathy, 1817. This station is situated on Tarbathy Hill, on the coast of Aberdeenshire, and was the southern extremity of the Belhelvie Base. No trace of the exact site of the station is now in existence.

Taur, 1834. This hill is about 7 miles north-east of King William's Town, on the confines of Cork and Kerry. The station is on the summit of the hill, which is flat or rather concave about the station; in the centre of this concavity there is a small mound of stones, and the centre of this mound is the station. The pile is about 6 feet high, and the centre stone will be found 4·5 feet under the surface,—an irregular shaped stone weighing between 80 and 90 pounds. The top of Taur hill is covered quite close to the station with good peat moss. There are no stones on the hill, with the exception of those about the station.

Tawnaghmore, 1843, is a long narrow-topped hill in the county of Mayo, Ireland, about 2 miles from the coast guard station at Belderg. The station, which is identical with the zenith sector station, is on the highest point of the hill, and is marked by the frame on which the zenith sector rested, which was left in the ground.

Telegraph Tower, 1850, is a station so called after the telegraph tower on which it is situated, in the island of St. Mary's, one of the Scilly Isles. The station is in the centre of the top of the tower.

Tharfield, 1843. This station is on the high land eastward of the village of Buckland, in Hertfordshire.

Thaxted Church Spire, 1844. This station is immediately over the centre of the coping-stone of Thaxted church spire. The instrument was supported by a scaffold erected from about the middle of the spire, and the observatory was supported by another scaffold erected from the top of the tower at the base of the spire.

Tilton, 1843. This station is situated in a pasture field, which is the second from and on the north side of the road leading from Tilton village towards Oakham, in Rutland, and at a distance of nearly 2 miles from the former. It is marked by a centre stone of irregular shape, measuring about 18 inches by 16 inches, having a hole, 3 inches in depth and 1 inch in diameter, bored in its centre, sunk 3·5 feet below the general surface of the hill. The square wooden frame on which the instrument rested was left in the ground, the top being nearly level with the surface; and a pile of turf, 12 feet in diameter and 10 feet in height, has been erected over it.

Tofts Tower, 1845. This tower is that of the parish church of Tofts, in the county of Norfolk. The station is in the centre of the roof of the tower.

Trevose Head, 1846. The station on this headland in the Bristol Channel, about 5 miles north-west of Padstow, in the county of Cornwall, is on the most prominent part, and 917 links east-south-east of the centre of the lighthouse.

TROSTAN, 1827. This mountain is in the county of Antrim, 6 miles south-west of Cushendall. The station is on the top of the mountain, and is marked by a pile of turf 15 feet in height, and 50 feet in circumference at base. The top of the mountain is long and flat, and exceedingly boggy and wet.

UPCOT DOWN, 1850. This station is situated in a cultivated field on Upcot Hill, in the northern part of Wiltshire, about 1·25 miles east of the church tower of Broad Hinton, and about 5 miles south of Swindon church tower. The site of the station is marked by a piece of freestone, measuring 15 inches by 12 inches and 6 inches thick, with a hole, 1 inch in diameter and 2 inches deep, bored in it. The upper part of the stone is about 1·3 feet below the general surface of the ground. A pole, 12 feet long, was placed perpendicularly over the station.

VICARS CARN, 1827, is on a cultivated hill, about 4 miles south-south-east of the city of Armagh. The station is on the summit of a large carn of small stones known by this name. It is about 24 feet in diameter at the top, and its height above the general surface is about 10 feet. The station is a little to the south-south-west of the present centre of the carn; and the centre stone, which has a well-cut hole, is 6 feet below the surface of the top of the carn, and rests on a bed of small stones.

WALPOLE ST. PETER'S, 1843. This station was upon the church tower of Walpole St. Peter's, in the county of Norfolk. The tower is square, strongly built, and has a very high roof, out of which rises a massive vane post covered with lead. This post was taken down, and the theodolite placed exactly in the centre of the tower.

WALTON TOWER 1844. This station is on the centre of Walton church tower, on the east coast of Essex.

WATER CRAG, 1852. This is a barren mountain in the North Riding of Yorkshire. The station is a boundary mark between Arkengarth Dale and Swale Dale, and is situated 1·5 miles south-east of William Gill Houses, 5 miles west of Arkengarth Dale, and 7 miles north-by-east of Muker. There is not an object on the hill from which measurements could be taken, but the station is well known to the inhabitants for miles around. The site of the station is marked by a pile of stone 14 feet high, and 36 feet in circumference at base.

WEEK DOWN, 1846. The trigonometrical station on Week Down, on the south side of the Isle of Wight, is identical with that from which the observations were made with Airy's zenith sector. The Down is about three quarters of a mile north-north-east of the village of St. Lawrence, and has several tumuli upon its summit. Three of these tumuli lie nearly north and south of each other; on the central and largest of them, called "The Frenchman's Grave," is the station: it is marked by a centre stone, having a hole bored in it, placed 2 feet below the surface.

WESTBURY DOWN, 1850. The station on Westbury Down in Wiltshire is about 2 miles north-east of Westbury church, and 1 mile south-west of Bratton church, and about 200 yards south of Bratton Castle. The site is on the side of the road leading from Bratton to Warminster over the Down, and near the junction of several small mounds which run at right angles, and serve as boundaries to the several farms. A stone, with a hole 4 inches deep, and 1·75 in diameter, is placed 1·6 feet below the surface; and a pile 9 feet high, with a pole 21·7 feet long, was erected above the stone.

WHITEHORSE HILL, 1844, is in the county of Berkshire, on a range of hills running east and west between Wantage and Lambourn.

The station is on the outer edge of a bank, apparently that of a fortified encampment on the east-south-east extremity of the hill; it is 12 feet from the edge of the bank, and 20 feet west of the path leading from Farringdon to Lambourne. The stone placed to mark the centre of the station is nearly level with the surface, and has a hole jumped in its centre, and the year 1799 inscribed upon it.

WHITTLE HILL, 1841, or as it is better locally known, Whittle Pike or Skelp Moor, is in Lancashire, 2 miles east of the village of Edenfield, 6 miles from Bury, 7 miles from Rochdale, and about half a mile from Knoll Moor, which is higher than Whittle Hill, and bounds about 90° of the view to the north-east. The station was on the highest point of the hill, and is marked by a pile chiefly built of turf. The wooden frame on which the instrument stood, was left in the pile, together with a centre stone, having a hole in it, and marked with a broad arrow, and the date of the year "1841."

WINGREEN, 1844. Two series of observations have been taken from the top of Wingreen Hill, situated about 2 miles south-east of the village of Charlton, in Dorsetshire, and about 5 miles south-east of the town of Shaftesbury. The observations taken in 1794 were from a point now distinguished as Wingreen Old Station, and those of 1844 from a spot called Wingreen New Station. The stations are about 17·5 feet apart, the new station being about 50 yards in a south-easterly and perpendicular direction from the road which passes over the hill, and 185·9 feet south-south-west of an old tree in a small circular plantation of stunted birch trees. The old station is 203·3 feet also south-south-west from the old tree. Both sites are marked by centre stones with holes in them, in each of which was placed a halfpenny of the reign of George III. The stone marking the new station is sunk 3 feet below the surface, that at the old station 10 inches.

WISP, 1809, 1816. This station is on the top of Wisp Hill, 2 miles south-west of Mosspaul Inn, between the counties of Roxburgh and Dumfries. The centre is covered with a turf pile.

WORDESLOW, 1846. The station is about a mile and a quarter north-east of Houghton-le-Spring, in the county of Durham; it is 94 feet from the highest point of the hill, in line with the middle range of houses in the village of Houghton-le-Spring.

WROTHAM, 1844. This station is in the southern corner of a field called "The Plains" upon Wrotham Hill, in the county of Kent.

YELL, 1821. This station is in the Island of Yell, one of the Shetland Isles; it is on the top of Comb Hill, or the "Long Cairn" as it is termed by the inhabitants, to the north of two piles on the same hill, and nearly on the same level. One of the piles is of turf, and the other of stone; from the former it is about a quarter of a mile, and from the latter half a mile. A stone sunk into the ground, and having a hole bored in it to the depth of 3 inches, marks the centre. A pole 20 feet long and 6 inches in diameter, supported by a pile of stone and turf, was erected over the site.

YORK MINSTER, 1846. This station is on the centre of the great tower whence it is named.

SECTION II.

DESCRIPTION OF INSTRUMENTS.

THE perfection of the results of geodetical operations, and the degree of importance to be attached to them, must be proportioned to the excellence of the individual observations, or to the mean error of an observation, which depends not only on the skill and carefulness of the observer, but on the perfection of his instruments. It is therefore desirable in a collection of such results to have descriptions of the instruments with which the observations were made; but, inasmuch as minute descriptions of all the chief instruments that have been used on the Ordnance Survey have been already published in the previous volumes of the Survey, it will not be necessary here to give more than a general description of each.

Description of Theodolites.

(1.) The Principal Triangulation for the Ordnance Survey of Great Britain and Ireland has been effected with four large theodolites, namely, two of 3 feet in diameter, one of 2 feet in diameter, and a fourth of 18 inches diameter. Besides these, however, subsidiary use has been made in a few cases, and for short distances only, of smaller instruments, having diameters of 12, 9, and 7 inches. The two first-mentioned instruments, measuring 3 feet in diameter, were constructed by the celebrated Ramsden, at the close of the last century. One belongs to the Honourable Board of Ordnance, the other is the property of the Royal Society. The only difference between these instruments is, that the horizontal circle of the former is divided to ten minutes, and read by three microscopes; while in the latter the circle is divided to quarter degrees, or parts of fifteen minutes, and is read by five microscopes: the respective advantage in each case being counterbalanced by the corresponding comparative disadvantage. Each instrument has an extra microscope not generally used, being one of the original pair at 180° from one another. They have, therefore, respectively four and six microscopes. The following general description, written with reference to the Ordnance theodolite, of which an engraving will be found in Plate 1, is applicable, with the exceptions just mentioned, to the other instrument.

(2.) *The Great Theodolite.*—The instrument consists of three distinct principal parts, not including its stand, which is a strong circular four-legged mahogany table, firmly braced.

The lower part consists of, and connects rigidly, the feet of the instrument, the vertical axis, and the microscopes for reading the horizontal circle.

The next part above consists of, and connects rigidly, the horizontal circle, the hollow vertical axis fitting on to the axis before mentioned, and the Y's for carrying the telescope.

The third part consists of the telescope, its axis, and the vertical circles attached.

The instrument rests on three feet, provided with levelling screws, which are firmly united at the place where they branch off by a strong cylindrical plate, or cylinder of bell-metal, about eight inches high and seven in diameter, co-axial with the instrument. From this cylinder, and very strongly connected with it, rises the axis of the instrument, which is of steel, conical in form, and about two feet high, having a bell-metal base, and terminating in a cast-steel pivot with sloping cheeks. From the upper part of the cylinder project three arms, carrying at their extremities the micrometer microscopes for reading the horizontal circle; each microscope is over one of the feet of the instrument. These arms are strongly braced to one another and to the feet of the instrument, so that the connection between the microscope, the feet of the instrument, and its axis, is as nearly as possible rigid and unalterable. Besides these three equidistant microscopes, there is a fourth bisecting the angle of 120° made by one pair. This microscope, with that one which is 180° distant from it, were the original pair of microscopes, the other two having been subsequently applied. One revolution of the micrometer, when in perfect adjustment, is equal to a minute, ten revolutions carrying the moveable wire over the space between two dots. The micrometer head is divided into 60 parts, so that single seconds are indicated, and tenths of seconds can be estimated by dividing mentally the space, about a twentieth of an inch, between two divisions of the micrometer head.

The horizontal circle, of brass, is 3 feet in diameter, divided by dots into spaces of ten minutes, and connected very firmly to the lower part of the hollow brass exterior axis by ten hollow conical radii. The exterior axis is 24 inches high, 5·5 inches in diameter at base, and 2·5 inches in diameter at the top. Within the base of this hollow axis a collar of cast steel is strongly driven, fitting exactly the bell-metal base of the steel axis: the upper end is terminated interiorly in a thick bell-metal plate, with sloping cheeks fitting accurately on to the extremity of the steel axis. Thus the exterior axis and circle, when placed upon the interior axis, revolve with the greatest possible smoothness, and without any shake. By means of an arrangement of screws on the top of the axis it is easy to regulate the pressure of the outer axis upon the inner, and consequently the ease of revolution. When the instrument is travelling, the outer axis and circle are let down completely upon the inner axis. In this state it could not be used for observing, as it would require force to move the circle; nor, on the other hand, could the instrument be used for observing if the pressure were made too light, as in that case there would arise a shake at the centre from want of sufficient contact. The degree of pressure or ease of revolution must therefore be regulated by the observer, but the limits are not wide. This excellent method of centering is one of the best points of the instrument, and has been much admired. Probably no other could have stood the immense wear and tear to which these instruments have been subjected, not only from constant use, but from travelling, necessarily in the roughest manner, and exposure to weather on the summits of the wildest mountains.

A horizontal flat bar 2 feet long, and projecting 1 foot on each side of the centre of the instrument, extends across the top of the outer axis. On its extremities are raised the Y's for the support of the pivots of the telescope. It is supported and made perfectly rigid by two ladder-formed braces springing from the lower part of the outer axis. The Y's are of such a height as to permit a circle of 6 inches radius attached to the axis of the transit to pass freely, and consequently the telescope to be directed to the stars at high elevation, not including of course the zenith.

The focal length of the telescope is 36 inches, aperture 2·5 inches, and the generally used magnifying power 54. In the focus of the eye-piece are the cross wires, with the usual apparatus for adjustment in collimation. Two of the wires intersect in an acute vertical angle; the third is horizontal. By means of a small screw, this horizontal wire has a vertical motion, and is intended for measuring small vertical angles, the amount being indicated by the revolutions and parts of revolutions of the screw. In astronomical observations the wires are illuminated as in ordinary transit instruments, and a reflecting eye-piece used. Concentric with the hollow axis of the telescope, are two vertical circles, similarly placed on each side of the telescope, their planes about 2 inches and a half apart. The diameter of each divided circle is 10·5 inches. They are read by two long horizontal micrometer microscopes passing through the short vertical arms carrying the Y's, each division of the micrometer head being equal to 3 seconds.

The perfect adjustment of the micrometers as to distinct vision and runs is very tedious, and can only be effected after much practice. The runs of the micrometer are easily altered, particularly by travelling. In this adjustment, however, as is also the case with others, if the precise amount of error is known, it is all that is necessary; for if $10+n$ revolutions of the micrometer go to 10 minutes, or the space between two dots, then n being known by observation, the values of a revolution and division become known.

The instrument has two long levels; one of them, 21 inches long, hangs by brackets at the extremities of the horizontal arm, when required in that position, and is also used for levelling the telescope. The other is the striding level, 24 inches long, for levelling the pivots of the telescope in astronomical observations: one division of this level is assumed as equal to one second, which is true only for moderate temperatures.

The whole body of the instrument admits of a small horizontal motion in any direction on its table, which is perforated in the centre. This motion, which is communicated by horizontal screws acting on the raised edge of the mahogany table or stand, is used only for the final adjustment of the centre of the instrument vertically over the centre-mark of the station.

The weight of the whole instrument is about 200 lbs.; it is carried, when travelling, in a four-wheeled spring-van.

(3.) *The Eighteen-inch Theodolite* was also made by Ramsden. It is of precisely the same general construction as the two large theodolites.

The circle is attached by eight conical radii to the base of the exterior hollow axis. This conical axis is 12 inches high, 2 inches in diameter at the base, and 1 inch at the top, across which is fixed the horizontal bar which supports at its extremities the Y's for receiving the telescope pivots. The bar is 12·5 inches long, and is braced to the lower part of the axis.

The focal length of the telescope is 19·5 inches, the aperture of the object glass 2 inches, and the ordinary observing power 30. To the hollow axis of the telescope is connected the vertical circle; it is placed about 4 inches from the centre of the telescope and 2 inches from one extremity of the axis, having a counterpoise at the other extremity. The diameter of this circle is 8 inches; it is read to 10 seconds by three verniers. The horizontal circle, which is divided by lines to every five minutes, is read by three micrometer microscopes in the same manner as in the large instruments.

The whole instrument rests upon a thick hexagonal plate of mahogany, being firmly connected with it as its base. The instrument is levelled by three screws passing through this plate and acting in grooves cut in metal bosses in the upper surface of the three-legged stool forming the stand of the instrument. As in the 3-foot instruments, the same level, 12 inches long, is used for levelling either the telescope or the horizontal arm carrying the Y's, hanging in each case by brackets. A striding level is also used for astronomical observation; the value of one division of this level, as in the large instruments, is a second.

(4.) *The Two-foot Theodolite* was made by Messrs. Troughton and Simms at the commencement of the Irish survey. It is of a different construction from Ramsden's theodolites, and more strictly an altitude and azimuth instrument, with a repeating table, which has been seldom used.

The horizontal circle is 2 feet in diameter and connected with six conical radii to the vertical hollow internal axis of the instrument. This axis is conical and of steel, about 2·5 inches in diameter, but is not so long in proportion as the axes of Ramsden's instruments.

A cylindrical drum of metal, about 8 inches in diameter, having six vertical microscopes attached by conical radii or arms, and an interior hollow axis, is placed upon the vertical interior axis just mentioned, and revolves smoothly round it; the pressure is regulated as in Ramsden's theodolites. From a metal plate on the upper surface of the drum rise two pillars supporting the Y's for the pivots of the telescope. The pillars are of sufficient height to allow the telescope to revolve freely on its axis in a vertical plane, without coming in contact with the upper surface of the drum.

The focal length of the telescope is 27 inches, and the aperture 2·125 inches. It is placed between two vertical circles of 15 inches diameter, firmly secured together by connecting bars, and concentric with the axis of rotation of the telescope. The horizontal and both vertical circles are divided by lines, on silver inlaid into brass, to every 5 minutes; the former is read by the vertical microscopes attached to the radii proceeding from the drum, and the latter by horizontal microscopes passing through the pillars supporting the telescope. As in the Royal Society's theodolite, five of the microscopes are equidistant, or 72° apart, while the sixth bisects one of these spaces.

It will be seen that this instrument differs entirely from Ramsden's, in this respect, that the microscopes for reading the horizontal circle move round with the telescope, while the circle itself remains clamped; in Ramsden's theodolites, on the contrary, the horizontal circle moves round with the telescope, while the microscopes are fixed to the lower part of the instrument. A full description of the instrument will be found in the "Account of the Measurement of the Lough Foyle Base," pp. 108 and 109.

(5.) The stores generally accompanying an observing party are as follows:—

1 Waggon for Instrument.
1 Observatory.
2 Wooden houses.
11 Five-inch theodolites.
13 Large heliostats.
11 Portable heliostats.
15 Brass rings for heliostats.
7 Pocket telescopes.
1 Chronometer.
1 Mountain barometer.
11 Pocket compasses.
10 Measuring tapes.

&c. &c. &c.

with all the necessaries for camp life, and full supplies of stationery, carpenters' tools, mathematical instruments, and any books that may be of use to the observer.

The strength of the party varies from 6 to 10 men.

(6.) The portable observatories used for the theodolites are hexagonal in plan, with a pyramidal skeleton roof, canvas covered. The wooden sides or panels are 5 feet high (4 feet for the 18-inch theodolite), a space of 2 feet, canvas covered, being left between their upper edge and the horizontal rafters of the roof, for the unobstructed use of the telescope. The six iron rods supporting the roof and resting on the tops of the six uprights at the corners of the hexagon, are so formed, being double, that by revolving round a vertical axis, any one of them can be twisted out of the line of sight of the telescope, should it be necessary. Framed wooden huts, having canvas covers to their tops, have now been many years in general use for the parties employed on the Ordnance Survey with the large theodolites and with the zenith sector. The experiment was first tried in 1840, on the recommendation of Major Robinson, R.E., who was then in charge of the Ordnance 3-foot theodolite, in the north of Scotland, and they have been found to be not only much more comfortable for the officers and men employed on this arduous and, at times, disagreeable duty, but also more economical than marquees and tents. They vary in size from 8 feet square to 9 feet by 12, and being constructed in parts they are easily packed or carried by pieces to the tops of mountains.

(7.) Heliostats are used for reflecting the sun's rays to distant stations. The smallest size is 5 inches in diameter, being merely a plane mirror set in brass, with a stand or handle terminated in a spike for sticking in the ground. The next size is 12 inches in diameter. The larger heliostats are rectangular; the largest measures 20 inches by 16. The surface of the mirror, in each case, can be moved with ease to follow the sun. The exact position of its surface is determined by throwing its light through a ring placed in the line between the heliostat, or the reflecting station, and the station reflected to. The ring, which lies in a vertical plane perpendicular to the line joining the two stations, and at a convenient distance from the heliostat, is placed in its true position by means of a small theodolite.

Method of placing the Instrument in Position.

(8.) In placing the theodolite over a station to be observed from, the first point to be attended to, and which has always been considered of the greatest importance, is that it shall rest upon a perfectly solid foundation. The method of obtaining this desideratum must in each case depend entirely on the nature of the ground. While at some stations the instrument has rested almost upon the solid rock; at others some difficulty has been experienced in getting a steady foundation, as for instance at the station on Holme Moss, where it was found that the bog extended to a depth of 9 feet, and below it was a layer of sand 6 feet thick, all of which had to be removed to a total depth of 15 feet before a solid foundation could be obtained, from which to build a strong scaffolding up to the ordinary surface of the ground. Even on the tops of mountains some trouble has occasionally been found in obtaining a firm foundation for the instrument to rest upon. In one or two instances the instrument has been sunk in the ground, so that the telescope only grazed the top of the hill; the excavation also served as a protection against storms. This method would of course be very objectionable if the surface of the hill were flat and of considerable extent.

The following description from the observation-book of the station on Ben Hutig, in Sutherlandshire, will serve to give a general idea of the method of preparing the station when the foundation is rocky:—Four holes were jumped in the rock, about 6 inches deep and 5 inches by 3 in length and breadth, at equal distances of 1·75 feet from the centre mark of the station, to receive four pieces of wood scantling, upon the heads of which the feet of the table for the instrument were to be screwed. These holes were run with lead, the tops of the scantling cut off and levelled accurately, and further secured against shaking by four horizontal braces nailed near the tops, and also two diagonal ones. Their tops were cut off at the level of the highest piece of the rock on which a corner of the observatory rested, and were thus also on a level with the lower edges of the six panels forming the sides of the observatory. The panels were supported by piling up stones so as to form a level all round with that of the top of the rock. A space of about 4 feet square was left in the centre of the flooring, by which the instrument and its stand were insulated, and not liable to be shaken by any motion above or around it. The flooring was laid upon joists, 6 inches above the level of the top of the posts. A batten of wood was nailed upon the extremity of the flooring round the centre space, to keep the feet of the observer from touching the legs of the table of the theodolite.

When the station is on ground of a soft and boggy nature, a hole of a sufficient diameter is sunk until a bed of gravelly earth is reached, which is levelled and firmly rammed. A strongly braced framework of wood of sufficient height, is then placed in the excavation, which is afterwards filled up with stones. Thus the instrument being also isolated from the flooring of the observatory, is rendered as nearly as possible perfectly free from any source of unsteadiness. In order also that the centre of the station may not be lost, four pickets are driven into the ground previous to the commencement of the excavation, in such positions that the intersection of diagonal lines joining them shall coincide exactly with the centre-mark of the station.

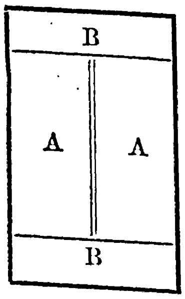
In exposed situations on the tops of mountains a wall of stone is generally built close round the lower part of the observatory, a precaution which at the station on Fashven saved the great instrument from certain destruction, as on the 11th of October 1838 a storm levelled all the marquees and tents, and moved the observatory from its true position, although thus protected by a wall 2 feet thick.

In some cases it has been necessary to raise the instrument above the surface, in order to obtain observations of distant points, not visible from the ground. The high stage used for the great instruments is 32 feet high, but has been only used in cases of absolute necessity. At the south end of the Base-line on Salisbury Plain this stage only barely sufficed to make the two extremities of the line mutually visible. Many instances of high scaffolding occurred in the triangulation of the Eastern Counties, where, the country being flat, the stations were over church-towers. One of the most remarkable was the station on Thaxted Church. The tower of this edifice is 79 feet high, and is surmounted by a spire of 93 feet, making a total height of 172 feet from the ground to the top of the spire. The scaffold for the observatory was carried from the base to the top of the spire; the scaffold for the supporting the instrument, which was interior to the other, was raised from a point of the spire 139 feet above the ground, having its bearing upon timbers passing through the spire at that height. Thus the instrument, at the height of 178 feet above ground, was insulated, and not affected by the movements of the observer, or the action of the wind upon the observatory. This was the 18-inch theodolite. The station, as will be seen on reference to the diagram, was indispensable to the connexion of the triangulation.

In placing the framework for receiving the feet of the instrument, every care has always been taken to have it as nearly concentric with the station as possible: the legs of the table of the instrument are then screwed to the framework. The circular mahogany tray upon which the instrument immediately rests, is then placed on the table, and accurately brought over the centre mark by means of a plumb line suspended from the orifice in its centre, and the motion communicated by horizontal screws acting on the edge of the table, as before explained.

(9.) *The Referring Object.*—The next step after the placing of the instrument in position is to select a spot for the referring-object; an object which from its position should be visible under all circumstances, and to which all other objects are to be referred as to their bearings.

The referring-object, or, as it is usually written in abbreviation, the R.O., generally used on mountain stations, consists of two similar and equal rectangular plates of metal, A A, in the same vertical plane, and with parallel edges, sliding upon horizontal cross-pieces B B. This being placed against the sky, a fine vertical line of light of any required breadth is obtained. When the opening cannot be backed by the sky, the same effect is easily produced by a plane mirror placed behind the opening, and inclined at an angle of 45°. The angular breadth of the opening generally adopted, and found most convenient for perfect bisection, is about 10″.



In the placing of the referring-object, two points have to be attended to; namely, that it must be as nearly as possible on the same level as the observer, and that it must not be either too close or too distant. If too close, it will not be visible to perfection without altering the focus of the telescope, which is to be avoided; and if too far off, it may be hidden, when required, by mist. The ordinary distance is between one and two miles; but upon the top of a mountain it is obvious that a selection cannot always be made, and the best position that offers must be taken. In some cases the distance has been as small as half a mile, and the effect of the proximity of the referring-object then tells with disadvantage upon the observations of the elongations of circumpolar stars for absolute azimuth. In these observations a lamp is placed behind the referring-object.

Method of Observing with the Theodolite.

(10.) All the adjustments of the theodolite are more or less liable to be deranged by travelling, and consequently the first operation at a station is to correct any deficiencies of this nature. In general, however, the mode of observing is calculated to eliminate as much as possible the effects of slight errors of adjustment.

The general adjustment consists in this: that the line of collimation of the telescope should be perpendicular to its axis of rotation, this axis perpendicular to the revolving axis of the instrument, and the latter perpendicular to the plane of the horizon. The runs of the micrometers should also agree with the divisions on the limb.

The observations then proceed as follows: the instrument being perfectly levelled, and the lower part or body firmly clamped to the table, the observer directs his telescope upon the referring-object, and having carefully clamped the upper limb, brings the intersection of the crosshairs, by the motion of the tangent screw, to bisect the vertical line of light in the referring-object. He then reads the degrees, minutes, and seconds given by the divided circle and each of the micrometer microscopes. The upper limb is then unclamped, and the telescope directed on the next point to be observed, which is bisected, and the readings recorded. Similarly, all the other points to be observed, and then finally the referring-object is bisected and read again. The agreement of this last reading with that at the commencement of the series is some test of the care of the observer and the steadiness of the instrument. Each such series is called an "arc," and they are numbered consecutively from the commencement to the close of the station.

The first arc being completed, the telescope is reversed in its Y's, the horizontal circle turned through 180°, and the levels adjusted, if necessary: the second arc is then proceeded with, precisely in the same manner as the first, closing as well as commencing with the referring-object.

The second arc being completed, the lower limb of the instrument is moved through a small angle, 20° or 30°, and clamped, so as to get readings on another part of the circle; and in this position the third arc is taken.

The third arc being completed, the telescope and horizontal circle are reversed as in the second arc; and so on.

It has generally been the custom to read both horizontal and vertical circles for each bisection of a point, but the vertical angles have not, perhaps, been measured with such care as the horizontal; and the readings of the vertical circles for the true level position of the telescope have not generally been observed at the close of each arc, although its variation during the time of observation of a complete arc is liable to be larger than the change in the readings of the referring-object.

All observations, when received in the office, are considered of equal weight, unless a remark of the observer against any observation condemns it as to be rejected. Observations under favourable circumstances are doubtless more valuable than observations under less favourable circumstances; but how to assign their relative numerical value is a question that admits of no general solution. Observations are seldom taken under decidedly unfavourable circumstances, if it can be avoided. "It appears that the longer time one is compelled to bestow, and does bestow, upon observations, under less favourable circumstances, in a great measure compensates external disadvantages, and that causes of errors of observation of which the observer himself has not been conscious often influence him no less than those which obtrude themselves upon him."—(*Bessel: Gradmessung in Ostpreussen.*) It has indeed been often noticed, that an observation to which the observer has attached a remark to the effect that the bisection was unsatisfactory, or that the light was bad, or any other expression of doubt, has been found to agree with singular precision with the general mean or probable truth.

The mode of observing described above does not strictly apply to the observations from the commencement of the survey until the year 1839, for the referring-object was first instituted in 1840 by Lieutenant-Colonel (then Captain) Yolland, both for the sake of convenience and of accuracy. Otherwise the method followed was much the same as that described above. The method of "arcs" has been always followed; though in the earlier operations an arc contained a smaller number of points. It has never been the practice to observe single and independent *angles*, as in the Indian and other geodetical operations.

(11.) As the readings of the microscopes at each observation are read out by the observer, they are recorded by the booker—invariably in ink—in the observation book. The following is the form:—

Royal Society's Theodolite. BEN NEVIS. 11th November 1846.

		Degrees and Minutes.		Seconds of Microscopes.					Means.
				A	B	C	D	E	
Arc 141.	Bar^{r} 25^{in}·466 Ther^{r} att^{d} 40°. Readings of Vert. Circles corresponding to Telescope Level, A 0° 0′ 2″ B 0° 0′ 0″.	°	′	″	″	″	″	″	″
Corp^{l} Winzer, R.S.M., Observer.	R. O.	121	22	21	19	15	21½	20½	19·40
P^{te} Bate, R.S.M., Booker,	Pile Scournalapich . . . . G^{d} dep. A 359° 35′ 42″ B 359° 35′ 39″	164	54	24	25	19	18	23½	21·90
Time 7½ A.M.	Pile Ben Wyvis . . . . . G^{d} dep. A 359° 28′ 51″ B 359° 28′ 49″	182	23	57	53	46	48	50	50·80
Clear in direction of Ben Lomond, Ben Wyvis, and Glashmeal; misty in direction of Ben Tuire, Ben Tartevil, Ben More South Uist, and Storr, and on all low hills.	Pile Ben Macdui . . . . . G^{d} dep. A 359° 39′ 14″ B 359° 39′ 14″	56	50	54½	55½	47	47	54½	51·70
	Pile 50 miles . . . . . G dep. A 359° 23′ 7″ B 359° 23′ 5″	49	24	16	16½	7½	10	14	12·80
	Pile Ben Amhlair . . . . . G^{d} dep. A 359° 32′ 25″ B 359° 32′ 21″	74	26	45	43	35	35	41	39·80
	Pile Ben Lawers . . . . . G^{d} dep. A 359° 39′ 57″ B 359° 39′ 54″	107	56	44	43	35	35	39½	39·30
	Pile Ben Dornish . . . . . G^{d} dep. A 359° 21′ 5″ B 359° 21′ 2″	161	2	45	41	35	32	38	38·20
	Pile Ben More in Cowal (v.h.)* . G^{d} dep. A 359° 18′ 6″ B 359° 18′ 0″	169	38	35	35½	26	26	32	30·90
	Pile Ben More in Mull (v.h.) . G^{d} dep. A 359° 28′ 16″ B 359° 28 13″	44	38	65	65	62	65	65½	64·50
	Pile Creachbheinn . . . . G^{d} dep. A 359° 1′ 30″ B 359° 1′ 27″	50	43	22	24	24½	23½	24½	23·70
	R. O. for Elongations . . . . G^{d} dep. A 357° 26′ 32″ B 357° 26′ 28″	69	49	56	58	55	55	56½	56·10
	R. O.	121	22	21	20	16	21	20	19·60

The column of means is not supplied by the observer; it is filled in in the calculation rooms, and each mean corrected if necessary for the runs of the microscopes, the values of which are occasionally observed and recorded. The original record of the observations is retained by the observer until the book is filled. A copy of the day's work is made every evening, and being carefully compared by the observer with the original, is transmitted to the head quarter office every two or three days, or in very remote stations as opportunities offer.

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* This, together with some other similar abbreviations, as (m.m.) (f.) are used to denote any atmospheric circumstance peculiar to the observation, as *very hazy*, *much motion*, *faint*.

In the later operations of the principal triangulation the observer has generally been furnished with a list of bearings of the principal points or heliostats that he is expected to observe, in order that no time or opportunities may be lost. When there is doubt as to whether any of these principal points may not be too low or too distant to be observed, the approximate depression is also given to the observer, so that on a fine day he may be able, by the observed depression of the furthest ground visible in the direction of the given point, to ascertain whether it is entirely cut off by this intervening ground, or whether it may not under peculiar atmospheric circumstances be visible.

(12.) A watch is always kept in the camp for distant heliostats. The first man that calls out "heliostat," when one of these star-like points shows itself, receives a shilling for his vigilance. In order also to incite watchfulness in the man directing the heliostat to the observing station, and that he might miss no gleam of sunshine, a pecuniary reward was proposed in 1840 by Captain Robinson, R.E., and sanctioned by Major General Colby, Superintendent of the Survey. The amount varied with the distance, being 6 pence for each time the heliostat was observed for a distance less than 10 miles, one shilling for a distance between 10 and 20 miles, and so on; whilst for a distance between 90 and 100 miles the allowance was 15 shillings, and for a distance over 100 miles, a guinea. But, subsequently to the establishment of this rule, the size of the heliostats used for long distances was increased, whereby the chances of observation were also increased; the allowance was therefore reduced, and for long distances half allowance was given for the second arc, a-third allowance for the third arc, and a shilling each for every subsequent observation on the same day. The propriety of this allowance will be readily admitted when it is remembered that as long a period as a month has very frequently occurred between consecutive observations of very distant points. The value of an observation under such circumstances becomes very great.

(13.) *Observations for Absolute Azimuth.*—The determination of the direction of the meridian at various points in the triangulation has in all cases been effected by observations of the elongations of the following circumpolar stars:—

α Ursæ Minoris.
δ Ursæ Minoris.
ε Ursæ Minoris.
λ Ursæ Minoris.
51 Cephei.

In the earlier observations the polar star alone was used. The time of the elongation was calculated, and the instrument carefully levelled for the observation. When the time arrived, the angle between the star and the referring lamp was taken; the telescope was then immediately taken out of its Y's, the horizontal circle turned through 180°, and the telescope replaced in a reversed position as regards its pivots and the collimation: the angle was then re-observed.

In subsequent observations, however, the exact error and rate of the chronometer have generally been unknown, as special observations for time were seldom made. The approximate time was then all that the observer had to assist him, and therefore the observation of the star was commenced about a quarter of an hour before the elongation, and continued until about ten minutes past the elongation, as judged by the motion in azimuth of the star. The instrument was kept carefully levelled, and the telescope and horizontal circle reversed at the apparent moment of elongation. The reading of the referring lamp was observed both before and after the elongation, in reversed positions of the telescope.

In the year 1844, at the suggestion of the Astronomer Royal, who had visited one of the observing parties, and made himself acquainted with the mode of observation, the instruments were fitted with striding or pivot levels, and the reversal of the horizontal circle at the elongation discontinued, as well as any attempts to eliminate mechanically the collimation error; the observed level error and the observed collimation error were thereafter recorded, and the observations corrected accordingly. The method of observation since then is as follows: the instrument being levelled, the referring lamp is first bisected; the telescope is then reversed in its Y's, and the lamp again bisected; the telescope is then directed to the star, which is observed every three or four minutes, and the readings of the circle and of the chronometer recorded at each bisection. When the star is stationary the telescope is reversed, and the observations of the star continued until about ten minutes past the elongation; the referring lamp is then again observed in two positions of the telescope. The error of collimation is inferred from the readings of the lamp. The striding level is also read and recorded six times previous to the elongation, in reversed positions, to eliminate the error of the level, and as many times immediately after the elongation.

It is necessary, even when the readings of a pivot level are recorded, to have the instrument as nearly level as possible, for the value in seconds of a division of the level is a very variable quantity, varying very sensibly with the temperature, which alters the curvature of the glass tube of the level. In some levels an increase of temperature increases the value of a division; in others it diminishes it. By experiments upon the level of the 3-feet theodolite, it was ascertained, that though at the ordinary temperature of 66° the value of a division was very nearly one second, yet at the temperature of 32° the value was increased to 5 seconds. It is therefore necessary that very little should depend upon the readings of the level, or that the instrument should be as nearly as possible level during the observations. This, unfortunately, has not always been attended to by the observers, and is doubtless the cause of many discrepancies otherwise unaccountable in the results of azimuthal observations.

In the observations taken at Ben More, South Uist, in 1851, the reversal of the telescope during the observations of the star was discontinued, as also in the few observations that have been made at other stations since that date. The weight of the telescope, and the danger of communicating a move to the circle during the reversal, make that practice objectionable.

This species of observation is very difficult and delicate, and discrepancies in the results, unavoidable even with the best observers, have been observed in all countries.

They necessarily increase with the latitude, and are probably due to some lurking level error and instrumental defects.

The terrestrial object observed in connexion with the star is generally a lamp placed behind the opening in the referring object before described, and detriment to the observations has doubtless often occurred by the necessity of having it too near the observer on the tops of mountains.

(14.) *Probable error of an Observation.*—The probable error of an observation is dependent upon the circumstances of the observation as much as on the excellence of the instrument or the eye of the observer, and still more on the nature of the object to be observed, or, as it is technically termed, bisected.

With a view to test the power of the telescope, and to determine what accuracy of observation might be expected on a fine object at a short distance, the following experiment was made:—A wooden framework was set up at a distance of about two miles and a quarter from the Ordnance theodolite when at Ben More, South Uist. It was so arranged that being placed against the sky, a fine vertical line of light, of which the breadth was regulated by the sliding of a board, was shown to the observer. The breadth of this opening was varied by half inches, from half an inch to six inches, during the observations, which are recorded as follows:—

BEN MORE, SOUTH UIST.

Ordnance 3-ft. Theodolite . . . 14th August 1855.

Observations made to an opening in a frame with a sliding board, fixed near the pile on Hecla.

No.	Width.	Side of Opening.	Readings of Microscopes.			Mean.
			A	B	C	
1	6·0	Left .	27·5	29·0	27·5	28·00
		Right .	36·5	39·0	37·0	37·50
2	5·5	Left .	28·0	30·0	27·5	28·50
		Right .	36·5	38·5	36·0	37·00
3	5·0	Left .	29·0	31·0	27·5	29·16
		Right .	36·5	39·0	36·0	37·16
4	4·5	Left .	30·0	31·5	29·0	30·16
		Right .	36·5	38·5	35·0	36·66
5	4·0	Left .	30·5	32·0	29·0	30·50
		Right .	37·0	38·5	36·0	37·16
6	3·5	Left .	31·5	32·5	29·5	31·16
		Right .	37·0	38·5	35·5	37·00
7	3·0	Left .	33·0	34·5	30·5	32·66
		Right .	37·0	38·0	35·5	36·83
8	2·5	Left .	34·0	35·0	31·5	33·50
		Right .	37·5	38·0	35·5	36·83
9	2·0	Left .	34·5	35·0	32·0	33·83
		Right .	37·0	39·0	35·0	37·00
10	1·5	Left .	36·0	37·5	33·0	35·50
		Right .	37·5	39·0	35·0	37·16

"The light of an inch opening and of half an inch opening are visible, but not sufficiently defined for observation.

"The centres of the spaces are bisected in the following observations:—

No.	Width.	Readings of Microscopes.			Mean.
		A	B	C	
1	1·0	37·5	38·0	35·5	37·00
2	1·5	37·0	37·5	34·0	36·16
3	2·0	36·0	37·5	33·5	35·66
4	2·5	36·0	37·5	33·0	35·50
5	3·0	35·5	37·0	33·0	35·16
6	3·5	35·0	36·5	33·0	34·83
7	4·0	35·0	35·5	32·5	34·33
8	4·5	33·5	35·0	32·0	33·50
9	5·0	33·5	35·0	31·5	33·33
10	5·5	33·5	35·0	31·0	33·16
11	6·0	33·5	35·0	31·0	33·16

"The observations were made under very favourable circumstances; the object was well defined, and the air clear of motion."

W. JENKINS, Corp., R.S.M.

If from the first set of these observations we would determine the most probable angular value of an opening of one inch, we should have the following equations, where x is the angular value required,

$$\begin{array}{rr} 6x - 9{\cdot}50 = 0 & 3{\cdot}5x - 5{\cdot}84 = 0 \\ 5{\cdot}5x - 8{\cdot}50 = 0 & 3x - 4{\cdot}17 = 0 \\ 5x - 8{\cdot}00 = 0 & 2{\cdot}5x - 3{\cdot}33 = 0 \\ 4{\cdot}5x - 6{\cdot}50 = 0 & 2x - 3{\cdot}17 = 0 \\ 4x - 6{\cdot}66 = 0 & 1{\cdot}5x - 1{\cdot}66 = 0 \end{array}$$

Multiplying each equation by its co-efficient of x, and adding, there results

$$161{\cdot}25x - 249{\cdot}74 = 0$$
$$\therefore x = 1''{\cdot}55$$

Now the individual values of x as derived from the observations are

(1) . . . . . 1·58
(2) . . . . . 1·56
(3) . . . . . 1·60
(4) . . . . . 1·44
(5) . . . . . 1·67
(6) . . . . . 1·67
(7) . . . . . 1·39
(8) . . . . . 1·33
(9) . . . . . 1·58
(10) . . . . . 1·11

And the true value of x as determined from the distance of the observed object, namely, 12462 feet, is

$$x = 1''{\cdot}38$$

It appears, then, that the opening as deduced by observation is, as might be expected, rather greater than its real value; the difference, however, is extremely small.

When compared with the true angular value of the aperture in each case, the observations show the following errors:—

(1)	+1″·22	(6)	+1·01
(2)	+0·91	(7)	+·03
(3)	+1·10	(8)	−·12
(4)	+0·29	(9)	+·41
(5)	+1·14	(10)	−·41

giving a mean error of 0″·79, or a probable error of 0″·53, for each observation of the angle. Now the observed value of the angle depends upon two equally good single observations; therefore the probable error of a single observation is the above probable error divided by the square root of 2, or = 0″·38.

In the second set of observations if $37''+x''$ be the reading of the right edge of the opening, then x will be found to be $x=+{\cdot}12$, and the resulting corrections to the bisections of the spaces will be,

(1)	−0″·57	(6)	−·12
(2)	−·07	(7)	+·03
(3)	+·08	(8)	+·52
(4)	−·10	(9)	+·34
(5)	−·11	(10)	+·17
		(11)	−·18

giving a mean error of 0″·27, or a probable error of 0″·18.

It is therefore clear that a streak of light can be bisected with extreme precision; the probable error of this species of observation being, as far as can be determined from these observations, about half that of the bisection of a dark edge against a strong light.

Description of the Zenith-Sector.

(15.) The instrument used for the determination of the latitudes of points fixed in the triangulation of Great Britain and Ireland, from the commencement of the survey up to the year 1836, was the zenith-sector constructed by Ramsden, described in the second volume of the "Account of the Trigonometrical Survey," and destroyed by fire in the Tower of London.

This instrument consisted of an arc of about 15½° with a radius of 8 feet, divided into spaces of 5′ each by dots on the heads of gold pins, and having a micrometrical division, by which quantities of about a tenth of a second could be judged. The focal length of the telescope was about 8 feet, and the diameter of the object-glass 4 inches, showing distinctly stars of the 3rd magnitude during daylight in clear weather. The instrument was read off by means of a plumb-line; and the apparatus for placing it truly vertical, for reversing its position, and for bringing the plumb-line over the centre of the instrument, were all very perfect. But, notwithstanding the general excellence of the instrument, it was not, from the complexity of its construction and the number of its parts, well adapted to constant transport from station to station; the necessary reversal of the instrument on successive nights, and the consequent delay during bad weather, was also a source of considerable inconvenience.

In consequence of these deficiencies, an application was made to the Astronomer Royal by Major General Colby for a design for a new zenith-sector, by which the observations of a single fine night might give an accurate determination of the latitude. The design was furnished, and eventually the construction of the instrument superintended by Mr. Airy.

The first principle in this instrument, now known as Airy's Zenith-Sector, was the arrangement for making successive observations in two positions of the instrument, face east and face west at the same transit. The second principle was the substitution of a level or system of levels for the usual plumb-line. The third principle was the casting in one piece, as far as practicable, of each of the different parts of the instrument, in order to avoid the great number of screws and fastenings with which most instruments are hampered, and to secure, if possible, perfect rigidity.

The lower part of this instrument, which is represented in outline in Plate 1 is a rectangular tray of cast-iron, which is screwed, if required, to the pier or framework on which the instrument may rest, by means of ears projecting from the plane of its base. Within this tray is placed a rectangular thin plate of iron, whose breadth is two inches less than that of the tray, and length one inch less. This plate has three bosses on its upper surface, one at the centre of one extremity of the rectangle, the other two at the other extremity. These bosses are cut to receive the footscrews of the instrument; and immediately corresponding to them on the lower surface are three other bosses, on which the plate rests, and by means of which the weight of the instrument is immediately transferred to the tray. By means of screws passing horizontally through the sides of the tray, an azimuthal motion is communicated to the plate, and by them also it is retained in the required plane of azimuth.

The instrument is in three parts; the Framework, the Revolving Frame, and the Telescope Frame.

The Framework is cast in four pieces; the lower part, an inverted rectangular tray with levelling footscrews; two uprights, with broad bearing pieces, very firmly screwed to the inverted tray; and a cross bar uniting the tops of these uprights, whose ends are cut as screws. Through the centre of this bar passes downwards a screw with a conical point, which together with the vertex of a cone rising from the centre of the inverted rectangular tray, determine the axis of revolution, and form the bearings of the Revolving Frame.

The Revolving Frame is cast of gun-metal, in one piece. It is also in the form of a tray, strongly ribbed at the back, having four lappets or ears acting as stops in the revolution, In the centre of the front of this frame is a raised ring of about nine inches diameter, forming the bearing plate of the Telescope Frame. Concentric with this ring at each end of the frame are the divided limbs, which have a radius of 20·5 inches, and are divided on silver to every five minutes; the divisions are numbered from 0° to 360°, interrupted by the portions of the circle which are wanting. There is also at each end a raised clamping-limb, roughly divided, to which the clamp for securing the Telescope Frame at the required zenith distance is attached; the graduation furnishing zenith distances on both sides of the zenith, and also circle readings corresponding with those of the divisions of the limb, the pointer-reading being given by a small index attached to the clamp. On the reverse side of the Revolving Frame are mounted three levels, the divisions of which are numbered from right to left.

The Telescope Frame revolves in a vertical plane by a horizontal axis or pivot, of 3 inches diameter, passing through a corresponding cylindrical hole in the Revolving Frame. Cast in one piece with the Telescope Frame, are, the ring for holding the object-glass-cell of the telescope, the four micrometer microscopes, which are afterwards bored through the metal, and the eye-piece. The micrometers are of the usual construction, the wires intersect in an acute angle, and have a range of about 10 minutes on the divided limb. The value of a division of the micrometers reading the limb is approximately a quarter of a second.

In the eye-piece of the telescope are five meridional wires carried by a fixed plate, and a single wire at right angles to them, moved by a micrometer-screw. The range of the screw admits of the wire being carried completely across the field of view: one division of the telescope micrometer is equal to 0″·43096. The tube of the telescope is merely a protection from dust, and carries no essential part of the instrument except a simple apparatus for regulating the amount of light illuminating the wires, which, by the turning of a screw, increases or diminishes the orifice through which the light enters.

The focal length of the telescope is 46 inches, the diameter of the object-glass 3·75 inches, and the magnifying power usually employed, about 70.

The cast-iron tray, forming the stand of the instrument, generally rests when in use on strong pickets or piles driven into the ground, or secured to the rock by jumper-holes. In some instances a strong rectangular frame of wood has been sunk into the ground; and this method of preparing the station is found the most expeditious and convenient, the frame being carried about with the observatory and stores.

The Revolving Frame is generally reversed in its position with regard to the pivots, once at each station, so that the reading of the zenith point as given by the pointer at the lower end of the telescope is for one part of the observations about 12°30′, and for the other part about 192°30′.

Method of observing with the Zenith-Sector.

(16.) The deviation of the plane of the instrument from the meridian, which is generally very small, being carefully ascertained by observations of the transits of northerly and southerly stars, and the axis being as nearly as possible vertical, the observer sets the Telescope Frame at the approximate zenith distance of the star to be observed, and clamps it. He then, before the star enters the field, reads the four micrometer microscopes, and immediately after them the levels on the reverse side of the Revolving Frame. He is then ready to observe the star, which is done by bringing the micrometer wire in the eye-piece of the telescope to bisect the star on one of the wires, the name of the wire being recorded with the reading of the telescope micrometer. This completes a single observation. The Telescope Frame is then immediately unclamped, and the Revolving Frame reversed by turning it through 180° on its vertical axis, so that the face which before was east is now west. The Telescope Frame is quickly set to the approximate zenith distance, and clamped, and the star again bisected by the telescope micrometer on one of the wires; generally the same one on which it was previously observed. The five micrometers are then read, and the levels on the reverse side. This completes the double observation.

The method of reduction is then, for each position of the instrument, to add together, the pointer reading, the mean of the readings of the microscopes, the correction for their runs, the reading of the telescope micrometer, and the mean indication of the three levels, besides the correction for azimuthal deviation, and for the wire on which the star is observed. This sum is called the corrected instrumental reading. The mean of the corrected instrumental readings for the same star observed face east and face west is the quantity corresponding to an observation of the zenith, and is called the zenith point. The zenith points must, when the instrument is carefully used, agree with great exactness, and therefore a comparison of their values affords a test of the accuracy of the observations.

The amount of the azimuthal deviation is ascertained by comparing the differences of the observed transits of northerly and southerly stars with the true differences of their meridian transits, or differences of right ascension. If Δ be the excess of the difference of right ascension of two stars over the observed difference of time of their transits, δ δ' their north polar distances, and λ the latitude of the instrument, then α, the azimuthal deviation, is,

$$\alpha = \frac{\Delta}{\cos\lambda} \quad \frac{\sin\delta \sin\delta'}{\sin(\delta - \delta')}$$

The correction to the zenith distance z, on account of this deviation, is,—

$$- \frac{\sin^2\alpha \sin z \cos\lambda}{\sin\delta \sin 2''}$$

in seconds, where δ is the north polar distance of the star.

The correction for the equatorial interval i from the meridian, of the wire on which the star is observed, is,

$$\pm \frac{i^2}{2} \cot\delta \sin 1''$$

the upper sign applying to south stars, the lower sign to north stars.

TABLE SHOWING THE STATIONS VISITED WITH AIRY'S ZENITH-SECTOR.

Year.	Stations.	Officer or N. C. O. in charge of the Instrument.	Strength of the Party.	Number of Nights on which Observations were made.	Number of single Observations.
1842–3	Blackdown	Lieuts. Hornby and Gosset, R.E.	7	20	1087
1843	Precelly	Lieuts. Hornby and Luykin, R.E.	5	17	674
1843	Forth Mountain	Lieut. Hornby, R.E.	7	12	659
1843	Hungry Hill	„ „	7	9	295
1843	Feaghmaan	„ „	7	9	395
1843	Taunaghmore	„ „	2	7	294
1843	S. end of L. Foyle Base	„ „	2	6	335
1844	Monach	Lieut. Gosset, R.E.	5	10	180
1844	Ben Hulig	„ „	6	10	480
1845	Hensbarrow	Corp. Steel, R.S.M.	4	6	290
1842	South Berule	Lieuts. Hornby and Gosset, R.E.	6	2	113
1845	South Berule	Corp. Steel, R.S.M.	3	2	114
1845	Ben Lomond	Corp. Steel, R.S.M.	4	11	635
1845	Ben Heynish	„ „	4	10	267
1846	Week Down	„ „	4	11	556
1846	Dunnose	„ „	4	13	643
1846	Boniface Down	„ „	4	7	356
1846	Port Valley	„ „	4	10	411
1846–7	Saxavord	„ „	4	20	566
1847	Gerth of Scaw	„ „	4	21	581
1847	Balta	„ „	4	20	732
1847	Cowhythe	„ „	4	18	641
1847–8	Southampton	Serg. Steel & Corp. Jenkins, R.S.M.	2	180	8730
1850	St. Agnes	Serg. Steel, R.S.M.	4	11	418
1850	Goonhilly Down	„ „	4	9	442
1850	North Rona	„ „	4	5	428
1850	Great Stirling	„ „	4	9	439

Three of the stations whose latitudes were determined with Ramsden's Zenith-Sector have been re-determined with Airy's Zenith-Sector. The comparisons are as follows:

Stations.	Airy.			Ramsden.[1]			Dif.
	°	′	″	°	′	″	″
Dunnose	50	37	6·98	50	37	7·09	0·11
Balta	60	45	1·68	60	45	2·31	0·63
Cowhythe	57	41	9·58	57	41	9·74	0·16

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[1] The latitudes here given are as published in the volume of the Ordnance Survey entitled "Astronomical Observations made with Ramsden's Zenith-Sector." In the present volume the resulting latitudes are slightly altered, according to the method of reduction explained in the note at page 199.

The latitude of Balta, given by Ramsden's Zenith-Sector, was deduced by amplitudes with Dunnose after an interval of 15 years, the observations at Dunnose having been made in 1802, those at Balta in 1817. But the latitude of Balta, deduced from all the observations then made, would have been 60° 45′ 1″·59, and similarly the latitude of Cowhythe, deduced from all the observations made in 1813, would have been 57° 41′ 9″·47, both of which are more nearly equal to the results obtained from Airy's Zenith-Sector.

There is no appreciable difference in the results given by the instrument in the two positions of the revolving frame. Thus, the latitude of Southampton with the pointer reading from 0° to 25°, by

1384 Observations of stars north of the zenith, is . . .	50° 54′ 46″·81
1196 „ „ south „	46″·68

and with the pointer reading from 180° to 205°,

1044 Observations of stars north of the zenith give . . .	50° 54′ 46″·80
749 „ „ south „	46″·55

For a detailed description of this instrument, and the method of observing, together with all the observations that have been made with it, the reader is referred to the last published volume of the Ordnance Survey, entitled "Astronomical Observations made with Airy's Zenith-Sector."

SECTION III.

REDUCTION OF OBSERVATIONS.

(1.) Until the year 1840 the mean results from the observation books were abstracted in the form of "included angles;" thus, supposing $\alpha_1\ \beta_1,\ \alpha_2\ \beta_2,\ \cdots\cdots\ \alpha_n\ \beta_n$ to be the reading of two objects A B, upon n different arcs, then the angle between A and B was taken as,

$$\frac{\beta_1 - \alpha_1 + \beta_2 - \alpha_2 + \cdots\cdots + \beta_n - \alpha_n}{n}$$

In consequence, however, of the imperfection of this method, and the great labour involved in it, it was abandoned by Captain Yolland, and the more perfect solution of the question of most probable bearings by the theory of probabilities substituted in its stead.

(2.) The theory of the reduction of the observed to the most probable mean bearings is as follows: Let the bearings of the objects observed, reckoned from the referring-object, be A B C $\cdots\cdots$ these being the most probable values to be determined, and let the first arc give the readings $m_1\ m_1^{(1)}\ m_1^{(2)}\ m_1^{(3)}\ \cdots\cdots\ m_1$ corresponding to the arbitrary reading of the referring-object, of which let x_1 be the true or most probable value, $m_1 - x_1$ being thus the error of observation of the referring-object; then we ought to have the equations,

$$m_1 - x_1 = 0;\ m_1^{(1)} - x_1 - A = 0;\ m_1^{(2)} - x_1 - B = 0 \cdots\cdots$$

The second and third arcs will give,

$$m_2 - x_2 = 0;\ m_2^{(1)} - x_2 - A = 0;\ m_2^{(2)} - x_2 - B = 0 \cdots\cdots$$
$$m_3 - x_3 = 0;\ m_3^{(1)} - x_3 - A = 0;\ m_3^{(2)} - x_3 - B = 0 \cdots\cdots$$

and so on. These equations would be identically true were the observations free from error. As it is, the left-hand members are the errors of observation, and therefore the sum of the squares of these quantities must be made a minimum in order to give the most probable values of A B C $\cdots\cdots$ and the arbitrary distances $x_1\ x_2\ x_3\ \cdots\cdots$ of the initial point of the instrument from the referring-object in the different arcs.

In order to make the result general, multiply these equations or errors by quantities $\sqrt{p_1}\ \sqrt{p_1^{(1)}}\ \sqrt{p_1^{(2)}}\ \cdots\ \sqrt{p_2}\ \sqrt{p_2^{(1)}}\ \sqrt{p_2^{(2)}}\ \cdots\cdots$ each of these to be equal to unity when there is an observation, and to zero when the corresponding observation is wanting. Then the sum of the squares of the errors is,

$$\begin{aligned}
&p_1\,(m_1 - x_1)^2 + p_1^{(1)}\,(m_1^{(1)} - x_1 - A)^2 + p_1^{(2)}\,(m_1^{(2)} - x_1 - B)^2 + \cdots\cdots \\
+\,&p_2\,(m_2 - x_2)^2 + p_2^{(1)}\,(m_2^{(1)} - x_2 - A)^2 + p_2^{(2)}\,(m_2^{(2)} - x_2 - B)^2 + \cdots\cdots \\
+\,&p_3\,(m_3 - x_3)^2 + p_3^{(1)}\,(m_3^{(1)} - x_3 - A)^2 + p_3^{(2)}\,(m_3^{(2)} - x_3 - B)^2 + \cdots\cdots \\
&\qquad\qquad\text{\&c.}\qquad\qquad \text{\&c.}\qquad\qquad \text{\&c.}
\end{aligned}$$

Making the differential co-efficients of this quantity with respect to $x_1\, x_2\, x_3 \cdot\cdot$ A B C $\cdot\cdot\cdot$ equal to zero, there result the equations,

$$\left.\begin{array}{l}
p_1m_1+p_1^{(1)} m_1^{(1)} +p_1^{(2)} m_1^{(2)} + \cdot\cdot\cdot = (p_1+p_1^{(1)} +p_1^{(2)} + \cdot\cdot\cdot)\, x_1+p_1^{(1)}\,\mathrm{A}+p_1^{(2)}\,\mathrm{B}+p_1^{(3)}\,\mathrm{C} + \cdot\cdot \\
p_2m_2+p_2^{(1)}m_2^{(1)}+p_2^{(2)}m_2^{(2)}+ \cdot\cdot\cdot = (p_2+p_2^{(1)}+p_2^{(2)}+ \cdot\cdot\cdot)\, x_2+p_2^{(1)}\mathrm{A}+p_2^{(2)}\mathrm{B}+p_2^{(3)}\mathrm{C} + \cdot\cdot \\
p_3m_3+p_3^{(1)}m_3^{(1)}+p_3^{(2)}m_3^{(2)}+ \cdot\cdot\cdot = (p_3+p_3^{(1)}+p_3^{(2)}+ \cdot\cdot\cdot)\, x_3+p_3^{(1)}\mathrm{A}+p_3^{(2)}\mathrm{B}+p_3^{(3)}\mathrm{C} + \cdot\cdot \\
\qquad\qquad\qquad\qquad\qquad\&c. \qquad \&c. \qquad \&c.
\end{array}\right\}(\alpha)$$

$$\left.\begin{array}{l}
p_1^{(1)}m_1^{(1)}+p_2^{(1)}m_2^{(1)}+p_3^{(1)}m_3^{(1)}+ \cdot\cdot = (p_1^{(1)}+p_2^{(1)}+p_3^{(1)}+ \cdot\cdot)\,\mathrm{A}+p_1^{(1)}x_1+p_2^{(1)}x_2+p_3^{(1)}x_3 \quad \cdot\cdot \\
p_1^{(2)}m_1^{(2)}+p_2^{(2)}m_2^{(2)}+p_3^{(2)}m_3^{(2)}+ \cdot\cdot = (p_1^{(2)}+p_2^{(2)}+p_3^{(2)}+ \cdot\cdot)\,\mathrm{B}+p_1^{(2)}x_1+p_2^{(2)}x_2+p_3^{(2)}x_3+ \cdot\cdot \\
p_1^{(3)}m_1^{(3)}+p_2^{(3)}m_2^{(3)}+p_3^{(3)}m_3^{(3)}+ \cdot\cdot = (p_1^{(3)}+p_2^{(3)}+p_3^{(3)}+ \cdot\cdot)\,\mathrm{C}+p_1^{(3)}x_1+p_2^{(3)}x_2+p_3^{(3)}x_3+ \cdot\cdot \\
\qquad\qquad\qquad\qquad\qquad\&c. \qquad \&c. \qquad \&c.
\end{array}\right\}(\beta)$$

The values of $x_1\, x_2\, x_3 \cdot\cdot\cdot$ being obtained from the equations (α), and thus eliminated from (β), there results a final system of equations for obtaining A B C $\cdot\cdot\cdot$

In these equations $x_1\, x_2\, x_3 \cdot\cdot\cdot$ differ from $m_1\, m_2\, m_3 \cdot\cdot\cdot$ by the errors of observation. Instead therefore of x_n, make x_n-m_n the unknown quantity, calling it still x_n, which symbol now represents the correction to the observation of the referring-object on the n^{th} arc, and is a small quantity. Substitute in the above equations, and instead of $m_q^{(r)}-m_q$ write $m_q^{(r)}$, which now does not mean the reading of the r^{th} object on the q^{th} arc, as recorded in the observation book, but that reading minus the reading of the referring-object on that arc. The equations then become,

$$\left.\begin{array}{l}
p_1^{(1)}m_1^{(1)}+p_1^{(2)}m_1^{(2)}+ \cdot\cdot\cdot = (p_1+p_1^{(1)}+p_1^{(2)}+ \cdot\cdot\cdot)\, x_1+p_1^{(1)}\,\mathrm{A}+p_1^{(2)}\,\mathrm{B}+p_1^{(3)}\,\mathrm{C} + \cdot\cdot\cdot \\
p_2^{(1)}m_2^{(1)}+p_2^{(2)}m_2^{(2)}+ \cdot\cdot\cdot = (p_2+p_2^{(1)}+p_2^{(2)}+ \cdot\cdot\cdot)\, x_2+p_2^{(1)}\,\mathrm{A}+p_2^{(2)}\,\mathrm{B}+p_2^{(3)}\,\mathrm{C} + \cdot\cdot\cdot \\
p_3^{(1)}m_3^{(1)}+p_3^{(2)}m_3^{(2)}+ \cdot\cdot\cdot = (p_3\quad p_3^{(1)}+p_3^{(2)}+ \cdot\cdot\cdot)\, x_3+p_3^{(1)}\,\mathrm{A}+p_3^{(2)}\,\mathrm{B}+p_3^{(3)}\,\mathrm{C} + \cdot\cdot\cdot \\
\qquad\qquad\qquad\qquad\qquad\&c. \qquad \&c. \qquad \&c.
\end{array}\right\}(\alpha')$$

$$\left.\begin{array}{l}
p_1^{(1)}m_1^{(1)}+p_2^{(1)}m_2^{(1)}+p_3^{(1)}m_3^{(1)}+ \cdot\cdot = (p_1^{(1)}+p_2^{(1)}+p_3^{(1)}+ \cdot\cdot)\,\mathrm{A}+p_1^{(1)}x_1+p_2^{(1)}x_2+p_3^{(1)}x_3+ \cdot\cdot \\
p_1^{(2)}m_1^{(2)}+p_2^{(2)}m_2^{(2)}+p_3^{(2)}m_3^{(2)}+ \cdot\cdot = (p_1^{(2)}+p_2^{(2)}+p_3^{(2)}+ \cdot\cdot)\,\mathrm{B}+p_1^{(2)}x_1+p_2^{(2)}x_2+p_3^{(2)}x_3+ \cdot\cdot \\
p_1^{(3)}m_1^{(3)}+p_2^{(3)}m_2^{(3)}+p_3^{(3)}m_3^{(3)}+ \cdot\cdot = (p_1^{(3)}+p_2^{(3)}+p_3^{(3)}+ \cdot\cdot)\,\mathrm{C}+p_1^{(3)}x_1+p_2^{(3)}x_2+p_3^{(3)}x_3+ \cdot\cdot \\
\qquad\qquad\qquad\qquad\qquad\&c. \qquad \&c. \qquad \&c.
\end{array}\right\}(\beta')$$

It is evident that when the number of observed points is very large it would be impossible to solve these equations directly. This, however, is generally, indeed always, the case; and therefore recourse must be had to the method of successive approximation.

First, then assume x_1, x_2, x_3, $\cdot\cdot\cdot$ individually equal to zero, then the second set of equations will give,

$$\left.\begin{array}{l}
\mathrm{A}' = \dfrac{p_1^{(1)}m_1^{(1)} + p_2^{(1)}m_2^{(1)} + p_3^{(1)}m_3^{(1)} + \cdot\cdot\cdot}{p_1^{(1)} + p_2^{(1)} + p_3^{(1)} + \cdot\cdot\cdot} \\[2ex]
\mathrm{B}' = \dfrac{p_1^{(2)}m_1^{(2)} + p_2^{(2)}m_2^{(2)} + p_3^{(2)}m_3^{(2)} + \cdot\cdot\cdot}{p_1^{(2)} + p_2^{(2)} + p_3^{(2)} + \cdot\cdot\cdot} \\[2ex]
\mathrm{C}' = \dfrac{p_1^{(3)}m_1^{(3)} + p_2^{(3)}m_2^{(3)} + p_3^{(3)}m_3^{(3)} + \cdot\cdot\cdot}{p_1^{(3)} + p_2^{(3)} + p_3^{(3)} + \cdot\cdot\cdot}
\end{array}\right\}\cdot\cdot\cdot\;(\gamma)$$

for the first approximate values of A B C $\cdot\cdot\cdot$

Substitute these values in equation (α^{I}), and we get,

$$\left.\begin{aligned}
x_1 &= \frac{p_1^{(1)}(m_1^{(1)} - A') + p_1^{(2)}(m_1^{(2)} - B') + p_1^{(3)}(m_1^{(3)} - C') + \cdots}{p_1 + p_1^{(1)} + p_1^{(2)} + p_1^{(3)} + \cdots} \\
x_2 &= \frac{p_2^{(1)}(m_2^{(1)} - A') + p_2^{(2)}(m_2^{(2)} - B') + p_2^{(3)}(m_2^{(3)} - C') + \cdots}{p_2 + p_2^{(1)} + p_2^{(2)} + p_2^{(3)} + \cdots} \\
x_3 &= \frac{p_3^{(1)}(m_3^{(1)} - A') + p_3^{(2)}(m_3^{(2)} - B') + p_3^{(3)}(m_3^{(3)} - C') + \cdots}{p_3 + p_3^{(1)} + p_3^{(2)} + p_3^{(3)} + \cdots} \\
&\quad \&\text{c.} \qquad \&\text{c.} \qquad \&\text{c.}
\end{aligned}\right\} \cdots (\delta)$$

These values substituted in equations (β') will give the next approximation to the values of A B C $\cdots$, which are,

$$\left.\begin{aligned}
A &= \frac{p_1^{(1)}(m_1^{(1)} - x_1) + p_2^{(1)}(m_2^{(1)} - x_2) + p_3^{(1)}(m_3^{(1)} - x_3) + \cdots}{p_1^{(1)} + p_2^{(1)} + p_3^{(1)} + \cdots} \\
B &= \frac{p_1^{(2)}(m_1^{(2)} - x_1) + p_2^{(2)}(m_2^{(2)} - x_2) + p_3^{(2)}(m_3^{(2)} - x_3) + \cdots}{p_1^{(2)} + p_2^{(2)} + p_3^{(2)} + \cdots} \\
C &= \frac{p_1^{(3)}(m_1^{(3)} - x_1) + p_2^{(3)}(m_2^{(3)} - x_2) + p_3^{(3)}(m_3^{(3)} - x_3) + \cdots}{p_1^{(3)} + p_2^{(3)} + p_3^{(3)} + \cdots} \\
&\quad \&\text{c.} \qquad \&\text{c.} \qquad \&\text{c.}
\end{aligned}\right\} \cdots (\varepsilon)$$

These values may be again substituted in equation (α'), and more correct values of x_1, x_2, x_3 $\cdots\cdot$ will be obtained.

These last values of A B C $\cdots\cdot$ have been used as the true or most probable bearings with reference to the referring-object.

Should the observation of this object be deficient in the r^{th} arc, then from equations (α)

$$m_r = \frac{p_r^{(1)}(m_r^{(1)} - A) + p_r^{(2)}(m_r^{(2)} - B) + p_r^{(3)}(m_r^{(3)} - C) + \cdots}{p_r^{(1)} + p_r^{(2)} + p_r^{(3)} + \cdots}$$

The practical working is as follows:—The means of the reading of the microscopes for each object are taken in the observation book, and to the reading of *each* object on any one arc a certain quantity is added, making the reading of the referring-object on that arc equal to a given quantity constant for the station. This constant reading of the referring-object is (for convenience) its calculated bearing from the south meridian line.

These results are abstracted for each station in the following form, in which the reading of the referring-object is supposed deficient on the fifth arc:—

1st. Abstract of Angles observed at —— with the —— Theodolite, commenced —— finished ——.

Date.	Observer.	Number of Arc.	R. O.	A.	B.	C.	D.	E.	F.	G.
		1	m	$m_1^{(1)}$	$m_1^{(2)}$	. .	. .	$m_1^{(5)}$	$m_1^{(6)}$	$m_1^{(7)}$
		2	m	$m_2^{(1)}$	. .	$m_2^{(3)}$	$m_2^{(4)}$	$m_2^{(5)}$	. .	. .
		3	m	$m_3^{(1)}$	$m_3^{(2)}$	$m_3^{(3)}$	. .	. .	$m_3^{(6)}$	$m_3^{(7)}$
		4	m	. .	$m_4^{(2)}$	$m_4^{(3)}$	$m_4^{(4)}$	. .	$m_4^{(6)}$	$m_4^{(7)}$
Means.			m	$m^{(1)}$	$m^{(2)}$	$m^{(3)}$	$m^{(4)}$	$m^{(5)}$	$m^{(6)}$	$m^{(7)}$
		5	. .	. .	. .	$n^{(3)}$	$n^{(4)}$	$n^{(5)}$	$n^{(6)}$	$n^{(7)}$

To each reading in the fifth arc the quantity

$$\tfrac{1}{5}\left(m^{(3)} - n^{(3)} + m^{(4)} - n^{(4)} + m^{(5)} - n^{(5)} + m^{(6)} - n^{(6)} + m^{(7)} - n^{(7)}\right)$$

is to be added, which will then alter them to $n_1^{(3)}\ n_1^{(4)}\ \ldots$ and the abstract will stand thus:

Number of Arc.	R. O.	A.	B.	C.	D.	E.	F.	G.
1	m	$m_1^{(1)}$	$m_1^{(2)}$	. .	. .	$m_1^{(5)}$	$m_1^{(6)}$	$m_1^{(7)}$
2	m	$m_2^{(1)}$	. .	$m_2^{(3)}$	$m_2^{(4)}$	$m_2^{(5)}$	. .	. .
3	m	$m_3^{(1)}$	$m_3^{(2)}$	$m_3^{(3)}$	. .	. .	$m_3^{(6)}$	$m_3^{(7)}$
4	m	. .	$m_4^{(2)}$	$m_3^{(3)}$	$m_4^{(4)}$	. .	$m_4^{(6)}$	$m_4^{(7)}$
5	. .	. .	. .	$n_1^{(3)}$	$n_1^{(4)}$	$n_1^{(5)}$	$n_1^{(6)}$	$n_1^{(7)}$
Means -	m	$m^{(1)}$	$m^{(2)}$	$m^{(3)}$	$m^{(4)}$	$m^{(5)}$	$m^{(6)}$	$m^{(7)}$

The quantities $x_1\ x_2\ \ldots$ are then calculated for each arc by equations (δ) thus:—

$$-x_1 = \tfrac{1}{6}\left(m^{(1)} - m_1^{(1)} + m^{(2)} - m_1^{(2)} + m^{(5)} - m_1^{(5)} + m^{(6)} - m_1^{(6)} + m^{(7)} - m_1^{(7)}\right)$$

and similarly $x_2\ x_3$. These quantities are applied to each reading in the respective arcs, and the new modified readings abstracted again in the same form. The means obtained from this last abstract are taken as the final value of the bearings.

The following numerical example will suffice to illustrate the process:—

Observer and Date.	Number of Arc.	R. O. 4° 21′	A. 11° 7′	B. 37° 34′	C. 97° 54′	D. 220° 3′	E. 271° 43′
	1	29·21	36·04	14·07	47·84	19·00	39·17
	2	29·21	35·91	. .	. .	18·18	38·22
	3	29·21	34·21	11·86	. .	. .	39·42
	4	29·21	32·41	10·71	46·05	16·30	. .
	5	29·21	. .	11·91	48·30	14·17	. .
	6	29·21	. .	. .	. .	18·59	41·04
	Means	29·21	34·64	12·14	47·40	17·25	39·46

The errors of the arcs will then stand thus:—

	R. O.	A.	B.	C.	D.	E.	Mean.
1	·00	+1·40	+1·93	+0·44	+1·75	−0·29	+0·87
2	·00	+1·27	. .	. .	+0·93	−1·24	+0·24
3	·00	−0·43	−0·28	. .	. .	−0·04	−0·19
4	·00	−2·23	−1·43	−1·35	−0·95	. .	−0·19
5	·00	. .	−0·23	+0·90	−3·08	. .	−0·60
6	·00	. .	. .	. .	+1·34	+1·58	+0·97

The mean error for each arc being then applied, with contrary sign, the second abstract will stand thus:—

Number of Arc.	R. O. 4° 21′	A. 11° 7′.	B. 37° 34′	C. 97° 54′	D. 220° 3′	E. 271° 43′
1	28·34	35·17	13·20	46·97	18·13	38·30
2	28·97	35·67	. .	. .	17·94	37·98
3	29·40	34·40	12·05	. .	. .	39·61
4	30·40	33·60	11·90	47·24	17·49	. .
5	29·81	. .	12·51	48·90	14·77	. .
6	28·24	. .	. .	. .	17·62	40·07
Means	29·19	34·71	12·42	47·70	17·19	38·99

These last means are taken as the resulting bearings for the station.

In this manner all the observations made with the two great instruments, and some with the 18-inch, have been reduced. All given in the abstracts in this volume have been thus calculated; the old as well as the new. It may perhaps be objected to this method, that it is not a sufficient approximation; but another approximation would have cost great time and labour, as, to be consistent, it must have been carried through the whole of the observations, and the advantage gained would, after all, be but small.

The weights have been calculated from the final abstract, the differences of the individual readings from their means being considered the errors, in the usual formula,

$$w = \frac{n^2}{2\,\Sigma\,(\varepsilon^2)}$$

When there are only two or three observations to a point, the weight thus obtained has not been generally used, as theoretically n is supposed large. In such cases the rule adopted by Captain Yolland was first to obtain the mean weight of a single observation at the station, and then to give to the mean of two observations twice this weight, and to the mean of three observations three times this weight. The mean weight of an observation is calculated thus: if $n_1\,w_1$, $n_2\,w_2$ . . . be the number of observations and weights corresponding to the different bearings, and w the mean weight of a single observation at the station in question, then

$$\frac{1}{w} = \frac{\frac{n_1}{w_1} + \frac{n_2}{w_2} + \frac{n_3}{w_3} + \cdots}{n_1 + n_2 + n_3 + \cdots}$$

but if the weight that would thus be assigned to a bearing should exceed that due to it by the usual formula, then the latter was made use of, and not that resulting from the mean weight of an observation at the station.

The weights would have been materially increased in many instances by rejecting what would appear *bad* observations; but the rule has been never to reject *any*, unless the observer has made a remark to the effect that it ought to be rejected.

(3.) The method of reduction of azimuthal observations is to take the last reading of the star before the reversal of the telescope at the elongation, and the first reading after it, and having corrected them for their respective level and collimation errors, to use their mean as the final result for that star. If l be the recorded level error corresponding to an observation as given in seconds by the striding level, west readings being considered positive (the scales are divided from the centre outwards) and east readings negative, then the correction for level is, z being the zenith distance of the star,

$$-l \cot z$$

If c be the error of collimation, as given by the referring object, then the correction for collimation is

$$c \operatorname{cosec} z$$

In the case of the stars above named, it is sufficient to take the colatitude of the place of observation for the zenith distance; though indeed the latter has been generally approximately computed.

If, then, R R′ be the observed theodolite readings, l l' the corresponding level errors, c the collimation error, then the reduced readings are

$$R + l \cot z \pm c \operatorname{cosec} z$$
$$R' + l' \cot z \mp c \operatorname{cosec} z$$

The collimation correction disappears in the result, but it is calculated in order to show the agreements of the individual results. The declinations of the stars are taken from the "Nautical Almanac," interpolating for quarter days. The small correction depending on the moon's longitude has not been regarded, as the observations have not sufficient precision to make it necessary.

Considering the great value of azimuthal determinations in deducing the final results from geodetical operations, some experimental calculations were made by the method of least squares for obtaining the most probable direction of the meridian as given by the collective observations of separate elongations. The method is as follows:—If a star be observed at a given time, t minutes, after or before an elongation, then its azimuth will differ from its azimuth when at the elongation by the quantity

$$(450\,t)^2 \sin 1'' \frac{\sin 2\delta}{\sin z} \pm \tfrac{1}{4}(900\,t)^3 \sin^2 1'' \frac{\sin 2\delta}{\sin z} \cot h$$

$$= \cdot 9817 \frac{\sin 2\delta}{\sin z} t^2 \pm \cdot 0043 \frac{\sin 2\delta}{\sin z} \cot h\, t^3$$

in seconds, where δ, z, h, are the declination, zenith-distance, and hour-angle of the star. The second term of this expression need not be used unless the star be 20 minutes or more from its elongation.

Let $\alpha_1, \alpha_2, \alpha_3 \cdot \cdot \cdot$ be the readings of the star at the times $t_1\ t_2\ t_3 \cdot \cdot \cdot$ respectively; $A, T,$ the azimuth and time corresponding to the greatest elongation, then for an eastern elongation these will be the equations:

$$\begin{aligned} \alpha_1 + \mu\,(T - t_1)^2 - A &= 0 \\ \alpha_2 + \mu\,(T - t_2)^2 - A &= 0 \\ \alpha_3 + \mu\,(T - t_3)^2 - A &= 0 \\ &\vdots \\ \alpha_n + \mu\,(T - t_n)^2 - A &= 0 \end{aligned}$$

Where

$$\mu = \cdot 9817\ \frac{\sin 2\ \delta}{\sin z}$$

the equations for a western elongation are formed by merely changing the sign of μ.

The left-hand members of these equations, being in general not equal to zero, are the errors of observation. We must therefore give T and A such values as will make the sum of the squares of these errors the least possible; that is, the quantity

$$U = \Big(\alpha_1 + \mu\,(T - t_1)^2 - A\Big)^2 + \Big(\alpha_2 + \mu\,(T - t_2)^2 - A\Big)^2 + \cdot \cdot \cdot \Big(\alpha_n + \mu\,(T - t_n)^2 - A\Big)$$

in which T and A are to be considered variables, must be a minimum.

Making the differential co-efficients of U with respect to A and T separately equal to zero, there results, after a little reduction

$$\mu\,\Big(2\ T + t_m\Big)\ \Big([t^2]_m - [t_m]^2\Big) = [\alpha \cdot t]_m - \alpha_m \cdot t_m + \mu\ \Big([t^3]_m - [t_m]^3\Big)$$

$$A = \alpha_m + \mu\ \Big(T^2 - 2\ t_m\ T + [t^2]_m\Big)$$

where $[t_m]^r$ denotes the r^{th} power of the mean of the quantities t, and $[t^r]_m$ denotes the mean of the r^{th} powers of the quantities t. These equations furnish the most probable values of the greatest azimuth, or that at the elongation. In the case of a western elongation the sign of μ in these equations must be changed.

In consequence, however, of the laborious nature of the calculation, and the often erroneous value of the time T resulting from the discordance frequently shown between the observations of the star in one position of the telescope previous to the instant of elongation and those taken in the reverse position after it, this method was not generally carried out. In those cases in which this calculation was made the result did not differ but by a very small fraction of a second from the result obtained in the usual manner.

(4.) In some cases it has been necessary to combine the observations taken at different times and with different instruments at the same point. The following is the method of calculation:—Let $\alpha\ \beta\ \gamma \cdot \cdot \cdot$, $\alpha'\ \beta'\ \gamma' \cdot \cdot \cdot$, be the two sets of bearings, of points common to both sets of observations, $w_1\ w_2\ w_3 \cdot \cdot \cdot\ w_1'\ w_2'\ w_3' \cdot \cdot \cdot$ the respective weights of their determinations; let ϕ be a quantity to add to the second set of bearings, in order to refer them to the same initial point of azimuth with the first observations, and let A B C . . . be the most probable bearings to be determined from both sets of observations, then, by the theory of probabilities, the quantity

$$w_1 (A-\alpha)^2 + w_2 (B-\beta)^2 + w_3 (C-\gamma)^2 + \cdots$$
$$+ w_1' (A-\alpha'-\phi)^2 + w_2' (B-\beta'-\phi)^2 + w_3' (C-\gamma'-\phi)^2 + \cdots$$

must be made a minimum, A B C . . . ϕ being considered independent variables. This will give the following equations:—

$$\begin{array}{l} (w_1 + w_1') A = w_1 \alpha + w_1' \alpha' + w_1' \phi \\ (w_2 + w_2') B = w_2 \beta + w_2' \beta' + w_2' \phi \\ \quad \vdots \qquad\qquad \vdots \\ (v_1 + v_2 + \cdots) \phi = v_1 (\alpha - \alpha') + v_2 (\beta - \beta') + \cdots \end{array}$$

Where

$$\frac{1}{v_1} = \frac{1}{w_1} + \frac{1}{w_1'}, \quad \frac{1}{v_2} = \frac{1}{w_2} + \frac{1}{w_2'} \quad \cdots$$

(5.) It may be as well, in connexion with the subject of bearings, to explain the method by which the position of a lost old station is calculated by means of the old bearings taken from it, and the new angles taken from a new station known to be very near the position of the old station.

Let d_1 d_2 d_3 . . . . be the distances of the points common to the two sets of observations, the nearer the better; α_1 α_2 α_3 . . . . . their bearings, with the weights w_1 w_2 w_3 . . . . as given in the new observations; α_1' α_2' α_3' . . . . . their bearings with the weights w_1' w_2' w_3' . . . . as given in the old observations:

Let x y be the co-ordinates of the old station, x being measured from the new point in the direction of the line from which the bearings α' are counted, and y perpendicular to it. Let also z be a quantity to be applied to all the old bearings, in order to make them comparable with the new bearings, z is then the angle between the directions of the line from which α_1 α_2 α_3 . . . . are measured, and of the line from which α_1' α_2' α_3' . . . . are measured: We then have the equations

$$\frac{x \sin \alpha_1 - y \cos \alpha_1}{d_1} = \alpha_1' + z - \alpha_1$$

$$\frac{x \sin \alpha_2 - y \cos \alpha_2}{d_2} = \alpha_2' + z - \alpha_2$$

$$\frac{x \sin \alpha_3 - y \cos \alpha_3}{d_3} = \alpha_3' + z - \alpha_3$$

and so on. If both sets of observations were perfect, then these equations would be identically true, and any three would give the true values of x y and z. But each equation being affected by errors of observation, x y and z must be so determined as to make the sum of the squares of the errors, each multiplied by the weight of its determination, a minimum. Now the weight of each equation depends on the weights of the bearings from which it is derived; that is, the weight of the n^{th} equation is—

$$v_n = \frac{w_n\, w_n'}{w_n + w_n'}$$

and therefore the quantity

$$\begin{aligned} U = {} & v_1 \left(x \frac{\sin \alpha_1}{\delta_1} - y_1 \frac{\cos \alpha_1}{\delta_1} - z + \alpha_1 - \alpha_1' \right)^2 \\ & + v_2 \left(x \frac{\sin \alpha_2}{\delta_2} - y \frac{\cos \alpha_2}{\delta_2} - z + \alpha_2 - \alpha_2' \right)^2 \\ & + v_3 \left(x \frac{\cos \alpha_3}{\delta_3} - y \frac{\cos \alpha_3}{\delta_3} - z + \alpha_3 - \alpha_3' \right)^2 \\ & + \quad \cdots\cdots\cdots\cdot \end{aligned}$$

where $\delta_n = d_n \sin 1''$, must be a minimum with respect to the quantities $x\ y\ z$. If we write $a_n\ b_n\ c_n$ for the co-efficients of x and y and the quantity $\alpha_n - \alpha_n'$ in the n^{th} parenthesis, then the equations for determining the minimum are

$$\begin{aligned} (v\,a^2)\,x + (v\,a\,b)\,y - (v\,a)\,z + (v\,a\,c) &= 0 \\ (v\,a\,b)\,x + (v\,b^2)\,y - (v\,b)\,z + (v\,b\,c) &= 0 \\ (v\,a)\,x + (v\,b)\,y - (v)\,z + (v\,c) &= 0 \end{aligned}$$

where the brackets have the usual signification of summation; that is

$$\begin{aligned} (v\,a^2) &= v_1\,a_1^2 + v_2\,a_2^2 + v_3\,a_3^2 + \cdots \\ (v\,a\,b) &= v_1\,a_1\,b_1 + v_2\,a_2\,b_2 + v_3\,a_3\,b_3 + \cdots \\ &\text{\&c.} \qquad \text{\&c.} \end{aligned}$$

These equations determine $x\ y\ z$ and the respective probable errors.

OBSERVATIONS,

TERRESTRIAL AND ASTRONOMICAL,

FOR THE

PRINCIPAL TRIANGULATION.

SECTION IV.

THE first part of this section contains extracts from the reduced theodolite observations of terrestrial objects. These extracts are confined chiefly, but not exclusively, to the principal points of the triangulation of Great Britain and Ireland. It has been considered sufficient to give the 'mean bearing' of each observed object, with the number of observations taken to it, their range, or the difference between the greatest and least observation, with the calculated weight of each mean bearing. The method of obtaining the mean bearings from the Observation Books is explained in the preceding section.

The corrections to the observed bearings when the trigonometrical point is not the object observed, are obtained from the measured distance and bearing of that object from the trigonometrical point. There are, however, two exceptions, namely, in the corrections for reducing the bearings of the old stations at Delamere and Easington to the new stations; the corrections in these cases were obtained from the distances and bearings of the old from the new stations, as calculated from a comparison of the old and new observations at those points. In these cases the weight of the corrected bearing has been erroneously assumed equal to the weight of the observed bearing, but the oversight is not of sufficient magnitude to affect the positions of those stations.

The bearings are referred to an approximate South Meridian Line.

The second part contains the principal azimuthal observations in detail, and the results of those which are less important, together with their range and probable error.

The third part contains the results of the observations made with Airy's and Ramsden's Zenith Sectors for the latitudes of various points in the triangulation.

EXTRACTS FROM THE GENERAL ABSTRACTS OF OBSERVATIONS.

ACKLAM WOLD.

18-in. Theodolite.

From 24th November 1841 to 26th March 1842. Observer: Corp. STEEL, R. S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Crowle	3 12 0·81	3	5·27	3·30	Great Whernside .	99 17 24·43	5	4·66	1·09
Hemingbro' Spire .	22 10 9·72	6	12·72	5·26	Hambleton Down .	127 47 57·76	5	3·45	0·73
Clifton Beacon . .	23 41 28·67	2	1·16	0·33	Black Hambleton .	133 24 33·60	6	9·82	2·84
Garforth Cliff . .	52 23 44·59	4	8·91	5·05	Botton Head . .	153 46 15·82	5	10·20	4·99
York Minster . .	63 18 4·36	7	18·90	10·43	Saltergate Brow .	191 26 2·87	8	16·82	6·63
Rumbles Moor . .	76 54 45·29	3	4·84	3·25	Wilton Beacon . .	346 20 1·31	8	29·85	31·57

ARBURY HILL.

3-ft. Theodolite, B.O.

From 20th January to 29th March 1843. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Epwell	48 2 48·63	7	6·88	1·12	Dunchurch Tower .	157 19 14·80	5	4·01	0·89
Broadway Tower[1] .	62 41 46·15	5	5·38	1·47	Bardon Hill . . .	172 16 8·40	15	5·50	0·25
Breedon Tower . .	72 54 32·47	3	1·55	0·27	Referring-object . .	185 7 13·94	102	7·39	0·03
Malvern	80 37 49·72	8	6·81	0·94	Naseby Ch. Tower .	218 13 34·00	7	4·39	0·98
Lord Plymouth's Monument . .	107 16 40·87	3	0·78	0·07	Hanslope Ch. Spire .	295 11 14·03	2	1·46	2·13
Bar Beacon . . .	129 24 36·25	8	9·43	1·76	Dunstable	310 37 15·24	10	2·95	0·23
Corley[2]	138 1 19·18	7	4·57	0·58	Wendover	329 55 17·18	6	3·84	0·53
					Brill	347 28 19·38	6	3·30	0·50

[1] A correction of + 3″·39 to be applied to this bearing, to reduce it to the trigonometrical station.

[2] A correction of + 9″·99 " " "

AXEDGE.

3-ft. Theodolite, R.S.

From 21st April to 21st June 1842. Observer: Lieut. PIPON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Longmount Pole	39 10 56·49	5	4·27	0·86	Ingleboro'	164 15 59·81	5	5·30	1·56
Ashley	39 46 26·05	5	2·23	0·24	Boulsworth	170 50 44·13	4	4·69	1·87
Long Mountain	49 58 30·76	2	0·78	0·15	Holme Moss	187 15 44·19	15	6·17	0·48
Mowcopt	53 32 25·03	11	8·89	1·11	Kinder Scout	194 50 53·66	18	9·54	0·68
Cyrn-y-Brain	75 43 38·14	8	2·78	0·19	Lord's Seat	211 8 0·19	13	7·07	0·54
Snowdon	83 32 43·92	4	2·00	0·26	Back Tor	218 38 2·57	12	5·82	0·45
Moel Famman	84 50 47·58	10	6·50	0·56	Heathersedge	243 34 45·64	10	3·52	0·27
Rivington	129 47 40·62	5	5·23	1·16	Lincoln Minster[1]	269 19 9·51	1	—	6·04
Whittle Hill	156 33 21·96	10	8·77	1·18	Bardon Hill	323 32 35·24	7	3·27	0·43
Pendle Hill	162 1 9·66	4	3·13	0·97	Weaver Hill, New	346 14 31·08	12	9·07	1·14

[1] A correction of − 2″·83 to be applied to this bearing, to reduce it to the station.

BACONSTHORPE TOWER.

2-ft. Theodolite.

From 5th August to 4th September 1843. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Hingham Tower	19 27 8·95	7	2·39	0·21	Happisburgh Tower	285 16 37·71	8	3·19	0·26
Swaffham Spire[1]	50 15 42·85	9	4·61	0·42	Norwich Spire	341 26 13·94	6	2·40	0·21
Docking Tower	92 41 53·13	8	2·13	0·12					

[1] A correction of + 2″·39 to be applied to this bearing, to reduce it to the observing station at Swaffham.

BALLYCREEN.

3-ft. Theodolite, B.O.

From 26th August to 24th September 1852. Observer: Corp. JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Coona	0 53 2·95	9	3·77	0·35	Mount Leinster	40 3 13·31	32	7·21	0·18
Forth Mountain	11 16 13·70	34	5·21	0·10	Knockanaffrin	50 6 6·84	18	5·08	0·16
Slieve Buy	15 54 12·95	23	5·20	0·14	Keeper	82 28 31·74	36	5·88	0·09
Blackstairs	35 59 12·43	17	7·33	0·36	Cullenagh	94 13 6·33	24	4·14	0·09

BALLYCREEN—*continued.*

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cadeen	104 11 54·30	13	5·15	0·32	Snowdon	262 53 15·19	12	4·27	0·19
Lugnaquillia	129 28 14·83	15	3·97	0·11	Collin	270 37 53·26	50	5·57	0·05
Mullaghslaban	172 33 42·02	18	7·49	0·28	Ballymile	296 43 48·45	19	4·25	0·14
Kippure	184 33 36·93	33	6·37	0·15	Precelly	314 20 29·80	10	3·81	0·30
Douce	199 42 45·03	21	4·51	0·13	Arklow Rock	318 42 35·26	34	7·69	0·12
Great Sugar Loaf	208 50 0·56	22	5·01	0·13	Tara	337 35 41·81	74	6·81	0·03
Dunrain	227 14 29·96	25	4·91	0·10	Croghan	348 10 53·70	28	5·85	0·12
Carrickmarino	241 43 15·59	35	4·24	0·06					

BALNASKERRISH.

3-ft. Theodolite, B.O.

From 1st to 9th October 1819. Observers: Major-Gen. COLBY, Lt.-Col. ROBE, and Capt. DAWSON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Vacher	51 43 4·52	5	7·70	3·62	Scarabin	197 8 19·50	3	5·63	4·15
Scournalapich, Old[1]	54 59 25·85	1	–	6·27	Bin of Cullen	289 46 27·60	6	4·92	0·75
Ben Wyvis	67 53 33·53	8	3·83	0·35	Knock	295 9 14·08	6	3·22	0·43
Airdross	76 57 14·14	6	3·03	0·45	Findlay Seat	311 31 36·93	6	3·77	0·62
Ben Lundie	138 21 20·88	6	3·02	0·36	Corryhabbie	326 38 29·86	10	5·53	0·59
Ben Horn	148 21 10·58	3	1·14	0·17	Ben Macdui	354 25 14·45	5	7·49	2·70

[1] A correction of + 16″·53 to be applied to this bearing, to reduce it to the trigonometrical station.

BALSHAM CHURCH TOWER.

2-ft. Theodolite.

From 23rd May to 23rd June 1844. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Tharfield	62 7 52·00	14	4·40	0·28	Peterboro' Cathedral	142 22 4·79	2	1·71	0·73
Royston	67 48 37·01	10	4·96	0·49	Balsham	150 59 14·94	91	4·89	0·01
Orwell	95 13 2·95	8	3·69	0·37	Ely Minster	172 41 47·36	14	4·30	0·19
Keysoe Spire	104 34 44·25	6	5·05	0·91	Swaffham Spire[1]	203 39 57·66	8	4·74	0·43
					Lawshall	264 49 10·32	10	2·50	0·11
Cambridge Observatory Dome	120 45 58·95	11	5·44	0·36	Thaxted Spire	355 22 26·44	9	2·93	0·28

[1] A correction of + 2″·87 to be applied to this bearing, to reduce it to the trigonometrical station.

BALTA.

3-ft. B.O. and 7-in. Theodolites.

From 10th to 27th August 1817. Observers: Major-Gen. COLBY and Mr. GARDNER.
From 24th to 26th February 1847. Observers: Privates JENKINS and CLARKE, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Fetlar	16 15 31·65	9	6·65	0·80	Saxavord . . .	160 45 39·21	11	4·87	0·51
Yell	36 2 23·70	10	4·94	0·31	Nive Hill . . .	181 20 56·07	35	86·66	15·12
Vallafield . . .	88 39 27·35	3	2·01	0·45	Gerth of Scaw . .	186 30 49·64	35	113·82	35·79

BANSTEAD.

3-ft. Theodolite, B.O.

From 7th August to 11th October 1848. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Leith Hill of 1822[1].	35 4 30·43	14	5·64	0·36	Wimbledon Spire .	181 12 25·00	11	6·24	0·47
Obelisk on Bagshot Heath	94 12 41·18	5	4·32	0·80	Berkhampstead . .	187 11 22·78	3	1·16	0·15
St. Anne's, Old . .	112 38 20·73	9	4·70	0·40	Highgate Spire . .	189 0 15·87	5	3·49	0·59
Ditto New . .	112 51 20·49	10	3·55	0·27	Westminster Abbey	196 24 57·78	32	6·64	0·17
Windsor Castle Staff	124 32 17·29	5	5·21	1·29	St. Paul's Cathedral	200 14 8·03	9	6·01	0·79
Hampton Church Tr.	136 13 29·22	8	5·04	0·67	Chingford . . .	202 32 16·59	18	7·10	0·42
King's Arbour . .	137 53 51·48	14	7·17	0·42	St. George's Church Tower, Ratcliffe .	206 33 21·83	5	2·65	0·31
Hampton Poor House	139 10 54·40	13	5·21	0·23	Epping	208 23 34·18	12	6·50	0·34
Harrow Spire . .	163 23 3·43	10	4·62	0·35	Severndroog Tower	228 51 8·69	13	8·87	1·08
Hanger Hill Tower[2] . .	165 38 58·79	16	8·78	0·68					

[1] A correction of −4′ 10″·54 applied to this bearing will give the bearing of the present station on the Tower.
[2] A correction of − 2″·92 ” ” ”

BARDON HILL.

3-ft. Theodolite, B.O.

From 6th July to 29th December 1842. Observers: Lieuts. ROBINSON and LUYKEN, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Broadway Tower[1] .	24 45 34·83	5	2·05	0·26	Bar Beacon . . .	68 45 20·63	4	2·72	0·53
Corley[2]	30 36 30·84	5	2·34	0·24	Brown Clee . . .	73 28 56·64	1	–	2·57
Malvern	46 0 58·31	4	3·25	0·74	Longmount Pole .	80 21 56·54	2	0·71	0·12
Lord Plymouth's Monument . .	51 26 20·57	2	0·64	0·10	Wrekin	87 2 6·67	4	0·74	0·03
					Castle Ring, New .	90 10 12·01	3	1·95	0·44

[1] A correction of − 0″·28 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of + 8″·81 ” ” ”

BARDON HILL—*continued.*

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ″		″			° ′ ″		″	
Ashley Heath . .	108 5 32·85	3	1·87	0·40	Referring-object .	213 47 12·42	147	6·02	0·02
Mowcopt	126 56 41·65	4	1·83	0·22	Lincoln Minster[1] .	222 0 59·43	6	3·03	0·37
Weaver Hill, New .	132 47 55·03	3	2·15	0·62	Stathhearne . . .	241 11 20·20	4	1·73	0·20
Axedge	144 2 31·59	3	4·02	2·18	Buckminster Spire .	257 16 35·87	3	2·64	0·77
Back Tor	161 54 48·02	2	2·74	1·87	Tilton	283 52 49·07	10	4·24	0·37
Heathersedge . .	165 21 43·38	3	2·91	0·94	Naseby Church Tr. .	327 24 16·62	11	4·25	0·18
Sutton	188 26 36·02	6	2·25	0·26	Arbury Hill . . .	352 11 37·59	8	6·00	1·00
Holland Hill . .	201 32 26·17	5	2·85	0·42	Dunchurch Tower .	356 58 37·34	2	2·09	1·09

[1] A correction of + 1″·86 to be applied to this bearing, to reduce it to the trigonometrical station.

BARROW HILL.

3-ft. Theodolite, B.O.

From 28th August to 26th September 1845. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Hensbarrow . . .	102 37 50·08	18	14·49	1·07	Ryder's Hill . .	155 44 51·53	20	4·01	0·16
Maker	109 44 32·42	6	4·35	0·75	Furland	212 25 46·79	6	2·89	0·39
Maker Tower . .	110 58 46·19	8	8·77	1·48	Pillesdon	223 9 27·32	18	6·30	0·33
Kit Hill	127 25 36·60	3	1·59	0·30	Golden Cape . .	227 11 13·18	16	4·51	0·27
Kit Hill Tower . .	127 28 40·02	8	12·84	3·53	Blackdown . . .	237 27 47·94	12	6·61	0·56
Butterton, New .	145 25 44·50	8	5·32	0·78	Referring-object .	352 32 29·98	90	11·53	0·05

BAURTREGAUM.

3-ft. Theodolite, B.O.

From 29th July to 5th October 1831. Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Knocknagante . .	11 50 47·01	5	5·69	1·83	Galtymore . . .	260 25 32·11	1	–	9·10
Knocknadober . .	44 59 17·14	9	3·41	0·25	Taur	265 35 25·62	25	7·01	0·23
Feaghmaan . . .	48 26 53·59	6	4·98	1·04	Knockanaffrin . .	265 42 45·05	5	3·56	0·70
Brandon	96 18 33·84	10	5·68	0·42	Knockmealdown .	268 13 45·65	1	–	9·10
Bencorr	180 59 22·50	3	4·69	2·44	Knocknaskagh . .	276 5 16·54	3	1·19	0·15
Knockanore . . .	203 14 14·41	6	6·10	1·42	Caherbarnagh . .	293 27 3·21	6	3·39	0·33
Slievecallan . . .	208 3 16·36	7	10·58	2·51	Mangerton . . .	317 56 43·36	22	9·44	0·45
Keeper	239 45 27·96	4	6·90	4·23	Hungry Hill . .	357 27 47·76	12	6·33	0·48
Knockfeerina . .	246 27 32·46	6	3·68	0·50					

BEACHY HEAD.

18-in. Theodolite.

From 8th March to 10th May 1845. Observer: Corp. Steel, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Dunnose	83 2 40·38	12	7·03	0·58	Brightling Observy	199 33 50·06	15	15·42	1·86
Rooks' Hill	103 48 42·47	1	–	23·68	Frittenfield	218 3 32·58	17	13·06	1·48
Ditchling	125 27 3·61	29	32·08	2·07	Fairlight	239 13 45·86	28	11·31	0·57
Leith Hill Tower	138 9 15·89	20	14·45	1·07	Dungeness Lighthouse	248 49 26·42	16	10·02	1·19
Referring-object	185 44 4·84	107	11·16	0·12					

BEACON HILL.

3-ft. Theodolite, R.S.

From 20th March to 17th August 1849. Observer: Serg. Donelan, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Queen's Manor	10 33 7·79	18	5·01	0·25	Lansdown Obelisk	151 12 49·34	8	9·05	1·68
Coringdon	17 5 11·41	16	4·96	0·21	Milk Hill	157 20 8·40	26	6·59	0·18
Swyre Barrow	22 18 16·60	14	7·17	0·54	Referring-object	212 52 57·12	19	6·83	0·23
Old Sarum	28 31 12·11	57	6·90	0·05	Inkpen	223 36 47·94	25	5·17	0·11
Old Sarum Castle	30 1 56·58	49	5·84	0·06	Butser	293 28 36·77	22	6·97	0·24
Wingreen	50 55 2·35	27	8·05	0·31	Old Lodge	316 17 13·40	31	7·40	0·25
Four-mile Stone	52 30 52·48	28	6·47	0·12	Dunnose	329 26 22·02	16	6·00	0·39
Mendip	94 11 6·28	16	4·83	0·18	Motteston	340 12 53·15	10	6·18	0·56
Westbury	106 19 7·59	16	4·10	0·19	Dean Hill	344 4 53·94	25	4·88	0·15
Stoke Hill	112 34 45·09	46	6·74	0·09	Thorney Down	354 57 54·85	28	9·00	0·21

BEACON HILL, TRESCOW.

18-in. Theodolite.

From 25th July to 14th August 1850. Observer: Corp. Wotherspoon, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
St. Agnes Lighthouse	2 17 56·64	14	9·86	1·00	Pertinny	251 41 41·57	20	6·66	0·27
Karnminnis	245 26 42·36	25	9·97	0·51	St. Martin's Head	263 8 29·24	16	13·39	1·44
Karn Galver	246 9 3·39	24	12·49	0·52	Telegraph Tower	322 47 27·69	17	7·30	0·48
					Peninnis Windmill	336 7 27·08	18	11·45	0·85

BELLEFIELD.

3-ft. Theodolite, B.O.

From 2nd to 9th June 1807. Observer: Major-Gen. COLBY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Delamere, Old[1] .	25 23 57·07	7	6·21	1·14	Rivington . . .	188 1 10·65	5	11·74	7·30
Cyrn-y-Brain .	48 42 2·93	1	–	15·99	Knoll Hill . .	214 29 26·08	2	7·79	15·17
Heswell . . .	86 27 35·79	4	4·81	1·66	Mowcopt . . .	317 4 38·60	4	3·69	0·94
Billinge . . .	152 17 13·65	6	6·85	1·79					

[1] A correction of −1″·07 to be applied to this bearing, to reduce it to the new station.

BEN CHEILT.

3-ft. Theodolite, B.O.

From 18th to 21st September 1819. Observers: Major-Gen. COLBY, Lieut. Col. ROBE, and Capt. DAWSON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Scarabin . . .	48 7 26·27	6	4·38	0·73	South Ronaldshay	206 5 32·83	6	9·02	2·75
Ben Clibrig . .	81 27 47·99	5	6·32	2·04	Knock	336 12 57·95	9	7·04	1·43
Ben Hope . . .	98 38 16·58	4	5·36	2·05	Bin of Cullen .	337 30 46·67	9	6·52	1·02
Ben Hutig . .	111 37 32·64	4	4·48	2·03	Corryhabbie . .	354 13 26·22	6	3·11	0·48
Wart Hill Hoy .	181 58 23·40	7	4·33	0·72					

BEN CLEUGH.

3-ft. Theodolite, R.S.

From 27th September to 13th November 1848. Observer: Corp. WINZER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cairnsmuir on Deugh . . .	15 16 14·94	11	8·49	1·02	Craigowl . . .	228 49 38·69	19	7·04	0·38
Merrick . . .	20 59 10·06	13	9·09	0·69	East Lomond .	259 16 5·55	22	7·42	0·31
Corkmulaw . .	49 5 47·13	21	5·33	0·30	Largo Law . .	263 47 37·58	18	7·55	0·37
Bin of Campsie .	50 28 41·24	22	7·28	0·49	Sayrs Law . .	298 22 1·08	16	5·57	0·27
Goat Fell . . .	55 34 43·16	8	2·79	0·26	Calton Observatory Dome[1] .	304 54 53·37	7	4·31	0·51
Hill of Stake . .	57 39 37·89	24	11·13	0·41	Arthur's Seat .	305 10 1·74	8	6·16	1·18
Ben Lomond . .	90 54 22·98	48	16·16	0·36	Allarmoor . . .	315 17 31·46	30	7·35	0·26
Ben Nevis . .	132 20 35·37	27	7·57	0·25	Carnethy Cairn .	320 40 53·75	30	10·27	0·57
Ben Lawers . .	145 16 23·84	38	10·40	0·26	Dunrich . . .	331 36 30·00	10	4·32	0·48
Ben Macdui . .	183 31 26·17	26	6·55	0·22	Hart Fell . . .	344 56 3·41	9	4·17	0·34
Glashmeal . .	197 41 34·89	21	7·32	0·32	Tinto	354 14 8·27	17	6·72	0·40
Mount Battock .	216 10 28·01	11	7·84	0·78					

[1] A correction of + 9″·57 to be applied to this bearing, to reduce it to the trigonometrical station.

BEN CLIBRIG.

3-ft. Theodolite, B.O.

From 5th August to 20th September 1839. Observers: Lieuts. ROBINSON, PIPON, and HAMLEY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Wyvis	9 11 42·13	25	7·19	0·41	Dunnet Head	231 1 33·23	6	4·64	0·95
Scournalapich[1]	22 3 58·93	13	5·44	0·35	South Ronaldshay	233 55 25·86	4	1·80	0·22
Storr	53 6 52·83	15	15·56	1·59	Ben Cheilt	260 35 6·21	16	8·96	0·94
Suilbheinn	72 58 30·60	5	8·58	3·64	Scarabin	271 36 34·34	36	14·78	0·50
Cuinnag	86 51 4·75	33	10·99	0·30	Bin of Cullen	304 13 29·71	6	9·77	4·92
Monach	97 42 21·30	16	14·23	1·15	Findlay Seat	316 3 30·17	3	4·05	1·82
Foinnebheinn	125 42 7·95	5	5·60	1·56	Duke of Sutherland's Monument	319 38 0·50	4	10·49	8·28
Fashven	142 1 38·22	20	10·92	0·57	Buck	320 17 3·91	3	9·61	12·70
Ben Hope	149 54 17·98	6	8·56	3·02	Ben Lundie	321 33 2·76	28	14·25	0·58
Ben Hutig	170 25 45·44	34	14·04	0·56	Corryhabbie	323 24 41·65	9	3·85	0·36
Fitty Hill	214 14 27·50	3	7·40	6·25	Ben Macdui	340 49 41·45	7	11·00	3·25
Wart Hill Hoy	219 35 36·56	7	9·44	2·54					

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[1] A correction of + 38″·45 to be applied to this bearing, to reduce it to the new Station.

BEN HEYNISH.

3-ft. Theodolite, B.O.

From 2nd to 17th October 1822. Observers: Captains DAWSON and VETCH, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Ival Barra	149 50 0·84	7	4·19	0·51	Ben More in Mull	273 5 2·18	27	6·62	0·17
Ben More, S. Uist	165 56 19·88	7	4·06	0·47	Jura	316 46 38·20	17	5·75	0·14
Storr	200 42 52·45	1	–	6·05	Ben Tartevil	339 38 12·05	16	6·54	0·43
Ben Nevis	251 18 47·90	2	0·33	0·02					

BEN HUTIG.

3-ft. Theodolite, B.O.

From 11th July to 17th August 1838. Observers: Major-Gen. COLBY, Lieut.-Col. ROBE, and Lieut. ROBINSON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Wyvis	2 19 38·89	3	6·69	5·61	Wart Hill Hoy	239 40 30·92	11	10·25	1·36
Ben Hope	19 50 9·65	7	6·84	1·38	South Ronaldshay	253 33 25·56	6	4·88	0·96
Foinnebheinn	54 31 8·17	4	3·08	0·60	Dunnet Head	258 18 17·93	12	10·78	1·48
Fashven	93 14 49·54	6	20·03	12·65	Ben Cheilt	290 39 32·96	13	5·25	0·40
Cnoc-ghiubhais	97 56 16·52	4	3·79	0·92	Scarabin	303 56 51·08	4	4·06	1·29
North Rona Island	130 44 23·76	4	4·51	1·28	Ben Clibrig	350 20 33·48	10	8·77	1·74
Fitty Hill	226 4 57·50	5	6·41	1·91					

BEN LAWERS.

3-ft. Theodolite, B.O.

From 12th October to 17th November 1850. Observer: Corp. JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Merrick . . .	5 46 14·67	16	2·72	0·06	Largo Law . .	292 52 54·37	24	4·90	0·17
Hill of Stake . .	22 56 28·65	23	6·88	0·28	East Lomond .	298 13 15·48	17	7·03	0·35
Goat Fell . . .	30 58 50·81	14	4·85	0·24	West Lomond .	299 55 24·57	15	5·80	0·31
Ben Lomond . .	32 59 47·29	34	6·34	0·12	Sayrs Law . .	308 16 47·99	16	3·27	0·10
Jura	57 51 39·38	22	5·97	0·26	Allarmoor . .	319 59 29·37	18	4·58	0·14
Ben More in Mull	83 51 24·12	22	6·97	0·20	Carnethy Cairn .	322 36 16·09	4	3·02	0·59
Ben Nevis . .	120 39 3·03	42	6·20	0·08	Ben Cleugh . .	324 54 22·33	31	5·02	0·07
Mamsuil . . .	146 34 10·51	26	4·29	0·11	Hart Fell . . .	337 39 45·05	12	2·20	0·08
Ben Amhlair . .	153 34 10·39	26	4·23	0·12	Tinto	341 39 15·31	6	3·94	0·55
Ben Macdui . .	209 43 24·94	40	6·93	0·10	Bin of Campsie .	355 5 56·71	31	6·23	0·11
Glashmeal . . .	234 40 8·80	41	7·19	0·09	Corkmulaw . .	356 40 38·61	36	5·20	0·09
Craigowl . . .	269 16 0·29	35	6·37	0·11					

BEN LOMOND.

3-ft. Theodolite, B.O.

From 10th to 19th August 1818. Observers: Major-Gen. COLBY and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Brown Carrick .	3 18 52·81	3	1·34	0·23	East Lomond .	265 37 47·83	11	5·66	0·64
Hill of Stake . .	11 1 47·10	13	8·18	0·80	Ben Cleugh . .	270 11 16·06	26	6·84	0·16
Goat Fell . . .	29 18 41·60	4	5·83	2·37	Calton Hill . .	285 34 57·62	4	3·74	1·46
Ben Tuirc . .	40 29 20·66	6	7·40	2·43	Allarmoor . . .	290 55 4·83	7	3·87	0·47
Knocklayd . .	42 19 7·32	3	1·46	0·27	Bin of Campsie .	302 22 59·85	30	6·36	0·10
Jura	70 0 37·61	4	4·83	1·63	Dunrich . . .	306 36 53·11	4	3·00	0·72
Ben More in Mull	107 33 54·39	6	6·25	1·62	Corkmulaw . .	306 40 28·15	9	7·33	1·19
Creachbheinn . .	135 37 21·03	3	2·47	0·70	Tinto	317 16 30·11	5	2·53	0·28
Ben Nevis . .	161 29 0·99	5	1·19	0·91	Hart Fell . . .	317 55 15·34	3	1·91	0·41
Ben Amhlair . .	188 24 21·86	4	5·90	2·85	Cairnsmuir on Deugh . . .	345 35 39·18	4	2·19	0·30
Ben Lawers . .	212 38 39·85	6	3·37	0·59					

BEN LUNDIE.

3-ft. Theodolite, B.O.

From 25th to 28th September 1819. Observers: Major-Gen. COLBY and Capt. DAWSON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Airdross . . .	37 22 42·71	11	5·44	0·33	Bin of Cullen .	296 58 28·48	6	5·47	0·92
Ben Wyvis . .	42 35 0·42	7	4·81	0·92	Knock	300 22 47·68	6	6·97	2·27
Cairn Cruniach .	57 1 21·10	7	7·93	1·66	Balnaskerrish . .	318 9 15·42	10	6·85	1·00
Cuinnag . . .	113 19 10·83	3	5·11	3·65	Corryhabbie . .	324 27 19·66	6	2·75	0·58
Ben Clibrig . .	141 51 38·94	8	8·37	1·34	Dornoch Spire .	355 27 18·38	3	0·72	0·06

BEN MACDUI.

3-ft. Theodolite, R.S.

From 6th June to 16th August 1847. Observer: Corp. WINZER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Cleugh . .	3 36 17·88	20	5·55	0·23	Ben Cheilt . .	186 57 14·99	19	5·60	0·21
Ben Lawers . .	30 10 26·15	41	7·23	0·13	Findlay Seat . .	204 16 29·18	11	6·40	0·72
Ben Amhlair . .	59 48 10·02	25	5·48	0·18	Bin of Cullen .	215 31 1·35	6	1·69	0·14
Ben Nevis . .	70 0 26·92	39	7·66	0·12	Corryhabbie . .	222 53 12·34	21	6·47	0·27
Mamsuil . . .	105 29 31·40	10	4·49	0·29	Buck	239 47 27·61	14	8·08	0·55
Scournalapich .	112 11 38·12	35	6·45	0·15	Dudwick . . .	247 9 35·91	14	5·68	0·24
Ben Wyvis . .	141 26 40·59	16	4·93	0·28	Caerlock . . .	277 29 6·28	14	6·98	0·43
Ben Clibrig . .	161 27 15·88	19	9·32	0·60	Mount Battock .	283 4 39·20	44	9·62	0·16
Ben Lundie . .	167 43 18·17	16	4·18	0·19	Glashmeal . . .	320 5 23·60	23	4·19	0·11
Balnaskerrish . .	174 32 13·09	4	3·68	0·93	Largo Law . .	333 36 26·41	2	2·67	1·78
Scarabin . . .	182 2 15·99	19	3·17	0·11	Sayrs Law . .	335 15 29·91	15	6·80	0·57

BEN MORE, MULL.

3-ft. Theodolite, B.O.

From 18th to 24th September 1822. Observers: Captains DAWSON and VETCH, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Oa	11 16 37·80	6	4·91	0·88	Ben Lawers . .	262 21 12·09	4	4·77	1·44
Ben Tartevil . .	19 10 1·72	15	6·76	0·37	Ben Lomond . .	286 24 57·96	10	9·58	1·48
Ben Heynish . .	93 50 28·62	15	6·34	0·61	Goat Fell . . .	329 42 28·99	3	0·82	0·09
Ben More, S. Uist	140 26 28·69	6	5·91	1·50	Ben Tuire . .	343 31 40·47	7	3·44	0·46
Storr	175 11 39·22	3	1·39	0·22	Carn na Leagh .	353 8 21·68	4	3·26	1·17
Creachbheinn .	231 10 45·55	4	2·76	0·60	Jura	359 23 32·82	23	6·20	0·19
Ben Nevis . .	235 51 3·35	9	7·40	1·36					

BEN MORE, SOUTH UIST.

3-ft. Theodolite, B.O.

From 5th June to 29th August 1851. Observer: Corp. JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cleisham . . .	199 57 56·17	42	3·87	0·04	Creachbheinn .	300 13 28·05	17	4·98	0·23
Storr	247 3 43·54	46	5·37	0·08	Referring-object .	305 41 44·45	34	5·80	0·01
Scournalapich .	263 50 15·08	27	5·23	0·12	Ben More in Mull	319 22 13·80	29	6·31	0·17
Mamsuil . . .	268 3 12·83	19	5·41	0·22	Jura	331 45 6·78	14	4·11	0·19
Elms Hill . . .	287 16 30·89	17	4·71	0·24	Ben Heynish . .	345 37 43·61	45	6·41	0·11
Ben Nevis . .	289 19 22·47	15	7·27	0·52					

BENCORR.

3-ft. Theodolite, B.O.

From 27th July to 31st August 1830. Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Baurtregaum . .	1 1 3·98	6	7·79	2·21	Knockalongy . .	221 9 30·66	2	4·44	4·92
Brandon . . .	12 35 49·98	5	7·44	2·45	Keeper . . .	308 44 41·66	13	8·92	0·84
Slieve More . .	162 39 3·07	8	4·22	0·50	Slievecarran . .	310 53 48·13	20	8·12	0·39
The Reek . . .	197 7 35·60	5	1·12	0·09	Slievecallan . .	334 36 0·28	18	8·77	0·68
Nephin . . .	206 10 36·40	15	11·77	0·86	Knockanore . .	353 25 46·20	3	2·41	0·64

BEN NEVIS.

3-ft. Theodolite, R.S.

From 1st August to 14th November 1846. Observer: Corp. WINZER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Jura	32 16 10·63	13	3·91	0·17	Scournalapich .	176 57 15·22	50	6·94	0·10
Ben More in Mull	56 41 57·25	28	9·47	0·30	Ben Wyvis . .	194 26 41·61	42	7·96	0·11
Creachbheinn .	62 46 13·65	18	8·93	0·42	Corryhabbie . .	240 9 13·00	14	7·87	0·46
Ben Heynish . .	72 55 15·04	23	6·46	0·22	Ben Macdui . .	248 53 44·10	35	10·05	0·20
Referring-object .	81 52 46·07	23	13·23	0·68	Ben Amhlair . .	266 29 31·38	19	3·83	0·15
Ben More, S. Uist	111 14 50·35	13	9·14	0·90	Ben Lawers . .	299 59 31·59	38	10·63	0·23
Elms Hill . . .	113 48 43·16	3	1·87	0·48	Ben Cleugh . .	311 19 5·44	16	7·40	0·39
Storr	138 25 45·76	14	5·14	0·25	Ben Lomond . .	341 10 43·04	19	7·55	0·49
Mamsuil . . .	172 30 10·01	8	2·94	0·19					

BEN TARTEVIL.

3-ft. Theodolite, B.O.

From 17th July to 2nd August 1822. Observers: Captains VETCH and DAWSON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Slieve Snaght .	44 3 58·65	6	6·98	2·01	Jura	234 19 29·88	40	7·90	0·13
Ben Heynish . .	160 2 5·73	9	3·92	0·29	Carn na Leagh .	317 20 48·58	16	8·27	0·54
Ben More in Mull	198 48 39·51	14	7·39	0·80	Oa *or* Cairnard .	317 52 44·00	25	6·72	0·25
Creachbheinn in Mull . . .	205 37 37·10	4	4·67	1·48	Trostan . . .	346 17 28·44	5	5·31	1·59
					Knocklayd . .	348 53 59·14	21	6·89	0·34

BEN WYVIS.

3-ft. Theodolite, B.O.

From 8th to 12th September 1819. Observers: Major-Gen. COLBY, Lt.-Col. ROBE, and Capt. DAWSON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Nevis	14 47 45·67	4	9·69	5·97	Balnaskerrish	247 14 22·90	3	5·99	4·03
Mamsuil	36 24 44·61	1	–	6·95	Bin of Cullen	270 8 3·86	3	4·70	2·46
Scournalapich[1]	40 3 26·95	5	2·41	0·32	Knock	274 49 51·43	3	2·51	0·75
Suilbheinn	146 1 36·20	5	2·50	0·31	Findlay Seat	277 25 48·39	2	0·70	0·12
Cuinnag	155 10 4·34	3	3·54	1·66	Corryhabbie	293 36 17·61	6	2·25	0·19
Ben Hutig	182 16 15·23	2	3·79	3·59	Ben Macdui	320 40 42·90	7	3·53	0·47
Ben Clibrig	189 3 7·63	8	7·83	1·08	Ben Amhlair	355 51 12·04	5	9·89	4·15
Scarabin	223 42 17·47	2	1·73	0·74					

[1] A correction of + 1′ 0″·54 to be applied to this bearing, to reduce it to the new station.

BERKHAMPSTEAD.

3-ft. Theodolite, B.O.

September 1823. Observers: Captains VETCH and DRUMMOND, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Leith Hill Tower	14 55 45·83	11	12·50	1·68	Chingford	327 24 25·29	21	7·40	0·19
Dunstable	113 2 37·76	16	4·63	0·16	Wrotham Staff	329 39 56·11	4	3·44	0·98
Thaxted Spire	235 28 15·26	3	6·62	4·87	Severndroog	338 14 49·47	5	6·49	2·01
High Beach	312 26 58·02	10	6·03	0·51	St. Paul's	356 3 1·60	5	7·31	3·02

BERYL.

3-ft. Theodolite, B.O.

From 28th May to 3rd June 1808. Observer: Major-Gen. COLBY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Black Comb	158 8 14·39	6	14·12	6·29	Billinge	329 42 25·44	3	4·61	3·02
Rivington	306 21 7·52	1	–	14·41	Go Hill	344 6 52·63	4	4·51	2·10

BESSY BELL.

3-ft. Theodolite, B.O.

From 22nd May to 16th June 1827. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cuilcagh	26 46 52·23	17	20·15	2·60	Sawel	236 41 41·39	36	6·62	0·14
Breesy	66 35 35·34	9	12·45	3·10	Slieve Gallion	262 31 8·50	8	4·69	0·45
Barnesmore Connell	97 8 25·36	14	6·99	0·62	Mullaghcarn	275 46 37·74	6	5·71	1·31
Slieve Snaght	183 53 40·09	18	10·26	0·57	Slieve Donard	299 50 38·87	1	–	12·51

BESSY BELL—*continued.*

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Vicar's Carn · ·	309 27 46·18	8	10·99	2·96	Shantavny · ·	315 47 48·43	21	9·33	0·38
Slieve Gullion ·	314 43 57·15	2	1·44	0·51	Mullyash · · ·	320 5 19·84	6	11·00	3·88
Armagh Breagh ·	314 45 46·95	2	0·82	0·16	Carnmore · · ·	350 18 8·95	17	4·07	0·21

BILLINGE.

3-ft. Theodolite, B.O.

From 15th to 21st June 1807. Observer: Major-Gen. COLBY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cyrn-y-Brain · ·	30 29 57·39	3	3·24	1·29	Go Hill · · ·	115 57 33·48	13	7·71	0·70
Heswell Hill · ·	53 3 15·27	3	6·62	5·04	Rivington · · ·	224 13 46·87	15	7·23	0·58
Everton Flagstaff	61 29 35·32	3	4·61	2·87	Mowcopt · · ·	322 14 58·07	5	4·03	0·87
Snowdon · · ·	62 14 30·04	1	–	18·46	Bellefield · · ·	332 10 49·66	5	2·58	0·41
Great Ormes Head	76 4 1·90	2	0·72	0·13	Delamere, Old[1] ·	356 16 8·02	10	9·17	1·69

[1] A correction of + 3″.05 to be applied to this bearing to reduce it to the new station.

BIN OF CAMPSIE.

3-ft. Theodolite, B.O.

From 2nd to 7th August 1818. Observers: Major-Gen. COLBY and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cairnsmuir on Deugh · · ·	3 10 54·04	4	1·67	0·22	Ben Cleugh · ·	230 10 35·16	14	5·49	0·25
Brown Carrick ·	28 19 17·89	3	3·55	1·43	Calton Hill · ·	275 53 26·14	4	3·15	0·79
Corkmulaw · ·	33 28 3·41	4	0·57	0·02	Allarmoor · ·	284 11 14·49	11	6·91	0·61
Hill of Stake · ·	62 15 44·25	5	4·83	1·10	Dunrich · · ·	309 1 7·87	4	2·73	0·47
Ben Lomond · ·	122 47 53·47	8	3·09	0·28	Hart Fell · · ·	325 11 3·98	3	3·83	1·87
Ben Lawers · ·	175 9 48·33	7	4·95	0·82	Tinto · · · ·	327 22 25·98	9	4·89	0·45

BIN OF CULLEN.

3-ft. Theodolite, B.O.

From 18th to 21st August, 1814. Observers: Major-Gen. COLBY and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Buck · · · ·	8 1 26·68	8	4·59	0·47	Balnaskerrish ·	110 34 1·23	7	4·63	0·56
Findlay Seat · ·	66 24 22·11	19	4·07	0·13	Ben Lundie · ·	117 58 6·23	8	2·06	0·12
Ben Wyvis · ·	91 34 39·35	7	1·91	0·12	Ben Cheilt · ·	157 56 32·33	5	3·39	0·56

BIN OF CULLEN—*continued.*

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Cowhythe . . .	260° 56′ 58·76″	6	5·02″	1·09	Manor Lee . .	296° 27′ 52·65″	5	6·08″	1·91
Mormonth . .	277 33 29·68	3	6·67	5·31	Dudwick . . .	297 12 19·38	4	5·00	2·06
Alva	282 3 4·08	3	5·44	3·31	Knock	327 5 18·84	12	8·45	1·07

BLACK COMB.

2-ft. Theodolite.

From 31st August to 29th October 1841. Observer: Lieut. DA COSTA, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Snowdon . . .	20° 46′ 45·34″	5	8·13″	4·41	Dent Hill . . .	159° 23′ 6·89″	11	14·48″	2·66
South Berule . .	82 42 22·61	10	6·76	0·83	Criffel	165 49 5·89	8	9·44	2·79
Snea Fell . . .	90 56 40·86	9	8·75	1·94	Sca Fell . . .	199 9 18·04	10	11·09	2·28
North Berule . .	93 24 58·62	8	9·19	2·21	Little Whernside	272 47 13·12	13	8·69	1·29
Cairnsmuir of Fleet	141 4 37·06	2	6·36	10·11	Ingleboro' . . .	279 7 41·59	11	12·41	3·05
Merrick . . .	143 38 30·78	1	–	22·10	Pendle Hill . .	302 17 49·00	1	–	22·10
Ben Cairn . .	151 3 28·87	5	8·46	3·33	Whittle Hill . .	312 54 35·48	2	2·72	1·84
Cairnsmuir on Deugh	153 14 10·25	3	3·78	2·01	Beryl	337 54 38·61	7	9·98	3·00

BLACKDOWN.

3-ft. Theodolite, B.O.

From 3rd November 1848 to 6th March 1849. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Barrow Hill . .	58° 20′ 41·98	22	8·22″	0·32	Bull Barrow . .	222° 2′ 32·87″	20	10·99″	0·55
Furland . . .	63 58 43·99	20	5·27	0·17	Horton's Gazebo .	244 56 25·96	3	3·17	1·14
Ryder's Hill . .	78 34 21·30	8	6·73	1·24	Nodes Beacon[1] .	271 24 28·53	13	2·55	0·07
Little Haldon .	79 30 37·05	12	3·25	0·16	Motteston . . .	271 39 26·84	12	5·40	0·37
High Willhays .	90 28 20·48	21	4·06	0·11	Dunnose . . .	274 2 20·86	25	5·41	0·11
Golden Cape . .	101 51 5·47	5	5·43	1·31	Coringdon . .	278 47 35·84	13	4·03	0·26
Dumpdon . . .	110 9 28·12	10	2·32	0·09	Swyre Barrow .	285 27 57·76	18	5·21	0·22
Pillesdon . . .	123 45 6·19	11	4·96	0·44	Referring-object .	316 31 58·96	235	6·81	0·01
Mintern . . .	190 37 11·53	20	3·82	0·08	Vern	329 28 39·19	18	7·78	0·46

[1] A correction of — 27″·91 to be applied to this bearing, to reduce it to the trigonometrical station.

BLACK HAMBLETON.

3-ft. Theodolite, R.S.

From 4th October to 1st December 1851. Observer: Corp. GROSE, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Great Whernside	71 55 6·20	22	6·84	0·19	Collier Law . .	138 0 7·64	5	3·23	0·48
Water Crag . .	104 51 54·16	5	2·10	0·23	Botton Head . .	223 47 26·18	11	6·42	0·63
Cross Fell . .	119 20 12·98	12	6·95	0·60	Saltergate Brow .	264 55 0·12	3	0·86	0·10

BLACKHEDDON.

3-ft. Theodolite, R.S.

From 23rd January to 2nd March 1846. Observer: Corp. WINZER, R.S.M.

Objects	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cheviot . . .	37 32 10·55	20	5·08	0·12	Lumsden . . .	149 57 58·11	19	5·05	0·14
Dunrich . . .	85 11 34·01	5	2·18	0·24	Mordington . .	154 43 54·27	13	2·19	0·08
Sayrs Law . .	116 48 33·68	23	5·16	0·14					

BLUE HILL.

3-ft. Theodolite, B.O.

From 13th to 21st July 1814. Observer: Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Caerlock . . .	61 2 12·06	15	6·53	0·31	Mormonth . . .	185 40 7·44	3	2·94	1·08
Mount Battock .	66 49 27·16	9	4·73	0·46	Dudwick . . .	188 13 33·18	11	8·02	0·92
Buck	113 16 12·42	7	4·71	0·54	Little Stirling .	204 57 13·84	4	1·84	0·28

BONIFACE DOWN.

18-in. Theodolite.

28th and 29th July 1846. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Littletown Down	25 53 51·85	8	18·10	9·81	Shanklin Down .	162 53 15·31	4	10·95	10·48
Week Down . .	76 30 33·31	8	27·62	17·97	Dunnose . . .	183 40 15·58	8	29·84	8·69
Wroxall Down .	77 59 22·75	6	9·68	3·53	Boniface S. E. .	324 45 4·14	8	14·32	6·08
Nodes Beacon[1] .	106 25 12·01	8	16·90	7·24					

[1] A correction of + 1′ 21″·60 to be applied to this bearing, to reduce it to the trigonometrical station.

BONIFACE, S.E.

18-in. Theodolite.

30th July 1846. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Littletown Down	48 3 47·08	6	12·08	6·01	Nodes Beacon[1]	106 39 15·47	6	10·87	4·24
Week Down	79 44 53·64	6	3·82	0·66	Boniface Down	144 44 54·87	6	15·16	11·14
Wroxall Down	90 52 31·47	6	29·84	32·31	High Port Cliff	329 37 51·08	6	12·78	9·82

[1] A correction of + 1′ 21″·20 to be added to this bearing, to reduce it to the trigonometrical station.

BOSTON CHURCH TOWER.

18-in. Theodolite.

From 16th July to 12th October 1842. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Easton Tower	40 13 37·58	10	2·83	2·21	Swaffham Spire[2]	307 1 15·02	2	2·83	0·47
Buckminster	66 19 22·23	13	10·56	3·04	Lynn Tower[3]	311 17 26·88	2	4·34	0·32
Lincoln Minster[1]	129 58 21·81	21	12·64	0·95	Walpole, St. Peter's	328 51 0·87	8	6·85	0·92
Docking Tower	280 54 0·97	7	12·26	6·14					

[1] A correction of + 9″·67 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of — 10″·26 ” ” ”
[3] A correction of + 0″·82 ” ” ”

BOTTON HEAD.

3-ft. Theodolite, R.S.

From 13th October to 27th November 1840. Observers: Lieuts. PIPON and COLLINSON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Black Hambleton	43 55 14·03	7	5·78	1·21	Brandon Down	135 36 54·74	6	5·67	1·48
Great Whernside	65 41 13·00	17	6·44	0·36	Wordeslow	156 9 2·57	4	6·79	3·90
Little Whernside	77 28 34·13	2	5·49	7·53	Easington[1]	221 46 55·28	7	3·10	0·45
Water Crag	93 20 35·67	6	6·64	1·30	Saltergate Brow	286 55 48·76	3	5·42	3·26
Cross Fell	110 35 30·25	3	0·93	0·10	Acklam Wold	333 31 15·73	1	—	8·79
Collier Law	125 33 41·52	7	5·51	0·73	York Minster	359 42 32·98	3	5·33	3·49

[1] A correction of — 15″·09 to be applied to this bearing, to reduce it to the trigonometrical station.

BRANDON.

12-in. Theodolite.

From 3rd to 10th June 1840. Observer: Mr. FINAGHTY.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Loop Head Lighthouse . . .	211 2 20·97	6	15·54	9·00	Baurtregaum . .	275 58 42·22	6	15·89	8·46
					Knocknadober .	348 34 48·24	5	14·64	13·06

BRANDON.

2-ft. Theodolite.

From 19th January to 12th March 1845. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Newmarket . .	36 24 43·76	8	5·79	0·53	Swaffham Spire[1] .	189 49 30·92	9	5·63	0·52
Tharfield . . .	46 18 8·16	11	6·93	0·95	Hingham Church Tower . . .	231 40 54·48	9	12·33	3·45
Cambridge Observatory Heliostat[2]	59 29 29·80	11	7·97	0·81	Bunwell Church Tower . . .	254 25 23·66	8	7·15	0·92
Ely Minster . .	88 16 26·66	16	7·54	0·66	South Lopham .	272 23 7·29	10	8·26	1·13
Peterborough Tr.	107 56 52·27	1	–	11·03	Lawshall Tower .	345 50 17·35	17	8·07	0·45
Referring-object .	126 15 30·04	62	6·99	0·10					

[1] A correction of + 11″·69 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of + 21″·92 " " "

BRANDON DOWN.

3-ft. Theodolite, R.S.

From 6th September to 31st October 1853. Observer: Corp. GROSE, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Great Whernside	17 33 21·12	5	3·03	0·43	Durham Observatory Dome . .	256 1 33·39	2	4·90	6·00
Water Crag . .	38 28 1·31	11	7·35	0·81	Easington . . .	291 3 2·83	2	0·92	0·21
Collier Law . .	95 27 35·45	10	5·53	0·86	Burleigh Moor .	296 10 39·88	8	2·77	0·18
Pontop Pike . .	155 44 58·80	12	6·45	0·55	Botton Head . .	315 7 51·95	10	4·51	0·44
Wordeslow . .	237 8 15·88	10	6·54	0·91	Merrington Church	327 59 28·56	6	7·57	2·16
Durham Observatory (Mark on)	256 0 41·40	2	1·94	0·94	Appleton Wiske .	329 44 41·76	8	2·22	0·16
					Black Hambleton	330 44 44·34	8	5·74	0·96

BRASSA.

3-ft. Theodolite, B.O.

From 25th to 29th August 1821. Observers: Major-Gen. COLBY and Capts. VETCH and DRUMMOND, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Fair Isle . . .	25 1 17·23	11	7·99	1·00	Yell	180 32 13·92	17	4·03	0·22
Foula	91 35 47·38	15	4·66	0·28	Saxavord . . .	190 8 22·41	6	3·42	0·47
Ronas	156 57 20·58	15	9·30	0·72	Fetlar	193 6 34·34	9	3·44	0·26

BRIMMOND.

3-ft. Theodolite, B.O.

17th June 1817. Observer: Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Mount Battock .	51 4 38·81	7	5·66	0·81	Over Hill . . .	220 31 35·83	9	4·14	0·34
Buck	106 34 44·57	8	6·92	1·34	Layton . . .	226 40 45·30	9	11·24	2·15
Knock	145 3 31·73	3	3·64	1·53	Tarbathy . . .	246 34 24·71	10	5·61	0·67
Mormonth . .	194 22 32·98	6	6·50	1·77	Blue Hill . . .	322 1 11·99	12	6·71	0·49
Dudwick . . .	202 49 27·81	5	5·60	1·66					

BROADWAY TOWER.

3-ft. Theodolite B.O.

From 3rd May to 9th June 1850. Observer: Corp. JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cleeve	45 15 43·29	27	7·13	0·16	Bar Beacon . .	175 13 2·31	26	8·69	0·28
May Hill . . .	70 23 40·37	23	4·71	0·14	Hampton Church	191 25 11·56	10	4·22	0·82
Cradle	85 18 34·88	31	5·15	0·11	Claverdon Tower	196 43 35·26	5	4·96	1·04
Breedon Tower .	104 17 54·89	15	6·14	0·35	Corley	200 43 23·58	6	2·20	0·18
Malvern . . .	104 45 52·28	51	6·19	0·09	Bardon Hill . .	204 20 53·08	23	4·83	0·08
Worcester Cathedral Tower .	124 54 20·17	9	5·83	0·73	Warwick Tower .	210 25 29·32	14	7·36	0·56
Brown Clee . .	134 13 57·60	10	9·21	1·49	Dunchurch Church Tower . . .	226 51 51·93	3	7·02	5·50
Walton Hill . .	158 41 58·10	20	9·67	0·29	Arbury Hill . .	242 12 45·70	20	6·76	0·22
Lord Plymouth's Monument . .	162 48 50·62	6	4·13	0·71	Epwell	257 18 2·74	47	7·46	0·08
Oversley Lodge Tr.	174 24 31·91	5	8·57	3·29	Stow on the Wold	326 27 5·08	19	5·26	0·21
					White Horse Hill	339 32 39·96	26	3·07	0·04

BROWN CARRICK.

3-ft. Theodolite, B.O.

From 16th to 25th September 1815. Observer: Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Saugh	18° 30′ 5″·59	15	6″·54	0·46	Hill of Stake . .	176° 32′ 27″·8 7	21	5″·43	0·24
Benereard . .	19 21 32·12	14	5·73	0·39	Ben Lomond . .	183 14 53·9 8	2	2·01	1·01
Knocklayd . .	75 1 43·11	3	5·34	3·86	Bin of Campsie .	207 50 35·30	2	1·84	0·84
Carn na Leagh .	81 33 23·61	14	4·50	0·22	Tinto	252 21 38·64	8	5·72	0·58
Goat Fell . . .	128 58 9·83	14	6·48	0·51	Cairnsmuir on Deugh	297 47 14·14	19	10·83	0·74

BROWN WILLY.

18-in. Theodolite.

From 3rd October to 17th November 1849. Observer: Corp. WOTHERSPOON, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Deadman . . .	19° 4′ 42″·14	12	5″·66	0·50	Efford Beacon .	186° 57′ 12″·00	14	6″·00	0·45
Hensbarrow . .	33 47 2·82	21	17·88	1·43	Paracombe . .	218 26 22·56	3	2·99	1·25
Karnbonellis . .	44 41 17·48	9	6·66	1·19	High Willhays .	255 39 12·20	18	12·57	1·51
Karnbrea Monument . . .	48 25 18·13	8	15·31	5·05	Ryder's Hill . .	280 18 1·04	10	17·32	4·21
St. Agnes Beacon	54 21 33·46	11	7·33	0·85	Kit Hill Tower .	289 49 1·57	31	15·13	0·80
Karnminnis . .	56 44 8·40	15	4·67	0·22	Butterton . . .	291 14 18·34	6	8·61	3·20
Trevose Head .	81 25 36·17	14	5·49	0·28	Eddystone Lighthouse . . .	332 8 13·53	8	19·73	7·76
Cadon Barrow .	134 27 31·56	9	19·73	8·49	Lansallos . . .	354 6 19·95	13	9·83	1·03
Lundy I. Lightho.[1]	175 35 43·65	13	9·37	1·11					

[1] A correction of + 2′ 17″·85 has to be applied to this bearing, to reduce it to the trigonometrical station.

BUNWELL CHURCH TOWER.

2-ft. Theodolite.

From 10th October to 16th November 1844. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Mickfield . . .	0° 48′ 40″·61	11	2″·49	0·20	Baconsthorpe . .	182° 46′ 49″·88	10	5″·73	0·43
South Lopham .	40 27 46·11	13	5·99	0·44	Norwich Spire .	216 41 42·28	14	5·28	0·28
Brandon . . .	74 52 7·68	10	4·61	0·31	Tofts	267 47 14·61	7	2·17	0·14
Swaffham Spire[1] .	120 33 55·91	11	5·82	0·62	Laxfield . . .	322 32 5·94	12	4·96	0·34
Hingham Tower .	134 32 42·50	10	3·62	0·26					

[1] A correction of + 17″·75 to be applied to this bearing, to reduce it to the trigonometrical station.

BURLEIGH MOOR.

3-ft. Theodolite, B.O.

July 1806. Observer: Mr. WOOLCOT.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Wordeslow . .	141 25 46·89	7	8·74	2·29	Easington[1]. . .	272 59 44·65	7	5·42	1·28

[1] The position of the present station at Easington is found, by comparison of the new and old observations at that point, to be 2·47 feet distant, and bearing 313° 24′ from the old station. A correction of — 7″·79 must therefore be applied to the reading of the old point to reduce it to the new.

BURNSWARK.

2-ft. Theodolite.

From 22nd March to 4th May 1847. Observer: Serg. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Criffel	52 39 13·18	13	6·07	0·37	Referring-object .	204 52 12·04	94	—	—
Cairnsmuir of Fleet	79 22 51·73	4	3·75	1·10	Wisp	223 5 56·99	10	4·04	0·24
Merrick . . .	94 11 32·41	22	4·75	0·12	Cross Fell . .	310 21 13·30	16	4·98	0·16
Cairnsmuir on Deugh	107 7 30·42	21	3·93	0·10	High Pike . .	341 54 38·36	4	3·92	1·38
Tinto	156 20 38·55	13	2·19	0·05	Sca Fell . . .	356 31 6·82	11	1·68	0·04
Hart Fell . . .	167 24 20·57	15	4·70	0·27					

BUTSER HILL.

3-ft. Theodolite, B.O.

From 1st December 1844 to 16th February 1845. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Dunnose . . .	21 8 51·87	22	6·27	0·23	Beacon Hill . .	114 3 33·78	10	2·69	0·16
Motteston . .	41 53 58·96	5	9·74	4·58	Inkpen	141 3 52·07	13	5·12	0·32
Nodes Beacon[1] .	49 2 14·30	4	5·04	2·03	Leith Hill Tower	242 24 51·00	12	6·33	0·58
Coringdon . . .	61 53 4·85	13	5·39	0·52	Crowborough . .	263 22 26·84	12	4·93	0·25
Wingreen . . .	91 2 16·56	18	7·08	0·23	Ditchling . . .	277 31 55·77	27	6·98	0·21
Dean Hill . . .	97 20 58·48	5	4·22	0·88	Rooks Hill . .	300 48 11·89	14	6·78	0·52
Referring-object .	108 11 14·96	136	—	—					

[1] A correction of + 20″·43 to be applied to this bearing, to reduce it to the trigonometrical station.

CAHERBARNAGH.

3-ft. Theodolite, B.O.

From 21st to 30th June 1832. Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Slieve Buoymore	35 44 58·75	13	9·30	0·96	Galtymore	240 59 39·77	3	6·20	4·78
Hungry Hill	48 5 44·97	11	9·89	1·37	Mount Hillary, North	249 6 49·40	31	10·37	0·28
Mangerton	72 17 25·34	17	9·10	0·84	Knockmealdown	255 17 9·97	1	–	7·95
Drung Hill	89 42 13·14	4	3·45	0·77	Knocknaskagh	260 17 34·31	9	10·42	1·86
Brandon	107 33 17·52	5	3·59	0·93	Mushramore	277 49 41·69	9	7·87	1·25
Baurtregaum	113 57 51·04	11	8·11	0·92	Doolieve	298 10 15·88	5	8·13	3·64
Crushylean	136 11 13·16	4	1·52	0·16	Mullaghcorrin	299 49 57·55	1	–	7·95
Taur	188 50 3·57	4	3·40	0·86	Clara Mountain	322 0 42·97	3	1·63	0·29
Knockfeerina	206 59 16·52	4	1·29	0·15	Carrigfadda	352 31 6·81	12	11·26	1·21
Keeper	217 32 40·65	3	2·29	0·60					

CALTON HILL.

3-ft. Theodolite, B.O.

From 3rd May to 17th June 1816. Observers: Major-Gen. COLBY and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Dunrich	0 8 12·22	14	8·24	0·79	East Lomond	175 49 22·40	34	8·37	0·19
Allarmoor	22 35 49·80	18	8·48	0·75	Craigowl	188 55 34·23	8	4·07	0·35
Corkmulaw	94 24 53·01	7	5·50	0·78	Inchkeith Light-house	198 40 35·87	45	9·98	0·22
Bin of Campsie	96 40 40·93	13	4·62	0·22	Largo Law	207 12 44·51	33	8·90	0·21
Ben Lomond	106 47 9·38	13	6·68	0·53	Kellie Law	217 22 46·42	14	10·75	1·15
Ben Cleugh	125 24 7·08	29	6·57	0·17					
Ben Lawers	136 2 30·17	5	12·65	6·80					

CARNETHY CAIRN.

3-ft. Theodolite, B.O.

From 8th July to 24th August 1850. Observers: Serg. DONELAN and Corp. GROSE, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Hart Fell	9 31 2·61	28	5·51	0·15	Bin of Campsie	109 37 42·87	7	3·96	0·45
Tinto	41 18 46·49	31	11·44	0·32	Ben Lomond	115 0 52·45	6	3·67	0·46
Middlefield	62 18 39·73	6	6·91	1·82	Ben Cleugh	141 5 32·65	14	9·68	0·74
Hill of Stake	89 36 48·83	5	7·02	2·35	Ben Lawers	143 22 55·84	15	7·88	0·78
Corkmulaw	107 1 12·12	16	6·94	0·44	Ben Macdui	170 1 35·21	3	6·67	4·94

CARNETHY CAIRN—*continued.*

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Glashmeal	177 5 49·83	11	7·32	0·56	Arthur's Seat	211 54 5·44	16	8·91	1·02
West Lomond	178 6 25·48	6	5·92	1·60	Kellie Law	214 5 40·37	20	5·78	0·29
East Lomond	184 8 28·01	10	4·77	0·43	North Berwick Law	236 37 25·82	6	7·75	2·03
Craigowl	191 26 42·31	11	4·29	0·43	Sayrs Law	269 13 12·11	18	9·84	0·66
Mount Battock	194 42 2·89	4	6·33	2·52	Cheviot	299 21 36·12	23	10·22	0·42
Largo Law	206 20 55·41	13	5·38	0·21	Dunrich	349 36 53·76	14	6·61	0·48
Allarmoor	207 32 59·24	9	6·98	0·99					

CARN CLONHUGH.

3-ft. Theodolite, B.O.

From 12th to 19th November 1828. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Longford Spire	33 17 7·17	3	2·40	1·64	Slieve na Callia	279 41 12·66	5	5·02	1·22
Slieve Bawn South	67 11 51·23	9	4·29	0·49	Ben of Fowre	292 20 17·12	5	4·07	1·05
Cuilcagh	171 54 3·00	6	4·89	0·88	Knockeyon	306 21 9·57	7	3·71	0·37
Carnmore	209 14 41·49	1	——	4·93	Croghan	330 22 8·06	3	5·18	3·07
Loughanleagh	257 48 24·99	2	0·51	0·06	Knockastia	352 10 54·25	4	5·27	2·24

CARN NA LEAGH.

3-ft. Theodolite, B.O.

From 1st to 9th September 1818. Observers: Major-Gen. COLBY and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Trostan	39 3 13·45	9	5·70	0·83	Ben Tuire	205 19 0·30	11	9·35	1·33
Carnaleagh (Ireland)	53 32 46·93	6	8·26	2·77	Goat Fell	226 33 45·23	5	1·65	0·17
Knocklayd	61 3 5·36	7	4·40	0·80	Pladda Lighthouse	253 3 28·61	4	5·53	1·97
Malin Head Lighthouse	94 58 17·70	4	7·00	3·18	Brown Carrick	260 40 38·75	5	2·32	0·24
Innistrahull Lighthouse	98 52 15·29	5	5·13	1·38	Cairnsmuir on Deugh	272 57 53·02	4	6·29	2·80
Oa	137 44 13·35	4	6·82	3·03	Ailsa Craig	278 42 32·93	6	5·71	1·20
Ben Tartevil	137 53 35·52	6	5·77	1·12	Benereard	296 18 5·02	10	4·19	0·35
Jura	167 55 24·86	4	7·70	4·42	Corsill Lighthouse	310 26 33·22	3	1·11	0·14
Ben More in Mull	173 19 49·86	4	5·25	2·18	Cairn Piot	316 39 50·83	7	5·27	0·93
					Copeland Lighthouse	346 28 29·77	5	3·67	0·61

CARRIGFADDA.

3-ft. Theodolite, B.O.

From 21st to 30th July 1832. Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cruckmaddee	72 52 41·06	5	4·34	1·14	Caherbarnagh	172 35 7·42	9	9·03	1·25
Mount Gabriel	74 22 40·11	8	5·75	1·07	Mushramore	190 45 26·31	4	1·58	0·16
Mount Kidd	81 54 23·13	6	4·51	0·78	Corrin	232 27 13·09	15	7·52	0·76
Cruckrowra	90 54 1·73	2	1·52	0·57	Mullaghcorrin	238 8 50·89	1	–	6·99
Hungry Hill	97 1 53·41	10	4·78	0·46	Doolieve	248 26 54·97	10	8·48	1·25
Mangerton	144 20 53·95	2	0·29	0·02	Ballyroe	263 3 20·55	4	9·45	6·36

CASTLECOO.

18-in. Theodolite.

From 11th to 23rd October 1826. Observer: Lieut. MURPHY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Kippure	3 51 34·56	2	4·27	4·55	Slieve Gullion	163 36 11·43	8	8·38	2·38
Garristown Windmill	19 16 11·47	3	9·85	11·17	Carlingford	186 9 25·18	12	15·01	1·98
Loughanleagh	108 30 51·50	4	13·12	12·25	Slieve Donard	206 53 8·17	5	13·05	8·79
Mullyash	148 41 45·57	1	–	25·25	Lambay	333 6 23·31	8	9·38	2·97

CHEVIOT.

2-ft. Theodolite.

From 16th June to 13th September 1846. Observer: Serg. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cross Fell	14 18 57·45	18	4·11	0·10	Blackheddon	217 20 55·77	22	8·82	0·38
Sea Fell	31 21 4·26	12	4·10	0·16	Referring-object	246 9 15·05	171	6·53	0·01
Wisp	67 49 9·02	16	4·28	0·25	Alnwick	285 39 56·36	18	8·29	0·41
Hart Fell	84 53 4·68	21	8·02	0·33	Rufflaw, New	319 40 48·30	18	3·71	0·08
Dunrich	99 27 31·60	17	4·59	0·15	Easington	320 6 40·56	14	4·10	0·17
Carnethy Cairn	120 17 24·45	18	4·92	0·12	Wordeslow	326 28 56·98	20	4·04	0·10
Allarmoor	123 40 25·41	10	2·93	0·15	Botton Head	329 55 39·52	15	3·47	0·09
Sayrs Law	141 22 39·87	16	4·46	0·23	Pontop Pike	340 18 31·55	16	6·11	0·23
Lumsden	175 26 34·37	12	6·01	0·58	Collier Law	352 7 34·67	17	6·23	0·24
Mordington	187 14 14·90	20	7·11	0·19					

CHINGFORD.

3-ft. Theodolite, B.O. and R.S.

From 21st July to 28th August 1823. Observers: Capts. KATER and DRUMMOND.
From 19th April to 29th May 1848. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Greenwich Obser^{y}.	0° 5′ 5″·65	26	10″·63	0·22	Hornsey Hill .	48° 53′ 28″·65	26	6″·57	0·18
Hundred Acres .	21 43 17·66	13	3·78	0·15	Berkhampstead[2] .	147 30 13·36	46	10·28	0·12
Banstead . . .	22 42 7·12	15	5·08	0·21	Wrotham . . .	330 38 38·42	52	8·18	0·12
St.Paul's Cathedral	26 14 16·40	45	9·24	0·12	Severndroog . .	347 13 40·48	104	12·32	0·04
Leith Hill (1822)[1]	26 51 17·78	18	5·54	0·19	Greenwich Obser-				
WestminsterAbbey	30 4 46·70	27	6·36	0·15	vatory Transit .	359 59 53·97	11	5·09	0·26

[1] A correction of − 1′ 19″·88 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of + 0″·29 „ „ „

CLEISHAM.

3-ft. Theodolite, B.O.

From 13th June to 27th August 1840. Observers: Lieuts. ROBINSON and HORNBY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Ben More, S. Uist	20° 22′ 12″·09	7	6″·82	1·32	Fashven . . .	238° 29′ 26″·47	4	12″·65	10·31
North Rona . .	203 49 54·14	2	0·67	0·11	Cuinnag . . .	254 10 59·52	3	6·33	4·83
Monach . . .	213 52 1·75	18	8·46	0·61	Ru Rea . . .	282 30 37·30	4	1·05	0·08
CapeWrathLight-					Scournalapich[1] .	301 34 44·23	3	5·14	3·38
house . . .	234 30 21·85	2	1·53	0·59	Mamsuil . . .	306 16 33·25	1	–	12·11
Cnoc-ghiubhais .	236 25 36·92	4	2·76	0·53	Storr	323 19 56·97	12	4·90	0·47

[1] A correction of + 33″·81 to be applied to this bearing to reduce it to the new station.

CLIFTON BEACON.

3-ft. Theodolite, B.O.

From 10th to 23rd August 1801. Observer: Mr. WOOLCOT.
From 26th April to 21st June 1842. Observer: Lieut. LUYKEN, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Referring-object .	21° 34′ 35″·89	71	7″·22	0·04	Wilton Beacon .	206° 30′ 7″·19	5	4″·56	0·88
Lord's Seat . .	73 32 15·70	6	3·98	0·76	Nunburnholme .	214 31 13·28	2	2·78	1·93
Back Tor . . .	81 50 50·99	6	5·53	1·03	Hunsley . . .	228 1 36·10	8	3·96	0·34
Holme Moss . .	101 50 38·15	6	11·70	4·77	Crowle	236 47 8·69	6	3·30	0·60
Great Whernside	147 5 56·08	2	1·58	0·62	North End Base .	248 57 57·20	9	12·66	2·89
Ledstone Beacon	169 26 45·54	2	1·43	0·51	South End Base .	269 45 20·49	6	15·48	7·89
York Minster . .	189 11 9·57	7	6·15	1·01	Gringley Beacon .	283 42 43·48	12	3·52	0·16
Acklam Wold .	203 20 26·08	4	2·96	0·65	Lincoln Minster[1] .	298 21 47·55	9	5·56	0·52

[1] A correction of — 8″·06 to be applied to this bearing, to reduce it to the trigonometrical station.

CNOC-GHIUBHAIS.

3-ft. Theodolite, B.O.

From 3rd to 18th September 1838. Observers: Lieuts. ROBINSON and PIPON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cuinnag	5 23 12·52	11	12·02	1·62	Ben Hutig	277 32 10·09	14	5·53	0·26
Suilbheinn	9 46 36·02	8	8·47	1·28	Fashven	297 18 35·49	11	13·02	3·12
Monach	72 22 28·43	8	16·06	5·84	Ben Hope	310 51 58·85	8	8·68	2·10
North Rona	141 36 47·01	3	12·68	17·95	Foinnebheinn	342 51 49·02	5	7·63	2·89
Dunnet Head	263 34 11·66	2	0·13	0·01					

COLLIER LAW.

3-ft. Theodolite, R.S.

From 22nd June to 11th August 1851. Observer: Corp. GROSE, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Water Crag	13 12 44·10	10	8·21	0·86	Durham Observatory Dome	270 33 42·27	9	5·84	1·11
Cross Fell	77 14 36·58	32	7·73	0·21	Brandon	275 12 46·83	9	8·68	1·44
Cheviot	172 15 48·84	22	9·74	0·41	Easington	286 56 11·03	22	4·51	0·11
Rathbury Crag	187 44 42·56	4	3·19	0·91	Burleigh Moor	289 51 5·29	14	7·00	0·39
Rufflaw, Old	195 16 52·13	7	7·05	1·55	Merrington Church	292 48 19·11	4	5·55	2·58
Rufflaw, New	195 19 21·02	12	7·29	0·53	Botton Head	304 49 56·87	24	7·35	0·23
Pontop Pike	230 26 16·25	5	4·67	0·98	Black Hambleton	317 24 16·61	6	6·35	1·69
Wordeslow	256 0 29·80	27	6·33	0·17					

CORINGDON.

3-ft Theodolite, B.O.

From 22nd February to 14th March 1845. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Swyre Barrow	69 50 19·45	14	5·55	0·39	Dean Hill	207 54 8·11	9	5·37	0·60
Blackdown	99 13 32·31	9	6·55	0·66	Butser Hill	241 5 44·05	15	9·77	0·67
Pillesdon	108 33 23·60	16	6·05	0·31	Nodes Beacon[1]	262 33 36·41	4	5·02	2·08
Wingreen	168 5 3·87	15	6·16	0·41	Motteston	264 56 11·06	3	1·87	0·43
Horton's Gazebo	184 56 44·56	4	3·55	0·93	Dunnose	271 4 16·31	16	5·00	0·31
Beacon Hill	196 52 41·31	9	6·44	0·80					

[1] A correction of $-58''\cdot76$ to be applied to this bearing, to reduce it to the trigonometrical station.

CORRYHABBIE.

3-ft. Theodolite, B.O.

From 6th September to 21st November 1850. Observer: Corp. JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Glashmeal	11 17 31·68	22	6·33	0·26	Referring-object	204 40 10·96	31	8·07	0·21
Ben Macdui	43 17 10·10	35	4·99	0·07	Bin of Cullen	208 20 34·63	31	5·53	0·10
Ben Nevis	61 39 59·11	20	6·29	0·28	Cowhythe	220 11 24·43	42	8·40	0·10
Mamsuil	87 15 34·51	27	7·96	0·24	Knock	223 15 15·06	38	6·81	0·13
Scournalapich	92 11 19·91	31	6·82	0·18	Alva	230 24 26·60	5	3·90	0·69
Ben Wyvis	114 46 22·01	31	5·19	0·09	Manor Lee	231 8 36·82	11	7·39	0·68
Ben Clibrig	144 26 28·89	40	5·00	0·08	Mormonth	247 8 17·88	47	6·25	0·09
Ben Lundie	145 10 30·27	20	5·98	0·28	Dudwick	261 41 42·88	25	7·19	0·32
Balnaskerrish	147 9 35·52	23	4·15	0·09	Buck	294 41 33·81	59	5·23	0·05
Scarabin	166 37 8·57	38	5·19	0·09	Caerlock	313 35 19·48	53	4·73	0·04
Findlay Seat	173 59 26·02	47	4·85	0·06	Mount Battock	327 48 37·11	59	6·26	0·05
Ben Cheilt	174 22 46·32	34	5·00	0·14					

COWHYTHE.

3-ft. Theodolite, B.O.

From 16th November 1846 to 8th February 1847. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Buck	23 6 5·17	10	8·76	1·26	Bin of Cullen	81 7 43·42	18	3·87	0·11
Knock Hill	31 57 1·24	41	9·09	0·22	Ben Lundie	113 10 18·25	14	9·02	0·68
Corryhabbie	40 38 28·86	22	8·09	0·36	Scarabin	137 44 25·66	24	14·56	0·60
Referring-object	66 27 31·99	133	4·98	0·03	Ben Cheilt	149 25 32·83	20	4·51	0·13

CRADLE.

3-ft. Theodolite, R.S.

From 1st August to 30th September 1843. Observers: Lieut. LUYKEN, R.E., Corp. COSGROVE, and Priv. STEWART, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Dunkerry	20 19 5·41	7	2·03	0·12	Cader Idris	147 31 15·31	7	5·66	1·27
Paracombe	30 57 14·80	8	2·95	0·31	Snowdon	152 51 22·89	13	4·94	0·29
Llangeinor	42 5 53·06	6	2·41	0·24	Llandinam	157 43 49·39	5	4·76	1·29
Precelly	90 19 41·91	9	3·37	0·21	Arrenig	158 42 28·89	3	4·64	2·90
New Inn	92 43 36·82	5	2·32	0·38	Radnor Forest	171 27 6·70	8	2·38	0·16
Referring-object	130 50 37·98	119	5·38	0·01	Long Mountain	181 44 18·51	5	6·23	2·05
Plynlimmon	142 1 54·68	9	5·67	0·65	Stow	190 0 66·69	7	5·63	1·04

CRADLE—*continued.*

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Longmount Pole	194 54 16·52	7	3·11	0·37	May Hill	278 18 38·14	4	0·80	0·04
Wrekin	205 47 33·04	7	1·68	0·08	White Horse Hill	290 42 9·08	10	2·01	0·07
Brown Clee	211 18 39·36	3	1·07	0·16	Trellig	308 41 49·05	2	1·93	0·93
Malvern	252 8 48·97	6	3·23	0·38	Dundry	331 33 3·59	5	5·27	2·02
Broadway Tower	264 17 55·98	4	1·37	0·12	Mendip	333 38 0·61	4	1·81	0·22
Cleeve, Old	272 13 26·80	6	2·37	0·18	Staple Hill	358 33 48·29	3	2·51	0·76

CRAIGOWL.

3-ft. Theodolite, B.O.

From 5th June to 4th July 1817. Observer: Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Calton	9 3 57·42	6	3·11	0·45	Mount Battock	200 22 31·59	9	8·27	0·99
Allarmoor	10 38 13·88	5	6·42	1·85	Caerlock	212 52 40·06	6	4·74	0·90
East Lomond	20 37 0·03	25	7·21	0·27	Broxy	227 4 10·96	4	0·09	0·00
Ben Cleugh	49 27 15·14	7	2·98	0·22	Red Head, Old	256 33 58·02	15	10·94	0·81
Ben Lawers	90 15 46·09	4	3·33	0·81	Kellie Law	336 25 9·77	14	6·71	0·59
Glashmeal	149 17 55·46	9	7·32	1·06	Largo Law	351 0 23·14	21	7·92	0·38

CRIFFEL.

3-ft. Theodolite, B.O.

From 23rd May to 8th June 1841. Observer: Lieut. HORNBY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
South Berule	37 49 46·62	1	–	13·11	Dunrich	201 41 12·50	6	6·90	1·88
Glasserton	65 18 19·66	3	5·61	3·53	Wisp Hill	227 32 10·07	6	9·05	3·28
Ben Cairn	67 9 45·68	4	2·04	0·43	Burnswark	232 22 17·80	2	0·52	0·06
Cairnsmuir of Fleet	95 9 29·16	11	8·40	0·81	Cross Fell	289 21 43·38	7	12·38	5·59
Merrick	112 43 29·77	12	6·24	0·41	High Pike	305 20 24·46	9	12·62	2·53
Goat Fell	128 9 45·25	3	3·17	1·18	Helvellyn	319 14 2·04	8	9·52	1·88
Cairnsmuir on Deugh	133 30 0·32	8	4·72	0·55	Sca Fell	333 25 22·23	9	6·35	0·92
Hart Fell	195 28 13·51	7	3·91	0·56	Black Comb	345 34 20·76	4	11·28	11·79
					Dent Hill	349 22 17·37	8	6·10	0·84

CROGHAN.

3-ft. Theodolite, B.O.

From 3rd November 1828 to 13th January 1829. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Keeper	45 16 30·10	6	12·96	5·26	Slieve Donard .	223 25 7·76	1	–	13·54
Knockastia . .	114 7 27·50	24	11·75	0·77	Lyons Hill . .	276 51 29·20	11	7·07	0·81
Slieve Bawn, South	128 38 30·64	1	–	13·54	Kippure . . .	286 7 9·66	5	7·36	2·36
Carn Clonhugh .	150 43 15·08	5	2·47	0·33	Hill of Allen . .	295 2 27·76	34	11·71	0·35
Cuilcagh . . .	159 56 18·91	3	9·05	10·56	Church Mountain	304 49 32·95	3	9·88	12·96
Knockeyon . .	177 44 48·49	21	11·33	0·65	Dunmurry . .	305 21 47·83	22	10·90	0·59
Ben of Fowre .	186 37 20·01	8	8·87	1·42	Lugnaquillia . .	307 26 58·39	6	4·48	0·79
Slieve-na-Callia .	193 52 51·52	13	7·02	0·64	Cadeen	313 3 15·93	5	4·88	1·33
Loughanleagh .	201 32 28·89	5	11·38	5·40	Mount Leinster .	337 21 56·46	2	2·59	1·67
Slieve Gullion .	212 31 31·85	2	2·99	2·23	Cullenagh . . .	358 16 10·67	10	6·28	0·70

CROSS FELL.

3-ft. Theodolite, R.S.

From 3rd October to 27th November 1841. Observer: Lieut. PIPON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Sca Fell . . .	59 42 28·14	12	7·68	0·83	Wisp	154 46 49·00	5	5·50	1·69
Helvellyn . . .	60 27 46·04	5	3·23	0·57	Dunrich . . .	155 34 44·83	1	–	8·52
High Pike . . .	90 38 39·84	3	1·85	0·42	Cheviot . . .	194 2 34·32	6	8·92	2·51
Ben Cairn . . .	102 1 22·25	2	1·84	0·84	Collier Law . .	256 50 10·58	3	3·79	1·90
Cairnsmuir of Fleet	105 2 34·70	2	0·27	0·01	Botton Head . .	289 27 25·84	9	9·72	1·69
Criffel	110 17 46·33	9	8·92	1·85	Black Hambleton	298 20 2·13	4	9·80	7·20
Merrick . . .	111 43 50·93	4	6·80	3·05	Water Crag . .	320 28 22·19	4	2·53	0·44
Cairnsmuir on Deugh . . .	119 49 31·27	2	3·47	3·01	Great Whernside	332 5 31·69	9	8·41	1·71
					Little Whernside	354 4 8·57	4	7·02	3·47
Burnswark . .	131 0 19·03	1	–	8·52	Ingleboro' . . .	354 26 22·07	4	4·32	1·31
Hart Fell . . .	143 42 25·97	5	6·89	2·00					

CROWBOROUGH.

3-ft. Theodolite, R.S.

From 19th to 22nd September 1822. Observers: Major-Gen. COLBY, Capt. KATER, and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Leith Hill of 1822[1]	110 17 19·98	23	3·58	0·08	Tolsford . . .	264 59 35·56	10	4·57	0·40
Wrotham . . .	197 22 34·93	23	3·27	0·05	Fairlight . . .	301 4 59·55	15	4·27	0·19
Stede Hill . .	239 20 55·76	16	2·82	0·09	Brightling Observatory Dome .	303 30 12·56	4	4·91	2·04
Frant Church .	239 33 36·94	22	3·67	0·05					

[1] A correction of — 59″·07 to be applied to this bearing, to reduce it to the trigonometrical station.

CROWLE.

18-in. Theodolite.

From 9th April to 2nd July 1842. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Gringley . . .	10 0 39·01	7	9·44	3·30	Hemingboro' Spire	149 59 17·29	7	8·28	1·78
Clifton Beacon .	57 6 22·58	7	7·94	1·51	Acklam Wold .	183 10 10·29	6	12·86	5·80
Back Tor . . .	69 54 15·90	1	–	11·23	Wilton Beacon .	185 31 9·35	3	5·50	4·02
Garforth Cliff .	118 13 57·76	7	6·07	1·33	Nunburnholme .	195 13 52·95	2	3·43	2·94
Ledstone Beacon	119 26 7·63	1	–	11·23	Hunsley . . .	218 5 45·90	5	7·63	2·41
Great Whernside	128 57 43·64	1	–	11·23	Lincoln Minster[1] .	335 9 31·57	4	3·82	1·14

[1] A correction of – 10″·12 to be applied to this bearing, to reduce it to the trigonometrical station.

CRUACH-NA-SLEAGH.

3-ft. Theodolite, B.O.

From 11th December 1847 to 17th January 1848. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Knocklayd . .	17 24 40·56	1	–	2·52	Ben Lomond . .	262 52 34·90	1	–	2·52
Jura	35 43 9·69	3	4·15	1·93	Ben Tuirc . .	350 37 41·46	2	1·11	0·30
Ben More in Mull	152 35 29·68	1	–	2·52					

CUILCAGH.

3-ft. Theodolite, B.O.

From 17th June to 12th September 1828. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Keeper . . .	10 40 26·23	5	2·50	0·44	Slieve Gallion .	228 37 19·00	10	5·88	0·42
Slieve Bawn South	17 35 59·45	12	9·46	1·31	Shantavny . .	234 36 5·48	28	8·97	0·35
Mullaghanoe . .	64 16 28·61	10	6·20	0·74	Carnmore . . .	257 35 33·56	38	12·33	0·36
The Reek . . .	68 43 5·24	7	4·50	0·65	Vicar's Carn . .	261 42 41·93	10	4·89	0·54
Nephin . . .	79 2 26·60	13	6·45	0·68	Slieve Donard .	270 18 14·77	14	10·27	0·82
Knockalongy . .	89 41 35·63	10	5·57	0·60	Mullyash . . .	271 49 25·44	7	6·68	1·09
Tawnaghmore .	95 49 57·93	1	–	10·85	Slieve Gullion .	275 1 35·50	14	7·24	0·55
Knocknarea . .	97 40 27·79	24	8·19	0·38	Carlingford . .	278 50 1·34	12	4·56	0·49
Slieve League .	131 8 25·03	32	11·95	0·38	Loughanleagh .	298 18 33·28	30	19·33	0·84
Breesy	157 48 9·92	21	7·98	0·32	Slieve-na-Callia .	317 37 17·58	18	8·78	0·62
Barnesmore Connell	170 22 28·50	4	2·75	0·55	Kippure . . .	318 45 13·93	2	8·73	19·05
Slieve Snaght .	195 19 23·89	15	10·01	1·32	Croghan . . .	339 30 33·70	6	8·98	2·68
Bessy Bell . .	206 26 33·05	30	9·73	0·35	Carn Clonhugh .	351 49 23·60	13	8·10	0·79
Sawel	215 39 52·24	32	13·98	0·49	Knockastia . .	351 57 33·04	3	3·39	1·56
Mullaghcarn . .	216 18 24·75	27	10·60	0·35					

CUNDTHAM.

2-ft. Theodolite.

July 8th and 9th 1829. Observers: Capt. DAWSON and Lieut. MURPHY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Slieve Snaght .	95 58 46·73	11	11·02	1·71	North End of Base	287 41 4·17	16	3·15	0·09
Mount Sandy .	271 3 29·92	16	6·57	0·36	South End of Base	334 20 52·79	16	6·23	0·29
Knocklayd . .	272 9 28·41	3	3·90	1·69	Sawel Mountain .	352 11 23·23	4	4·41	1·70

CYRN-Y-BRAIN.

18-in. Theodolite.

From 25th October 1852 to 24th January 1853. Observers: Serg. STEEL and Corp. WOTHERSPOON, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Arrenig . . .	70 52 37·26	25	15·94	1·38	Axedge . . .	254 44 24·51	8	11·32	3·89
Snowdon . . .	93 32 7·78	17	25·20	3·90	Mowcopt . . .	262 4 49·51	14	12·98	2·05
Llanelian . . .	123 13 54·09	17	22·25	3·82	Longmount Pole	339 0 47·16	16	26·71	8·24
Delamere . . .	237 49 28·74	20	29·61	5·86					

DANBURY SPIRE.

18-in. Theodolite.

From 17th July to 15th September 1844. Observers: Corps. STEEL and BEATON, and Privates FRANCIS and VALLY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Gad's Hill Obelisk[1]	12 46 14·15	2	1·30	0·42	Lawshall . . .	191 50 44·13	5	18·14	17·39
Wrotham . . .	24 24 27·90	10	22·38	8·14	Naughton . . .	211 3 22·62	5	6·88	2·64
Langdon Hill .	30 9 3·88	14	17·41	2·61	Stoke Tower . .	215 35 51·15	19	30·90	4·13
Severndroog . .	52 13 39·31	10	6·42	0·92	Walton Tower .	251 14 59·53	13	6·80	0·69
Epping Cupola .	87 42 45·97	14	11·62	1·79	St. Peter's Ch. Tr.[2]	303 22 17·85	13	23·57	6·49
Hatfield Broadoak	118 19 30·56	4	2·53	0·45	Norwood . . .	331 45 61·21	19	13·42	1·64
Thaxted Spire .	148 53 5·33	19	9·53	0·78	Frittenfield . .	342 13 16·97	12	8·94	0 ·95

[1] A correction of + 5′ 28″·57 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of + 2″·59 „ „ „

DEADMAN.

3-ft. Theodolite, B.O.

From 27th January to 5th March 1846. Observer: Serg. **Donelan**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Goonhilly Down	54° 18′ 17″·13	18	7″·18	0·33	High Wilhays	227° 7′ 11″·79	25	9″·82	0·39
Karnbonellis	82 3 27·37	14	9·26	0·90	Kit Hill Tower	227 17 32·14	8	6·50	1·02
Karnbrea Monument.	90 5 1·09	11	3·17	0·18	Lansallos	232 15 20·90	10	7·68	0·88
St. Agnes	107 54 51·68	14	8·88	1·07	Ryder's Hill	243 36 48·61	25	7·17	0·17
Hensbarrow	176 3 3·20	18	9·11	0·62	Butterton	251 28 13·69	6	3·94	0·55
Brown Willy	198 55 44·82	15	6·66	0·31	Eddystone Lighthouse	276 30 47·40	14	6·83	0·49

DEAN HILL.

3-ft. Theodolite, R.S.

From 14th January to 17th March 1850. Observer: Serg. **Donelan**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Coringdon Hill	28° 10′ 12″·81	12	4″·85	0·36	Beacon Hill	164° 8′ 32″·26	32	5″·29	0·09
Swyre Barrow	33 31 58·37	9	5·09	0·63	Old Lodge	181 54 26·67	9	5·21	0·45
Horton's Gazebo	48 38 39·34	6	7·08	1·86	Inkpen	200 5 37·03	21	6·48	0·19
Wingreen	81 12 15·66	15	5·98	0·48	Butser Hill	276 49 40·99	22	6·20	0·21
Old Sarum Castle	123 13 41·97	17	7·25	0·48	Ordnance Map Office	306 27 43·15	9	5·85	0·54
Four Mile Stone	124 39 30·90	15	5·33	0·31	Dunnose	324 54 39·78	19	10·33	0·50
Westbury Down	126 52 18·39	23	6·51	0·20	Motteston	338 43 28·24	9	4·54	0·54
Queen's Manor	132 8 26·62	25	6·02	0·24	Nodes Beacon[1]	348 59 17·13	3	0·64	0·04
Stoke Hill	133 17 43·98	7	4·15	0·76					
Milk Hill	160 18 49·27	8	5·45	0·60					

[1] A correction of – 23″·53 to be applied to this bearing, to reduce it to the trigonometrical station.

DEERNESS.

3-ft. Theodolite, B.O.

From 5th to 7th November 1821. Observers: Capts. **Vetch** and **Drummond**, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
South Ronaldshay	30° 34′ 14″·33	9	2″·66	0·11	Fitty Hill	158° 59′ 5″·67	5	2″·53	0·34
Wart Hill Hoy	80 46 20·50	9	4·50	0·37	Stronsay	216 39 42·15	12	3·25	0·15

DELAMERE.

2-ft. Theodolite, B.O.

From 24th April to 10th June 1842. Observer: Lieut. DA COSTA, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Long Mountain	23 1 51·44	1	–	6·78	Lord's Seat	255 48 10·51	5	2·16	0·27
Cyrn-y-Brain	58 13 25·51	8	3·40	0·32	Mowcopt	290 26 12·34	7	9·07	2·23
Garreg[1]	101 20 56·27	4	4·62	1·73	Mowcopt Tower	290 47 46·50	2	5·17	6·68
Heswell	114 13 40·96	8	8·77	1·75	Nantwich Tower	327 15 38·40	4	7·01	3·33
Billinge	176 17 23·65	6	10·53	3·71	Ashley Heath	328 34 36·67	6	5·93	1·33
Rivington	193 53 47·61	5	3·25	0·53	Wrekin	351 36 15·06	6	2·66	0·25
Bellefield	205 19 16·17	5	4·18	0·88	Hawkestone Obelisk	354 23 0·43	1	–	6·78
Whittle	209 12 18·18	4	0·97	0·07	Brown Clee	355 58 39·69	2	1·64	0·67
Holme Moss	236 13 26·23	3	1·52	0·25					
Kinder Scout	251 27 37·85	7	2·16	0·12					

[1] The bearing of this point is inferred from the old observations at Delamere, and corrected for the difference of stations. *See* Description of Delamere.

DOOLIEVE.

3-ft. Theodolite, B.O.

From 31st May to 14th June 1832. Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ballyroe	31 17 19·93	7	6·45	1·32	Rock Hill	156 59 5·39	4	1·76	0·27
Carrigfadda	68 56 49·46	10	3·43	0·19	Knocknaskagh	182 45 25·82	13	5·75	0·40
Corrin	74 12 22·60	10	5·42	0·67	Galtymore	196 31 10·22	4	4·51	1·56
Clara Mountain	98 9 20·14	11	6·02	0·56	Knockmealdown	217 10 35·67	4	1·92	0·27
Mangerton	106 16 31·80	2	0·18	0·00	Knockanaffrin	226 55 9·65	3	14·85	24·51
Mullaghcorrin	111 31 26·20	8	3·59	0·27	Bunnaloo	229 41 37·18	6	2·00	0·17
Caherbarnagh	118 44 15·15	6	8·76	2·72	Slieve Grian	240 33 18·08	5	2·04	0·21
Mushramore	125 4 40·02	7	5·53	0·80					

DITCHLING.

3-ft. Theodolite, B.O.

From 16th October to 23rd November 1844. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Recip.
	° ′ ″		″			° ′ ″		″	
Dunnose	68 9 23·96	17	6·35	0·28	Brightling Observatory Dome	258 42 6·09	8	5·37	0·68
Butser Hill	98 12 15·73	11	6·20	0·43	Fairlight	272 44 54·21	17	7·13	0·47
Leith Hill Tower	148 55 18·87	17	7·19	0·32	Referring-object	274 23 17·16	96	5·55	0·03
Wrotham	210 36 22·82	12	9·77	0·89	Beachy Head	305 10 19·43	11	3·56	0·30
Crowborough	226 58 17·92	9	8·39	1·34					
Frant Ch. Tower	230 19 50·67	4	3·62	1·08					

DIVIS.

3-ft. Theodolite, B.O.

From 24th July to 11th November 1825. Observers: Major-Gen. COLBY, Capt. ORD, and Lieuts. HENDERSON, PORTLOCK, DRUMMOND, and MURPHY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Carlingford	11 36 46·69	17	8·12	0·53	Benercard	233 52 0·53	14	7·20	0·51
Slieve Gullion	26 30 41·09	20	8·68	0·45	Merrick	238 47 55·20	5	5·76	2·01
Vicar's Carn	47 21 16·71	14	4·71	0·27	Cairn Piot	245 23 13·03	12	15·03	2·43
Shantavny	80 11 33·39	4	2·65	0·44	Cairnsmuir of Fleet	248 41 39·27	6	3·72	0·48
Mulloughcarn	95 41 53·07	9	2·22	0·15	Mull of Galloway	267 28 50·03	15	7·63	0·70
Slieve Gallion	105 56 46·91	53	12·84	0·17	Scrabo Hill	279 57 0·35	13	11·58	1·52
Sawel	109 50 24·48	15	7·74	0·54	North Berule	288 4 41·05	7	7·44	1·70
Slieve Snaght	128 11 45·91	18	6·85	0·24	Snea Fell	290 20 52·19	9	10·00	1·72
Benyevenagh	134 30 7·02	5	6·27	1·99	South Berule	299 50 21·52	6	6·32	1·38
Knocklayd	166 24 47·61	23	11·06	0·50	Calf of Man	308 0 51·84	3	2·06	0·48
Trostan	169 44 31·59	6	4·80	0·95	Slieve Donard	352 27 5·77	24	14·67	0·73
Goat Fell	204 41 46·71	1	–	14·24	Slieve Croob	354 27 44·78	8	12·77	4·21

DOCKING CHURCH TOWER.

2-ft. Theodolite.

From 31st May to 23rd July 1843. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Referring-object	40 47 20·10	35	–	–	Boston Tower[1]	101 25 25·87	6	3·30	0·40
Lynn Old Tower	42 14 21·87	7	2·95	0·22	Baconsthorpe	272 16 3·47	6	4·10	0·65
Walpole St. Peter's	54 43 28·17	5	2·28	0·27					

[1] A correction of – 7″·86 to be applied to this bearing, to reduce it to the trigonometrical station.

DRUNG POINT.

2-ft. Theodolite.

From 11th to 14th July 1829. Observers: Capt. DAWSON and Lieut. MURPHY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ′		″	
Mount Sandy	256 53 42·00	16	5·28	0·25	South End of Base	323 31 41·39	15	5·02	0·31
North End of Base	270 39 53·11	16	5·18	0·29					

DUBLIN OBSERVATORY.

3-ft. Theodolite, B.O.

From 9th February to 30th June 1830. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Lyons' Hill	50 39 55·52	11	10·42	1·25	Dalkey Obelisk	311 47 53·03	2	3·63	3·29
Maynooth Obelisk	82 22 32·13	4	3·42	0·80	Brayhead Staff	323 43 41·66	5	5·17	1·25
Knockbrack	194 25 16·54	5	5·63	1·83	Fairy Hill	339 15 17·41	5	4·82	1·45
Howth	274 51 53·68	12	5·62	0·39	Kippure	358 55 17·12	8	5·63	0·71
Poolbeg Lightho.	291 54 37·49	3	0·44	0·04					

DUDWICK.

3-ft. Theodolite, B.O.

From 10th to 12th June 1817. Observer: Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Tarbathy	4 44 10·26	7	4·89	0·86	Corryhabbie	82 40 10·12	4	3·81	1·42
Blue Hill	8 18 9·13	12	7·77	0·59	Knock	111 25 43·69	5	2·03	0·27
Over Hill	12 18 58·36	8	5·28	0·70	Bin of Cullen	117 54 31·29	5	5·90	1·47
Brimmond	22 59 38·95	6	3·70	0·45	Manor Lee	118 14 16·27	4	2·40	0·41
Caerlock	29 26 26·74	8	4·71	0·64	Mormonth	180 41 45·33	7	4·37	0·61
Mount Battock	38 47 11·26	4	4·31	1·47	Little Stirling	256 42 38·12	7	3·80	0·41
Buck	74 40 1·24	9	10·38	1·89	Layton	357 52 36·86	8	5·87	1·07

DUNKERY.

3-ft. Theodolite, R.S.

From 11th October to 4th December 1844. Observer: Corp. STEWART, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
High Wilhays	29 29 12·14	10	6·66	0·76	Ogmoor	177 57 0·96	12	6·14	0·49
Kit Hill	35 5 2·52	3	1·40	0·28	Llangeinor	181 3 25·39	9	7·30	0·81
Brown Willy	48 43 18·23	7	4·23	0·62	Cradle	199 57 12·28	13	8·53	0·72
Cadon Barrow	54 34 50·26	4	3·01	0·73	Dundry	247 58 11·47	5	3·92	0·73
Referring-object	84 41 9·98	78	5·78	0·04	Mendip	264 43 37·42	12	4·49	0·29
Paracombe	94 23 24·27	11	8·06	0·77	Staple Hill	304 16 18·87	6	5·51	1·31
Precelly	137 4 47·00	4	2·58	0·50	Pillesdon	306 31 49·10	18	7·82	0·47
Cefn Bryn	140 31 7·90	8	6·97	1·34	Dumpdon	321 37 50·84	7	6·61	1·91
Llannon	149 14 10·20	1	–	6·32	Little Haldon	356 31 27·90	3	1·56	0·29
Margam	170 4 0·35	3	1·98	0·45					

DUNMURRY.

3-ft. Theodolite, B.O.

From 4th to 6th October 1846. Observer: Capt. **Portlock**, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cullenagh	39 1 48·78	2	3·03	2·29	Maynooth Obelisk	232 24 6·93	1	–	5·01
Cushina	88 19 52·22	3	1·56	0·27	Hill of Allen	232 59 54·91	4	3·57	0·80
Ballyduff	105 47 14·60	1	–	5·01	Lyons' Hill	248 42 7·53	4	5·00	1·71
Knockastia	120 17 39·40	1	–	5·01	Kippure	272 57 26·21	1	–	5·01
Croghan	125 38 20·96	5	3·21	0·49	Slieveroe	277 33 23·54	4	2·46	0·51
Hill of Carbury	171 55 36·56	3	6·94	6·08	Lugnaquillia	309 10 59·41	1	–	5·01

DUNNET HEAD.

3-ft. Theodolite, B.O.

From 5th to 18th November 1838. Observers: Lieut.-Col. **Robe** and Lieuts. **Robinson** and **Pipon**, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Cheilt	0 37 52·94	8	2·41	0·11	Fashven	83 0 19·40	2	2·77	1·91
Scarabin	14 29 35·80	3	3·16	1·38	Wart Hill Hoy	184 0 0·45	5	3·64	0·63
Ben Clibrig	51 54 43·99	6	2·80	0·37	Deerness	228 29 3·01	1	–	3·21
Ben Hutig	79 16 48·32	5	2·91	0·39	South Ronaldshay	242 55 15·92	5	3·67	0·64

DUNNOSE.

3-ft., R.S., and 18-in. Theodolites.

From 6th to 23rd July 1844. Observers: Corps. **Stewart** and **Cosgrove**, R.S.M.;
4th and 5th August 1846. Observer: Corp. **Steel**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Boniface Down	3 40 12·69	9	12·60	3·38	Dean Hill	145 15 29·58	17	4·89	0·15
Wroxall Down	20 31 49·19	9	20·34	7·11	Beacon Hill	149 50 52·11	6	4·26	1·17
Week Down	48 50 33·30	2	0·30	0·02	Ordnance Map Office	156 21 32·09	20	6·55	0·31
Swyre Barrow	89 5 44·64	9	7·88	0·98	Inkpen	167 12 27·96	19	5·56	0·26
Shanklin Down	90 11 43·37	9	15·52	6·11	Butser Hill	200 58 27·85	20	6·68	0·30
Coringdon	91 41 3·56	12	6·47	0·49	Rooks' Hill	225 42 56·53	6	3·67	0·48
Blackdown	95 5 6·88	11	4·61	0·33	Ditchling	247 18 44·02	13	6·14	0·39
Nodes Beacon[1]	102 38 58·26	2	2·16	1·16	Beachy Head	261 55 24·63	19	7·91	0·53
Motteston	106 24 44·53	6	5·27	0·85	Referring-object	339 12 19·99	101	7·62	0·03
Horton's Gazebo	116 57 5·54	2	2·04	1·04					
Wingreen	122 50 5·64	13	5·37	0·37					

[1] A correction of +1′ 22″·57 to be applied to this bearing, to reduce it to the trigonometrical station.

DUNRICH.

3-ft. Theodolite, R.S.

From 7th September to 3rd November 1850. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Criffel	22 2 43·28	20	7·87	0·40	East Lomond .	178 16 58·45	26	8·41	0·34
Hart Fell . . .	36 53 25·12	12	10·42	1·20	Calton Hill[1] . .	180 8 13·41	3	2·67	0·87
Tinto	94 20 55·61	11	5·79	0·54	Arthurs Seat . .	182 0 55·67	5	8·68	4·52
Hill of Stake . .	106 47 32·53	4	1·82	0·27	Sayrs Law . .	226 23 12·69	24	13·54	0·56
Dungoyne . .	126 22 53·01	3	1·85	0·42	Referring-object .	242 28 42·12	45	–	1·64
Ben Lomond . .	127 48 46·80	17	5·15	0·19	Mordington . .	249 0 32·50	1	–	8·19
Bin of Campsie .	129 48 7·44	7	6·21	1·15	Blackheddon . .	264 8 52·39	15	5·29	0·22
Ben Cleugh . .	152 5 27·10	34	10·10	0·27	Cheviot . . .	278 36 11·46	13	11·38	1·45
Carnethy Cairn .	169 41 15·19	16	6·57	0·37	Cross Fell . .	335 0 4·58	20	5·21	0·16
Allarmoor . . .	174 36 53·36	19	3·69	0·10	Wisp	336 38 43·84	9	5·02	0·57

[1] A correction of – 4″·64 to be applied to this bearing, to reduce it to the trigonometrical station.

DUNSTABLE.

3-ft. Theodolite, B.O.

From 15th April to 29th June 1843. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Scutchamfly . .	59 14 19·03	8	4·06	0·41	Keysoe Spire . .	189 44 10·41	9	4·60	0·50
White Horse Hill	66 7 52·12	10	3·64	0·15	Royston . . .	239 26 9·78	7	4·09	0·63
Brill	84 46 8·25	5	6·98	2·33	Tharfield . . .	243 29 49·83	9	5·33	0·63
Epwell, Old . .	109 54 3·20	8	3·25	0·24	Epping Cupola .	291 4 35·15	4	2·08	0·35
Arbury Hill . .	131 9 52·31	8	5·56	0·82	Berkhampstead .	292 43 21·72	8	8·45	1·35
Hanslope Church					Wrotham . . .	316 37 14·86	7	5·91	0·96
Spire . . .	144 14 43·59	5	5·47	1·71	Leith Hill Tower	351 25 21·66	11	6·10	0·55
Naseby Church Tr.	152 33 59·26	7	6·34	0·96	Leith Hill (1822)	351 25 56·30	10	5·60	0·53

EASINGTON.

2-ft. Theodolite.

From 25th February to 12th April 1846. Observer: Serg. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Loosehoe . . .	15 45 15·47	7	5·62	1·15	Pontop Pike . .	120 0 22·67	6	2·63	0·31
Botton Head . .	41 58 40·39	12	5·50	0·30	Referring-object .	128 38 55·03	75	4·92	0·02
Water Crag . .	80 40 59·20	2	2·72	1·84	Wordeslow . .	130 16 57·13	18	1·30	0·01
Burleigh Moor .	93 9 30·36	16	5·57	0·37	Cheviot . . .	141 10 55·97	11	2·80	0·14
Collier Law . .	107 52 2·37	6	1·70	0·14	Saltergate Brow .	338 32 51·35	3	3·88	1·76

EAST LOMOND.

3-ft. Theodolite, B.O.

From 31st May to 14th June 1818. Observers: Major-Gen. COLBY and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Allarmoor	1 25 43·16	12	5·07	0·29	Caerlock	207 58 15·47	1	–	8·04
Tinto	21 4 16·75	10	5·83	0·71	Red Head, Old	226 58 15·57	3	2·62	0·86
Ben Cleugh	79 43 17·45	16	7·15	0·53	Kellie Law	268 26 28·86	7	5·71	1·44
Ben Lomond	86 48 9·32	5	3·91	0·94	Largo Law	272 48 1·55	13	10·97	1·28
Ben Lawers	119 2 37·87	6	9·13	3·25	Lumsden	300 6 32·98	4	5·02	1·90
Glashmeal	172 43 23·66	9	6·95	0·87	Sayrs Law	321 48 56·96	10	7·28	0·74
Mount Battock	200 18 26·17	4	4·89	1·52	Calton	355 47 29·10	13	3·89	0·21
Craigowl	200 26 40·18	17	8·48	0·55	Dunrich	358 15 11·87	5	4·36	1·31

EASTON CHURCH TOWER.

18-in. Theodolite.

From 12th July to 22nd August 1843. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Naseby Tower	51 38 26·02	8	12·07	3·84	St. Peter's Church, Walpole	257 16 40·54	6	9·09	3·00
Tilton	93 56 59·86	8	8·49	1·51	Ely Minster	295 58 12·89	4	5·48	2·06
Buckminster Spire	145 28 58·47	11	9·99	2·27	Keysoe Spire	352 42 52·02	9	4·59	0·71
Boston Tower[1]	219 50 24·05	10	9·03	1·60					

[1] A correction of + 5″·31 to be applied to this bearing, to reduce it to the trigonometrical station.

ELMS HILL.

3-ft. Theodolite, R.S.

From 18th March to 1st May 1848. Observers: Corp. WINZER and Private BATE, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben More in Mull	0 54 10·27	27	10·47	0·30	Ben Ival, North Uist	128 10 54·89	2	3·12	2·43
Ben More, South Uist	108 21 55·30	28	6·30	0·21	Ben Nevis	292 58 35·71	31	11·57	0·37

ELY MINSTER.

2-ft. Theodolite.

From 23rd July to 29th August 1845. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Tharfield . . .	25° 44′ 5″·10	15	6″·44	0·32	Walpole, St. Peter's	175° 48′ 2″·86	11	5″·80	0·36
Cambridge Observatory Dome .	29 11 5·49	7	3·43	0·31	Grey Friars' Tr., Lynn . . .	193 12 15·24	11	4·99	0·34
Keysoe Spire . .	70 55 58·69	15	3·60	0·17	Swaffham Spire[1] .	226 5 7·33	12	2·48	0·07
Easton Tower .	116 35 30·13	25	5·02	0·12	Brandon . . .	267 59 24·18	21	6·73	0·18
					Newmarket . .	325 1 8·10	7	6·89	1·72
Peterboro' Cathedral	119 45 33·18	6	3·52	0·51	Balsham Tower .	352 39 30·66	16	8·37	0·42

[1] A correction of — 1″·53 to be applied to this bearing, to reduce it to the trigonometrical station.

EPPING POORHOUSE CUPOLA.

2-ft. Theodolite.

From 15th July to 18th August 1844. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Severndroog Castle	9° 22′ 54″·73	8	6″·16	0·92	Dunstable . . .	111° 35′ 39″·86	12	2″·27	0·09
					Tharfield . . .	162 43 60·14	15	5·80	0·39
Greenwich Observatory Transit .	18 55 43·20	7	2·70	0·22	Thaxted Spire .	208 25 13·79	19	11·31	0·85
Leith Hill Tower	30 30 33·72	13	4·34	0·31	Danbury Spire .	267 22 1·54	14	12·03	1·09
St. Paul's, London	35 55 6·22	7	6·91	1·28	Hollingbourne .	321 55 58·07	9	3·49	0·20
Westminster Abbey	37 27 26·30	1	–	7·16	Gads Hill Obelisk[2]	323 41 36·34	2	6·79	11·52
Highgate Spire .	51 33 38·11	8	3·75	0·39	Referring-object .	342 30 4·97	100	–	–
Harrow Spire .	65 43 10·32	10	6·36	0·55	Wrotham . . .	345 19 50·54	15	7·35	0·46
Berkhampstead[1] .	108 47 45·54	8	7·02	1·14					

[1] A correction of — 1″·88 to be applied to this bearing, to reduce it to the trigonometrical station.

[2] A correction of — 2′ 35″·55 " " "

FAIR ISLE.

3-ft. Theodolite, B.O.

From 17th May to 3rd July 1821. Observers: Major-Gen. COLBY and Capts. VETCH and DRUMMOND, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Stronsay . . .	46° 16′ 36″·41	7	2″·22	0·12	Foula	158° 46′ 53″·90	13	12″·55	1·23
Start Lighthouse .	55 3 23·42	13	4·64	0·28	Ronas	185 19 43·66	4	2·10	0·29
					Brassa	204 33 28·32	13	7·02	0·49
North Ronaldshay Lighthouse . .	67 2 55·78	9	7·89	1·06	Noss	207 44 50·54	10	3·86	0·44
Fitty Hill . . .	70 12 21·71	9	5·79	0·86					

FAIRLIGHT.[1]

3-ft. Theodolite, B.O.

From 5th September to 6th October 1844, and from 23rd to 29th August 1822. Observers. Serg. DONELAN and Priv. VALLE, R.S.M. Capt. KATER and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Beachy Head . .	59 30 38·86	14	5·16	0·27	Frittenfield . .	202 26 8·98	20	6·53	0·30
Ditchling . . .	93 18 32·69	16	100·88	0·82	Tolsford . . .	231 57 5·49	49	6·88	0·08
Brightling Observatory Dome .	119 5 43·13	19	6·67	0·25	Paddlesworth . .	234 0 14·67	16	4·60	0·23
Crowborough . .	121 26 34·51	36	8·15	0·18	Folkstone . . .	238 47 20·84	53	8·27	0·11
Frant Church . .	135 10 51·47	19	7·76	0·45	Lydd Church . .	247 38 28·55	6	4·40	0·73
Wrotham . . .	154 32 66·38	40	5·04	0·06	Blancnez . . .	265 36 39·21	8	6·51	0·85
Goudhurst Church	157 13 62·20	12	11·62	2·51	Referring-object .	276 5 8·93	105	–	–
Hollingbourne .	184 6 13·85	13	3·46	0·23	Napoleon's Monument	281 45 15·87	7	5·96	0·95
Stede Hill . . .	186 29 10·79	38	10·21	0·29	Montlambert . .	283 16 4·38	6	5·10	0·91
Tenterden Church	192 37 18·29	7	5·50	0·77					

[1] General Roy's station of 1787 bears 308° 4′ 0″, and is distant 87·73 feet from this station.

FASHVEN.

3-ft. Theodolite, B.O.

From 25th September to 19th October 1839. Observers: Lieut.-Col. ROBE and Lieuts. ROBINSON and PIPON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cuinnag	12 48 33·30	14	5·15	0·31	Wart Hill Hoy .	246 40 30·42	7	12·78	4·23
Suilbheinn . . .	15 38 33·34	6	7·03	2·39	Dunnet Head .	261 42 1·64	3	8·58	9·29
Cleisham	60 6 53·40	2	1·02	0·26	Ben Hutig . .	272 54 56·79	13	8·98	0·69
Monach	75 2 8·95	4	6·45	3·34	Ben-Hope . .	314 1 23·19	7	5·32	0·92
Cnocghiubhais .	117 22 59·36	5	12·83	10·20	Ben Clibrig . .	321 36 40·47	16	9·83	1·13
North Rona . .	140 6 24·27	1	–	21·36	Foinnebheinn .	356 25 56·31	5	8·31	3·87

FEAGHMAAN.

12-in. Theodolite.

From 18th to 21st August 1832. Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″					° ′ ″			
Bray Head Watch Tower . . .	52 47 6·23	4	Repeated Angles.	60·12	Brandon . . .	190 14 40·67	3	Repeated Angles.	4·67
Bray Head . .	55 33 48·73	4		8·94	Baurtregaum . .	228 2 14·02	2½		73·16
Mount Eagle . .	164 51 24·20	3		22·49	Knocknadober .	236 16 53·63	3		243·50
					Knocknagante .	276 21 26·22	4½		2·05

FETLAR.

3 ft. Theodolite, B.O.

From 20th to 22nd August 1821. Observers: Major-Gen. **Colby** and Capts. **Vetch** and **Drummond**, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip of Weight.
	° ′ ″		″			° ′ ″		″	
Noss Head . .	8 17 39·27	6	3·88	0·57	Vallafield . . .	166 9 45·84	1	–	3·04
Brassa	13 18 41·21	14	5·93	0·39	Saxavord . . .	183 7 30·85	10	3·47	0·30
Yell	56 18 50·78	21	3·46	0·08	Balta	196 11 26·00	10	4·33	0·27
Ronas	73 25 42·34	13	3·72	0·16					

FINDLAY SEAT.

3-ft. Theodolite, B.O.

From 24th to 27th August 1814. Observers: Major-Gen. **Colby** and Mr. **Gardner**.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight
	° ′ ″		″			° ′ ″		″	
Ben Macdui . .	24 38 3·54	3	0·37	0·01	Ben Chielt . .	174 27 40·15	5	2·15	0·24
Ben Wyvis . .	98 33 33·31	4	5·46	2·21	Bin of Cullen . .	246 5 34·14	18	5·43	0·17
Balnaskerrish . .	132 0 19·08	9	9·27	1·34	Knock	268 25 52·94	17	5·57	0·35
Ben Lundie . .	134 10 42·64	9	4·72	0·52					

FITTY HILL.

3-ft. Theodolite, B.O.

From 31st August to 21st September 1821. Observers: Major-Gen. **Colby** and Capts. **Vetch** and **Drummond**, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Wart Hill Hoy .	24 24 6·46	15	4·52	0·20	North Ronaldshay Lighthouse .	252 51 35·60	7	7·09	1·73
Ben Hutig . .	47 22 36·28	5	0·81	0·05	Start Lighthouse	271 17 27·27	7	5·13	0·70
Foula	207 49 40·42	12	5·28	0·33	Stronsay . . .	309 9 19·18	37	6·04	0·08
Fair Isle . .	249 1 34·28	14	4·60	0·28	Deerness . . .	338 46 8·75	13	4·56	0·30

FOLKSTONE.[1]

3-ft. Theodolite, R.S.

From 31st August to 27th October 1822. Observers: Major-Gen. **Colby**, Capt. **Kater**. Mr. **Gardner**, and Mons. **Arago**.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Dungeness Lighthouse . . .	37 59 31·30	11	6·28	0·55	South Foreland Lighthouse . .	248 1 6·29	3	5·89	4·38
Fairlight . . .	59 14 17·42	17	4·75	0·22	Notre Dame, Calais	288 4 42·67	4	3·37	0·80
Tolsford . . .	95 32 13·80	21	4·49	0·11	Blancnez . . .	297 26 4·50	24	6·35	0·24
Lydd	231 45 23·07	3	5·37	3·32	Montlambert . .	322 27 50·98	24	5·53	0·13
Dover Castle . .	246 48 13·93	14	5·23	0·29					

[1] General Roy's station of 1795 is distant 3 feet from Captain Kater's of 1822, and bears from it 43° 55′ south-west.

FORTH.

3-ft. Theodolite, B.O.

From 16th October to 2nd November 1829. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Slieve Grian	69 22 53·07	3	1·01	0·11	Vinegar Hill	181 40 26·47	6	6·42	1·45
Knockanaffrin	87 55 7·13	9	8·19	1·37	Lugnaquillia	185 10 57·20	3	2·53	0·71
Slievecoiltia	95 56 53·83	14	10·10	0·93	Slieve Buy	187 26 13·24	17	5·29	0·32
Slievenaman	100 43 49·88	7	4·78	0·81	Ballycreen	191 7 5·46	5	14·24	10·07
Carrickbyrne	112 1 51·77	19	7·22	0·47	Croghan	196 45 39·44	7	4·82	0·77
Brandon	127 43 41·22	2	0·84	0·17	Carrickrew	201 16 46·76	12	6·74	0·47
Blackstairs	147 32 57·91	12	6·12	0·55	Tara	208 38 21·85	20	7·32	0·30
Mount Leinster	156 18 9·36	13	3·99	0·17	Precelly	287 52 45·05	4	3·07	0·83
Bree Hill	162 34 48·35	8	5·69	0·74					

FOULA.

3-ft. Theodolite, B.O.

From 10th to 30th July 1821. Observers: Major-Gen. COLBY and Capts. VETCH and DRUMMOND, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Start Lighthouse	9 27 53·31	1	–	4·72	Yell	230 25 11·38	6	2·94	0·31
North Ronaldshay Lighthouse	10 31 54·75	10	8·93	1·41	Noss Head	269 33 49·44	8	14·71	4·45
Fitty Hill	28 36 39·41	7	4·36	0·73	Brassa	270 43 44·62	18	4·24	0·12
Ronas	219 1 41·38	12	7·75	0·49	Fair Isle	338 22 52·82	20	8·53	0·42

FOUR-MILE STONE.

3-ft. Theodolite, B.O.

From 27th November 1849 to 6th January 1850. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Wingreen	49 59 50·78	15	8·03	0·59	Thorney Down	276 36 9·94	21	12·72	0·63
Westbury Down	128 12 43·21	16	6·03	0·38	Queen's Manor	296 3 21·18	30	6·70	0·24
Stoke Hill	139 49 2·08	17	10·21	0·81	Old Sarum Gun	302 26 40·85	27	6·66	0·18
Milk Hill	180 46 9·28	10	6·77	0·65	Dean Hill	304 29 45·94	36	8·66	0·20
Beacon Hill	232 24 45·97	24	10·55	0·34	Old Sarum Castle	308 28 58·60	48	8·79	0·15
Old Lodge	263 8 33·75	13	7·08	0·62	Salisbury Spire	325 19 51·80	9	8·53	1·27

FRITTENFIELD.

3-ft. Theodolite, B.O.

From 17th June to 21st July 1844. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Fairlight Down	22 35 51·56	11	9·87	1·30	Wrotham	108 4 55·33	15	9·08	0·71
Tenterden Church Tower	34 15 10·37	10	9·62	1·74	Stede Hill	116 46 22·86	4	6·50	2·92
Beachy Head	38 30 13·78	12	9·27	0·88	Danbury Spire	162 25 27·96	9	7·81	1·08
Brightling Observatory Dome	49 55 50·43	13	4·31	0·40	Norwood	179 46 36·71	7	4·94	0·84
Goudhurst Ch. Tr.	68 45 26·86	11	5·94	0·68	Walton Tower	203 2 35·14	10	3·72	0·26
Crowborough	70 56 27·95	11	5·24	0·32	Referring-object	231 8 10·11	105	3·71	0·02
Frant Church Tr.	73 23 29·99	13	6·43	0·57	St. Peter's Tower	246 11 41·31	11	7·58	0·89
Leith Hill Tower	88 15 15·73	18	6·86	0·27	Paddlesworth	295 34 15·78	15	7·49	0·60
					Tolsford	303 45 51·94	7	5·52	0·95
					Lydd Church Tr.	349 51 32·33	9	7·06	1·50

GAD'S HILL.

2-ft. Theodolite.

From 11th September to 25th October 1845. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Severndroog	101 50 59·30	19	2·19	0·03	Danbury Spire	192 46 53·69	14	3·87	0·19
St. Paul's	106 1 5·63	2	1·39	0·42	Norwood	269 54 51·79	19	2·49	0·04
Epping Cupola	143 54 55·62	16	3·69	0·17	Referring-object	307 11 50·07	71	–	–
Langdon	169 21 38·34	8	6·02	0·78	Stroud Hill	312 59 56·63	5	5·66	1·65

GARFORTH CLIFF.

3-ft. Theodolite, B.O.

From 26th December 1841 to 12th April 1842. Observers: Lieut. HORNBY, R.E., and Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Woburnstand Monument	1 56 23·12	8	11·21	4·68	Acklam Wold	231 54 50·20	15	10·32	1·05
Stain Cross	23 45 54·10	11	11·25	1·71	Wilton Beacon	238 48 8·01	2	0·00	0·00
Holme Moss	51 35 8·07	5	2·88	0·38	Nunburnholme	251 30 0·92	5	1·65	0·14
Boulsworth	94 50 20·54	4	9·37	6·43	Hunsley Beacon	267 38 58·19	5	6·71	3·18
Great Whernside	135 54 5·89	7	10·74	4·14	Ledstone Beacon	284 34 38·60	18	9·88	0·95
Black Hambleton	187 39 35·85	3	3·26	1·29	Crowle	297 46 53·93	5	5·12	1·41
York Minster	223 18 35·78	22	11·68	0·79	Gringley Beacon	324 59 15·64	1	–	14·69

GARREG.

3-ft. Theodolite, B.O

1805. Observer: Major-Gen. COLBY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Arrenig	35 33 31·50	7	5·25	0·91	Gwaunysgaer	116 2 46·61	15	3·28	0·12
Snowdon	64 27 35·39	6	7·47	1·89	Billinge	238 22 36·66	4	2·44	0·61
Llanelian	81 44 5·83	14	7·12	0·68	Delamere[1]	280 51 23·44	4	4·79	1·90
Orme's Head	96 56 29·26	8	8·82	1·75	Cyrn-y-Brain	343 20 34·91	12	7·77	0·92

[1] A correction of + 5″·46 to be applied to this bearing, to reduce it to the new trigonometrical station.

GERTH OF SCAW.

7-in. Theodolite.

From 30th January to 2nd February, and on 19th April, 1847. Observers: Corp. STEEL and Private JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Balta	186 29 8·34	30	70·00	20·59	Saxavord	288 7 3·93	29	110·00	39·98
Nive Hill	195 9 12·51	31	93·34	30·82					

GLASHMEAL.

3-ft. Theodolite, B.O.

From 17th June to 2nd July 1818. Observers: Major-Gen. COLBY and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Cleugh	18 1 33·48	7	5·86	1·25	Mount Battock	257 12 37·68	14	7·73	0·56
Bin of Campsie	26 45 28·01	4	4·11	1·13	Red Head, Old	297 46 35·20	4	3·36	0·94
Ben Lawers	55 22 18·84	6	3·84	0·56	Craigowl	329 0 10·37	14	6·66	0·38
Ben Amhlair	84 49 45·20	6	3·41	0·55	Kellie Law	332 17 59·88	2	0·13	0·00
Ben Macdui	140 20 35·39	3	3·12	1·31	Largo Law	338 53 28·67	10	5·37	0·63
Corryhabbie	191 8 49·89	9	10·42	1·70	East Lomond	352 36 3·15	15	10·04	0·77

GLASSERTON.

3-ft. Theodolite, B.O.

5th and 6th August 1815. Observer: Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
South Berule	11 6 56·53	2	0·53	0·70	Cairnsmuir of Fleet	196 45 16·14	8	3·89	0·36
Mull of Galloway	71 4 47·96	8	3·21	0·32	Bencairn	243 40 30·31	9	5·09	0·65
Cairn Piot	115 14 12·32	4	7·56	4·01	Criffel	244 35 58·33	4	0·95	0·06
Benereard	144 39 33·77	6	5·43	1·07	North Berule	353 3 19·39	4	6·16	2·97
Merrick	180 59 59·80	4	5·04	2·08	Snea Fell	358 30 23·64	4	2·35	0·58

GOAT FELL.

3-ft. Theodolite, R.S.

From 12th August to 2nd November 1852. Observers: Colour-Serg. DONELAN and Corp. GROSE, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Slieve Donard .	16 30 36·61	6	1·62	0·09	Ben Cleugh . .	234 24 10·01	18	4·53	0·15
Divis	25 22 29·25	18	6·51	0·22	Bin of Campsie .	236 20 13·92	14	6·46	0·42
Trostan . . .	43 50 10·81	12	2·95	0·17	West Lomond .	238 59 45·49	8	5·50	0·66
Carn-na-Leagh .	47 2 59·84	15	10·86	1·17	Carnethy Cairn .	257 49 14·81	12	3·35	0·16
Knocklayd . .	52 52 33·93	12	5·07	0·50	Tinto Cairn, New	271 37 38·56	16	4·50	0·17
Slieve Snaght .	71 28 55·91	8	4·31	0·39	Hart Fell . . .	281 21 39·62	20	8·26	0·37
Ben Tuirc . . .	74 39 36·69	15	6·32	0·37	Cairnsmuir on Deugh . . .	303 12 22·15	16	7·69	0·34
Oa	90 58 35·09	19	6·96	0·43	Criffel	306 52 36·27	16	3·89	0·18
Jura	121 28 14·23	19	7·50	0·33	Brown Carrick .	308 34 40·87	14	6·38	0·53
Ben More in Mull	150 23 32·91	19	4·33	0·15	Merrick . . .	319 27 16·93	16	6·04	0·25
Creachbheinn . .	171 24 38·29	18	3·48	0·09	Benereard . . .	344 30 12·43	13	4·15	0·23
Ben Lomond . .	208 51 9·33	31	5·43	0·11	South Berule . .	348 15 25·73	11	3·65	0·22
Ben Lawers . .	210 10 9·45	24	8·05	0·29	Cairn Piot . .	353 49 41·75	15	6·17	0·29
Hill of Stake . .	230 5 32·75	15	4·30	0·19					
Referring-object .	233 2 2·31	47	–	–					

GOONHILLY.

3-ft. Theodolite, B.O.

From 5th December 1845 to 17th January 1846. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Grade Church Tr.	8 38 31·11	14	5·36	0·32	Tregonning . .	122 51 34·24	20	5·58	0·23
Lizard Lighthouse, East . . .	10 1 57·07	10	5·68	0·46	Karnbonellis . .	166 35 14·75	14	4·57	0·21
Lizard Lighthouse, West . . .	10 24 15·68	9	7·98	0·86	Hensbarrow . .	214 14 51·86	14	6·40	0·52
The Wolf Rock .	76 17 4·58	9	2·38	0·14	St. Just, Windmill	218 55 55·95	6	3·95	0·69
Pertinney . . .	101 30 59·84	22	4·21	0·10	Deadman . . .	234 1 27·22	13	5·62	0·73
Karn Galver . .	113 10 27·42	5	2·91	0·44	St. Kevern's Spire	265 55 22·77	18	6·15	0·20
					Referring-object .	342 58 11·99	77	–	–

GORLESTON CHURCH TOWER.

18-in. Theodolite.

From 22nd December 1843 to 3rd February 1844. Observers: Corp. STEEL and Private M^c^NALLY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Southwold . .	6 24 13·15	2	6·98	12·18	Norwich Spire .	101 49 10·48	9	8·19	1·80
Toft's Tower . .	49 58 29·60	10	14·50	4·02	Happisburgh . .	154 33 14·66	16	12·61	1·21

GREAT STIRLING.

7-in. Theodolite.

From 7th to 14th November 1850. Observer: Serg. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Little Stirling .	68 45 1·07	14	64·36	36·81	Peterhead Old Windmill . .	177 5 41·90	12	60·78	34·94
Mormonth . .	136 46 14·82	6	28·61	39·01	Reform Monument	169 56 21·60	13	45·04	42·95

GREAT WHERNSIDE.

3-ft. Theodolite, R.S.

From 5th to 20th September 1840. Observer: Lieut. PIPON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Boulsworth . .	10 41 56·66	10	6·65	0·56	Cross Fell . .	152 29 1·83	4	3·53	0·85
Whittle Hill . .	17 51 18·30	6	3·36	0·45	Water Crag . .	166 38 6·29	9	3·13	0·26
Pendle Hill . .	31 16 42·51	4	3·09	0·68	Botton Head . .	244 56 47·85	7	5·28	0·66
Ingleboro' . . .	91 29 49·88	5	3·71	0·70	Black Hambleton	251 18 31·61	4	3·10	1·04
Little Whernside	105 54 53·14	2	0·13	0·00	Wilton Beacon .	282 11 33·60	2	1·47	0·54
Sca Fell . . .	112 57 10·45	5	4·06	0·88	York Minster .	289 51 55·72	3	5·31	3·46
Helvellyn . . .	121 59 38·12	3	3·67	1·64	Holme Moss . .	353 43 22·54	3	1·49	0·30

GRINGLEY.

12-in. and 3-ft. Theodolites, B.O.

From 21st July to 26th September 1843. Observer: Mr. CAMPBELL.
1801. Observer: Mr. WOOLCOTT.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Sutton	34 28 6·68	3	4·34	2·38	North End of Base	174 26 57·33	5	11·44	6·13
Laughton Spire .	84 22 43·81	4	9·94	6·72	Axholme Church	195 10 16·40	4	9·00	6·24
Clifton	103 58 34·28	9	4·36	0·38	Crowle . . .	189 57 14·69	17	11·66	0·84
South End of Base	155 9 40·68	6	14·97	7·83	Lincoln Minster[1]	309 37 48·29	64	24·89	0·77

[1] A correction of — 14″·21 to be applied to this bearing, to reduce it to the trigonometrical station.

GWAUNYSGAER.

3-ft. Theodolite, B.O.

1st October 1805. Observer: Mr. WOOLCOTT.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		′			° ′ ″		″	
East End of Base	52 14 31·79	6	11·51	4·20	West End of Base	76 28 29·29	6	7·09	1·68
Moelfre Issa . .	54 48 4·23	7	5·24	0·90	Orme's Head . .	92 30 39·52	3	3·24	1·16

HAMPTON POORHOUSE.

3-ft. Theodolite, B.O.

1792. Observer: Gen. MUDGE.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
St. Anne's	73 51 37·07	8	4·57	0·52	Hanger Hill[1]	203 54 38·87	7	4·56	0·69
King's Arbour	135 18 11·63	2	0·87	0·18					

[1] A correction of — 40′ 55″·71 to be applied to this bearing, to reduce it to the Tower on Hanger Hill.

HANGER HILL TOWER.

3-ft. Theodolite, B.O.

From 15th February to 7th April 1848. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Leith Hill Tower	7 26 59·75	26	5·25	0·11	St. Paul's Cathedral	274 15 33·12	13	5·58	0·28
Hampton Ch. Tr.	19 30 36·70	5	7·83	2·54	Westminster Abbey	282 29 30·79	23	5·96	0·21
Hampton Poorho.	23 16 46·19	9	4·88	0·46	Severndroog	284 2 13·13	12	6·50	0·52
St. Anne's	48 20 42·95	12	3·06	0·15	Forest Hill	297 9 52·51	17	10·15	0·46
King's Arbour	65 19 25·17	2	4·50	5·06	Wrotham	299 15 20·10	12	3·60	0·15
Windsor Castle Flag-staff	78 27 48·70	6	3·08	0·33	Banstead	345 34 44·15	10	3·64	0·20
					Kew Pagoda	357 41 32·65	6	4·65	0·61

HAPPISBURGH CHURCH TOWER.

3-ft. Theodolite, B.O.

From 8th to 30th September 1843. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Norwich Spire	35 51 46·73	9	7·52	1·09	Referring-object	145 28 59·61	34	7·62	0·16
Baconsthorpe Ch. Tower	105 34 39·45	4	2·52	0·50	Gorleston Tower	334 24 44·62	4	3·48	0·93
					Tofts Tower	355 25 38·61	3	5·97	4·20

HART FELL.

2-ft. Theodolite.

From 30th September 1846 to 4th March 1847. Observer: Serg. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Criffel	15 39 21·01	17	9·52	0·53	Cairnsmuir on Deugh	72 8 50·71	4	8·10	4·16
Ben Cairn	29 0 49·92	10	8·63	1·26	Goat Fell	102 50 17·50	2	2·90	2·10
Cairnsmuir of Fleet	51 37 52·45	3	2·87	0·95	Hill of Stake	119 19 4·25	1	–	8·14
Merrick	66 38 0·10	20	9·28	0·60	Ben Lomond	138 56 34·90	13	6·62	0·62

HART FELL—*continued.*

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Tinto	141 9 23·99	20	8·77	0·38	Cheviot . . .	263 51 24·56	24	9·11	0·59
Bin of Campsie .	145 47 36·68	8	5·08	0·61	Wisp	296 23 49·33	21	7·80	0·33
Ben Lawers . .	158 20 12·23	15	9·85	0·65	Cross Fell . .	322 57 24·32	24	9·90	0·38
Ben Cleugh . .	165 14 30·78	13	7·17	0·74	High Pike . .	344 13 30·01	5	3·24	0·53
Carnethy Cairn .	189 24 58·51	15	6·60	0·38	Burnswark . .	347 18 27·56	11	3·52	0·23
Allarmoor . . .	191 0 50·92	9	8·55	1·07	Referring-object .	350 20 29·89	161	8·57	0·04
Dunrich . . .	216 43 0·16	24	7·40	0·40	Sea Fell . . .	353 23 17·07	24	12·30	0·49
Sayrs Law . .	222 59 54·81	13	6·99	0·37					

HENSBARROW.

3-ft Theodolite, R.S.

From 18th April to 4th June 1845. Observer: Corp. STEWART, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Goonhilly . . .	34 30 53·52	11	3·93	0·28	Brown Willy . .	213 37 15·93	10	13·20	2·29
Karnbonellis . .	52 42 13·22	10	5·27	0·46	High Willhays .	239 19 58·22	18	5·72	0·18
Carnbrea Monument . . .	59 36 12·21	3	3·87	1·90	Kit Hill . . .	247 50 56·34	7	5·71	1·36
Pertinney . . .	62 43 56·99	10	4·27	0·35	Butterton . . .	266 51 24·70	15	5·35	0·24
Karnminnis . .	67 52 6·09	15	4·36	0·20	Maker Church Tower . . .	274 52 44·68	4	5·49	2·00
St. Agnes .	73 22 46·10	10	3·81	0·21	Maker	275 38 6·80	6	7·90	2·67
Referring-object .	78 25 55·01	119	5·93	0·02	Barrow Hill . .	281 45 46·62	10	2·27	0·16
Trevose Head .	140 28 53·94	8	6·50	1·02	Lansallos . . .	284 48 40·10	6	4·97	1·07
Lundy Island Lighthouse[1] .	186 40 24·11	2	6·09	9·27	Eddystone Lighthouse . . .	299 32 15·24	7	6·02	0·91
Cadon Barrow .	195 22 26·65	7	3·19	0·38	Deadman . . .	356 2 14·28	6	2 18	0·18

[1] A correction of + 1′ 38″·57 to be applied to this bearing, to reduce it to the trigonometrical station.

HIGH PORT CLIFF.

18-in. Theodolite.

From 20th to 22nd July 1846. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Littletown Down	123° 2′ 6″·33	11	18″·99	4·75	Boniface, S.E. .	149° 37′ 54″·63	11	12″·20	1·87

HIGH WILHAYS.

3-ft. Theodolite, B.O.

From 9th June to 20th July 1845. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Eddystone Lighthouse	17 59 8·89	5	7·10	3·51	Precelly	159 31 15·74	14	6·22	0·32
Lansallos	45 42 42·32	5	4·33	0·90	Paracombe	190 53 16·75	12	6·78	0·61
Kit Hill	47 20 32·08	5	8·15	3·27	Dunkery	209 9 24·23	10	6·89	0·64
Deadman	47 43 44·26	4	2·22	0·42	Mendip	239 29 58·61	12	5·97	0·39
Hensbarrow	59 57 17·73	13	8·31	0·97	Staple Hill	245 50 43·55	6	2·31	0·19
Brown Willy	76 6 47·23	11	7·12	0·65	Dumpdon	254 31 50·91	4	3·53	1·16
Cadon Barrow	86 3 48·54	7	3·42	0·53	Pillesdon	260 17 49·55	17	4·37	0·15
Bradbury Castle	122 23 38·45	4	1·88	0·22	Black Down	269 20 20·00	6	2·46	0·30
					Swyre Barrow	272 59 12·19	8	5·99	0·95
Lundy Island Lighthouse	139 17 21·39	5	12·76	6·67	Vern	276 16 51·17	8	4·28	0·46
					Little Haldon	290 56 24·94	8	4·17	0·52
Lundy Island Station	139 18 57·28	15	4·19	0·13	Ryder's Hill	336 54 31·64	15	6·34	0·39

HILL OF STAKE.

3-ft. Theodolite, B.O.

From 11th to 18th October 1816. Observers: Major-Gen. COLBY and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Benercard	7 0 49·11	2	0·57	0·08	Glasgow Observ^{y}	262 1 3·49	7	6·12	1·01
Goat Fell	50 26 51·47	3	1·55	0·26	Dunrich	285 29 40·86	1	–	6·18
Jura	96 28 22·01	1	–	6·18	Tinto	290 33 59·69	4	5·97	3·16
Ben Lomond	190 55 39·29	12	8·48	1·18					
Bin of Campsie	241 44 41·58	21	4·52	0·13	Cairnsmuir on Deugh	331 27 28·14	5	2·33	0·47
Corkmulaw	243 33 58·59	8	7·12	1·18	Brown Carrick	356 30 17·30	8	5·71	0·74

HINGHAM CHURCH TOWER.

18-in. Theodolite.

From 17th May to 3rd July 1843. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Brandon	51 59 7·87	5	8·87	3·40	Norwich Spire	254 45 1·51	11	10·58	1·90
Swaffham Spire[1]	111 16 5·52	10	20·13	5·04	Bunwell Tower	314 24 6·53	8	13·38	4·80
Baconsthorpe	199 19 53·10	9	7·83	1·63	South Lopham Tr.	357 19 21·58	7	9·86	2·94

[1] A correction of + 27″·64 to be applied to this bearing, to reduce it to the trigonometrical station.

HOLME MOSS.

18-in. Theodolite.

From 24th August to 11th November 1841. Observer: Corp. Steel, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Kinder Scout · ·	0 1 53·60	10	13·64	2·62	Little Whernside·	156 13 15·79	3	4·49	2·45
Axedge · · ·	7 18 51·72	5	12·98	7·26	Great Whernside·	173 49 12·25	8	6·35	0·86
Cyrn-y-Brain Tr.[1]	57 37 52·53	2	1·92	0·92	Garforth Cliff· ·	231 10 47·00	2	2·32	1·34
Whittle Hill · ·	119 29 7·50	8	8·72	1·57	Staincross · · ·	256 25 28·78	3	4·54	2·29
Pendle Hill · ·	143 25 17·21	7	10·23	3·23	Clifton · · · ·	281 18 34·65	3	2·33	0·72
Boulsworth · ·	154 28 14·44	12	19·53	4·43	Back Tor · · ·	318 59 43·84	5	13·49	8·65

[1] A correction of — 49″·75 to be applied to this bearing, to reduce it to the trigonometrical station.

HOWTH.

3-ft. Theodolite, B.O., and 7-in. Theodolite.

From 29th November to 19th December 1829. Observer: Lieut. Portlock, R.E.
From 28th August to 10th September 1844. Observer: Mr. Gordon.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Bray Head · ·	2 22 22·04	20	5·14	0·19	Garristown Windmill · · · ·	134 39 59·21	12	10·57	1·62
Great Sugar Loaf	12 33 4·81	17	6·57	0·25	Loughanleagh ·	137 33 32·11	6	8·58	2·73
Douce Mountain or Joice's Hill ·	22 57 3·07	9	8·20	1·13	Knockbrack · ·	150 24 37·13	5	5·04	1·19
Kingstown Observatory · · ·	24 56 19·96	15	100·89	279·88	Slieve Gullion ·	164 3 51·83	2	2·18	1·18
Fairy Hill · · ·	38 10 38·44	18	6·97	0·42	Carlingford · ·	172 39 28·30	5	0·96	0·04
Kippure · · ·	39 0 46·25	22	8·66	0·38	Slieve Donard ·	186 9 16·29	4	4·33	1·62
PoolbegLighthouse	57 34 6·57	10	5·82	0·63	Lambay Island ·	194 35 27·94	48	5·20	0·08
Lyons Hill · ·	73 38 46·50	15	4·90	0·25	Holy Head · ·	273 33 3·24	7	7·75	2·12
Maynooth Obelisk	89 25 59·58	7	2·54	0·24	The Rivel · · ·	291 26 27·92	3	1·94	0·45
Dome of Dublin Observatory ·	95 4 50·46	8	3·73	0·45	Collin · · · ·	359 34 19·55	7	7·11	1·18

HUNGRY HILL.

3-ft. Theodolite, B.O.

From 11th to 16th July 1832. Observer: Capt. Portlock, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Mizen Hill · ·	2 7 40·28	5	4·25	0·77	Baurtregaum · ·	177 29 31·14	6	6·44	1·61
Bolus Mountain ·	109 29 33·63	7	2·98	0·27	Carn Thuil · ·	185 36 29·08	5	3·94	0·76
Knocknaskerita ·	119 40 3·43	5	1·64	0·11	Mangerton· · ·	213 50 28·36	6	3·72	0·52
Knocknadober ·	142 23 56·96	4	2·97	0·55	Caherbarnagh	227 36 47·11	10	6·40	0·97
Brandon · · ·	152 40 53·60	2	3·32	2·75	Slieve Buoymore·	241 47 12·34	12	1·99	0·05
Knocknagante ·	156 47 46·92	5	2·31	0·26	Carrigfadda · ·	276 29 2·19	11	4·40	0·32
Colly Mountain ·	159 2 13·74	5	3·97	0·85	Mount Gabriel ·	309 44 6·62	9	5·99	0·64

INKPEN.

3-ft. Theodolite, R.S.

From 2nd May to 24th June 1844. Observers: Corps. **Stewart** and **Cosgrove**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Nodes Beacon[1]	4 3 29·88	2	1·50	0·56	Brill	206 55 58·75	11	11·82	1·54
Dean Hill	20 13 57·92	4	10·86	7·73	Nettlebed	232 32 33·81	6	13·09	6·44
Beacon Hill	43 48 44·83	8	6·51	0·97	Windsor Castle Flagstaff	255 56 0·00	5	7·64	2·86
Wingreen	48 0 1·60	19	12·59	0·87	Leith Hill Tower	283 58 7·57	14	4·47	0·24
Westbury	78 1 32·35	8	4·55	0·50	Referring-object	319 9 39·94	120	8·77	0·04
Mendip	79 13 55·85	6	1·20	0·07	Butser Hill	320 40 46·05	13	5·80	0·42
Upcot	120 43 44·37	1	–	11·68	Dunnose	346 59 50·48	15	10·48	0·89
Whitehorse Hill	164 3 57·29	15	9·50	0·77	Motteston	357 58 35·85	10	8·19	0·97
Scutchamfly	199 57 55·74	4	5·80	2·17					

[1] A correction of – 6″·13 to be applied to this bearing, to reduce it to the trigonometrical station.

JURA.

3-ft. Theodolite, B.O.

From 30th July to 12th November 1847. Observer: Serg. **Donelan**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Trostan	5 47 24·11	15	5·77	0·34	Ben Nevis	211 26 1·32	31	6·19	0·15
Knocklayd	10 48 52·09	10	5·53	0·54	Cruach-na-Sleagh	215 29 41·27	17	8·87	0·57
Oa	31 10 27·44	12	6·78	0·45	Ben Lawers	236 22 15·88	21	8·36	0·36
Mount Sandy	36 32 47·43	11	8·25	0·76	Ben Lomond	248 52 28·43	12	6·06	0·51
Slieve Snaght	47 26 18·39	19	3·69	0·13	Hill of Stake	275 26 26·70	5	4·97	1·13
Ben Tartevil	54 41 18·95	12	6·67	0·48	Goat Fell	300 47 51·72	13	5·35	0·36
Ben Heynish	137 32 32·73	28	8·07	0·19	Merrick	310 34 52·92	14	3·04	0·11
Ben More, S. Uist	152 49 42·60	9	4·01	0·31	Ben Tuirc	323 28 53·55	14	9·61	1·12
Ben More in Mull	179 24 7·32	21	9·65	0·36	Carn-na-Leagh	347 44 21·69	5	1·36	0·10
Creachbheinn	200 55 11·74	6	1·61	0·10					

KARNBONELLIS.

3-ft. Theodolite, B.O.

1796. Observer: Gen. **Mudge**.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Buryan	67 20 29·93	5	0·88	0·03	Hensbarrow	232 23 16·62	1	–	3·87
Karnminnis	93 43 28·24	7	7·64	1·49	Deadman	261 43 41·41	3	2·79	1·00
St. Agnes	183 26 59·87	8	2·86	0·23					

KARN GALVER.

18-in. Theodolite.

From 24th August to 12th October 1850. Observer: Corp. WOTHERSPOON, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Pertinny . . .	19 29 14·99	22	14·56	1·06	Trevose Head .	223 52 6·61	15	13·57	1·37
St. Agnes Lighthouse . . .	60 15 3·65	12	11·37	2·16	Brown Willy . .	236 15 34·86	3	6·36	4·49
Peninnis Windmill	60 16 58·20	12	9·09	1·62	Karnminnis . .	239 51 33·54	25	14·55	1·14
Telegraph Tower	62 16 2·95	12	8·34	1·11	Hensbarrow . .	246 28 33·39	16	19·96	2·76
St. Martin's Head	64 57 37·51	12	12·20	1·68	Carnbrea Monument . . .	256 26 24·81	37	15·86	0·61
Beacon Hill, Trescow, Watchhouse[1]	66 41 47·82	11	15·36	3·46	Karnbonellis . .	265 52 9·59	21	7·55	0·31
					Goonhilly . . .	292 50 1·10	17	10·28	0·75

[1] A correction of + 1′ 3″·15 to be applied to this bearing, to reduce it to the trigonometrical station.

KARNMINNIS.

3 ft. Theodolite, B.O.

From 7th to 27th October 1845. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Pertinney . . .	39 10 57·56	13	5·20	0·27	Carnbrea Monumt	261 42 51·32	7	4·28	0·48
Karn Galver . .	59 55 4·87	7	5·86	0·93	Karnbonellis . .	273 29 24·80	13	8·76	0·79
St. Martin's Daymark[1] . . .	64 26 7·15	6	6·32	1·37	St. Keverne Spire	296 27 9·87	6	2·55	0·21
Trevose Head .	221 58 46·52	10	5·98	0·66	Tregonning . .	303 5 26·60	14	5·52	0·38
Cadon Barrow .	228 53 39·16	2	5·22	6·81	Grade Church Tr.	313 36 20·33	17	8·95	0·48
Brown Willy . .	236 1 12·03	2	0·80	0·16	Lizard Lighthouse, East	317 33 37·24	7	4·77	0·82
St. Agnes Beacon	240 59 26·70	10	3·83	0·25	Lizard Lighthouse, West . . .	317 39 17·48	8	2·24	0·16
Hensbarrow . .	247 19 1·71	14	8·60	0·70					

[1] A correction of + 39″·58 to be applied to this bearing, to reduce it to the trigonometrical station.

KEEPER.

3-ft. Theodolite, B.O.

From 16th September to 29th December 1830, and from 29th May to 10th July 1831.
Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Knocknaskagh .	9 20 44·75	9	5·74	0·83	Bencorr . . .	129 58 10·94	3	2·82	0·88
Caherbarnagh .	38 16 20·51	3	5·64	4·17	Nephin . . .	152 45 54·75	6	4·95	1·17
Taur	46 8 20·94	2	2·21	1·22	Slieve Bawn, S. .	186 43 19·14	1	—	10·24
Baurtregaum . .	61 0 2·04	6	10·91	4·69	Cuilcagh . . .	190 18 42·04	3	2·32	0·61
Knockanore . .	74 59 26·98	4	4·57	1·69	Knockastia . .	208 35 12·30	6	5·81	1·29
Meelick . . .	83 39 17·13	15	9·69	0·72	Croghan . . .	224 29 21·65	9	6·60	0·70
Slievecallan . .	98 42 7·94	13	6·29	0·46	Arderin . . .	231 32 30·78	24	18·86	0·92
Gortourka . .	106 10 3·96	8	4·50	0·54	Kippure . . .	249 6 44·84	3	8·33	8·78
Slievecarran . .	127 36 19·24	25	9·57	0·28	Cullenagh . . .	251 4 23·56	18	9·93	0·57

KEEPER—*continued.*

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Devil's Bit	251 27 52·19	89	8·61	0·06	Slievenaman	306 56 58·70	21	13·43	0·71
Lugnaquillia	258 4 1·37	3	5·43	3·87	Knockanaffrin	317 52 7·12	28	11·84	0·45
Mount Leinster	277 50 2·59	9	6·97	1·10	Knockmealdown	337 54 57·82	21	10·02	0·56
Blackstairs	282 17 45·29	2	1·11	0·30	Galtymore	352 33 0·52	31	10·33	0·31

KELLIE LAW.

3-ft. Theodolite, R.S.

From 15th December 1846 to 6th March 1847. Observer: Corp. WINZER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Carnethy Cairn	34 30 5·44	11	6·80	0·68	Mount Battock	181 42 45·29	22	5·64	0·22
Allarmoor	35 8 20·27	7	4·86	0·71	Caerlock	191 45 36·06	7	8·18	1·72
Arthur's Seat	35 11 52·83	14	7·73	0·67	Red Head	203 21 47·11	15	8·94	0·60
Calton Hill[1]	37 42 43·20	6	6·34	1·38	Isle of May Lighthouse	296 38 22·40	14	7·45	0·54
Largo Law	80 3 7·79	17	8·36	0·54	Lumsden	316 27 40·61	31	8·37	0·18
East Lomond	88 48 23·45	25	12·74	1·35	Sayrs Law	351 14 7·12	30	13·05	0·43
Glashmeal	152 47 14·05	17	7·91	0·67					
Craigowl	156 36 47·70	19	6·39	0·27					

[1] A correction of −1″·81 to be applied to this bearing, to reduce it to the trigonometrical station.

KEYSOE CHURCH SPIRE.

18-in. Theodolite.

From 9th March to 8th May 1843. Observers: Corp. STEEL and Private McNALLY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Dunstable	9 49 19·77	7	4·08	0·55	Easton Tower	172 47 11·20	5	4·66	1·08
Hanslope Spire	60 57 14·97	8	14·49	6·84	Ely Minster	250 22 54·76	11	14·37	2·64
Naseby Church Tr.	113 22 1·45	6	4·48	0·90	Royston	312 17 37·54	6	12·59	6·16
Tilton	146 2 19·03	13	4·43	0·21	Tharfield	313 41 2·06	6	1·02	0·17

KING'S ARBOUR.

3-ft. Theodolite, B.O.

May 1792. Observer: Gen. MUDGE.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
St. Anne's	29 28 45·94	5	2·00	0·21	Banstead, Old[1]	317 42 22·70	3	0·97	0·10
Hampton Poor-ho.	315 14 12·06	5	2·50	0·35					

[1] A correction of + 10″·04 to be applied to this bearing to reduce it to Banstead, New.

KINGSTOWN OBSERVATORY.

7-in. Theodolite.

1st and 2nd October 1844. Observer: Mr. H. GORDON.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Poolbeg Lighthouse . . .	166 48 11·58	12	117·13	159·56	Howth . . .	204 53 37·52	12	37·19	14·83

KIPPURE.

3-ft. Theodolite, B.O.

From 10th June to 16th July 1829. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ballycreen . .	4 35 21·03	10	6·13	0·54	Slieve Gullion .	176 23 26·53	2	6·46	10·43
Lugnaquillia . .	20 46 22·26	30	7·84	0·19	Dublin Observʸ .	178 55 36·47	13	4·41	0·36
Knockanaffrin[1] .	40 58 38·24	8	7·08	0·97	Castlecoo . . .	183 48 19·58	4	3·64	0·94
Galtymore . .	54 47 43·41	4	5·47	2·37	Slieve Donard .	193 31 23·96	12	8·31	0·87
Cullenagh . . .	68 33 57·24	4	5·87	2·28	Howth	218 48 5·63	34	10·03	0·20
Keeper . . .	70 39 14·01	7	7·05	1·48	South Berule .	224 48 35·40	2	0·33	0·02
Slieveroe . . .	86 39 39·00	3	7·67	6·85	Moel Rhydladd .	258 40 49·72	4	3·21	1·06
Dunmurry . .	93 26 26·96	8	6·45	1·13	Holy Head . .	261 35 19·02	4	5·21	1·79
Croghan . . .	106 52 36·51	18	12·75	1·04	Snowdon . . .	273 43 11·82	4	2·61	0·49
Knockastia . .	109 7 17·74	2	3·62	3·27	Rhiw	287 55 59·56	1	—	8·40
Lyons Hill . .	132 39 15·38	22	14·66	1·00	Precelly . . .	321 46 7·25	7	5·47	0·90
Cuilcagh . . .	139 56 47·42	3	2·89	2·53	Collin	328 13 17·36	3	1·23	0·18
Loughanleagh .	155 16 50·91	9	11·06	2·13	Tara	351 45 12·93	7	5·07	0·60

[1] A correction of — 6″·25 to be applied to this bearing, to reduce it to the trigonometrical station.

KNOCK.

3-ft. Theodolite, B.O.

From 4th to 14th August 1814. Observers: Major-Gen. COLBY and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Buck	19 43 39·92	16	7·07	0·47	Cowhythe . . .	211 51 9·52	12	3·75	0·24
Findlay Seat . .	88 49 34·18	12	5·98	0·49	Manor Lee . .	262 30 22·43	8	5·67	0·83
Balnaskerish . .	116 1 41·72	1	–	8·72	Mormonth . .	267 2 10·33	13	7·36	0·84
Ben Lundie . .	121 27 18·73	2	0·84	0·17	Dudwick . . .	290 48 21·93	13	4·60	0·35
Bin of Cullen .	147 10 15·30	11	5·51	0·56	Caerlock . . .	346 0 56·59	9	11·62	2·45
Ben Cheilt . .	156 43 39·68	5	4·54	0·87	Mount Battock .	358 20 12·72	5	11·61	5·90

KNOCKALONGY.

18-in. Theodolite.

From 8th to 15th September 1828. Observer: Lieut. Murphy, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Knockma . . .	9 36 17·20	2	14·13	49·91	Truskmore . .	231 28 35·17	8	9·91	3·24
The Reek . .	51 3 30·33	7	20·62	10·77	Cuilcagh . . .	268 55 22·98	6	14·05	7·48
Nephin . . .	63 23 43·33	10	20·16	5·87	Slieve Bawn, South	319 7 24·77	1	–	33·90
Tawnaghmore .	101 53 12·60	4	10·52	7·22	Mullaghanoe . .	355 22 45·87	3	5·72	4·46
Slieve League .	183 51 57·16	11	11·48	3·04					

KNOCKANAFFRIN.

3-ft. Theodolite, B.O.

From 3rd August to 4th October 1829. Observer: Lieut. Portlock, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Slieve Grian . .	21 50 30·97	13	5·64	0·32	Cullenagh . . .	196 19 53·37	19	8·30	0·39
Doolieve . . .	47 36 42·49	6	2·24	0·20	Kippure . . .	219 58 51·93	9	12·26	2·32
Knockmealdown .	73 30 34·99	34	11·41	0·23	Lugnaquillia . .	224 38 7·21	10	5·25	0·44
Baurtregaum . .	87 29 20·21	3	3·28	1·26	Ballycreen . .	229 8 15·52	4	5·49	2·40
Taur	87 32 38·00	4	4·06	1·14	Mount Leinster .	235 46 55·53	23	7·87	0·39
Galtymore . .	102 7 14·18	21	7·25	0·29	Mount Brandon .	239 3 33·91	2	2·01	1·01
Keeper . . .	138 24 26·57	35	8·58	0·19	Blackstairs . .	240 58 44·64	19	6·00	0·24
Devilsbit . . .	159 15 20·14	17	9·57	0·62	Carrickbyrne . .	260 16 50·01	10	5·18	0·43
Arderin . . .	176 41 7·04	9	10·03	1·82	Slievecoiltia . .	262 33 20·40	14	6·39	0·32
Slievenaman . .	185 7 44·74	36	7·77	0·20	Forth Hill . .	267 6 37·33	8	3·92	0·39

KNOCKASTIA.

3-ft. Theodolite, B.O.

From 21st to 24th September 1836. Observer: Capt. Portlock, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Corr Hill . . .	40 55 58·85	2	0·68	0·11	Knockeyon . .	227 19 8·57	2	1·81	0·81
Monksland . .	84 47 23·14	3	4·84	2·60	Croghan . . .	293 50 26·08	5	9·02	4·42
Slieve Bawn, South	137 5 23·95	1	–	3·82	Ballyduff . . .	335 41 59·34	3	4·47	2·39
Carn Clonhugh .	172 14 56·67	2	2·22	1·23					

KNOCKLAYD.

3-ft. Theodolite, B.O.

From 8th July to 17th August 1827. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Slieve Gallion	34 31 21·18	17	9·21	0·80	Jura	190 36 34·79	22	8·57	0·34
Carntogher	47 53 10·36	11	7·52	1·19	Ben Lomond	220 58 54·41	6	5·83	1·41
Sawel	53 15 36·63	31	13·00	0·46	Ben Tuirc	223 38 7·85	34	11·46	0·32
Benyevenagh	83 3 35·29	61	9·32	0·12	Goat Fell	232 0 6·81	4	1·50	0·15
Scalp	84 0 33·00	15	8·37	0·81	Carn-na-leagh	240 39 52·96	42	15·46	0·46
Eskaheen	84 50 25·17	10	6·70	0·63	Cairnsmuir on Deugh	264 33 30·14	2	3·05	2·32
Mount Sandy	93 11 12·09	17	5·27	0·17	Merrick	270 33 19·86	1	–	12·45
Slieve Snaght	93 36 47·49	31	7·86	0·21	Benereard	276 38 33·29	12	14·43	2·15
Magilligan Tower	94 33 1·52	23	13·18	1·24	Cairn Piot	292 52 32·93	9	8·33	1·23
Ben Tartevil	169 3 29·57	31	10·12	0·39	Nachore	312 1 43·37	5	6·30	1·94
Mull of Kinoe	173 46 28·91	7	14·24	5·36	Trostan	334 43 48·07	45	16·65	0·34
Oa	177 3 53·87	41	12·92	0·39	Divis	346 13 22·81	31	13·67	0·48
Scarribh	184 13 45·61	13	6·08	0·44					

KNOCKNADOBER.

12-in. Theodolite.

From 21st to 31st July 1840. Observer: Corp. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Feaghmaan	56 24 57·72	7	10·09	3·76	Knocknagante	303 48 24·70	7	20·82	13·92
Brandon	168 38 13·26	5	6·44	2·54	Hungry Hill	322 5 59·12	2	2·97	2·20
Baurtregaum	224 43 0·66	6	12·97	6·49					

KNOCKNAGANTE.

12-in. Theodolite.

From 5th to 13th May 1840. Observer: Mr. BEALE.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Feaghmaan	96 41 45·16	5	21·69	29·37	Baurtregaum	191 46 25·13	9	50·38	44·70
Knocknadober	124 0 21·73	6	13·72	6·69	Hungry Hill	336 41 35·61	6	27·41	39·88

KNOCKNASKAGH.

3-ft. Theodolite, B.O.

From 21st to 30th October 1831. Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Doolieve . . .	2 46 33·79	6	7·70	2·01	Taur	107 23 42·51	8	10·59	2·63
Carrigfadda . .	41 13 12·39	5	9·04	3·77	Slievecallan . .	145 27 44·52	1	–	13·87
Clara Mountain .	47 51 52·35	4	9·98	10·51	Knockfeerina .	148 7 17·38	9	8·66	1·45
Slieve Buoymore	64 22 28·79	4	5·27	2·52	Keeper . . .	189 12 33·46	10	8·58	1·84
Mushramore . .	74 41 58·30	6	5·31	1·04	Galtymore . .	211 1 4·96	8	13·26	4·00
Mangerton . .	78 29 22·87	2	1·19	0·35	Slievenaman . .	238 43 20·05	3	1·85	0·40
Caherbarnagh .	80 52 49·09	7	8·54	2·32	Knockmealdown .	249 1 37·95	7	6·10	1·04
Mount Hillary, S.	89 11 23·51	11	6·73	0·80	Slieve Grian . .	279 21 42·05	9	16·35	4·23
Mount Hillary, N.	90 51 37·00	12	5·75	0·46	Bunnaloo . . .	288 11 27·47	7	5·14	0·82
Baurtregaum . .	97 11 18·52	4	11·33	8·16					

LAWSHALL CHURCH TOWER.

3-ft. Theodolite, B.O.

From 27th November 1843 to 28th January 1844. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Thaxted Spire .	49 41 51·60	3	5·63	3·84	Hingham . . .	200 21 19·80	10	4·31	0·52
Balsham Church Tower . . .	85 8 58·81	5	1·32	0·07	South Lopham .	214 41 21·26	8	4·02	0·37
Brandon . . .	165 55 22·95	12	5·75	0·50	Mickfield[2] . . .	256 43 50·82	2	1·44	0·51
Swaffham Spire[1] .	177 36 32·73	4	2·43	0·41	Naughton . . .	290 47 5·94	12	12·39	1·94
					Stoke Tower . .	328 3 57·76	12	13·05	1·50

[1] A correction of + 7″·73 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of + 0″·35 „ „ „

LAXFIELD CHURCH TOWER.

2-ft. Theodolite.

From 10th May to 30th June 1845. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Otley	30 25 22·53	10	4·67	0·37	Norwich Spire .	173 10 33·90	15	6·28	0·46
Mickfield[1] . . .	59 4 35·11	12	8·16	0·73	Tofts Tower . .	212 21 10·70	16	2·62	0·07
South Lopham .	112 38 17·73	11	5·59	0·54	Southwold Tower	262 4 15·86	19	5·60	0·17
Bunwell . . .	142 40 19·91	19	7·78	0·33	Orford Castle .	333 58 50·37	15	5·69	0·35
Referring-object .	145 54 30·03	75	–	–					

[1] A correction of + 0″·41 to be applied to this bearing, to reduce it to the trigonometrical station.

LAYTON.

3-ft. Theodolite, B.O.

From 7th to 9th June 1817. Observer: Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Blue Hill . . .	16 23 0·06	6	2·72	0·36	Over Hill . . .	60 16 46·18	6	5·31	1·58
Tarbathy . . .	19 3 41·39	6	7·68	1·72	Dudwick . . .	177 53 9·14	6	4·56	0·90
Brimmond . .	46 51 26·22	5	2·18	0·25					

LEITH HILL TOWER.

3-ft. Theodolite, B.O. and R.S.

From 3rd to 5th June 1792. Observer: Gen. MUDGE.
From 10th to 17th October 1822. Observers: Major-Gen. COLBY, Capt. KATER, and Mr. GARDNER.
From 13th February to 29th April 1844. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Chanctonbury Ring	1 27 11·01	10	5·01	0·46	St. Paul's Cathedral	206 45 28·16	18	6·81	0·42
Rook's Hill . .	40 26 39·51	9	5·47	0·63	Epping Cupola .	210 7 12·94	10	3·75	0·19
Butser Hill . .	62 52 47·27	10	6·18	0·90	Banstead, Old[1] .	214 52 35·85	3	2·59	0·79
Inkpen	104 49 16·76	13	5·45	0·57	Referring-object .	221 32 45·05	179	–	–
Whitehorse Hill .	118 31 54·72	11	5·26	0·36	Severndroog . .	222 46 41·24	45	5·98	0·06
Bagshot . . .	131 3 34·51	2	4·34	4·71	Wrotham . . .	250 54 15·19	40	4·16	0·05
Windsor Castle					Wrotham, Old .	251 48 50·24	5	2·54	0·26
Flagstaff . .	154 42 18·98	2	1·25	0·39	Hallingbourn . .	261 23 18·18	11	6·01	0·54
St. Anne's . . .	156 34 49·69	1	–	1·96	Frittenfield . .	267 19 12·35	8	6·49	0·84
Wendover . . .	156 42 11·04	6	3·71	0·55	Frant Church .	280 43 30·98	10	3·14	0·49
Dunstable . . .	171 32 56·47	9	4·57	0·36	Crowboro' . . .	289 51 45·80	31	6·61	0·09
Hanger Hill Tower	187 23 42·91	31	6·73	0·13	Blackheath Windm.	295 50 5·37	1	–	1·96
Berkhampstead .	194 44 10·01	11	6·04	0·44	Beachy Head . .	317 40 1·56	8	4·45	0·51
Westminster Abbey	205 6 29·10	26	9·47	0·30	Ditchling . . .	328 42 47·76	9	8·99	1·46

[1] A correction of + 12″·47 to be applied to this bearing, to reduce it to Banstead, New.

LINCOLN MINSTER.

2-ft. Theodolite.

From 29th June to 24th September 1842. Observer: Lieut. DA COSTA, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ′		″			° ′ ″		″	
Buckminster Spire	12 38 7·20	4	1·30	0·15	Sutton Hill . .	73 55 38·32	5	6·40	1·88
Stathern . . .	26 45 39·55	4	3·80	1·58	Axedge . . .	90 26 29·90	4	2·60	0·55
Bardon Hill . .	42 38 8·84	8	9·49	2·72	Back Tor . . .	104 58 54·40	1	–	11·00
Holland Hill . .	59 54 16·15	4	4·39	1·36	Referring-object .	109 23 54·80	110	19·10	0·12

LINCOLN MINSTER—*continued.*

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Clifton Beacon .	118 53 55·72	6	6·95	1·78	Flint Hill . . .	262 2 27·90	6	4·00	0·51
Crowle	155 22 27·67	6	5·39	1·60	Greetham . . .	273 7 11·04	4	9·35	5·49
Normanby . .	211 20 55·65	4	4·09	1·42	Boston Tower[1] .	309 34 0·52	8	15·14	4·89
Maidenwell . .	259 0 21·62	7	6·34	1·30	Wellingore Spire	359 36 49·25	5	4·02	0·82

[1] A correction of +5″·54 to be applied to this bearing, to reduce it to the trigonometrical station.

LITTLE STIRLING.

3-ft. B.O., and 7-in. Theodolites.

From 5th to 7th July 1814. Observer: Mr. GARDNER.
From 7th to 23rd November 1850. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Blue Hill . . .	25 13 11·01	3	4·07	1·91	Peterhead, Old Windmill . .	190 13 32·49	2	17·75	78·76
Caerlock . . .	38 18 17·65	3	3·82	1·93	Reform Monument	192 3 36·72	21	62·13	30·59
Dudwick . . .	76 53 62·07	6	3·80	0·48	Great Stirling .	248 44 27·81	23	37·54	7·14
Mormonth . .	140 28 10·89	6	5·72	1·50					

LITTLETOWN DOWN.

18-in. Theodolite.

From 31st July to 1st August 1846. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ventnor Ch. Spire	26 40 49·89	13	–	–	Boniface Down .	205 52 42·56	9	12·98	3·37
Week Down . .	84 31 36·46	9	7·83	0·99	Boniface, S.E. .	228 2 4·08	9	13·55	3·87
Wroxall Down .	123 59 19·39	5	49·84	129·30	High Port Cliff .	302 59 44·16	9	20·07	10·71

LLANELIAN.

3-ft. Theodolite, B.O.

1805. Observer: Major-Gen. COLBY.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Arrenig . . .	1 30 32·25	4	7·02	3·68	Garreg . . .	261 22 48·81	4	4·50	1·41
Orme's Head . .	136 30 39·84	16	11·28	0·69	Moelfre Issa . .	275 16 27·19	4	0·99	0·06
Billinge . . .	247 4 30·33	4	8·90	5·43	Cyrn-y-Brain .	302 47 3·21	7	6·08	1·38
Gwaunysgaer .	251 38 33·34	1	–	–					

LONGMOUNT POLE.

3-ft. Theodolite, R.S.

From 10th October to 9th November 1843. Observers: Lieut. LUYKEN, R.E., and Priv. STEWART, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cradle	15 6 20·66	20	8·32	0·31	Ashley . . .	217 56 31·33	7	3·11	0·42
Stow	25 25 17·65	7	3·72	0·51	Axedge . . .	218 26 52·05	6	2·50	0·28
Radnor . . .	36 39 16·65	5	4·41	1·15	Bardon . . .	259 8 5·90	4	1·54	0·15
Llandinam . .	70 40 26·64	10	7·66	0·65	Lord Plymouth's Monument . .	287 46 20·50	1	–	5·61
Plynlimmon . .	82 28 39·68	14	7·32	0·40	Brown Clee . .	292 59 37·40	4	6·90	3·56
Cader Idris . .	104 6 29·26	12	3·64	0·19	Cleeve	319 31 18·79	8	9·21	1·41
Snowdon . . .	125 57 62·92	5	5·69	1·67	Malvern . . .	323 36 60·20	9	2·93	0·17
Cyrn-y-Brain .	159 15 37·13	7	3·79	0·41	May Hill . . .	338 19 39·71	3	3·79	1·86
Mowcopt . . .	214 23 42·65	1	–	5·61					

LOUGHANLEAGH.

12-in. Theodolite.

From 19th September to 17th October 1828. Observer: Lieut. HENDERSON, R.E.

Objects.	Bearings.	No. of Repetitions.	No. of Simple Arcs.	Recip. of Weight.	Objects.	Bearings.	No. of Repetitions.	No. of Simple Arcs.	Recip. of Weight.
	° ′ ″					° ′ ″			
Croghan . . .	21 50 40·23	57		0·09	Mullyash . . .	207 10 56·46	15		0·41
Knockastia . .	43 3 24·80	19		0·14	Slieve Gullion .	232 5 59·42	39		3·36
Slieve Bawn South	75 6 48·17	12		0·60	Slieve Donard .	244 23 35·62	15		3·41
Carn Clonhugh .	78 27 51·01	30		0·08	Castlecoo . . .	287 59 56·76	45		0·01
Cuilcagh . . .	119 2 41·12	98		0·08	Hill of Howth .	316 53 11·62	12		0·03
Carnmore . . .	148 58 19·23	51		0·75	Kippure . . .	334 49 16·84	61		0·14

LOUGH FOYLE BASE, NORTH END.

2-ft. Theodolite.

From 23rd November 1827 to 25th January 1828. Observers: Lieuts. HENDERSON, MURPHY, and MOULD, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Sawel	8 55 54·08	2	2·99	2·24	Cundtham . . .	107 49 47·66	11	9·03	1·02
South End of Base	17 12 28·36	6	5·68	1·21	Magilligan Tower	168 43 44·59	4	0·72	0·04
Drung Point . .	90 49 46·56	10	4·52	0·29	Mount Sandy .	197 12 24·41	20	4·16	0·14
Slieve Snaght .	101 37 42·67	9	6·23	0·89					

LOUGH FOYLE BASE, SOUTH END.

2-ft. Theodolite.

From 8th November 1828 to 14th January 1829. Observers: Capt. PRINGLE and Lieuts. MURPHY, HENDERSON, and MOULD, R.E.

Objects.	Bearings.	No. of Repe-titions.	No. of Simple Arcs.	Recip. of Weight.	Objects.	Bearings.	No. of Repe-titions.	No. of Simple Arcs.	Recip. of Weight.
	° ′ ″					° ′ ″			
Slieve Snaght .	129 31 56·89	27	25	0·08	Cundtham . . .	154 26 43·58	17	18	0·16
Ballykelly Church Tower . . .	140 4 59·08	–	2	–	Mount Sandy .	197 9 34·20	11	4	0·34
Drung Point . .	143 38 42·61	20	25	0·21	North End of Base	197 9 35·12	5	17	0·23

LUMSDEN.

3-ft. Theodolite, R.S.

From 20th April to 17th May 1846. Observer: Corp. WINZER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Sayrs Law . .	77 8 24·02	13	5·69	0·38	Craigowl . . .	145 12 15·14	4	2·04	0·31
East Lomond . .	120 57 6·35	4	3·15	0·68	Glashmeal . . .	146 45 28·95	6	4·80	0·84
Largo Law . .	129 20 20·27	7	2·06	0·13	Mount Battock .	164 19 13·28	6	2·72	0·33
Kellie Law . .	136 56 20·64	13	3·50	0·14	Caerlock . . .	171 29 4·50	1	–	2·88
Isle of May Lighthouse . . .	144 53 47·86	7	–	–	Blackheddon . .	329 43 41·07	9	3·87	0·29
					Cheviot . . .	355 23 36·69	26	5·92	0·17

LUNDY ISLAND.

3-ft. Theodolite, R.S.

From 17th February to 9th April 1845. Observer: Corp. STEWART, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cadon Barrow .	2 6 49·87	4	3·43	0·78	Referring-object .	194 25 50·99	77	4·25	0·01
Hensbarrow . .	6 48 31·46	11	1·87	0·05	Llannon . . .	208 54 0·84	10	6·52	0·73
Trevose Head .	20 21 44·54	11	6·01	0·52	Cefn Bryn . .	218 42 21·27	6	1·95	0·13
St. Ann's . . .	149 42 30·58	8	2·49	0·15	Margam Down .	234 55 24·22	9	7·14	0·91
Highgate . . .	161 26 20·25	5	1·97	0·23	Llangeinor . .	235 0 26·02	4	3·10	0·64
Newton Down .	165 1 46·99	4	1·50	0·16	Paracombe . .	268 43 51·51	11	4·82	0·36
Precelly . . .	175 22 41·09	21	5·31	0·17	High Wilhays .	318 47 54·03	19	5·63	0·22
Marros Beacon .	184 17 42·47	6	6·74	1·63	Brown Willy . .	355 34 34·88	11	3·22	0·20

LYNN TOWER.

2-ft. Theodolite.

From 12th March to 14th May 1843. Observer: Corp. Bay, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″	″		° ′ ″		″	
Ely Minster . .	13 18 15·66	11	4·42	0·30	Great Massingham Church . . .	261 51 31·67	2	0·73	0·13
Walpole, St. Peter's	78 4 55·21	5	4·75	1·30	Swaffham Spire[2] .	300 10 57·50	7	4·25	0·73
Boston Tower[1] .	131 37 53·21	8	4·85	0·56					
Docking Church Tower . . .	222 3 31·04	9	4·91	0·43					

[1] A correction of + 6″·42 has been applied to the observed bearing, to reduce it to the trigonometrical station.
[2] A correction of — 26″·95 „ „ „

LYON'S HILL.

3-ft. Theodolite, B.O.

From 18th to 23rd January 1829. Observer: Lieut. Portlock, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″	″		° ′ ″		″	
Cadeen . . .	4 41 48·89	5	3·94	0·78	Dublin Observatory . . .	230 30 27·65	9	7·77	1·19
Church Mountain	7 17 9·50	3	2·70	1·00	Howth . . .	253 16 17·71	9	6·77	0·77
Cullenagh . . .	52 38 42·48	1	–	6·47	Poolbeg Lighthouse . . .	257 11 11·09	2	0·86	0·18
Dunmurry . .	69 1 16·14	11	5·95	0·32	Kippure . . .	312 29 27·32	13	12·11	1·98
Hill of Allen . .	72 54 2·15	14	3·19	0·13	Lugnaquillia . .	352 26 29·94	5	3·34	0·65
Croghan . . .	97 27 11·52	4	1·96	0·24					
Garristown Windmill	197 13 33·19	6	4·69	0·99					

MAKER CHURCH TOWER.

3-ft. Theodolite, B.O.

From 8th May to 5th June 1846. Observer: Serg. Donelan, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Eddystone Lighthouse . . .	17 19 0·85	8	5·46	0·87	Kit Hill Tower .	158 4 21·63	8	7·52	1·39
Maker	64 45 31·87	102	5·52	0·02	Black Down . .	189 21 41·09	6	5·30	1·04
Deadman . . .	72 34 40·50	21	9·34	0·52	High Wilhays .	198 7 4·33	11	3·78	0·22
Lansallos . . .	88 23 12·80	7	7·52	1·56	Butterton, New .	250 37 0·91	7	5·19	1·06
Hensbarrow . .	95 22 0·47	32	6·92	0·16	Butterton, Old .	250 40 33·42	2	5·34	7·12
					Barrow Hill . .	290 36 1·87	25	7·77	0·38

MALVERN.

3-ft. Theodolite, R.S.

From 28th November 1843 to 27th January 1844. Observers: Corp. COSGROVE and Private STEWART, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Mendip . . .	8 18 15·31	2	3·37	2·83	Lord Plymouth's Monument . .	216 26 32·22	1	–	2·35
May Hill . . .	16 37 20·51	9	3·52	0·32	Bardon	225 12 34·95	2	0·48	0·06
Trellig	32 31 32·67	6	2·81	0·30	Arbury . . .	259 45 8·01	8	2·79	0·19
Cradle	72 45 56·74	7	3·94	0·50	Broadway Tower	284 22 17·73	8	1·80	0·07
Radnor . . .	107 23 40·59	1	–	2·35	Breedon Tower .	284 45 37·31	1	–	2·35
Stow	122 44 35·65	4	1·66	0·20	Cleeve[1]	311 51 16·28	8	2·48	0·25
Longmount Pole	144 2 9·46	9	5·14	0·54	Whitehorse Hill .	317 37 6·20	7	4·76	0·58
Brown Clee . .	156 47 59·22	9	3·36	0·22					

[1] A correction of −1″·04 to be applied to this bearing, to reduce it to the trigonometrical station.

MAMSUIL.

3-ft. Theodolite, R.S.

From 29th July to 31st August 1848. Observer: Corp. WINZER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Creachbheinn .	17 34 57·66	17	9·79	0·66	Ben Wyvis . .	215 57 42·07	34	8·52	0·25
Ben More in Mull	30 9 24·09	9	4·76	0·50	Corryhabbie . .	265 38 38·35	29	6·83	0·21
					Ben Macdui . .	284 16 43·37	19	11·64	0·58
Ben More, South Uist	89 52 66·71	21	10·15	0·38	Glashmeal . . .	292 21 18·68	4	4·97	2·07
Storr	112 3 36·22	32	6·45	0·17	Ben Amhlair . .	322 14 8·46	21	7·11	0·26
Cleisham . . .	127 42 37·92	14	6·44	0·37	Ben Lawers . .	325 48 38·03	24	9·37	0·37
Scournalapich .	200 15 29·23	44	8·38	0·17	Ben Nevis . . .	352 24 15·99	27	10·28	0·46

MENDIP.

3-ft. Theodolite, R.S.

From 6th September to 6th October 1844. Observer: Corp. STEWART, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Pillesdon . . .	23 59 8·40	14	5·74	0·31	Symonds Hall .	198 14 24·33	6	8·79	2·21
Staple Hill . .	51 14 48·51	5	6·13	1·60	Whitehorse Hill .	239 20 40·27	7	4·06	0·42
High Wilhays .	60 38 4·12	6	6·15	1·50	Inkpen . . .	258 23 8·23	10	5·70	0·62
Dunkery . . .	85 32 6·21	12	4·19	0·24	Westbury . .	260 26 28·88	11	4·19	0·21
Referring-object .	114 54 10·06	95	–	–	Beacon Hill . .	273 32 18·81	12	8·51	0·72
Cradle	154 4 52·05	11	7·10	0·66	Wingreen . . .	310 9 18·32	15	5·67	0·39
Dundry . . .	161 0 42·69	7	4·76	0·65	Bull Barrow . .	338 38 30·61	8	4·02	0·39
Malvern . . .	188 8 10·86	14	10·78	0·99	Mintern . . .	355 46 46·64	5	3·96	0·74

MERRICK.

3-ft. Theodolite, B.O.

From 25th May to 2nd August 1852. Observer: Corp. JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Glasserton . .	1 0 47·50	16	5·37	0·21	Corkmulaw . .	191 18 24·25	5	5·55	1·37
South Berule .	6 47 45·72	35	6·24	0·11	Bin of Campsie .	191 53 31·22	5	5·83	1·97
Mull of Galloway	24 8 34·59	16	6·88	0·35	Ben Cleugh . .	200 24 27·10	10	5·46	0·52
Slieve Donard .	41 52 49·08	24	6·23	0·19	Tinto	225 2 52·22	22	8·02	0·38
Cairn Piot . .	50 40 46·00	21	7·04	0·36	Cairnsmuir on Deugh . . .	231 15 32·31	37	6·03	0·11
Divis	60 3 52·46	11	9·83	1·06	Hart Fell . . .	245 45 7·87	28	5·11	0·14
Referring-object .	71 27 0·36	24	–	–	Wisp	259 47 3·98	15	3·34	0·07
Benereard . .	74 26 21·96	28	7·80	0·19	Burnswark . .	273 12 28·29	2	6·49	10·53
Trostan . . .	85 10 12·67	32	5·20	0·06	Cross Fell . .	290 6 10·11	10	5·29	0·55
Knocklayd . .	92 0 50·72	26	5·56	0·15	Criffel	292 1 53·45	20	4·72	0·23
Carn-na-leagh .	103 32 48·27	11	9·44	1·05	Ben Cairn . .	310 7 15·36	19	7·95	0·38
Saugh . . .	116 15 51·02	39	6·65	0·10	Sea Fell . . .	312 48 54·59	32	6·30	0·13
Ben Tuirc . .	124 47 55·94	14	6·01	0·36	Black Comb . .	322 42 25·75	36	6·52	0·12
Jura	131 50 45·62	18	7·41	0·31	Cairnsmuir of Fleet . . .	336 8 51·54	24	4·74	0·11
Goat Fell . . .	140 2 48·31	12	4·51	0·26	North Berule . .	357 3 53·75	5	3·73	0·83
Ben More in Mull	146 31 5·11	13	5·33	0·35	Snea Fell . . .	359 44 21·66	5	4·54	1·35
Hill of Stake . .	166 43 6·57	20	7·51	0·40					
Ben Lomond . .	175 0 40·00	29	5·59	0·08					
Ben Lawers . .	185 33 23·73	19	4·94	0·18					

MERRINGTON CHURCH TOWER.

12-in. Theodolite.

From 19th June to 15th July 1854. Observer: Mr. CAHILL.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Water Crag . .	50 59 25·98	4	11·77	11·69	Durham Observatory	182 43 5·14	2	8·79	19·31
Collier Law . .	112 38 40·42	19	17·09	2·04	Wordeslow . .	209 11 47·10	15	16·11	2·97
Brandon Down .	147 35 7·62	25	25·49	2·94					

MICKFIELD CHURCH TOWER.

18-in. Theodolite.

From 30th January to 25th February 1845. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Naughton . . .	43 45 14·22	21	22·29	2·34	Bunwell . . .	180 45 20·14	28	8·77	0·44
Lawshall . . .	77 2 6·28	21	15·38	1·65	Laxfield . . .	238 53 3·53	20	8·13	0·44
South Lopham .	157 3 39·76	12	7·38	0·84	Otley Tower . .	316 54 32·70	27	17·10	0·82

MILK HILL.

3-ft. Theodolite, R.S.

From 27th March to 13th May 1850. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Four-mile stone ·	0 46 6·98	30	9·05	0·17	Inkpen · · ·	275 28 53·21	14	6·24	0·47
Wingreen · · ·	22 33 36·80	23	11·36	0·47	Old Lodge · ·	332 6 26·50	22	8·79	0·39
Stoke Hill · ·	51 55 28·32	17	7·35	0·54	Beacon Hill · ·	337 13 60·15	40	7·40	0·08
Westbury Down ·	57 46 10·98	30	10·03	0·37	Dean Hill · · ·	340 8 59·05	21	5·89	0·22
Mendip · · ·	70 20 3·65	22	6·95	0·23	Thorney Down ·	341 53 54·93	28	16·87	0·60
Cleeve Hill · ·	170 0 32·47	11	4·30	0·39	Queen's Manor ·	347 45 43·15	27	8·44	0·23
Upcot Down · ·	194 49 34·51	23	5·17	0·18	Old Sarum Castle	354 12 7·45	20	3·42	0·08
Whitehorse Hill ·	221 32 44·62	3	5·86	3·84					

MISTERTON CARR BASE, NORTH END.

3-ft. Theodolite, B.O.

1801. Observer: Mr. WOOLCOTT.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
South End of Base	8 56 43·00	5	2·68	0·32	Gringley · · ·	354 27 0·10	5	4·15	1·05
Clifton Beacon ·	69 14 0·76	4	4·42	1·64					

MISTERTON CARR BASE, SOUTH END.

3-ft. Theodolite, B.O.

1801. Observer: Mr. WOOLCOTT.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Clifton Beacon ·	90 1 14·21	6	5·50	1·49	Gringley · · ·	335 9 40·46	9	9·84	2·15
North End of Base	188 56 39·04	8	8·88	2·04					

MOELFRE ISSA.

3-ft. Theodolite, B.O.

1806. Observers: Gen. COLBY and Mr. WOOLCOTT.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Llanelian · · ·	95 24 3·23	4	5·94	2·66	Gwaunysgaer ·	234 39 39·69	5	3·46	0·49
Orme's Head · ·	117 13 28·34	5	6·29	2·06	East End of Base	236 41 33·59	5	6·96	2·86
West End of Base	172 39 20·30	7	5·50	0·75	Cyrn-y-Brain · ·	310 51 38·29	4	4·25	1·17

MONACH.

3-ft. Theodolite, B.O.

From 23rd October to 6th September 1839. Observers: Lieuts. ROBINSON and HORNBY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cleisham . . .	34 17 34·22	28	10·23	0·35	Ben Clibrig . .	276 5 27·91	11	19·87	4·43
North Rona . .	198 23 18·91	6	7·16	2·28	Suilbheinn . .	290 44 45·41	13	0·59	5·99
Cape Wrath Lighthouse . . .	248 1 17·64	8	8·36	2·28	Scournalapich[1] .	325 25 55·65	6	9·51	3·50
Cnoc-ghiubhais .	251 14 38·60	1	–	21·44	Ru Rea . . .	330 49 47·21	7	9·02	2·32
Fashven . . .	253 50 2·38	13	7·22	0·86	Storr	355 23 32·70	20	6·93	0·32

[1] A correction of + 38″·61 to be applied to this bearing, to reduce it to the new station.

MORDINGTON.

3-ft. Theodolite, R.S.

From 6th March to 15th April 1846. Observer: Corp. WINZER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cheviot . . .	7 18 14·94	41	6·11	0·06	Sayrs Law . .	96 52 19·37	16	6·15	0·38
Wisp Hill . . .	44 31 36·96	13	4·43	0·28	Referring-object .	148 27 44·92	89	–	–
Dunrich . . .	69 55 53·87	24	7·56	0·20	Blackheddon . .	334 36 31·82	11	5·06	0·43

MORMONTH.

3-ft. Theodolite, B.O.

From 22nd February to 22nd April 1847. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Dudwick . . .	0 41 51·73	23	9·35	0·39	Manor Lee . .	88 56 8·24	7	3·40	0·35
Blue Hill . . .	5 44 51·69	10	6·62	0·86	Scarabin . . .	127 19 52·89	20	4·53	0·13
Brimmond . .	14 32 48·95	16	6·95	0·60	Ben Cheilt . .	135 41 6·01	19	4·43	0·18
Caerlock . . .	22 21 37·06	29	6·68	0·15	Peterhead Old Windmill . .	305 1 22·02	5	2·56	0·40
Mount Battock .	30 44 12·83	28	6·82	0·22	Reform Monument, Peterhead	311 11 51·15	11	6·56	0·73
Buck	58 32 58·00	10	4·53	0·35	Little Stirling .	320 16 52·92	22	9·07	0·35
Corryhabbie . .	68 6 52·11	37	6·35	0·10					
Referring-object .	84 55 17·02	150	–	–					
Knock	87 39 35·32	13	4·92	0·30					

MOUNT BATTOCK.

3-ft. Theodolite, B.O.

From 10th March to 3rd July 1847. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Kellie Law	1 44 36·83	8	2·87	0·21	Mormonth	210 8 29·70	17	5·06	0·23
Largo	8 8 44·90	2	1·26	0·39	Dudwick	218 11 31·52	9	2·73	0·18
Carnethy Cairn	15 8 24·21	10	4·01	0·29	Brimmond	230 39 12·00	18	5·29	0·30
Craigowl	20 36 1·99	16	7·46	0·35	Referring-object	238 28 21·50	11	–	–
East Lomond	20 42 15·68	13	4·29	0·27	Blue Hill	246 18 30·84	18	6·19	0·25
Ben Cleugh	37 1 40·95	16	6·49	0·41	Caerlock	256 17 57·64	17	4·90	0·26
Glashmeal	77 43 54·99	28	5·53	0·10	Broxy	278 15 57·36	12	5·20	0·26
Ben Macdui	103 51 13·60	33	6·06	0·11	Red Head	337 33 33·05	14	5·65	0·56
Corryhabbie	148 11 18·51	25	4·97	0·16	Lumsden	343 52 19·28	20	3·74	0·12
Buck	159 53 9·51	24	6·62	0·21	Sayrs Law	357 54 26·38	14	2·66	0·11
Knock	178 21 48·49	14	3·16	0·13					

MOUNT SANDY.

2-ft. Theodolite.

From 24th June to 6th July 1829. Observers: Capt. DAWSON and Lieuts. HENDERSON and MURPHY, R.E.

Objects.	Bearings.	No. of Repetitions.	No. of Simple Arcs.	Recip. of Weight.	Objects.	Bearings.	No. of Repetitions.	No. of Simple Arcs.	Recip. of Weight.
	° ′ ″					° ′ ″			
Sawel	9 39 17·31	–	16	0·64	Slieve Snaght	93 46 32·44	–	17	0·38
North End of Base	17 13 11·94	12	22	0·06	Magilligan Tower	121 4 13·12	–	1	–
South End of Base	17 13 15·38	–	15	0·21	Ben Tartevil	206 51 3·00	–	11	2·25
Drung Point	77 4 22·13	–	17	0·22	Jura	215 46 50·53	–	1	–
Cundtham	91 13 1·16	5	17	0·19	Knocklayd	272 37 40·18	–	16	0·37

MOWCOPT.

3-ft. Theodolite, B.O.

From 11th March to 18th May 1851. Observer: Corp. JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Referring-object	280 37 6·87	27	6·07	–	Cyrn-y-Brain	82 51 5·27	12	8·35	0·75
Brown Clee	20 17 50·68	12	4·72	0·33	Snowdon	88 22 37·53	4	2·83	0·66
Black Bank	24 6 54·56	29	6·96	0·17	Delamere	110 48 33·86	20	7·14	0·21
Ashley Heath	27 14 25·41	28	7·81	0·20	Bellefield	137 22 24·55	14	7·79	0·54
Longmount Pole	34 54 58·85	27	7·22	0·24	Whittle Hill	176 42 54·73	26	5·14	0·12
					Holme Moss	204 49 21·81	18	2·64	0·07
Hawkestone Obelisk	44 26 24·55	10	6·16	0·66	Axedge	233 19 29·94	40	8·46	0·17
					Weaver Hill	295 6 49·36	26	8·38	0·28
Nantwich Church Tower Vane	75 38 1·62	15	6·89	0·57	Bardon Hill	306 13 50·05	16	4·67	0·20

NASEBY CHURCH TOWER.

18-in. Theodolite.

From 24th October 1842 to 3rd March 1843. Observers: Corp. STEEL and Priv. McNALLY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Arbury Hill	38 24 50·73	11	17·71	3·64	Easton Tower	231 16 3·01	6	15·84	10·05
Bardon Hill	147 39 59·43	5	10·75	6·48	Keysoe Spire	292 55 18·90	4	9·78	7·49
Tilton	196 36 28·05	10	11·96	1·96	Hanslope Ch. Spire	340 42 0·84	6	8·51	3·09

NAUGHTON CHURCH TOWER.

18-in. Theodolite.

From 9th November 1844 to 10th January 1845. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Stoke Tower	18 22 2·83	15	7·85	0·62	South Lopham	185 20 15·80	1	–	11·45
Danbury Spire	31 21 38·20	3	5·69	3·69	Mickfield[1]	223 37 18·02	17	15·43	1·73
Thaxted Spire	68 49 53·38	11	6·55	0·45	Otley Tower	254 1 5·98	7	7·14	1·15
Lawshall Tower	110 57 22·84	19	6·24	0·36	Walton Tower	318 39 1·17	18	7·20	0·39

[1] A correction of −1″·31 to be applied to this bearing, to reduce it to the trigonometrical station.

NEPHIN.

3-ft. Theodolite, B.O.

From 25th September to 2nd November 1828. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Bencorr	26 31 13·51	6	10·90	3·94	Cuilcagh	257 46 51·47	11	11·95	2·25
The Reek	34 20 55·15	17	5·90	0·27	Mullaghanoe	281 0 52·90	4	3·11	0·70
Slieve More	89 46 54·48	12	16·60	2·89	Slieve Bawn, South	290 28 0·17	2	3·21	2·57
Tawnaghmore	154 28 24·12	6	3·65	0·70	Keeper	331 52 45·74	2	0·91	0·20
Slieve League	210 56 29·82	11	14·71	3·49	Knockma	335 28 50·67	7	2·54	0·17
Knockalongy	242 54 14·74	18	9·29	0·83	Slieve Carran	346 43 19·93	2	1·31	0·43

NIVE HILL.

7-in. Theodolite.

From 27th February to 6th March 1847. Observer: Priv. JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Balta	1 20 56·07	18	65·00	21·27	Gerth of Scaw	195 10 43·57	19	68·67	32·88
Saxavord	141 43 10·15	19	69·08	31·22					

NODES BEACON.

18-in. Theodolite.

From 17th May to 23rd June 1845. Observers: Corps. WINZER and STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Swyre Barrow	80 22 42·41	12	10·33	1·73	Sussex Lodge	210 47 5·77	7	10·23	3·21
Coringdon	82 53 22·23	15	7·03	0·52	Butser	228 36 6·16	21	14·31	0·90
Black Down	92 10 44·82	6	7·47	1·83	Rooks Hill	245 25 16·40	14	9·22	0·62
Wingreen	131 48 4·83	13	10·06	0·98	Motteston	274 41 10·72	11	9·25	1·68
Dean Hill	169 3 35·12	8	6·30	0·92	Dunnose	282 24 15·43	18	15·97	1·84
Inkpen	183 59 53·04	8	8·62	1·75	Week Down	289 21 38·81	4	9·03	5·28
Southampton	199 29 5·59	10	6·19	0·92					

NORWICH CATHEDRAL SPIRE.

18-in. Theodolite.

From 9th November to 8th December 1843. Observers: Corp. STEEL and Priv. M^cNALLY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Bunwell Tower	36 49 0·53	7	8·80	2·42	Gorleston Tower	281 31 22·57	5	17·38	13·25
Hingham Tower	75 0 56·67	7	7·28	1·87	Tofts Tower	308 11 26·39	2	4·29	4·60
Baconsthorpe	161 34 58·78	4	3·24	0·92	Laxfield Tower	353 9 38·24	1	–	18·01
Happisburgh	215 42 35·41	2	3·68	3·38					

NORWOOD.

18-in. Theodolite.

From 30th April to 31st May 1844. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Stede Hill	29 30 5·94	1	–	13·32	Gads Hill Obelisk[1]	89 58 50·27	2	1·33	0·44
Hollingbourne	36 28 8·56	16	12·70	1·29	Langdon Hill	118 34 15·96	11	10·94	1·51
Deptling	53 11 3·79	15	16·31	2·09	Danbury Spire	151 58 21·96	14	10·25	1·12
Wrotham	74 23 7·50	4	0·66	0·03	Walton Tower	211 57 41·08	14	9·72	1·34
					St. Peter's Ch.	277 22 43·36	12	10·72	1·41
Chatham Observatory	84 45 32·67	4	5·66	2·52	Frittenfield	359 46 51·42	13	9·87	1·14

[1] A correction of + 13′ 8″·06 to be applied to this bearing, to reduce it to the trigonometrical station.

OA or CAIRNARD.

3-ft. Theodolite.

From 25th June to 14th July 1822. Observers: Major-Gen. Colby, Capts. Vetch and Dawson, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Benyevenagh . .	34 40 51·04	5	2·87	0·50	Goat Fell . . .	270 3 60·58	3	1·72	0·34
Slieve Snaght .	54 7 27·34	3	2·05	0·55	Ben Tuirc . .	278 36 36·31	9	6·51	0·73
Innistrahul Lighthouse . . .	69 47 60·00	4	3·07	0·66	Carn-na-Leagh .	317 18 52·03	13	5·79	0·44
Ben Tartevil . .	138 0 9·29	19	7·01	0·29	Cantyre Lighthouse . . .	318 50 57·58	2	0·66	0·11
Ben More in Mull	191 2 46·82	2	3·50	3·06	Trostan . . .	352 17 3·95	3	1·63	0·34
Jura	210 56 3·97	12	8·20	0·67	Knocklayd . .	357 1 49·61	7	6·50	1·18
Ben Varn . . .	221 38 51·93	25	6·81	0·21					

OLD LODGE BEACON.

3-ft. Theodolite, B.O.

From 26th October to 6th December 1849. Observer: Corp. Jenkins, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Dean Hill . . .	1 54 22·83	18	3·58	0·13	Four-mile Stone .	83 18 13·92	11	8·55	1·00
Salisbury Spire .	53 38 34·38	39	8·48	0·13	Westbury Down .	112 10 56·81	23	5·45	0·19
Thorney Down .	56 59 59·75	48	6·73	0·10	Stoke Hill . .	117 57 34·29	42	5·65	0·09
Wingreen . . .	62 57 58·43	17	4·74	0·18	Beacon Hill . .	136 20 45·04	47	5·78	0·06
Old Sarum Castle	67 30 27·76	27	5·82	0·19	Milk Hill . . .	152 16 11·39	42	7·41	0·16
Old Sarum Gun .	67 45 53·27	28	3·81	0·09	Inkpen	207 39 39·95	27	8·08	0·29

OLD SARUM CASTLE.

3-ft Theodolite, B.O.

From 19th July to 27th August 1849. Observer: Serg. Donelan, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Four-mile Stone .	128 30 42·12	17	9·08	0·57	Thorney Down .	255 41 0·51	18	7·37	0·44
Stoke Hill . .	137 55 26·77	15	5·03	0·23	Queen's Manor .	281 21 23·26	23	6·66	0·23
Milk Hill . . .	174 13 50·91	3	1·02	0·12	Dean Hill . . .	5 33·97	13	6·44	0·59
Beacon Hill . .	209 57 27·71	22	4·35	0·17	Salisbury Cathedral Spire . .	349 51 48·43	10	4·61	0·51
Old Sarum Gun .	242 3 55·68	16	15·65	1·62					
Old Lodge . .	247 22 24·20	15	4·59	0·28					

OLD SARUM GUN.

3-ft. Theodolite, B.O. and R.S.

From 18th June to 8th July 1849. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Old Sarum Castle	62 4 59·82	19	4·06	0·11	Old Lodge . .	247 38 51·96	22	4·97	0·18
Four-mile Stone .	122 29 26·42	49	8·71	0·09	Thorney Down .	256 54 2·47	44	7·35	0·09
Beacon Hill . .	208 27 46·74	50	6·29	0·04	Queen's Manor .	287 10 59·48	52	7·09	0·08

ORFORD CASTLE.

3-ft. Theodolite, B.O.

From 14th October to 14th November 1843. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Walton Tower .	33 14 27·11	11	7·42	0·69	Laxfield . . .	154 8 48·25	12	3·78	0·20
Referring-object .	68 51 29·77	27	–	–	Southwold . .	201 9 38·31	8	7·00	0·91
Otley	106 4 4·50	10	4·50	0·42					

OTLEY CHURCH TOWER.

18-in. Theodolite.

From 30th September to 31st October 1844. Observers: Corps. STEEL and BEATON, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Stoke Tower . .	51 57 31·35	14	7·37	0·67	Referring-object .	207 24 10·00	57	–	–
Naughton . . .	74 13 32·86	14	7·56	0·73	Orford Castle . .	285 47 7·61	23	13·16	0·71
Mickfield[1] . . .	136 59 14·11	13	14·25	2·50	Walton Tower .	351 33 20·80	22	9·91	0·46
Laxfield . . .	210 18 24·05	16	11·95	1·33					

[1] A correction of — 5″·46 to be applied to this bearing, to reduce it to the trigonometrical station.

OVER HILL.

3 ft. Theodolite, B.O.

15th June 1817. Observer: Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Blue Hill . . .	3 46 37·68	5	3·38	0·60	Dudwick . . .	192 15 24·02	5	7·35	2·22
Caerlock . . .	38 6 51·49	4	2·94	0·60	Little Stirling .	217 58 54·48	3	2·60	0·77
Brimmond . .	40 38 13·93	5	7·28	2·42	Layton . . .	240 12 44·04	6	3·15	0·33
Mount Battock .	48 42 10·40	5	5·53	1·79	Tarbathy . . .	335 9 4·80	6	5·40	1·10
Mormonth . . .	186 34 1·54	4	3·00	0·58					

PADDLESWORTH.

3-ft. Theodolite, B.O.

From 10th May to 9th June 1844. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Lydd Church Tr.	42 8 24·13	14	7·69	0·75	Goudhurst Church Tower . . .	90 14 34·84	12	7·48	0·59
Fairlight . . .	54 24 0·36	13	4·71	0·24	Frittenfield . .	115 48 18·61	8	4·51	0·80
Brightling Observatory Dome .	72 43 24·38	4	1·67	0·20	Walton Tower .	187 2 5·85	10	3·83	0·27
Tolsford . . .	73 55 27·94	6	4·35	0·69	St. Peter's Church Tower . . .	214 50 43·80	12	9·76	0·95
Crowboro' . . .	85 0 24·73	12	7·45	0·58	Folkstone . . .	296 43 0·05	14	4·63	0·27
Frant Church Tr.	88 41 47·20	5	3·75	0·75					

PARACOMBE

3-ft. Theodolite, R S.

From 8th December 1844 to 8th February 1845. Observer: Corp. STEWART, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
High Wilhays .	11 0 11·11	11	6·64	0·79	Precelly . . .	143 53 29·43	10	5·48	0·68
Kit Hill . . .	22 48 39·72	2	0·95	0·22	Cefn Bryn . .	156 44 24·03	4	2·94	0·69
Bradbury Castle .	22 58 10·97	4	1·13	0·08	Llannon . . .	162 33 54·99	7	5·39	0·84
Brown Willy . .	39 0 50·94	11	12·06	1·76	New Inn . . .	164 43 16·77	3	2·74	1·01
Cadon Barrow .	45 49 45·36	6	3·91	0·51	Margam Down .	193 15 2·75	7	4·06	0·40
Referring-object .	87 45 7·06	19	6·95	–	Llangeinor . .	201 4 4·29	10	8·30	0·88
Lundy Island Lighthouse[1] .	89 21 53·42	10	12·32	2·37	Ogmoor Down .	208 49 51·90	9	6·35	0·91
Highgate . . .	126 9 38·73	2	3·64	3·31	Cradle	210 22 18·29	12	4·45	0·30
Marros Beacon .	140 59 8·31	5	6·32	1·96	Dunkery . . .	274 10 27·46	16	10·98	0·91

[1] A correction of + 0″·62 to be applied to this bearing, to reduce it to the trigonometrical station.

PENDLE HILL.

2-ft. Theodolite.

From 28th October 1841 to 7th January 1842. Observer: Lieut. DA COSTA, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Rivington . . .	28 26 23·54	6	7·53	2·01	Little Whernside	170 22 40·38	10	8·22	0·98
Go Hill . . .	50 8 6·49	3	5·95	4·35	Water Crag . .	189 34 22·95	1	–	16·63
Snowdon . . .	53 41 41·55	4	9·83	6·67	Great Whernside	211 2 24·71	8	8·45	2·54
Beryl Hill . . .	86 30 49·34	4	3·69	1·17	Boulsworth . .	294 34 0·05	9	13·76	4·75
Black Comb . .	123 7 50·48	9	11·17	2·63	Holme Moss . .	323 5 15·60	7	10·95	3·76
Sca Fell . . .	137 55 15·62	8	6·76	1·47	Kinder Scout . .	333 3 4·52	6	5·42	1·35
Helvellyn . . .	147 39 54·53	3	5·16	3·58	Axedge . . .	341 44 19·30	4	6·16	2·87
Ingleboro' . . .	168 56 41·80	9	9·09	1·82	Whittle . . .	354 35 19·80	8	10·96	2·92

PENINNIS WINDMILL.

18-in. Theodolite.

From 29th April to 21st May 1850. Observer: Corp. WOTHERSPOON, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
St. Agnes' Lighthouse . . .	59 10 45·05	62	17·37	0·36	St. Martin's Day-mark[1] . . .	203 17 45·41	12	19·50	5·24
Beacon Hill, Trescow	156 9 12·81	12	28·76	9·21	Karn Galver . .	239 44 56·14	18	16·08	1·51
Telegraph Tower	182 37 45·99	11	24·36	10·29	Pertinny . . .	244 42 15·92	17	16·15	2·10
					Wolf Rock . .	263 9 32·80	5	10·57	5·96

[1] A correction of — 7′ 28″·99 to be applied to this bearing, to reduce it to the trigonometrical station.

PERTINNY.

3-ft. Theodolite, B.O.

From 31st October to 30th November 1845. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Wolf Rock . .	33 15 3·73	8	9·30	2·33	Hensbarrow . .	242 5 44·94	21	3·28	0·07
Lighthouse, Scilly Island . . .	64 52 26·05	9	4·97	0·52	Karnbrea Monument . . .	245 40 41·60	8	8·12	1·18
Peninnis Windmill	65 12 32·99	8	4·64	0·55	Karnbonellis . .	254 2 57·34	10	7·46	0·79
Telegraph Tower	67 29 40·74	8	4·81	0·79	Tregonning . .	265 12 34·54	9	10·63	1·72
St. Martin's Day-mark[1] . . .	70 54 37·63	10	5·19	0·53	St. Keverne Spire	278 38 46·86	8	4·86	0·66
Beacon Hill, Trescow Watch-house[2]	72 12 43·32	3	5·62	4·25	Goonhilly . . .	281 8 53·93	29	7·27	0·22
Centre of Tor on Western Scilly	73 55 52·80	10	4·68	0·38	Grade Church Tr.	292 31 24·68	10	7·53	0·83
Karn Galver . .	199 27 37·85	7	2·99	0·33	Lizard Lighthouse, East	296 59 47·32	7	4·80	0·75
Karnminnis . .	219 5 50·74	18	6·86	0·33	Lizard Lighthouse, West	297 3 51·07	10	6·23	0·71
					Referring-object .	31 21 40·02	51	5·19	—

[1] A correction of + 43″·28 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of + 1′ 4″·02 " " "

PETERHEAD OLD WINDMILL.

7-in. Theodolite.

12th December 1850. Observer: Serg. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Reform Monument	7 11 41·29	9	46·22	41·80	Great Stirling Sector Station .	357 5 23·51	9	36·61	35·95
Little Stirling .	10 13 45·73	9	37·80	26·98					
Mormonth . .	125 12 40·73	9	67·58	74·18					

PILLESDON.

3-ft. Theodolite, B.O.

From 12th April to 1st June 1845. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Golden Cape . .	4 24 23·16	9	3·42	0·23	Staple Hill . .	131 17 9·02	8	9·11	1·88
Barrow Hill . .	43 49 37·80	6	1·99	0·15	Mendip . . .	203 46 21·98	15	8·37	0·65
Furland . . .	46 37 6·67	15	5·85	0·29	Wingreen . . .	248 29 4·97	19	6·60	0·26
Little Haldon .	61 48 16·19	10	6·78	0·74	Coringdon . .	287 54 35·33	18	9·80	0·59
High Willhays .	81 13 1·25	13	6·95	0·61	Swyre Barrow .	292 54 23·77	14	3·50	0·17
Dumpdon . . .	96 14 36·17	2	0·16	0·00	Black Down . .	303 32 13·12	23	16·63	1·51
Dunkery . . .	127 7 23·57	19	7·15	0·39					

POOLBEG LIGHTHOUSE.

7-in. Theodolite.

11th September 1844. Observer: Mr. H. GORDON.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Howth	237 30 9·01	6	33·40	35·42	Kingstown Observatory . . .	346 46 46·48	6	28·33	28·85

PRECELLY.

3-ft. Theodolite, R.S.

From 12th April to 19th July 1843. Observer: Lieut. LUYKEN, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Highgate . . .	19 46 15·25	5	6·04	1·64	Trigarron . . .	242 57 9·15	8	6·75	1·10
St. Ann's . . .	44 9 11·62	2	0·09	0·00	New Inn . . .	264 20 15·82	7	8·79	2·13
Forth	109 17 7·20	4	1·94	0·35	Cradle	269 1 18·00	6	2·52	0·25
Mount Leinster .	119 25 49·10	1	–	5·52	Llangeinor . .	291 46 37·13	7	6·67	1·23
Tara	130 58 32·43	7	3·41	0·37	Lannon . . .	296 36 17·48	4	6·80	3·49
Croghan . . .	132 27 1·81	4	3·89	1·05	Margam . . .	298 4 22·48	6	9·61	2·70
Kippure . . .	143 0 5·47	2	0·54	0·07	Cefn Bryn . .	312 40 32·60	6	6·30	1·47
Rhiw	185 35 16·00	8	7·33	1·41	Dunkery . . .	316 8 41·50	7	2·68	0·30
Snowdon . . .	200 30 37·13	12	4·54	0·26	Paracombe . .	323 10 24·48	5	2·23	0·24
Pengarn . . .	211 33 49·64	9	7·62	1·03	Marros Beacon .	332 27 6·43	3	1·72	0·36
Referring-object .	211 58 30·09	127	–	–	High Willhays .	338 55 14·87	6	2·73	0·41
Cader Idris . .	214 48 14·20	14	5·45	0·29	Lundy Island .	355 17 52·94	6	5·36	1·05
Aberystwith . .	221 29 36·20	5	1·16	0·09	Lundy Lighthouse	355 19 36·21	5	5·09	1·37
Plynlimmon . .	229 4 38·06	13	8·46	0·67	Cadon Barrow .	357 58 19·73	6	2·22	0·18

QUEEN'S MANOR.

3-ft. Theodolite, R.S.

From 9th September to 2nd October 1849. Observer: Colour-Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Wingreen . . .	65 54 34·44	10	4·94	0·53	Milk Hill . . .	167 48 29·79	1	–	4·72
Old Sarum Castle	101 22 29·74	40	8·28	0·15	Beacon Hill . .	190 29 44·66	43	6·73	0·10
Old Sarum Gun .	107 11 3·62	51	9·67	0·14	Thorney Down .	228 31 29·56	43	13·09	0·19
Four-mile Stone .	116 6 4·60	25	6·28	0·17	Butser Hill . .	282 15 13·92	15	2·87	0·08
Westbury Down .	125 18 13·53	4	3·01	0·64	Dean Hill . . .	312 1 23·27	14	5·95	0·42
Stoke Hill . .	133 32 4·13	14	7·65	0·43					

RED HEAD.[1]

3-ft. Theodolite, R.S.

From 22nd March to 29th April 1847. Observer: Corp. WINZER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Sayrs Law . .	7 25 41·40	15	8·35	0·76	Glashmeal . .	118 30 14·67	15	11·01	0·89
Kellie Law . .	23 35 59·24	20	5·39	0·35	Mount Battock .	157 45 55·95	30	7·07	0·20
Largo Law . .	32 22 5·03	14	5·01	0·23	Caerlock . . .	179 15 59·54	28	9·82	0·26
East Lomond .	47 33 34·46	13	4·30	0·25	Broxy	200 4 21·07	19	6·34	0·26
Craigowl . . .	76 58 48·56	23	6·70	0·24	Lumsden . . .	347 12 37·14	11	3·41	0·24

[1] This point is not identical with that observed in the old observations: the old station is calculated from these observations, and those taken at the old station in 1814 to have been 48·57 ft. distant from the new, with the bearing 293° 11′.

RHUDDLAN BASE, EAST END.

3-ft. Theodolite, B.O.

1806. Observer: Mr. WOOLCOTT.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Moelfre Issa . .	56 46 27·47	5	5·58	1·37	Orme's Head . .	101 37 20·84	2	0·03	0·00
West End of Base	96 58 59·85	9	3·91	0·26	Gwaunysgaer .	232 11 0·86	8	10·94	2·40

RHUDDLAN BASE, WEST END.

3-ft. Theodolite, B.O.

1806. Observer: Mr. WOOLCOTT.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Orme's Head . .	103 24 47·23	5	6·88	1·99	East End of Base	276 54 3·23	5	6·87	2·55
Gwaunysgaer .	256 20 1·63	5	5·15	1·27	Moelfre Issa . .	352 39 23·95	6	8·28	2·81

RONAS.

3-ft. Theodolite, B.O.

From 4th to 10th August 1821. Observers: Maj.-Gen. COLBY, and Capts. VETCH and DRUMMOND, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Foula	39 35 37·90	9	6·71	0·80	Yell	265 14 21·12	21	7·30	0·31
Saxavord . . .	224 59 30·31	13	7·65	0·69	Noss Head . .	3[illegible]0 47 9·38	4	–	–
Vallafield . . .	229 21 48·15	1	–	–	Brassa	336 39 5·90	9	3·81	0·24
Fetlar	252 55 19·67	10	6·54	0·70					

RU REA.

3-ft. Theodolite, R.S.

From 22nd May to 26th June 1848. Observer: Corp. WINZER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Storr	34 21 52·82	31	6·64	0·16	Monach . . .	151 17 46·10	16	7·23	0·50
Cleisham . . .	103 24 1·39	22	4·79	0·21	Cnoc-ghiubhais .	208 33 9·90	7	4·54	0·75
Barvas	137 15 53·37	3	1·93	0·47					

RYDER'S HILL.

3-ft. Theodolite, B.O.

From 26th July to 22nd August 1845. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Eddystone Lighthouse . . .	36 33 46·55	8	5·79	0·84	Pillesdon . . .	245 29 16·65	15	5·49	0·33
Goonhilly Down .	61 23 1·16	20	5·65	0·22	Golden Cape . .	251 22 5·27	9	6·89	0·82
Deadman . . .	64 19 4·83	7	2·17	0·14	Little Haldon .	255 2 16·56	11	7·40	0·66
Lansallos . . .	69 4 41·09	7	4·53	0·57	Black Down . .	257 32 16·59	12	3·32	0·14
Hensbarrow . .	78 41 28·21	15	8·19	0·44	Swyre Barrow .	264 19 37·28	9	3·69	0·29
Kit Hill Tower .	93 6 59·16	5	4·58	1·49	Vern	265 51 24·26	14	5·15	0·23
Brown Willy . .	100 51 18·82	12	6·82	0·57	Furland . . .	302 35 46·65	5	6·35	1·99
Cadon Barrow .	106 11 43·81	15	7·01	0·42	Referring-object .	310 0 44·97	138	6·16	0·01
High Wilhays .	157 0 20·43	14	6·41	0·49	Barrow Hill . .	335 35 49·97	11	4·68	0·41

ST. AGNES' BEACON.

3-ft. Theodolite, B.O.

From 20th to 23rd May 1797. Observer: Gen. MUDGE.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Karnbonellis	3 27 26·26	1	–	7·79	Hensbarrow	253 4 18·68	7	1·13	0·03
Karnminnis	61 14 4·20	7	4·70	0·50	Deadman	287 35 38·79	3	6·63	5·25
Trevose Head	205 54 16·89	6	7·40	1·94					

ST. ANNE'S.

3-ft. Theodolite, B.O.

April 1792. Observer: Gen. MUDGE.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
King's Arbour	209 8 55·69	1	–	0·68	Shooter's Hill	258 38 25·00	2	0·00	0·00
Hanger Hill[1]	228 10 7·99	11	3·92	0·18	Banstead (Old)[2]	292 6 54·18	7	2·54	0·25
Hampton Poor-house	253 27 47·75	5	2·21	0·25	Leith Hill (1792)[3]	336 9 56·72	2	1·18	0·35

[1] A correction of — 16′ 20″·56 to be applied to this bearing, to reduce it to the station on the Tower.

[2] A correction of + 14″·97 " " Banstead, New.

[3] A correction of + 1′ 18″·38 " " the station on the Tower.

ST. MARTIN'S HEAD.

18-in. Theodolite.

From 29th June to 13th July 1850. Observer: Corp. WOTHERSPOON, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Peninnis Windmill	23 11 57·05	19	8·69	0·53	Karnminnis	243 53 1·85	13	4·51	0·31
Telegraph Tower	32 33 34·75	17	6·17	0·51	Karn Galver	244 27 23·25	25	12·51	0·66
Beacon Hill Trescow, Watchhouse[1]	83 3 29·78	8	10·57	2·48	Pertinny	250 26 49·18	19	6·29	0·43
					Wolf Rock	273 55 34·10	7	7·12	1·37

[1] A correction of + 8′ 32″·97 to be applied to this bearing, to reduce it to the trigonometrical station.

ST. PETER'S CHURCH TOWER.

18-in. Theodolite.

From 14th February to 15th April 1844. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Folkstone . . .	27 35 32·17	5	6·79	2·43	Deptling . . .	83 0 23·37	21	11·38	0·96
Paddlesworth . .	35 4 14·13	10	4·84	0·38	Norwood . . .	97 50 10·63	1	–	9·33
Frittenfield . .	66 39 16·78	5	4·66	1·13	Danbury Spire[1] .	124 2 9·17	5	4·39	0·98
Hollingbourne .	78 54 46·17	10	5·36	0·57	Walton Tower .	170 48 35·69	24	9·34	0·50

[1] A correction of + 9″·93 to be applied to this bearing, to reduce it to the trigonometrical station.

SAWEL.

3-ft. Theodolite, B.O.

From 24th August to 20th September 1827. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Shantavny . .	4 42 58·36	8	4·50	0·48	Benyevenagh .	193 28 38·66	31	6·48	0·15
Carnmore . . .	13 58 38·27	6	6·18	1·44	Knocklayd . .	232 36 55·14	14	7·46	0·63
Cuilcagh . . .	36 17 40·47	8	11·45	4·19	Trostan . . .	245 42 3·63	9	3·92	0·39
Bessy Bell . .	56 59 9·35	23	7·31	0·31	Divis	289 0 24·91	11	6·12	0·57
					Slieve Gallion .	298 32 21·29	12	6·26	0·61
Barnesmore Connell	80 22 38·67	2	12·89	41·54	Slieve Donard .	314 2 12·06	3	2·21	0·69
Eskaheen . . .	151 49 37·31	15	5·95	0·33	Slieve Gullion .	332 55 40·85	1	–	7·22
Slieve Snaght .	155 54 24·72	11	5·21	0·39	Vicar's Carn . .	333 33 36·69	2	4·47	4·99
North End of Base	188 51 27·97	26	8·07	0·19	Armagh Breagh .	337 8 22·70	1	–	7·22
Mount Sandy .	189 34 6·97	10	3·39	0·19	Loughanleagh .	354 55 29·64	6	2·05	0·16

SAXAVORD.

3-ft. B.O. and 7-in. Theodolites.

28th August 1817. Observer: Mr. GARDNER.

From 28th January to 23rd February 1847. Observers: Corp. STEEL and Priv. TURNER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Fetlar	3 8 42·54	10	5·61	0·45	Gerth of Scaw .	288 5 32·77	27	102·85	48·96
Brassa	10 21 43·01	5	3·14	0·59	Nive Hill . . .	321 40 27·16	28	99·63	26·75
Yell	23 27 52·27	10	6·93	0·66	Balta	340 42 47·94	9	3·90	0·35
Ronas	45 31 7·16	8	6·39	1·00					

SAYRS LAW.

3-ft. Theodolite, R.S.

From 27th May to 5th July 1846. Observer: Corp. WINZER, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Wisp	16 47 9·91	6	5·77	1·20	Largo Law	159 49 32·06	8	3·79	0·40
Hart Fell	43 35 51·65	3	1·40	0·22	Craigowl	164 48 50·73	6	3·59	0·43
Dunrich	46 48 48·76	6	3·97	0·49	Kellie Law	171 19 46·26	7	3·35	0·23
Allarmoor	96 35 7·71	11	3·74	0·22	Mount Battock	177 58 14·17	3	3·02	1·13
Corkmulaw	100 31 37·88	1	–	4·89	Caerlock	184 40 24·48	8	3·39	0·30
Bin of Campsie	102 3 8·08	1	–	4·89	Isle of May Lighthouse	190 32 11·94	5	–	–
Ben Lomond	108 9 17·31	3	5·26	3·13	Lumsden	256 45 27·61	14	6·63	0·30
Arthur's Seat	109 34 6·66	5	2·58	0·26	Mordington	276 22 26·10	10	5·59	0·40
Referring-object	113 50 20·00	85	8·67	–	Blackheddon	296 11 22·70	9	9·74	1·50
Ben Cleugh	119 16 40·33	8	4·02	0·35	Alnwick	309 42 11·58	1	–	4·89
Ben Lawers	129 33 34·10	9	6·59	0·90	Cheviot	320 56 49·39	14	7·66	1·23
East Lomond	142 16 32·82	8	4·33	0·38					
Glashmeal	159 36 21·83	9	4·06	0·33					

SCA FELL.

3-ft. Theodolite, R.S.

From 8th July to 20th September 1841. Observers: Lieuts. PIPON and CRAIGIE, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Blackcomb	19 14 52·80	10	3·03	0·23	Criffel	153 45 43·29	11	9·53	1·37
Snowdon	20 38 31·85	3	4·83	2·93	Hart Fell	173 32 25·97	5	6·71	2·48
Llanclian	31 45 38·86	4	14·91	16·11	Wisp	189 31 25·96	6	7·68	2·59
Holyhead	37 45 50·86	2	4·32	4·66	High Pike	199 23 59·43	7	3·58	0·38
South Berule	70 55 36·88	6	5·14	0·97	Cheviot	210 28 54·36	1	–	9·54
Snea Fell	75 51 16·39	7	5·76	0·90	Helvellyn	237 7 18·55	7	4·36	0·64
North Berule	77 7 0·84	8	3·16	0·36	Cross Fell	239 6 52·22	11	7·06	0·63
Mull of Galloway	101 23 34·47	3	4·30	2·20	Water Crag	271 6 11·28	5	7·96	3·25
Dent Hill	107 11 2·27	5	3·81	0·71	Calf	281 41 13·39	6	9·30	3·56
Glasserton	109 41 21·28	4	4·22	1·18	Great Whernside	291 58 11·83	10	6·13	0·61
Benereard	122 26 36·47	3	3·06	1·05	Little Whernside	295 16 15·73	10	10·73	2·56
Cairnsmuir of Fleet	128 59 39·77	6	7·61	2·50	Ingleboro'	300 50 23·35	7	13·54	6·36
Merrick	133 50 41·88	7	8·52	1·88	Boulsworth	314 4 59·20	2	1·02	0·26
Ben Cairn	135 45 48·06	2	1·12	0·31	Pendle	317 10 43·86	5	4·52	0·91
Cairnsmuir on Deugh	144 37 56·91	4	5·67	2·03	Whittle Hill	324 23 43·98	6	4·75	0·89
					Beryl	350 43 43·12	5	3·62	0·73

SCARABIN.

3-ft. Theodolite, B.O.

From 11th June to 23rd July 1839. Observers: Lieuts. HORNBY, ROBINSON and PIPON, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Macdui	2 6 15·65	2	5·32	7·07	South Ronaldshay	210 50 55·07	17	12·01	0·98
Duke of Sutherland's Monument	42 47 51·61	3	4·36	2·11	Ben Cheilt	227 56 36·28	41	14·22	0·57
Ben Wyvis	44 32 34·66	18	10·84	0·97	Mormonth	306 0 53·52	7	4·93	0·86
Ben Clibrig	92 18 27·60	34	14·25	0·56	Cowhythe	316 57 3·70	6	6·22	1·19
Foinnebheinn	106 18 46·26	4	4·02	1·41	Manor Lee	319 46 57·21	4	2·54	0·59
Ben Hope	110 15 1·08	6	6·31	1·45	Bin of Cullen	325 10 23·42	7	9·90	3·35
Ben Hutig	124 43 58·81	33	10·22	0·25	Knock	325 19 56·99	10	13·07	2·38
Wart Hill Hoy	190 48 26·60	16	6·93	0·49	Findlay Seat	343 24 9·16	1	–	14·15
Dunnet Head	194 18 25·00	28	11·25	0·48	Corryhabbie	346 17 4·05	10	8·69	1·37

SCOURNALAPICH.

3-ft. Theodolite, B.O.

From 15th July to 25th October 1846. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Mamsuil	20 18 32·79	12	4·13	0·30	Ben Wyvis	219 40 27·55	24	6·84	0·30
Ben More, South Uist	85 43 13·33	20	3·38	0·10	Balnaskerrish	233 56 37·36	22	6·47	0·27
Cleisham	123 4 25·94	18	6·21	0·28	Balrinnis	267 15 13·01	11	7·27	0·90
Monach	146 30 20·95	11	4·70	0·34	Corryhabbie	270 37 21·55	32	6·75	0·15
Glashmeal, Ross	166 51 15·51	9	5·65	0·52	Ben Macdui	291 1 46·95	31	9·93	0·24
Ben Clibrig	201 31 57·87	11	8·97	0·86	Referring-object	351 58 39·99	240	–	–
					Ben Nevis	356 54 21·54	34	7·91	0·23

SEVERNDROOG.

3-ft. Theodolite, B.O. and R.S.

From 25th October to 3rd November 1822. Observers: Major-Gen. COLBY, Capt. KATER, and Mr. GARDNER.
From 20th January to 29th March 1848. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Leith Hill Tower	43 7 6·16	80	12·10	0·08	Greenwich Observatory Dome[1]	106 21 22·94	11	5·40	0·46
Banstead, New	49 4 5·68	4	3·86	1·02	St. Paul's Cathedral	115 33 25·03	33	6·18	0·13
Hundred Acres	50 48 36·69	4	4·44	1·53	Berkhampstead	158 23 44·85	22	5·59	0·24
St. Anne's, Old	79 31 51·50	9	6·60	0·89	Chingford	167 16 45·76	94	10·49	0·06
Windsor Castle Tower	92 39 20·67	4	7·17	3·66	Epping Poor-house Cupola	189 20 1·08	19	5·83	0·31
Hanger Hill Tower	104 19 21·66	39	7·85	0·10	Gad's Hill	281 32 14·53	14	6·59	0·44
Westminster Abbey Pole	105 42 10·09	21	6·66	0·33	Wrotham	316 42 58·45	112	10·86	0·06

[1] A correction of — 7′ 48″·57 to be applied to this bearing, to reduce it to the transit.

SHANKLIN DOWN.

18-in. Theodolite.

8th August 1846. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Week Down	40 58 55·14	8	8·84	1·87	Boniface Down	342 53 10·23	8	6·74	1·11
Dunnose	270 11 34·98	8	3·30	0·31					

SHANTAVNY.

3-ft. Theodolite, B.O.

From 4th to 15th May 1827. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Cuilcagh	55 11 31·47	3	8·59	9·59	Slieve Donard	294 39 36·88	3	0·68	0·05
Bessy Bell	136 2 58·73	15	25·45	4·30	Vicar's Carn	305 0 52·84	7	8·78	2·15
Mullaghcarn	157 35 25·17	13	11·82	2·10	Carlingford	311 27 50·62	2	1·78	0·79
Sawel	184 40 40·12	9	9·57	1·53	Armagh Breagh	314 19 16·10	3	6·84	6·89
Slieve Gallion	218 47 46·75	15	8·22	0·60	Slieve Gullion	314 28 17·96	3	3·12	1·08
Divis	259 19 20·79	2	0·79	0·15	Mullyash	323 4 49·15	5	13·41	7·31

SLIEVECALLAN.

3-ft. Theodolite, B.O.

From 22d September to 2nd October 1837. Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″			
Baurtregaum	28 29 39·12	1	–	8·26	Meelick	289 10 5·75	1	–	8·26
Slievecarran	211 42 58·75	1	–	8·26	Newtown, Limerick	299 43 3·46	1		8·26
Gortourka	245 38 56·57	6	7·09	1·80	Clonoula	338 3 31·63	1	–	8·26
Mooghane	283 30 50·58	1	–	8·26					

SLIEVECARRAN.

3-ft. Theodolite, B.O.

From 9th to 18th August 1837. Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Gortourka	12 11 40·62	9	9·30	1·77	Newtown, Galway	152 51 1·66	4	2·34	0·39
Slievecallan	31 55 30·19	5	2·79	0·46	Gorathorna	297 36 28·97	3	2·71	0·95
Black Head	106 58 43·15	5	3·55	0·58	Keeper	307 0 29·56	3	3·79	1·92
Bencorr	131 31 23·17	1	–	5·05					

SLIEVE DONARD.

3-ft. Theodolite, R.S.

From 19th August to 18th December 1845. Observers: Corps. FORSYTH and WINZER, and Priv. STEWART, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Howth	6 16 22·49	12	7·33	0·57	Trostan	171 9 49·62	24	6·92	0·21
Kippure	13 51 12·00	26	11·33	0·51	Divis	172 31 46·07	40	15·02	0·28
Garristown Wd. mill	24 32 45·41	5	5·34	1·71	Benercard	212 48 32·88	11	6·00	0·53
Castlecoo	27 9 44·81	5	14·73	13·15	Cairn Piot	216 14 0·42	14	7·11	0·48
Croghan	44 30 43·65	10	4·62	0·39	Merrick	220 41 42·50	8	1·96	0·07
Loughanleagh	65 11 21·20	2	5·37	7·21	Mull of Galloway	233 18 29·34	4	3·25	0·78
Slieve Gullion	79 13 27·26	9	10·52	2·48	Criffel	239 20 50·09	12	4·73	0·36
Mullyash	89 31 13·86	5	13·42	7·95	Sca Fell	259 5 10·02	13	6·84	0·46
Cuilcagh	91 50 10·05	7	2·52	0·24	North Berule	262 22 21·01	2	1·07	0·28
Vicar's Carn	106 53 2·45	11	11·30	2·69	Snea Fell	263 51 34·86	2	1·97	0·97
Referring Object	130 5 13·06	161	–	–	South Berule	271 53 16·88	19	7·75	0·37
Sawel	134 56 41·86	34	6·55	0·18	Snowdon	314 38 39·43	21	6·40	0·22

SLIEVE LEAGUE.

3-ft. Theodolite, B.O.

From 14th November 1827 to 5th January 1828. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Knockalongy	3 54 32·99	13	22·30	4·18	Slieve Snaght	234 51 13·82	2	3·75	3·51
Nephin	31 28 44·93	7	6·35	1·41	Barnesmore Connell	259 55 18·33	8	10·74	2·68
The Reek	32 28 15·92	3	11·26	17·85	Breesy	288 5 53·78	4	8·51	5·73
Slieve More	51 28 45·90	2	4·40	4·84	Cuilcagh	310 24 46·20	5	5·04	1·57
Tawnaghmore	55 51 17·92	6	16·87	8·71	Knocknarea	348 49 38·45	14	8·38	1·13

SLIEVE MORE.

12-in. Theodolite.

From 17th to 19th August 1831. Observer: Serg. DOULL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″					° ′ ″			
Tawnaghmore	223 20 56·16	10	Repeated Angles.	0·07	The Reek	316 29 54·14	27	Repeated Angles.	0·27
Nephin	269 13 10·97	10		0·07	Bencorr	342 25 55·61	7		4·84

SLIEVE SNAGHT.

3-ft. Theodolite, B.O.

From 29th September to 25th October 1827. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Bessy Bell . .	3 56 43·66	8	5·72	0·83	Mount Sandy . .	273 26 45·74	14	5·05	0·28
Cuilcagh . . .	15 42 47·17	6	9·59	2·90	Curidtham . . .	275 48 30·68	15	6·65	0·41
Barnesmore Connell .	37 52 38·29	4	6·14	2·59	North End of Base	281 18 43·61	13	5·81	0·36
Slieve League .	55 58 38·50	4	4·77	1·53	Trostan . . .	282 5 30·75	6	7·25	1·82
Ben Tartevil . .	223 20 1·33	3	2·92	1·03	Divis	307 7 15·09	3	6·99	5·48
Jura	226 20 30·56	1	–	7·18	South End of Base	309 15 51·55	8	8·18	1·70
Carn-na-leagh .	261 55 29·69	2	1·54	0·59	Slieve Gallion .	324 24 17·73	5	3·55	0·60
Knocklayd . .	272 43 29·64	5	4·51	1·08	Sawel	335 39 50·08	21	8·51	0·31

SNOWDON.

3-ft. Theodolite, R.S.

From 16th July to 21st November 1842. Observers: Lieuts. PIPON and STANLEY, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Precelly . . .	21 4 12·58	6	2·47	0·24	Beryl	218 16 40·48	2	1·79	0·80
Rhiw	55 6 6·42	5	7·04	2·05	Ingleboro' . . .	221 35 53·74	4	4·26	1·14
Tara	74 56 27·38	3	1·82	0·37	Moelfre Issa . .	239 15 23·27	7	8·88	2·18
Kippure . . .	95 31 34·54	2	1·54	0·59	Whittle . . .	240 21 10·17	5	4·39	1·01
Howth	105 5 44·54	2	0·54	0·07	Billinge . . .	241 8 48·83	7	5·50	0·69
Holyhead . . .	124 28 18·91	6	5·06	0·91	Heswell . . .	245 18 29·91	3	4·21	2·47
Slieve Donard .	136 7 51·72	4	4·91	2·17	Axedge . . .	261 50 24·62	3	6·56	4·79
Moel Rhydladd .	137 43 39·85	4	4·96	1·70	Cyrn-y-Brain . .	272 49 0·91	8	5·88	0·64
Llanelian . . .	158 11 20·74	7	8·92	2·14	Longmount Pole .	305 0 16·42	4	7·03	3·14
South Berule . .	162 10 9·13	4	10·92	10·10	Plynlimmon . .	343 25 1·29	4	3·48	0·79
Snea Fell . . .	169 18 44·44	3	3·24	1·17	Cader Idris . .	344 34 56·30	16	8·91	0·66
North Berule . .	171 20 59·85	2	6·20	9·61	Tregarron . . .	350 24 40·73	2	0·59	0·08
Sca Fell . . .	199 56 40·20	3	3·47	1·35	Pengarn . . .	356 8 19·58	5	4·73	1·01
Black Comb . .	200 10 26·25	1	–	7·11	Aberystwith . .	359 44 47·72	3	0·53	0·03

SOUTHAMPTON.

10-in. Theodolite.

From 16th to 24th May 1845. Observer: Priv. SCOTT, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Motteston . .	3 19 37·77	10	27·65	14·28	Dean Hill . . .	126 38 5·75	10	45·84	40·66
Nodes Beacon[1] .	19 34 36·61	8	31·41	37·83	Dunnose . . .	336 11 3·28	17	44·77	12·06

[1] A correction of + 2″·85 to be applied to this observation, to reduce it to the trigonometrical station.

SOUTH BERULE.

3-ft. Theodolite, R.S.

From 20th June to 6th August 1845. Observers: Corps. STEEL and FORSYTH, and Priv. MOUNTAYNE, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Kippure	46 9 17·55	16	5·58	0·27	Glasserton	190 58 15·09	15	7·03	0·43
Howth	47 28 38·11	8	6·62	1·00	Cairnsmuir of Fleet	192 48 41·09	3	4·28	2·35
Slieve Donard	92 54 35·32	24	10·23	0·60	Criffel	216 58 59·61	18	5·63	0·21
Divis	120 56 26·59	9	5·63	0·60	Snea Fell	226 51 29·99	18	11·12	0·86
Trostan	136 41 52·20	17	14·85	1·25	North Berule	228 48 19·09	11	11·25	2·00
Carn-na-leagh	151 32 26·80	4	3·34	0·79	Dent	242 22 49·92	13	7·02	0·67
Cairn Piot	162 58 20·61	17	12·43	1·02	Sca Fell	249 44 48·27	17	6·92	0·49
Referring-object	167 21 27·42	20	–	–	Black Comb	261 37 15·82	24	12·02	0·47
Mull of Galloway	167 21 38·50	15	6·38	0·40	Ingleboro'	268 21 27·13	10	13·00	3·76
Benereard	171 2 8·05	6	10·18	3·52	Snowdon	341 41 50·13	20	11·04	0·61
Merrick	186 38 21·37	20	11·99	0·81					

SOUTH LOPHAM CHURCH TOWER.

2-ft. Theodolite.

From 26th November 1844 to 10th January 1845. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Lawshall Tower	34 53 47·71	11	5·58	0·39	Bunwell Tower	220 18 32·90	13	7·41	0·61
Brandon	92 40 41·48	10	7·56	1·15	Laxfield	292 20 51·65	9	6·70	0·73
Swaffham Spire[1]	143 35 8·07	4	2·50	0·52	Stradbrook Tower	294 24 53·65	10	4·76	0·35
Hingham Tower	177 18 42·69	6	2·18	0·17	Mickfield[2]	336 57 51·06	10	3·74	0·36
Referring-object	190 11 6·82	55	–	–					

[1] A correction of + 17″·41 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of + 2″·54 " " "

SOUTH RONALDSHAY.

3-ft. Theodolite, B.O.

From 19th to 26th October 1821. Observers: Capts. VETCH and DRUMMOND, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Cheilt	26 27 48·42	12	5·55	0·58	Ben Hutig	74 53 54·61	1	–	4·27
Scarabin	31 24 6·43	6	3·50	0·54	Wart Hill Hoy	120 8 45·22	17	6·07	0·30
Ben Clibrig	55 10 38·21	3	3·70	1·53	Deerness	210 24 19·25	20	4·42	0·12
Dunnet Head	63 17 12·86	3	2·80	1·05					

SOUTHWOLD CHURCH TOWER.

2-ft. Theodolite.

From 7th to 30th September 1844. Observer: Corp. **Bay**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Orford Castle	21 14 33·59	9	4·82	0·45	Toft's Tower	159 38 41·41	10	4·89	0·43
Laxfield Tower	82 19 13·84	11	7·61	0·99	Gorleston Tower	186 22 18·04	10	1·28	0·03

STOKE CHURCH TOWER.

2-ft. Theodolite.

From 19th January to 2nd May 1844. Observer: Corp. **Bay**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Danbury Spire	35 51 33·69	13	5·85	0·39	Otley	231 42 24·60	10	1·00	0·02
Lawshall Tower	148 11 41·39	13	6·10	0·45	Referring-object	272 34 49·95	89	–	–
Naughton Tower	198 19 25·77	10	5·02	0·41	Walton Tower	296 49 49·72	14	3·32	0·12

STOKE HILL.

3-ft. Theodolite, B.O.

From 17th December 1849 to 4th February 1850. Observer: Corp. **Jenkins**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Wingreen	5 41 47·79	24	8·33	0·35	Thorney Down	306 26 46·13	6	5·34	0·84
Westbury	76 57 42·08	48	10·32	0·14	Dean Hill	312 58 2·92	22	6·37	0·21
Milk Hill	231 45 36·38	24	6·51	0·28	Queen's Manor	313 19 26·04	11	4·36	0·28
Inkpen	257 38 22·93	26	4·49	0·11	Old Sarum Castle	317 43 52·81	13	4·94	0·32
Beacon Hill	292 18 42·30	25	8·35	0·21	Four-mile Stone	319 39 8·95	27	9·95	0·33
Old Lodge	297 37 58·45	12	5·54	0·49					

STORR.

3-ft. Theodolite, R.S.

From 26th September 1847 to 26th February 1848. Observer: Corp. **Winzer**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Heynish	21 20 18·84	6	3·85	0·71	Cnoc-ghiubhais	210 2 36·12	19	14·32	0·91
Ben More, South					Ru Rea	214 0 45·81	19	4·80	0·16
Uist	68 0 2·91	24	9·70	0·37	Ben Clibrig	231 37 12·21	17	8·95	0·80
Cleisham	143 52 7·42	33	7·23	0·24	Mamsuil	291 9 52·14	22	7·87	0·36
Monach	175 30 21·81	25	8·88	0·23	Ben Nevis	317 26 17·03	41	11·33	0·29
Ru Stoir	209 24 37·72	13	5·65	0·40	Ben More in Mull	355 3 25·22	17	8·83	0·66

STRONSAY.

3-ft. Theodolite, B.O.

From 28th October to 1st November 1821. Observers: Captains **VETCH** and **DRUMMOND**, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Deerness . . .	36 50 18·97	23	5·66	0·18	North Ronaldshay Lighthouse .	196 57 18·31	5	1·88	0·19
Wart Hill Hoy .	65 8 16·50	12	8·51	0·77	Start Lighthouse	205 3 47·23	8	6·71	1·18
Fitty Hill . . .	129 32 53·86	17	6·55	0·50	Fair Isle . . .	225 29 33·49	9	2·93	0·21

SWAFFHAM CHURCH SPIRE.

18-in. Theodolite.

From 7th September to 30th October 1843. Observer: Corp. **STEEL**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Brandon . . .	9 53 5·37	8	10·64	3·21	Boston Tower[2] .	127 36 2·48	5	10·98	6·24
Cheveley Tower .	17 53 49·20	6	8·80	3·73	Baconsthorpe Tr.	229 53 32·38	4	8·31	5·69
Newmarket . .	21 35 31·04	8	12·17	3·08	Hingham Tower .	291 1 41·58	22	18·61	1·44
Ely Minster . .	46 25 29·88	13	10·49	1·19	S. Lopham Tower	323 21 12·11	5	10·17	5·01
St. Peter's, Walpole	106 1 49·75	12	13·86	3·38	Lawshall Tower .	357 34 51·63	8	11·50	2·82
Lynn, Old Tower[1]	120 25 26·42	15	12·04	1·88					

[1] A correction of — 3″·07 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of — 4″·35 „ „ „

SWYRE BARROW.

3-ft. Theodolite, B.O.

From 18th March to 8th April 1845. Observer: Serg. **DONELAN**, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Vern	79 16 13·65	12	4·64	0·26	Horton's Gazebo .	198 43 48·36	4	7·81	4·89
Little Haldon .	88 7 13·50	7	3·15	0·32	Beacon Hill . .	202 0 54·31	10	7·32	0·80
High Wilhays .	94 28 10·93	10	8·57	1·36	Dean Hill . . .	213 10 55·52	12	3·79	0·20
Black Down .	105 48 57·51	13	9·76	0·83	Coringdon . .	249 45 25·96	12	6·86	0·68
Pillesdon . . .	113 28 15·01	10	6·88	0·88	Nodes Beacon[1] .	259 57 50·01	2	5·28	6·97
Wingreen . . .	178 41 18·76	13	5·33	0·42	Dunnose . . .	268 24 0·15	15	10·41	0·65

[1] A correction of — 46″·30 to be applied to this bearing, to reduce it to the trigonometrical station.

TARA.

3-ft. Theodolite, B.O.

From 8th to 20th November 1829. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Forth	28 54 40·75	17	8·98	0·85	Ballycreen	157 42 46·84	10	5·94	0·65
Vinegar Hill	46 11 14·38	10	4·73	0·57	Kippure	171 50 34·71	10	3·42	0·28
Slievecoiltia	50 45 20·52	4	2·34	0·35	Collin Hill	203 13 23·67	10	5·06	0·45
Blackstairs	67 29 12·94	7	5·81	1·08	Snowdon	253 13 56·80	3	2·64	0·85
Mount Leinster	76 55 9·48	8	8·87	1·37	The Rivel	254 56 19·09	4	7·91	4·20
Croghan	146 14 14·54	10	4·65	0·51	Precelly	309 50 13·36	6	6·61	1·51

TARBATHY.

3-ft. Theodolite, B.O.

From 7th to 30th May 1817. Observer: Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Blue Hill	14 44 38·95	7	1·18	0·46	Over Hill	155 10 57·80	6	3·18	0·62
Caerlock	45 37 0·62	5	3·66	0·64	Dudwick	184 42 29·38	4	2·97	0·61
Mount Battock	54 36 7·01	3	3·95	2·18	Layton	199 1 31·92	6	3·34	0·34
Brimmond	66 42 54·85	8	2·76	0·16	Little Stirling	209 21 52·14	4	18·26	27·57

TAUR.

3-ft. Theodolite, B.O.

From 5th to 13th August 1832. Observer: Capt. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Caherbarnagh	8 52 32·41	22	6·55	0·22	Galtymore	257 12 58·92	6	4·46	0·99
Mangerton	39 37 21·31	8	5·91	1·02	Knockanaffrin	266 19 27·99	3	5·32	3·82
Colly Mountain	62 15 21·04	4	4·25	1·40	Knockmealdown	270 19 21·55	3	0·63	0·05
Baurtregaum	86 8 43·94	7	2·35	0·17	Knocknaskagh	286 50 54·31	22	5·51	0·18
Brandon	90 11 7·77	1	–	3·99	Rock Hill	303 18 35·46	2	1·72	0·73
Knockanore	134 21 36·10	7	10·80	2·58	Mount Hillary	305 54 29·88	29	5·68	0·09
Meelick	207 45 0·75	2	1·43	0·51	Musheramore	337 54 45·92	4	1·33	0·15
Keeper	225 27 10·44	7	7·11	1·53					

TAWNAGHMORE.

12-in. Theodolite.

From 10th to 13th September 1829. Serg. DOULL, R.S.M.

Objects.	Bearings.	No. of Repetitions.	No. of simple Arcs.	Recip. of Weight.	Objects.	Bearings.	No. of Repetitions.	No. of simple Arcs.	Recip. of Weight.
	° ′ ″					° ′ ″			
Slieve More	43 43 31·68	–	14	2·06	Knockalongy	281 12 32·06	20	–	0·31
Slieve League	235 7 43·90	28	1	0·68	Nephin	334 17 16·39	52	3	0·31

TELEGRAPH TOWER.

18-in. Theodolite.

From 28th May to 19th June 1850. Observer: Corp. WOTHERSPOON, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Peninnis Windmill	2° 37′ 47″·78	17	12″·69	1·52	St. Martin's Head	212° 31′ 55″·52	22	12″·60	0·88
St. Agnes' Lighthouse . . .	36 40 30·90	24	17·23	1·43	Karn Galver . .	241 44 7·18	26	11·87	0·68
Referring-object	40 30 20·03	79	–	–	Pertinny . . .	246 59 25·08	14	5·16	0·34
Beacon Hill Trescow Watchhouse[1]	142 54 24·28	12	14·21	3·20	Wolf Rock . .	266 49 56·50	15	11·90	1·54

[1] A correction of – 5′ 11″·84 to be applied to this bearing, to reduce it to the trigonometrical station.

THARFIELD.

3-ft. Theodolite, B.O.

From 15th July to 23rd August 1843. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Berkhampstead .	12° 8′ 12″·89	9	5″·95	0·62	Ely Minster . .	205° 29′ 37″·98	13	8″·18	0·76
Dunstable . . .	63 53 20·36	19	6·76	0·25	Brandon . . .	225 46 36·62	6	4·03	0·67
Hanslope Ch. Spire	101 21 54·48	4	5·40	2·04	Newmarket . .	230 57 31·17	12	5·17	0·50
Referring-object .	111 38 29·96	112	7·29	–	Balsham . . .	236 35 11·89	9	3·59	0·27
Keysoe Spire . .	133 59 28·92	13	7·67	0·88	Balsham Tower .	241 51 5·61	8	9·57	2·87
Royston . . .	152 2 25·46	5	5·40	1·57	Thaxted Spire .	285 3 32·32	6	4·50	0·76
Orwell	184 6 15·29	3	1·36	0·21	Epping Cupola .	342 36 18·64	5	1·52	0·10

THAXTED CHURCH SPIRE.

18-in. Theodolite.

From 8th June to 11th July 1844. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
Epping Cupola .	28° 35′ 24″·75	10	6″·59	1·03	Lawshall Tower .	229° 23′ 14″·25	18	23″·81	3·11
Berkhampstead .	55 50 18·47	22	20·22	1·85	Stoke Tower .	263 52 14·33	8	9·00	2·07
Tharfield . . .	105 21 26·21	11	14·96	4·10	Danbury Spire .	328 42 30·38	18	17·95	2·81
Balsham Tower .	175 23 35·86	19	18·82	1·91					

THORNEY DOWN.

3-ft. Theodolite, B.O.

From 12th September to 15th October 1849. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Coringdon	20 18 20·92	10	3·82	0·23	Four-mile Stone	96 42 49·94	16	10·38	0·62
Horton's Gazebo	32 11 1·93	8	7·38	1·08	Westbury Down	119 26 8·41	6	4·76	0·79
Queen's Manor	48 35 27·29	29	8·02	0·21	Stoke Hill	126 43 22·92	24	6·12	0·19
Salisbury Spire	51 15 11·47	19	9·24	0·45	Milk Hill	162 0 36·02	3	8·28	7·80
Wingreen Beacon	63 56 42·59	7	2·94	0·42	Beacon Hill	174 58 29·71	33	10·28	0·21
Old Sarum Castle	75 46 1·80	18	5·80	0·20	Inkpen	212 17 23·71	3	1·28	0·19
Old Sarum Gun	76 58 2·12	19	4·87	0·19	Old Lodge	236 56 57·85	15	8·64	0·79

TILTON.

3-ft. Theodolite, R.S.

From 29th January to 23rd March 1843. Observer: Corp. MULLIGAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Naseby Tower	16 42 1·07	10	7·38	0·82	Sutton	155 29 25·69	6	2·12	0·15
Bardon Hill	104 14 9·40	5	6·25	1·67	Holland Hill	159 0 48·14	4	1·61	0·17
Breedon Hill Tr.	116 36 46·59	7	4·68	0·51	Stathern	184 46 17·67	7	4·98	0·70
Weaver Hill	121 56 6·52	1	–	6·04	BuckminsterSpire	213 56 20·90	8	3·88	0·33
Loughborough Tr.	122 21 5·94	7	12·90	4·21	Easton Tower	273 40 5·20	1	–	6·04
Elvaston Tower	127 48 11·22	1	–	6·04	Keysoe Spire	325 41 11·43	8	4·33	0·45

TOFTS CHURCH TOWER.

2-ft. Theodolite.

From 29th March to 26th April 1845. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Laxfield Ch. Tr.	32 30 49·78	11	14·21	2·34	Happisburgh	175 26 58·64	9	5·93	0·55
Stradbrook Tower	45 13 54·27	7	4·19	0·67	Gorleston Tower	229 51 20·02	11	10·09	1·40
Bunwell Tower	88 5 9·24	16	11·73	0·78	Referring-object	273 34 59·98	36	–	–
Norwich Spire	128 22 3·55	12	3·94	0·26	Southwold Tower	339 33 26·02	15	11·62	1·08

TOLSFORD.

3-ft. Theodolite, R.S.

From 6th to 8th September 1822. Observers: Major-Gen. COLBY, Capt. KATER, and Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Fairlight . . .	52 18 29·86	19	5·02	0·17	Notre Dame, Calais	286 10 59·10	7	2·84	0·26
Tenterden . . .	82 16 40·80	3	3·18	1·14	Blancnez . . .	293 43 40·80	4	1·49	0·14
Crowborough . .	85 42 37·26	7	6·37	1·26	Fienne	299 0 12·64	5	1·19	0·06
Stede Hill . .	121 26 28·51	16	3·38	0·12	Montlambert . .	316 30 28·18	3	0·79	0·07
Folkestone . .	275 26 43·23	18	4·15	0·09					

TREVOSE HEAD.

3-ft. Theodolite, B.O.

From 12th March to 25th April 1846. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Carnbrea Monumᵗ	22 57 13·10	7	5·78	0·87	Lundy Island .	200 5 19·86	33	8·27	0·18
St. Agnes . . .	26 3 1·96	12	6·57	0·68	Cadon Barrow .	243 27 30·31	13	6·07	0·51
Karnminnis . .	42 22 3·16	23	5·89	0·20	Brown Willy . .	261 5 57·71	22	9·46	0·44
Karn Galver . .	44 18 55·31	8	3·73	0·49	Hensbarrow . .	320 19 6·73	13	8·78	0·83

TROSTAN.

3-ft. Theodolite, B.O.

From 23rd June to 5th July 1827. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Slieve Gullion .	10 1 51·09	2	2·13	1·13	Jura	185 39 49·05	6	6·14	1·66
Shantavny . .	44 54 2·66	4	6·91	3·32	Ben Tuirc . .	212 32 59·70	12	14·25	2·22
Slieve Gallion .	48 17 57·21	11	8·99	1·38	Carn-na-leagh .	218 44 45·90	19	8·51	0·48
Sawel	66 25 24·98	9	13·52	2·60	Goat Fell . . .	223 2 30·11	3	2·51	0·73
Eskaheen . . .	95 26 19·39	2	0·81	0·16	Cairnsmuir on Deugh . . .	258 29 58·85	3	5·22	3·25
Benyevenagh . .	99 10 59·70	12	5·55	0·68	Merrick . . .	263 47 11·74	4	6·03	3·28
Slieve Snaght . .	103 3 32·60	8	11·68	3·40	Benereard . . .	267 51 16·10	8	4·38	0·64
Magillagan Tower	107 54 51·33	14	13·95	1·78	Cairn Piot . .	285 30 23·76	12	9·03	1·00
Knocklayd . .	154 48 31·92	31	8·23	0·25	Divis	349 37 46·12	30	14·46	0·52
Ben Tartevil . .	166 31 44·13	13	11·30	1·39	Slieve Donard .	350 58 27·08	10	11·80	2·21
Oa	172 23 54·32	16	7·55	0·53					

UPCOT DOWN.

3-ft. Theodolite, B.O.

From 24th March to 23rd April 1850. Observer: Corp. JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Milk Hill	14 51 54·54	21	4·53	0·13	Cleeve	164 27 21·49	23	4·03	0·10
Stoke Hill	37 46 43·97	13	4·76	0·28	Whitehorse Hill	237 8 50·65	29	5·77	0·12
Westbury Down	44 0 21·71	14	6·07	0·37	Inkpen	300 27 54·62	12	7·36	0·78
Mendip	60 51 44·82	17	2·43	0·06					

VICAR'S CARN.

3-ft. Theodolite, B.O.

From 17th to 22nd April 1827. Observer: Lieut. PORTLOCK, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Armagh Breagh	9 1 44·68	8	8·01	1·41	Armagh Transit	150 32 25·65	2	6·08	9·24
Mullyash	18 56 33·12	10	7·49	1·46	Sawel	153 55 11·64	2	4·38	4·79
Cuilcagh	82 41 54·37	1	–	15·51	Slieve Gallion	167 37 31·14	6	6·65	1·73
Carnmore	86 2 26·09	6	7·66	2·59	Divis	226 52 58·79	7	4·34	0·41
Shantavny	125 24 38·89	8	8·94	2·49	Slieve Croob	263 12 56·18	5	19·17	15·89
Bessy Bell	130 6 42·12	1	–	15·51	Slieve Donard	286 20 10·15	7	8·47	2·13
Armagh Observatory Dome	150 25 1·67	9	12·60	2·41	Carlingford	318 30 47·53	4	6·56	3·59
					Slieve Gullion	331 26 35·10	6	8·19	2·39

WALPOLE ST. PETER'S CHURCH TOWER.

2-ft. Theodolite.

From 23rd September to 20th October 1843. Observer: Corp. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Peterboro' Cathedral	61 6 48·27	5	3·55	0·99	Docking Tower	234 24 16·49	12	11·98	1·45
Easton Tower	77 51 50·32	8	2·67	0·15	Lynn Old Tower[2]	257 56 36·49	15	9·95	0·75
Boston Tower[1]	149 3 14·03	9	6·31	0·88	Swaffham Spire[3]	285 39 25·45	10	4·55	0·26
Referring-object	194 47 29·96	50	–	–	Ely Minster	355 45 47·77	9	6·47	0·81

[1] A correction of — 4″·66 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of + 4″·78 „ „
[3] A correction of — 17″·25 „ „

WALTON CHURCH TOWER.

2-ft. Theodolite.

From 4th November 1843 to 9th January 1844. Observer: Corp. Bay, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Stede Hill	31 41 50·95	1	–	6·03	Otley Church Tr.	171 36 43·14	6	3·80	0·41
Danbury Spire	71 49 1·23	8	6·61	0·80	Orford Castle	213 0 56·81	7	5·14	0·65
Stoke Church Tr.	117 8 14·77	9	5·83	0·63	St. Peter's Church[1]	350 42 37·60	3	7·05	60·0
Naughton Ch. Tr.	138 54 50·06	7	2·57	0·21					

[1] A correction of + 2″·40 to be applied to this bearing, to reduce it to the trigonometrical station.

WART HILL HOY.

3-ft. Theodolite, B.O.

From 25th September to 16th October 1821. Observers: Capts. Vetch and Drummond, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Ben Cheilt	2 0 18·57	7	4·26	0·65	North Ronaldshay Lighthouse	225 25 25·74	2	0·16	0·00
Dunnet Head	4 1 38·43	4	6·99	3·32	Start Lighthouse	232 20 24·74	3	6·85	5·27
Scarabin	11 1 13·03	2	1·21	0·36	Stronsay	244 27 18·54	8	5·86	0·84
Ben Clibrig	40 30 26·48	12	6·57	0·81	Deerness	260 16 2·78	12	5·84	0·47
Fashven	68 0 28·54	1	–	5·77	Copinshay	270 20 10·10	5	4·07	0·77
Fitty Hill	204 6 42·60	9	3·21	0·17	South Ronaldshay	299 48 21·29	23	6·13	0·16

WATER CRAG.

3-ft. Theodolite, R.S.

From 17th April to 5th June 1852. Observer: Corp. Grose, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Little Whernside	39 18 26·48	7	8·13	1·85	Merrington Church	231 2 55·87	3	3·80	1·85
Sca Fell	91 59 43·45	13	6·60	0·52	Easington	259 38 56·97	13	4·20	0·16
Cross Fell	140 46 26·40	18	6·05	0·34	Appleton Wiske	267 40 28·97	11	4·81	0·28
Mickle Fell	148 10 28·13	9	3·86	0·27	Botton Head	272 30 37·13	11	6·04	0·80
Collier Law	193 6 23·85	8	4·46	0·59	Black Hambleton	284 9 48·26	13	12·63	1·52
Brandon Down	218 6 57·29	11	6·41	0·48	Kirklington Church	301 5 48·44	15	3·43	0·16
Wordeslow	223 41 28·60	8	5·84	0·70	Great Whernside	346 32 39·53	9	6·16	0·88

WEEK DOWN.

18-in. Theodolite.

From 11th to 14th August 1846. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Nodes Beacon[1]	109 36 9·08	6	9·22	2·80	Boniface Down	256 30 25·57	9	11·54	2·93
Motteston	117 25 28·78	7	9·93	3·70	Boniface, S.E.	259 44 6·83	9	9·31	1·70
Shanklin Down	220 58 59·31	9	17·22	5·57	Littletown Down	264 32 33·50	9	16·45	5·72
Dunnose	228 50 28·76	9	12·44	3·62					

[1] A correction of + 1′ 29″·30 to be applied to this bearing, to reduce it to the trigonometrical station.

WESTBURY DOWN.

3-ft. Theodolite, B.O.

From 8th February to 17th March 1850. Observer: Corp. JENKINS, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Mendip	80 45 7·31	35	7·94	0·15	Milk Hill	237 32 12·76	20	4·03	0·12
Dundry Church Tr.	114 20 39·41	12	6·64	0·68	Stoke Hill	256 53 37·21	32	4·74	0·12
Lansdown Monument	136 45 23·61	10	4·34	0·34	Inkpen	257 29 26·19	22	5·95	0·20
Farley Down Monument	144 17 36·85	13	5·81	0·43	Beacon Hill	285 59 0·42	14	4·85	0·29
Marshfield Church Tower	151 58 19·74	8	6·37	0·99	Old Lodge	291 47 15·98	13	4·15	0·26
Lord Somerset's Monument	160 25 57·51	8	6·09	0·91	Thorney Down	299 5 29·05	3	5·40	3·24
Lansdown Obelisk	218 59 33·35	23	5·94	0·23	Queen's Manor	305 1 30·56	15	5·43	0·27
Upcot Down	223 44 2·22	33	5·51	0·08	Dean Hill	306 28 34·73	19	3·87	0·14
					Four-mile Stone	307 58 42·72	7	5·11	0·80
					Salisbury Spire	311 45 34·76	13	5·34	0·31
					Wingreen	355 22 15·76	16	11·44	0·93

WHITEHORSE HILL.

3-ft. Theodolite, R.S.

From 1st March to 27th April 1844. Observers: Corp. COSGROVE and Priv. STEWART, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Upcot Down	57 19 37·83	1	–	5·43	Wytham	216 12 47·43	6	4·23	0·83
Mendip	60 6 36·08	3	3·09	1·08	Brill	229 58 40·44	8	4·03	0·35
Symond's Hall	101 2 37·11	3	4·97	2·75	Shotover	233 13 34·81	7	3·14	0·35
Cradle	111 55 5·33	10	4·29	0·34	Dunstable	245 19 2·79	13	4·59	0·18
Malvern	138 13 11·85	7	10·27	2·47	Wendover	250 3 34·49	10	10·03	1·25
Cleeve	141 50 46·75	12	4·51	0·29	Leith Hill Tower	297 35 43·70	14	6·89	0·52
Broadway Tower	159 45 9·47	11	8·38	0·91	Inkpen	343 58 57·42	15	5·56	0·31
Arbury Hill	198 35 49·68	9	4·10	0·39					

WHITTLE.

3-ft. Theodolite, R.S.

From 14th May to 23rd June 1841. Observers: Lieuts. PIPON and CRAIGIE, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Delamere . . .	29 32 13·49	1	–	3·10	Ingleboro' . . .	171 13 39·18	4	5·09	1·66
Cyrn-y-Brain .	41 12 30·09	1	–	3·10	Pendle Hill . .	174 36 40·88	4	5·33	2·27
Billinge . . .	59 26 52·31	3	3·10	1·12	Great Whernside .	197 38 22·25	7	5·35	0·76
Snowdon . . .	61 48 15·06	1	–	3·10	Boulsworth . .	211 53 50·66	2	2·26	1·27
Rivington . . .	76 8 33·76	6	2·85	0·30	Holme Moss . .	299 10 30·23	7	3·53	0·34
Beryl Hill . .	110 54 54·72	5	3·12	0·53	Kinder Scout . .	321 21 3·08	5	4·15	1·00
Black Comb . .	133 45 53·87	3	1·85	0·39	Axedge . . .	336 17 54·36	4	3·08	0·60
Sea Fell . . .	145 9 32·50	4	1·19	0·10	Mowcopt . . .	356 40 16·87	1	–	3·10
Hamilton Hill .	162 37 31·98	2	0·22	0·01					

WINGREEN.

3-ft. Theodolite, R.S.

From 29th July to 1st September 1844. Observers: Corps. STEWART and COSGROVE, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Vern	26 22 37·00	11	9·03	1·04	Dean Hill . . .	260 50 25·63	8	4·62	0·50
Mintern . . .	61 11 50·51	10	5·85	0·40	Butser Hill . .	270 9 8·74	15	8·65	0·66
Pillesdon . . .	69 2 19·30	8	2·91	0·17	Dunnose . . .	302 7 29·47	13	6·38	0·34
Referring-object .	85 58 34·88	90	–	–	Motteston . .	306 39 34·37	13	7·94	0·64
Staple Hill . .	86 36 16·57	10	7·87	1·02	Nodes Beacon[1] .	311 22 8·77	7	5·35	1·72
Mendip . . .	130 29 49·49	14	7·73	0·59	Horton's Gazebo	322 44 54·77	10	5·35	0·48
Westbury . . .	175 24 9·52	8	2·87	0·20	Coringdon . . .	347 59 24·08	21	5·13	0·15
Inkpen	227 29 52·86	20	9·82	0·59	Swyre Barrow .	358 40 33·02	20	5·65	0·19
Beacon Hill . .	230 36 54·15	5	3·66	0·83					

[1] A correction of – 35″·06 to be applied to this bearing, to reduce it to the trigonometrical station.

WISP.

3-ft. Theodolite, B.O.

From 5th to 8th August 1809. Observer: Mr. GARDNER.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Criffel	48 4 22·05	11	11·15	1·96	Dunrich . . .	156 49 30·17	23	6·11	0·20
Cairnsmuir of Fleet	69 10 14·00	5	6·77	2·38	Sayrs Law . .	196 32 24·06	4	8·01	6·80
Cairnsmuir on Deugh . . .	88 13 34·55	7	7·54	1·52	Cheviot . . .	247 8 41·44	10	7·04	0·98
Hart Fell . . .	116 44 57·78	10	4·98	0·57	Cross Fell . . .	334 22 50·93	2	9·13	20·83

WORDESLOW.

2-ft. Theodolite.

From 18th April to 28th May 1846. Observer: Serg. BAY, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Great Whernside	26 5 17·98	15	4·11	0·18	Pontop	96 12 0·94	9	6·63	0·83
Merrington Church	29 48 38·67	3	3·25	1·23	Cheviot	147 4 28·49	22	4·75	0·12
Water Crag	44 14 57·45	4	4·49	1·73	Rufflaw New	152 35 57·90	14	4·79	0·25
Brandon	57 20 42·80	6	3·80	0·46	Easington	309 48 15·17	16	4·39	0·19
Referring-object	57 51 4·98	104	–	–	Burleigh Moor	321 7 1·67	8	4·00	0·34
Collier Law	76 27 41·24	10	2·78	0·18	Botton Head	335 52 26·81	13	5·54	0·38

WROTHAM

3-ft. Theodolite, B.O. and R.S.

From 10th to 17th October 1822. Observers: Capt. KATER and Mr. GARDNER.
From 28th July to 30th August 1844. Observer: Serg. DONELAN, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Frant Church	2 29 54·40	16	8·33	0·71	Chingford	150 52 13·87	18	3·36	0·09
Crowborough	17 28 51·14	50	6·80	0·10	Epping Cupola	165 27 25·06	12	4·05	0·23
Ditchling	30 54 42·18	19	7·13	0·38	Danbury Spire	204 11 22·14	14	4·02	0·19
Leith Hill Tower	71 25 8·83	41	6·42	0·11	Gad's Hill Obelisk[2]	229 12 11·12	6	5·04	0·91
Stede Hill	104 12 1·93	38	6·93	0·11	Norwood	253 57 42·19	11	8·16	0·69
Hanger Hill	119 43 0·46	11	2·53	0·10	Referring-object	274 56 45·11	144	–	–
Wendover	123 52 20·07	9	3·06	0·16	Deptling	275 25 46·86	7	4·70	0·67
					Hollingbourne	280 59 8·09	10	5·04	0·56
Westminster Abbey, left Tower	125 23 35·90	4	6·72	3·22	Frittenfield	287 39 45·50	17	7·43	0·33
Westminster Abbey, right Tower[1]	125 25 14·07	4	6·28	2·54	Tenterden Church Tower	314 19 37·34	11	7·30	0·69
St. Paul's	129 34 14·51	12	6·16	0·58					
Severndroog	136 53 29·64	65	7·44	0·07	Goudhurst Church Tower	331 19 38·70	8	2·49	0·16
Dunstable	137 15 56·29	8	4·83	0·60	Fairlight	334 17 42·21	38	4·73	0·08
Berkhampstead	149 59 23·59	11	7·76	1·20	Brightling Obser^y	350 37 34·47	13	6·13	0·50

[1] A correction of + 2″·62 to be applied to this bearing, to reduce it to the trigonometrical station.
[2] A correction of —20′ 5″·54 „ „ „

WROXALL.

18-in. Theodolite.

29th July 1846. Observer: Corp. STEEL, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Shanklin Down	181 32 37·33	2	2·05	1·05	Boniface S.E.	270 52 13·67	2	7·28	13·24
Dunnose	200 31 52·00	2	2·28	1·29	Littletown Down	304 0 51·50	2	3·62	3·27
Boniface Down	257 59 20·33	2	4·05	4·10					

YELL.

3-ft. Theodolite, B.O.

From 12th to 15th August 1821. Observers: Major-Gen. Colby, Capts. Vetch and Drummond, R.E.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Brassa	0 32 36·94	14	8·10	0·67	Saxavord	203 14 49·53	11	7·47	0·79
Foula	51 17 43·32	10	5·67	0·44	Balta	215 46 33·68	10	6·61	0·67
Ronas	86 3 1·02	16	5·25	0·24	Fetlar	236 7 6·20	15	8·37	0·51
Vallafield	200 59 28·18	2	–	–					

YORK MINSTER.

2-ft. Theodolite.

From 7th November 1845 to 14th February 1846. Observer: Serg. Bay, R.S.M.

Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.	Objects.	Bearings.	No. of Obs.	Range.	Recip. of Weight.
	° ′ ″		″			° ′ ″		″	
Clifton	9 17 37·29	14	3·69	0·14	Acklam Wold	243 3 28·71	13	9·47	1·12
Garforth Cliff	43 32 51·27	20	5·84	0·27	Calais Beacon	258 42 41·79	3	4·36	2·33
Great Whernside	110 36 32·14	12	6·84	0·39	Referring-object	260 26 25·03	150	–	–
Botton Head	179 42 48·99	14	9·33	0·64	Hunsley	298 29 12·62	11	4·39	0·32

AZIMUTHAL OBSERVATIONS.

BALSHAM.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1844:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
June 9	Polaris E.	230 48 52·40 81·40	+13·82 − 6·21	66·22 75·19	199 19 25·40 28·40	2 28 57·24	150 59 13·43
” 10	” ”	202 22 28·80 60·00	+ 6·85 − 0·84	35·65 59·16	170 52 59·00 62·00	57·25	10·34
” 14	” ”	180 0 16·00 13·40	+ 3·37 − 5·20	19·37 8·20	148 30 27·80 32·20	57·71	13·92
” 15	” ”	178 32 20·00 17·00	+ 1·24 + 0·69	21·24 17·69	147 2 35·40 36·00	57·71	13·94
” 16	” ”	173 28 7·80 1·80	− 1·37 − 0·56	6·43 1·24	141 58 18·00 16·20	57·71	10·97
” 22	” ”	124 1 52·80 49·60	− 2·99 + 4·28	49·81 53·88	92 31 67·40 64·00	58·20	12·05
” 11	Polaris W.	197 24 68·60 52·60	+ 2·79 − 2·25	71·39 50·35	170 52 66·60 71·60	57·43	10·80
” 22	” ”	141 40 12·60 10·00	− 0·25 + 4·21	12·35 14·21	115 8 29·40 27·00	58·20	16·72

Mean of Eastern Elongations 150° 59′ 12″·44 General Mean 150° 59′ 12″·77
” Western ” ” ” 13″·76 Probable Error ± 0″·48

BARROW HILL.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1845:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
Aug. 31	Polaris E.	105 33 53·33 22·00	−18·58 +13·66	34·75 35·66	95 43 64·33 39·67	2 22 10·60	352 32 27·39
Sept. 2	” ”	62 22 57·00 7·33	−27·52 +27·61	29·48 34·94	52 32 68·00 31·67	9·66	27·28
” 6	” ”	124 51 8·67 39·00	+14·71 −21·63	23·38 17·37	115 1 29·00 51·67	7·47	27·42
” 7	” ”	91 20 35·00 57·33	+10·08 − 8·78	45·08 48·55	81 30 57·67 69·67	7·00	23·85
” 9	” ”	58 43 58·33 75·00	+ 7·22 −19·12	65·55 55·88	48 54 19·00 27·33	5·90	28·35
Aug. 28	Polaris W.	177 37 89·67 56·00	−12·34 +29·38	77·33 85·38	172 32 67·33 39·00	11·77	20·04[1]
” 29	” ”	163 33 24·67 15·67	−11·78 + 2·46	12·89 18·13	158 27 63·67 52·33	11·30	31·19
” 30	” ”	151 29 46·67 9·67	−26·40 +27·70	20·27 37·37	146 23 84·67 51·67	10·83	28·52
Sept. 6	” ”	120 7 8·00 39·33	+10·82 −12·63	18·82 26·70	115 1 51·00 62·67	7·23	26·84
” 7	” ”	86 36 34·33 36·00	− 8·96 − 2·88	25·37 33·12	81 31 5·00 0·33	6·73	26·70
” 8	” ”	71 24 45·00 77·67	+10·35 −16·06	55·35 61·61	66 19 28·67 41·00	6·14	30·21
” 9	” ”	53 59 44·33 42·67	− 2·97 − 4·66	41·36 38·01	48 54 14·33 12·33	5·67	27·98

Mean of Eastern Elongations 352° 32′ 26″·86 General Mean 352° 32′ 27″·79
” Western ” ” ” 28″·57 Probable Error ± 0″·36

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[1] A considerable discrepancy in the readings of the microscopes: rejected in the mean.

BEACON HILL.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^{n}.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1849:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
April 5	δ Urs. Min. E.	108 0 84·00 49·60	− 7·20 + 5·74	76·80 55·34	135 27 43·30 33·80	5 26 19·87	212 52 52·35
" "	Polaris "	104 57 68·60 54·40	−10·25 − 5·41	58·35 48·99	135 27 48·20 38·00	2 23 11·08	60·51
" 20	" "	150 55 27·00 14·70	−16·43 + 5·54	10·57 20·24	1 24 72·00 58·50	2 23 18·11	67·95[1]
" 26	δ Urs. Min. "	45 11 38·50 40·80	+ 5·13 − 5·23	43·63 35·57	72 38 15·60 25·40	5 26 15·60	56·50
" "	λ " "	41 34 61·00 57·80	+12·31 − 6·50	73·31 51·30	72 38 14·80 25·70	1 49 38·44	56·38
" 28	δ " "	149 57 50·20 27·40	− 5·42 + 3·55	44·78 30·95	177 24 30·40 24·80	5 26 14·97	64·70
" 30	" " "	112 34 67·80 46 40	−14·25 +17·37	53·55 63·77	140 1 52·20 36·50	5 26 14·33	60·02
May 4	" " "	159 11 69·00 71·20	+10·94 − 4·17	79·94 67·03	6 38 58·00 61·60	5 26 12·91	59·23
" "	λ " "	155 35 43·00 36·60	+ 0·34 + 8·89	43·34 45·49	6 38 55·20 55·80	1 49 37·57	48·65
" 10	Polaris "	2 21 39·00 27·60	− 4·89 + 1·58	34·11 29·18	32 51 4·00 0·20	2 23 27·23	57·68
April 5	Polaris W.	100 11 41·80 22·00	− 9·33 +13·68	32·47 35·68	135 27 43·00 30·60	2 23 11·32	51·41
" "	51 Cephei "	98 11 107·00 62·00	− 7·52 + 6·06	99·48 68·06	135 27 47·20 36·10	4 22 30·89	46·99
" 16	Polaris "	4 34 65·80 61·40	−10·29 − 4·95	55·51 56·45	39 51 11·40 9·20	2 23 17·22	57·10
" 26	51 Cephei "	35 22 50·40 50·80	+ 5·61 −10·07	56·01 40·73	72 38 14·60 26·10	4 22 34·64	57·34
" 28	Polaris "	154 4 25·60 17·20	− 3·84 +10·06	21·76 27·26	9 20 47·47 36·30	2 23 22·65	54·73
" "	51 Cephei "	140 8 79·50 54·00	− 5·65 + 3·78	73·85 57·78	177 24 33·30 24·70	4 22 35·16	48·02
" 30	" "	102 45 86·40 64·00	−12·32 +15·44	74·08 79·44	140 1 51·80 38·60	4 22 35·71	52·73
May 3	Polaris "	177 36 47·20 35·80	− 5·91 − 2·17	41·29 33·63	321 52 62·20 59·90	2 23 24·84	58·73
" 4	51 Cephei "	149 23 22·80 27·00	+10·61 − 3·85	33·41 23·15	6 38 57·20 60·00	4 22 37·00	53·32
" 6	Polaris "	127 24 46·40 34·40	− 0·08 + 4·53	46·32 38·93	162 41 2·00 0·40	2 23 25·95	52·63
June 2	" "	158 11 64·60 52·50	−28·33 − 5·58	36·27 46·92	13 27 75·00 59·80	2 23 34·06	51·74
" 9	" "	37 37 66·80 76·60	−15·40 −27·64	51·40 48·96	72 54 22·27 20·06	2 23 35·02	55·97

Mean of Eastern Elongations 212° 52′ 57″·34
" Western " " " 53″·97

General Mean 212° 52′ 55″·08
Probable Error ± ″·62

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[1] Cloudy: rejected in the mean.

BEN MACDUI*.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n^.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1847:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
July 4	Polaris E.	38 4 24·90 4.00	− 2·52 +11·67	22·38 15·67	197 6 8·80 10·20	2 46 39·07	161 48 29·55
„ 13	„ „	38 3 42·50 27·70	−12·62 + 6·96	29·88 34·66	197 5 18·15 14·65	37·64	21·77
„ 16	„ „	25 33 72·10 61·40	−10·15 − 1·32	61·95 60·08	185 35 60·65 60·00	37·09	36·40
„ 20	„ „	84 56 62·40 43·20	−14·98 − 2·15	47·42 41·05	243 58 38·45 35·75	36·21	29·08
Aug. 13	„ „	99 45 35·30 48·10	+ 4·34 − 3·04	39·64 45·06	258 47 32·90 42·35	27·56	22·84
„ „	51 Cephei „	102 2 69·00 48·40	−13·90 + 3·67	55·10 52·07	258 47 46·85 46·65	5 3 37·15	30·32
„ 14	Polaris „	191 34 9·10 0·00	+ 6·31 +10·73	15·41 10·73	350 36 9·60 13·20	2 46 27·19	25·52
„ „	51 Cephei „	193 50 88·00 58·50	− 1·11 +21·78	86·89 80·28	350 36 21·90 15·95	5 3 38·13	33·47
„ 15	Polaris „	191 34 19·90 20·60	+ 7·41 + 6·76	27·31 27·36	350 36 27·35 37·20	2 46 26·68	31·62
„ „	51 Cephei „	193 51 44·70 21·60	− 0·87 +12·61	43·83 34·21	350 36 32·80 32·70	5 3 38·95	32·68
„ 16	Polaris „	178 40 59·20 36·70	+ 0·37 +13·93	59·57 50·63	337 42 62·75 59·85	2 46 26·13	32·33
„ „	51 Cephei „	180 58 19·70 17·10	− 2·18 +12·90	17·52 30·00	337 43 17·35 17·10	5 3 38·91	32·38
July 20	Polaris W.	79 23 35·10 41·40	− 9·68 − 9·13	25·42 32·27	243 58 30·85 34·90	2 46 36·40	27·63
Aug. 13	δ Urs. Min. „	90 43 67·80 40·60	− 7·99 + 2·56	59·81 43·16	258 47 45·75 46·35	6 15 32·35	22·21[1]
„ 14	„ „	182 32 22·50 −9·70	− 2·44 +16·54	20·06 6·84	350 36 21·70 15·55	6 15 31·98	33·20
„ 15	„ „	182 32 33·50 12·60	− 4·19 +17·65	29·31 30·25	350 36 37·60 35·30	6 15 31·61	35·06
„ „	Polaris „	186 1 50·40 34·70	− 3·09 +12·36	47·31 47·06	350 36 43·45 38·55	2 46 26·41	27·41
„ 16	δ Urs. Min. „	169 39 16·30 3·10	− 4·01 +12·04	12·29 15·14	337 43 12·70 12·30	6 15 31·24	27·55

Mean of Eastern Elongations 161° 48′ 29″·83 General Mean 161° 48′ 29″·96
„ Western „ „ „ 30″·17 Probable Error ± 0″·65

[1] Imperfect observation, Elongation doubtful: rejected in the mean.

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* The bearing of the Referring-object at this station, omitted at page 81, is 161° 48′ 21″·98 observed 181 times, range 8″·23.

BEN MORE S. UIST.

Date.		Name and Elongation of Star.		Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
				° ′ ″			° ′ ″	° ′ ″	° ′ ″
1851: June	29	Polaris	E.	20 42 25·83	+28·95	54·78	143 39 13·25 7·50	2 45 19·09	305 41 34·69
July	3	„	„	72 34 25·83	−14·79	11·04	195 30 35·42 39·08	18·90	45·11
„	22	„	„	172 5 9·50	+ 5·99	15·49	295 1 43·83 38·67	15·24	41·00
Aug.	4	„	„	174 47 44·67	−13·79	30·88	297 44 9·58 2·75	10·85	46·14
„	5	51 Cephei	„	177 7 50·83	− 4·96	45·87	297 44 4·58 11·92	5 5 16·08	38·46
„	„	Polaris	„	112 38 26·67	−15·85	10·82	235 34 46·25 42·17	2 45 10·48	43·87
„	6	„	„	96 24 15·00	− 4·23	10·77	219 20 40·17 38·83	2 45 10·11	38·84
„	7	51 Cephei	„	98 44 15·50	− 1·37	14·13	219 20 33·00 36·50	5 5 16·51	37·13
„	„	Polaris	„	174 17 71·67	−22·33	49·34	297 14 18·33 18·67	2 45 9·60	38·76
„	8	51 Cephei	„	176 37 50·83	+ 6·07	56·90	297 14 18·42 18·59	5 5 17·36	38·97
„	„	Polaris	„	163 48 62·33	− 8·64	53·69	286 45 26·17 30·00	2 45 9.18	43.57
„	9	51 Cephei	„	166 8 50·17	+ 0·29	50·46	286 45 20·67 18·59	5 5 17·80	46·97
„	„	Polaris	„	154 12 36·00	+13·27	49·27	277 9 24·33 27·25	2 45 8·68	45·20
July	20	Polaris	W.	25 38 33·17	+ 8·68	41·85	154 5 36·58 33·83	2 45 16·03	37·33
„	24	„	„	128 8 58·50	+ 6·00	64·50	256 35 60·00 53·17	2 45 14·78	37·30
Aug.	5	δ Urs. Min.	„	165 44 76·17	−18·42	57·75	297 44 10·17 5·00	6 17 29·07	40·76
„	„	Polaris	„	160 44 43·50	−21·96	21·54	289 11 23·42 17·42	2 45 10·66	48·22
„	6	δ Urs. Min.	„	103 35 53·50	−26·46	27·04	235 34 47·33 42·50	6 17 28·65	49·23
„	„	Polaris	„	107 7 67·50	−15·86	51·64	235 34 42·33 43·42	2 45 10·29	40·95[1]
„	7	δ Urs. Min.	„	87 21 38·00	−12·94	25·06	219 20 37·50 34·58	6 17 28·15	42·83
„	„	Polaris	„	79 24 22·67	− 8·39	14·28	207 51 10·42 10·67	2 45 9·88	46·39[1]
„	8	δ Urs. Min.	„	165 15 15·83	− 5·37	10·46	297 14 19·25 18·17	6 17 27·77	40·48
„	9	δ „	„	154 46 3·33	+11·86	15·19	286 45 18·25 20·92	6 17 27·40	37·00

Mean of Eastern Elongations 305° 41′ 41″·44 General Mean 305° 41′ 41″·70
„ Western „ „ „ 42″·05 Probable Error ± 0″·56

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[1] Great motion during this arc.

BEN NEVIS.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1846:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
Aug. 5	51 Cephei E.	155 7 30·00 13·80	− 8·51 − 6·44	21·49 7·36	51 58 18·20 22·45	5 1 15·16	81 52 21·06[1]
” 21	Polaris ”	122 45 35·80 41·20	+ 2·49 −17·67	38·29 23·53	21 52 14·50 28·95	2 45 41·51	32·33[2]
” 24	” ”	182 46 6·20 19·80	+14·12 − 6·35	20·32 13·45	81 52 49·70 65·45	2 45 39·90	20·59
” ”	51 Cephei ”	185 1 59·40 45·80	− 4·76 +16·77	54·64 62·57	81 52 55·45 53·05	5 1 22·45	18·10
” 25	Polaris ”	182 45 61·20 84·00	+ 6·47 −21·64	67·67 62·36	81 52 46·40 65·90	2 45 39·35	30·48
” 26	” ”	172 39 72·10 75·00	− 2·53 −18·51	69·57 56·49	71 46 43·45 55·05	2 45 38·81	25·03
” 27	” ”	172 39 66·20 82·00	+ 0·69 −22·87	66·89 59·13	71 46 43·90 60·15	2 45 38·26	27·28[3]
” 28	” ”	148 53 42·50 33·00	− 8·88 − 9·21	33·62 23·79	48 0 13·55 19·45	2 45 37·71	25·51
” ”	51 Cephei ”	151 9 15·70 23·80	+ 3·82 −13·53	19·52 10·27	48 0 7·85 21·80	5 1 23·76	23·69
Sept. 12	” ”	179 31 21·00 38·40	+24·42 + 3·32	45·42 41·72	76 22 29·05 45·25	5 1 27·17	20·75
” 13	” ”	175 2 6·40 18·30	+ 9·22 − 4·62	15·62 13·68	71 53 2·05 12·00	5 1 27·33	19·71
” 14	Polaris ”	172 45 16·80 41·10	+27·41 −13·14	44·21 27·96	71 52 23·65 46·95	2 45 27·66	26·88
” ”	51 Cephei ”	175 1 8·70 37·10	+17·99 − 0·22	26·69 36·88	71 52 18·90 36·75	5 1 27·50	23·54
Aug. 24	δ Urs. Min. W.	173 47 51·00 29·00	− 3·19 +15·99	47·81 44·99	81 52 53·90 53·35	6 12 48·44	18·79
” 28	Polaris ”	167 8 60·70 60·40	− 9·26 − 4·63	51·44 55·77	71 46 50·40 52·40	2 45 37·43	20·36
” ”	δ Urs. Min. ”	139 55 5·60 20·60	+ 3·63 −17·85	9·23 2·75	48 0 6·00 24·50	6 12 47·19	22·07
Sept. 12	” ”	168 17 17·60 28·20	+23·93 + 4·51	41·53 32·71	76 22 28·15 44·05	6 12 43·89	15·09
” ”	λ ” ”	172 24 32·60 55·60	+20·31 + 3·01	52·91 58·61	76 22 29·65 42·40	2 5 19·24	21·03[4]
” 13	δ ” ”	163 47 48·00 60·70	+14·13 − 6·56	62·13 54·14	71 53 0·75 15·45	6 12 43·70	26·26
” ”	λ ” ”	167 55 25·90 42·40	+ 1·79 − 8·16	27·69 34·24	71 53 0·35 13·35	2 5 18·79	17·09
” ”	Polaris ”	167 15 1·90 25·60	+14·87 − 4·04	16·77 21·56	71 52 55·75 70·35	2 45 27·93	15·95
” 14	δ Urs. Min. ”	163 47 2·20 30·90	+17·27 − 7·27	19·47 23·63	71 52 20·15 37·60	6 12 43·52	23·81
” ”	λ ” ”	167 54 28·10 50·00	+20·39 − 5·10	48·49 44·90	71 52 17·30 37·40	2 5 18·33	22·32

Mean of Eastern Elongations 81° 52′ 24″·23 General Mean 81° 52′ 22″·51
” Western ” ” ” 20″·28 Probable Error ± 0″·60

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[1] Cloudy and misty.
[2] Cloudy.
[3] Very misty; partial sun.
[4] Very clear in all directions.

BLACK DOWN.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R. O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1848: Nov. 8	Polaris E.	139 7 35·67 35·33	+ 5·02 +13·31	40·69 48·64	93 18 9·00 12·00	2 21 33·71	316 31 59·55
" 10	" "	127 12 12·00 29·33	+ 8·43 + 8·17	20·43 37·50	41 22 43·27 43·77	32·65	47·20[1]
" 13	" "	94 3 58·67 69·00	− 9·00 + 4·73	49·67 73·73	48 14 44·77 38·27	31·07	70·90
" 21	" "	87 53 37·33 32·67	− 3·11 +16·97	34·22 49·64	42 4 16·83 8·00	27·08	57·57
" 24	" "	69 13 41·67 42·00	+12·14 + 2·33	53·81 44·33	23 24 20·33 23·67	25·78	58·71
" 27	" "	51 34 71·67 48·00	−16·68 +23·50	54·99 71·50	5 45 47·83 24·00	24·40	57·07
1849: Feb. 9	" "	100 39 47·00 47·67	−12·22 + 8·15	34·78 55·82	54 50 37·33 30·00	16·50	64·87
" 10	" "	100 39 57·33 42·67	− 3·74 − 0·76	53·59 41·91	54 50 33·67 29·33	16·78	60·53[2]
" 13	" "	159 17 69·00 69·33	−10·07 + 3·55	58·93 72·88	113 28 52·30 50·30	17·73	63·12
" 16	" "	40 44 66·67 61·67	− 2·57 − 2·02	64·10 59·65	174 55 44·30 40·30	18·52	58·94[3]
" 17	" "	24 1 57·00 57·00	− 5·78 − 1·55	51·22 55·45	158 12 35·70 36·03	18·83	61·36[4]
" 22	" "	63 12 33·67 28·00	−12·18 + 3·72	21·49 31·72	17 23 7·80 3·00	20·57	59·36
1848: Nov. 7	Polaris W.	144 7 16·67 6·67	− 7·89 + 9·73	8·78 16·40	283 0 53·67 47·67	33·95	64·13
" 15	" "	73 53 50·67 42·00	+ 3·02 +12·97	53·69 54·97	212 47 31·17 20·67	30·32	61·27
" 24	" "	64 30 47·33 56·00	+ 8·21 − 3·32	55·54 52·68	203 24 16·17 19·34	25·58	58·07[5]
1849: Jan. 26	" "	147 16 33·33 47·00	+ 4·97 − 5·79	38·30 41·21	286 9 54·84 52·50	14·05	59·86
Feb. 8	" "	95 56 53·67 95·00	+15·49 −18·05	69·16 76·95	234 50 19·50 40·17	16·42	60·36[6]
" 11	" "	5 46 56·67 46·00	− 5·81 +11·71	50·86 57·71	144 40 21·84 8·67	16·94	64·04
" 12	" "	168 14 12·67 20·33	+ 1·99 + 3·01	14·66 23·34	307 7 44·51 41·34	17·25	66·68
" 13	" "	154 35 32·33 25·00	−12·50 + 0·58	19·83 25·58	293 28 48·63 45·30	17·57	66·70[7]
" 17	" "	36 2 39·67 21·33	− 9·33 + 0·37	30·34 21·70	174 55 48·76 43·27	18·60	61·40
" 27	" "	137 14 28·00 15·00	− 8·90 +11·35	19·10 26·35	276 7 44·84 41·17	22·31	57·97

Mean of Eastern Elongations 316° 32′ 1″·09 General Mean 316° 32′ 1″·55
" Western " " " 2″·05 Probable Error ± 0″·52

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[1] Referring-object in haze: rejected in the mean.
[2] Referring-object in very bad light; very unsatisfactory.
[3] Referring-object in haze.
[4] Referring-object difficult to observe.
[5] Unsatisfactory, in consequence of clouds.
[6] The star was not satisfactorily observed, the wind being too strong for the axis light.
[7] Referring object was not observed for two hours after the star, owing to mist.

BURNSWARK.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1847:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
March 28	δ Urs. Min. E.	102 46 57·40	+ 4·02	61·42	121 41 20·10	5 57 41·47	204 52 0·15
,, ,,	λ ,, ,,	98 49 47·40	+ 5·77	53·17	121 41 19·40	2 0 38·66	4·89
,, 29	,, ,, ,,	95 22 14·80	+ 4·37	19·17	118 13 47·05	2 0 38·89	6·77
,, ,,	δ ,, ,,	99 19 11·20	+12·25	23·45	118 13 46·55	5 57 41·47	4·57
April 1	,, ,, ,,	270 25 6·80	+ 5·02	11·82	289 19 44·10	5 57 41·29	13·57
,, ,,	λ ,, ,,	266 28 5·80	+ 3·79	9·59	289 19 42·30	2 0 39·63	12·34
,, 2	δ ,, ,,	267 47 53·60	+ 2·44	56·04	286 42 26·80	5 57 41·29	12·05
,, ,,	λ ,, ,,	263 50 49·00	+ 2·54	51·54	286 42 23·55	2 0 39·72	11·73
,, 5	δ ,, ,,	85 51 13·40	+ 1·48	14·88	104 45 45·05	5 57 40·81	10·98
,, ,,	λ ,, ,,	81 54 12·20	+ 0·95	13·15	104 45 44·05	2 0 40·09	10·99
,, 8	δ ,, ,,	264 9 37·00	+ 3·62	40·62	283 4 9·25	5 57 40·45	9·08
,, 9	λ ,, ,,	260 12 36·40	+ 2·30	38·70	283 4 10·10	2 0 40·58	11·98
,, ,,	δ ,, ,,	78 47 50·40	+ 1·15	51·55	97 42 20·50	5 57 40·28	9·23
,, 10	λ ,, ,,	74 50 52·40	— 1·88	50·52	97 42 19·45	2 0 40·70	9·63
,, 12	δ ,, ,,	76 1 35·00	+ 0·21	35·21	94 56 3·05	5 57 39·75	7·59
,, 13	λ ,, ,,	72 4 31·20	+ 1·72	32·92	94 56 1·30	2 0 40·98	9·36
,, 17	δ ,, ,,	245 14 14·60	+ 1·30	15·90	264 8 42·10	5 57 38·70	4·90
,, 18	λ ,, ,,	241 17 16·60	— 2·38	14·22	264 8 40·90	2 0 41·16	7·84
,, 19	δ ,, ,,	64 3 0·20	+ 0·22	0·42	82 57 28·75	5 57 38·22	6·55
,, ,,	λ ,, ,,	60 5 58·80	+ 1·73	60·53	82 57 28·35	2 0 41·33	9·15
,, ,,	Polaris ,,	240 43 21·60	+ 0·16	21·76	262 57 28·95	2 37 59·47	6·66
,, 21	δ Urs. Min. ,,	242 17 33·00	— 2·05	30·95	261 12 1·15	5 57 37·70	7·90
,, ,,	λ ,, ,,	238 20 33·00	— 0·36	32·64	261 12 2·00	2 0 41·26	10·62
,, ,,	Polaris ,,	58 57 55·40	+ 2·16	57·56	81 12 2·75	2 38 0·52	5·71
,, 23	λ Urs. Min. ,,	53 58 29·80	+ 3·88	33·68	76 49 58·85	2 0 41·14	6·31
,, ,,	δ ,, ,,	57 55 27·20	+ 4·25	31·45	76 49 58·45	5 57 37·00	4·00
,, 24	,, ,, ,,	222 20 61·60	— 4·86	56·74	241 15 28·55	5 57 36·64	8·45
May 4	,, ,, ,,	8 12 69·60	+ 6·04	75·64	27 7 45·15	5 57 33·14	2·65
,, ,,	λ ,, ,,	4 16 16·80	+ 2·38	19·18	27 7 45·95	2 0 40·51	7·28
March 28	51 Cephei W.	92 1 44·00	+ 3·46	47·46	121 41 20·00	4 47 28·16	4·38
,, 29	,, ,,	88 34 6·60	+ 5·50	12·10	118 13 46·30	28·20	6·00
,, 31	,, ,,	259 40 2·60	+ 1·30	3·90	289 19 45·35	28·23	13·22
April 1	,, ,,	257 2 45·20	+ 4·55	49·75	286 42 26·75	28·28	8·72
,, 4	,, ,,	75 6 6·00	+ 1·23	7·23	104 45 45·30	28·65	9·42
,, 8	,, ,,	253 24 34·40	— 0·27	34·13	283 4 9·75	29·12	6·50
,, 9	,, ,,	68 2 47·00	+ 0·34	47·34	97 42 20·65	29·26	4·05
,, 12	,, ,,	65 16 32·40	— 2·47	29·93	94 56 5·20	29·72	5·55
,, 17	,, ,,	234 29 8·20	— 0·79	7·41	264 8 42·85	30·69	4·75
,, 19	Polaris ,,	55 27 29·40	+ 0·71	30·11	82 57 32·75	2 37 59·73	2·91
,, ,,	51 Cephei ,,	53 17 54·00	+ 0·39	54·39	82 57 29·20	4 47 31·07	3·74
,, 21	Polaris ,,	53 41 57·60	+ 2·71	60·31	81 12 2·75	2 38 0·50	1·94
,, ,,	51 Cephei ,,	231 32 23·80	+ 0·35	24·15	261 12 1·55	4 47 31·47	5·93
,, 23	,, ,,	47 10 21·40	+ 2·66	24·06	76 49 58·65	31·97	2·62
,, 24	,, ,,	211 35 53·00	— 4·82	48·18	241 15 28·80	31·97	8·65
May 4	,, ,,	357 28 3·00	+ 5·44	8·44	27 7 46·65	35·64	2·57

Mean of Eastern Elongations 204° 52′ 8″·03　General Mean 204° 52′ 7″·20

,, Western ,, ,, ,, 5″·68　Probable Error ± 0″·33

BUTSER HILL.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1844: Dec. 5	Polaris E.	172 18 59·67 31·33	−27·98 +10·49	31·69 41·82	98 5 34·00 37·00	2 23 62·30	108 10 61·05[1]
1845: Jan. 20	„ „	59 58 59·67 67·00	+ 6·71 − 3·93	66·38 63·07	165 45 57·00 64·67	55·34	51·45
„ 27	„ „	51 17 21·67 30·67	− 6·19 + 1·22	15·48 31·89	157 4 27·67 20·00	55·95	56·10
„ 28	„ „	24 53 60·00 40·00	+ 5·13 − 1·33	65·13 38·67	130 40 39·83 40·33	56·11	44·29[2]
„ 29	„ „	11 1 66·67 56·67	+ 2·47 +12·14	69·14 68·81	116 49 13·00 6·00	56·26	56·78
Feb. 1	„ „	6 19 56·00 84·67	+12·90 − 9·81	68·90 74·86	112 7 1·67 14·00	56·85	52·80
„ 11	„ „	113 37 46·67 64·67	+ 6·26 − 6·46	52·93 58·21	39 24 45·00 53·00	59·08	52·51
„ 14	„ „	54 36 9·67 16·00	+ 5·36 + 2·15	15·03 18·15	160 23 4·67 11·67	60·03	51·61[3]
1844: Dec. 5	λ Urs. Min. W.	168 5 34·00 43·67	− 7·50 − 8·74	26·50 34·93	98 5 62·33 57·67	1 49 29·78	59·50
1845: Jan. 20	Polaris „	64 38 48·00 67·00	+ 9·70 − 9·19	57·70 57·81	175 13 36·33 46·33	2 23 55·34	48·24[4]
„ 21	„ „	55 11 7·67 18·00	+ 4·15 − 2·92	11·82 15·08	165 46 0·67 6·67	55·34	54·88
„ 24	„ „	167 57 30·33 62·67	+12·79 −14·03	43·12 48·64	98 32 28·67 46·33	55·55	56·07[5]
„ 26	„ „	146 2 57·17 86·33	+ 6·38 −11·94	63·55 74·39	76 37 58·33 70·00	55·71	59·49
„ 27	„ „	46 29 27·67 35·83	+ 0·05 − 2·01	27·72 33·82	157 4 24·00 23·17	55·87	56·94
Feb. 2	„ „	166 12 35·67 58·67	+ 4·60 −16·02	40·27 42·65	96 47 26·67 39·33	56·98	54·56
„ 4	„ „	134 28 58·67 81·00	+ 5·10 −10·04	63·77 70·96	65 3 58·33 66·67	57·30	57·83[6]
„ 6	„ „	26 58 36·67 41·67	+ 7·46 + 4·37	44·13 46·04	137 33 33·00 37·33	57·77	52·32[7]
„ 7	„ „	12 34 3·33 39·00	+20·66 − 3·27	23·99 35·73	123 9 13·00 25·67	57·97	51·50

Mean of Eastern Elongations 108° 10′ 54″·61 General Mean 108° 10′ 54″·92
„ Western „ „ „ 55″·13 Probable Error ± 0″·55

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[1] The level out of adjustment.
[2] Rejected in the mean.
[3] Star clouded from 19^m 27^s to 24^m 13^s.
[4] Weather stormy.
[5] Strong wind caused the light on referring-object to be unsteady.
[6] The light on referring-object unsatisfactory.
[7] Wind very strong, observation unsatisfactory.

CHEVIOT.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1846:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
July 4	Polaris E.	164 14 16·40 54·40	+ 9·18 −15·60	25·58 38·80	227 43 40·90 40·30	2 40 20·79	246 9 29·20
August 2	” ”	49 48 2·80 47·40	+10·81 + 0·34	13·61 47·74	113 17 52·90 47·70	14·17	33·79
” 3	” ”	55 10 58·20 67·00	+ 5·96 − 0·02	64·16 66·98	118 40 21·30 19·80	13·83	28·81
” 4	” ”	56 6 12·20 39·60	+ 5·79 − 4·95	17·99 34·65	119 35 42·60 41·20	13·46	29·04
” 5	” ”	58 3 9·80 20·40	+ 0·96 + 0·50	10·76 20·90	121 32 34·70 32·90	12·98	30·96
” ”	51 Cephei ”	60 13 66·60 76·60	+ 0·60 − 2·90	67·20 73·70	121 32 34·30 34·70	4 51 4·19	28·24
” 6	Polaris ”	59 9 60·40 68·80	− 4·38 + 1·10	56·02 69·90	122 39 22·60 19·70	2 40 12·58	30·77
” 18	” ”	2 29 6·00 22·20	+ 5·17 − 0·44	11·17 21·76	65 58 38·80 39·30	2 40 7·51	30·09
” ”	51 Cephei ”	4 39 67·40 89·60	+ 1·28 − 3·60	68·68 86·00	245 58 43·50 40·70	4 51 9·25	34·01
” 19	Polaris ”	359 13 58·20 70·20	+ 3·38 − 0·35	61·58 69·85	62 43 28·60 29·00	2 40 6·98	30·06
” 21	” ”	358 32 45·60 48·60	+ 7·02 + 4·26	52·62 52·86	62 2 20·00 20·40	2 40 6·05	33·51
” 24	” ”	357 51 9·20 17·60	+ 0·63 − 0·86	9·83 16·74	61 20 39·10 38·30	2 40 4·48	29·90
” ”	51 Cephei ”	0 2 18·40 25·40	− 3·77 − 4·13	14·63 21·27	61 20 41·80 38·50	4 51 11·23	33·43
” 25	Polaris ”	356 28 17·80 29·60	+ 2·51 − 4·33	20·31 25·27	59 57 47·70 48·20	2 40 3·98	29·14
” ”	51 Cephei ”	358 39 25·20 30·60	− 0·10 − 3·80	25·10 26·80	59 57 47·20 48·20	4 51 11·55	33·30
” 26	Polaris ”	354 44 58·20 56·40	− 0·27 + 1·59	57·93 57·99	58 14 23·60 24·40	2 40 3·45	29·49
” ”	51 Cephei ”	356 55 65·80 68·40	+ 1·14 − 7·09	66·94 61·31	58 14 23·70 25·40	4 51 11·87	32·29
Sept. 2	” ”	337 35 54·60 16·00	−15·20 +15·93	39·40 31·93	38 53 72·50 57·10	13·75	42·88[1]
” 9	” ”	305 35 27·80 28·60	− 0·15 + 3·92	27·65 32·52	6 53 48·00 40·40	15·32	29·43
” 10	” ”	307 56 32·20 29·80	− 7·08 − 1·19	25·12 28·61	9 14 43·20 35·20	15·48	27·81
” 12	” ”	308 54 65·00 55·20	− 9·06 + 8.33	55·94 63·53	10 13 19·80 10·10	15·79	31·01
August 5	δ Urs. Min. W.	49 22 38·80 50·00	+ 0·98 − 2·08	39·78 47·92	121 32 34·10 34·10	6 0 18·59	31·66
” 18	” ” ”	353 48 48·80 62·80	+ 3·78 − 7·69	52·58 55·11	65 58 39·20 42·10	13·50	33·30
” 24	” ” ”	349 10 50·00 61·00	+ 0·06 − 0·43	50·06 60·57	61 20 40·50 38·10	11·37	32·61
” 25	” ” ”	347 47 55·80 70·20	+ 4·36 − 3·38	60·16 66·82	59 57 48·50 48·60	11·07	33·99
” 26	” ” ”	346 4 43·20 47·80	+ 1·13 − 4·17	44·33 43·63	58 14 23·30 25·40	10·84	29·53
” ”	λ ” ” ”	350 3 39·80 42·00	+ 0·20 + 0·16	40·00 42·16	58 14 25·50 23·80	2 1 13·91	29·66
Sept. 2	δ ” ” ”	326 44 36·40 10·20	−12·17 +11·08	24·23 21·28	38 53 71·20 56·50	6 0 9·07	32·02

[1] Rejected in mean : Star not observed until 20^m past elongation, owing to clouds.

CHEVIOT—*continued.*

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1846 : Sept. 9	λ Urs. Min. W.	298 42 60·40 56·60	+ 0·59 + 8·98	60·99 65·58	6 53 44·90 37·20	2 1 6·92	246 9 30·85
„ 10	δ „ „	297 4 68·80 53·80	−10·32 +10·07	58·48 63·87	9 14 41·40 32·10	6 0 7·35	28·22
„ „	λ „ „	301 3 64·40 53·60	− 3·22 +11·22	61·18 64·82	9 14 40·90 33·00	2 1 6·48	27·47
„ 11	δ „ „	302 52 61·40 55·40	− 0·23 + 6·45	61·17 61·85	15 2 46·00 37·20	6 0 7·17	32·92
„ 12	„ „ „	298 3 43·40 30·40	− 7·83 + 7·10	35·57 37·50	10 13 18·20 9·90	6 0 6·99	30·52
„ „	λ „ „	302 2 44·80 34·60	− 7·57 + 7·46	37·23 42·06	10 13 18·30 8·90	2 1 5·59	28·36
„ 13	δ „ „	287 45 39·20 24·20	− 4·83 +10·32	34·37 34·52	359 55 12·10 2·80	6 0 6·82	26·19

Mean of Eastern Elongations 246° 9′ 30″·71 General Mean 246° 9′ 30″·63
„ Western „ „ „ „ 52 Probable Error ± 0″·24

COWHYTHE.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1846 : Dec. 2	Polaris E.	124 26 63·33 78·00	− 1·85 − 8·31	61·48 69·69	188 6 5·50 4·67	2 48 38·56	66 27 38·06
„ 28	„ „	10 36 24·67 22·33	− 7·25 + 1·98	17·42 24·31	74 15 33·17 21·33	29·53	35·92[1]
„ 30	„ „	157 56 20·33 28·00	−14·08 − 7·29	6·25 20·71	221 35 29·50 18·25	28·97	39·36
1847 : Jan. 10	„ „	126 9 14·00 16·33	+ 5·48 + 2·97	19·48 19·30	189 48 30·17 24·33	27·80	35·66
„ 11	„ „	116 57 19·67 26·67	− 0·52 − 0·67	19·15 26·00	180 36 35·00 30·50	27·80	37·98
„ 12	„ „	106 45 15·00 28·33	+ 4·11 − 1·07	19·11 27·26	170 24 31·67 28·17	27·80	34·53
„ 14	„ „	69 23 53·67 60·67	+ 1·93 − 4·82	55·60 55·85	133 3 6·67 6·33	27·80	38·58
„ 22	„ „	9 1 11·00 20·67	− 2·58 + 0·60	8·42 21·27	72 40 26·00 16·33	28·17	34·49
1846 : Dec. 20	Polaris W.	41 14 6·67 28·33	+ 9·51 −12·14	16·18 16·19	110 30 16·67 23·17	31·45	32·28
„ 29	„ „	152 19 13·00 23·67	+ 0·15 − 2·78	13·15 20·89	221 35 29·67 20·67	29·25	38·90
1847 : Jan. 12	„ „	101 8 13·33 32·00	− 2·48 + 1·30	10·85 33·30	170 24 33·33 21·66	27·80	37·62[2]
„ 13	„ „	72 8 10·67 18·00	− 2·20 − 2·95	8·47 15·05	141 24 16·83 9·00	27·80	33·36
„ 18	δ Urs. Min. „	163 16 19·33 39·33	−15·27 −10·45	4·06 28·88	236 6 39·83 32·33	6 22 36·53	43·08

Mean of Eastern Elongations 66° 27′ 36″·82 General Mean 66° 27′ 36″·91
„ Western „ „ „ 37″·05 Probable Error ± 0″·52

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[1] The weather was rather cloudy, and some delay occurred before the star could be found on the 2d arc.
[2] Referring-object observed only on one arc.

DUNKERY.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n^.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1844:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
Oct. 21	Polaris E.	69 22 52·90 60·00	+ 6·73 − 6·88	59·63 53·12	151 39 6·80 14·20	2 25 0·26	84 41 14·39
" 22	" "	69 22 49·60 63·10	+ 6·83 − 8·23	56·43 54·87	151 39 7·90 20·10	2 24 59·74	18·09
" "	51 Cephei "	71 20 52·20 58·60	+13·53 −13·89	65·73 44·71	151 39 3·80 17·50	4 22 54·06	9·49
" 23	Polaris "	5 6 68·70 61·60	− 5·36 + 3·86	63·34 65·46	87 23 20·50 18·40	2 24 59·14	14·19
" "	51 Cephei "	7 4 55·90 52·30	+ 1·36 + 1·02	57·26 53·32	87 23 17·50 20·30	4 22 53·90	17·51
Nov. 17	" "	66 28 46·10 53·40	+10·30 − 7·02	56·40 46·38	146 47 4·80 18·60	4 22 47·84	8·15
Oct. 22	δ Urs. Min. W.	61 32 18·20 29·70	+ 0·71 −10·48	18·91 19·22	151 39 2·80 12·60	5 25 33·75	14·89
" 23	λ " "	65 8 7·90 10·70	− 5·39 − 7·81	2·51 2·89	151 39 10·20 15·90	1 49 53·68	16·67
" 24	Polaris "	0 16 59·70 63·80	+ 8·26 − 4·84	67·96 58·96	87 23 17·40 28·40	2 24 58·83	20·61
Nov. 17	δ Urs. Min. "	56 40 0·30 24·60	+15·23 −14·56	15·53 10·04	146 46 53·60 73·70	5 25 40·66	10·20
" "	λ " "	60 15 55·20 69·40	+10·46 − 6·68	65·66 62·72	146 46 63·90 78·70	1 49 53·07	14·04
" 18	Polaris "	59 41 16·20 12·30	+ 2·59 − 3·63	18·79 8·67	146 47 23·40 31·20	2 24 44·98	28·59[1]
" 21	λ Urs. Min. "	60 16 14·10 15·80	+ 2·98 − 0·85	17·08 14·95	146 47 21·10 26·00	1 49 53·74	13·80
" 22	Polaris "	59 41 31·80 19·50	− 6·64 + 6·21	25·16 25·71	146 47 27·40 21·30	2 24 42·91	16·00

Mean of Eastern Elongations 84° 41′ 13″·64 General Mean 84° 41′ 14″·46
" Western " " " 15″·17 Probable Error ± 0″·64

[1] Rejected in the mean.

DUNNOSE.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n^.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1844:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
July 15	Polaris E.	65 20 42·70 37·10	− 8·52 − 3·30	34·18 33·80	42 8 41·40 45·10	2 24 5·01	339 12 14·27
" 16	" "	8 22 23·80 25·30	−18·29 − 1·29	5·51 24·01	165 10 21·90 24·60	4·74	13·23
" 21	" "	62 22 53·60 50·60	+ 9·48 −19·38	63·08 31·22	219 10 55·30 59·00	3·64	13·64[1]
" 20	Polaris W.	166 59 13·90 2·80	− 4·37 +19·31	9·53 22·11	148 35 38·20 31·40	3·99	14·99[2]
" 21	" "	153 29 58·90 53·40	+ 7·81 + 9·80	66·71 63·20	135 6 15·10 19·50	3·79	8·56
" 23	" "	57 34 42·80 36·90	− 6·44 + 1·24	36·36 38·14	219 10 53·50 49·40	3·36	10·84

Mean of Eastern Elongations 339° 12′ 13″·71 General Mean 339° 12′ 12″·59
" Western " " " 11″·46 Probable Error ± 0″·61

[1] Levels were not read until the termination of the arc; observation not satisfactory. [2] Much motion.

DUNRICH.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1850:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
Sept. 15	Polaris E.	65 48 37·30 65·80	+ 8·13 − 3·86	45·43 61·94	125 39 45·77 46·07	2 38 11·90	242 28 64·14
„ 17	„ „	71 42 86·00 43·00	−10·25 − 2·27	75·75 40·73	131 33 50·42 42·82	10·78	59·16
„ 22	„ „	164 33 54·10 23·80	− 9·99 + 1·96	44·11 25·76	224 24 13·91 18·16	7·47	48·57
„ 24	„ „	84 25 72·00 34·40	−13·10 + 6·76	58·90 41·16	144 16 31·40 23·90	6·23	43·85
Oct. 9	„ „	113 42 16·80 26·50	+13·41 + 3·14	30.21 29·64	173 33 27·90 27·90	2 37 56·14	54·11
„ 11	„ „	62 41 60·40 60·50	+ 6·51 − 3·22	66·91 57·28	122 33 3·40 2·55	2 37 54·85	55·73
„ „	51 Cephei „	64 56 13·20 26·80	− 0·93 −10·04	12·27 16·76	122 32 55·90 58·70	4 52 12·53	55·31
„ 20	Polaris „	67 5 22·10 39·80	+ 6·88 −13·22	28·98 26·58	126 56 29·70 43·35	2 37 48·83	57·57
„ 23	„ „	158 19 23·30 17·00	+ 0·67 + 9·19	23·97 26·19	38 10 40·05 32·90	2 37 46·89	58·29
„ „	51 Cephei „	160 33 46·80 36·80	+ 0·77 + 7·02	47·57 43·82	38 10 37·50 34·80	4 52 10·98	61·43
„ 26	Polaris „	98 6 46·90 34·70	− 2·95 +14·64	43·95 49·34	157 57 70·00 59·50	2 37 44·82	62·92
„ „	51 Cephei „	100 21 29.90 11·40	−13·36 +12·50	16·54 23·90	157 58 13·70 4·00	4 52 10·39	59·02
„ 28	Polaris „	48 53 15·30 29·10	+ 3·34 − 3·10	18·64 26·00	108 44 34·80 35·20	2 37 43·40	56·08
„ 29	„ „	179 42 62·00 52·90	− 3·64 + 5·70	58·36 58·60	59 34 13·75 12·45	2 37 42·82	57·44
Sept. 24	δ Urs. Min. W.	75 47 16·00 12·80	−18·73 + 3·32	−2·73 16·12	144 16 28·80 27·40	6 0 49·21	32·19[1]
Oct. 9	Polaris „	108 26 30·20 35·90	+ 2·89 + 6·37	33·09 42·27	173 33 28·25 23·40	2 37 55·79	52·35
„ 11	δ Urs. Min. „	54 3 17·10 28·80	+ 3·30 −11·85	20·40 16·95	122 32 58·10 60·65	6 0 49·96	50·74
„ „	Polaris „	57 26 10·20 15·10	+ 2·54 − 2·91	12·74 12·19	122 33 2·55 8·05	2 37 54·55	58·28
„ 14	„ „	40 52 25·30 26·40	− 1·03 − 1·53	24·27 24·87	105 59 11·60 14·70	2 37 52·43	56·15
„ 20	„ „	26 46 47·60 43·10	+ 3·05 + 3·64	50·65 46·74	91 53 34·95 32·90	2 37 48·53	56·70
„ 23	δ Urs. Min. „	149 40 52·00 48·80	− 2·30 + 2·68	49·70 51·48	38 10 38·60 34·85	6 0 51·92	54·21
„ „	λ „ „	153 41 41·80 18·90	− 3·83 + 5·40	37·97 24·30	38 10 40·00 34·15	2 0 3·74	62·19
„ 26	δ „ „	89 28 32·80 13·20	−11·30 +12·29	21·50 25·49	157 58 15·40 7·05	6 0 52·66	55·08
„ „	λ „ „	93 28 70·10 55·80	− 6·06 +13·00	64·04 68·80	157 58 10·60 0·85	2 0 3·42	55·89
„ „	Polaris „	92 51 36·90 10·50	− 7·73 + 6·75	29·17 17·25	157 58 14·85 9·00	2 37 44·46	64·25
„ 29	δ Urs. Min. „	171 4 31·90 10·10	−21·97 + 5·60	9·93 15·70	59 34 11·50 4·10	6 0 53·20	61·78

Mean of Eastern Elongations 242° 28′ 56″·69 General Mean 242° 28′ 56″·85
„ Western „ „ „ 57″·06 Probable Error ± 0″·63

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[1] Rejected in the mean.

EASINGTON.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^{n}.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1846:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
Feb. 27	δ Urs. Min. E.	2 6 57·00 33 40	− 8·10 +16·58	48·90 49·98	124 52 45·70 39·20	5 53 2·80	128 38 55·81
” ”	λ ” ”	178 12 54·20 45·20	−11·88 +11·29	42·32 56·49	304 52 47·70 39·50	1 59 13·15	67·35
” 28	δ ” ”	179 1 19·40 31·40	+ 2·00 + 0·12	21·40 31·52	301 47 19·30 19·70	5 53 3·00	56·04
March 1	” ” ”	174 27 51·00 70·60	+ 6·62 − 1·78	57·62 68·82	297 13 56·00 61·60	5 53 3·32	58·90
” ”	λ ” ”	170 34 2·60 21·00	+ 4·86 − 6·75	7·46 14·25	297 13 55·50 61·00	1 59 14·11	61·50
” 3	δ ” ”	171 21 60·40 77·80	+ 6·99 − 3·92	67·39 73·88	294 8 5·00 8·30	5 53 3·72	59·74
” 5	” ” ”	164 21 18·20 38·20	+ 3·63 − 1·40	21·83 36·80	287 7 25·30 26·10	5 53 4·06	60·44
” ”	” ”	160 27 24·20 45·80	+ 3·79 − 7·18	27·99 38·62	107 7 19·90 23·20	1 59 15·91	64·16
” 6	δ ” ”	161 44 35·40 61·60	+ 3·72 − 0·89	39·12 60·71	284 30 42·00 39·90	5 53 4·23	55·26
” ”	λ ” ”	157 50 56·60 66·60	− 2·20 − 1·43	54·40 65·17	284 30 40·50 40·80	1 59 16·34	57·21
” 7	” ” ”	145 44 59·00 75·00	− 1·40 − 0·93	57·60 74·07	272 24 52·50 50·60	1 59 16·70	62·41
” 8	δ ” ”	138 38 28·20 40·60	− 2·05 + 1·23	26·15 41·83	261 24 27·70 27·50	5 53 4·75	58·36
” ”	λ ” ”	134 44 35·00 49·40	+ 1·75 + 1·19	36·75 50·59	261 24 28·30 27·60	1 59 17·10	61·38
” 9	δ ” ”	123 14 66·00 78·40	+ 3·74 − 6·79	69·74 71·61	246 0 63·30 65·30	5 53 4·93	58·56
” ”	λ ” ”	119 21 22·60 20·40	− 0·27 − 3·59	22·33 16·81	246 0 64·40 65·30	1 59 17·50	62·78
” ”	Polaris ”	119 58 3·60 9·00	+ 1·52 + 2·00	5·12 11·00	246 0 63·40 69·10	2 36 2·26	60·45
” 12	δ Urs. Min. ”	116 26 32·40 33·20	+ 0·60 − 1·42	33·00 31·78	239 12 25·60 31·60	5 53 5·32	61·53
” ”	λ ” ”	112 32 47·20 39·40	− 6·78 + 9·24	40·42 48·64	239 12 26·30 27·00	1 59 18·69	60·81
” ”	δ ” ”	112 54 52·20 47·80	− 2·90 + 4·42	49·30 52·22	235 40 43·30 47·30	5 53 5·45	59·99
” 18	” ” ”	99 35 68·20 59·40	− 6·51 +10·64	61·69 70·04	222 21 62·20 59·70	5 53 6·01	61·09
” 23	” ” ”	90 4 49·60 43·40	−12·42 +10·17	37·18 53·57	212 50 42·60 37·10	5 53 6·35	60·83
” ”	λ ” ”	86 10 56·40 50·00	− 5·29 + 7·16	51·11 57·16	212 50 39·00 36·00	1 59 22·36	65·72
” 24	δ ” ”	78 35 34·20 45·20	+ 5·41 −16·83	39·61 28·37	201 21 30·60 29·10	5 53 6·35	62·21
” 26	” ” ”	74 44 62·40 64·40	− 6·58 + 3·53	55·82 67·93	197 30 56·30 52·90	5 53 6·22	58·95
” ”	λ ” ”	70 51 20·00 19·60	− 9·14 − 0·82	10·86 18·78	197 30 60·20 58·70	1 59 23·07	67·70
” 27	δ ” ”	70 33 13·60 17·40	− 0·58 + 0·69	13·02 18·09	193 19 10·90 9·00	5 53 6·18	60·57
” ”	λ ” ”	66 39 28·40 33·80	− 5·52 + 1·06	22·88 34·86	193 19 10·90 9·80	1 59 23·29	64·77
” 28	δ ” ”	59 57 34·80 35·20	−10·48 − 0·47	24·32 34·73	182 43 28·40 25·40	5 53 6·18	63·56

EASINGTON—*continued.*

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1846: March 28	λ Urs. Min. E.	56 3 40·00 39·60	+ 8·20 + 3·40	48·20 43·00	182 43 26·70 26·70	1 59 23·52	128 38 64·62
April 3	δ " "	49 20 34·40 30·80	−10·00 + 3·65	24·40 34·45	172 6 21·50 18·10	5 53 5·83	56·20
" 4	λ " "	45 26 54·00 38·60	−21·60 + 3·21	32·40 41·81	172 6 26·20 19·50	1 59 24·93	70·68
Feb. 25	Polaris W.	177 1 35·80 47·80	− 2·63 − 4·41	33·17 43·39	308 16 34·70 28·80	2 35 57·50	55·97
" 26	" "	173 37 43·60 50·60	− 3·57 + 1·94	40·03 52·54	304 52 44·20 34·90	2 35 58·00	55·26
" 27	51 Cephei "	171 29 70·20 52·20	−12·79 +13·38	57·41 65·58	304 52 46·20 40·10	4 43 40·00	61·66
" 28	" "	168 24 36·40 49·00	− 2·75 − 2·18	33·65 46·82	301 47 19·40 19·20	39·71	59·35
March 3	" "	160 45 14·60 38·80	+ 3·70 − 2·06	18·30 36·74	294 8 4·00 9·10	38·84	60·19
" 4	" "	158 35 46·40 56·20	+ 3·07 − 1·08	49·47 55·12	291 58 29·40 31·60	38·65	59·56
" 5	" "	153 44 33·40 52·00	+ 1·54 − 1·19	34·94 50·81	287 7 24·00 25·10	38·46	63·21
" 6	" "	151 7 53·80 75·60	+ 6·36 − 7·31	60·16 68·29	284 30 41·80 41·00	38·27	58·91
" 8	" "	128 1 43·60 58·20	+ 0·73 − 1·91	44·33 56·29	261 24 26·50 26·80	37·89	58·45
" 9	" "	112 38 16·80 40·60	+ 4·47 − 2·47	21·27 38·13	246 0 64·50 64·50	37·68	57·12
" 11	Polaris "	107 57 28·20 33·80	− 2·63 − 2·65	25·57 31·15	239 12 25·40 28·60	2 36 4·06	54·58
" 12	51 Cephei "	105 49 47·60 58·80	+ 1·85 − 1·97	49·45 56·83	239 12 26·20 31·20	4 43 37·13	58·43
" "	Polaris "	104 25 40·00 43·80	− 2·70 − 0·11	37·30 43·69	235 40 45·80 48·10	2 36 4·54	61·91
" 13	51 Cephei "	102 18 7·20 8·20	− 3·64 + 4·46	3·56 12·66	235 40 42·80 45·80	4 43 36·97	59·22
" 17	Polaris "	91 6 59·60 44·00	− 7·86 +10·67	51·74 54·67	222 21 62·20 60·70	2 36 7·17	61·08
" 18	51 Cephei "	88 59 22·60 22·40	− 7·52 + 9·04	15·08 31·44	222 21 61·90 59·60	4 43 36·54	60·95
" 23	" "	79 27 62·60 64·20	−11·07 +10·24	51·53 74·44	212 50 41·40 35·80	36·13	59·48
" 24	" "	67 58 39·00 75·20	+ 7·53 −16·45	46·53 58·75	201 21 29·70 27·40	36·15	59·76
" 25	" "	64 8 19·80 21·00	− 4·93 − 0·48	14·87 20·52	197 30 55·60 53·70	36·18	60·78
" 26	" "	59 56 25·60 38·80	− 4·44 + 3·03	21·16 41·83	193 19 10·00 6·50	36·20	60·55
" 27	" "	49 20 44·80 64·00	− 5·16 − 3·43	39·64 60·57	182 43 26·20 25·00	36·21	59·29

Mean of Eastern Elongations 128° 38′ 61″·28
" Western " " " 59″·32

General Mean 128° 38′ 60″·49
Probable Error ± 0″·30

FAIRLIGHT DOWN.

Date.		Name and Elongation of Star.		Reading of Star.	Computed Level and Col^n^.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1844 :				° ′ ″			° ′ ″	° ′ ″	° ′ ″
Sept.	11	Polaris	E.	84 57 14·00 16·00	− 3·35 − 2·71	10·65 13·29	178 37 57·67 48·67	2. 24 30·30	276 5 11·50
„	13	„	„	23 11 50·67 69·33	+ 6·54 + 0·77	57·21 70·10	116 52 51·00 43·33	29·14	12·65
„	19	„	„	177 14 20·67 46·00	+ 8·14 − 9·58	28·81 36·42	90 55 20·67 20·33	25·81	13·69
„	20	„	„	149 5 21·67 21·00	− 4·54 + 2·50	17·13 23·50	62 46 7·00 5·67	25·23	11·24
„	24	„	„	144 31 57·33 54·33	− 0·65 + 0·03	56·68 54·36	58 12 42·67 43·67	22·85	10·50
„	25	„	„	107 13 13·00 1·67	− 0·17 + 9·40	12·83 11·07	20 53 59·67 57·00	22·28	8·66
„	26	„	„	71 38 30·33 31·00	− 6·02 + 0·01	24·31 31·01	165 19 16·33 14·00	21·69	9·20
„	27	„	„	36 52 38·00 28·00	− 7·19 + 5·51	30·81 33·51	130 33 21·67 12·00	21·10	5·77
„	28	„	„	27 26 18·33 21·67	+ 2·10 + 2·27	20·43 23·94	121 7 11·00 10·33	20·47	8·95
„	30	„	„	157 45 8·67 8·17	− 1·92 + 0·69	6·75 8·86	71 26 6·00 5·00	19·31	17·00
Oct.	4	„	„	51 3 56·33 55·00	+ 0·71 + 4·41	57·04 59·41	144 44 53·33 53·33	16·82	11·92
Sept.	10	Polaris	W.	94 59 57·33 60·67	−11·38 + 9·04	45·95 69·71	13 29 47·33 31·67	30·62	11·05
„	25	„	„	102 24 20·67 17·00	+ 2·04 + 5·44	22·71 22·44	20 54 1·50 0·83	21·93	16·66
„	26	„	„	66 49 52·17 41·67	− 6·52 + 2·52	45·65 44·19	165 19 19·33 14·33	21·42	10·49
„	29	„	„	22 37 39·33 35·67	− 4·18 + 1·84	35·15 37·51	121 7 11·00 9·33	19·55	14·29

Mean of Eastern Elongations 276° 5′ 11″·01
„ Western „ „ „ 13″·12

General Mean 276° 5′ 11″·57
Probable Error ± 0″·51

GADS HILL.

Date.		Name and Elongation of Star.		Reading of Star.	Computed Level and Col^n^.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1845 :				° ′ ″			° ′ ″	° ′ ″	° ′ ″
Sept.	19	51 Cephei	E.	54 51 69·20 52·00	− 9·12 +12·58	60·08 64·58	177 39 30·00 15·00	4 24 30·34	307 11 50·51
„	26	„	„	27 10 45·20 60·40	+ 1·87 − 4·99	47·07 55·41	149 58 14·00 7·20	30·90	50·26
„	27	„	„	23 49 63·60 46·80	+ 0·38 + 8·95	63·98 55·75	146 37 26·00 21·00	30·98	54·61
„	29	„	„	13 43 51·80 69·60	+ 1·27 − 5·57	53·07 64·03	136 31 23·00 18·20	31·08	53·13
Oct.	4	„	„	0 3 38·20 46.40	+ 4·15 +13·23	42·35 59·63	122 51 28·80 23·80	31·08	66·39

GADS HILL—*continued.*

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1845: Oct. 5	51 Cephei E.	355 28 18·60 23·00	+ 5·38 +10·65	23·98 33·65	118 15 44·80 43·40	4 24 31·08	307 11 46·36
„ 7	„ „	343 20 52·60 22 41·20	+46·71 −45·67	99·31 115·53	106 8 36·60 96·20	31·08	50·06[1]
„ 9	„ „	328 37 42·80 37·40	− 4·04 + 6·14	38·76 43·54	91 24 59·00 52·40	31·03	45·58
„ 10	Polaris „	311 0 78·60 56·40	− 3·61 +13·65	74·99 70·05	75 47 39·00 30·20	2 25 27·45	49·53
„ 11	51 Cephei „	313 0 26·60 15·40	− 7·29 + 8·54	19·31 23·94	75 47 44·40 32·20	4 24 30·90	47·57
„ 13	„ „	90 1 54·60 59·40	+17·69 − 3·35	72·29 56·05	212 49 21·60 30·20	4 24 30·77	52·50
„ 14	Polaris „	261 55 40·40 31·80	− 5·51 + 9·32	34·89 41·12	26 41 66·60 57·20	2 25 25·17	49·06
„ „	51 Cephei „	263 54 53·60 32·00	−12·28 +14·80	41·32 46·80	26 41 69·00 58·40	4 24 30·71	50·35
„ 16	„ „	75 42 38·00 47·20	− 3·19 − 1·74	34·81 45·46	198 29 64·20 55·80	30·61	50·47
„ 21	„ „	68 27 13·40 28·40	− 4·81 − 5·03	8·59 23·37	191 14 39·00 33·40	29·98	50·20
„ 23	„ „	63 21 49·20 44·40	− 2·97 + 3·40	46·23 47·80	6 8 59·20 70·60	29·66	47·54
„ 24	Polaris „	61 22 43·60 19·60	−17·34 +16·40	26·26 36·00	186 8 71·80 58·00	2 25 19·05	52·82
„ „	51 Cephei „	57 17 39·40 19 40	−18·86 +14·98	20·54 34·38	180 4 54·60 39·60	4 24 29·50	49·14
„ 25	„ „	53 24 43·60 34·60	− 1·36 +11·10	42·24 45·70	176 11 68·80 59·80	4 24 29·34	49·67
Sept. 19	δ Urs. Min. W.	45 0 33·20 22·60	−10·38 + 6·50	22·82 29·10	177 39 30·00 18·00	5 27 12·26	45·78
„ 26	„ „ „	17 19 10·60 12·00	− 2·12 + 3·38	8·48 15·38	149 58 14·00 3·40	5 27 11·75	45·02
„ „	λ „ „	20 56 4·60 4·80	+ 0·17 + 1·40	4·77 6·20	149 58 11·60 6·60	1 50 14·69	48·92
„ 27	δ „ „	13 58 26·80 24·80	+ 1·02 + 0·87	27·82 25·67	146 37 26·00 21·00	5 27 10·65	46·10
„ „	λ „ „	17 35 34·40 24·60	− 1·07 − 1·33	33·33 23·27	146 37 27·00 22·80	1 50 14·65	41·95
„ 28	δ „ „	3 52 17·40 7·00	− 1·24 +11·84	16·16 18·84	136 31 23·00 14·80	5 27 11·67	49·73
„ 29	λ „ „	7 28 56·40 67·40	+ 7·02 +16·61	63·42 84·01	136 31 22·80 17·60	1 50 14·55	51·93
Oct. 5	δ „ „	345 36 70·40 29·00	− 0·87 + 7·80	69·53 36·80	118 15 44·80 43·20	5 27 11·71	39·12
„ „	λ „ „	349 13 45·40 53·80	− 2·53 − 5·09	42·87 48·71	118 15 47·40 43·20	1 50 12·83	46·68
„ 7	δ „ „	333 29 12·20 30 58·20	+45·35 −47·55	57·55 70·65	106 8 36·60 93·80	5 27 11·82	49·28
„ 8	Polaris „	325 10 60·20 36·40	−25·75 −10·70	34·45 25·70	94 47 60·80 49·80	2 25 28·41	56·81
„ 9	δ Urs. Min. „	318 45 53·60 49·00	−11·12 + 8·07	42·48 57·07	91 24 59·00 47·40	5 27 11·99	51·43
„ „	λ „ „	322 22 59·20 59·20	− 4·58 + 7·08	54·62 66·28	91 24 59·80 52·40	1 50 11·43	44·22
„ 11	δ „ „	303 8 27·40 29·20	+ 1·21 +11·28	28·61 40·48	75 47 44·40 29·20	5 27 11·94	50·31

[1] This star was observed with an accidental collimation error of 58″·40.

GADS HILL—*continued.*

Date.		Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1845 :			° ′ ″			° ′ ″	° ′ ″	° ′ ″
Oct.	11	λ Urs. Min. W.	306 45 40·40 26·00	+ 1·85 +12·67	42·25 38·67	75 47 42·20 32·00	1 50 11·41	307 11 45·23
„	„	Polaris „	306 10 23·80 21·00	− 1·49 + 7·86	22·31 28·86	75 47 43·00 33·60	2 25 26·65	46·26
„	13	δ Urs. Min. „	260 10 19·60 7·60	+ 1·90 +16·57	21·50 24·17	32 49 30·20 18·60	5 27 12·10	49·47
„	„	λ „ „	263 47 9·80 28·20	+ 7·41 + 1·36	17·21 29·56	212 49 19·40 28·40	1 50 11·02	49 50
„	„	Polaris „	263 11 55·60 58·60	+ 9·08 +14·34	64·68 72·94	32 49 30·40 18·00	2 25 26·00	49·39
„	14	δ Urs. Min. „	254 2 69·40 65·20	−13·15 + 4·33	56·25 69·53	26 41 69·00 60·20	5 27 12·35	49·36
„	„	λ „ „	257 40 13·60 15·20	−11·44 + 0·36	2·16 15·56	26 41 69·20 61·20	1 50 10·83	45·51
„	„	Polaris „	257 4 56·00 56·00	− 7·48 + 6·44	48·52 62·44	26 41 69·20 60·00	2 25 24·89	44·23
„	16	δ Urs. Min. „	65 50 83·60 52·00	−23·46 + 8·36	60·14 60·36	198 29 64·20 52·60	5 27 12·67	45·48
„	„	λ „ „	69 36 57·60 34·60	−28·50 +10·63	29·10 45·23	198 38 42·60 27·00	1 50 10·44	47·19
„	23	δ „ „	53 29 79·40 59·60	−20·84 + 2·80	58·56 62·40	186 8 70·60 57·20	5 27 13·64	49·78
„	24	„ „ „	47 25 50·00 37·40	−13·03 + 9·43	36·97 46·83	180 4 54·60 41·00	5 27 13·80	52·10
„	„	λ „ „	51 2 62·80 45·00	−12·84 +10·96	49·96 55·96	180 4 53·00 39·60	1 50 9·40	43·94
„	25	δ „ „	43 32 66·80 55·60	− 7·03 + 7·46	59·77 63·06	176 11 68·80 59·40	5 27 13·88	48·80
„	„	λ „ „	47 9 74·00 58·40	−12·95 +17·97	61·05 76·37	176 11 78·20 58·80	1 50 9·29	50·50

Mean of Eastern Elongations 307° 11′ 50″·83
„ Western „ „ „ 47″·72

General Mean 307° 11′ 48″·95
Probable Error ± 0″·38

GOAT FELL.

Date.		Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1852 :			° ′ ″			° ′ ″	° ′ ″	° ′ ″
Sept.	5	Polaris E.	67 40 47·60 32·50	−11·46 + 1·70	36·14 34·20	118 5 7·85 10·90	2 37 22·57	233 1 56·78
„	6	„ „	51 17 65·80 45·80	− 6·74 +10·76	59·06 56·56	101 42 30·05 34·55	22·08	56·57
„	13	„ „	172 25 48·70 47·80	+ 2·87 + 1·21	51·57 49·01	42 50 31·20 37·55	17·69	61·77
„	14	51 Cephei „	106 15 63·80 60·60	+ 7·25 − 1·13	71·05 59·47	154 25 23·90 34·55	4 52 34·72	58·69
„	16	„ „	29 49 39·60 30·40	−16·94 + 5·45	22·66 35·85	77 58 56·70 55·10	35·01	61·65

GOAT FELL—*continued.*

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1852 : Sept. 17	Polaris E.	162 0 90·20 61·20	−16·89 + 5·44	73·31 66·64	32 25 57·25 49·75	2 37 15·03	233 1 58·86
" "	51 Cephei "	164 16 30·00 12·50	−10·82 + 8·85	19·18 21·35	32 25 49·10 48·75	4 52 35·15	63·81
" 21	Polaris "	96 56 39·00 18·00	−23·98 − 2·23	15·02 15·77	147 21 7·00 3·60	2 37 12·51	62·41
" 27	" "	123 24 62·40 49·80	− 3·69 + 0·40	58·71 50·20	173 49 38·15 41·30	2 37 8·61	53·88
" "	51 Cephei "	125 40 15·60 12·80	− 2·66 +10·36	12·94 23·16	173 49 39·40 39·30	4 52 35·88	57·18
Oct. 5	Polaris "	2 38 71·40 46·40	− 9·41 + 5·39	61·99 51·79	53 3 50·70 45·87	2 37 3·16	54·56
" 8	" "	132 3 67·70 34·00	− 7·37 +12·86	60·33 46·86	2 28 51·87 42·60	2 37 1·03	54·67
" 11	" "	79 50 21·40 4·40	− 9·86 + 9·97	11·54 14·37	130 14 71·30 61·70	2 36 58·91	52·46
" "	51 Cephei "	82 5 51·00 34·50	− 3·70 +14·11	47·30 48·61	130 14 67·30 61·20	4 52 35·51	51·80
" 17	Polaris "	47 14 75·60 53·50	− 1·43 + 1·80	74·17 55·30	97 39 61·40 63·50	2 36 54·83	52·55
Dec. 1	" "	167 56 56·20 48·00	− 5·59 + 5·59	50·61 53·59	38 22 37·34 27·60	2 36 27·49	67·86
Sept. 10	Polaris W.	147 49 67·80 44·80	−13·20 + 6·98	54·60 51·78	23 29 13·35 10·15	2 37 19·86	58·70
" 14	δ Urs. Min. "	95 21 70·80 63·60	− 6·47 − 2·72	64·33 60·88	154 25 23·45 32·30	6 1 23·50	61·77
" "	Polaris "	81 14 30·40 38·30	− 1·10 − 4·75	29·30 33·55	136 53 46·90 50·90	2 37 16·67	60·80
" 16	" "	48 2 23·20 22·50	− 4·46 + 3·96	18·74 26·46	103 41 34·80 33·35	2 37 16·10	55·38
" "	δ Urs. Min. "	18 55 41·00 32·90	− 9·76 + 8·29	31·24 41·19	77 58 58·10 53·60	6 1 23·28	56·35
" "	λ " "	22 56 58·80 56·40	− 7·48 + 7·98	51·32 64·38	77 58 53·40 52·10	1 59 54·09	60·81
" 17	δ " "	153 22 42·20 32·00	− 9·24 − 0·56	32·96 31·44	32 25 50·10 48·80	6 1 23·14	54·11
" 21	" "	88 17 62·20 34·30	−16·97 + 9·62	45·23 43·92	147 20 66·20 61·30	6 1 22·96	56·22
" 27	Polaris W.	71 3 47·40 63 20	− 8·50 − 6·50	38·90 56·70	126 42 58·15 60·10	2 37 8·88	62·44
" "	δ Urs. Min. "	114 46 23·10 18·20	− 4·83 + 0·91	18·27 19·11	173 49 38·90 41·60	6 1 22·79	58·77
" "	λ " "	118 47 55·70 38·60	− 5·24 + 8·16	50·46 46·76	173 49 36·50 36·50	1 59 50·16	57·73
Oct. 11	δ " "	71 11 49·40 33·80	− 3·87 + 5·96	45·53 39·76	130 15 8·10 4·70	6 1 23·85	59·91
" "	λ " "	75 13 28·20 16·80	− 5·12 + 9·51	23·08 26·31	130 15 7·00 1·75	1 59 46·56	53·12

Mean of Eastern Elongations 233° 1′ 57″·83
" Western " " " 58″·16

General Mean 233° 1′ 57″·98
Probable Error ± 0″·48

HART FELL.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1846: Oct. 1	51 Cephei E.	130 9 55·00 80·00	+11·33 + 4·79	66·33 84·79	295 39 39·10 38·20	4 50 46·30	350 20 9.39
” 9	” ”	313 38 57·20 89·20	+ 9·50 − 9·50	66·70 79·70	299 8 43·00 39·80	40·48	8·68
” 10	” ”	310 21 25·20 35·80	+ 6.60 +14·87	31·80 50·67	115 51 6·30 4·30	40.12	4·18
Dec. 15	Polaris ”	259 21 54·80 51·80	+ 1·63 − 0·42	56·43 51·38	67 3 16·00 17·30	2 38 41·88	4·62
1847: Jan. 11	δ Urs. Min. ”	251 1 30·40 20·80	− 1·81 + 5·57	28·59 26·37	55 21 36·70 30·40	6 0 6·59	12·66
” 12	” ”	242 49 59·60 69·80	+ 1·16 − 0·43	60·76 69·37	47 10 6·70 6·90	6 0 7·22	8·96
Feb. 3	Polaris ”	189 56 50·60 54·60	+ 1·68 − 1·32	52·28 53·28	357 38 17·70 19·20	2 38 39·06	4·73
” 6	δ Urs. Min. ”	187 54 68·60 69·60	+ 1·37 − 4·77	69·97 64·83	352 15 1·50 6·90	6 0 20·52	17·32
” 7	λ ” ”	183 55 61·80 53·20	+ 4·78 − 2·60	66·58 50·60	352 15 2·60 7·20	2 1 14·84	21·15
” 23	δ ” ”	5 52 18·00 9·80	+ 0·61 + 0·60	18·61 10·40	350 11 59·90 61·70	6 0 27·24	13·53
” ”	λ ” ”	181 52 59·40 72·40	+ 2·13 − 4·68	61·53 67·72	350 11 60·30 62·30	2 1 23·32	19·99
” 24	δ ” ”	182 59 34·40 52·60	− 3·48 − 0·53	30·92 52·07	347 19 27·70 26·10	6 0 27·38	12·78
1846: Nov. 9	δ ” W.	302 48 46·80 66·40	+11·43 −10·58	58·23 55·82	119 8 42·30 43·90	5 59 33·67	12·40
” ”	λ ” ”	306 47 28·80 55·00	+27·26 − 3·07	56·06 51·93	119 8 35·00 42·20	2 0 39·48	5·12
” ”	Polaris ”	306 9 34·80 39·40	+ 5·87 + 4·53	40·67 43·93	119 8 44·80 45·00	2 38 58·94	3·66
” 10	δ Urs. Min. ”	299 31 2·80 32·60	+18·44 −13·83	21·24 18·77	115 51 6·70 6·40	5 59 34·02	12·53
Dec. 1	λ ” ”	301 20 52·20 69·40	− 3·85 − 3·54	48·35 65·86	113 41 43·90 43·10	2 0 41·61	4·78
” ”	Polaris ”	300 42 44·20 50·20	− 0·93 − 5·12	43·27 45·08	113 41 42·60 43·60	2 38 47·17	11·75
” 2	λ Urs. Min. ”	298 30 1·40 14·80	+ 1·48 − 0·39	2·88 14·41	110 50 64·60 64·50	2 0 42·91	12·99
” ”	Polaris ”	297 52 4·80 8·20	+ 0·13 + 0·60	4·93 8·80	110 50 66·40 66·00	2 38 45·64	13·69
” 3	λ Urs. Min. ”	297 51 48·20 50·40	− 3·53 + 2·56	44·67 52·96	110 12 45·20 42·30	2 0 43·16	11·77
” ”	Polaris ”	297 13 42·20 57·40	− 1·80 − 3·05	40·40 54·35	110 12 42·40 42·70	2 38 46·24	8·93
” 4	λ Urs. Min. ”	296 27 8·00 33·40	+12·35 + 2·16	20·35 35·56	108 48 24·70 21·30	2 0 43·39	11·65
” 5	” ” ”	294 46 47·40 63·40	+ 0·99 + 0·58	48·39 63·98	107 7 55·10 52·10	43·65	13·76
” 7	” ” ”	293 50 49·40 57·20	− 2·87 + 2·75	46·53 59·95	106 11 51·50 46·60	44·15	11·66
” 10	” ” ”	291 18 24·40 5·80	−18·47 + 7·95	5·93 13·75	103 38 67·80 57·00	45.22	7·34
” 12	” ” ”	256 56 60·80 63·40	+ 1·23 + 0·11	62·03 63·51	69 17 58·70 59·40	45·92	10·36
” 13	Polaris ”	256 18 55·60 64·80	+ 2·21 + 3·37	57·81 68·17	69 17 59·80 57·50	2 38 42·41	13·25

HART FELL—*continued.*

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1846: Dec. 14	Polaris W.	254 4 19·40 35·80	+ 9·45 −11·75	28·85 24·05	67 3 11·30 24·50	2 38 42·06	350 20 9·39
„ 17	„ „	249 40 56·00 69·20	+ 4·82 − 4·58	60·82 64·62	62 39 54·20 58·30	41·18	12·35
1847: Jan. 11	„ „	242 22 46·20 43·80	− 3·78 + 3·53	42·42 47·33	55 21 39·50 35·50	36·86	15·76
„ „	51 Cephei „	240 11 11·40 11·20	− 5·00 + 5·48	6·40 16·68	55 21 36·50 30·70	4 50 9·67	12·39
„ 12	Polaris „	234 11 11·60 18·60	− 0·32 + 0·44	11·28 19·04	47 10 7·90 7·40	2 38 36·86	15.63
„ „	51 Cephei „	231 59 38·20 56·00	+ 3·14 − 1·92	41·34 54·08	47 10 6·60 7·40	4 50 9·12	10·17
„ 13	Polaris „	237 35 8·00 12·20	− 0·26 − 0·83	7·74 11·37	50 33 58·00 57.30	2 38 36·86	11·23
Feb. 3	„ „	184 39 34·20 16·40	− 3·33 +11·07	30·87 27·47	357 38 25·50 18·70	39·21	13·72
„ 4	„ „	182 10 32·80 44·60	+ 4·38 − 0·75	37·18 43·85	355 9 30·40 32·90	39·38	11·75
„ 6	„ „	180 28 66·60 60·80	−12·08 +12·92	54·52 73·72	353 27 53·40 53·70	39·55	9·88
„ 7	„ „	179 16 22·60 25·40	+ 3·69 − 3·08	26·29 22·32	352 15 11·90 16·50	39·95	9·94
„ „	51 Cephei „	177 4 66·80 56·20	+ 1·65 −10·38	68·45 45·82	352 15 1·60 10·90	4 49 56·33	12·78
„ 21	Polaris „	177 58 4·00 7·20	+ 0·79 − 4·42	4·79 2·78	350 56 56·00 57·30	2 38 45·70	7·16
„ 23	„ „	357 12 52·60 54·00	− 2·29 + 6·29	50·31 60·29	350 11 50·90 50·90	45·46	10·14
„ „	51 Cephei „	175 1 59·49 59·60	+ 1·11 − 3·12	60·51 56·48	350 11 61·00 61·10	4 49 49·65	12·90
„ 24	„ „	172 8 57·80 96·40	+ 2·53 − 3·99	60·33 92·41	347 19 15·50 15·90	49·47	9·86

Mean of Eastern Elongations 350° 20′ 11″·50 General Mean 350° 20′ 11″·11
„ Western „ „ „ 10″·96 Probable Error ± 0″·38

LEITH HILL TOWER.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R. O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1844: Mar. 6	Polaris E.	56 38 14·33 21·33	The level was not read at this Station.		95 45 41·33 58·33	2 25 12·43	221 32 44·43[1]
„ 8	„ „	39 30 9·33 22·33			78 37 46·33 66·67	13·23	53·90[2]
„ 21	„	84 50 7·33 25·33			123 57 43·33 62·00	19·30	55·64

[1] Referring-object could not be observed for 1½ hours after the star, owing to mist.

[2] Morning cloudy, and mist about referring-object made the observation unsatisfactory.

LEITH HILL TOWER—*continued.*

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1844:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
March 24	δ Urs. Min. E.	45 40 57·00 95·67			81 47 24·00 47·33	5 26 43·12	221 32 62·46
,, 29	,, ,, ,,	0 58 8·67 33·00			37 4 19·00 39·00	5 26 42·96	51·13
,, 30	Polaris ,,	127 55 48·33 74·00			167 3 23·33 39·67	2 25 23·76	54·09[3]
,, 31	δ Urs. Min. ,,	113 15 41·00 4·00			149 21 24·33 41·67	5 26 42·80	53·30[4]
,, ,,	Polaris ,,	110 13 55·67 87·00			149 21 23·33 44·67	2 25 24·36	47·03[5]
April 1	δ Urs. Min. ,,	70 20 59·00 94·33			106 27 21·33 43·33	5 26 42·64	58·30
,, ,,	Polaris ,,	67 19 58·33 80·67			106 27 26·33 41·33	2 25 24·88	49·21
,, 2	δ Urs. Min. ,,	55 0 23·67 49·67			91 6 42·33 45·33	5 26 42·64	49·80
,, ,,	Polaris ,,	51 59 4·33 24·00			91 6 43·33 46·33	2 25 25·36	56·02
,, 3	δ Urs. Min. ,,	43 19 35·00 70·33			79 26 4·33 11·33	5 26 42·48	57·65
,, 4	Polaris ,,	23 56 39·33 67·67			63 4 18·00 15·33	2 25 26·32	49·49
,, 5	δ Urs. Min. ,,	146 18 22·67 26·66			2 24 32·83 34·67	5 26 42·32	51·40
,, 8	Polaris ,,	131 2 18·50 25·83	+ 1·68 +13·45	20·18 39·28	170 9 51·33 48·00	2 25 28·75	48·69
,, ,,	δ Urs. Min. ,,	69 30 31·00 44·33	− 2·28 −12·70	28·72 31·63	105 36 41·33 41·33	5 26 41·88	53·04
,, 9	Polaris ,,	66 29 9·33 11·00	− 0·33 + 5·51	9·00 16·51	105 36 47·67 44·00	2 25 29·35	62·43
,, 10	,, ,,	46 53 31·00 17·33	− 5·99 + 4·12	25·01 21·45	86 0 43·67 42·67	29·75	49·69[6]
,, 11	,, ,,	34 36 13·67 17·00	− 0·36 −12·60	13·31 4·40	73 43 33·67 38·67	30·31	57·63
,, 16	δ Urs. Min. ,,	32 15 4·00 4·33	+ 3·49 + 4·00	7·49 8·33	68 21 18·67 16·00	5 26 40·16	49·58
,, 17	Polaris ,,	29 13 73·00 54·33	− 5·99 + 5·58	67·01 59·91	68 21 26·67 10·33	2 25 33·18	48·22
,, ,,	δ Urs. Min. ,,	13 8 47·33 37·67	−15·75 − 4·22	31·58 33·45	49 14 56·00 41·00	5 26 39·84	55·83
,, 19	Polaris ,,	10 7 9·33 7·33	− 1·36 +10·08	7·97 17·41	49 14 36·67 25·00	2 25 34·14	52·28
,, 23	,, ,,	169 9 28·00 19·67	+ 0·60 + 8·27	28·60 27·94	28 16 46·00 39·33	36·05	50·45
,, 25	,, ,,	58 43 55·33 48·67	+ 0·15 + 7·00	55·48 55·67	97 51 13·33 5·33	36·85	50·61
,, 26	,, ,,	53 1 21·67 12·50	− 2·46 +10·39	19·21 22·89	92 8 36·00 24·33	37·33	46·45
,, 28	,, ,,	25 38 72·33 58·67	−16·30 + 3·85	56·03 62·52	64 46 21·33 9·67	38·13	54·35
Feb. 20	Polaris W.	33 20 8·00 17·33			77 18 21·33 18·33	6·73	60·43

[3] Referring-object very hazy.
[4] Observation to the star taken at midnight. Referring-object not before six in the morning owing to mist.
[5] Referring-object not observed for one hour after the star, owing to mist.
[6] Referring-object very indistinct.

LEITH HILL TOWER—*continued.*

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1844:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
March 3	Polaris W.	69 9 14·33 51·00			113 7 37·33 56·67	2 25 11·23	221 32 63·10[7]
„ 5	„ „	51 47 27·00 66·67			95 45 42·67 63·33	12·19	53·98
„ 9	„ „	171 4 42·00 57·33			35 2 50·00 57·33	13·95	50·05
„ 10	„ „	154 10 11·00 26·33			18 8 23·67 36·33	14·43	56·90
„ 21	„ „	64 24 41·67 75·33			108 22 53·33 74·33	19·57	45·76
„ 23	„ „	37 48 57·67 88·67			81 47 18·67 39·67	20·65	55·35
„ 28	„ „	173 5 47·67 83·00			37 4 12·33 33·67	23·04	54·63
„ 29	„ „	123 4 56·33 83·67			167 3 13·67 32·00	23·53	49·30
„ 30	„ „	105 22 57·00 76·33			149 21 16·33 30·67	24·04	52·79
„ 31	„ „	62 29 6·67 33·67			106 27 21·33 40·67	24·64	46·19
April 1	„ „	47 8 21·33 40·00			91 6 50·00 47·33	25·12	52·88
„ 3	„ „	19 5 36·00 58·67			63 4 18·00 9·00	26·08	60·09
„ 4	„ „	11 52 38·00 40·17			55 50 55·67 59·67	26·56	52·03
„ 5	„ „	138 26 10·50 13·50			2 24 31·33 30·00	27·03	51·64
„ 8	„ „	61 38 30·00 38·66	− 1·06 − 5·37	28·94 33·29	105 36 49·33 51·00	29·11	49·94
„ 9	„ „	42 2 29·00 38·00	+ 0·29 − 7·75	29·29 30·25	86 0 52·67 56·67	29·59	55·31[8]
„ 10	„ „	29 45 25·83 26·17	+ 1·20 + 1·91	27·03 28·08	73 43 48·67 49·00	30·07	51·20
„ 17	„ „	5 16 37·67 28·67	−18·48 + 2·61	19·19 31·28	49 14 63·33 51·67	33·42	58·85
„ 23	„ „	116 45 65·00 52·00	− 4·96 + 9·94	60·04 61·94	160 44 38·67 29·33	36·25	56·76
„ 25	„ „	53 52 52·00 44·33	− 6·17 − 3·78	45·83 40·55	97 51 19·00 11·67	37·07	55·07

Mean of Eastern Elongations	221° 32′ 52″·61	General Mean	221° 32′ 53″·17
„ Western „	„ „ 53″·92	Probable Error	± 0″·43

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[7] Storm acting on axis light as well as on the light of the referring-object made the observation rather unsatisfactory.

[8] Much motion.

LUNDY ISLAND.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n^.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1845: Feb. 18	Polaris E.	90 44 38·30 29·70	− 3·32 + 3·38	34·98 33·28	102 46 7·10 6·00	2 24 36·49	194 25 68·91
March 11	” ”	61 37 9·70 14·10	+ 1·56 − 7·88	11·26 6·22	73 38 24·40 34·60	44·87	65·63
” 12	δ Urs. Min. ”	143 59 5·00 4·60	− 1·21 − 1·23	3·79 3·37	152 58 37·20 41·80	5 26 30·10	66·02
” 15	” ”	104 46 62·00 79·50	+10·00 − 5·22	72·00 74·28	113 46 40·20 51·80	30·29	63·15
” 21	” ”	178 28 23·10 5·40	+ 2·17 + 7·97	25·27 13·37	7 27 55·10 54·30	30·61	65·99
April 6	” ”	32 58 64·90 55·20	− 5·83 +11·91	59·07 67·11	41 58 42·80 35·00	29·14	64·95
” ”	λ ” ”	29 23 27·00 10·90	− 9·71 +12·86	17·29 23·76	41 58 44·00 34·50	1 50 44·64	63·37
” 7	Polaris ”	29 57 37·70 31·50	−10·37 + 2·91	27·33 34·41	41 58 41·10 37·00	2 24 58·28	66·46
March 12	Polaris W.	136 7 46·00 52·20	− 1·76 − 2·84	44·24 49·36	152 58 31·20 36·20	45·59	61·31
” 14	” ”	96 55 56·90 51·20	+ 2·11 − 4·44	59·01 46·76	113 46 41·20 45·80	46·55	64·06
April 6	” ”	25 6 60·70 55·70	− 0·69 +10·91	60·01 66·61	41 58 41·40 38·40	58·04	98·55[1]

Mean of Eastern Elongations 194° 26′ 5″·56 General Mean 194° 26′ 4″·99
” Western ” ” ” 2″·69 Probable Error ± 0″·43

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[1] Rejected in the mean.

MORDINGTON.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n^.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1846: March 8	Polaris E.	108 26 20·10 24·50	− 6·23 − 6·67	13·87 17·83	74 13 0·15 4·05	2 40 59·72	148 27 45·97
” 12	δ Urs. Min. ”	95 11 53·20 60·10	− 4·39 + 3·52	48·81 63·62	57 35 13·20 16·85	6 4 19·21	38·02
” 21	λ ” ”	61 0 87·50 46·40	− 8·68 +38·02	78·82 84·42	27 25 64·50 44·40	2 3 9·11	41·94
” 23	δ ” ”	65 2 56·80 1·00	−20·91 +35·34	35·89 36·34	65·50 42·50	6 4 20·28	38·16
” ”	λ ” ”	61 1 45·40 0 49·60	−19·87 +37·55	25·53 27·15	65·25 41·40	2 3 9·45	36·44
” ”	Polaris ”	61 38 93·20 45·50	−11·98 +33·47	81·22 78·97	63·40 41·25	2 40 67·20	39·43
” 24	δ Urs. Min. ”	25 47 36·80 14·80	− 9·40 + 8·54	27·40 23·34	168 10 45·55 45·80	6 4 20·28	40·59

MORDINGTON—*continued.*

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1846:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
Mar. 11	Polaris W.	86 26 34·00 23·30	− 6·18 + 6·79	27·82 30·09	57 35 14·65 17·50	2 40 61·05	148 27 46·07
,, 12	51 Cephei ,,	84 14 59·70 56·00	− 3·99 + 5·84	55·71 61·84	57 35 14·35 16·50	4 52 37·73	38·92
,, 14	Polaris ,,	46 4 73·60 33·90	− 6·28 +15·00	67·32 48·90	17 13 44·25 37·20	2 40 62·48	40·14
,, 21	51 Cephei ,,	54 5 48·40 15·60	− 8·33 +21·25	40·07 36·85	27 25 57·55 50·70	4 52 36·77	38·90
,, 22	Polaris ,,	56 16 92·00 37·90	−24·20 +23·37	67·80 61·27	27 25 64·35 44·30	2 40 66·93	42·86
,, ,,	51 Cephei ,,	54 5 54·50 3·20	−21·95 +35·98	32·55 39·18	27 25 65·80 42·85	4 52 36·69	41·77
,, 24	,, ,,	14 50 39·60 27·60	− 9·20 + 8·70	30·40 36·30	168 10 45·10 45·20	4 52 36·73	35·07
,, 25	Polaris ,,	17 1 64·10 44·90	− 2·36 +15·38	61·74 60·28	168 10 49·60 43·50	2 40 68·53	37·00
,, 27	,, ,,	169 16 39·50 29·40	− 1·93 + 7·34	37·57 36·74	140 25 29·25 27·50	69·60	41·62
,, 28	,, ,,	32 4 60·20 56·80	− 3·31 + 4·29	56·89 61·09	3 13 49·35 48·95	70·14	40·02
,, 30	,, ,,	115 26 46·00 61·50	+ 3·59 − 2·49	49·59 59·01	86 35 47 80 55·87	71·20	46·34

Mean of Eastern Elongations 148° 27′ 40″·08 General Mean 148° 27′ 40″·51
,, Western ,, ,, ,, 40″·80 Probable Error ± 0″·51

MORMONTH.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1847:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
Feb. 25	Polaris E.	2 48 33·67 60·67	+ 4·47 −20·52	38·14 40·15	84 56 7·00 11·67	2 48 14·10	84 55 44·29
,, 27	,, ,,	129 20 21·00 73·00	+18·82 −16·45	39·82 56·55	211 28 14·17 25·17	15·22	46·70
Mar. 19	,, ,,	150 57 17·00 23·00	−10·06 − 5·18	6·94 17·82	233 4 32·33 31·83	26·05	45·75
,, 18	Polaris W.	145 20 17·00 21·00	+ 1·54 − 5·09	18·54 15·91	233 4 29·67 33·00	25·77	48·34
,, 19	,, ,,	132 14 12·00 35·33	+ 5·29 − 8·20	17·29 27·13	219 58 34·33 37·33	26·33	47·29
,, 22	,, ,,	56 44 39·00 54·33	− 4·31 −22·25	34·69 32·08	144 28 51·17 53·17	28·01	50·77

Mean of Eastern Elongations 84° 55′ 45″·58 General Mean 84° 55′ 47″·19
,, Western ,, ,, ,, 48″·80 Probable Error ± 0″·56

MOUNT BATTOCK.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1847:	·	° ′ ″			° ′ ″	° ′ ″	° ′ ″
May 30	λ Urs. Min. E.	40 53 39·00 64·33	− 4·72 −10·16	34·28 54·17	97 16 5·83 6·50	2 6 28·62	238 28 50·56
„ „	Polaris „	41 33 13·33 46·00	+11·27 −16·01	24·60 29·99	97 16 6·00 17·17	2 46 3·48	47·77
June 1	„ „	139 43 55·00 100·33	+14·29 −10·49	69·29 89·84	15 26 51·33 61·17	2 46 4·03	40·71
„ 2	λ Urs. Min. „	127 50 35·33 61·33	+13·75 + 0·48	49.08 61·81	4 13 10·33 12·33	2 6 27·81	43·69
„ „	Polaris „	128 30 20·00 41·67	+ 5·49 −13·83	25·49 27·84	4 13 6·50 15·83	2 46 4·21	48·71
„ 30	„ „	72 4 2·33 12·33	+16·72 +13·82	19·05 26·15	127 46 56·17 64·17	6·82	44·39
July 2	„ „	45 28 48·33 64·67	+ 3·05 − 4·33	51·38 60·34	101 11 33·00 37·17	6·67	45·90
June 29	Polaris W.	66 31 58·33 91·67	− 1·73 −17·24	56·60 74·43	127 46 50·81 56·48	6·82	41·31
„ 30	„ „	52 2 54·67 80·33	+ 9·44 −14·06	64·11 66·27	113 17 50·48 61·48	6·82	43·97
July 1	„ „	39 56 30·00 56·33	+ 5·91 −13·16	35·91 43.17	101 11 20·98 27·98	6·77	38·17
„ 2	„ „	22 14 24·33 5·67	−17·17 + 9·21	7·16 14·88	83 28 68·31 50·31	6·63	41·66

Mean of Eastern Elongations 238° 28′ 45″·96 General Mean 238° 28′ 44″·26
„ Western „ „ „ 41″·28 Probable Error ± 0″·73

MOWCOPT.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1851:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
April 10	δ Urs. Min. E.	42 13 16·17	+10·01	26·18	137 9 36·98	5 40 51·43	280 36 62·23
„ 15	„ „ „	67 49 50·67	−12·25	38·42	162 45 50·67	5 40 50·00	62·25
„ 18	λ „ „	160 1 53·67	−11·09	42·58	78 44 41·87	1 54 2·56	61·85
„ 21	δ „ „	158 5 26·00	+ 2·51	28·51	73 1 38·25	5 40 49·15	58·89
„ „	λ „ „	154 18 28·67	+12·44	41·11	73 1 37·88	1 54 2·64	59·41
„ 25	δ „ „	97 42 24·17	+ 1·37	25·54	12 38 36·13	5 40 48·14	58·73
„ 26	„ „ „	69 50 58·33	−13·49	44·84	164 47 6·54	47·43	69·13
May 1	„ „ „	113 5 18·17	−17·34	0·83	28 1 22·33	46·14	67·64
, 3	„ „ „	90 17 60·33	− 4·53	55·80	5 14 9·71	45·51	59·42
„ „	λ „ „	86 31 10·33	+ 2·05	12·38	5 14 9·54	1 54 1·71	58·87
„ 13	δ „ „	19 31 56·00	+ 2·05	58·05	114 28 18·37	5 40 41·49	61·81
„ „	λ „ „	15 45 12·67	+ 0·84	13·51	114 28 19·33	1 53 59·89	65·71
„ „	δ „ „	160 15 56·67	− 3·76	52·91	75 12 12·62	5 40 41·12	60·83
„ 14	λ „ „	125 32 50·17	+ 6·16	56·33	44 16 7·09	1 53 59·65	70·41

MOWCOPT—*continued.*

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1851:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
April 18	51 Cephei W.	153 33 22·17	+ 2·91	25·08	78 44 45·29	4 34 13·72	280 36 66·49
„ 19	„ „	58 35 51·83	− 6·70	45·13	163 46 55·33	13·89	56·31
„ 21	„ „	147 50 27·33	− 3·73	23·60	73 1 37·42	13·98	59·84
„ 25	„ „	87 27 13·00	+ 3·00	16·00	12 38 35·88	15·36	64·52
„ 26	„ „	59 35 51·00	− 6·52	44·48	164 47 7·67	15·62	67·57
May 1	„ „	102 50 2·83	− 3·46	0·63	28 1 23·42	16·97	67·08
„ 13	„ „	9 16 57·67	+ 4·21	61·88	114 28 17·54	21·13	54·53

Mean of Eastern Elongations 280° 37′ 2″·66 General Mean 280° 37′ 2″·55
„ Western „ „ „ 2″·33 Probable Error ± 0″·63

PRECELLY.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1843:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
May 29	Polaris E.	168 6 21·00 15·30	− 3·49 + 1·89	17·51 17·19	197 36 14·53 10·63	2 28 47·45	211 58 42·68
June 11	„ „	123 0 15·40	+ 3·32	18·72	152 30 11·53 14·63	49·45	43·81[1]
„ 16	„ „	48 37 49·70 56·10	+ 2·86 − 2·54	52·56 53·56	78 7 49·73 55·23	50·10	49·52
„ 23	„ „	24 1 56·40 63·00	+ 4·24 − 5·51	60·64 57·49	53 31 56·13 63·13	50·10	50·67
„ 24	„ „	152 24 28·70 44·40	+12·59 −10·36	41·29 34·04	181 54 22·23 39·13	50·10	43·11
„ 25	„ „	152 24 30·00 46·30	+ 4·47 −11·82	34·47 34·48	181 54 22·83 35·03	50·10	44·55
„ 26	„ „	29 10 33·80 46·30	+ 8·04 − 7·72	41·84 38·58	58 40 24·83 35·13	50·10	39·87
„ 28	„ „	135 21 41·00 46·80	+ 1·41 − 4·13	42·41 42·67	164 51 37·83 42·53	50·14	47·78
May 28	„ W.	133 56 30·40 21·50	− 2·14 − 1·85	28·26 19·65	168 23 62·50 57·30	47·09	48·85[2]
„ 29	„ „	70 38 39·10 53·90	+ 8·02 − 9·46	47·12 44·44	105 6 12·50 21·60	47·37	43·90[2]
June 17	„ „	43 39 69·40 79·70	−11·16 + 3·32	58·24 83·02	78 7 56·80 52·50	50·10	53·93[2]
„ 26	„ „	147 26 61·00	− 7·93	53·07	1 54 35·30 26·92	50·10	47·94[2]

Mean of Eastern Elongations 211° 58′ 45″·25 General Mean 211° 58′ 46″·38
„ Western „ „ „ 48″·65 Probable Error ± 0″·75

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[1] Collimation error inferred.

[2] Much motion in referring-object.

SOUTH LOPHAM TOWER.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1844:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
Nov. 26	Polaris E.	25 6 62·20 24·00	−15·99 +17·72	46·21 41·72	32 48 77·80 54·60	2 28 41·64	190 10 63·87
„ 27	„ „	19 1 69·60 63·60	− 5·54 + 8·47	64·06 72·07	26 44 37·40 21·40	41·15	62·48
„ 30	„ „	2 49 51·20 25·00	− 1·75 +21·57	49·45 46·57	10 31 80·20 56·80	39·81	60·30
Dec. 5	„ „	328 26 56·60 41·80	−10·00 + 3·51	46·60 45·31	336 9 20·20 8·40	37·71	66·05
„ 6	„ „	317 16 66·00 54·00	−18·07 +12·65	47·93 66·65	324 59 34·20 13·40	37·22	63·73
„ 21	„ „	266 45 62·00 39·80	− 1·89 +13·58	60·11 53·38	274 28 27·60 13·80	32·43	56·38
„ 28	„ „	251 3 64·60 29·00	−10·16 +23·15	54·44 52·15	258 46 29·00 9·20	31·07	56·87
Nov. 26	λ Urs. Min. W.	20 44 48·60 34·00	− 2·21 +10·55	46·39 44·55	32 48 62·80 30·00	1 52 57·06	63·87
„ „	Polaris „	20 8 58·00 38·80	+ 2·55 +23·87	60·55 62·67	32 48 53·40 25·20	2 28 41·40	56·29
„ 27	λ Urs. Min. „	14 39 45·60 42·40	+ 4·95 +16·16	50·55 58·56	26 43 59·00 35·20	1 52 57·27	55·27
Dec. 4	Polaris „	339 54 34·00 26·20	− 2·76 + 7·97	31·24 34·17	352 34 23·20 9·00	2 28 37·95	65·44
„ „	λ Urs. Min. „	340 30 24·00 7·20	−11·36 +22·73	12·64 29·93	352 34 27·40 10·20	1 52 58·76	58·75
„ 5	δ „ „	320 22 42·80 34·20	+ 3·68 + 3·92	46·48 38·12	336 8 49·20 38·60	5 34 51·53	70·07
„ „	λ „ „	324 4 44·20 34·40	− 6·48 +10·27	37·72 44·67	336 8 51·80 32·80	1 52 58·98	62·12
„ „	Polaris „	323 28 65·80 60·40	− 9·25 +13·80	56·55 74·20	336 8 54·60 35·00	2 28 37·46	61·96
„ 6	δ Urs. Min. „	309 12 54·00 53·60	− 9·02 +10·42	44·98 64·02	324 58 63·60 41·80	5 34 52·02	66·18
„ „	Polaris „	312 18 52·80 48·60	− 1·88 +11·53	50·92 60·13	324 58 44·40 23·60	2 28 37·02	61·45
„ 16	„ „	297 31 30·20 11·40	−11·14 +16·87	19·06 28·27	310 10 76·00 50·40	33·74	65·79
„ 20	„ „	266 34 50·80 39·00	−11·32 +17·06	39·48 56·06	279 14 38·60 17·00	32·59	67·44
„ 21	„ „	256 46 37·00 14·00	−14·28 +20·78	22·72 34·78	269 25 80·40 53·20	32·30	65·75
„ 23	„ „	246 7 69·40 34·40	−16·30 +21·50	53·10 55·90	258 47 47·00 18·40	31·97	66·23

Mean of Eastern Elongations 190° 11′ 1″·38 General Mean 190° 11′ 2″·68
„ Western „ „ „ 3″·33 Probable Error ± 0″·59

WORDESLOW.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^n^.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1846 :		° ′ ″			° ′ ″	′ ″	° ′ ″
April 20	δ Urs. Min. E.	159 37 48·40 57·40	−13·06 + 2·11	35·34 59·51	31 33 39·30 25·00	5 55 32·01	57 51 16·73
„ 28	„ „	125 22 7·00 35·00	+15·62 − 8·00	22·62 27·00	357 18 12·50 15·40	5 55 29·28	18·42
„ „	λ Urs. Min. „	121 27 0·20 25·00	+10·37 − 7·89	10·57 17·11	357 18 15·10 14·90	2 0 15·93	17·08
May 7	Polaris „	76 1 20·80 11·20	−16·28 +13·79	4·52 24·99	311 14 63·40 55·30	2 37 39·52	24·11
„ 8	δ Urs. Min. „	67 21 64·80 59·20	− 2·08 +13·98	62·72 73·18	299 17 63·80 58·70	5 55 25·62	18·92
„ „	λ „ „	63 26 53·00 45·40	− 5·65 +15·13	47·35 60·53	299 17 65·50 59·80	2 0 14·53	23·25
„ „	Polaris „	64 4 37·40 14·20	−26·12 +10·24	11·28 24·44	299 17 63·40 55·70	2 37 39·97	21·66
„ 10	δ Urs. Min. „	63 21 53·80 59·40	+ 3·13 + 7·34	56·93 66·74	295 17 59·50 47·40	5 55 24·60	16·22
„ „	λ „ „	59 26 48·00 47·60	− 0·77 +10·13	47·23 57·73	295 17 63·70 50·00	2 0 14·07	18·44
„ „	Polaris „	60 4 11·00 14·80	+11·58 +15·78	22·58 30·58	295 17 64·40 57·00	2 37 40·83	14·95
„ 11	δ Urs. Min. „	55 25 41·20 61·60	+ 1·87 − 2·11	43·07 59·49	287 21 48·60 45·70	5 55 24·22	20·09
„ „	λ „ „	51 30 35·00 51·00	+ 2·93 − 4·46	37·93 46·54	287 21 45·80 44·40	2 0 13·93	16·80
„ 14	δ „ „	53 10 58·60 84·20	+12·40 − 4·90	71·00 79·30	285 7 8·30 7·10	5 55 22·88	15·44
„ 15	„ „	50 33 46·00 56·20	+10·19 + 0·05	56·19 56·25	282 29 45·90 45·30	5 55 22·35	11·73
„ „	λ „ „	46 38 29·20 51·40	+10·01 − 0·30	39·21 51·10	282 29 48·10 46·00	2 0 13·04	14·93
„ „	Polaris „	47 16 5·20 15·20	+ 8·80 + 1·14	14·00 16·34	282 29 52·60 49·50	2 37 42·75	18·63
„ 18	δ Urs. Min. „	48 40 51·60 68·80	+17·53 − 2·18	69·13 66·62	280 36 60·50 64·10	5 55 21·08	15·50
„ „	λ „ „	44 45 35·20 67·40	+12·06 −11·59	47·26 55·81	280 36 60·40 67·20	2 0 12·28	24·55
„ 21	„ „	22 1 57·20 76·60	+ 8·74 − 5·06	65·94 71·54	257 53 16·80 18·40	2 0 11·50	20·36
„ „	Polaris „	22 39 27·00 58·60	+ 7·50 − 4·79	34·50 53·81	257 53 16·90 16·40	2 37 44·83	17·32
„ 29	λ Urs. Min. „	341 58 56·80 82·40	+ 9·03 − 3·17	65·83 79·23	217 50 24·70 25·90	2 0 8·59	21·36
„ „	Polaris „	342 36 47·00 62·40	+ 6·64 − 2·62	53·64 59·78	217 50 31·70 29·80	2 37 47·09	21·12
April 20	51 Cephei W.	328 56 50·20 36·40	− 0·71 − 1·30	49·49 35·10	31 33 37·80 27·70	4 45 39·58	10·87
„ 28	„ „	114 41 5·20 19·60	+13·58 − 1·82	18·78 17·78	357 18 13·20 15·70	41·82	14·35
May 8	„ „	56 40 52·60 38·80	− 5·05 +16·34	47·55 55·14	299 17 63·00 57·20	45·42	23·34
„ 10	„ „	52 40 38·20 48·20	+ 2·49 +10·09	40·69 58·29	295 17 59·60 47·60	46·19	17·92
„ 11	„ „	44 44 36·40 46·20	+ 3·04 − 0·55	39·44 45·65	287 21 48·50 46·40	46·57	18·33
„ 15	„ „	39 52 35·20 44·60	+ 8·35 + 1·62	43·55 46·22	282 29 46·10 46·00	48·30	12·87

Mean of Eastern Elongations 57° 51′ 18″·53 General Mean 57° 51′ 18″·05
„ Western „ „ „ 16″·28 Probable Error ± 0″·45

WROTHAM.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1844:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
July 29	Polaris E.	162 50 67·00 50·67	−13·00 +12·18	54·00 62·85	75 21 39·33 12·67	2 26 12·44	274 56 40·02[1]
Aug. 4	" "	0 35 9·00 16·67	− 0·15 + 3·28	8·85 19·95	93 5 46·00 37·33	10·32	37·59
" 7	" "	87 28 39·67 44·67	− 5·91 − 5·45	33·76 39·22	179 59 6·67 2·67	9·56	37·74[2]
" 8	" "	71 17 10·00 3·33	− 3·08 − 3·06	6·92 0·27	163 47 37·33 34·00	9·24	41·31
" 10	" "	170 1 40·00 12·00	−14·13 +17·78	25·87 29·78	82 31 70·33 47·33	8·31	39·32
" 13	" "	148 5 36·33 36·00	− 2·94 + 2·21	33·39 38·21	60 36 15·00 6·00	7·16	41·86[3]
" 17	" "	179 55 17·00 18·00	− 2·48 + 9·18	14·52 27·18	92 25 57·33 45·00	5·56	35·88[4]
" 23	" "	18 59 31·00 31·67	− 4·52 + 1·61	26·48 33·28	111 29 68·00 57·00	2·77	35·39
" 28	" "	147 31 55·33 75·00	+ 6·26 −10·42	61·59 64·58	60 2 35·67 38·00	0·47	34·22
" 29	" "	135 56 35·33 46·67	+ 4·42 − 7·23	39·75 39·44	48 27 11·00 10·67	0·00	31·23
" 20	Polaris W.	107 0 61·67 52·00	− 6·54 + 8·84	55·13 60·84	24 23 46·67 34·33	4·07	38.45
" 23	" "	158 50 20·67 30·67	− 1·05 − 7·19	19·62 23·48	76 13 3·67 0·67	2·63	37·99
" 28	" "	142 40 17·00 29·33	+ 2·40 −17·18	19·40 12·15	60 2 51·00 54·00	0·23	36·49

Mean of Eastern Elongations 274° 56′ 37″·46 General Mean 274° 56′ 37″·50
" Western " " " 37″·64 Probable Error ± 0″·52

[1] One of the level readings altered 10 divisions.
[2] Level observations made before the star was commenced.
[3] Referring-object not observed for half an hour after the star.
[4] One of the cross levels altered East for West.

YORK MINSTER.

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Colⁿ.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
1846:		° ′ ″			° ′ ″	° ′ ″	° ′ ″
Jan. 5	δ Urs. Min. E.	117 29 29·80 47·40	+18·33 + 6·99	48·13 54·39	192 8 46·50 35·60	5 47 30·36	260 26 20·15
Feb. 9	" " "	90 41 33·00 55·80	− 7·11 −11·09	25·89 44·71	165 20 15·90 27·00	48·44	34·59
" 11	" " "	80 25 28·80 59·20	+ 5·61 −26·44	34·41 32·76	155 4 11·10 19·30	49·34	30·95[1]
" 14	" " "	48 30 43·40 61·40	− 6·43 − 9·11	36·97 52·29	123 9 22·00 23·50	50·54	28·66

[1] Clouded over for several minutes.

YORK MINSTER—*continued.*

Date.	Name and Elongation of Star.	Reading of Star.	Computed Level and Col^{n}.	Corrected Reading of Star.	Reading of R.O.	Computed Azimuth of Star at Elongation.	Resulting Azimuth of R.O.
		° ′ ″			° ′ ″	° ′ ″	° ′ ″
1845: Dec. 9	λ Urs. Min. W.	19 21 62·40 61·60	+ 8·13 +24·53	70·53 86·13	101 45 38·30 26·70	1 56 51·11	260 26 23·06
„ 30	Polaris „	124 44 57·60 61·00	+ 7·62 +12·80	65·22 73·80	207 45 13·00 5·10	2 33 33·51	26·03
1846: Jan. 1	„ „	113 51 8·00 19·00	− 1·78 +18·55	6·22 37·55	196 51 31·80 14·30	33·17	27·99
„ 4	„ „	109 8 29·40 51·00	+18·13 −16·99	47·53 34·01	192 8 40·70 33·70	32·83	23·60
„ 5	51 Cephei „	107 2 8·20 9·20	+16·57 +34·07	24·77 43·27	192 8 45·30 33·70	4 39 55·82	9·91[1]
„ 27	Polaris „	179 25 28·60 75·00	+12·24 −14·76	40·84 60·24	262 25 46·20 49·40	2 33 33·17	24·09
„ 28	„ „	166 29 26·80 73·00	+12·09 −17·01	38·89 55·99	69 29 38·90 43·40	2 33 33·34	20·37
„ 29	51 Cephei „	164 23 15·20 55·00	+20·60 −21·98	35·80 33·02	249 29 40·80 45·30	4 39 43·43	25·21
Feb. 7	Polaris „	103 35 53·00 86·20	− 2·58 −11·75	50·42 74·45	186 36 0·30 9·80	2 33 35·34	27·27
„ 8	„ „	82 20 7·00 30·20	− 2·09 − 9·26	4·91 20·94	165 20 12·00 23·70	2 33 35·68	29·25
„ 9	51 Cephei „	80 14 12·20 30·00	− 2·90 − 3·31	9·30 26·69	165 20 18·10 25·90	4 39 38·27	25·73
„ „	Polaris „	75 41 50·40 65·40	−16·20 − 8·69	34·20 56·71	158 41 53·50 59·80	2 33 35·98	35·22
„ 10	„ „	72 3 58·40 81·00	+10·94 − 2·79	69·34 78·21	155 4 11·10 18·30	36·19	24·73
„ 11	„ „	67 27 9·80 50·00	+15·32 −15·10	25·12 34·90	150 27 26·50 36·70	36·53	25·06
„ 12	„ „	64 17 57·20 95·20	+18·02 −16·19	75·22 79·01	147 18 14·70 27·80	36·87	27·27
„ 13	„ „	40 9 17·60 24·00	− 4·82 − 4·01	12·78 19·99	123 9 19·10 19·50	37·21	25·71
„ 14	51 Cephei „	38 3 15·60 33·60	− 4·71 − 4·01	10·89 29·59	123 9 23·00 26·50	4 39 36·78	27·73

Mean of Eastern Elongations 260° 26′ 28″·59
„ Western „ „ „ 26″·15

General Mean 260° 26′ 26″·63
Probable Error ± 0″·58

[1] Rejected in mean.

RESULTS OF AZIMUTHAL OBSERVATIONS

AT

STATIONS NOT INCLUDED AMONGST THE FOREGOING.

Name.	Azimuth of R.O.	No. of Obs.	Range.	Probable Error.
				±
	° ′ ″		″	″
Arbury Hill . . .	185 7 50·69	19	25·19	1·15
Bardon Hill . . .	213 47 10·13	28	27·37	0·81
Brandon	126 15 11·35	42	29·52	0·66
Bunwell Tower . .	183 20 28·23	16	21·70	1·08
Clifton Beacon . . .	21 34 22·58	20	18·45	0·82
Corryhabbie . . .	204 40 1·59	17	16·52	0·82
Cradle	130 50 37·59	8	14·70	1·04
Ditchling	274 23 10·05	7	10·04	0·76
Docking Tower . .	40 47 21·26	7	8·10	0·70
Epping Poorhouse . .	342 29 56·23	14	12·20	0·59
Frittenfield	231 8 35·44	6	10·41	0·95
Goonhilly	342 57 35·14	7	8·47	0·73
Happisburgh . . .	145 27 39·70	11	16·10	1·05
Hensbarrow . . .	78 25 49·82	17	25·12	1·15
Inkpen Beacon . . .	319 9 45·78	8	12·78	1·01
Laxfield Tower . .	145 54 34·09	14	22·77	1·05
Lumsden	144 53 59·95	8	15·76	0·84
Mendip	114 54 24·91	14	14·73	0·78
Merrick	71 27 6·48	13	31·59	1·50
Orford Castle . . .	68 49 23·65	8	7·98	0·72
Paracombe	87 45 11·24	7	6·76	0·56
Pertinny	31 21 43·70	8	13·09	0·93
Ryders Hill . . .	310 0 36·12	8	11·89	0·87
Sayrs Law	113 50 21·61	11	12·35	0·74
Scournalapich . . .	351 58 35·14	5	5·79	0·61
Slieve Donard . . .	130 5 23·07	22	24·59	1·46
Stoke Tower . . .	272 34 28·67	13	18·75	1·12
Tharfield	111 38 39·58	13	16·78	0·85
Tofts Tower . . .	273 35 14·96	17	18·59	0·75
Walpole, St. Peter's .	194 47 31·91	6	5·25	0·49
Wingreen	85 58 44·84	18	15·86	0·72

RESULTS OF OBSERVATIONS

MADE WITH THE ZENITH-SECTORS,

FOR THE

LATITUDES OF VARIOUS POINTS IN THE TRIANGULATION.[†]

Date.	Stations.	Latitudes Direct.	No. of Obs.	Amplitudes.	Latitudes Inferred.	Instrument.
		° ′ ″		° ′ ″	° ′ ″	
1802	Greenwich	51 28 38·52	47	—	51 28 38·30	Ramsden's Zenith-Sector.
1802	Dunnose[1]	50 37 6·99	232	0 51 31·38	50 37 6·72	,, ,,
1802	Clifton Beacon	53 27 30·44	157	1 58 51·94	53 27 30·24	,, ,,
1802	Arbury Hill	52 13 27·13	152	0 44 49·05	52 13 27·35	,, ,,
1806	Delamere	53 13 18·47	84	1 44 40·29	53 13 18·59	,, ,,
1806	Burleigh Moor	54 34 19·45	70	3 5 41·45	54 34 19·75	,, ,,
1813	Kellie Law	56 14 50·33	273	—	56 14 50·33	,, ,,
1813	Cowhythe	57 41 9·47	258	1 26 18·94	57 41 9·27	,, ,,
1817	Balta	60 45 1·59	382	4 30 11·37	60 45 1·70	,, ,,
1836	Greenwich[2]	51 28 39·30	–	—	—	,, ,,
1842	South Berule	54 8 56·85	111	3 14 10·12	54 8 56·80	Airy's Zenith Sector.
1842	Black Down	50 41 8·90	1062	0 13 37·79	50 41 8·89	,, ,,
1843	Precelly	51 56 46·06	660	1 1 59·59	51 56 46·27	,, ,,
1843	Forth Mountain	52 18 57·84	659	1 24 11·23	52 18 57·91	,, ,,
1843	Hungry Hill	51 41 10·23	294	0 46 23·58	51 41 10·26	,, ,,
1843	Feaghmaan	51 55 22·85	393	1 0 36·17	51 55 22·85	,, ,,
1843	Tawnaghmore	54 17 41·56	293	3 22 54·66	54 17 41·34	,, ,,
1843	South End of L.F. Base	55 2 38·78	334	4 7 52·06	55 2 38·74	,, ,,
1844	Monach	58 21 20·77	179	7 26 34·16	58 21 20·84	,, ,,
1844	Ben Hutig	58 33 6·44	479	7 38 19·79	58 33 6·47	,, ,,
1845	Hensbarrow	50 23 1·15	286	0 31 45·55	50 23 1·13	,, ,,
1845	South Berule[3]	54 8 55·38	114	3 14 9·31	54 8 55·26	,, ,,
1845	Ben Lomond	56 11 26·41	624	5 16 39·59	56 11 26·27	,, ,,
1845	Ben Heynish	56 27 16·86	266	5 32 30·20	56 27 16·88	,, ,,
1846	Week Down	50 35 51·19	550	0 18 55·26	50 35 51·42	,, ,,
1846	Dunnose	50 37 6·98	638	0 17 39·53	50 37 7·15	,, ,,
1846	Boniface Down	50 36 10·43	352	0 18 36·13	50 36 10·55	,, ,,

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[1] The point observed from in 1802 was 6·5 feet south of that observed from in 1836.

[2] The point observed from in 1836 was 50 feet north of the mural circles.

[3] The point observed from in 1845 was 74 feet south of that observed from in 1842.

RESULTS OF ZENITH-SECTOR OBSERVATIONS—*continued*

Date.	Stations.	Latitudes Direct.	No. of Obs.	Amplitudes.	Latitudes Inferred.	Instrument.
		° ′ ″		° ′ ″	° ′ ″	
1846	Port Valley . . .	50 35 45·61	407	0 19 1·16	50 35 45·52	Airy's Zenith Sector.
1846	Saxavord	60 49 38·83	563	9 54 51·90	60 49 38·58	,, ,,
1847	Gerth of Scaw . . .	60 48 56·27	576	9 54 9·75	60 48 56·43	,, ,,
1847	Balta	60 45 1·68	730	9 50 15·07	60 45 1·75	,, ,,
1847	Cowhythe	57 41 9·58	638	6 46 22·65	57 41 9·33	,, ,,
1847	Southampton . . .	50 54 46·68	8687	—	50 54 46·68	,, ,,
1850	St. Agnes	49 53 32·99	411	1 1 13·32	49 53 33·36	,, ,,
1850	Goonhilly Down . .	50 2 49·93	431	0 51 56·61	50 2 50·07	,, ,,
1850	North Rona . . .	59 7 15·49	427	8 12 28·72	59 7 15·40	,, ,,
1850	Great Stirling . . .	57 27 49·20	439	6 33 2·44	57 27 49·12	,, ,,

The Latitudes Direct are obtained for each Station, by giving to the latitude resulting from the observations of any one star a weight equal to the number of observations of that star: the resulting latitudes given by different stars are then combined with respect to their weights.

In the observations made with Ramsden's Zenith Sector, the amplitudes are obtained as follows:—Let x_1 x_2 x_3 . . . . be the successive amplitudes of the stations A A_1 A_2 A_3 . . . . and let a observations of a certain star at A_m give its latitude λ_m, and b observations of the same star at A_n give its latitude λ_n, c observations of the same star at A_r give its latitude λ_r, and so on; then we have these equations,—

$$x_{m+1} + \dots + x_n = \lambda_n - \lambda_m\text{; weight} = \frac{ab}{a+b}$$

$$x_{n+1} + \dots + x_r = \lambda_r - \lambda_n\text{; weight} = \frac{bc}{b+c}$$

$$\vdots$$

From which, by the method of least squares, x_1 x_2 x_3 . . . . are obtained. The six stations first on the list, observed at between 1802 and 1806, are combined by themselves; the other three, Kellie Law, Cowhythe, Balta, observed at between 1813 and 1817, are treated separately. The amplitudes of the first set are applied to the latitude of Greenwich as determined by the Astronomer Royal, namely, 51° 28′ 38″·30. In the observations made with Airy's Zenith Sector, the amplitudes are all taken with Southampton directly.

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† The Zenith-Sector Station was not in every case coincident with the Trigonometrical Station; the following table contains the consequent corrections to be applied to the observed latitudes.

Station.	Correction.	Station.	Correction.	Station.	Correction.
	″		″		″
Clifton	+0·03	Cowhythe	−0·41	Port Valley	−2·31
Arbury	−0·33	S. Berule (1845) . . .	+0·73	St. Agnes	+0·57
Delamere	−0·03	Precelly	−1·09	Southampton	+0·29
Kellie Law	+1·10	Hensbarrow	+0·71	North Rona	−0·21

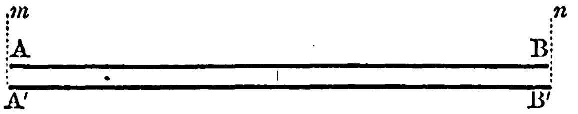
SECTION V.

MEASUREMENT OF BASE LINES.

To determine by actual measurement on the ground the length of one side of a network of triangulation wherefrom to infer the lengths of all the other sides, is not the least difficult operation in a trigonometrical survey. To effect this with all the accuracy necessary for the purposes of topography presents indeed no formidable difficulty; but when the ultimate end of the operations is to contribute towards the knowledge of the figure and actual dimensions of the earth, and when the problem is stated thus,—To determine the number of times that a certain standard or unit of length is contained between two finely marked points on the surface of the earth at a distance of some miles asunder, so that the error of the result may be pronounced with certainty to lie between certain very narrow limits plus or minus,—then the question requires and has always obtained serious consideration. In the first place, the representation of the unit of length, an abstract idea, by means of the distance between two finely engraved dots or lines upon the surface of a bar of metal at a certain temperature, is never itself free from uncertainty and probable error, from the difficulty of ascertaining at any moment the precise temperature of the bar, and also in a less degree from the circumstance that such bars or scales have in the course of time altered perceptibly in length. Omitting this last consideration, the transference of the unit of length or a certain multiple of it to a measuring bar will be affected, not only with the unavoidable errors of observation, but with errors arising from the uncertainty of temperature of both bars. It is therefore necessary that the ratio of the multiple bar to the standard, as well as the coefficients of expansion of both, should be the objects of very numerous and careful observations. This multiple of the standard is however seldom used in actual measurement, but serves as a secondary standard for the examination of the rods or bars with which the actual measuring is performed. The construction of the measuring rods must then be such that they may admit of easy and direct comparison with the working standard, that they may be aligned with great exactness between the extremities of the line of measurement, that they may be made individually perfectly level, and that, with respect to the alterations due to variations of temperature, they may be either eliminated by the principle of compensation or any other mechanical means, or that the temperature of the bars may be precisely ascertainable at any given time.

Compensation Bars.

(1.) The compensation bars, invented by Major-General Colby with the view of eliminating the errors resulting from our ignorance of the exact temperature of the measuring rods at a given time, are constructed on the following simple and beautiful principle:—Two bars of different metals and different rates of expansion are laid parallel and close to each other as $A\,B$, $A'\,B'$; these are firmly connected at the centre, from or to which point they are free to expand and contract. At a given temperature they are taken of the same length, and in this state suppose lines $A'\,A\,m$, $B'\,B\,n$, to be drawn through their extremities and perpendicular to their lengths, so that $A\,m = B\,n$; make also the ratio of $A\,m$ to $A'\,m$ or of $B\,n$ to $B'\,n$ equal to the ratio of the expansion of the bar $A\,B$ to the expansion of the bar $A'\,B'$. Now if we suppose both bars to receive an equal increment positive or negative of temperature and in this position lines to be drawn through their extremities, these lines will evidently pass through the points $m\,n$ as already determined. This, indeed, is not mathematically or strictly speaking true, but the difference would be quite imperceptible.



(2.) In the actual construction the bar $A\,B$ is of iron, the bar $A'\,B'$ of brass, $A'\,m$, $B'\,n$, are flat steel tongues at the extremities of these bars moving freely on conical brass pivots, allowing them to be inclined at small angles with the lines perpendicular to $A\,B$, $A'\,B'$. At the temperature of 62° Fahrenheit the bars are assumed to be precisely the same length, and the tongues consequently at right angles to the bars.

It is evidently necessary for the construction that in order to the fulfilment of the required compensation, the bars must always be of the very same temperature, but as they are necessarily subjected to constant variations of temperature it is also clear that they will not generally be of the same temperature unless they have the same rates of changing temperature. It was necessary therefore to ensure this equality, and two modes presented themselves, either to alter the dimensions of one until its rate was equal to that of the other or to produce the same effect by varying the surface of the one while the other remained constant. The latter method was adopted, and after a number of experiments, in which the brass bars were bronzed and varnished and remained unaltered, while the iron bars were browned (as the barrels of fowling pieces), lacquered, and smoked in different degrees, the desired equality was obtained.

(3.) The compensation bar consists of two bars, each 10 feet 1·5 inch long, 0·5 inch broad, and 1·5 inch deep, placed 1·125 inch apart, and firmly connected at their centres by two small transverse steel cylinders not quite in contact. At each extremity of the bars is a metal tongue (corresponding to the lines $A'm$ $B'n$) so connected by pivots to the bars as to admit freely of any expansion, and yet to be quite immoveable otherwise. These tongues are each 6·2 inches long, and on a silver pin at the extremity of each is marked the compensation point. This compound bar is placed in a deal box (resting more immediately upon two brass rollers in the bottom of it), and is kept from moving lengthways by means of a brass stay firmly fixed to the bottom of the box at the centre, and projecting upwards between the two small steel cylinders before mentioned. The long level for levelling the bars is fixed to the upper surface of the brass bar, and is read by means of a glass covered opening W., Pl. II., in the top of the box. The tongues carrying the compensation points project beyond the box, but are carefully protected. These points, it is to be noted, and not the bars themselves, lie in the line of measurement.

The complete set contains six bars, distinguished by the letters A, B, C, D, E, G. Each box when in use is supported at one-fourth and three-fourths of its length by means of strong brass tripods, having rollers *o o* (Plate II. fig. 2) on their upper surface. One of the tripods is provided with a tangent screw *g* for communicating a longitudinal motion to the bar, and both have screws *f*, for communicating a transverse motion, and an elevating screw *e*, for the final adjustment of the bar as to level. These tripods rest on trestles T T′, which are of various lengths from 6 to 30 inches, and are selected as the nature of the ground requires.

(4.) The interval between the adjacent compensation points of two bars lying in line is brought to exactly six inches by means of a compensation microscope which is constructed in the following manner: Two microscopes of two inches focal distance, lying parallel to one another, and six inches apart, are connected by two bars, one of brass and the other of iron, in such a manner that the outer foci of their object glasses are compensated points at exactly six inches apart. Through the centre of these two bars passes a telescopic microscope parallel to the other two. The microscopes revolve on the axis of this centre or telescopic microscope in a vertical tube attached to the upper surface of a tripod with levelling screws, and screws for communicating motion in a longitudinal or transverse direction. The tripod of the compensation microscope rests on a three armed grooved stand *s s′ s″* (Fig. 3, Pl. II.) carried by the brass plate *t t t t*, screwed to the end of the box. To the brass bar of each compensation microscope is attached the level, and on the other side is a small telescope moveable in a vertical plane for the alignement. The measurement of the six inch interval is effected by bringing one of the outer microscopes of the compensation microscope when in perfect adjustment, alignement, and level, to bisect the compensation point of one of the bars, and then bringing the compensation point of the other bar (by tangent screw motion) to be bisected by the other outer microscope.

Seven microscopes, distinguished by the letters M, N, O, P, Q, R, S, are thus required for a set of bars, so that between the telescopic microscopes of the first and last compensation microscopes of a complete set of bars arranged in the line of measurement, the distance is 63 feet.

(5.) The end of each series of bars when used in measurement is transferred to the ground by means of a point carrier; this simple apparatus consists of a massive triangular cast-iron plate of about 15·5 inches side, with a short, flat, hollow cylinder of brass screwed to the centre of its upper surface, enclosing a moveable disc having a fine point engraved on its surface, this dot is adjusted to the required position by means of three small equi-distant screws passing through the cylinder. The point carrier is placed under the telescopic microscope, so that the dot shall be very nearly in the line of its optical axis, and the final adjustment of the dot is then made by means of the three adjusting screws. This dot forms the initial point for the next set of bars.

Standards.

(6.) *The Ten-Feet Ordnance Standards*, designated as O_1 and O_2, are two wrought-iron bars 122·15 inches long, 1·45 inch broad, and 2·5 inches deep (constructed for the Ordnance Survey Department by Messrs. Troughton and Simms, in 1826–7), supported at one-fourth and three-fourths of their length on rollers secured to the bottom of the wooden box by which the bars are encased, and having a slight pin attached to the box and passing into a small hole in the centre of the under surface of the bars, to prevent their moving longitudinally in the box. The mode of supporting them on rollers was devised to allow them to contract or expand freely from the effects of temperature, some of the preliminary experiments having shewn that the length of standards would be sensibly affected by the manner in which they were supported throughout their length.

The precaution was taken of cutting away the ends of the bars to half their depth, to obviate the effects of flexure, consequent on the mode adopted for supporting the bars at two points only; and by this method, by which the dots marking the standard of length are placed in the axes of the bars, it is believed that no material flexure will exist, and from their depth and massiveness it is more than probable that, whatever it may be, it is not likely to vary in amount, which is, after all, the great desideratum in a standard measure.

Platina pins were then inserted at each end, and the distance, 10 feet having been set off at the temperature of 62° Fahrenheit, dots were engraved on them to indicate the standard of measure. On the upper surface of each bar, over the supporting brass rollers, were two cylindrical holes, made large enough to admit the bulbs of two thermometers, for registering the temperature of the bar, the metal and the bulbs of the thermometer being brought into contact by immersing the bulbs in oil or mercury. The bars are retained in their proper positions in their boxes by small pieces of brass at the top and sides, to prevent their being displaced with reference to the rollers.

The ends of the bars are protected by circular brass caps screwed on to the ends of the boxes when not required to be used.

Six-inch Standard.—The distance between the outer foci of the compensation microscopes is referred to a brass standard measure 11·5 inches long, 1·5 inch broad, and 0·55 inch deep, supported at each end by small pieces of brass screwed on to the under surface, the standard measure being defined by dots engraved on silver pins, let into the brass, 6 inches apart. The distance between the outer foci of the microscopes was adjusted to the length between these dots, the temperature of the standard being ascertained by means of a thermometer having its bulb let into an orifice at one end.

(7.) *Comparisons of Standards.*—Extensive comparisons, given in detail in the "Account of the Measurement of the Lough Foyle Base"* were made at Southampton in the years 1844–5–6, in order to obtain the relations between these standards and the following: Indian Bar *B*, Cape Bar *B*, Ramsden's Prismatic Bar, Roy's Scale, Messrs. Troughton and Simms' Brass Scale, and the Indian 6-inch standard. The first of these bars differs from the Ordnance Standard only in the dimensions of the section which are rather smaller, the second bar differs only in having gold pins instead of platina. Ramsden's Prismatic Bar is of cast-iron, 21 feet in length; the section is an equilateral triangle of 1·25 inch side. The 20 feet measure is divided into parts of 40 inches marked by dots on brass pins let into the bar; these distances were laid off at the temperature of 54° by Ramsden from his Brass Scale. General Roy's scale is of brass 42·8 inches long, ·55 inch broad, and ·22 inch deep, divided by lines, which are now much worn, into inches and tenths of inches; it has a vernier at each end, one dividing the inch into 1000, the other into 500 parts. Messrs. Troughton and Simms' Brass Scale is 66 inches long, 1·5 inch broad, and ·5 inch deep; it is divided by lines on silver into inches and tenths of inches, numbered from 0 to 60; it is also divided into inches by a scale of dots on silver pins let into the brass opposite the linear divisions of inches, these dots were used in the comparisons.

(8.) The following table contains the principal results of these comparisons; the third column contains the factor by which any results given in terms of the several scales are to be multiplied, in order to reduce them to mean feet of O_1; the fourth column contains the corresponding logarithm.

Standard Measure.	Mean Feet of O_1.	Factor.	Logarithm.
Ordnance Standard O_2	9·9999972	0·99999972	9·9999999
Ramsden's 20-feet Bar	20·0007656	1·00003828	0·0000166
Roy's Scale 0—40 in.	3·3333368	1·00000105	0·0000005
Roy's Scale 0—36 in.	3·0000281	1·00000937	0·0000041
Cape Bar *B*	9·9998498	0·99998498	9·9999935
Indian Bar *B*	9·9999279	0·99999279	9·9999969
Troughton and Simms 0—60 in. .	4·9999925	0·99999850	9·9999993
Troughton and Simms 0—40 in. .	3·3333458	1·00000375	0·0000016
Troughton and Simms 0—36 in. .	3·0000018	1·00000060	0·0000003
Ordnance 6-inch Standard . . .	0·4999778	0·99995550	9·9999807
Indian 6-inch Standard *B* . . .	0·4999652	0·99993030	9·9999697

(9.) The method of making these comparisons was essentially the same as followed in the measurement of the Salisbury Plain Base, page 217. If we suppose x y to be the coefficients of expansion of the two bars under comparison, z their true difference of length at the standard temperature of 62°, whilst $62° + h°$, $62° + k°$, and n, are the observed temperatures of the bars and their difference of length, then we shall have

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* Published by the Honourable Board of Ordnance.

$$\begin{aligned} h_1 x - k_1 y + z + n_1 &= 0 \\ h_2 x - k_2 y + z + n_2 &= 0 \\ h_3 x - k_3 y + z + n_3 &= 0 \\ h_r x - k_r y + z + n_r &= 0 \end{aligned}$$

each comparison giving an equation: hence, by the method of least squares,

$$\begin{aligned} (h^2)\,x - (h\,k)\,y + (h)\,z + (h\,n) &= 0 \\ -\,(h\,k)\,x + (k^2)\,y - (h)\,z - (k\,n) &= 0 \\ (h)\,x - (k)\,y + r\,z + (n) &= 0 \end{aligned}$$

The following table contains the coefficients of expansion obtained in this manner:—

Year.	Standards compared.	O_1	O_2	Cape Bar *B*.	Indian Bar *B*.	Ramsden's Bar.	Troughton & Simms'.	No. of Comp.
1844	O_1 and Cape Bar *B* . .	·000006130	. . . .	·000005918	. . . .	. . . .	. . . .	101
1844	O_1 and Troughton & Simms'	·000006069	. . . .	. . . .	. . . .	. . . .	·000009749	100
1845	O_1 and Ramsden's Bar .	·000006196	. . . .	. . . .	. . . .	·000005848	. . . .	100
1846	O_1 and Indian Bar *B* . .	·000005921	. . . .	. . . .	·00006013	. . . .	. . . .	20
1831	O_2 and Indian Bar *B* . .	. . . .	·000006508	. . . .	. . . .	. . . .	. . . .	25

(10.) The coefficient of expansion of the standard O_1 resulting from the whole of these comparisons, is somewhat different from that obtained by Captain Drummond, R.E., in 1827. In his experiment, the length of the standard when in a natural state was read by a microscope placed over each dot, one having a micrometer, and the temperature of the bar registered. The bar was then carried to an adjoining room, in which the temperature was about 120°, and, acquiring thus a considerable increase of temperature, was quickly replaced under the microscope, and its length again ascertained. The mean of eight values of the coefficient of expansion thus obtained gave ·000006522; and it will be seen from the comparison made at the base line on Salisbury Plain, that the coefficient of expansion at that time was apparently intermediate to this value obtained by Captain Drummond in 1827, and that resulting from the comparisons at Southampton in 1844–6, as contained in the above Table.

(11.) In 1834 a series of 54 comparisons was made by Lieut. Murphy, R.E., between two lengths of the Royal Astronomical Society's Scale and the standard O_2, by which it appeared that the former exceeded the latter by ·00195 of an inch (see "Lough Foyle Base," page [29] Appendix); and, therefore, since $O_1 - O_2 = \cdot000034$ of an inch, and since the whole length of the Royal Astronomical Society's Scale is equal to 59·999712 mean inches of the centre yard (see vol. ix, page 100, R. A. S. Memoirs), therefore

$O_1 = 119{\cdot}997508$ mean inches of centre yard of R. A. S. Scale.

Measured Lines.

(12.) The three volumes, entitled "An Account of the Trigonometrical Survey,"* contain all the particulars relating to the measurement of the base lines on Hounslow Heath, latitude 51° 27′, longitude E. 0° 27′, by General Roy, assisted by Lieut.-Colonel Pringle, R. E., in 1784, and by Lieut.-Colonel Williams and General Mudge in 1791; on Salisbury Plain, latitude 51° 10′, longitude 1° 45′, by the same officers, in 1794; on Misterton Carr, latitude 53° 30′, longitude 0° 55′, by General Mudge, in 1801; on Rhuddlan Marsh, latitude 53° 10′, longitude 3° 30′, by the same officer, in 1806. A base line, of which no account has been published, was measured, in 1817, by Major-General Colby, with the assistance of Mr. Gardner, on Belhelvie Sands, latitude 57° 15′, longitude 2° 10′, near Aberdeen in Scotland; of this proceeding there is no written account, further than the note-books used in the operation, containing the readings of the thermometers, &c., together with the result of Mr. Berge's comparisons of the chains with Ramsden's Prismatic Bar and the "Reduction of the Base," by Major-General Colby. In the present work the absolute distances are derived from the bases measured with the compensation apparatus at Lough Foyle, in 1827 and 1828, and on Salisbury Plain in 1849, the others being considered as bases of verification. A general account of these bases will now be given, referring for the details of those previous to the date 1806 to the volumes of the "Trigonometrical Survey" and the engravings given in them; and for the details of the Lough Foyle Base, to the account published by the Honourable Board of Ordnance in 1847.

HOUNSLOW HEATH BASE.

The measurement of the base line on Hounslow Heath, by General Roy, was the first operation of the trigonometrical survey of Great Britain. The ground was selected from the extraordinary evenness of its surface, its great extent, without any local obstructions to the measurement of a base line, and also from its advantageous vicinity to London and Greenwich.

The first measurement which was made in June 1784 with a steel chain of 100 feet in length, constructed by Ramsden, was considered merely experimental, and gave for the length of the line, after due corrections for temperature, 27408·22 feet.

The bases which had previously to that time been measured in other countries, with the appearance of the greatest care and exactness, had been mostly effected with deal rods, whose actual lengths were compared with a metal standard; and it was therefore determined to measure the base line on Hounslow Heath with such rods, very carefully constructed of the finest material. The rods, which were 3 in number, were each 20 feet 3 inches long between the extremities of the bell-metal tips, by the contact of which the measurement was finally made; the lengths were carefully laid off by Ramsden, from General Roy's scale.

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* Vols. I. & II. were first published in the Philosophical Transactions.

In the course of the measurement, however, it became obvious that the deal rods were affected by the variations of the hygrometric state of the atmosphere to a degree that was quite unexpected, and threw at last such doubts upon the correctness of the operation that, before the measurement was even completed, it was considered a failure. In order to throw additional light on the subject of the expansion of deal rods, several experiments were afterwards made, which led to the conclusion that they were quite unfit to be used in the measurement of a base line. It does not, however, appear that the surfaces of the rods were prepared in any manner by oiling or varnishing, in order to diminish the effect of the humidity of the atmosphere.

The result of this measurement gave 27406·26 for the distance between the centres of the pipes which marked the extremities of the line, reduced to the level of the lowest, in the temperature of 63°, being that of the brass scale when the lengths of the deal rods were laid off.

At the same time that it became clear that the rods of deal would not furnish a result that could be finally relied upon, it also became evident that the measurement must necessarily be repeated with rods of some other material. At the suggestion of Lieut.-Colonel Calderwood of the Royal Horse Guards, F.R.S., General Roy determined to remeasure the line with tubes of glass.

These tubes, which were each 20 feet in length, were firmly secured in deal cases eight inches deep, of the same width in the middle, and tapering from thence in a curvilinear manner towards each end. Through two small orifices in the top of each case passed downwards the bent tubes of two thermometers, whose bulbs were within the deal case. Thus the temperature of each rod was inferred from the readings of two thermometers. The manner of securing the rods in their cases was such that they were free to expand or contract to and from their centres, which parts were rigidly and permanently fixed in the cases. The ends of each tube were ground perfectly smooth and truly at right angles to the axis of the bore, and projected respectively seven and nine-tenths of an inch beyond the case, the former called the fixed, the latter the moveable end. Into the former end was tightly fixed a cork of the very best material, into which was inserted a small brass tube co-axial with the cork. Into this tube again was inserted, screwing into the inner end of the brass tube, a steel pin carrying a button at its exterior end. The inside of the button fitted very closely the ground end of the glass tube to which the outer surface of the button, made as nearly as possible a perfect plane, was parallel. The apparatus at the other extremity of the glass tube was somewhat similar, except that the button, which had a spherical surface the radius of which was two inches, was moveable in the direction of the axis of the tube, by means of a spring acting outwards; and the pin, instead of screwing into the small brass tube in the cork, passed freely through it, and carried at its inner extremity a small ivory scale with a fine intersection cut upon it. Thus, two rods being brought into contact, and the fixed button of one being pressed against the moveable button of the other, the mark on the ivory scale of the latter was pushed backward until it coincided with a diamond line cut on the interior and upper surface of the glass tube, whose length was so adjusted that, when the coincidence was perfect, the distance between the plane surface of the one button and the spherical surface of the other was exactly 20 feet of General Roy's Scale, at the temperature of 68°. Each case had at each extremity a pair of wheels, two inches in diameter, connected by a common steel axis, and also a pair of small cross feet projecting four or five inches outwards from the centre on both sides. By the turning of a screw, the axis of either pair of wheels could be raised or depressed with reference to the case, and thus the case might as required rest either immoveably upon its feet, or have its weight transferred to its four rollers and so become moveable in order to perfect adjustment in the measurement. Also from the upper surface of each end of each case projected upwards a small brass piece two inches long, by means of which, and a slow motion screw connecting them, the rods were drawn together to the required distance; that is, until the mark on the ivory scale coincided with the mark on the inner surface of the glass tube as before described.

The final length of the base line, deduced from the measurement with these rods, when reduced to the temperature of 62°, and to the mean level of the sea, was 27404·0137 feet. The coefficient of the expansion of the glass tubes used in the reduction of the base, namely ·0000043, was obtained from a single observation made with a microscopic pyrometer, of the total expansion of a five foot tube for 180°, the difference of 32° and 212° Fahrenheit.

The principal defect in this measure is probably the manner in which the rods were supported, or rather left unsupported. Each rod rested on two trestles, one at each extremity, the same length supporting the extremity of two contiguous rods; it therefore is probable that the rods were subject to considerable flexure, and also that in removing from the trestles any one of the rods, the trestle which still supported the end of the contiguous rod would, from the change in the force acting upon it, be liable to some small motion. This last source of error was indeed experimentally investigated and found to be real, and produce an observable motion.

These considerations, together with that of the novelty of the method, demanded some verification of the result of the glass rod measurement; it was therefore determined by his Grace the Duke of Richmond, Master-General of the Ordnance in 1791, after the death of General Roy, to re-measure the same line with the steel chain, to which method General Roy had expressed his preference.

Two steel chains of 100 feet in length, designated as A and B, were constructed by Ramsden for this purpose. Each chain consisted of 40 links 2·5 feet long and half an inch square in section, the handles were of brass perfectly flat on the under side, so that the coincidences between the arrows at the extremities of the chains and the division of scales on the brass register heads were easily observed. The chain A was used for measuring, while B was preserved as a standard.

As in the operations with the deal and glass rods, the measurement was made in hypothenusal lengths which were afterwards reduced for inclination. The alignement of the base was effected with a species of transit instrument, which was constructed by Ramsden for that purpose.

At every 100 feet, as measured off in the line of the base with a 20-feet deal rod, was driven a beech post or very stout picket having on its upper surface a brass register head carrying a graduated slider, moved in the direction of the line of the base by a slow-motion screw. This post supported no part of the apparatus, but served merely to indicate, by a division of the scale, the extremity of one chain, and thus give the initial point for the next succeeding. The chain was laid out in a succession of five deal coffers resting upon trestles, the extreme points of support being within about ten inches of the extremity of the chain, or of the register head pickets, so that each handle of the chain rested upon the sliding scale of one of those pickets. The forward end of the chain was then drawn with a weight of twenty-eight pounds, while the rear end was drawn back by means of a screw which had its bearing from a post used for that purpose, until the mark on the handle indicating that extremity of the chain, coincided with that division on the register head scale which indicated the forward extremity of the chain in its previous position. The register head scale at the forward end was then adjusted by its screw, until one of its divisions coincided with the end of the chain, which division again became the initial point for the next chain. The apparatus for the ends of the chain can only be understood in all details by a reference to the engravings in the original account.

Five thermometers were laid close by the chain in each position, and suffered to remain until they showed nearly the same temperature, which took generally from 7 to 15 minutes. When the sun shone out, the chain was covered with a white linen cloth, the ends of which were put over the openings of the first and last coffers to exclude the circulation of the air.

The result of this measurement gave for the length of the base line, after the due correction for the inclinations of the several hypothenuses, reduction to the temperature of 62° Fahrenheit and to the mean level of the sea, 27404·24 feet, exceeding the result given by the glass rods by ·21 feet or two inches and a half, and falling short of that by the deal rods by 2·02 feet; the coefficient of expansion used in making the reduction for temperature was ·00000625, which was deduced from the mean of nine observations made on the chain directly.

The lengths of the chains were ascertained by comparison with Ramsden's Prismatic Bar. The length of this bar, 20 feet, was laid off at the temperature of 54° from his brass scale, which he had compared with that of the Royal Society and found to be of precisely the same length. In the comparison of the chains with the bar, the latter was supported in twenty-one points in a perfectly straight line, as nearly as could be ascertained; part of the chain was then placed on rollers in a horizontal line parallel to the bar and a few inches below it, and whilst it was fastened to an adjusting screw near one end of the bar, it was kept straight on the rollers by a weight of fifty-six pounds. From the extremities of the 20 feet on the edge of the bar, two fine wires with plummets were suspended which were immersed in vessels of water, the wires hanging so as nearly to touch the chain. One end of the chain being then brought under its wire by means of the adjusting screw, a fine point was made on the chain coinciding with the other wire. This part of the chain was then shifted, and so the whole measured in five successive portions.

It is not stated that the operation was repeated more than once for each chain. The result, when corrected for temperature to 54°, gave for the length of the chains—

A = 100·00952 feet of the Prismatic Bar.
B = 100·00485 " "

It is to be feared, from the manner in which the comparison as above described was made, and from the probability of their being the results of one observation only, that the lengths of the chains as thus obtained are affected with a large probable error; for it is now well known that in these matters the truth cannot be arrived at by a small number of observations.

It may be necessary to observe that the length of the base as given above is in terms of feet of Ramsden's Brass Scale at 62°. The extremities of this line are marked by iron guns sunk in the ground, the centre of the bore being in each case the precise end of the base.

SALISBURY PLAIN.

This base of verification was measured in the summer of 1794, by Lieutenant-Colonel Williams and Captain William Mudge of the Royal Artillery. The measurement was effected with the same chains that had been used at Hounslow Heath.

Previous to the measurement, the chains were compared with one another, and were found to have the same difference of length as when measured by Ramsden subsequent to the measurement of the Hounslow Heath base.

The operation was conducted in the same manner as at Hounslow Heath, with some trifling improvements in the method of aligning with the transit; the inclinations of the different hypothenuses were observed with the transit, which had a small vertical arc for the purpose. An examination of the chains after the operation showed that the measuring chain A had stretched $\frac{1}{200}$ part of an inch. The final length of the base when corrected for the wear of the chain, reduced to the temperature of 62°, and to the level of the station on Beacon Hill which was one extremity of the base, was 36575·401 feet of the brass scale. Now the height of Beacon Hill is 669·5 feet, as obtained by levelling, therefore the length of the base reduced to the level of the sea becomes 36574·232. The extremities of the line were marked by iron guns sunk in the earth, the centre of the bore of each marking the precise end of the base.

MISTERTON CARR.

The apparatus used in the measurement of this base in 1801 by Major Mudge was the same as that employed on Hounslow Heath and Salisbury Plain, with the addition of a steel chain of fifty feet in length of exactly the same construction with the longer chains.

The resulting length of the base line was 26342·19 feet when reduced to the temperature of 62° of the brass scale and to the mean level of the sea;* after the operation the chains were compared by Mr. Berge, successor to Mr. Ramsden, with the Prismatic Bar, from which comparisons it was found that at 54°—

A = 100·01104 feet of the Prismatic Bar.
B = 100·00648 „ „

The 50-feet chain C was also carefully compared with the standard B, which gave—

$$2\,C = B + \cdot 00786 \text{ feet.}$$

This chain only measured four lengths or two hundred feet of the line. The ends of the base were marked by two blocks of oak with square holes in their upper surface, these holes were run in with lead which was filed off even with the surface of the wood and the diagonals drawn. The intersections of the diagonals indicated the precise ends of the base. These oak blocks are not now to be found.

. . . . .

RHUDDLAN MARSH.

This base line was measured in the summer of 1806 on an extensive and level piece of ground about four miles north-west of St. Asaph in Flintshire, North Wales. The measurement was made with the same apparatus and in the same manner as in the former lines; the chains were very carefully compared before and after the operation with the standard B. The final result for the length of the base, after reducing to the temperature of 62°, was 24514·26 feet of Ramsden's Brass Scale.

The extremities of the line were marked as at Misterton Carr with blocks of wood, but of these unfortunately every trace has been lost.

BELHELVIE SANDS.

The ground selected for this base line is situated near Aberdeen. The same chains and method of measurement were employed as on all former occasions. The chains were compared with the points in the cast-iron prismatic bar both before and after the measurement by Mr. Berge, and the results of the comparisons gave, when corrected to the temperature of 54°,

A = 100·01270 feet of the Prismatic Bar
B = 100·00547 „ „ } Before the measurement.
C = 50·00127 „ „
A = 100·01441 „ „
B = 100·00631 „ „ } After the measurement.
C = 50·00123 „ „

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* In the reduction of the base in the original account there is a slight error in the reduction from 54° to 62°; the quantity should evidently be 2·172 feet.

The "Reduction of the Base" by Major-General Colby is as follows:—

Reduction of the Base Line.

	Feet.
The apparent length of the base line, when reduced from the several hypothenuses (to mean level of the sea), according to the Table, is	26514·2653
The mean length of the measuring chain A, as found by Mr. Berge's comparisons with the points on the cast iron bar, previously and subsequently to the measurement, was, when reduced to the temperature of 54° Fahr., 100·01355 feet, whence $\cdot01355 \times 261$	3·5365
The mean length of the fifty-feet chain was 50·00125 feet, whence $\cdot00125 \times 8$	·0100
The sum of all the degrees shown on the thermometers was $70699\cdot5 + 1314\cdot25$, wherefore $\left(\frac{70699\cdot5}{5} + \frac{1314\cdot25}{6} - 54^\circ \times 265\cdot18\right) \cdot \frac{\cdot0075}{12} = \cdot0245$ of a foot, which is the correction for the mean heat at which the base was measured above 54°, the temperature at which the chains were laid off	·0245
Finally, we have $\frac{\cdot01237 \times 265 \times 8}{12} = 2\cdot1854$ feet for the reduction of the base to the temperature of 62°	−2·1854
	26515·6509

In explanation of the reduction of the thermometer readings to obtain the correction ·0245 feet, it should be stated that the 50-feet chain had three thermometers to record its temperature, so that two 50-feet chains had six thermometers, whence the number 6 in the denominator of the fraction whose numerator is 1314·25, which is the sum of the readings of the thermometers for the eight positions of the 50-feet chain. The quantity ·01237 in the last correction is the expansion in inches of 100 feet of the brass scale for 1° of Fahrenheit. The reduction to the mean level of the sea is included in the first term. This line was measured between May 5th and June 6th 1817 by the late Major-General Colby with the assistance of Mr. Gardner.

To reduce these base lines into terms of Ramsden's Bar at 62° we must multiply them by the ratio of the length of a foot of the brass scale to that of the prismatic bar when both are at that temperature. Now both standards are equal at 54°, therefore if $e\ e'$ be their respective coefficients of expansion, their relative lengths at 62° will be as $1 + 8e$ to $1 + 8e'$, or $1 + 8(e - e')$ to unity, and using the values given in the original accounts, namely $e = \cdot0000103$ and $e' = \cdot0000062$, the factor for multiplying the bases becomes ·0000328, also the factor for converting measures in terms of Ramsden's Bar into feet of O_1 is 1·0000383. The following table contains the length of the old bases according to the different standards of measure:—

Date.	Base.	Length in feet of		
		Brass Scale.	Prismatic Bar.	Ordnance O_1.
1791	Hounslow Heath . . .	27404·24	27405·14	27406·19
1794	Salisbury Plain	36574·23	36575·43	36576·83
1801	Misterton Carr	26342·19	26343·05	26344·06
1806	Rhuddlan Marsh . . .	24514·26	24515·06	24516·00
1817	Belhelvie Sands . . .	26515·65	26516·52	26517·53

LOUGH FOYLE.

(13.) The ground chosen for the measurement of the Lough Foyle base lies on the eastern border of the lake of that name in the north of Ireland. The northern extremity of the line is about two miles and three quarters south of the martello tower on Magilligan Point, in the county of Londonderry, the southern extremity is a quarter of a mile south-east of the church at Ballykelly. The ground is admirably adapted for the purpose, and is at a mean height of eighteen feet above the mean level of the sea.

The measurement commenced on the 6th of September 1827 and was continued until the 25th of October, during which time several partial remeasurements were made in order to test the accuracy of the compensation apparatus and the stability of the point carriers. The results of these remeasurements were always very satisfactory. The measurement was resumed on the 7th of July in the following year and continued until the 25th of the same month, when it was suspended in consequence of the crops being on the ground. It was again resumed on the 13th of September and finally concluded at the south end of the base on the 20th of November 1828. The measurement was thus effected in three periods amounting in the total to sixty days, the average daily progress five hundred feet.

The mean lengths of the bars applicable to these three periods in terms of the compensation bar A, were determined by numerous comparisons, and are as follows, the bars being individualized by the letters A, B, C, D, E, G:

Bar.	1st Period.	2d Period.	3d Period.
B =	$A + 33{\cdot}0\,d$	$A + 39{\cdot}3\,d$	$A + 46{\cdot}1\,d$
C =	$+38{\cdot}3\,d$	$+41{\cdot}0\,d$	$+48{\cdot}8\,d$
D =	$+14{\cdot}2\,d$	$+\ 7{\cdot}9\,d$	$+11{\cdot}1\,d$
E =	$+\ 9{\cdot}0\,d$	$+16{\cdot}8\,d$	$+23{\cdot}5\,d$
G =	$+11{\cdot}5\,d$	$+\ 6{\cdot}8\,d$	$+\ 9{\cdot}4\,d$

when d is one division of the micrometer and equal to $\frac{1}{245400}$ of a foot. The total length measured by the bars is then found by summation equal to

$$3966\,A + 86696{\cdot}8\,d$$

Similarly, by frequent comparisons of the compound microscopes with the Ordnance 6-inch Standard, and using their mean values for 5 different periods, the total length of the part measured with microscopes was found equal to

$$5208{\cdot}5\ O^{6\ \mathrm{inch}} + 23735{\cdot}2\ d' - {\cdot}708 \text{ feet}$$

$O^{6\ \mathrm{inch}}$ being the length of the Ordnance 6-inch Standard at 62°: d' the value of a division of the micrometer $= \frac{1}{251808}$ of a foot; ·708 feet was the distance measured *back* from the dot marking the termination of the last series, to the dot on platina wire marking the south end of the base. This measurement was made with two beam compasses of ebony.

The values of the compensation bar A, and the 6-inch Standard in terms of the mean feet of O_1 were determined as

$$A = 10 \text{ feet} - 24{\cdot}3\ d$$
$$O^{6\ \mathrm{inch}} = 0{\cdot}5 \text{ feet} - {\cdot}56\ d'$$

The length of the base reduced to feet of O_1 at the temperature of 62° is thus found to be 41640·9228 feet, and the reduction to the mean level of the sea being ·0355 feet, the length finally is 41640·8873 feet.

The base was then verified by means of intermediate points in its line by a Comparative Triangulation, which will be found in the original account. The conduct of this measurement was principally entrusted to Captain Drummond, R.E.

SALISBURY PLAIN.

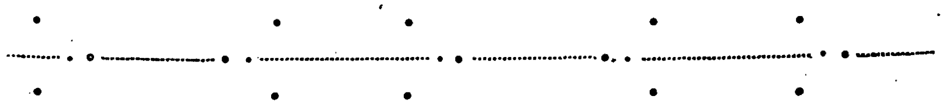
(14.) In the year 1849 the base line on Salisbury Plain was remeasured with the compensation bars. As the old measured distance could not be reproduced with sufficient accuracy by calculation from the Lough Foyle base, Lieut.-Col. (then Captain) Yolland directed, with the approval of Colonel Hall, then Superintendent of the Ordnance Survey, the remeasurement of this line with the apparatus used in the measurement of the Irish base. This operation was conducted entirely by a party of the Royal Sappers and Miners under Serjeant Steel of the same corps, who was selected for the duty by Captain Yolland as a man quite capable of doing justice to this difficult undertaking.

The north end of the base is on Beacon Hill, three miles west of the village of Amesbury; the south end near Old Sarum Castle, a mile and a half from Salisbury. The guns marking the extremities of the old base were easily found, and having retained their vertical position, and being very firm, it is highly probable they have not moved sensibly from the position in which they were placed in 1794. As it was found that the extremities of the line were not reciprocally visible, it became necessary to elevate the theodolite at the south end by a scaffolding 32 feet high. The choice of this point in the first instance as the extremity of the base line is rather singular, as, a few yards further on, a higher piece of ground affords a much more satisfactory terminus. The muzzles of both guns were fitted tightly with a cylindrical plug of wood, the centre being very carefully marked with a brass pin. Over the Beacon Hill centre was brought the Ordnance 3-foot theodolite with its observatory, the instrument being centred in the usual manner with a plummet. The centreing of the Royal Society's theodolite over the south end was a little troublesome, on account of the height of the instrument above the gun; but, finally, by protecting carefully the plummet from currents of air, it was satisfactorily accomplished. These instruments served to align the base and connect it with the triangulation. The observer was Serjeant Donelan of the Royal Sappers and Miners.

The party proceeded from the Map Office, Southampton, on the 1st of May 1849, and pitched camp on Arrow Down, near Amesbury. The first three weeks were spent in tracing the line, clearing it of trees, furze, &c., driving pickets for the alignement and comparisons of bars. The pickets were driven at short intervals at first, and afterwards the observer at the ends of the line, taking advantage of particularly favorable circumstances of atmosphere, selected those which were more exactly bisected by the vertical plane of the base, and the others were rejected. The pickets were from 1·5 to 2·5 inches square, with the centre marked on the upper surface.

The intermediate alignement was made with a portable transit instrument, the telescope of which had a magnifying power of 40. This instrument was brought into the base line by means of one of the "point carriers" left on the ground by the bars in their measurement, and the signal at the end of the base. As the point carrier was at a very short distance and was bisected by means of a small straight-edge erected vertically and precisely over the dot, it is evident that the transit was by this means brought with the greatest precision into the base line; if the extremity of the line was not visible, the furthest picket was used in conjunction with the point carrier, and the final adjustment of the transit into line was effected by screws, which gave the instrument a lateral motion on its stand. The transit thus adjusted and scrupulously levelled, aligned the centre of the telescopic microscope of each compensation microscope (a brass pin being erected over that centre for perfect bisection), when the set of bars and microscopes were approximately laid on the trestles. Each microscope was aligned by its own directing telescope on a point marked on the transit at a distance from the plane of the axis of its telescope equal to the distance between the plane of the axes of a compensation microscope and the axis of its directing telescope.

(15.) The method of measurement was much the same as described at Lough Foyle. Preparatory to the laying of a set of bars, a party of two men lay off distances in the line of the base of 5 feet and 5 feet 6 inches alternately, which points are to be the centres of equilateral triangles, of 17 inches side, formed by driving pickets thus,—



They then level carefully each set of pickets, and place on each a wooden triangular frame (R R′, Plate II.); having levelled these triangles, they next place on each a trestle, selecting them according to length if the ground be uneven, so that their upper surfaces may be in a horizontal plane nearly. A second party of two men then place on each trestle a camel or tripod, L L′, which are aligned by signals from the observer at the transit, and carefully levelled, each in itself and with respect to all the others, so that the bars may require but little final adjustment with respect to level. The bars are then laid on the camels, and levelled individually by a third party. The microscopes M are laid by the principal operator, and in the meantime the whole is covered by a fourth party with the base tents.

Suppose now the bars and microscopes thus approximately arranged to be in the usual order,

A B C D E G
M N O P Q R S

The principal first brings the telescopic or centre microscope of M over the point to be measured from (on the ground), aligns and levels the microscope, and brings the following dot of bar A under the advanced microscope of M. In the meantime one assistant has brought the telescopic microscope of N into the base line by signals from the transit, and another has brought the preceding end of the bar A into level with the following end. The principal then proceeds to the second microscope N, aligns and levels it, and bisects the contiguous dots on A and B, whilst the bar B is being levelled, and the microscope O, or rather the axis of its telescopic microscope, brought into the line of the base by the two assistants; and so on to the end, when a point carrier is brought accurately to bisection under the telescopic microscope of S. The principal then goes over the whole, and revises, if necessary, every bisection, level, and alignement. The bars then move forward to the trestles which are ready for them in their next position.

All the different operations here mentioned were invariably performed in the same order, for regularity is indispensable in the use of such a complicated apparatus.

The measurement commenced at the Beacon Hill or northern extremity, and owing to the steepness of the hill for some distance, only one or two bars could be laid at a time. The slope has a maximum value of about 11°, and disappears at the distance of about 700 to 800 yards. This was the only troublesome part of the measurement as regards ground. Salisbury Plain is very well adapted for the measurement of a base, and a longer line might have been selected on it; but it is certain that little if any advantage is gained by the measurement of a base of more than six or seven miles, provided it be surrounded with very careful triangulation.

(16.) The comparisons of the compensation bars with each other and with the standard O_1, were conducted in the same manner as the comparisons made at the Ordnance Map Office, Southampton, in 1845 and 1846, and described in the "Measurement of the Lough Foyle Base.". Two stone pillars (A A′, Plate III.), about five feet long and one foot square, were sunk two feet into very firm soil, which was rammed as tightly as possible round them, leaving however the upper surface of the adjacent ground not in contact with them, so that the movement of the observer could have no effect upon their stability. To the upper surface of these pillars, the centres of which were ten feet apart, brass arms $i\,i'$, for carrying the microscopes, were fixed by vertical screws $s\,s'$, clamping them to the ends of truncated brass cones sunk into the pillars, and run in with plaster of Paris. In order to ensure the stability of the bars when under observation, a strong wooden frame B C D, braced in the most solid manner, seven feet long, three and a half broad, and rather more than a foot high, and having a firm resting on the earth, was placed between the pillars and to the front, the length being exactly parallel to the line of the pillars. The end beams of this frame, $b\,c$, $b'\,c'$, were made to act as rails, on which a strong plank E, seven feet nine inches long, ten inches broad and three inches thick, was made to slide in a direction transverse to itself, and therefore perpendicular to the line of the pillars. This motion was communicated by a lever $d\,d'$, without communicating the slightest jar or shake to the apparatus. Equidistant from the extremities of this plank, and five feet apart, were placed the tripods $F\,F'$ for carrying the bars, their elevation screws passing down through circular holes cut in the plank. The bar being then placed on the camels and levelled, could be brought by the sliding of the plank, with the greatest ease under the microscopes, the ultimate adjustment of its position being effected by the screws $f\,f'$ $e\,e'$ of the tripods, the bar having a longitudinal motion communicated to it by a tangent screw $g\,g$, working on one of the tripods. The microscopes $a\,a'$, one of which had a fixed wire and the other a micrometer, were at the commencement of the operations brought to focus over the dots of the bar when perfectly level, and firmly secured in that position by means of their screws (acting on the supporting arm). In order to bring the surface, carrying the dot, of each of the other bars into precisely the same position or focal distance, the levers used in the comparisons in 1844–46 were also used in the comparisons on the base line. The arrangement may be described as follows: to each of the stone pillars was fixed a bracket $o\,o'$, having a knife edge on its upper surface, on which as a fulcrum rested a lever $l\,l'$, the short arm being counterpoised rested with a metallic point on the surface carrying the dot, whilst the exact position of the extremity of the longer arm, which was six times the length of the shorter, was indicated by a fine line on a post erected for the purpose. Thus after one bar was finally adjusted, all the others were brought to exactly the same position as regards height and focal distance by bringing the outer extremity of each of the levers to its mark on the post.

The standard O_1, was in every case first brought under the microscope, then the six compensation bars in succession, and then the standard again. The temperatures of the bars as well as the readings of the micrometer were registered. The standard bar has the bulbs of two mercurial thermometers let into it, and the interstice being filled with oil, the temperature of the air is avoided.

With respect to the constancy of the distance between the microscopes during the comparison of one set of bars, it is probable that, mathematically speaking, it was always varying, especially with any rapid change of temperature, but within such a short space of time as one set of comparisons (reading of eight bars) would take up, it is probable that the variation would be of so small an amount as not to materially affect any results. Besides, care was taken in the selection of the days for observing, so that changes of temperature within a short space of time were avoided, and the whole apparatus was of course protected by the base tents. The following table contains the comparisons of the bars with the standard O_1.

Date. 1849.		MICROMETER READINGS.								Observed Temperature of O_1.		Mean Temperature reduced to Standard.
		O_1	A	B	C	D	E	G	O_1	At Commencement.	At Close.	
May	17	77·0	93·5	154·0	159·0	107·0	142·0	82·0	69·0	60°·00	59°·55	60°·02
,,	18	87·0	192·0	256·0	252·0	202·0	243·0	184·0	97·0	51·60	52·20	51·87
,,	,,	0·0	90·0	150·0	152·0	98·0	138·0	81·0	8·0	53·00	53·35	53·08
August	2	148·0	84·5	143·0	143·0	100·0	136·0	78·0	136·5	69·55	68·10	69·28
,,	3	92·0	105·0	160·0	164·0	118·0	156·0	97·0	94·0	61·60	61·40	62·04
,,	,,	94·0	107·0	163·0	164·0	119·0	157·0	94·0	94·0	61·25	61·25	61·66
,,	,,	94·0	106·0	164·0	167·0	117·0	158·0	97·0	95·0	61·05	61·10	61·32
,,	4	5·0	113·0	171·0	173·0	126·0	163·0	107·0	13·0	51·20	51·65	51·81
,,	,,	15·0	116·0	173·0	173·0	125·0	164·0	105·0	22·0	51·90	52·65	52·21
,,	,,	40·0	116·0	165·7	174·0	122·0	160·0	101·0	66·0	54·65	57·20	56·06
,,	,,	84·0	115·0	168·0	173·0	122·0	160·0	103·0	101·0	59·60	60·90	60·53
,,	,,	5·0	14·0	73·0	76·0	23·0	60·0	2·5	12·5	61·40	62·70	62·29
,,	,,	117·0	112·0	165·0	169·0	123·0	160·0	96·0	127·0	63·00	63·90	63·58
,,	,,	32·0	12·0	65·0	70·0	22·0	56·0	0·0	32·0	64·60	65·15	65·00
,,	,,	35·0	14·0	63·0	68·0	25·0	58·0	0·0	33·0	65·40	65·15	65·49
,,	6	26·0	113·0	169·0	174·0	124·5	166·0	106·0	33·0	53·05	53·80	53·33
,,	,,	38·0	114·0	170·0	174·0	124·0	161·0	102·5	54·0	54·35	55·40	54·88
,,	,,	74·0	116·0	169·0	173·0	119·0	157·0	102·0	100·0	58·45	60·35	59·63
,,	,,	39·0	14·0	65·0	69·0	18·0	59·0	0·0	50·0	66·00	66·95	66·87
,,	,,	158·0	112·0	163·0	168·0	119·0	156·0	95·0	164·0	67·60	68·45	68·45
,,	,,	178·0	110·0	165·0	167·0	118·0	160·0	98·0	179·0	69·35	69·95	70·02
,,	,,	178·0	114·0	166·0	168·0	122·0	155·0	97·5	178·0	70·30	70·30	70·63
,,	,,	181·0	112·0	166·0	169·0	119·0	158·0	97·5	181·5	70·35	70·50	70·76
September	18	62·5	51·0	131·0	129·0	75·0	109·0	59·0	62·0	63·10	62·50	63·03
,,	,,	51·0	59·0	141·0	132·0	81·0	117·0	60·0	48·0	61·15	61·00	61·35
,,	19	91·0	266·0	349·0	339·0	285·5	325·0	269·5	109·0	43·60	45·05	44·20
,,	,,	20·5	158·5	236·0	228·0	174·0	210·5	155·0	32·5	47·55	48·75	47·79
,,	,,	52·0	154·0	229·0	225·5	166·5	207·0	146·0	65·0	51·70	52·90	52·24
,,	,,	81·0	144·0	225·0	218·0	161·0	202·0	146·0	93·5	55·40	56·15	55·90
,,	,,	99·0	179·0	256·5	254·0	202·0	237·0	182·0	146·0	—	57·85	58·07
,,	,,	51·0	77·0	159·0	149·0	99·0	136·5	79·5	57·0	59·00	59·50	59·48
,,	,,	59·0	79·5	160·0	149·0	99·0	136·0	80·5	60·5	59·85	60·05	60·21
,,	20	89·0	183·5	261·0	253·0	201·0	239·0	183·5	93·0	52·50	52·90	52·62
,,	,,	98·0	176·0	255·0	248·0	193·0	236·0	180·0	102·0	53·50	54·80	54·08
,,	,,	6·0	71·0	153·0	145·0	93·5	129·5	74·5	10·0	54·60	55·00	54·81
,,	,,	18·0	70·0	148·0	140·0	87·0	127·0	69·5	24·5	56·35	56·90	56·72
,,	,,	27·0	67·0	145·5	139·0	85·0	124·0	65·0	28·0	57·65	57·90	57·83
,,	,,	30·0	64·0	145·0	135·0	86·0	125·0	64·0	34·5	58·20	58·60	58·69
,,	,,	33·0	69·0	145·5	134·0	82·0	121·5	66·0	34·5	58·45	58·55	58·78
,,	,,	35·0	67·0	144·5	134·5	86·5	123·0	64·0	30·0	58·30	58·00	58·45
,,	21	58·0	176·0	257·0	250·0	197·5	234·0	179·0	60·0	49·50	50·00	49·78
,,	,,	68·0	175·0	253·0	246·0	196·0	233·0	175·0	72·0	50·75	51·25	50·70
,,	,,	83·5	174·0	253·0	243·0	194·0	232·0	175·5	93·0	52·20	52·90	52·47
,,	,,	96·0	169·0	248·0	236·0	187·0	228·0	172·0	106·0	53·95	55·20	54·54
October	18	63·0	63 0	138·0	124·0	75·0	116·0	60·0	67·0	61·25	—	61·42

Date.	MICROMETER READINGS.								Observed Temperature of O_1.		Mean Temperature reduced to Standard.
	O_1	A	B	C	D	E	G	O_1	At Commencement.	At Close.	
October 18	68·0	55·0	133·0	116·0	72·0	109·0	53·0	72·0	62°·85	63°·25	63°·27
„ „	71·5	57·5	136·0	116·0	70·5	106·5	49·0	71·0	63·70	63·65	63·77
„ „	67·0	50·0	125·0	111·0	71·0	106·5	46·0	63·0	63·75	63·15	63·58
„ 19	38·0	100·5	180·0	169·0	122·0	161·0	101·0	48·0	54·40	55·35	54·89
„ „	55·0	93·5	175·0	162·0	117·0	155·0	95·0	68·0	56·95	58·40	57·79
„ „	82·0	92·0	175·0	159·5	108·0	153·0	93·5	85·0	59·60	60·90	60·53
„ „	104·0	91·0	173·0	158·0	111·0	152·0	93·0	108·0	62·85	63·40	63·30
„ „	108·0	90·0	171·0	159·0	115·0	151·0	96·0	104·0	63·00	63·30	63·32
„ „	101·0	91·0	168·0	161·0	110·0	147·0	88·0	96·0	63·05	62·40	62·83
„ 20	25·0	71·5	152·0	140·0	93·0	135·0	72·5	25·0	56·55	56·50	56·63
„ „	26·0	71·0	151·0	141·0	91·0	130·0	74·0	25·0	56·50	56·70	56·70
„ „	27·0	68·0	152·0	139·5	93·0	133·0	72·0	29·0	56·65	57·00	56·91
„ „	30·0	69·0	148·0	136·0	90·0	130·5	72·0	31·0	57·10	57·30	57·23
„ „	36·0	67·0	144·0	134·0	90·0	126·0	69·0	40·0	58·15	58·00	58·38
„ „	42·0	65·0	144·5	136·0	89·0	128·0	70·5	44·0	58·35	58·95	58·92
„ „	45·0	67·0	147·0	134·0	91·0	127·0	68·0	43·0	58·85	58·90	59·12
„ „	41·0	64·0	146·0	135·5	91·0	130·0	71·0	41·0	58·20	58·55	58·66
„ 22	12·0	206·0	286·0	275·0	223·0	263·0	207·0	20·0	41·60	42·10	41·85
„ „	36·0	199·0	282·0	272·0	223·0	263·0	205·0	48·5	43·95	45·05	44·29
„ „	61·0	201·0	281·5	271·0	219·0	261·0	204·5	74·0	46·45	47·75	46·19

The last column in this Table contains the mean of the thermometer readings of the standard bar reduced to the corresponding reading of the standard thermometer used in the measurement of the Lough Foyle Base.

(17.) These observations have been reduced in the following manner.

Let X be the distance between the fixed wire of one microscope and that point in the other from which the readings of the micrometer commence; then if m be the mean of the readings of the standard (which was observed the first and last of each set); t the mean of the four thermometer readings; $m_1 \ldots\ldots m_6$ the readings of the compensation bars; x the expansion for 1° Fahrenheit of O_1 in micrometer divisions

$$\begin{aligned} O_1 &= X + m + (62 - t)\,x \\ A &= X + m_1 \\ B &= X + m_2 \\ &\vdots \qquad \vdots \\ G &= X + m_6 \end{aligned}$$

Now put $X - O_1 = y$, $O_1 - A = x_1$ $O_1 - B = x_2 \ldots\ldots O_1 - G = x_6$; then each set of comparisons will give the seven equations following:—

$$\begin{aligned} 0 &= m + ax + y \\ 0 &= m_1 + x_1 + y \\ 0 &= m_2 + x_2 + y \\ &\vdots \qquad \vdots \\ 0 &= m_6 + x_6 + y \end{aligned}$$

The quantity x, or the expansion of O_1 has been shown to be variable in the "Measurement of the Lough Foyle Base," the earlier determinations giving an expansion of ·0000065 on unity, and the later ·0000061. On using this last determination in the above equation for obtaining the differences between the compensation bars and the standard, it was obvious at once, from the resulting discrepancies, that the assumed expansion was too small. As no observations of the expansion were made about the time of the measurement, the best that can be done is to use that expansion which will introduce the least discrepancies.

If, therefore, there be n sets of comparisons on the same day, or on two or three successive days, there will be $7n$ equations for the determination of the $7+n$ quantities x, $x_1 \ldots\ldots x_6$, $y\ y' \ldots\ldots$ the quantity y being assumed different for each set of comparisons.

If we solve these equations by the method of least squares, and put

μ = the mean of the seven readings in each set successively,
m = the reading of the standard in each set,
μ_1 = the mean of the n readings of A,
μ_2 = the mean of the n readings of B, and so on,
σ = the mean of all the quantities m or readings of the standard,
β = the mean of the $6n$ readings of the compensation bars,

then we shall have, the brackets ( ) signifying summation,

$$
\begin{aligned}
0 &= 7\,(a[\mu-m]) + 6\,(a)\left(\sigma-\beta\right) + 6\left(\frac{(a)^2}{n} - (a^2)\right) x \\
0 &= x_1 + \mu_1 - \sigma - \frac{(a)}{n}\,x \\
0 &= x_2 + \mu_2 - \sigma - \frac{(a)}{n}\,x \\
&\vdots \\
0 &= x_6 + \mu_6 - \sigma - \frac{(a)}{n}\,x \\
y &= -\mu - \frac{6}{7}\left(\sigma-\beta\right) - \frac{1}{7}\left(a + 6\,\frac{(a)}{n}\right) x \\
y' &= -\mu' - \frac{6}{7}\left(\sigma-\beta\right) - \frac{1}{7}\left(a' + 6\,\frac{(a)}{n}\right) x \\
\vdots\ \ & \quad\ \vdots
\end{aligned}
$$

$$
\begin{aligned}
x_1 &= -m_1 - y & \qquad x_2 &= -m_2 - y \\
&= -m'_1 - y' & &= -m'_2 - y' \qquad \&c. - - - \\
&= -m''_1 - y'' & &= -m''_2 - y'' \\
&\ \ \vdots & &\ \ \vdots
\end{aligned}
$$

The value of the expansions of the standard bar thus determined, though it may not have the same accuracy as if direct experiments had been made for determining it, is certainly more near the truth at the time than the old determination. Comparisons of the bars were made in May, August, September, and October. The comparisons in May are not sufficiently numerous to obtain a value of x. Twenty comparisons in August give

$$4350x = 39477 \quad , \quad x = 9{\cdot}07$$

Twenty-one comparisons in September give

$$2677x = 24712 \quad , \quad x = 9{\cdot}23$$

Twenty-one comparisons in October give

$$4755x = 44204 \quad , \quad x = 9{\cdot}29$$

Combining these we get the expansion equal to 9·20 divisions of the micrometer for 1° Fahrenheit. Now one inch is equal to 12040 divisions, therefore, 9·20 divisions are equal to ·000037 of a foot, and the bar being 10 feet long, gives an expansion of

·00000637 on unity.

It seems, therefore, in accordance with these comparisons, that the expansion of the standard O_1 must have increased since 1846, and is approaching again its former expansion, namely, ·0000065.

Using the coefficient of expansion as determined above, namely, ·00000637, the values of the compensation bars at the different periods of comparison will be as follows :

Difference.	MAY.		AUGUST.		SEPTEMBER.		OCTOBER.	
	Micr. Division.	Probable Error.	Micr. Division.	Probable Error.	Micr. Division.	Probable Error.	Micr. Division.	Probable Error.
$A-O_1$	+4·1	0·021	+8·7	0·031	+1·3	0·027	−4·4	0·038
$B-O_1$	65·6	0·069	63·1	0·017	80·6	0·020	75·4	0·027
$C-O_1$	66·6	0·078	66·5	0·019	72·6	0·029	63·1	0·029
$D-O_1$	14·6	0·068	18·0	0·031	20·4	0·024	16·2	0·034
$E-O_1$	53·3	0·051	55·7	0·029	58·6	0·019	55·2	0·026
$G-O_1$	−5·4	0·073	−3·3	0·027	+2·1	0·028	−3·0	0·023

One micrometer division = $\frac{1}{12040}$ inch.

(18.) The compensation microscopes were compared with the Ordnance 6-inch Standard every week and adjusted if necessary, the readings of each being always recorded. As the microscopes were generally very near the state of adjustment in regard to the measure of six inches, the total correction of the base line is very small. It is to be regretted that these comparisons were not made oftener in the measurement of the base at Lough Foyle as it is probable that from the neglect of this in the first part of the measurement in 1827, an error of an inch as a maximum might result in the estimate of the length. The compensation microscope is a delicate apparatus and even uncareful handling will cause motion in the wires; hence the necessity for frequent comparison with the dots of the six-inch scale.

In correcting the length of the brass scale in the following table of microscope equivalents the expansion for one degree of Fahrenheit has been taken at ·0000097.

MICROSCOPE EQUIVALENTS.

Sets.		M		N		O		P		Q		R		S	
From	To.	Value.	No.	Value.	No.	Value.	No.	Value.	No.	Value.	No.	Value.	No.	Value.	No.
1	11	+0·53	4	+0·27	7·5	—	—	—	—	—	—	—	—	−13·53	5·5
12	38	−0·04	16	+0·98	13·5	—	—	—	—[1]	+0·26	4·5	—	—[1]	− 0·27	17
39	88	+2·19	48	+2·71	24·5	− 3·80	22	−7·79	7	−0·27	30·5	−0·80	6	− 0·10	45
89	172	−2·20	84	+2·77	82	− 3·86	42	+7·53[2]	84	−0·54	83	−1·85	77	+ 0·44	50
173	207	−5·22	35	+0·67	35	− 3·89	17·5	+2·71	35	−0·97	35	−2·40	35	+ 1·26	17·5
208	240	−3·03	33	+1·36	33	—	—	+2·95	33	−1·10	33	−0·83	33	—	—
208	216	—	—	—	—	− 3·48	4·5	—	—	—	—	—	—	—	—
217	—	—	—	—	—	− 9·47[3]	0·5	—	—	—	—	—	—	—	—
218	240	—	—	—	—	+28·82	11·5	—	—	—	—	—	—	—	—
208	231	—	—	—	—	—	—	—	—	—	—	—	—	+ 1·54	12
232	—	—	—	—	—	—	—	—	—	—	—	—	—	−31·14[4]	0·5
233	240	—	—	—	—	—	—	—	—	—	—	—	—	− 0·92	4
241	259	−3·73	19	+2·18	19	+ 3·61	9·5	+3·12	19	−1·42	19	−0·18	19	− 1·21	9·5
260	268	−1·82	9	+2·52	9	+ 1·65	6·5	−2·43	7	+0·	9	+0·05	9	+ 0·50	4·5
269	313	−3·16	45	+1·49	45	− 0·29	45	−8·01	22·5	−1·49	45	−0·64	45	+ 0·72	22·5
314	360	−3·29	48	+1·96	48	− 1·16	45	+6·74[5]	24	−1·54	40	+1·00	37	− 0·43	24
361	408	−3·47	49	+1·91	49	− 1·93	45	+7·18	24·5	−2·08	41	+2·74	41	− 2·04	24·5
409	456	−3·61	48	+0·43	48	− 0·63	48	+6·76	24	−1·95	45	+2·75	40	− 0·99	24
457	493	−3·12	37	−0·37	37	− 0·37	37	+6·66	18·5	−2·36	37	+2·12	37	− 1·57	18·5
494	574	−2·12	73	−0·70	82	+ 0·26	82	+6·08	41	−2·14	82	+2·07	82	− 0·37	41
575	621	−2·95	31	−1·01	48	+ 1·41	47	+5·98	24	−1·79	45	+2·22	45	+ 1·73	24
622	638	−4·66	17	−0·70	17	+ 1·16	17	+5·66	8	−2·02	16	+2·06	15	− 1·64	8·5

[1] The microscopes P. R. fell from their feet to their sides when the box was being removed down the hill, but were not injured.

[2] The result of some accidental disturbance of the wire plate screws subsequent to June 18th.

[3] This value applies to set 217 only. Between sets 216 and 217 microscope O was deranged by the wind blowing down the lid of its box.

[4] Applies to set 232 only. Microscope S was disturbed between 231 and 232; after 232 and before 233 it was compared.

[5] A change took place in P between the evening of the 18th and morning of the 19th of August.

9102·66 Micrometer divisions = one inch.

(19.) The following Table contains an abstract of the number of bars and microscopes, used in the measurement of the line.

ABSTRACT OF THE BARS AND MICROSCOPES IN THE BASE LINE MEASUREMENT ON SALISBURY PLAIN.

Date.	Sets.	Bars.						Microscopes.						
		A	B	C	D	E	G	M	N	O	P	Q	R	S
May 23 to 24	1 to 4	4	4	—	—	—	—	2	4	—	—	—	—	2
,, 24	5	1	1	—	—	—	—	1	0·5	—	—	—	—	0·5
,, 25	6 and 7	2	—	—	—	—	—	—	1	—	—	—	—	1
,, ,,	8	1	1	—	—	—	—	1	0·5	—	—	—	—	0·5
,, 26	9 to 20	12	—	—	—	—	—	—	6	—	—	—	—	6
,, 30	21 to 24	4	4	—	—	—	—	4	2	—	—	—	—	2
,, ,,	25	1	—	—	—	—	—	—	0·5	—	—	—	—	0·5
,, 31	26 to 29	4	4	—	—	—	—	4	2	—	—	—	—	2
June 1	30 to 37	8	8	8	—	—	—	8	4	—	—	4	—	8
,, 2	38	1	—	—	—	—	—	—	0·5	—	—	0·5	—	—
,, 4	39 to 49	11	11	11	—	—	—	11	5·5	—	—	5·5	—	11
,, 6	50	1	—	1	—	—	—	1	0·5	—	—	0·5	—	—
,, 7	51 to 53	3	3	3	—	—	—	3	1·5	—	—	1·5	—	3
,, ,,	54	1	1	1	1	—	—	1	0·5	1	—	0·5	—	1
,, 8	55	1	—	1	—	—	—	1	0·5	—	—	0·5	—	—
,, ,,	56	1	1	1	1	—	—	1	0·5	1	—	0·5	—	1
,, 9	57	1	1	1	—	—	—	1	0·5	—	—	0·5	—	1
,, ,,	58 to 61	4	4	4	4	—	—	4	2	4	—	2	—	4
,, ,,	62	1	—	—	—	—	—	—	0·5	—	—	0·5	—	—
,, 11	63 to 66	4	4	4	4	—	—	4	2	4	—	2	—	4
,, 12	67	1	1	1	—	—	—	1	0·5	—	—	0·5	—	1
,, ,,	68 to 69	2	2	2	2	—	—	2	1	2	—	1	—	2
,, ,,	70	1	—	1	—	—	—	1	0·5	—	—	0·5	—	—
,, ,,	71	1	1	1	—	—	—	1	0·5	—	—	0·5	—	1
,, 13	72 to 75	4	4	4	4	—	—	4	2	4	—	2	—	4
,, 14	76 to 79	4	4	4	4	—	—	4	2	2	—	4	—	4
,, ,,	80	1	1	1	1	1	—	1	0·5	0·5	1	1	—	1
,, 15	81	1	1	1	1	—	—	1	0·5	0·5	—	1	—	1
,, ,,	82 to 92	10	10	10	10	10	10	10	5	5	10	10	10	10
,, 21	93 to 104	12	12	12	12	12	12	12	12	6	12	12	6	12
,, 25	105	1	1	1	—	—	1	1	1	0·5	1	—	—	0·5
August 10	106 to 264	159	159	159	159	159	159	159	159	79·5	159	159	159	79·5
,, 22	265 to 344	80	80	80	80	80	80	80	80	80	40	80	80	40
,, ,,	345 and 346	2	2	2	2	—	2	2	2	2	1	2	—	1
,, 24	347	1	1	—	—	—	1	1	1	—	0·5	—	—	0·5
,, ,,	348 and 349	2	2	2	—	—	2	2	2	2	1	—	—	1
,, ,,	350	1	1	—	—	—	1	1	1	—	0·5	—	—	0·5
,, ,,	351	1	1	1	—	—	1	1	1	1	0·5	—	—	0·5
,, ,,	352 and 353	2	2	2	2	2	2	2	2	2	1	2	2	1
,, 25	354	1	1	1	1	—	1	1	1	1	0·5	1	—	0·5
,, ,,	355	1	1	1	—	—	1	1	1	1	0·5	—	—	0·5
,, ,,	356	1	1	—	—	—	1	1	1	—	0·5	—	—	0·5
,, ,,	357	1	1	1	—	—	1	1	1	1	0·5	—	—	0·5
,, ,,	358 to 383	27	27	27	27	27	27	27	27	27	13·5	27	27	13·5
,, ,,	384 to 385[a]	3	3	3	—	—	3	3	3	3	1·5	—	—	1·5
,, 30	386 to 389	4	4	—	—	—	4	4	4	—	2	—	—	2
September 1	390 to 405	16	16	16	16	16	16	16	16	16	8	16	16	8
,, ,,	406	1	1	1	—	—	1	1	1	1	0·5	—	—	0·5
,, ,,	407 and 408	2	2	2	2	2	2	2	2	2	1	2	2	1
,, 3	409	1	1	1	—	—	1	1	1	1	0·5	—	—	0·5
,, ,,	410 to 414	5	5	5	5	5	5	5	5	5	2·5	5	5	2·5
,, 4	415	1	1	1	—	—	1	1	1	1	0·5	—	—	0·5
,, ,,	416 to 418	3	3	3	3	—	3	3	3	3	1·5	3	—	1·5
,, ,,	419	1	1	1	1	1	1	1	1	1	0·5	1	1	0·5

Abstract of the Bars and Microscopes in the Base Line Measurement on Salisbury Plain—*continued.*

DATE.	SETS.	BARS.						MICROSCOPES.						
		A	B	C	D	E	G	M	N	O	P	Q	R	S
September 4	420 and 421	2	2	2	2	—	2	2	2	2	1	2	—	1
" "	422	1	1	1	—	—	1	1	1	1	0·5	—	—	0·5
" 5 to Oct. 5	423 to 565	144	144	144	144	144	144	144	144	144	72	144	144	72
October 6	566 to 591	26	—	26	26	26	26	—	26	26	13	26	26	13
" 10	592 to 606	16	16	16	16	16	16	16	16	16	8	16	16	8
" "	607	1	1	1	—	—	1	1	1	1	0·5	—	—	0·5
" "	608	1	1	—	—	—	1	1	1	—	0·5	—	—	0·5
" "	609	1	1	1	—	—	1	1	1	1	0·5	—	—	0·5
Oct. 12 to 16	610 to 635	26	26	26	26	26	26	26	26	26	13	26	26	13
" 16	636	1	1	1	—	—	1	1	1	1	0·5	—	—	0·5
" "	637	1	1	1	1	1	1	1	1	1	0·5	1	1	0·5
" "	638	1	1	1	1	—	1	1	1	1	0·5	1	—	0·5
		641	595	602	558	528	560	596	597·5	480	372	565	521	352·5

Summation of the Measurement.

Between May 23d and August 2d we have, putting S for a complete set

$$176\,S + 82\,A + 62\,B + 51\,C + 22\,D + E + G.$$

Where the values of the bars are

$$A = O_1 + 6{\cdot}40\,d. \qquad D = O_1 + 16{\cdot}30\,d.$$
$$B = O_1 + 64{\cdot}35\,d. \qquad E = O_1 + 54{\cdot}50\,d.$$
$$C = O_1 + 66{\cdot}65\,d. \qquad G = O_1 + 4{\cdot}35\,d.$$

Between August 10th and September 18th

$$209\,S + 27\,A + 27\,B + 20\,C + 8\,D + 27\,G.$$

Where the values of the bars are

$$A = O_1 + 5{\cdot}00\,d. \qquad D = O_1 + 19{\cdot}20\,d.$$
$$B = O_1 + 71{\cdot}85\,d. \qquad E = O_1 + 57{\cdot}15\,d.$$
$$C = O_1 + 69{\cdot}55\,d. \qquad G = O_1 - 0{\cdot}60\,d.$$

Between September 21st and October 18th

$$116\,S + 31\,A + 5\,B + 30\,C + 27\,D + 26\,E + 31\,G.$$

Where the values of the bars are

$$A = O_1 - 1{\cdot}55\,d. \qquad D = O_1 + 18{\cdot}30\,d.$$
$$B = O_1 + 78{\cdot}00\,d. \qquad E = O_1 + 56{\cdot}90\,d.$$
$$C = O_1 + 67{\cdot}85\,d. \qquad G = O_1 - 0{\cdot}45\,d.$$

So that the whole distance measured by the bars is equal to

$$3484\,O_1 + 123956\,d.$$

And the value of one division of the micrometer is $\frac{1}{12040}$ of an inch, so that the above distance is equal to

$$34840 \cdot 8579 \text{ feet of } O_1.$$

The summation of the distance measured by the microscopes is

$$3484 \text{ (6-inch Standard)} + 101\, d.$$

The micrometer divisions are equal $\frac{1}{9102 \cdot 66}$ of an inch; so that the above is equal to

$$1742 \cdot 0009 \text{ feet of 6-inch Standard,}$$

which, converted into feet of the standard O_1 by the factor ·99995550, given at page 101 of the "Measurement of the Lough Foyle Base," is

$$1741 \cdot 9234 \text{ feet of } O_1.$$

The whole measured distance is therefore 36582·7813 feet. From this must be deducted 4·2938 feet measured *back* to the extremity of the base from the point carrier at the extremity of the 3484th bar. This measure was carefully made with a beam compass.

Reduction to Mean Level of the Sea.

(21.) If h_n be the height of any bar above the mean level of the sea, the correction to it is—$10 \cdot 5 \frac{h_n}{R}$; so that the whole reduction to the mean level of the sea is

$$- 10 \cdot 5 \frac{\Sigma (h_n)}{R} \text{ feet.}$$

The quantity $\Sigma (h_n)$ is obtained from the levelled heights and the recorded differences of heights of the different sets of bars; it is equal to 1254220·38 feet. The logarithm of the radius of curvature for the mean latitude (51°· 8′· 21″) and mean bearing (28°· 29′· 16″) of the base is 7·3206329; hence, the reduction to the level of the sea is —·6294 of a foot.

The following Table contains the height above the mean level of the sea of each set of bars used in the measurement.

Abstract of the Heights of the Successive Parts of the Line above Mean Level of the Sea.

No. of Set.	Height.	No. of Set.	Height.	No. of Set.	Height.	No. of Set.	Height.	No. of Set.	Height.	No. of Set.	Height.	No. of Set.	Height.	No. of Set.	Height.	No. of Set.	Height.
1	671·01	73	489·05	146	442·89	218	380·47	290	370·80	361	332·86	432	349·94	504	323·52	575	233·41
2	668·44	74	485·96	147	440·67	219	380·95	291	369·96	362	333·86	433	350·60	505	322·55	576	232·47
3	666·14	75	482·82	148	438·89	220	381·52	292	369·97	363	333·68	434	350·47	506	321·49	577	230·83
4	663·24	76	479·59	149	437·00	221	382·08	293	370·23	364	333·71	435	350·42	507	320·99	578	230·79
5	660·06	77	476·52	150	435·13	222	383·66	294	371·53	365	333·69	436	350·18	508	320·03	579	230·05
6	657·00	78	473·37	151	433·68	223	384·12	295	372·62	366	332·83	437	350·06	509	319·10	580	229·74
7	654·50	79	470·20	152	432·07	224	384·42	296	374·02	367	332·58	438	349·87	510	318·30	581	229·02
8	651·97	80	467·96	153	430·28	225	385·54	297	375·99	368	332·56	439	349·18	511	317·96	582	228·10
9	649·11	81	464·93	154	428·95	226	386·16	298	376·83	369	331·66	440	347·64	512	317·52	583	227·76
10	645·98	82	460·93	155	427·39	227	386·33	299	376·71	370	330·80	441	346·10	513	317·67	584	227·42
11	643·27	84	458·97	156	425·96	228	386·86	300	376·72	371	329·94	442	345·27	514	317·16	585	226·53
12	641·33	85	456·45	157	423·50	229	386·94	301	376·57	372	329·38	443	343·71	515	316·63	586	226·45
13	639·33	86	456·94	158	421·32	230	387·08	302	376·63	373	328·81	444	342·81	516	316·17	587	226·39
14	636·60	87	456·43	159	419·14	231	387·24	303	376·56	374	328·27	445	340·47	517	315·28	588	225·57
15	634·43	88	453·29	160	417·70	232	387·28	304	374·72	375	327·73	446	338·94	518	314·32	589	225·65
16	632·64	89	453·82	161	416·40	233	386·91	305	374·07	376	326·05	447	337·31	519	312·42	590	225·42
17	630·61	90	454·30	162	413·89	234	386·83	306	373·18	377	325·43	448	335·76	520	312·36	591	226·03
18	628·55	91	454·18	163	412·30	235	387·26	307	371·99	378	324·89	449	333·67	521	311·88	592	226·74
19	626·91	92	453·55	164	410·72	236	387·66	308	369·76	379	323·25	450	331·36	522	310·99	593	227·78
20	624·52	93	432·31	165	409·33	237	388·35	309	368·02	380	321·71	451	329·21	523	309·31	594	229·32
21	624·92	94	453·70	166	408·71	238	389·07	310	367·13	381	320·86	452	328·27	524	308·37	595	230·30
22	624·17	95	453·60	167	407·08	239	389·04	311	366·19	382	318·49	453	327·38	525	307·44	596	231·43
23	621·29	96	452·35	168	405·05	240	389·56	312	365·30	383	316·36	454	327·69	526	305·99	597	232·52
24	618·62	97	451·43	169	403·54	241	390·61	313	364·00	384	313·22	455	327·98	527	304·40	598	234·06
25	616·34	98	450·77	170	402·74	242	391·64	314	363·29	385	309·98	456	328·76	528	303·69	599	233·97
26	614·44	99	450·09	171	401·70	243	392·01	315	362.66	385*a*	306·59	457	330·37	529	302·65	600	233·63
27	612·15	100	449·62	172	400·83	244	392·67	316	362·35	386	303·26	458	330·90	530	300·90	601	234·04
28	610·34	101	449·39	173	398·25	245	393·64	317	361·46	387	300·06	459	331·25	531	299·89	602	233·28
29	608·13	102	448·76	174	396·47	246	394·74	318	361·16	388	296·77	460	331·30	532	297·71	603	231·69
30	606·06	103	447·63	175	393·50	247	395·80	319	360·26	389	293·78	461	330·46	533	295·67	604	229·54
31	603·35	104	449·86	176	393·97	248	396·83	320	360·03	390	290·44	462	329·61	534	293·78	605	227·92
32	600·60	105	451·83	177	393·04	249	397·75	321	359·44	391	287·55	463	328·76	534	292·24	605*a*	227·59
33	597·56	106	455·23	178	391·42	250	398·62	322	358·81	392	287·03	464	328·25	535	290·36	606	229·56
34	594·90	107	457·10	179	389·72	251	399·32	323	357·95	393	287·42	465	327·39	536	288·12	607	232·14
35	592·16	108	457·79	180	388·82	252	399·89	324	357·68	394	288·68	466	327·30	537	286·52	608	234·64
36	589·57	109	458·57	181	387·93	253	400·32	325	356·85	395	289·57	467	326·86	538	284·41	609	237·61
37	586·78	110	459·82	182	387·25	254	400·60	326	356·14	396	290·46	468	325·84	539	282·17	610	240·32
38	584·18	111	459·49	183	385·78	255	400·71	327	355·63	397	290·95	469	324·89	540	280·59	611	241·93
39	583·12	112	460·06	184	384·32	256	400·33	328	355·59	398	290·85	470	325·54	541	279·64	612	244·42
40	580·46	113	460·44	185	382·71	257	400·07	329	355·09	399	290·31	471	325·90	542	277·77	613	245·56
41	578·12	114	460·09	186	382·23	258	403·03	330	354·24	400	288·63	472	326·51	543	275·81	614	246·77
42	575·60	115	460·25	187	380·72	259	402·63	331	353·35	401	286·71	473	327·78	544	274·74	615	248·10
43	573·26	116	459·40	188	379·05	260	402·17	332	352·49	402	284·18	474	329·71	545	272·47	616	249·15
44	570·86	117	458·85	189	378·10	261	401·92	333	351·55	403	281·57	475	331·05	546	270·54	617	249·79
45	568·01	118	458·84	190	376·82	262	401·55	334	350·99	404	278·72	476	331·95	547	268·78	618	250·16
46	565·33	119	458·39	191	376·37	263	399·93	335	350·10	405	280·07	477	333·41	548	266·29	619	250·97
47	563·03	120	458·51	192	376·17	264	398·69	336	348·55	406	282·48	478	335·16	549	263·85	620	251·56
48	560·50	121	459·48	193	376·34	265	397·25	337	347·61	407	285·96	479	336·62	550	263·06	621	251·56
49	557·54	122	461·24	194	375·79	266	396·87	338	346·67	408	288·81	480	339·35	551	262·82	622	252·17
50	554·64	123	462·72	195	376·07	267	395·60	339	344·89	409	291·94	481	342·31	552	262·35	623	251·56
51	551·94	124	463·45	196	377·25	268	393·52	340	343·11	410	295·15	482	343·52	553	263·21	624	251·84
52	550·73	125	464·37	197	378·62	269	392·22	341	341·52	411	298·11	483	344·23	554	263·17	625	251·85
53	547·92	126	464·90	198	379·39	270	390·51	342	339·53	412	301·58	484	345·46	555	262·64	626	251·74
54	545·06	127	465·28	199	380·11	271	388·53	343	337·62	413	304·69	485	346·78	556	261·90	627	251·15
55	542·58	128	465·51	200	380·74	272	387·64	344	335·00	414	307·65	486	347·35	557	261·14	628	250·75
56	539·76	129	465·65	201	381·10	273	387·66	345	331·66	415	311·06	487	347·23	558	260·36	629	249·36
57	536·99	130	465·45	202	381·45	274	386·35	346	328·12	416	313·78	488	347·83	559	258·67	630	248·57
58	534·91	131	465·21	203	381·76	275	385·84	347	324·62	417	316·79	489	348·29	560	256·95	631	247·63
59	531·22	132	464·87	204	382·11	276	385·68	348	321·38	418	319·47	490	347·70	561	255·25	632	245·69
60	528·24	133	464·48	205	382·29	277	385·49	349	317·99	419	322·48	491	346·72	562	254·26	633	243·43
61	525·26	134	463·70	206	382·54	278	384·46	350	314·56	420	325 41	492	344·61	563	252·38	634	243·09
62	522·26	135	462·93	207	382·86	279	383·57	351	311·24	421	328·61	493	344·26	564	251·55	635	243·95
63	520·38	136	461·01	208	382·38	280	382·80	352	309·20	422	331·91	494	343·04	565	249·45	636	244.57
64	517·13	137	459·54	209	381·80	281	382·47	353	310·09	423	335·27	495	341·24	566	247·83	637	243·01
65	514·03	138	459·81	210	382·00	282	381·23	354	313·37	424	337·99	496	339·88	567	246·17	638	244.29
66	510·94	139	455·91	211	381·32	283	380·82	355	316·78	425	340·23	497	338·21	568	244·23		
67	507·57	140	454·19	212	380·62	284	379·23	356	320·04	426	342·41	498	335·47	569	242·30		
68	504.86	141	452·53	213	380·31	285	378·77	357	323·38	427	345·42	499	332·83	570	241·29		
69	501·28	142	450·86	214	380·18	286	376·98	358	326·55	428	347·02	500	330·34	571	239·75		
70	498·03	143	449·17	215	380·05	287	375·45	358*a*	329·17	429	348·23	501	327·51	572	238·21		
71	495·26	144	446·68	216	380·13	288	374·55	359	330·73	430	348·81	502	325·40	573	236·43		
72	492·96	145	444·92	217	380·11	289	372·84	360	331·94	431	349·53	503	324·45	574	234·43		

Mean Height = 360·00 feet.

The final length of the line stands therefore thus:

SALISBURY PLAIN BASE.

1849.

Measured with Compensation Bars		34840·8579
Measured with Compensation Microscopes		1741·9234
Measured back with Beam Compass	—	4·2938
Reduction to Level of Sea	—	0·6294
Length of Base Line in feet of O_1		36577·8581

Log = 4·5632182708

SECTION VI.

PRINCIPLES OF CALCULATION.

In a triangulation in which the distances are short and the triangles very small, the necessary calculations can be made with very little labour, as the terms of the formulæ employed depending on the higher powers of the distances become insensible; but when, as in the triangulation of Great Britain and Ireland, the distances are long, often approaching to and sometimes exceeding 100 miles, it becomes necessary to examine into the sufficiency of the formulæ employed, and to make sure that the small terms neglected cannot lead to any final defect. The present section contains an exposition of those principles upon which all the methods of calculation are based.

§ I.

1. Let a and b be the semi-axes of the spheroid, so that the equation of the meridian is,

$$u = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0,$$

x and y being the distances of any point in the meridian from the polar axis and from the plane of the equator; then, if λ be the latitude of the point $x\,y$,

$$\frac{du}{dx} = \left(\frac{du^2}{dx^2} + \frac{du^2}{dy^2}\right)^{\frac{1}{2}} \cos\lambda \qquad \frac{du}{dy} = \left(\frac{du^2}{dx^2} + \frac{du^2}{dy^2}\right)^{\frac{1}{2}} \sin\lambda$$

$$\therefore\ \frac{x}{a^2} = \left(\frac{x^2}{a^4} + \frac{y^2}{b^4}\right)^{\frac{1}{2}} \cos\lambda \qquad \frac{y}{b^2} = \left(\frac{x^2}{a^4} + \frac{y^2}{b^4}\right)^{\frac{1}{2}} \sin\lambda$$

Multiply these last equations by a and b, and by x and y, and add:

$$a^2 \cos^2\lambda + b^2 \sin^2\lambda = \left(\frac{x^2}{a^4} + \frac{y^2}{b^4}\right)^{-1} = (x\cos\lambda + y\sin\lambda)^2$$

and by substitution in the preceding,

$$x = \frac{a^2\cos\lambda}{(a^2\cos^2\lambda + b^2\sin^2\lambda)^{\frac{1}{2}}} \qquad y = \frac{b^2\sin\lambda}{(a^2\cos^2\lambda + b^2\sin^2\lambda)^{\frac{1}{2}}}$$

or, if we make use of the excentricity, and put $1 - e^2\sin^2\lambda = \Delta^2$, then the values of x and y are,

$$x = \frac{a}{\Delta}\cos\lambda \qquad y = \frac{a}{\Delta}(1 - e^2)\sin\lambda \qquad (1)$$

If ϱ be the radius of curvature of the meridian at the same point, then $\varrho\, d\lambda = d\, s$, s being the length of the meridian curve, and we have

$$\frac{dx}{d\lambda} = -\varrho \sin\lambda \qquad \frac{dy}{d\lambda} = \varrho \cos\lambda$$

multiplying these equations by $-y$ and x, and adding

$$\frac{x\dfrac{dy}{d\lambda} - y\dfrac{dx}{d\lambda}}{x\cos\lambda + y\sin\lambda} = \varrho = \frac{x^2\dfrac{d}{d\lambda}\left(\dfrac{y}{x}\right)}{(a^2\cos^2\lambda + b^2\sin^2\lambda)^{\frac{1}{2}}}$$

But expressing the ratio of x to y, we find

$$\frac{d}{d\lambda}\left(\frac{y}{x}\right) = \frac{b^2}{a^2}\sec^2\lambda$$

$$\therefore \quad \varrho = \frac{a^2 b^2}{(a^2\cos^2\lambda + b^2\sin^2\lambda)^{\frac{3}{2}}} = \frac{a(1-e^2)}{\Delta^{\frac{3}{2}}} \tag{2}$$

The logarithm of Δ, which is a quantity of frequent occurrence, may be obtained as follows: if we put

$$n = \frac{a-b}{a+b}$$

then it follows that

$$(1+n)\Delta = (1 + n^2 + 2n\cos 2\lambda)^{\frac{1}{2}} \tag{3}$$

$$= (1 + n\varepsilon^{2\lambda\sqrt{-1}})^{\frac{1}{2}}(1 + n\varepsilon^{-2\lambda\sqrt{-1}})^{\frac{1}{2}}$$

$$\log(1+n)\Delta = \frac{M}{2}\left\{n(\varepsilon^{2\lambda\sqrt{-1}} + \varepsilon^{-2\lambda\sqrt{-1}}) - \frac{n^2}{2}(\varepsilon^{4\lambda\sqrt{-1}} + \varepsilon^{-4\lambda\sqrt{-1}}) + \ldots\right\}$$

$$\therefore \quad \log\Delta = -\log(1+n) + M\left(n\cos 2\lambda - \frac{n^2}{2}\cos 4\lambda + \ldots\right) \tag{4}$$

the term of this series in n^3 can only affect the tenth figure of the logarithm, and is consequently not required.

The portion of the normal to the curve at the point $x\,y$, intercepted between that point and the minor axis, is evidently equal to $x\sec\lambda$, so that if we call this quantity N,

$$N = \frac{a}{\Delta}$$

2. If the ordinate of the point $x\,y$ be produced to meet a circle described upon the major axis of the ellipse as diameter, and a straight line be drawn joining this point of intersection, with the centre of the ellipse, the inclination of this line to the major axis is called the reduced latitude; and if we designate it by u, the following relations exist between u and λ:—

$$\left.\begin{aligned} (1-e^2)^{\frac{1}{2}}\sin\lambda &= \Delta\sin u \\ \cos\lambda &= \Delta\cos u \\ \Delta_1\Delta &= (1-e^2)^{\frac{1}{2}} \end{aligned}\right\} \tag{5}$$

Where Δ_1 is the same function of u that Δ is of λ.

§ II.

On the mutual relation of two points on the surface of a spheroid.

3. Let α, α' be the reciprocal azimuths* of two points P Q on a spheroid, k the chord line joining them, or their rectilinear distance; μ, μ', the angles made by the chord with the normals of P Q, so that $90° - \mu$, $90° - \mu'$, are the mutual depressions of these points, λ, λ', ω, their latitudes and difference of longitude, and

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} - 1 = 0$$

the equation of the surface: then if the plane of $x z$ be made to pass through P, the co-ordinates of P and Q will be

$$x = \frac{a}{\Delta} \cos \lambda \qquad y = 0 \qquad z = \frac{a}{\Delta} (1 - e^2) \sin \lambda$$

$$x' = \frac{a}{\Delta} \cos \lambda' \cos \omega \qquad y' = \frac{a}{\Delta} \cos \lambda' \sin \omega \qquad z' = \frac{a}{\Delta} (1 - e^2) \sin \lambda'$$

Let now $f\,g\,h$ be the direction-cosines of the normal to that plane which contains the line of the vertical at P and passes through Q, and whose inclination to the meridian of P is α; let also $l\,m\,n$, $l'\,m'\,n'$, be the direction-cosines of the normal at P, and of the tangent to the surface at P, which is contained in the plane passing through Q: then since the first line $(f\,g\,h)$ is perpendicular to each of the other two, and to the chord line k whose direction-cosines are proportional to $x' - x$, $y' - y$, $z' - z$, we have these three equations:—

$$\begin{aligned} f(x'-x) + gy' + h(z'-z) &= 0 \\ fl + gm + hn &= 0 \\ fl' + gm' + hn' &= 0 \end{aligned}$$

Now if we eliminate $f\,g\,h$ from these equations and substitute the following values,

$$l = \cos \lambda \qquad m = 0 \qquad n = \sin \lambda$$

$$l' = - \sin \lambda \cos \alpha \qquad m' = \sin \alpha \qquad n' = \cos \alpha \cos \lambda$$

there will result

$$(x'-x) \sin \lambda \sin \alpha + y' \cos \alpha - (z'-z) \cos \lambda \sin \alpha = 0 \quad .$$

The substitution of the values of $x\,z$, $x'\,y'\,z'$, in this equation will give immediately the value of $\cot \alpha$; and if we put $\zeta\,\zeta'$, for the corresponding azimuths on a sphere, or on the supposition $e = 0$, the following equations will exist:—

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* In the figure page (236) let PM, QN, be the normals to the surface at the points P, Q, meeting the axis of revolution OC in M, N. Join PQ; draw MH, NK parallel respectively to QN, PM, and join PH, QK. Then α, the azimuth of Q at P, is the inclination of the planes PMQ, PMO, and α', the azimuth of P at Q, is the inclination of the planes QNK, QNO. The inclination of the planes PMH, PMO, is the azimuth ζ in equation (6); the inclination of the planes QNH, QNO, is equal to ζ' in the same set of equations. The angle QPM $= \mu$, PMN $= \mu'$.

$$\left.\begin{aligned}\cot\alpha - \cot\zeta &= e^2\frac{\cos\lambda}{\cos\lambda'}\cdot\frac{Q}{\Delta}\\ \cot\alpha' - \cot\zeta' &= -e^2\frac{\cos\lambda'}{\cos\lambda}\,\frac{Q}{\Delta'}\\ \Delta'\sin\lambda - \Delta\sin\lambda' &= \sin\omega\cdot Q\end{aligned}\right\}\qquad(6)$$

4. These azimuths* may also be obtained in the following manner: from Q let fall a perpendicular upon the meridian plane of P, and from P let fall a perpendicular upon the meridian plane of Q; then the following equations will become evident:—

$$\left.\begin{aligned}h\sin\mu\sin\alpha &= \frac{a}{\Delta'}\cos\lambda'\sin\omega\\ h\sin\mu'\sin\alpha' &= \frac{a}{\Delta}\cos\lambda\sin\omega\end{aligned}\right\}\qquad(7)$$

Now in any surface, $u = 0$, we have

$$h^2 = (x'-x)^2 + (y'-y)^2 + (z'-z)^2$$

$$-\cos\mu = \frac{(x'-x)\frac{du}{dx} + (y'-y)\frac{du}{dy} + (z'-z)\frac{du}{dz}}{h\left(\frac{du^2}{dx^2} + \frac{du^2}{dy^2} + \frac{du^2}{dz^2}\right)^{\frac{1}{2}}}$$

$$\cos\mu' = \frac{(x'-x)\frac{du}{dx'} + (y'-y)\frac{du}{dy'} + (z'-z)\frac{du}{dz'}}{h\left(\frac{du^2}{dx'^2} + \frac{du^2}{dy'^2} + \frac{du^2}{dz'^2}\right)^{\frac{1}{2}}}$$

In the present instance therefore, if we put $1 - \frac{xx'}{a^2} - \frac{zz'}{b^2} = U$,

$$\frac{h^2}{a^2} = 2U - e^2\left(\frac{z'-z}{b}\right)^2$$

$$\cos\mu = \frac{a}{h}\Delta U$$

$$\cos\mu' = \frac{a}{h}\Delta' U$$

---

* From the convexity of the earth, the supposition of two points on the surface being mutually visible is imaginary. If, instead of Q, a point S vertically above it, at the height h, be observed at P, then the influence of this upon the observed azimuth may be thus ascertained:—Join S with M, the foot of the normal at P (see figure, page 236). This line, being sensibly parallel to QM, will intersect the meridian of Q at the distance $h \times \sin$ MQN from Q; but if s, α, be the length and azimuth of the line, λ the mean latitude, $\sin \mathrm{MQN} = e^2\frac{s}{a}\cos\alpha\cos^2\lambda$. The influence of this small meridian distance on the azimuth at P is equal $h \times \sin \mathrm{MQN} \times \sin\alpha \div s$, or $= \frac{1}{2}e^2\frac{h}{a}\sin 2\alpha\cos^2\lambda$. This quantity is very small; in the latitude of Great Britain its maximum amount is about an eightieth of a second for every thousand feet of height.

By putting v* for the third side of a spherical triangle, of which two sides are $90° - u$, $90° - u'$, and the included angle ω, and using a subsidiary angle ϕ, such that

$$\sin\phi \sin\frac{v}{2} = e \sin\frac{u' - u}{2} \cos\frac{u' + u}{2}$$

we obtain from the preceding three equations and (7) the following set,

$$\left.\begin{aligned} k &= 2\, a \sin\frac{v}{2} \cos\phi \\ \cos\mu &= \Delta \sin\frac{v}{2} \sec\phi \\ \cos\mu' &= \Delta' \sin\frac{v}{2} \sec\phi \\ \sin\mu \sin\alpha &= \frac{a}{k} \cos u' \sin\omega \\ \sin\mu' \sin\alpha' &= \frac{a}{k} \cos u \sin\omega \end{aligned}\right\} \qquad (8)$$

These equations determine the distance, mutual depressions and azimuths of any two points on the surface of a spheroid.†

5. Hitherto we have considered only the chord or rectilinear distance of P and Q, we proceed now to obtain the distance as measured along either of the curves formed by the intersection of the vertical planes at P and Q with the surface. If we take the curve formed by the plane vertical at P, and measure x along the tangent to the curve at P, and y parallel to the vertical line at P, then the equation of the curve is

$$x^2\left(1 + \frac{e^2}{1 - e^2}\cos^2\lambda \cos^2\alpha\right) - 2\,xy\,\frac{e^2}{1 - e^2}\sin\lambda\cos\lambda\cos\alpha + y^2\left(1 + \frac{e^2}{1 - e^2}\sin^2\lambda\right) - 2\,\frac{a}{\Delta}\,y = 0$$

---

* By substituting the values of x, x', z, z', in terms of u and u' in the value of U,

$$U = 1 - \cos u \cos u' \cos\omega - \sin u \sin u'$$
$$= 1 - \cos v = 2 \sin^2 \frac{1}{2} v$$

† The direct distance between two points on a spheroid whose latitudes and difference of longitude are $\lambda, \lambda', \omega$, is most readily calculated by the following formula :—Put

$$Q^2 = \cos\lambda \cos\lambda' \sin_2 \frac{\omega}{2}$$

$$\tan\phi = \frac{1 - e^2}{Q\,\Delta^2_0} \sin\frac{\lambda' - \lambda}{2}$$

then

$$\frac{k}{a} = \frac{2\,Q}{(\Delta\Delta')^{\frac{1}{2}}} \sec\phi$$

which is very convenient for calculation when tables of log Δ are at hand, and is true to 9 places of decimals for a distance of more than 2°. Δ_0 is the value of Δ corresponding to the mean latitude of the two stations.

From this equation we may deduce the value of the radius of curvature of the vertical section at the point P, for it is the limit of the ratio $\frac{x^2}{2y}$ as x and y become zero; therefore if R be this radius,

$$\frac{1}{R} = \frac{\Delta}{a}\left(1 + \frac{e^2}{1 - e^2}\cos^2\alpha\cos^2\lambda\right) \qquad (9)$$

which gives the curvature of the surface for any latitude and azimuth.

. . .

We may write the equation of the curve in the form

$$r + f\sin^2\theta \cdot r + h\sin 2\theta \cdot r - g\sin\theta = 0$$

and then assuming

$$\theta = \alpha r + \beta r^2 + \gamma r^3 + \quad \cdots\cdot$$

we shall have, by forming the values of $\sin\theta$, $\sin 2\theta$, $\sin^2\theta$, and substituting them in the equation of the curve,

$$\alpha = \frac{1}{g} \;; \qquad \beta = \frac{2h}{g^2}$$

$$\gamma = \frac{1 + 6f + 24h^2}{6g^3}$$

Also, since $ds^2 = dr^2 + r^2\,d\theta^2$, we have $\frac{ds}{dr} = 1 + \frac{1}{2}\left(r\frac{d\theta}{dr}\right)^2 - \frac{1}{8}\left(r\frac{d\theta}{dr}\right)^4$

$$s = r + \int_0^r\left(\frac{1}{2}\left(r\frac{d\theta}{dr}\right)^2 - \frac{1}{8}\left(r\frac{d\theta}{dr}\right)^4\right)$$

$$r\frac{d\theta}{dr} = \alpha r + 2\beta r^2 + 3\gamma r^3 + \cdots\cdot$$

Substituting this last equation in the preceding, we obtain s in terms of $\alpha\,\beta\,\gamma\,\cdots$; and finally, by substituting for $\alpha\,\beta\,\gamma$ their values given above,

$$s = r + \frac{r^3}{6g^2} + \frac{3r^5}{40g^4} + \left(h + \frac{3f}{5}\frac{r}{g}\right)\frac{r^4}{g^3}$$

$$s = k + \frac{k^3}{24R^2} + \frac{3k^5}{640R^4} - \frac{e^2}{16}\frac{k^4}{R^3}\cos\alpha\sin 2\lambda + \cdots\cdots \qquad (10)$$

in which last equation k is the chord P Q, and s the length of the elliptical arc. Similarly, if s' be the length of the elliptic curve formed by the plane which is vertical at Q and passes through P, λ' the latitude of Q, and $180° - \alpha'$ the azimuth of P at Q,

$$s' = k + \frac{k^3}{24R'^2} + \frac{3k^5}{640R'^4} + \frac{e^2}{16}\frac{k^4}{R'^3}\cos\alpha'\sin 2\lambda' + \cdots\cdots \qquad (11)$$

these curves may be taken as equal in length, differing only from the *shortest* line by a quantity quite imperceptible for arcs of only a small number of degrees in length.

6. The term in $e^2 k^4$ in either of these series is very small, it may amount to 2 feet in 500 miles: we may therefore reject both the terms in e^2, and consider the remainder of the expression in either equation as the true length of the curve. The term in $e^2 k^4$ will, however, disappear by adding the two equations, and if R_0 be the radius of curvature of the surface for the mean latitude $\lambda_0 = \frac{1}{2}(\lambda + \lambda')$, and the mean azimuth $\alpha_0 = \frac{1}{2}(\alpha + \alpha')$, the length of the curve will be

$$s = k + \frac{k^3}{24 R_0^2} + \frac{3 k^5 (1 + ne^2)}{640 R_0^4} + \cdots \tag{12}$$

Where n is a function of α and λ. As in every case of actual calculation, we may put $n = 0$, it follows that

$$\log\left(\frac{s}{k}\right) = \frac{M}{24}\left(\frac{k}{R_0}\right)^2 \left\{1 + \frac{11}{120}\left(\frac{k}{R_0}\right)^2\right\} \tag{13}$$

$$\log\frac{M}{24} = \bar{2}\cdot2575731$$

7. We shall illustrate these formulæ in the calculation of an exact spheroidal triangle which may be useful in the examination of approximate methods of calculation:—Let it be required to find the relative distances, azimuths, and depressions of three points A B C, given by their astronomical positions as follows:—

$\lambda_1 = 52^\circ\ 0'$ $\omega_1 = 0^\circ\ 0'$
$\lambda_2 = 52^\circ\ 30'$ $\omega_2 = 3^\circ\ 15'$
$\lambda_3 = 53^\circ\ 30'$ $\omega_3 = 4^\circ\ 30'$

Let the chords $AB = k_3$, $BC = k_1$, $CA = k_2$, the corresponding curve distances being c, a, b. Let the azimuth of $AB = \gamma$, of $BA = \gamma'$, of $BC = \alpha$, of $CB = \alpha'$, of $CA = \beta$, of $AC = \beta'$. Let also the complements of the depressions (μ), reckoned in the same order as the azimuths, be $\mu_1\ \mu'_1\ \mu_2\ \mu'_2\ \mu_3\ \mu'_3$.

Let the semi-axes of the surface be 20923713 and 20853810 feet; then we have

$\log a = 7\cdot32063887544$; $\log b = 7\cdot31918854123$; $\log c = 8\cdot9120804964$
$\log \Delta_1 = 9\cdot9990986767$; $\log \Delta_2 = 9\cdot9990863878$; $\log \Delta_3 = 9\cdot9990619836$.

If u_1, u_2, u_3, be the reduced latitudes, then, since $a \tan\lambda = b \tan u$, we find,

$u_1 = 51^\circ\ 54'\ 24''\cdot9900$ $u_2 = 52^\circ\ 24'\ 26''\cdot4893$ $u_3 = 53^\circ\ 24'\ 29''\cdot7927$

and if v_1 be the third side of a spherical triangle, of which two sides are $90^\circ - u_2$ and $90^\circ - u_3$, including the angle $\omega_2 - \omega_3$; v_2, v_3 having a similar meaning, we shall find by spherical trigonometry

$\log \sin\frac{v_1}{2} = 8\cdot0387964746$ $\log \sin\frac{v_2}{2} = 8\cdot4341719716$ $\log \sin\frac{v_3}{2} = 8\cdot2537431052$

The subsidiary angles ϕ equation (8) are next found, giving

$\log\cos\phi_1 = 9\cdot9996635530$ $\log\cos\phi_2 = 9\cdot9998760943$ $\log\cos\phi_3 = 9\cdot9999676821$

The rectilinear distances of the three points are now derived immediately from the equation

$$k = 2a \sin \frac{v}{2} \cos \phi$$

adding the corresponding logarithms, there results

$$\log k_1 = 5{\cdot}6601287778 \qquad \log k_2 = 6{\cdot}0557168159 \qquad \log k_3 = 5{\cdot}8754027338$$

Also from the equations

$$\cos \mu = \Delta \sin \frac{v}{2} \sec \phi : \qquad \cos \mu' = \Delta' \sin \frac{v}{2} \sec \phi$$

by adding the corresponding logarithms, we find

$$\begin{array}{lll} 90 - \mu_1 = 0^\circ\ 37'\ 32''{\cdot}4392 : & 90 - \mu_2 = 1^\circ\ 33'\ 15''{\cdot}4660 : & 90 - \mu_3 = 1^\circ\ 1'\ 32''{\cdot}5164 \\ 90 - \mu_1' = 0^\circ\ 37'\ 32''{\cdot}3136 : & 90 - \mu_2' = 1^\circ\ 33'\ 15''{\cdot}9389 : & 90 - \mu_3' = 1^\circ\ 1'\ 32''{\cdot}4102 \end{array}$$

which are the reciprocal depressions: the first line gives the depressions of C at B, A at C, and B at A; the second line the reverse depressions, namely of B at C, C at A, and A at B.

Equations (8) give

$$\sin \alpha = \frac{a \cos u'}{k \sin \mu} \sin \omega \qquad \sin \alpha' = \frac{a \cos u}{k \sin \mu'}, \sin \omega$$

from which we have finally, counting the azimuths from north round by east and south,

$$\begin{array}{lll} \alpha = 36^\circ\ 31'\ 19''{\cdot}744 & \beta = 243^\circ\ 1'\ 21''{\cdot}577 & \gamma = 74^\circ\ 38'\ 48''{\cdot}125 \\ \alpha' = 217\ \ 31\ \ 13\ {\cdot}793 & \beta' = 59\ \ 26\ \ 22\ {\cdot}812 & \gamma' = 257\ \ 13\ \ 0\ {\cdot}211 \end{array}$$

If R_1, R_2, R_3 be the radii of curvature for the mean latitude and mean azimuth of each line,

$$\log R_1 = 7{\cdot}3208918 \qquad \log R_2 = 7{\cdot}3213011 \qquad \log R_3 = 7{\cdot}3214816$$

whence by means of equation (13) we get

$$\log \frac{a}{k_1} = {\cdot}0000086311 \qquad \log \frac{b}{k_2} = {\cdot}0000532748 \qquad \log \frac{c}{k_3} = {\cdot}0000231964$$

and thence

$$\log a = 5{\cdot}6601374089 \qquad \log b = 6{\cdot}0557700907 \qquad \log c = 5{\cdot}8754027338$$

The angles and sides of the spheroidal triangle are, therefore,

$$\begin{array}{llll} & \circ \quad ' \quad '' & & \\ A = & 15\ \ 12\ \ 25{\cdot}314 & a = & 457232{\cdot}83 \text{ feet.} \\ B = & 139\ \ 18\ \ 19{\cdot}533 & b = & 1137025{\cdot}20 \text{ ,,} \\ C = & 25\ \ 30\ \ \ 7{\cdot}784 & c = & 750589{\cdot}93 \text{ ,,} \end{array}$$

Dalby's Theorem.

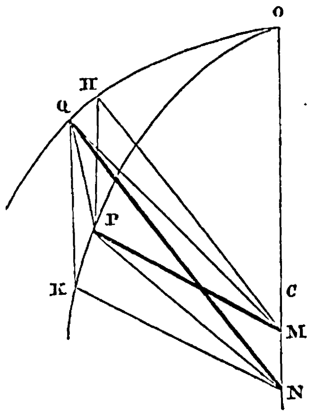
8. By a series of reductions from the equations (6), the following result may be obtained:—

$$\alpha + \alpha' = \zeta + \zeta' + \frac{e^4}{4} \omega (\lambda' - \lambda)^2 \cos^4 \lambda \sin \lambda \qquad (14)$$

A short computation will show that the small quantity on the right-hand side of this equation can never amount to even the ten-thousandth part of a second, which is, practically speaking, zero; consequently the sum of the spheroidal is equal to the sum of the spherical azimuths, whence we obtain the following important theorem:—If λ λ' be the latitudes of two points on the surface of a spheroid, ω their difference of longitude, α α' their reciprocal azimuths,

$$\tan\frac{\omega}{2} = \frac{\cos\frac{\lambda'-\lambda}{2}}{\sin\frac{\lambda'+\lambda}{2}}\cot\frac{\alpha'+\alpha}{2} \qquad {}^*(15)$$

9. From the inaccuracies of the results obtained from the use of this equation in the determination of the difference of longitude of Beachy Head and Dunnose, in the first volume of the "Account of the Trigonometrical Survey," and the imperfection of the demonstration there given, doubts have been thrown upon the truth of this equation, and therefore the following proof is here given:



Let PQ be the two points, their verticals intersecting the axis of revolution OC in M, N. Draw MH, NK parallel to QN, PM respectively, and join QM, PN, QP, HP, QK: then the spheroidal azimuths are QPMO and PQNO; the former exceeds the corresponding spherical azimuth HPMO by the angle QPMH, while the latter is less than the corresponding spherical azimuth KQNO by the angle PQNK: therefore the sum of the spheroidal azimuths exceeds the sum of the spherical azimuths by the excess of the angle QPMH above the angle PQNK. Let now D be the distance of the parallel planes PMH, QKN, and let U, V, be the lengths of the perpendiculars from Q upon PM and from P upon QN respectively, then, if E be the quantity in question,

$$\sin \text{QPMH} = \frac{\text{D}}{\text{U}};\ \sin \text{PQNK} = \frac{\text{D}}{\text{V}}$$

$$\therefore\ \text{E} = \text{D}\left(\frac{1}{\text{U}} - \frac{1}{\text{V}}\right)$$

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[*] This equation has been the subject of considerable discussion, and has been characterised by a high mathematical authority as the "greatest delusion that has ever prevailed in practical mathematics." In a series of papers published in the Philosophical Magazine for July, October, and December 1828, Mr. Ivory denies the truth of the theorem, and maintains that the longitude obtained by this method should be multiplied by $1 + \frac{1}{2} e^2 \cos^2 \frac{1}{2}(\lambda + \lambda')$. Dr. Tiarks, in the same journal for the month of November, proves the theorem true; and in January and February of the following year Mr. Ivory corrects his own error, and proves the theorem generally true for other surfaces of revolution.

But D is less than MN which is equal to e^2 ($\sin \lambda' - \sin \lambda$), taking the semi-axis of the spheroid as unity, and neglecting higher powers of e^2, therefore D is less than $e^2 k$, and consequently, since $U = k \sin \mu$, $V = k \sin \mu'$,

$$E < e^2 (\sin \mu' - \sin \mu) < \frac{1}{2} e^2 k (\mu' - \mu)$$

because $\cos \mu = \frac{1}{2} k$ approximately. But since from equations (8) $\Delta' \cos \mu = \Delta \cos \mu'$, we have $\Delta' (1 + (\mu' - \mu) \tan \mu) = \Delta$

$$\therefore \quad \mu' - \mu = \frac{1}{2} k (\Delta - \Delta')$$

but $\Delta - \Delta'$ is clearly less than $\frac{1}{2} e^2 k$, and therefore

$$E < \frac{1}{8} e^4 k^3.$$

which for a distance k of 150 miles is less than ·00005 of a second.

§ III.

Theory of Spheroidal Triangles.

10. In order that a spheroidal triangle may be rigorously defined, it is necessary that the nature of the lines joining the three points forming the vertices of the triangle should be expressed. It is evident that if we take two points A, B, on a spheroid, and make one plane to contain the vertical line through A and to pass through B, and make another plane containing the vertical line through B to pass through A, these planes, being inclined to each other at a definite angle, will cut the surface in two distinct plane curves. A variety of curves, very nearly plane curves, may be drawn between the two points and contained between the two plane curves just mentioned; such, for instance, is the curve which may be theoretically imagined to be traced in measuring a base line, in which the plane which is determined at each point by the direction of measurement and the vertical at that point, passes through the extremity of the base measured to. If we designate by P and Q respectively the curves formed by the intersection with the surface of the vertical planes at A and B, then the curve line just considered will touch P at A and Q at B. If a line were traced in such a manner that at every point in it the sum of the azimuths of the fixed extremities should be exactly 180°, or, which is the same thing, that the point should be apparently in a direct line between the stations, the curve so formed, being the intersection of a hyperbolic cylinder with the spheroid, will also touch P at A and Q at B: it has also this property, that it is nearer at every point to the straight line joining A and B than any other curve, but yet is not the shortest line that can be traced on the surface. If we suppose the celestial zeniths of A and B to be joined by a great circle, and take all those points on the surface which have their zeniths in this great circle, a curve different from either of the preceding will be formed. This curve has been sometimes confounded

with the geodetic or shortest curve between A and B, it is, however, a plane curve; for since at each point in it the normal to the surface is parallel to a given plane, the tangent plane to the surface at every point must be parallel to a given line, and therefore generates a cylindrical surface whose line of contact with the spheroid lies in a plane passing through the centre of the latter. The curve does not, however, lie between P and Q.

11. But the curve which has generally been considered in reference to spheroidal triangles is the geodetic or shortest line. A distinguishing property of this curve is, that at every point the sine of the azimuth of the line is inversely proportional to the distance of that point from the axis of revolution. It lies entirely between P and Q, and, if we neglect quantities of the order $e^2 \theta^3$, θ being the distance of A and B, makes at A an angle with P equal to the angle it makes with Q at B, each of these angles being equal to one third of the angle of intersection of P and Q. The difference of length of the geodetic line and either of the curves P, Q is

$$\frac{s}{360}\left(e^2 \frac{s^2}{a^2} \cos^2 \lambda \sin 2\alpha\right)^2$$

where α is the azimuth of the line, s its length, λ the latitude, and a the semi-axis major (*Bessel*, *Astronomische Nachrichten*, *No.* 330). This quantity is so minute as to be quite inappreciable; therefore, as far as length is concerned, either of the plane curves P, Q may be taken as the side of the triangle.

12. The greatest distance apart of the curves P, Q may be easily obtained thus: if λ be the mean latitude of A and B, ϕ the difference of latitude, the distance apart of the intersections of their normals with the minor axis is, neglecting e^4, $= 2\, a e^2 \sin \frac{1}{2} \phi \cos \lambda$: let now a meridian plane be drawn bisecting the chord AB, and let the portion of this meridian intercepted by the curves P, Q be x, then it will be evident, by considering the lines of intersection of this meridian plane with the two vertical planes at A and B, that

$$\frac{x}{a \operatorname{vers} \frac{1}{2} \theta} = 2\, e^2 \sin \frac{1}{2} \phi \cos^2 \lambda$$

and since we may put with sufficient approximation $\frac{s}{a} \cos \alpha = \sin \frac{1}{2} \phi$, α being the mean azimuth of the line,

$$x = \frac{s}{8} e^2 \frac{s^2}{a^2} \cos^2 \lambda \cos \alpha$$

the greatest distance of the curves is $x \sin \alpha$, and therefore equal to

$$\frac{s}{16} e^2 \frac{s^2}{a^2} \cos^2 \lambda \sin 2\alpha$$

This quantity is so small, that for a line of 100 miles in length, inclined at 45° to the meridian in the latitude of Great Britain, it will only amount to half an inch, whilst for a line of 50 miles it cannot surpass the sixteenth part of an inch.

13. We now proceed to find the angle subtended at the point H of intersection of the vertical line AH with the axis of revolution, by any given distance s measured from A along the curve of intersection of the plane AHB with the surface of the spheroid.

Let P be the extremity of the arc s, and let the required angle AHP $= \theta$. Let also α be the azimuth of the plane AHB, λ the latitude of A, and ϕ the angle between HP and the plane of the equator; put HA $= v$, HP $= r$, then from properties of the ellipse it may be shown that

$$r^2 - v^2 = -\frac{e^2}{1 - e^2}(r \sin \phi - v \sin \lambda)^2$$

From this equation we may obtain, neglecting quantities of the order $e^4 (\phi - \lambda)^3$,

$$r = v - \frac{v}{2} \frac{e^2}{1 - e^2} (\sin \phi - \sin \lambda)^2$$

also, by spherical trigonometry,

$$\sin \phi = \sin \lambda \cos \theta + \cos \lambda \sin \theta \cos \alpha$$

and by substitution in the preceding equation we get

$$\left.\begin{aligned} \frac{r}{v} &= 1 + \mathrm{P}\,\theta^2 + \mathrm{Q}\,\theta^3 + \dots \\ \mathrm{P} &= -\frac{1}{2} \frac{e^2}{1 - e^2} \cos^2 \alpha \cos^2 \lambda \\ \mathrm{Q} &= \ \ \frac{1}{4} \frac{e^2}{1 - e^2} \cos \alpha \sin 2\lambda \end{aligned}\right\} \tag{16}$$

which determines the nature of the curve.

Now

$$s = \int_0^\theta \left(r^2 + \frac{dr^2}{d\theta^2}\right)^{\frac{1}{2}} = r\,\theta + \frac{1}{2r} \int_0^\theta \frac{dr^2}{d\theta^2} + \dots$$

$$\frac{1}{2r} \left(\frac{dr}{d\theta}\right)^2 = 2\,v\,\mathrm{P}^2\,\theta^2 + 6\,v\,\mathrm{PQ}\,\theta^3$$

$$\therefore \quad \frac{s}{v} = \theta + \frac{1}{3}\mathrm{P}\,(1 + 2\,\mathrm{P})\,\theta^3 + \frac{1}{4}\mathrm{Q}\,(1 + 6\,\mathrm{Q})\,\theta^4 + \dots \tag{17}$$

and by reversing the series,

$$\theta = \frac{s}{v} - \frac{\mathrm{P}}{3}(1 + 2\mathrm{P})\frac{s^3}{v^3} + \dots \tag{18}$$

14. We must now find an expression for the value of a perpendicular from the extremity of the curve s to the normal v, in terms of s and its ratio to ϱ, the mean radius of curvature of the surface at A. We have already shown the value of the radius of curvature on the meridian to be $a\,(1 - e^2)\,\Delta^{-\frac{3}{2}}$, and that perpendicular to the meridian is equal to the normal, for if an indefinitely small arc be drawn perpendicular to the

meridian the normals at its extremities will intersect on the axis of revolution; therefore if ϱ be a mean proportional between the principal radii of curvature,

$$\varrho^2 = \frac{a^2(1-e^2)}{(1-e^2\sin^2\lambda)^2}$$

The perpendicular in question is equal to

$$r\sin\theta = v\,(1 + P\,\theta^2 + Q\,\theta^3 + \cdots)\sin\left(\frac{s}{v} - \frac{P}{3}\frac{s^3}{v^3}\right)$$

omitting the higher powers of e^2; continuing the reduction

$$r\sin\theta = s - \frac{s^3}{2\cdot3\cdot v^2}(1-4P) + \frac{s^5}{2\cdot3\cdot4\cdot5\cdot v^4} - \cdots$$

$$\varrho\sin\frac{s}{\varrho} = s - \frac{s^3}{2\cdot3\cdot\varrho^2} + \frac{s^5}{2\cdot3\cdot4\cdot5\cdot\varrho^4} - \cdots\cdot$$

$$r\sin\theta - \varrho\sin\frac{s}{\varrho} = \frac{s^3}{2\cdot3}\left(\frac{1}{\varrho^2} - \frac{1-4P}{v^2}\right) + \frac{s^5}{2\cdot3\cdot4\cdot5}\left(\frac{1}{v^4} - \frac{1}{\varrho^4}\right) + \cdots\cdot$$

$$= -\frac{s^3}{2\cdot3\cdot\varrho}\,e^2\cos^2\lambda\cos2\alpha + \cdots$$

$$r\sin\theta = \varrho\sin\frac{s}{\varrho}\left(1 - \frac{e^2 s^2}{6\varrho^2}\cos^2\lambda\cos2\alpha\right) \qquad (19)$$

in which expression terms of the order $e^2 s^4$ are neglected; also, if terms of the order $e^2 s^3$ be neglected, as they may, ϱ may be considered evidently as belonging to any point whose distance from A is of the order s.

15. Let A B C be three points on the surface of a spheroid, a b c their mutual distances, which may be considered as measured in each case either along the shortest line or along one of the plane curves. Supposing all azimuths measured from north round by east, let γ γ' be the azimuths of B at A, and A at B, α α' the azimuths of C at B, and of B at C, β β' the azimuths of A at C, and of C at A.

Let H K be the intersections, with the minor axis, of the normals of A and B, and let B K C $= \theta$, A H C $= \theta'$, K C $= r$, H C $= r'$, H A $= v$, K B $= v'$; let also h be the perpendicular from C upon the plane which is vertical at A passing through B, h' the perpendicular from C upon the plane which is vertical at B passing through A; then

$$h = r'\sin\theta'\sin A\ ;\ h' = r\sin\theta\sin B$$

$$\therefore\ \frac{\sin A}{\sin B} = \frac{h\,r\,\sin\theta}{h'\,r'\sin\theta'} \qquad (20)$$

From equation (19.), if we take ϱ for the centre of the triangle,

$$r\sin\theta = \varrho\sin\frac{a}{\varrho}\left(1 - \frac{e^2a^2}{6\varrho^2}\cos^2\lambda_0\cos2\alpha\right)$$

$$r'\sin\theta' = \varrho\sin\frac{b}{\varrho}\left(1 - \frac{e^2b^2}{6\varrho^2}\cos^2\lambda_0\cos2\beta\right)$$

$$\therefore\ \frac{\sin A}{\sin B} = \frac{h}{h'}\,\frac{\sin\frac{a}{\varrho}}{\sin\frac{b}{\varrho}}\left(1 - \frac{e^2}{6\varrho^2}(a^2\cos2\alpha - b^2\cos2\beta)\cos^2\lambda_0\right) \qquad (21)$$

in which λ_0 is the latitude of the centre of the triangle. If $e = 0$

$$\sin A \sin \frac{b}{\varrho} = \sin B \sin \frac{a}{\varrho}$$

16. It remains now to find the ratio of the perpendiculars h h'. Taking the point A on the plane of x z, let the equation of the vertical plane at A passing through B be

$$\frac{x}{f} + \frac{y}{g} + \frac{z}{k} = 1$$

then if x' y' z' be the co-ordinates of C, and λ the latitude of A,

$$h = \frac{1 - \left(\frac{x}{f} + \frac{y}{g} + \frac{z}{k}\right)}{\left(\frac{1}{f^2} + \frac{1}{g^2} + \frac{1}{k^2}\right)^{\frac{1}{2}}}$$

$$\frac{1}{f} = \frac{1}{ve^2 \cos\lambda} \quad ; \quad \frac{1}{k} = \frac{-1}{ve^2 \sin\lambda} \quad ; \quad \frac{1}{g} = \left(\frac{1}{f^2} + \frac{1}{k^2}\right)^{\frac{1}{2}} \cot\gamma$$

$$\therefore\ h = \left(1 - \frac{x'}{f} - \frac{y'}{g} - \frac{z'}{k}\right) e^2 v \cos\lambda \sin\lambda \sin\gamma$$

A similar expression gives h'. If now we put σ σ' for the spherical azimuths corresponding to γ γ', we shall find, after some lengthened reductions,

$$\frac{h}{h'} = \frac{\sin\gamma \sin\sigma'}{\sin\gamma' \sin\sigma} (1 + Q c^2)$$

$$Q = (\sin\lambda' - \sin\lambda) \left\{ \frac{\sin(\sigma - A)}{\sin c \sin A} \cos\lambda - \frac{\sin\sigma \cos\lambda}{\sin a \sin B} + \frac{\sin(\sigma' - B)}{\sin c \sin B} \cos\lambda' \right\}$$

from which finally

$$\frac{h}{h'} = 1 - \frac{e^2 c^2}{4\varrho^2} \sin 2\gamma \cos^2\lambda_0 \cot C \tag{22}$$

This same result may be obtained, and more readily, thus: Let a plane pass through the three points A, B, C, intersecting the surface in an ellipse of very small excentricity, and which may for the present purpose be taken as a circle; the centre of this circle will be distant $\frac{1}{2} \cdot c \cdot \cot C$ from A B, so that if μ be the inclination of this intersecting plane to either of the vertical planes at A and B,

$$\cot\mu = -\frac{c}{2\varrho} \cot C.$$

also, if i be the inclination of the vertical planes to each other,

$$i = e^2 \frac{c}{2\varrho} \sin 2\gamma \cos^2\lambda$$

But the ratio of the perpendiculars is determined by the relation $h = h' (1 + i \cot\mu)$, and if in this expression we substitute the values of i and μ, equation (22) is reproduced.

17. Having now obtained an expression for the value of the ratio $h : h'$, equation (21) becomes by the substitution of this expression

$$\frac{\sin A}{\sin B} = \frac{\sin \frac{a}{\varrho}}{\sin \frac{b}{\varrho}} \left\{ 1 + \frac{c^2 \cos^2 \lambda_0}{6\varrho^2} \left(- a^2 \cos 2\alpha + b^2 \cos 2\beta - \frac{3}{2} c^2 \sin 2\gamma \cot C \right) \right\}$$

Now in the small quantity within the inner bracket we may neglect all quantities of a smaller order, so that we may take

$$\begin{aligned} C &= -\alpha + \beta + 180^\circ \\ B &= -\gamma + \alpha - 180^\circ \\ A &= -\beta + \gamma + 180^\circ \end{aligned}$$

supposing C to be on the north side of A B and intermediate to A and B in longitude. By means of these equations the quantity within the bracket may be transformed to

$$\frac{c^2}{2 \sin C} \left(\sin 2\gamma \cos (\alpha - \beta) + 2 \sin (\alpha + \beta) \right)$$

Hence we have the following set of equations :—

$$\left.\begin{aligned} \frac{\sin A}{\sin B} &= \frac{\sin a}{\sin b} \left\{ 1 + \mu c \left(\cos (\alpha - \beta) \sin 2\gamma + 2 \sin (\alpha + \beta) \right) \right\} \\ \frac{\sin B}{\sin C} &= \frac{\sin b}{\sin c} \left\{ 1 + \mu a \left(\cos (\beta - \gamma) \sin 2\alpha + 2 \sin (\beta + \gamma) \right) \right\} \\ \frac{\sin C}{\sin A} &= \frac{\sin c}{\sin a} \left\{ 1 + \mu b \left(\cos (\gamma - \alpha) \sin 2\beta + 2 \sin (\gamma + \alpha) \right) \right\} \end{aligned}\right\} \qquad (23)$$

where

$$\mu = \frac{c^2}{12} \frac{c}{\sin C} \cos^2 \lambda_0$$

the unit of length being the radius of mean curvature.

18. From these fundamental equations of Spheroidal Trigonometry we may deduce the errors of the two sides a and b, as determined by Spherical Trigonometry from a given side c, and the angles of a spheroidal triangle A B C ; they are in fact

$$\begin{aligned} e_a &= - \mu \, ab \left\{ \cos (\gamma - \alpha) \sin 2\beta + 2 \sin (\gamma + \alpha) \right\} \\ e_b &= + \mu \, ab \left\{ \cos (\beta - \gamma) \sin 2\alpha + 2 \sin (\beta + \gamma) \right\} \end{aligned}$$

which again may be put in the form

$$\left.\begin{aligned} e_a &= - \mu \, a b k \sin (2\gamma + \phi) \\ e_b &= + \mu \, a b k' \sin (2\gamma + \phi') \end{aligned}\right\} \qquad (24)$$

in which

$$k = (4\cos^2 A + \cos^2 B + 4\cos^2 C)^{\frac{1}{2}}$$
$$k' = (\cos^2 A + 4\cos^2 B + 4\cos^2 C)^{\frac{1}{2}}$$
$$k \sin\phi = \cos B \sin 2A - 2\sin B$$
$$-k' \sin\phi' = \cos A \sin 2B - 2\sin A$$

19. We have now reduced the expressions for the errors in the solution of spheroidal triangles by spherical trigonometry to their simplest form, and it remains to examine the limits within which these errors are confined. If we take any triangle of which the sides are $a\ b\ c$ and the angles A B C, having also a mean latitude λ_0, and place it with respect to the meridian in such a position that $2\gamma + \phi = \frac{1}{2}\pi$ or $= \frac{3}{2}\pi$, then the resulting error of the side a, as calculated by spherical trigonometry from the side c, is

$$\frac{e^2}{12}\frac{abc}{\sin C}\cos^2\lambda_0 \cdot k$$

if the same triangle were so placed in azimuth that $2\gamma + \phi' = \frac{1}{2}\pi$ or $= \frac{3}{2}\pi$, then the side b would be affected with the greatest error, which is equal to

$$\frac{e^2}{12}\frac{abc}{\sin C}\cos^2\lambda_0 \cdot k'$$

Therefore a limit to which the error can never amount is (since $k < 3$)

$$\frac{e^2}{4}\frac{abc}{\sin C}\cos^2\lambda_0$$

For Great Britain the limit of error may be taken at

$$\frac{e^2}{12}\frac{abc}{\sin C}$$

which depends solely upon the length of the sides and the acuteness or obtuseness of the angle C.

In order to obtain an idea of the actual magnitude of this quantity, let us suppose a triangle of larger dimensions and more ill-shaped than any that would probably ever be actually solved, for example $A = 21°$, $B = 146°$, $C = 13°$, $a = 124$ miles, $b = 196$ miles, $c = 80$ miles, then the error in calculating either of the long sides from the shorter side could not amount to 1·6 feet. If we had taken a triangle of the same shape but smaller dimensions, say 93, 147, 60 miles, the limit of error would be found to be under 8 inches; but even this triangle is more unfavourable than any in the triangulation of Great Britain.

In general the spheroidal error varies as the square of the cosine of the latitude, it increases with the departure of the triangle from the "well-conditioned" or equilateral form, and in two similar triangles the errors are as the cubes of the homologous sides.

The following Table exhibits the greatest errors that could result in the calculation by Spherical Trigonometry of several large triangles :—

No.	Angles.			Sides in Miles.			Error in Feet in	
	A	B	C	a	b	c	a	b
1	30°	39°	111°	64	79	119	·09	·09
2	59	92	29	104	122	59	·22	·20
3	85	42	53	143	97	116	·20	·27
4	30	110	40	94	176	120	·50	·41
5	23	140	17	90	146	66	·57	·54
6	35	118	27	120	184	95	·81	·71
7	17	138	25	130	303	194	3·46	3·19
8	112	41	27	324	228	162	3·83	4·36
9	17	153	10	190	302	120	7·82	6·67

Legendre's Theorem.

20. Having now proved that the errors resulting from the solution of spheroidal triangles by Spherical Trigonometry are inconsiderable, we proceed to consider the solution of spherical triangles.

In the earlier operations of the Trigonometrical Survey, the triangles were calculated by the chord method; this has been for many years discontinued, and in place of it Legendre's theorem has been exclusively adopted. This simple and elegant method, though of less primâ facie accuracy than the chord method, is sufficiently near the truth for calculations of almost any degree of exactitude, and is of very easy application.

In large triangles, however, the accuracy of the result will both depend on the value of the spherical excess adopted, and on the shape and magnitude of the triangle.

If we put μ for one third of the spherical excess *adopted*, then x, the correction to the side a calculated from the side c, will be

$$x = \sin^{-1}\left(\sin c \frac{\sin A}{\sin C}\right) - c\,\frac{\sin (A - \mu)}{\sin (C - \mu)}$$

and expanding into series the quantities

$$\frac{\sin a}{\sin c}, \quad \frac{\sin (A - \mu)}{\sin (C - \mu)}$$

and substituting for cot C − cot A its value

$$\frac{a^2 - c^2}{ab \sin C}\left(1 - \frac{-a^2 + b^2 + c^2}{12}\right)$$

there results

$$x = \mu \frac{c^2 - a^2}{ab \sin C} \cdot a - \frac{c^2 - a^2}{6} \left(1 - \frac{3a^2 - 7c^2}{60}\right) a$$

$$= \frac{1}{6} a (c^2 - a^2) \left(\frac{6\mu}{ab \sin C} + \frac{3a^2 - 7c^2}{60} - 1\right) \tag{25}$$

If now we calculate μ by the formula

$$\mu = \frac{1}{6} \frac{ab}{r^2} \sin C \tag{α}$$

(r being the radius of the sphere which we have hitherto assumed to be unity) the errors of the sides a and b thus calculated will be

$$\left.\begin{aligned} e_a &= -a (c^2 - a^2) \frac{3a^2 - 7c^2}{360} \\ e_b &= -b (c^2 - b^2) \frac{3b^2 - 7c^2}{360} \end{aligned}\right\} \tag{26}$$

If C′ be the angle C reduced by $\frac{1}{3}$ of the spherical excess, then

$$\frac{\sin C'}{\sin C} = 1 - \frac{a^2 + b^2 - c^2}{12}$$

and consequently if μ be calculated by the formula

$$\mu = \frac{1}{6} \frac{ab}{r^2} \sin C' \tag{β}$$

the errors in a and b will be

$$\left.\begin{aligned} e_a &= a (c^2 - a^2) \frac{2a^2 + 5b^2 + 2c^2}{360} \\ e_b &= b (c^2 - b^2) \frac{2b^2 + 5a^2 + 2c^2}{360} \end{aligned}\right\} \tag{27}$$

Again, if the true spherical excess be used,

$$\sin \frac{3\mu}{2} = \frac{\sin \frac{a}{2} \sin \frac{b}{2} \sin C}{\cos \frac{c}{2}}$$

$$\therefore\ \mu = \frac{1}{6} ab \sin C \left(1 - \frac{a^2 + b^2}{24} + \frac{c^2}{8}\right)$$

$$= \frac{1}{6} ab \sin C' \left(1 + \frac{a^2 + b^2 + c^2}{24}\right) \tag{γ}$$

if this value of μ, which is true to the order of terms included, be used, the errors in a and b will be

$$\left.\begin{aligned} e_a &= -a (c^2 - a^2) \frac{a^2 - 5b^2 + c^2}{720} \\ e_b &= -b (c^2 - b^2) \frac{b^2 - 5a^2 + c^2}{720} \end{aligned}\right\} \tag{28}$$

These errors are rather smaller than in either of the preceding cases, and may be altogether neglected even in triangles of sides of 200 or 300 miles. This supposes the spherical excess to be true to terms of the order of c^4.

21. To take a numerical case, suppose the side $c = 60$ miles, $a = 220$ miles, $b = 180$ miles, then, using a radius of 4,000 miles,

$$a = \cdot 055 \qquad b = \cdot 045 \qquad c = \cdot 015$$

$$\frac{a}{360}(c^2 - a^2) = -9\cdot036 \text{ feet}, \quad \frac{b}{360}(c^2 - b^2) = -4\cdot752 \text{ feet},$$

and the errors in the three cases expressed in feet will be as follows:

(α)	(β)	(γ)
$e_a = + \cdot 068$	$e_a = - \cdot 150$	$e_a = - \cdot 031$
$e_b = + \cdot 026$	$e_b = - \cdot 093$	$e_b = - \cdot 030$

If we had taken a triangle whose sides are 120, 440, and 360 miles, the errors would have been evidently the above multiplied by 2^5 or 32, giving

(α)	(β)	(γ)
$e_a = + 2\cdot176$	$e_a = - 4\cdot800$	$e_a = - 0\cdot992$
$e_b = + 0\cdot832$	$e_b = - 2\cdot976$	$e_b = - 0\cdot960$

From these considerations it is clear that Legendre's Theorem may be applied without appreciable error to spheroidal triangles of much larger size than it is ever necessary to compute.

§. IV.

Latitudes, Longitudes, and Azimuths.

22. Let λ be the latitude of the given point A; s, α, the distance and azimuth of a point B, whose latitude and longitude are required, together with the meridional bearing of the given point A. Let H be the point of intersection of the vertical at A with the axis of revolution, and let $AH = v$, $BH = r$, $AHB = \theta$; then if ω be the difference of longitude, ϕ the inclination of r to the plane of the equator, the three lines HA, HB, HP, (P being the north pole of the earth,) will form a spherical triangle of which the sides are $90° - \lambda$, $90° - \phi$, θ; the angle included by the first and second is ω, that by the first and third α, that by the second and third is nearly but not quite equal to the azimuth α' of A at B.

By equation (18) we have, neglecting P^2, which is quite admissible,

$$\left.\begin{aligned} \theta &= \frac{s}{v} + \frac{1}{6}\frac{e^2}{1-e^2}\cos^2\lambda\cos^2\alpha\left(\frac{s}{v}\right) \\ \log\theta &= \log\frac{s}{v} + \frac{M}{6}\frac{e^2}{1-e^2}\cos^2\lambda\cos^2\alpha\left(\frac{s}{v}\right)^2 \end{aligned}\right\} \tag{29}$$

which determines θ with accuracy sufficient for any purpose; indeed, the second term is so small that for a distance of a degree along the meridian it only corresponds to a quantity less than six tenths of an inch, therefore for ordinary distances $s = v\,\theta$.

Now the two sides θ and $90° - \lambda$ with the included angle α will give by spherical trigonometry ϕ and ω, and we may find λ' the latitude of B thus: join B with the centre of the spheroid and let λ_1 be the geocentric latitude of B, then

$$\frac{\tan\lambda_1}{\tan\phi} = \frac{r\sin\phi - v\,e^2\sin\lambda}{r\sin\phi}$$

$$\therefore\quad \tan\lambda' = \frac{a^2}{b^2}\tan\phi\left(1 - e^2\frac{v\sin\lambda}{r\sin\phi}\right) \tag{30}$$

This is the only direct expression for λ', and the ratio of v to r may be assumed equal to unity, unless great accuracy is required, when from (16)

$$\log\frac{v}{r} = \frac{M}{2}\frac{e^2\,\theta^2}{1-e^2}\cos^2\lambda\cos^2\alpha$$

which is a minute quantity, and quickly estimated.

Having thus obtained λ' and ω, the value of α' is expressed by equation (15), viz.:

$$\cot\frac{\alpha'+\alpha}{2} = \frac{\sin\dfrac{\lambda'+\lambda}{2}}{\cos\dfrac{\lambda'-\lambda}{2}}\tan\frac{\omega}{2}$$

23. For actual calculation, however, the following method is preferable: Let $\alpha' + \zeta$ be the third angle of the spherical triangle just considered, then if $\varkappa = 90° - \lambda$, we have by spherical trigonometry

$$\left.\begin{aligned} \tan\tfrac{1}{2}(\alpha' + \zeta + \omega) &= \frac{\cos\frac{1}{2}(\varkappa - \theta)}{\cos\frac{1}{2}(\varkappa + \theta)}\cot\tfrac{1}{2}\alpha \\ \tan\tfrac{1}{2}(\alpha' + \zeta - \omega) &= \frac{\sin\frac{1}{2}(\varkappa - \theta)}{\sin\frac{1}{2}(\varkappa + \theta)}\cot\tfrac{1}{2}\alpha \\ \tan\tfrac{1}{2}(\phi - \lambda) &= \frac{\sin\frac{1}{2}(\alpha + \zeta - \alpha)}{\sin\frac{1}{2}(\alpha' + \zeta + \alpha)}\tan\tfrac{1}{2}\theta \end{aligned}\right\} \tag{31}$$

We may determine the quantity ζ in the following manner: Let K be the intersection of the normal of B with the axis of revolution, then from the spherical triangle determined by the lines B A, B H, B K,

$$\frac{\sin(\alpha' + \zeta)}{\sin\alpha'} = \frac{\sin \mathrm{ABK}}{\sin \mathrm{ABH}} = \frac{\mathrm{HB}}{\mathrm{HA}}\frac{\sin \mathrm{ABK}}{\sin \mathrm{BAH}}$$

$$\therefore \quad 1 + \zeta \cot\alpha' = \frac{r}{v}\frac{\sin\mu'}{\sin\mu}$$

Now neglecting quantities of the order $e^2\,\theta^3$, $\sin\mu' = \sin\mu$; therefore from equation (16)

$$\zeta = \frac{1}{4}\frac{e^2\,\theta^2}{1 - e^2}\cos^2\lambda\,\sin 2\,\alpha \qquad (32)$$

This quantity ζ is so small as not to amount to a tenth of a second even in a distance of a hundred miles. It remains now to find the difference of latitudes $\lambda' - \lambda$, which depends upon the

Distance of Parallels.

24. From equation (29) it follows that the distance of parallels of A and B is equal to $v\,(\phi - \lambda)\left\{1 - n\,(\phi - \lambda)^2\right\}$. The quantity $\phi - \lambda$ is obtained from the third equation of (31); writing it in the form

$$\tan\tfrac{1}{2}(\phi - \lambda) = k\tan\tfrac{1}{2}\theta$$

and substituting for the tangents their equivalent series, we find

$$\frac{\phi - \lambda}{\theta} = k\left(1 + \frac{\theta^2}{12}(1 - k^2) + \frac{\theta^4}{240}(1 - k^2)(2 - 3\,k^2)\right)$$

The term in θ^4 will be generally insensible, and therefore we may put

$$\phi - \lambda = k\,\theta\left(1 + \frac{\theta^2}{12}(1 - k^2)\right)$$

and since $s = v\,\theta\left\{1 - n\,(\phi - \lambda)^2\right\}$ we shall have, by replacing the value of k, and putting S for the distance of the parallels of A and B,

$$S = s\,\frac{\sin\frac{1}{2}(\alpha' + \zeta - \alpha)}{\sin\frac{1}{2}(\alpha' + \zeta + \alpha)}\left(1 + \frac{\theta^2}{12}\cos^2\frac{\alpha' - \alpha}{2}\right) \qquad (33)$$

This equation is also equivalent to the following:—

$$\left.\begin{aligned} S &= s\,\frac{\sin\frac{1}{2}(\alpha' - \alpha)}{\cos\frac{1}{2}(\alpha' + \alpha)}\left(1 + \frac{\theta^2}{12}\cos^2\frac{\alpha' - \alpha}{2}\,\beta\right) \\ \beta &= 1 + \frac{3\,e^2}{1 - e^2}\cos^2\lambda \end{aligned}\right\} \qquad (34)$$

25. The difference of latitudes may be obtained by dividing the meridian distance by the radius of curvature of the meridian for the mean of the extreme latitudes. To prove

this, let a distance $s_{,}$ be taken along the meridian from a point whose latitude is λ to a point whose latitude is $\lambda + \frac{1}{2}\alpha$, then

$$s_{,} = \left(\frac{ds}{d\lambda}\right)\frac{\alpha}{2} + \left(\frac{d^2 s}{d\lambda^2}\right)\frac{\alpha^2}{1\cdot 2\cdot 2^2} + \left(\frac{d^3 s}{d\lambda^3}\right)\frac{\alpha^3}{1\cdot 2\cdot 3\cdot 2^3} + \dots$$

and if s' be the length of the meridian curve measured in the opposite direction to a point whose latitude is $\lambda - \frac{1}{2}\alpha$,

$$-s' = -\left(\frac{ds}{d\lambda}\right)\frac{\alpha}{2} + \left(\frac{d^2 s}{d\lambda^2}\right)\frac{\alpha^2}{1\cdot 2\cdot 2^2} - \left(\frac{d^3 s}{d\lambda^3}\right)\frac{\alpha^3}{1\cdot 2\cdot 3\cdot 2^3} + \dots$$

$$\therefore \; s_{,} + s' = \left(\frac{ds}{d\lambda}\right)\alpha + \frac{1}{24}\left(\frac{d^3 s}{d\lambda^3}\right)\alpha^3 + \dots$$

But

$$\left(\frac{ds}{d\lambda}\right) = \varrho, \quad \left(\frac{d^3 s}{d\lambda^3}\right) = 3\, e^2 \varrho \cos 2\lambda + \dots$$

neglecting the higher powers of e^2; therefore if s be the length of an arc of meridian whose amplitude is α, ϱ the radius of curvature for λ the mean of the terminal latitudes,

$$\left.\begin{aligned} s &= \varrho\alpha + \varrho\left(\frac{\alpha}{2}\right)^3 e^2 \cos 2\lambda \\ \alpha &= \frac{s}{\varrho} - \left(\frac{\alpha}{2}\right)^3 e^2 \cos 2\lambda \end{aligned}\right\} \qquad (35)$$

This correction is generally extremely small, vanishing altogether at the latitude of 45°. In an arc of n degrees whose middle latitude is 60°, it amounts only to $n^3 \times \cdot 046$ feet, and it is therefore seldom necessary to use the term in e^2.

26. The following set of equations, therefore, contains the whole solution of the problem of geodetic latitudes and longitudes when great accuracy is required:—

$$\left.\begin{aligned} \tan \tfrac{1}{2}(\alpha' + \omega + \zeta) &= \frac{\cos \frac{1}{2}(\varkappa - \theta)}{\cos \frac{1}{2}(\varkappa - \theta)} \cot \frac{\alpha}{2} \\ \tan \tfrac{1}{2}(\alpha' - \omega + \zeta) &= \frac{\sin \frac{1}{2}(\varkappa - \theta)}{\sin \frac{1}{2}(\varkappa + \theta)} \cot \frac{\alpha}{2} \\ \lambda' - \lambda &= \frac{s}{\varrho}\frac{\sin \frac{1}{2}(\alpha' - \alpha + \zeta)}{\sin \frac{1}{2}(\alpha' + \alpha + \zeta)}\left(1 + \frac{\theta^2}{12}\cos^2 \tfrac{1}{2}(\alpha' - \alpha)\right) \\ \theta &= \frac{s}{v}\left(1 + \frac{1}{6}\,\frac{e^2\theta^2}{1 - e^2}\cos^2\lambda \cos^2\alpha\right) \\ \zeta &= \frac{1}{4}\,\frac{e^2\theta^2}{1 - e^2}\cos^2\lambda \sin 2\alpha\,^* \end{aligned}\right\} \qquad (36)$$

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* By extending the value of this quantity, so as to include terms of the order $e^2\,\theta^3$, it becomes

$$\zeta = \frac{1}{4}\,\frac{e^2\theta^2}{1 - e^2}\cos^2 \tfrac{1}{2}(\lambda' + \lambda)\sin(\alpha' - \alpha)$$

due to Mr. O'Farrell of this office.

The equation $s = v \cdot \theta$ may generally be taken without correction. The small quantity ζ in seconds may be obtained from the following table, by multiplying the quantity corresponding to the given latitude and azimuth by the square of the number of degrees in θ.

LATITUDE λ	Values of $\frac{e^2}{1 - e^2} 5 \pi \cos^2 \lambda \sin 2 \alpha$										
	AZIMUTH α.										
	2° | 88°	4° | 86°	8° | 82°	12° | 78°	16° | 74°	20° | 70°	25° | 65°	30° | 60°	35° | 55°	40° | 50°	45° | 45°
°	″	″	″	″	″	″	″	″	″	″	″
49	0·003	0·006	0·013	0·018	0·024	0·029	0·035	0·039	0·043	0·045	0·045
50	0·003	0·006	0·012	0·018	0·023	0·028	0·033	0·038	0·041	0·043	0·044
51	0·003	0·006	0·012	0·017	0·022	0·027	0·032	0·036	0·039	0·041	0·042
52	0·003	0·006	0·011	0·016	0·021	0·026	0·031	0·035	0·038	0·039	0·040
53	0·003	0·005	0·011	0·016	0·020	0·025	0·029	0·033	0·036	0·038	0·038
54	0·003	0·005	0·010	0·015	0·019	0·023	0·028	0·032	0·034	0·036	0·036
55	0·002	0·005	0·010	0·014	0·018	0·022	0·027	0·030	0·033	0·034	0·035
56	0·002	0·005	0·009	0·013	0·017	0·021	0·025	0·029	0·031	0·032	0·033
57	0·002	0·004	0·009	0·013	0·017	0·020	0·024	0·027	0·029	0·031	0·031
58	0·002	0·004	0·008	0·012	0·016	0·019	0·023	0·026	0·028	0·029	0·030
59	0·002	0·004	0·008	0·011	0·015	0·018	0·021	0·024	0·026	0·028	0·028
60	0·002	0·004	0·007	0·011	0·014	0·017	0·020	0·023	0·025	0·026	0·026
61	0·002	0·003	0·007	0·010	0·013	0·016	0·019	0·021	0·023	0·024	0·025

In illustrating the working of these formulæ, we shall take the side A C of the spheroidal triangle worked in Art. (7.), and assuming the length of the side, namely 1137025·201 feet (214 miles); the latitude of A 52° 0′; and the azimuth of C 59° 26′ 22″·8116; from these data determine the latitude and longitude of C and the bearing of A at C.

We have then given

$$\lambda = 52^\circ\ 0' \qquad \alpha = 59^\circ\ 26'\ 22''\!\cdot\!8116 \qquad \log s = 6\cdot0557700907$$

to find λ' α' ω. If we express θ in seconds, then

$$\log \theta = \log \frac{s}{v \sin 1''} + \cdot0004862\ (\Delta \lambda \cos \lambda)^2 \text{*}$$

Any approximate knowledge of $\Delta \lambda$, the difference of latitude, is sufficient for this calculation; in the present instance $\Delta \lambda = 1^\circ\ 30'$, the work then stands thus:—

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* The logarithms of $(v \sin 1'')^{-1}$ and $(\varrho \sin {}'')^{-1}$ are tabulated in the "Account of the Measurement of the Lough Foyle Base, by Lieut.-Col. Yolland, R.E.;" the semi-axes of the spheroid being 20923713 and 20853810 feet.

$$\begin{array}{rl} \text{Const.}\ \cdot\cdot & \bar{4}\cdot 6868 \\ \sin^2\Delta\lambda\ \cdot\cdot & 6\cdot 8359 \\ \cos^2\lambda\ \cdot\cdot & \underline{9\cdot 5787} \\ \log 1262 = & 3\cdot 1013 \end{array} \qquad \begin{array}{rr} \log s = & 6\cdot 0557700907 \\ (v \sin 1'')^{-1}\ \cdot\cdot & 7\cdot 9928850557 \\ & \underline{1262} \\ & 4\cdot 0486552726 \\ \theta = & 3^\circ\ 6'\ 25''\cdot 4967 \end{array}$$

$$\begin{array}{rl} \tfrac{1}{2}(\varkappa - \theta) = & 17^\circ\ 26\ 47''\cdot 2517 \\ \tfrac{1}{2}(\varkappa + \theta) = & 20\ \ 33\ \ 12\cdot 7484 \\ \tfrac{1}{2}\alpha = & 29\ \ 43\ \ 11\cdot 4058 \\ & \end{array} \qquad \begin{array}{rl} \cos\ \cdot\cdot & 9\cdot 9795472525 \\ \sec\ \cdot\cdot & 0\cdot 0285643261 \\ \cot\ \cdot\cdot & \underline{0\cdot 2434788810} \\ \tan\ \cdot\cdot & 0\cdot 2515904596 \end{array} \qquad \begin{array}{rl} \sin\ \cdot\cdot & 9\cdot 4768525580 \\ \cos\ \cdot\cdot & 0\cdot 4545907919 \\ \cot\ \cdot\cdot & \underline{0\cdot 2434788810} \\ \tan\ \cdot\cdot & 0\cdot 1749222309 \end{array}$$

$$\begin{array}{rl} \tfrac{1}{2}(\alpha' + \zeta + \omega) = & 60^\circ\ 44'\ 19''\cdot 3689 \\ \tfrac{1}{2}(\alpha' + \zeta - \omega) = & 56^\circ\ 14'\ 19''\cdot 3687 \end{array} \qquad \begin{array}{rl} \text{Tab. } \zeta = & \cdot 035 \\ \zeta = & \cdot 336 \end{array} \qquad \begin{array}{rl} \alpha' = & 116^\circ\ 58'\ 38''\cdot 4016 \\ \omega = & 4^\circ\ 30'\ 0''\cdot 0002 \end{array}$$

$$\begin{array}{rl} \tfrac{1}{2}(\alpha' + \zeta - \alpha) = & 28^\circ\ 46'\ 7''\cdot 9630 \\ \tfrac{1}{2}(\alpha' + \zeta + \alpha) = & 88\ \ 12\ \ 30\cdot 7746 \end{array} \qquad \begin{array}{rl} \sin\ \cdot\cdot & 9\cdot 6823956659 \\ \cos\ \cdot\cdot & 0\cdot 0002123196 \\ s\ \cdot\cdot & 6\cdot 0557700907 \\ & 0\cdot 0000817777 \\ (\varrho \sin 1'')^{-1} & \underline{7\cdot 9939338561} \\ & 3\cdot 7323937100 \end{array} \qquad \begin{array}{rl} \text{Const.}\ \cdot\cdot & \bar{3}\cdot 9297529 \\ \theta^2\ \cdot\cdot & 8\cdot 0973105 \\ \cos^2 \tfrac{1}{2}(\alpha' - \alpha)\ \cdot\cdot & \underline{9\cdot 8855714} \\ \cdot\ \ \cdot\ \ \cdot\ \ \cdot & 5\cdot 9126348 \end{array}$$

$$\lambda' - \lambda = 1^\circ\ 29'\ 59''\cdot 9994 \qquad \therefore \qquad \lambda' = \underline{53^\circ\ 29'\ 59''\cdot 9994}$$

27. In the case of a long line running very nearly in the meridian, such as from Dunnose to Arbury, the simplest method is to add a small correction to the actual length to reduce it to the meridian distance; for when α is small, if ζ be the projection of θ on the meridian plane of A,

$$\theta - \xi = \tan^2\frac{\alpha}{2}\sin 2\theta = \psi$$

Also putting $-x = \phi - \lambda - \theta$, we have

$$\sin\phi = \sin(\lambda + \xi)\frac{\cos\theta}{\cos\xi}$$

$$\sin(\lambda + \theta - x) = \sin(\lambda + \theta - \psi)\,(1 - \psi\tan\xi)$$

$$x = \psi\Big(1 + \tan\theta\tan(\lambda + \theta)\Big)$$

therefore, S being the distance of the parallels

$$S = s - 2\,v\tan^2\frac{\alpha}{2}\,\frac{\sin\theta\cos\lambda}{\cos(\lambda + \theta)} \tag{37}$$

the longitude and azimuth α' will then follow from the relations

$$\left.\begin{aligned} \omega &= \frac{\sin\theta\sin\alpha}{\cos(\lambda + \theta - x)} \\ 180^\circ - \alpha - \alpha' &= \omega\,\frac{\sin\frac{1}{2}(\lambda' + \lambda)}{\cos\frac{1}{2}(\lambda' - \lambda)} \end{aligned}\right\} \tag{38}$$

28. When the distance s does not exceed 40 or 50 miles, the following method is used. Take a point P_1 on the meridian of A, so that the angle $B P_1 A$ shall be a right angle, and let λ_1 be the latitude of P_1; take also on the same meridian another point P having the same latitude λ' with B. Let $A P_1 = q$, $B P_1 = p$, and let r be the radius of mean curvature for the triangle $A B P_1$; then

$$\sin \alpha = \frac{\sin \frac{p}{r}}{\sin \frac{s}{r}} = \frac{p}{s}\left(1 + \frac{s^2 - p^2}{6 r^2}\right)$$

$$\cos \alpha = \frac{\tan \frac{q}{r}}{\tan \frac{s}{r}} = \frac{q}{s}\left(1 - \frac{s^2 - q^2}{3 r^2}\right)$$

$$p = s \sin \alpha \left(1 - \frac{s^2}{6 r^2} \cos^2 \alpha\right); \quad q = s \cos \alpha \left(1 + \frac{s^2}{3 r^2} \sin^2 \alpha\right)$$

And if ω be the difference of longitude of A and B, we have

$$\sin \omega = \frac{p}{v' \cos \lambda'}\left(1 - \frac{p^2}{6 v^2}\right) = \omega \left(1 - \frac{\omega^2}{6}\right)$$

$$\therefore \quad \omega = \frac{s \sin \alpha}{v' \cos \lambda'}\left(1 - \frac{s^2}{6 r^2} + \frac{s^2 \sin^2 \alpha}{6 r^2} \sec^2 \lambda'\right) \qquad (\alpha)$$

It is easily shown that the difference of latitude of P and P_1 is

$$\lambda_1 - \lambda' = \frac{s^2}{2 r^2} \sin^2 \alpha \tan \lambda_1.$$

if therefore ϱ be the radius of curvature of the meridian for the middle point between A and P_1

$$\lambda' - \lambda = \frac{s}{\varrho} \cos \alpha \left(1 + \frac{s^2}{3 r^2} \sin^2 \alpha\right) - \frac{s^2}{2 r^2} \sin^2 \alpha \tan \lambda_1 \qquad (\beta)$$

From equation (10), if ν be the convergence of meridians,

$$\tan \frac{\nu}{2} = \frac{\sin \frac{1}{2}(\lambda' + \lambda)}{\cos \frac{1}{2}(\lambda' - \lambda)} \tan \frac{\omega}{2}$$

$$\therefore \quad \frac{\nu}{\omega} = \frac{\sin \frac{1}{2}(\lambda' + \lambda)}{\cos \frac{1}{2}(\lambda' - \lambda)} \left(1 + \frac{\omega^2 - \nu^2}{12}\right)$$

$$\therefore \quad \nu = \omega \sin \frac{\lambda' + \lambda}{2} \left(1 + \frac{s^2}{8 r^2} - \frac{s^2 \sin^2 \alpha}{24 r^2}\right) \qquad (\gamma)$$

Now, M being the modulus of the common system of logarithms, put

$$\frac{\text{M} s^2}{6 r^2} = f \; ; \quad f \sin^2 \alpha = g \; ; \quad g \sec^2 \lambda' = h$$

then

$$\left.\begin{aligned}\log(\lambda_1 - \lambda) &= \log\frac{s\cos\alpha}{\varrho\sin 1''} + 2g\\ \lambda' - \lambda_1 &= -\frac{s^2\sin^2\alpha\tan\lambda_1}{2r^2\sin 1''}\\ \log\omega &= \log\left(\frac{s\sin\alpha}{v'\cos\lambda'\sin 1''}\right) - f + h\\ \log\nu &= \log\left(\omega\sin\frac{\lambda'+\lambda}{2}\right) + \frac{3f-g}{4}\end{aligned}\right\} \qquad (39)$$

29. The following is an example of the working of this formula :— Given the latitude of A, and the distance and azimuth of B,

$$\begin{aligned}\lambda &= 52^\circ\ 42'\ 51''\cdot617\\ \log s &= 5\cdot3700394\\ \alpha &= 35^\circ\ 57'\ 27''\cdot147\end{aligned}$$

to find the latitude and longitude, λ', ω of B and the back azimuth α' of A ; the work stands thus:—

$\log s$ ··	5·3700394		1·2185	$\log s$ ··	5·3700394		0·37122
$\cos\alpha$ ··	9·9081913	$\log s^2$ ··	0·7401	$\sin\alpha$ ··	9·7687753	$s^2\sin^2\alpha$ ··	0·27763
$(\varrho\sin 1'')^{-1}$ ··	7·9938649			$\sec\lambda'$ ··	0·2228849	$\tan\lambda_1$ ··	0·12658
$2g$	62	$\log f$ ··	1·9586	$(v'\sin 1'')^{-1}$ ··	7·9928023		
		$\sin^2\alpha$ ··	1·5376	$-f+h$	-3	$5''\cdot963$ ··	0·77543
	3·2721018						
$\lambda_1 - \lambda =$	$0^\circ\ 31'\ 11''\cdot121$	$\log g$ ··	1·4962		3·3545016	···· $\omega =$	$0^\circ\ 37'\ 42''\cdot047$
	$-5\cdot963$	$\sec^2\lambda'$	·4458	$\sin\frac{1}{2}(\lambda'+\lambda)$	9·9021965		
$\lambda' - \lambda =$	$0^\circ\ 31'\ 5''\cdot158$	$\log h$ ··	1·9420	$\frac{1}{4}(3f-g)$	60		
					3·2567401	···· $\nu =$	$0^\circ\ 30'\ 5''\cdot943$

The logarithm 1·2185 may be taken constant for all latitudes; the logarithm 0·37122 is the logarithm of $(2r^2\sin 1'')^{-1}$: it varies slowly with the latitude, but may be taken as constant for all latitudes when great accuracy is not required.

§ V.

Combination of Observations made at different Stations.

30. It is essential that, in every triangulation from which accurate results are required, the number of bearings observed should considerably exceed the number that is absolutely necessary for the fixation of all the points, and every observed bearing over and above that number must be considered as adding weight to the determinations of the relative positions and distances of the stations, and to the general results that may be deduced. If the observations were free from error, then any supernumerary bearings would be quite unnecessary, as all the points being once determined, the distance of any two, or the angle subtended by any two at a third point, could be calculated. But when the observations are increased so as to admit of a result being obtained by using different sets of observed quantities, then each different method of calculation will afford a different value of the quantity sought. The values thus obtained will exhibit differences which are functions of the errors of observation, and the magnitude of which will afford a general estimate of the accuracy, or rather inaccuracy, of the triangulation. And inasmuch as every result that can be arrived at by more than one series of observed quantities will be affected in a greater or less degree by the same discrepancies, the question becomes, in general, how to render the triangulation consistent with itself, or, to find that consistent triangulation which shall most nearly represent or be represented by all the observations; the question thus turns upon greatest probability, and is solved by the method of least squares, a result of the theory of probabilities.

31. In by far the greater portion of the extensive triangulations that have been effected in connection with the determination of the figure of the earth, no general method has been followed in dispersing the discrepancies, so that the results are within certain limits arbitrary. It has, however, in almost every instance been customary to make the sum of the three observed angles of a triangle equal to 180°, or rather, if the triangle be very large, to calculate the spherical excess and then to alter the observed angles until their sum shall be equal 180° + the "spherical excess." In some cases these alterations or corrections to the three angles of each triangle have been made according to the relative degrees of dependence—in the opinion of the observer—to be placed upon the observed values of the angles, and in others this arbitrary apportionment has been avoided, and the residual error of each triangle divided evenly among the angles. In the earlier calculations of the triangulation for the Indian arc by Colonel Lambton, the error of each triangle was so divided that each angle received a correction proportioned to its magnitude, a method that cannot be supported on any principle. In more modern calculations, the errors of each triangle have generally been divided more correctly into parts inversely proportional to the calculated weights of the angles.

But the adjustment of each triangle in itself is not the solution of the problem, as there are other conditions to be fulfilled which present a practical difficulty not so easily overcome as the adjustment of triangles considered individually. Besides the necessity that the sum of the angles of every triangle should make up a known quantity, it is as necessary that the sum of the observed angles* round the horizon at any point should together make up 360°, and also that the length of any side may be the same by whatever route it may be calculated.

32. The complete solution of the problem, to determine the most probable corrections to the observed bearings or angles of a network of triangulation, which appears to be due to Bessel, is an application of the "method of least squares" invented by Gauss. From the extremely laborious nature of the calculations, very few applications of the theory to actual triangulations have been yet made; the principal instances will be found in Bessel's account of his measurement of an Arc in Prussia, and in the calculation of the Arc in Lapland by Rosenberger, in No. 121 and 122 of the Astronomische Nachrichten.

Theory of most probable Corrections.

33. The following results of the theory of probabilities are those most applicable to the calculation of geodetical results:—

(A.) If a series of direct and independent observations be made under similar circumstances of a quantity x, then the *weight* of the result is equal to the number of observations, divided by twice the mean of the squares of the differences of the individual observations from the mean or average of all the observations; thus, suppose $m_1\, m_2\, m_3 \ldots . m_n$ to be the observed values of x, and m their mean, then the weight of this determination of x, is

$$w = \frac{\frac{1}{2}n^2}{(m-m_1)^2 + (m-m_2)^2 + \ldots . (m-m_n)^2} = \frac{n}{2\mu}$$

where μ is the mean square of the errors. The corresponding probable error is

$$p = \cdot 674 \left(\frac{\mu}{n}\right)^{\frac{1}{2}} = \cdot 477 \left(\frac{1}{w}\right)^{\frac{1}{2}}$$

so that the weight is inversely as the square of the probable error. The nature of the 'probable error' may be defined as being such that it is an even chance that the truth will lie between the limits $m + p$ and $m - p$, or that the chances of the error exceeding or falling short of p without respect to sign, are equal.

(B.) The probable error of a quantity which is a function of several observed elements, is equal to the square root of the sum of the squares of the probable errors that would arise from each of the observed elements taken separately. Thus if u be a function of the observed quantities $x_1\, x_2\, x_3 \ldots.$ of which the probable errors are $e_1\, e_2\, e_3 \ldots$ then the probable errors that would result in u from each of the x's taken separately would be

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* That is, supposing the angles to be *independently* observed, which is not the case in the Ordnance Survey.

$$\frac{du}{dx_1} e_1 \qquad \frac{du}{dx_2} e_2 \qquad \frac{du}{dx_2} e_3 \ldots.$$

and the probable error of u,

$$e = \left(\frac{du^2}{dx_1^{\,2}} e_1^{\,2} + \frac{du^2}{dx_2^{\,2}} e_2^{\,2} + \frac{du^2}{dx_3^{\,2}} e_3^{\,2} + \ldots.\right)^{\frac{1}{2}}$$

If $u = x_1 + x_2 + x_3 \ldots.$ then we should have

$$e = (e_1^{\,2} + e_2^{\,2} + e_3^{\,2} + \ldots.)^{\frac{1}{2}}$$

and if w be the weight of the determination of u it will follow that

$$\frac{1}{w} = \frac{1}{w_1} + \frac{1}{w_2} + \frac{1}{w_3} + \ldots.$$

(C.) If $\alpha\, \beta\, \gamma \ldots$ be the observed values of the quantities $A\ B\ C \ldots$ which are connected by certain necessary relations, then the most probable values of $A\ B\ C$ .. are those which render the quantity

$$U = w_1\,(A-\alpha)^2 + w_2\,(B-\beta)^2 + w_3\,(C-\gamma)^2 + \ldots$$

an absolute minimum; $w_1\, w_2\, w_3 \ldots.$ being the weights of $\alpha\, \beta\, \gamma \ldots.$ respectively.

From (B) we may draw immediately the following conclusions:—The weight of an angle which is the sum or difference of two observed angles is the reciprocal of the sum of the reciprocals of the weights of the observed angles. If two base lines be measured under similar circumstances and with the same apparatus, the probable errors of the measurement will be as the square roots of the lengths, and the weights will be inversely as the lengths.

As an example of the application of this principle, suppose it required to find the probable errors in the sides $a\ b$ of a triangle as calculated from the measured base c and the observed angles A and B, and let $\gamma\ \alpha\ \beta$ be the respective probable errors of the three last quantities; then

$$a = c\frac{\sin A}{\sin (A+B)}$$

$$\frac{da}{dc} = \frac{a}{c} \qquad \frac{da}{dA} = \frac{b}{\sin C} \qquad \frac{da}{dB} = \frac{a \cos C}{\sin C}$$

and therefore the probable error of a is equal to

$$\left(\frac{a^2}{c^2}\gamma^2 + \frac{b^2\,\alpha^2}{\sin^2 C} + \frac{a^2\,\beta^2 \cos^2 C}{\sin^2 C}\right)^{\frac{1}{2}}$$

consequently if we put $\varkappa$ for the ratio of the probable error of c to c, the probable errors of a and b will be

$$e_a = \left((a\varkappa)^2 + (\alpha b \operatorname{cosec} C)^2 + (a\beta \cot C)^2\right)^{\frac{1}{2}}$$

$$e_b = \left((b\varkappa)^2 + (a\beta \operatorname{cosec} C)^2 + (\alpha b \cot C)^2\right)^{\frac{1}{2}}$$

the errors therefore increase rapidly as the angle C approaches zero or 2π.

Again; let it be required to determine the probable errors of x and y as obtained from the equations

$$\begin{aligned} a x + b y + m &= 0 \\ a' x + b' y + m' &= 0 \\ a'' x + b'' y + m'' &= 0 \\ \cdots\cdots\cdots\cdots \end{aligned}$$

$m\ m'\ m'' \ldots$ being the results of independent observations. The most probable values of x and y are

$$x = \frac{(ab)\,(bm) - (b^2)\,(am)}{(a^2)\,(b^2) - (ab)^2}\ ; \quad y = \frac{(ab)\,(am) - (a^2)\,(bm)}{(a^2)\,(b^2) - (ab)^2}$$

which, making $(a^2)\,(b^2) - (ab)^2 = \mathrm{Q}$, may be put in the form

$$x = \frac{1}{\mathrm{Q}}\Big(m\{(ab)\,b - (b^2)\,a\} + m'\{(ab)\,b' - (b^2)\,b'\} + \cdots\cdots\Big)$$

$$y = \frac{1}{\mathrm{Q}}\Big(m\{(ab)\,a - (a^2)\,b\} + m'\{(ab)\,a' - (a^2)\,a'\} + \cdots\cdots\Big)$$

Now, assuming the probable errors of the observed quantities m to have an average value e_m, the probable errors of x and y are obtained by summing the squares of the coefficients of m, thus we get

$$e_x = \frac{e_m}{\mathrm{Q}}\left((a^2)\,(b^2)^2 - (ab)^2\,(b^2)\right)^{\frac{1}{2}} = e_m\left((a^2) - \frac{(ab)^2}{(b^2)}\right)^{-\frac{1}{2}}$$

$$e_y = \frac{e_m}{\mathrm{Q}}\left((a^2)^2\,(b^2) - (ab)^2\,(a^2)\right)^{\frac{1}{2}} = e_m\left((b^2) - \frac{(ab)^2}{(a^2)}\right)^{-\frac{1}{2}}$$

From (C) we may derive immediately the following well-known principle:—Suppose one series of observations of a quantity x give the value m with a weight w, while others give the values $m'\ m'' \ldots$ with the weights $w'\ w'' \ldots.$, then the most probable value of x is found by multiplying each value by its weight, and dividing the sum of these quantities by the sum of the weights; for the quantity to be made a minimum is

$$2u = w(x - m)^2 + w'(x - m')^2 + w''(x - m'') + \cdots\cdots$$

$$\therefore \quad \frac{du}{dx} = w(x - m) + w'(x - m') + w''(x - m'') + \cdots\cdots = 0$$

$$\therefore \quad x = \frac{w'm' + wm + w''m'' + \cdots\cdots}{w + w' + w'' + \cdots\cdots}$$

If observations made upon two quantities x and y, between which there existed the necessary relation $ax + by = c$, gave the values $m\ m' \ldots$ for x, and $m_1\ m_1' \ldots$ for y, with the respective weights $w\ w' \ldots$ and $w_1\ w_1' \ldots$, then we should have

$$2u = w(x-m)^2 + w'(x-m')^2 + \ldots + w_1(y-m_1)^2 + w_1'(y-m_1')^2 + \ldots$$

$$\therefore 0 = \Big(w(x-m) + w'(x-m') + \ldots\Big)\,dx + \Big(w_1(y-m_1) + w_1'(y-m_1') + \ldots\Big)\,dy$$

but from the necessary connection of x and y we have $adx = -bdy$, and therefore, using brackets to denote summation,

$$0 = bx(w) - ay(w_1) + a(w_1m_1) - b(w\,m)$$

which, together with the equation of condition between x and y, will serve to determine those quantities.

34. The principle (C) is of great importance, and enables us to obtain that particular system of corrections to the observed angles in a triangulation (out of an infinite number that will satisfy all the required geometrical conditions) which has the greatest probability. For instance, suppose A, B, C, to be the true angles of a triangle, and that the mean of the observations of the angle A gives a value α with a weight w_1, and let β, γ be the observed values of B, C, their weights being w_2, w_3; then if ϵ be the error in the sum of angles of the triangle, the equation of condition is

$$A - \alpha + B - \beta + C - \gamma = \epsilon$$

and the most probable values that can be given for A, B, C, result from the condition that

$$w_1(A-\alpha)^2 + w_2(B-\beta)^2 + w_3(C-\gamma)^2$$

shall be a minimum, A, B, C, being the variables. Differentiating these equations, we have

$$\begin{aligned} w_1(A-\alpha)\,dA + w_2(B-\beta)\,dB + w_3(C-\gamma)\,dC &= 0 \\ dA \quad + \quad dB \quad + \quad d &= C0 \end{aligned}$$

from which by eliminating dC there results

$$\Big(w_1(A-a) - w_3(C-\gamma)\Big)\,dA + \Big(w_2(B-\beta) - w_3(C-\gamma)\Big)\,dB = 0$$

the differentials dA, dB, being indeterminate, their coefficients in this equation are necessarily zero; we have therefore for the determination of A, B, C, these three equations:—

$$\begin{aligned} A - \alpha + B - \beta + C - \gamma &= \epsilon \\ w_1(A-\alpha) - w_3(C-\gamma) &= 0 \\ w_2(B-\beta) - w_3(C-\gamma) &= 0 \end{aligned}$$

which give the following values

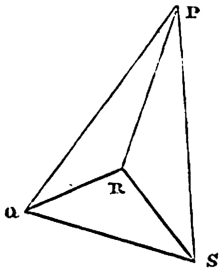
$$A - \alpha = \frac{\frac{1}{w_1}}{\frac{1}{w_1} + \frac{1}{w_2} + \frac{1}{w_3}} \cdot \varepsilon$$

$$B - \beta = \frac{\frac{1}{w_2}}{\frac{1}{w_1} + \frac{1}{w_2} + \frac{1}{w_3}} \cdot \varepsilon$$

$$C - \gamma = \frac{\frac{1}{w_3}}{\frac{1}{w_1} + \frac{1}{w_2} + \frac{1}{w_3}} \cdot \varepsilon$$

so that to rectify the error of any triangle taken without any further connection, this error must be divided into three parts proportional to the reciprocals of the weights of the angles.

35. Or again, suppose it be required to fix a point P in connection with a triangulation already completed, and so calculated that its results will admit of no further modification. Let Q, R, S, be three points of this triangulation, and suppose the angles observed for the connection to be PQR, QPR, QPS, RPS, PSR, each angle being observed independently. Let α, β, μ, γ, δ be the observed values of the angles with the weights w_1 w_2 w_3 w_4 w_5, the true or most probable values being $A = \alpha + x_1$, $B = \beta + x_2$, $B + C = \mu + x_3$, $C = \gamma + x_4$, $D = \delta + x_5$.



Now we know the angle QRS, and therefore the sum of the angles A, B, C, D; this gives the equation

$$x_1 + x_2 + x_4 + x_5 = f \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (1)$$

where f is known from the angle QRS. It is also evident that

$$x_2 + x_4 - x_3 = \mu - \beta - \gamma = g \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (2)$$

The triangles QRP and RSP will give two means of calculating the line RP, and these two values must be equal; therefore

$$\text{QR}\frac{\sin A}{\sin B} = \text{RS}\frac{\sin D}{\sin C}$$

$$\therefore \frac{\sin A \sin C}{\sin B \sin D} = \frac{\text{RS}}{\text{QR}} = h$$

where h is known, since RS, RQ, are given. Substituting for A, B, C, D, their assumed values,

$$\frac{\sin(\alpha + x_1)\sin(\gamma + x_4)}{\sin(\beta + x_2)\sin(\delta + x_5)} = h$$

Now $\sin(\alpha + x_1)$, supposing x_1 to be a very small quantity, is nearly equal to $\sin\alpha \cdot (1 + x_1 \cot\alpha)$; if we substitute this and similar expressions for the other sines in the above equation, we shall have, since the squares and products of the quantities x are neglected,

$$1 + x_1 \cot\alpha + x_4 \cot\gamma - x_2 \cot\beta - x_5 \cot\delta = h \frac{\sin\beta \sin\delta}{\sin\alpha \sin\gamma}$$

or, to abbreviate,

$$ax_1 - bx_2 + cx_4 - ex^5 = k \tag{3}$$

These three equations are necessary relations between the quantities x, and there are no other geometric conditions to be fulfilled. Then to obtain the most probable values of $x_1\ x_2 \ldots$ they must be such as to render a minimum the quantity $w_1 x_1^2 + \ldots\ldots w_5 x_5^2$; hence

$$w_1 x_1 \cdot dx_1 + w_2 x_2 \cdot dx_2 + w_3 x_3 \cdot dx_3 + w_4 x_4 \cdot dx_4 + w_5 x_5 \cdot dx_5 = 0 \tag{4}$$

Now were the quantities dx_1, $dx_2 \ldots$ all independent, this equation would simply resolve itself into $x_1 = 0$, $x_2 = 0$, &c., but by differentiating the equations (1) (2) (3), we get

$$\begin{aligned} dx_1 + dx_2 + dx_4 + dx_5 &= 0 \\ dx_2 - dx_3 + dx_4 &= 0 \\ adx_1 - bdx_2 + cdx_4 - edx_5 &= 0 \end{aligned}$$

and, by transforming,

$$\begin{aligned} (a + e)\, dx_1 - (b + c)\, dx_2 + (c + e)\, dx_3 &= 0 \\ (a + e)\, dx_1 - (b - e)\, dx_2 + (c + e)\, dx_4 &= 0 \\ -(a - c)\, dx_1 + (b + c)\, dx_2 + (c + e)\, dx_5 &= 0 \end{aligned}$$

thus the differentials dx_3, dx_4, dx_5, can be eliminated from equation (4), which will then take the form

$$\mathrm{P}\, dx_1 + \mathrm{Q}\, dx_2 = 0$$

and these differentials being arbitrary, we have $\mathrm{P} = 0$, $\mathrm{Q} = 0$; or, substituting their values,

$$\begin{aligned} w_1 (c + e)\, x_1 - w_3 (a + e)\, x_3 - w_4 (a + e)\, x_4 + w_5 (a - c)\, x_5 &= 0 \\ w_2 (c + e)\, x_2 + w_3 (b + c)\, x_3 + w_4 (b - e)\, x_4 - w_5 (b + c)\, x_5 &= 0 \end{aligned}$$

These two equations, taken with the equations of condition,

$$\begin{aligned} ax_1 - bx_2 + cx_4 - ex_5 &= k \\ x_1 + x_2 + x_4 + x_5 &= f \\ x_2 - x_3 + x_4 &= g \end{aligned}$$

are sufficient to determine the five unknown corrections.

36. We proceed now to consider the question more generally. The equations of condition of a triangulation are those which exist between the supernumerary observed quantities and their calculated values. For after there are just sufficient observations to fix

all the points, then any angle that may be subsequently observed can be compared with its calculated value. If a triangulation consist of $n + 2$ points, two of which determine the base line, or the magnitude and azimuthal position of the network, then the remaining n points will require $2n$ observed angles for their perfect fixation, so that if m be the actual number of observed angles then $m - 2n$ will be the number of equations of condition.

The manner of obtaining the equations is as follows:—Suppose a number of points A B C... already fixed, and that a new point P is observed from and observes reciprocally m of these points, then there will be formed $m - 1$ triangles, PAB, PBC.... in each of which the sum of the observed angles must equal 180° + the spherical excess; this gives at once $m - 1$ equations of condition, which are generally termed "angle equations." The $m - 2$ distances will each afford an equation of the form

$$\frac{PC}{PB} \cdot \frac{PB}{PA} \cdot \frac{PA}{PC} = 1$$

not, however, necessarily limited to three factors; equations of this form are called "side equations." Should P observe and be observed from only two points, then there will be but one equation of condition, an angle equation. When m is not less than 2, every other bearing not reciprocal, whether from P to the fixed points or from the fixed points to P, will give a side equation. Should $m = 1$, and P observe besides n points, there will be no angle equation, but $n - 1$ side equations; if $m = 0$, and r points observe P, there will be thus formed $r - 2$ side equations; and if $m = 0$, and P observes r' points, $r' - 3$ side equations must result, for it requires three observations from P to fix itself.

When independent angles are the observed quantities, then another species of equation must enter arising from this consideration, that at every point where all the angles round the horizon have been observed the sum of these angles must be equal to four right angles.

37. Finally, if M be the total number of observed bearings, N the number of observing points, then there will be $M - N$ angles for fixing $N - 2$ points, which require only $2N - 4$ angles, so that the number of equations of condition is $M - 3N + 4$. If there be also P points at which there are no observations, it is easy to see that, since two observations fix such a point, the number of equations of condition will be

$$M - 3N - 2P + 4$$

38. To these equations should be added those that may arise from the existence of more than one measured line in the triangulation, for if there be n bases there will exist $n - 1$ side equations, in order that there may be no discrepancies between the different measurements and the calculated lengths.

39. The form of the equations of condition is simple. If $x_1\ x_2 \ldots x_n$ be the corrections to the observed bearings, then since the angle equations arise from making the sums and

differences of a number of observed bearings with their corrections attached equal to a certain quantity, their form must be

$$0 = a \pm x_1 \pm x_2 \pm \,.\,.\,.\,.\,.$$

and in the side equations, if for the ratios of different sides we substitute the corresponding ratios of sines, the equation takes the form

$$\frac{\sin(A + x_1)\sin(B + x_2)\;.\,.\,.\,.}{\sin(C + x_3)\sin(D + x_4)\;.\,.\,.\,.} = 1$$

but since x_1 x_2...are very small quantities we may neglect their squares and higher powers, so that

$$\sin(A + x_1) = \sin A + x_1 \cos A = \sin A\,(1 + x_1 \cot A)$$

and similarly for the other sines; substituting this and the corresponding values in the above equation, it becomes

$$x_1 \cot A + x_2 \cot B - x_3 \cot C - x_4 \cot D + \,.\,.\,.\,. = \frac{\sin C \sin D\;.\,.\,.}{\sin A \sin B\;.\,.\,.} - 1$$

so that all equations of condition are of the form

$$0 = a + a_1 x_1 + a_2 x_2 + a_3 x_3 + \,.\,.\,.\,.$$

40. The problem then is this:—Given n equations,

$$\left.\begin{array}{l} 0 = a + a_1x_1 + a_2x_2 + a_3x_3 + \dots a_mx_m \\ 0 = b + b_1x_1 + b_2x_2 + b_3x_3 + \dots b_mx_m \\ 0 = c + c_1x_1 + c_2x_2 + c_3x_3 + \dots c_mx_m \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ 0 = q + q_1x_1 + q_2x_2 + q_3x_3 + \dots q_mx_m \end{array}\right\} \quad (1)$$

between m, $(m > n)$, unknown quantities, x_1, x_2, ... x_m, which are the corrections to the observed bearings, to determine these quantities so that they shall satisfy the n equations, and make the quantity

$$2u = w_1 x_1^2 + w_2 x_2^2 + \,.\,.\,.\,.\, w_m x_m^2$$

a minimum. The corrections thus determined must evidently give a consistent triangulation, and also that one which is most probable or requires the least total amount of alteration of the observations, having respect to the different weights or probable errors of each mean bearing.

Now in order that u may be a minimum, we must have $du = 0$, or

$$0 = w_1x_1 \cdot dx_1 + w_2x_2 \cdot dx_2 + \dots\dots + w_mx_m \cdot dx^m \qquad (2)$$

but by differentiating the equations (1) we have the following:—

$$\left.\begin{array}{l} 0 = a_1dx_1 + a_2dx_2 + a_3dx_3 + \dots + a_mdx_m \\ 0 = b_1dx_1 + b_2dx_2 + b_3dx_3 + \dots + b_mdx_m \\ 0 = c_1dx_1 + c_2dx_2 + c_3dx_3 + \dots + c_mdx_m \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ 0 = q_1dx_1 + q_2dx_2 + q_3dx_3 + \dots + q_mdx_m \end{array}\right\} \quad (3)$$

which establish n relations between the m differentials; we may therefore determine n of them, in terms of the remaining $m-n$, and thus the number of differentials in (2) is reduced by substitution from m to $m-n$, which are perfectly arbitrary; and therefore making their coefficients respectively equal to zero, there will result $m-n$ equations between the quantities x, which, together with the n equations of condition, will suffice to determine these corrections.

But the method practically adopted is that of independent multipliers, which is as follows:—Multiply the equations (3) by indeterminate quantities $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$, and adding their sum to the equation (2) multiplied by -1, we have the following system of equations:—

$$\left.\begin{aligned} 0 = \;\; & (-w_1x_1 + \lambda_1a_1 + \lambda_2b_1 + \lambda_3c_1 + \ldots \lambda_nq_1)\,dx_1 \\ & + (-w_2x_2 + \lambda_1a_2 + \lambda_2b_2 + \lambda_3c_2 + \ldots \lambda_nq_2)\,dx_2 \\ & + (-w_3x_3 + \lambda_1a_3 + \lambda_2b_3 + \lambda_3c_3 + \ldots \lambda_nq_3)\,dx_3 \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ & + (-w_mx_m + \lambda_1a_m + \lambda_2b_m + \lambda_3c_m + \ldots \lambda_nq_m)\,dx_m \end{aligned}\right\} \qquad (4)$$

Now as there are n arbitrary quantities λ, we may assume n of these coefficients of the differentials equal to zero; the quantities λ being thus determined, and then substituted in the remaining coefficients, the expression will take the form

$$0 = \mathrm{P}\,dx + \mathrm{Q}\,dx_1 + \ldots$$

containing $m-n$ differentials; and these being independent, it follows that $\mathrm{P}=0$, $\mathrm{Q}=0$, &c. . . . which amounts to putting at once each of the coefficients of the m differentials in (4) separately equal to zero; therefore we have the following m equations:—

$$\left.\begin{aligned} w_1x_1 &= \lambda_1a_1 + \lambda_2b_1 + \lambda_3c_1 + \ldots \lambda^nq_1 \\ w_2x_2 &= \lambda_1a_2 + \lambda_2b_2 + \lambda_3c_2 + \ldots \lambda_nq_2 \\ w_3x_3 &= \lambda_1a_3 + \lambda_2b_3 + \lambda_3c_3 + \ldots \lambda_nq_3 \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ w_mx_m &= \lambda_1a_m + \lambda_2b_m + \lambda_3c_m + \ldots \lambda_nq_m \end{aligned}\right\} \qquad (5)$$

Now substitute $x_1, x_2 \ldots x_m$ as derived from these equations in the n equations of condition; and we obtain the following set,

$$\left.\begin{aligned} 0 &= a + \lambda_1\left(\frac{a^2}{w}\right) + \lambda_2\left(\frac{ab}{w}\right) + \lambda_3\left(\frac{ac}{w}\right) + \ldots\ldots \lambda_n\left(\frac{aq}{w}\right) \\ 0 &= b + \lambda_1\left(\frac{ab}{w}\right) + \lambda_2\left(\frac{b^2}{w}\right) + \lambda_3\left(\frac{bc}{w}\right) + \ldots\ldots \lambda_n\left(\frac{bq}{w}\right) \\ 0 &= c + \lambda_1\left(\frac{ac}{w}\right) + \lambda_2\left(\frac{bc}{w}\right) + \lambda_3\left(\frac{c^2}{w}\right) + \ldots\ldots \lambda_n\left(\frac{cq}{w}\right) \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 0 &= q + \lambda_1\left(\frac{aq}{w}\right) + \lambda_2\left(\frac{bq}{w}\right) + \lambda_3\left(\frac{cq}{w}\right) + \ldots\ldots \lambda_n\left(\frac{q^2}{w}\right) \end{aligned}\right\} \qquad (6)$$

From these n equations, the n quantities λ must be eliminated, and their numerical values obtained. Finally, by the substitution of their values in (5), the numerical values of $x_1, x_2 \ldots\ldots x_m$ will result.

41. The different steps of the process are then as follows:—

First, the obtaining of the geometrical equations of condition supplied by the connexion of the triangulation.

Second, the substitution in these equations, of the observed bearings, each with its unknown correction appended. In taking the logarithm of a side equation such as

$$\frac{\sin(A + x_1)\sin(B + x_2)\ldots}{\sin(C + x_3)\sin(D + x_4)\ldots} = 1$$

if the quantities x are to be expressed in seconds, then

$$\log\sin(A + x_1) = \log\sin A + M\sin 1''\cot A\cdot x$$

Third, the equations of condition being written out in their algebraic form, and unknown multipliers assumed, the equations (5) are to be formed.

Fourth, from these equations the corrections must be obtained in terms of the multipliers, and substituted in the equations of condition.

Fifth, the n equations of condition being in terms of the n multipliers, these equations must be numerically solved and the actual values of the multipliers obtained.

Sixth, the substitution of the values of the multipliers in the equations (5), whereby the values of the corrections are immediately obtained.

Seventh, the verification of the work by the substitution of the corrections in the equations of condition, and also by the working out of the whole triangulation. The work may also be examined in the following manner:—Supposing $x_1, x_2, x_3 \ldots$ to be the corrections to the bearings round a point, whose weights are $w_1, w_2, w_3 \ldots$ respectively, then it is clear that a system of corrections $x_1 + z, x_2 + z, x_3 + z \ldots$ would have satisfied the requirements of the figure quite as well as $x_1, x_2, x_3 \ldots$ Now in order that

$$w_1(x_1 + z)^2 + w_2(x_2 + z)^2 + w_3(x_3 + z)^2 + \ldots$$

should be a minimum, we must have

$$z(w_1 + w_2 + w_3 + \ldots) + w_1x_1 + w_2x_2 + w_3x_3 \ldots = 0$$

but as the above quantity will not, after $x_1, x_2, x_3 \ldots$ are calculated, admit of further diminution consistent with the figure, z must be $= 0$, and therefore

$$w_1x_1 + w_2x_2 + w_3x_3 + \ldots = 0$$

or the sum of corrections to all the bearings round any point, each multiplied by the weight of the bearing, must $= 0$.

§ VI.

Meridian Arcs.

42. If s be the length of an arc of the meridian contained between the latitudes λ_1 and λ_2, and if $2\lambda = \lambda_1 + \lambda_2$, $\phi = \lambda_2 - \lambda_1$, then by equation (33)

$$\phi = \frac{s}{\varrho} - e^2\left(\frac{\phi}{2}\right)^3 \cos 2\lambda$$

Where the value of ϱ is

$$\frac{1}{\varrho} = \frac{(1 - e^2 \sin^2 \lambda)^{\frac{3}{2}}}{a(1 - e^2)}$$

If we suppose a and e^2 to receive small increments δa and δe^2, then $\frac{s}{\varrho}$ becomes

$$\frac{s}{\varrho} + f \cdot \frac{\delta a}{a} + g \cdot \delta e^2$$

$$f = -\frac{s}{\varrho} \quad : \quad g = \frac{s}{\varrho} \frac{1 - \frac{3 - e^2}{2} \sin^2 \lambda}{(1 - e^2)(1 - e^2 \sin^2 \lambda)}$$

If therefore we consider a, e^2, as approximate elements of the elliptic arc, and $a + \delta a$, $e^2 + \delta e^2$, to be the true, the value of ϕ will be

$$\phi = \frac{s}{\varrho} - e^2 \left(\frac{\phi}{2}\right)^3 \cos 2\lambda + fu + gv$$

putting $\delta e^2 = v$ and $\delta a = a u$.

If also λ_2, λ_1, be the observed latitudes, $\lambda_2 + x_2$, $\lambda_1 + x_1$, the true or most probable latitudes, then these last quantities must be substituted in the above equation. In so doing we may neglect quantities of the order $e^2 x$, and therefore

$$x_2 - x_1 = \frac{s}{\varrho} - \phi - e^2 \left(\frac{\phi}{2}\right)^3 \cos 2\lambda + fu + gv \qquad (1)$$

in which s is the measured length of the arc, ϱ the radius of curvature calculated with the approximate elements for the mean of the observed terminal latitudes, and ϕ the observed amplitude.

43. If besides the extreme latitudes of the arc there are also given the observed latitudes of several intermediate points, together with the corresponding measured lengths, then there will exist the following set of equations, in which x_1 is the correction to the latitude of the southern station of the arc:—

$$\begin{aligned} x_2 &= x_1 + h_1 + f_1 u + g_1 v \\ x_3 &= x_1 + h_2 + f_2 u + g_2 v \\ x_4 &= x_1 + h_3 + f_3 u + g_3 v \\ &\cdots\cdots\cdots\cdots \end{aligned}$$

Now in order to obtain those values of u and v which most nearly represent the observations made at the different points in the arc, or which require the smallest amount of alteration of the observations, the quantity $x_1^2 + x_2^2 + x_3^2 + \cdots\cdots$ must be a minimum; that is,

$$U = x_1^2 + (x_1 + h_1 + f_1 u + g_1 v)^2 + (x_1 + h_2 + f_2 u + g_2 v)^2 + \cdots\cdots$$

must be a minimum by the variation of x_1, u, v: therefore if n be the number of observed latitudes in the arc,

$$\begin{aligned} n x_1 + (h) + (f)\,u + (g)\,v &= 0 \\ (f)\,x_1 + (fh) + (f^2)\,u + (fg)\,v &= 0 \\ (g)\,x_1 + (gh) + (fg)\,u + (g^2)\,v &= 0 \end{aligned}$$

by making the differential coefficients of U with respect to x_1, u, and v respectively $= 0$. If in the last two of these equations we substitute the value of x_1 as derived from the first, we get these two equations :—

$$\begin{aligned} &\left\{(f^2) - \frac{(f)^2}{n}\right\} u + \left\{(fg) - \frac{(f)\,(g)}{n}\right\} v + (f\,h) - \frac{(f)\,(h)}{n} = 0 \\ &\left\{(fg) - \frac{(f)\,(g)}{n}\right\} u + \left\{(g^2) - \frac{(g)^2}{n}\right\} v + (g\,h) - \frac{(g)\,(h)}{n} = 0 \end{aligned}$$

The quantities u and v thus obtained, determine the elliptic elements which best accord with the observed and measured amplitudes in the arc. If the terminal latitudes only, be observed, these two equations will become identical, and u and v consequently indeterminate, though connected by a necessary relation; that is, if any arbitrary value be assigned to u, a necessary value of v must follow, and vice versâ.

In combining a number of arcs of meridian in order to determine that spheroid which is most nearly represented by all the observations, the sum of the squares of all the alterations of the latitudes must be made the least possible; that is,

$$\begin{aligned} U = x_1^2 &+ (x_1 + h_1 + f_1\,u + g_1\,v)^2 + (x_1 + h_2 + f_2\,u + g_2\,v)^2 + \ldots\ldots \\ + x_1'^2 &+ (x_1' + h_1' + f_1'u + g_1'\,v)^2 + (x_1' + h_2' + f_2'u + g_2'v)^2 + \ldots\ldots \\ + x_1''^2 &+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

must be a minimum; u_1, v_1, x_1, x_1', x_1'' . . . . being the independent variables. Making the differential coefficients of U with respect to these quantities respectively $= 0$, we get

$$\begin{aligned} n\;x_1 + (h) + (f)\,u + (g)\,v &= 0 \\ n'\,x_1' + (h') + (f')\,u + (g')\,v &= 0 \\ \ldots\ldots\ldots\ldots\ldots \end{aligned}$$

and finally, by substituting the values of x_1, x_1' . . . . obtained from these equations in the differential coefficients of U with respect to u and v,

$$\left.\begin{aligned} &\Sigma\left\{(f^2) - \frac{(f)^2}{n}\right\} u + \Sigma\left\{(fg) - \frac{(f)\,(g)}{n}\right\} v + \Sigma\left\{(fh) - \frac{(f)\,(h)}{n}\right\} = 0 \\ &\Sigma\left\{(fg) - \frac{(f)\,(g)}{n}\right\} u + \Sigma\left\{(g^2) - \frac{(g^2)}{n}\right\} v + \Sigma\left\{(g\,h) - \frac{(g)\,(h)}{n}\right\} = 0 \end{aligned}\right\} \quad (2)$$

44. The following is the method of development given by Bessel in the Astronomische Nachrichten, No. 333. If ϱ be the radius of curvature of the meridian, s the length of the arc increasing with the latitude,

$$\frac{d\,s}{d\,\lambda} = \varrho = \frac{a\,(1 - e^2)}{(1 - e^2 \sin^2\lambda)^{\frac{3}{2}}}$$

Now putting

$$n = \frac{a - b}{a + b} \qquad \therefore \quad e^2 = \frac{4n}{(1 + n)^2}$$

$$\varrho = a(1 - n)(1 - n^2)(1 + 2n \cos 2\lambda + n^2)^{-\frac{3}{2}}$$

which may be put in the form

$$\varrho = a(1 - n)(1 - n^2)(1 + nz)^{-\frac{3}{2}}(1 + nz^{-1})^{-\frac{3}{2}}$$

By the application of the binomial theorem, the following general result obtains:—

$$\begin{aligned}(1 + nz)^p(1 + nz^{-1})^p = 1 + (pn)^2 &+ \left(\frac{p(p-1)}{1\cdot 2}n^2\right)^2 + \left(\frac{p(p-1)(p-2)}{1\cdot 2\cdot 3}n^3\right)^2 + \ldots\ldots \\ &+ \left(z + \frac{1}{z}\right)\left(pn + \frac{p^2(p-1)}{1\cdot 2}n^3 + \frac{p^2(p-1)^2(p-2)}{1\cdot 2^2\cdot 3}n^5 + \ldots\ldots\ldots\right) \\ &+ \left(z^2 + \frac{1}{z^2}\right)\left(\frac{p(p-1)}{1\cdot 2}n^2 + \frac{p^2(p-1)(p-2)}{1\cdot 2\cdot 3}n^4 + \ldots\ldots\ldots\ldots\right) \\ &+ \left(z^3 + \frac{1}{z^3}\right)\left(\frac{p(p-1)(p-2)}{1\cdot 2\cdot 3}n^3 + \frac{p^2(p-1)(p-2)(p-3)}{1\cdot 2\cdot 3\cdot 4}n^5 + \ldots\right) \\ &\ldots\ldots\ldots\end{aligned}$$

of which the law is sufficiently obvious. In the present instance $p = -\frac{3}{2}$; therefore if we put

$$\begin{aligned}N &= 1 + \left(\frac{3}{2}\right)^2 n^2 + \left(\frac{3\cdot 5}{2\cdot 4}\right)^2 n^4 + \ldots\ldots \\ N\alpha &= \frac{3}{2}n + \frac{3\cdot 5}{2\cdot 4}\cdot\frac{3}{2}n^3 + \ldots\ldots \\ N\alpha' &= \frac{3\cdot 5}{2\cdot 4}n^2 + \frac{3\cdot 5\cdot 7}{2\cdot 4\cdot 6}\cdot\frac{3}{2}n^4 + \ldots\ldots \\ N\alpha'' &= \frac{3\cdot 5\cdot 7}{2\cdot 4\cdot 6}n^3 + \ldots\ldots\end{aligned}$$

we shall have

$$\varrho = a(1 - n)(1 - n^2)\,N\left(1 - \alpha\cdot 2\cos 2\lambda + \alpha'\cdot 2\cos 4\lambda - \alpha''\cdot 2\cos 6\lambda + \ldots\ldots\right)$$

If now s be the length of a meridian arc terminated by the latitudes λ_1, λ_2,

$$s = \int_{\lambda_1}^{\lambda_2} \varrho\cdot d\lambda$$

and therefore, putting $2\lambda = \lambda_1 + \lambda_2$, $\phi = \lambda_2 - \lambda_1$, $a(1 - n)(1 - n^2)N = a'$

$$s = a'\left(\phi - 2\alpha\sin\phi\cos 2\lambda + \alpha'\sin 2\phi\cos 4\lambda - \frac{2}{3}\alpha''\sin 3\phi\cos 6\lambda + \ldots\ldots\right)$$

The problem requires that the observed latitudes λ_1, λ_2 . . . . should be brought to accord with the measured distances by the application of small corrections x_1, x_2 . . . . such that $x_1^2 + x_2^2 + \ldots\ldots$ shall be a minimum, the variable quantities being the elements of the figure of the earth, or a and n. Those values of a and n which make this function

$x_1^2 + x_2^2 + \cdot \cdot \cdot$ a minimum, determine the most probable figure of the earth, as far as depends on the arcs under consideration. Putting, therefore, $\lambda_1 + x_1$, $\lambda_2 + x_2$, as the true latitudes in the above equation, we have

$$\frac{s}{a'} = \phi - 2\,\alpha \sin\phi \cos 2\,\lambda + \alpha' \sin 2\,\phi \cos 4\,\lambda - \frac{2}{3}\alpha'' \sin 3\,\phi \cos 6\,\lambda + \cdot \cdot \cdot \cdot + \mu\,(x_2 - x_1)$$

$$\mu = 1 - 2\,\alpha \cos\phi \cos 2\,\lambda$$

Instead of making a the unknown linear dimension, we may make the length of a *mean degree* of the meridian the required quantity: if we call this g, then $180\,g = \pi\,a'$

$$\therefore \quad x_2 - x_1 = \frac{1}{\mu}\left(\frac{s\,\pi}{180\,g} - \phi + 2\,\alpha \sin\phi \cos 2\,\lambda - \alpha' \sin 2\,\phi \cos 4\,\lambda + \cdot \cdot \cdot \cdot\right)$$

If now g_1 and α_1 be approximate values of g and α, we may put

$$\frac{1}{g} = \frac{1}{g_1}\,(1 + u), \qquad \alpha = \alpha_1\,(1 + v)$$

u and v being two small quantities to be determined.

By means of the values of $N\,\alpha\,\alpha' \cdot \cdot \cdot \cdot$ in terms of n, we derive the following:—

$$n = \frac{2}{3}\,\alpha + \frac{1}{9}\,\alpha^3$$

$$\alpha' = \frac{5}{6}\,\alpha^2 + \frac{5}{54}\,\alpha^4$$

When therefore α_1 becomes $\alpha_1 + \alpha_1 v$, $\alpha_1{}'$ becomes $\alpha_1{}' + \dfrac{d\,\alpha_1}{d\,\alpha_1}\;\alpha_1 v$

$$\therefore \quad \alpha' = \alpha_1{}' + v\left(\frac{5}{3}\,\alpha_1^2 + \frac{10}{27}\,\alpha_1^4\right)$$

Therefore the expression for $x_2 - x_1$ becomes

$$\begin{aligned} x_2 - x_1 = {} & \frac{1}{\mu}\left\{\frac{\pi\,s}{180\,g_1} - \phi + 2\,\alpha_1 \sin\phi \cos 2\,\lambda - \frac{5}{6}\,\alpha_1^2 \sin 2\,\phi \cos 4\,\lambda\right\} \\ & + \frac{1}{\mu}\left\{\frac{\pi\,s}{180\,g_1}\right\} u + \frac{1}{\mu}\left\{2\,\alpha_1 \sin\phi \cos 2\,\lambda - \frac{5}{3}\,\alpha_1^2 \sin 2\,\phi \cos 4\,\lambda\right\} v \end{aligned} \qquad (3)$$

in which we have omitted the terms in α_1^4, as they are so small as to be insensible.

The equation of condition thus formed corresponds with equation (1); the unknown quantities $u\,v$ are not, however, the same in both equations.

§ VII.

Logarithms.

45. Though logarithms to seven places of decimals are generally sufficient for ordinary calculations, yet when considerable accuracy is required for geodetical calculations,

eight places of decimals must be used. It is not always or generally possible to have the last figure of the resulting logarithm after any lengthened calculations true. Errors of one and often of two units in the last place are very probable, and therefore there should be at least one superfluous decimal.

In the absence of any printed tables to eight places, Vlacq's ten-place logarithms have been used. The method of interpolating by differences has been avoided as more troublesome than that supplied by the usual series

$$\log \frac{H}{K} = 2M \left\{ \frac{H-K}{H+K} + \frac{1}{3}\left(\frac{H-K}{H+K}\right)^3 + \dots \right\}$$

The second term within the bracket is always so small as not to be required; in those cases in which it might be required, other methods present themselves.

If we make successively $H = \sin(A+x)$, $K = \sin A$, and $H = \tan(A+x)$, $K = \tan A$, and substitute these values in the above series, omitting the second term, we shall have

$$\log \frac{\sin(A+x)}{\sin A} = 2M \frac{\tan \frac{1}{2}x}{\tan(A+\frac{1}{2}x)}$$

$$\log \frac{\tan(A+x)}{\tan A} = 2M \frac{\sin x}{\sin(2A+x)}$$

Also if we take N to represent any large number and n a very small number,

$$\log \frac{N+n}{N} = 2M \frac{n}{(2N+n)}$$

whence we have the following rules:—

$$\left.\begin{aligned} \log \sin(A+x) &= \log \sin A + Mx \cot\left(A + \frac{x}{2}\right) \\ \log \tan(A+x) &= \log \tan A + 2Mx \operatorname{cosec}(2A+x) \\ \log(N+n) &= \log N + \frac{Mn}{N+\frac{1}{2}n} \end{aligned}\right\} \qquad (\alpha)$$

This method is found much more manageable than that of differences when second differences are sensible.

The logarithms of the functions of very small arcs are more easily calculated directly than obtained by interpolation. For this purpose the series

$$\left.\begin{aligned} \log \sin x &= \log x - \frac{M}{6}x^2 - \frac{M}{180}x^4 \\ \log \cos x &= - \frac{M}{2}x^2 - \frac{M}{12}x^4 \\ \log \tan x &= \log x + \frac{M}{3}x^2 + \frac{7M}{90}x^4 \\ \log x &= \log \sin x + \frac{M}{6}\sin^2 x + \frac{11M}{180}\sin^4 x \\ \log x &= \log \tan x - \frac{M}{3}\tan^2 x + \frac{13M}{90}\tan^4 x \end{aligned}\right\} \qquad (\beta)$$

are useful.* The remaining terms of these series are seldom required, but may be obtained from the general expressions

$$\log \frac{\sin x}{x} = - \mathrm{M} \Sigma \frac{2^{2n-1}}{n} \mathrm{B}_{2n-1} \frac{x^{2n}}{1 \ldots 2n}$$

$$\log \cos x = - \mathrm{M} \Sigma \frac{2^{2n-1}(2^{2n}-1)}{n} \mathrm{B}_{2n-1} \frac{x^{2n}}{1 \ldots 2n}$$

$$\log \frac{\tan x}{x} = + \mathrm{M} \Sigma \frac{2^{2n}(2^{2n}-1)}{n} \mathrm{B}_{2n-1} \frac{x^{2n}}{1 \ldots 2n}$$

n taking the successive values 1, 2, 3 . . . . and B representing the numbers of Bernoulli,

$$\mathrm{B}_1 = \frac{1}{6},\ \mathrm{B}_3 = \frac{1}{30},\ \mathrm{B}_5 = \frac{1}{42},\ \mathrm{B}_7 = \frac{1}{30},\ \mathrm{B}_9 = \frac{5}{66}$$

---

* The following Table gives the quantity to be applied to the last figures of logarithms to ten places, for the terms in x^4, from 1° to 2°.

TERM.	1°	$1^\circ\ 10'$	$1^\circ\ 15'$	$1^\circ\ 20'$	$1^\circ\ 25'$	$1^\circ\ 30'$	$1^\circ\ 35'$	$1^\circ\ 40'$	$1^\circ\ 42'$	$1^\circ\ 44'$	$1^\circ\ 46'$	$1^\circ\ 48'$	$1^\circ\ 50'$	$1^\circ\ 52'$	$1^\circ\ 54'$	$1^\circ\ 56'$	$1^\circ\ 58'$	$2^\circ\ 0'$
$\frac{1}{180}\mathrm{M}x^4$	2	4	6	7	9	11	14	17	19	20	22	23	25	27	29	31	33	36
$\frac{1}{12}\mathrm{M}x^4$	33	61	82	106	135	169	211	289	280	303	327	352	379	408	438	469	502	537
$\frac{7}{90}\mathrm{M}x^4$	31	58	77	99	126	159	197	242	262	283	305	329	354	380	408	438	468	501
$\frac{11}{180}\mathrm{M}x^4$	24	45	60	78	99	124	155	190	206	222	240	258	278	299	321	344	368	394
$\frac{13}{90}\mathrm{M}x^4$	58	108	142	184	234	294	366	449	486	525	567	611	657	706	758	813	870	930

SECTION VII.

REDUCTION OF THE TRIANGULATION.

The results of a triangulation so well connected as that of the British Isles, extending from latitude 49° 53′ to 60° 50′, and from longitude 1° 40′ east to 10° 20′ west, must have much weight in the determination of the general figure and dimensions of the earth, though the more immediate and precise result will be the knowledge of the nature and curvature of the surface within the limits of the triangulation.

The general figure of the earth may be considered as already determined with much accuracy, the mean errors of Bessel's determination of the linear dimensions being one twenty thousandth part; and of the ellipticity, one sixty-fifth part. The magnitude of this error depends principally upon the existence of those small disturbances of the direction of gravity which may be expected at any point on the surface of the earth, and of which the mean value may be taken at about $2''\cdot 6$. The only method of eliminating the effect of this unavoidable source of error is the multiplication of the number of points having observed latitudes, the observations being of the best kind, and very numerous. The *general* figure of the earth, or more strictly speaking, that spheroidal surface which makes the sum of the squares of the apparent deflections a minimum for the *whole* of the astronomically determined points on the surface of the earth as far as connected by triangulation, being determined, the next point is to determine the variations of the surface or the osculating ellipsoid for as many parts of the surface as possible: and in order that this may be accomplished in each case with precision, it is necessary that the triangulation should in the first place be effected with the best instruments; in the second place, the connections of the various points with one another should be as numerous as possible—that is, the number of superfluous observations or of equations of condition should be abundant, as increasing the probability of the results; and thirdly, the triangulation should be calculated so that the results should be strictly the *most probable*, and unaffected by any arbitrary method. The first two of these conditions appear to be well fulfilled in the present triangulation, and the necessity for the fulfilment of the third is obvious.

Acting upon these considerations, Lieut.-Colonel Yolland directed that the whole triangulation should be reduced according to the method of least squares. It will be evident from the explanation given in the preceding Section of the application of the results of the Theory of Probabilities to the determination of the proper system of corrections to be applied to the observed bearings of the various points in a triangulation, that the principal, and indeed only, obstacle to its being applied to any actual survey lies in the extremely voluminous nature of the calculations.

There are in what may be called the principal triangulation 218 points, 1554 observed bearings, and therefore an equal number of corrections to be determined, and 920 equations of condition: if, therefore, the whole were to be reduced in one mass, the solution of an equation of 920 unknown quantities would form part of the work. It is evident, therefore, that some method of approximation must be adopted; and this indeed is sufficient, provided that method be unobjectionable. Let

$$
\begin{aligned}
0 &= a_1 + b_1 x_1 + c_1 x_2 + e_1 x_3 + \cdots \\
0 &= a_2 + b_2 x_1 + c_2 x_2 + e_2 x_3 + \cdots \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
0 &= a_i + b_i x_k + c_i x_{k+1} + e_i x_{k+2} \cdots\cdot \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
0 &= a_n + b_n x_p + c_n x_{p+1} + e_n x_{p+2} \cdots\cdot \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
0 &= a_r + b_r x_q + c_r x_{q+1} + e_r x_{q+2} \cdots \\
&\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
$$

be the equations of condition, then if the geometrical positions of the equations (i), (n), (r), as referred to the triangulation, be some hundred miles apart, with many intervening points, the equations containing x will not contain x_k, nor will those containing x_k contain x_p, nor will those containing x_p contain x_q. The corrections that appear in the first equations of condition gradually disappear and are succeeded by others, which in their turn give way to others, as the equations proceed; which is merely in accordance with the obvious fact, that the position of a point is dependent upon all the observations made round it to a considerable distance: the influence, however, diminishes with the distance, and at a sufficient distance becomes almost imperceptible. It is then quite possible to obtain a satisfactory approximation, as widely separated equations are in a manner independent; and the following is the method that has been actually carried out:—The triangulation was divided into a number of parts or figures, each affording a not unmanageable number of equations of condition. One of these being corrected or computed independently of all the rest, the corrections so obtained were substituted (as far as they entered) in the equations of condition of the next figure, and the sum of the squares of the remaining corrections in that figure made a minimum. The corrections thus obtained for the second figure were substituted in the third, and so on. Four of the divisions of the triangulation are independent commencements, having no corrections from adjacent figures carried into them, but having the corrections determined in them carried out to the contiguous divisions.* By this means, however, the ratios of the measured bases have not entered into the equations of condition, no two bases falling into the same division of the triangulation; and in an extended triangulation in which the bases have been measured with equal care, the conditions of agreements between the bases should exist to prevent the accumulation of error; but the neglect of these conditions will bring out in the comparison of bases a good test of the accuracy of the theodolite work.

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* This method of approximation is not the best that might have been adopted, and is liable to some objections; but it is certain that the results so obtained, when the observations are not bad, must be very close to the truth: and without a further expenditure of labour, better results could not, by any method perhaps, be had.

Division of the Triangulation.

Omitting in the first place the extremities of base lines and several sector points, which are connected to the general work by small and special triangulations, the remainder of the principal points were divided into 21 figures, which are indicated in Plate XVIII., and are as follows:—

Figure 1. (Plate IV.)

N. E. Lough Foyle Base.
Drung Point.
Knocklayd.
Divis.

S. E. Lough Foyle Base.
Cundtham.
Trostan.
Vicar's Cairn.
Cuilcagh.

Mount Sandy.
S. Snaght.
Sawel.
S. Donard.

Figure 2. (Plate IV.)

S. Snaght.
S. League.
Bencorr.
Kippure.

Cuilcagh.
Keeper.
Knockanaffrin.

S. Donard.
Nephin.
Croghan.
Baurtregaum.

Figure 3. (Plate IV.)

Baurtregaum.
Knockmealdown.
Taur.
Caherbarnagh.

Keeper.
Kippure.
Knocknaskagh.
Hungry Hill.

Knockanaffrin.
Galtymore.
Doolieve.
Carrigfadda.

Figure 4. (Plate V.)

Knocklayd.
S. Donard.
Merrick.
Sca Fell.
Great Whernside.
Mowcopt.

Trostan.
Howth.
S. Berule.
Blackcomb.
Pendle Hill.
Axedge.
Ingleborough.

Divis.
Kippure.
Criffel.
Snowdon.
Whittle Hill.
Holme Moss.

Figure 5. (Plate V.)

Great Whernside.
Hartfell.
Cross Fell.
Sayrs Law.
Lumsden.
Botton Head.

Sca Fell.
Merrick.
Water Crag.
Cheviot.
Collier Law.

Criffel.
Wisp.
Dunrich.
Blackheddon.
Wordeslow.
Easington.

FIGURE 6. (Plate VI.)

Foula.
Ronas.
Saxavord.
Fair Isle.
Yell.
Brassa.
Fetlar.
Balta.

FIGURE 7. (Plate VI.)

Ben Clibrig.
Scarabin.
Wart Hill.
Stronsay.
Foula.
Fashven.
Ben Cheilt.
S. Ronaldshay.
Fitty Hill.
N. Ronaldshay.
Ben Hutig.
Dunnet Head.
Deerness.
Fair Isle.
Start Lighthouse.

FIGURE 8. (Plate VII.)

Ben Clibrig.
Cnoc-ghiubhais.
Ru Rea.
Scournalapich.
Ben Nevis.
Jura.
Fashven.
North Rona.
Monach.
Ben More, S. U.
Ben Heynish.
Ben Hutig.
Cleisham.
Storr.
Mamsuil.
Ben More, Mull.
Ben Tartevil.

FIGURE 9. (Plate VII.)

Ben Cheilt.
Scournalapich.
Jura.
Ben Wyvis.
Knock.
Ben Lawers.
Scarabin.
Ben Hutig.
Mamsuil.
Ben Macdui.
Cowhythe.
Glashmeal.
Ben Clibrig.
Ben Nevis.
Ben More, Mull.
Corryhabbie.
Mormonth.
Mount Battock.

FIGURE 10. (Plate VII.)

Mount Battock.
Knock.
Dudwick.
Corryhabbie.
Blue Hill.
Mormonth.
Ben Macdui.
Little Stirling.

FIGURE 11. (Plate VIII.)

Mount Battock.
Ben Lawers.
Jura.
Knocklayd.
S. Donard.
Criffel.
Sayrs Law.
Ben Lomond.
Glashmeal.
Ben Nevis.
Ben Tartevil.
Trostan.
S. Berule.
Hartfell.
Goat Fell.
Ben Macdui.
Ben More, Mull.
S. Snaght.
Divis.
Merrick.
Dunrich.
Lumsden.
Ben Cleugh.

Figure 12. (Plate IX.)

Cradle.
Paracombe.
Pillesdon.
Barrow Hill.
Trevose.
Precelly.
Dunkery.
High Wilhays.
Maker.
Hensbarrow.
Lundy Island.
Mendip.
Ryder's Hill.
Brown Willy.
Deadman.

Figure 13. (Plate X.)

Hensbarrow.
St. Agnes.
Karn Galver.
Wolf Rock.
Peninnis.
Trevose.
Karnbonellis.
Goonhilly.
St. Martin's.
Beacon Hill, Trescow.
Deadman.
Karnminnis.
Pertinny.
St. Agnes Lighthouse.
Telegraph Tower.

Figure 14. (Plate X.)

Beacon Hill.
Westbury.
Inkpen.
Dean Hill.
Stoke Hill.
Wingreen.
Milk Hill.
Upcot Down.

Figure 15. (Plate XI.)

Milk Hill.
Beacon Hill.
Westbury.
Barrow Hill.
Blackdown.
Corringdon.
Upcot Down.
Dean Hill.
Mendip.
High Wilhays.
Swyre Barrow.
Horton's Gazebo.
Inkpen.
Wingreen.
Pillesdon.
Ryder's Hill.
Dunnose.
Butser.

Figure 16. (Plate XI.)

Cradle.
Milk Hill.
Whitehorse.
Leith Hill.
Crowborough.
Dunstable.
Berkhampstead.
St. Paul's.
Mendip.
Inkpen.
Dunnose.
Malvern.
Ditchling.
Arbury.
Chingford.
Upcot Down.
Butser.
Broadway Tower.
Beachy Head.
Wrotham.
Epping.
Fairlight.
Severndroog.

Figure 17. (Plate XII.)

Kippure.
Snowdon.
Axedge.
Croghan.
Whittle Hill.
Longmount.
Tara.
Plynlimmon.
Knockanaffrin.
Cradle.
Malvern.
Broadway Tower.
Holme Moss.
Bardon.
Cader Idris.
Precelly.
Mowcopt.
Arbury.
Keeper.
Ballycreen.
Cyrn-y-brain.
Forth.
Mount Leinster.

FIGURE 18. (Plate XIII.)

Epping.
Leith Hill.
Paddlesworth.
Norwood.
Frittenfield.

Severndroog.
Beachy Head.
St. Paul's.
Danbury.

Wrotham.
Fairlight.
Gad's Hill.
St. Peter's.
Walton Tower.

FIGURE 19. (Plate XIII.)

Dunstable.
Danbury.
Thaxted.
Otley.
Balsham.
S. Lopham.

Berkhampstead.
Tharfield.
Stoke.
Mickfield.
Ely Minster.

Epping.
Walton.
Naughton.
Lawshall.
Swaffham.
Brandon.

FIGURE 20. (Plate XIV.)

Walton.
S. Lopham.
Orford Castle.
Bunwell.
Tofts.
Gorleston.

Otley.
Brandon.
Lawshall.
Hingham.
Norwich.

Mickfield.
Swaffham.
Laxfield.
Southwold.
Baconsthorpe.
Happisburgh.

FIGURE 21. (Plate XIV.)

Botton Head.
Axedge.
Dunstable.
Ely Minster.
Acklam Wold.
Back Tor.
Lincoln Minster.
Docking.
Tilton.
Lynn.

Great Whernside.
Bardon.
Tharfield.
Swaffham.
York Minster.
Clifton.
Boston.
Easton.
Naseby.

Holme Moss.
Arbury.
Balsham.
Baconsthorpe.
Garforth Cliff.
Crowle.
Walpole, St. Peter's.
Buckminster.
Keysoe.
Hanslope.

It will be observed that with the exception of Figures 1, 6, 7, 12, 14, each figure has some names in italics; the points so printed in each case have been fixed in a previous figure, and the remaining points are those which are fixed in the figure in question. Thus, for instance, in Figure 11 the three points, Goat Fell, Ben Lomond, Ben Cleugh, are fixed dependently upon twenty other points which have been fixed in previous figures—namely, 1, 4, 5, 8, and 9. Figures 1, 6, 7, 12, and 14 are independent.

The following Table shows the mutual relations of the different figures, the number of points fixed in each, the number of bearings for the fixation of these points, and the number of equations of condition.

Figures	Dependent upon Figures	Number of Points.		Number of Bearings.	Number of Equations.
		Observing.	Non-observing.		
1	—	13	—	74	39
2	1	8	—	40	16
3	2	6	2	52	30
4	1, 2	13	1	100	59
5	4	13	—	103	64
6	—	8	—	37	17
7	—	11	2	82	45
8	7	13	1	91	50
9	7, 8	9	—	87	60
10	9	3	—	21	12
11	1, 4, 5, 8, 9	3	—	90	77
12	—	15	—	81	40
13	12	10	2	88	54
14	—	8	—	45	25
15	12, 14	5	1	77	56
16	12, 13, 14	15	1	103	56
17	2, 4, 12, 16	6	3	75	47
18	16	7	—	52	31
19	16, 18	12	—	76	40
20	19	10	—	69	39
21	4, 5, 16, 17, 19, 20	14	3	111	63
TOTALS	—	202	16	1,554	920

Here if we put

M = the number of observed bearings,
N = the number of observing points,
P = the number of non-observing points,

$$\begin{aligned} 3P &= 606 \\ 2Q &= \underline{32} \\ 3P + 2Q &= 638 \\ M + 4 &= \underline{1558} \\ M - 3P - 2Q + 4 &= 920 = \text{the number of equations of condition.} \end{aligned}$$

The circumstances which determined in each particular figure the boundary points and lines were in general, that each division of the work should contain as many points and bearings as possible; and in the selection of the points forming the boundary line, the guiding principle was, that they should all be points well fixed by observations to and from them in every direction.

Method of Calculation exemplified.

We proceed now to give in detail the calculation of the corrections in Figure 1. The first step is the obtaining of the Geometrical Equations of Condition, which we shall deduce successively.

The points A and B are the extremities of the base line. The point C is determined by the bearings from A and B, and the observed angle at C gives the equation of condition

$$0 = A + B + C - 180^\circ - \epsilon_1$$

The point E is fixed by bearings from A and C, and the observed angle CEA gives the equation

$$0 = A + C + E - 180^\circ - \epsilon_2$$

But the point E also observes and is observed from B: these two bearings furnish two equations of condition, for we have by the fixation of the point E the means of calculating the angles CEB and CBE. The ratio of the lines CE, CB expresses the ratio of the sines of CBE and CEB, and the ratio of the lines AE, AB expresses the ratio of the sines of AEB and ABE, which determine symmetrically the bearings of E at B, and of B at E. But a more simple equation suggests itself, for the angles of the newly-formed triangles CEB, ABE must fulfil the condition that their sum in each case = 180° + ϵ. We cannot, however, use both these equations, as one of them is not independent of the other; but one of them

$$0 = B + C + E - 180^\circ - \epsilon_3$$

together with the equation

$$\frac{CB}{CE}\,\frac{CE}{CA}\,\frac{CA}{CB} = 1$$

will serve to determine the new bearings.

We next take the point D. It is fixed by bearings from A and E; and the angle ADE, being observed, gives the equation of condition

$$0 = A + D + E - 180^\circ - \epsilon_4$$

But D also observes, and is observed from B. The two new angles thus formed give two equations of condition: one we may take as

$$0 = A + B + D - 180^\circ - \epsilon_5$$

For the other we may take

$$\frac{DA}{DB}\,\frac{DB}{DE}\,\frac{DE}{DA} = 1$$

This equation involves the sines of the angles DBA and DBE, which are very nearly right angles; and this is a little objectionable for the reason that the co-efficient of the unknown correction to either of these angles will be small. Instead, therefore, of this equation, we shall take the equation

$$\frac{AB}{AC}\frac{AC}{AE}\frac{AE}{AD}\frac{AD}{AB} = 1$$

which contains the ratio of the sines of ADB, ABD.

Take next the point F: this point is fixed by the bearings from A and E, and the observed angle at F gives the condition

$$0 = A + E + F - 180^\circ - \epsilon_6$$

But F also observes and is observed from B and D; this gives four equations of condition, two of which are

$$0 = A + B + F - 180^\circ - \epsilon_7$$

$$0 = A + D + F - 180^\circ - \epsilon_8$$

and the other two

$$\frac{AB}{AF}\frac{AF}{AD}\frac{AD}{AB} = 1$$

$$\frac{AD}{AF}\frac{AF}{AE}\frac{AE}{AD} = 1$$

Next take the point G: it is fixed by bearings from E and F, and the observed angle at G gives the equation

$$0 = E + F + G - 180^\circ - \epsilon_9$$

The reciprocal observation of B at G and G at B gives these two equations:—

$$0 = B + F + G - 180^\circ - \epsilon_{10}$$

and the side equation

$$\frac{FB}{FG}\frac{FG}{FE}\frac{FE}{FB} = 1$$

But there is also a bearing to G observed at D, which affords the equation

$$\frac{GD}{GE}\frac{GE}{GF}\frac{GF}{GD} = 1$$

Taking now the point H, it may be considered fixed by the bearings from F and G, and the observed angle at H gives the equation

$$0 = F + G + H - 180^\circ - \epsilon_{11}$$

The reciprocal bearings observed at H and E furnish two equations of condition; these are

$$0 = E + G + H - 180^\circ - \varepsilon_{12}$$

$$\frac{GE}{GF}\frac{GF}{GH}\frac{GH}{GE} = 1$$

But H is also observed from D, therefore there is another equation of condition—namely,

$$\frac{GE}{GH}\frac{GH}{GD}\frac{GD}{GE} = 1$$

Taking next the point K, and supposing it fixed by the bearings from G and H, the observed angle at K gives the equation of condition

$$0 = G + H + K - 180^\circ - \varepsilon_{13}$$

But K also observes and is observed from F: these bearings afford two more equations of condition, which are,

$$0 = G + F + K - 180^\circ - \varepsilon_{14}$$

$$\frac{GH}{GF}\frac{GF}{GK}\frac{GK}{GH} = 1$$

Take next the point L, and, supposing it first fixed by the observations from G and K, the observed angle at L gives

$$0 = G + K + L - 180^\circ - \varepsilon_{15}$$

But L observes and is observed from H and F, forming four new bearings, which must give as many equations of condition; they are

$$0 = G + H + L - 180^\circ - \varepsilon_{16}$$

$$0 = F + G + L - 180^\circ - \varepsilon_{17}$$

$$\frac{GK}{GF}\frac{GF}{GL}\frac{GL}{GK} = 1$$

$$\frac{GK}{GL}\frac{GL}{GH}\frac{GH}{GK} = 1$$

The point N may be first fixed by the bearings from L and G, and the observed angle at N will give the equation of condition

$$0 = G + L + N - 180^\circ - \varepsilon_{18}$$

But N also observes and is reciprocally observed by K, hence two additional equations; these are

$$0 = G + K + N - 180^\circ - \varepsilon_{19}$$

$$\frac{GL}{GN}\,\frac{GN}{GK}\,\frac{GK}{GL} = 1$$

Take next the point M, and let it first be fixed by the bearings from G and N; then the angle observed at M gives the equation

$$0 = G + M + N - 180^\circ - \varepsilon_{20}$$

But M also observes and is reciprocally observed by L; hence the two equations,

$$0 = G + L + M - 180^\circ - \varepsilon_{21}$$

$$\frac{GN}{GM}\,\frac{GM}{GL}\,\frac{GL}{GN} = 1$$

Lastly, the point O being considered first fixed by observations from F and G, the observed angle at O gives

$$0 = F + G + O - 180^\circ - \varepsilon_{22}$$

But O also observes and is observed from M and N; hence there must from the four new bearings arise the following four equations of condition:—

$$0 = G + N + O - 180^\circ - \varepsilon_{23}$$

$$0 = G + M + O - 180^\circ - \varepsilon_{24}$$

$$\frac{GM}{GN}\,\frac{GN}{GO}\,\frac{GO}{GM} = 1$$

$$\frac{GF}{GK}\,\frac{GK}{GN}\,\frac{GN}{GO}\,\frac{GO}{GF} = 1$$

The 39 equations thus deduced, being equal to the number of superfluous bearings, are all the necessary geometrical relations of the figure—or more strictly speaking, they contain all the necessary relations; for many of them might be altered, and others substituted in their places, which would equally well contain the necessary relations.

Each of the side equations, or those of the form

$$\frac{AB}{AC}\,\frac{AC}{AD}\,\frac{AD}{AE}\,\cdots\cdot$$

might have been written in the form

$$\frac{\sin ACB}{\sin ABC}\,\frac{\sin ADC}{\sin ACD}\,\frac{\sin AED}{\sin ADE}\,\cdots\cdot$$

in which form they are actually used in the calculation.

When the observed angles are substituted in these equations of condition, they should, but that they are all affected by errors, make all these equations identically true. As it is, however, none of them will be exactly satisfied, and the problem is, to apply to the observed bearings a system of small corrections that will, when the corrected angles are substituted in the equations of condition, render them identically true, and will also be the most probable system of corrections. For it is evident that the number of equations being less than the number of bearings or corrections, an indefinite number of systems of corrections might be obtained to fulfil the equations of condition, but that system which is required must make the function

$$w\,x^2 + w'\,x'^2 + w''\,x''^2 + \cdots$$

the least possible, $x\ x'\ x'' \cdots$ being the corrections to be determined, $w\ w'\ w'' \cdots$ the weights of the bearings to which they are to be attached.

Spherical Excess.

In the calculation of the side equations, it is usual to subtract from each angle one-third of the spherical excess of the triangle to which it belongs, in accordance with Legendre's Theorem.

The spherical excess is calculated from the formula

$$\varepsilon = \frac{a\,b \sin C}{2r^2 \sin 1''}$$

Where a, b are two sides of the triangle, C the angle included by those sides, and r the radius of curvature of a vertical section of the surface in azimuth 45° for the mean latitude of the triangle.*

---

* In a former section r was taken a mean proportional between the principal radii of curvature; the difference is insensible in the calculation of spherical excess, for the ratio of these two radii is

$$1 + \tfrac{1}{2}\left\{\left(\frac{\varrho}{\varrho'}\right)^{\frac{1}{4}} - \left(\frac{\varrho'}{\varrho}\right)^{\frac{1}{4}}\right\}^2$$

where $\varrho\ \varrho'$ are the principal radii of curvature: also

$$\frac{\varrho}{\varrho'} = 1 - e^2 \cos^2 \lambda$$

Consequently the ratio of the radii is $1 + \frac{1}{8} e^4 \cos^4 \lambda$ which can only affect the 7th figure of the logarithm.

The slowly varying quantity $\log (2\, r^2 \sin 1'')^{-1}$ computed with the elements of the figure of the earth given by the Astronomer Royal in the Encyclopædia Metropolitana, viz.:

Semiaxis-major = 20923713 feet.
Semiaxis-minor = 20853810 feet.

is shown in the following table:—

Latitude.	u	Latitude.	u	Latitude.	u
° ′		° ′		° ′	
49 0	·3717188	53 0	·3713216	57 0	·3709399
10	·3717020	10	·3713053	10	·3709244
20	·3716853	20	·3712891	20	·3709090
30	·3716685	30	·3712729	30	·3708936
40	·3716518	40	·3712567	40	·3708783
50	·3716351	50	·3712405	50	·3708630
50 0	·3716184	54 0	·3712244	58 0	·3708477
10	·3716018	10	·3712083	10	·3708325
20	·3715851	20	·3711922	20	·3708174
30	·3715685	30	·3711762	30	·3708023
40	·3715519	40	·3711602	40	·3707872
50	·3715353	50	·3711442	50	·3707721
51 0	·3715187	55 0	·3711283	59 0	·3707572
10	·3715022	10	·3711124	10	·3707422
20	·3714856	20	·3710965	20	·3707273
30	·3714691	30	·3710807	30	·3707125
40	·3714526	40	·3710649	40	·3706977
50	·3714362	50	·3710492	50	·3706829
52 0	·3714197	56 0	·3710334	60 0	·3706682
10	·3714033	10	·3710177	10	·3706536
20	·3713869	20	·3710021	20	·3706390
30	·3713705	30	·3709865	30	·3706244
40	·3713542	40	·3709709	40	·3706099
50	·3713379	50	·3709554	50	·3705955
53 0	·3713216	57 0	·3709399	61 0	·3705811

$$\log \varepsilon = (a\, b \sin C) + u - 10$$

The spherical excesses of the triangles are thus found to be

$''$
$\varepsilon_1 = 0{\cdot}3942$
$\varepsilon_2 = 0{\cdot}5037$
$\varepsilon_3 = 0{\cdot}1095$
$\varepsilon_4 = 0{\cdot}4857$
$\varepsilon_5 = 0{\cdot}3804$
$\varepsilon_6 = 1{\cdot}0224$
$\varepsilon_7 = 0{\cdot}8001$
$\varepsilon_8 = 0{\cdot}5010$

$''$
$\varepsilon_9 = 2{\cdot}6370$
$\varepsilon_{10} = 2{\cdot}3667$
$\varepsilon_{11} = 7{\cdot}1598$
$\varepsilon_{12} = 4{\cdot}4664$
$\varepsilon_{13} = 2{\cdot}2464$
$\varepsilon_{14} = 7{\cdot}2129$
$\varepsilon_{15} = 7{\cdot}5069$
$\varepsilon_{16} = 9{\cdot}2973$

$''$
$\varepsilon_{17} = 5{\cdot}9265$
$\varepsilon_{18} = 7{\cdot}5864$
$\varepsilon_{19} = 14{\cdot}8086$
$\varepsilon_{20} = 5{\cdot}5590$
$\varepsilon_{21} = 8{\cdot}0157$
$\varepsilon_{22} = 8{\cdot}6094$
$\varepsilon_{23} = 21{\cdot}6768$
$\varepsilon_{24} = 12{\cdot}3897$

The following also enter into the side equations:—

(DEG) . . $\varepsilon_{25} = 1''{\cdot}2600$ (DFG) . . $\varepsilon_{26} = 1''{\cdot}3413$ (DHG) . . $\varepsilon_{27} = 5''{\cdot}6931$

Formation of the Algebraic Equations of Condition.

We must now append an unknown correction to each of the observed bearings, and then substitute them in the equations of condition. For these unknown corrections, the notation

$$(1), (2), (3), (4) \cdots\cdots (n)$$

is adopted as convenient: that is, the bearings being written out consecutively, the n^{th} bearing has the quantity $+ (n)$ attached to it.

The following Table contains the observed bearings for all the points in this figure:—

Names of Stations.		Bearing.	Reciprocal of Weight.
From	To		
		° ′ ″	
SOUTH END OF BASE	Slieve Snaght	129 31 56·89 + (1)	0·080
	Drung Point	143 38 42·61 + (2)	0·206
	Cundtham	154 26 43·58 + (3)	0·158
	Mount Sandy	197 9 34·20 + (4)	0·338
	North end of base	197 9 35·12 + (5)	0·226
NORTH END OF BASE	Sawel	8 55 54·08 + (6)	3·4355
	South end of base	17 12 28·36 + (7)	1·211
	Drung Point	90 49 46·56 + (8)	0·292
	Slieve Snaght	101 37 42·67 + (9)	0·886
	Cundtham	107 49 47·66 + (10)	1·023
	Mount Sandy	197 12 24·41 + (11)	0·141
DRUNG POINT	Mount Sandy	256 53 42·00 + (12)	0·251
	North end of base	270 39 53·11 + (13)	0·285
	South end of base	323 31 41·39 + (14)	0·314
CUNDTHAM HILL	Slieve Snaght	95 58 46·73 + (15)	1·714
	Mount Sandy	271 3 29·92 + (16)	0·359
	Knocklayd	272 9 28·41 + (17)	1·694
	North end of base	287 41 4·17 + (18)	0·091
	South end of base	334 20 52·79 + (19)	0·291
	Sawel	352 11 23·23 + (20)	1·695
MOUNT SANDY	Sawel	9 39 17·31 + (21)	0·638
	North end of base	17 13 11·94 + (22)	0·064
	South end of base	17 13 15·38 + (23)	0·206
	Drung Point	77 4 22·13 + (24)	0·218
	Cundtham	91 13 1·16 + (25)	0·193
	Slieve Snaght	93 46 32·44 + (26)	0·383
	Knocklayd	272 37 40·18 + (27)	0·371

NAMES OF STATIONS.		Bearing.	Reciprocal of Weight.
From	To		
		° ′ ″	
SLIEVE SNAGHT	Cuilcagh	15 42 47·17+(28)	2·9005
	Knocklayd	272 43 29·64+(29)	1·0861
	Mount Sandy	273 26 45·74+(30)	0·2808
	Cundtham	275 48 30·68+(31)	0·4105
	North end of base	281 18 43·61+(32)	0·3671
	Trostan	282 5 30·75+(33)	1·8257
	Divis	307 7 15·09+(34)	5·4849
	South end of base	309 15 51·55+(35)	1·7072
	Sawel	335 39 50·08+(36)	0·3139
SAWEL	Cuilcagh	36 17 40·47+(37)	4·1919
	Slieve Snaght	155 54 24·72+(38)	0·3935
	North end of base	188 51 27·97+(39)	0·1972
	Mount Sandy	189 34 6·97+(40)	0·1944
	Knocklayd	232 36 55·14+(41)	0·6304
	Trostan	245 42 3·63+(42)	0·3952
	Divis	289 0 24·91+(43)	0·5746
	Slieve Donard	314 2 12·06+(44)	2·4091
	Vicar's Cairn	333 33 36·69+(45)	4·9953
KNOCKLAYD	Sawel	53 15 36·63+(46)	0·4652
	Mount Sandy	93 11 12·09+(47)	0·1752
	Slieve Snaght	93 36 47·49+(48)	0·2178
	Trostan	334 43 48·07+(49)	0·3475
	Divis	346 13 22·81+(50)	0·4856
TROSTAN	Sawel	66 25 24·98+(51)	2·6068
	Slieve Snaght	103 3 32·60+(52)	3·4087
	Knocklayd	154 48 31·92+(53)	0·2530
	Divis	349 37 46·12+(54)	0·5222
	Slieve Donard	350 58 27·08+(55)	2·2144
DIVIS	Vicar's Cairn	47 21 16·71+(56)	0·2775
	Sawel	109 50 24·48+(57)	0·5482
	Slieve Snaght	128 11 45·91+(58)	0·2427
	Knocklayd	166 24 47·61+(59)	0·5023
	Trostan	169 44 31·59+(60)	0·9509
	Slieve Donard	352 27 5·77+(61)	0·7301
VICAR'S CAIRN	Cuilcagh	82 41 54·37+(62)	15·5191
	Sawel	153 55 11·64+(63)	7·7595
	Divis	226 52 58·79+(64)	0·4175
	Slieve Donard	286 20 10·15+(65)	2·1358
SLIEVE DONARD	Cuilcagh	91 50 10·05+(66)	0·2432
	Vicar's Cairn	106 53 2·45+(67)	2·6985
	Sawel	134 56 41·86+(68)	0·1815
	Trostan	171 9 49·62+(69)	0·2167
	Divis	172 31 46·07+(70)	0·2886
CUILCAGH	Slieve Snaght	195 19 23·89+(71)	1·3281
	Sawel	215 39 52·24+(72)	0·4924
	Vicar's Cairn	261 42 41·93+(73)	0·5455
	Slieve Donard	270 18 14·77+(74)	0·8247

In the substitution of these bearings in the equations of condition, we shall take first the angle equations in the order in which they have been deduced, and after them the side equations in the order in which they have been deduced. The substitution will then be as follows:—

I.

$$\begin{array}{rrrrl} & {}^\circ & ' & '' & \\ A = & 53 & 30 & 52{\cdot}51 & -\ (2) + (5) \\ B = & 73 & 37 & 18{\cdot}20 & -\ (7) + (8) \\ C = & 52 & 51 & 48{\cdot}28 & - (13) + (14) \\ \hline & 179 & 59 & 58{\cdot}99 & \\ 180 + \epsilon = & 180 & 0 & 0{\cdot}3942 & \\ \hline & & & -\ 1{\cdot}4042 & \end{array}$$

$$0 = -1{\cdot}4042-(2)+(5)-(7)+(8)-(13)+(14)$$

II.

$$\begin{array}{rrrrl} & {}^\circ & ' & '' & \\ A = & 53 & 30 & 51{\cdot}59 & -\ (2) + (4) \\ E = & 59 & 51 & 6{\cdot}75 & - (23) + (24) \\ C = & 66 & 37 & 59{\cdot}39 & - (12) + (14) \\ \hline & 179 & 59 & 57{\cdot}73 & \\ 180 + \epsilon = & 180 & 0 & 0{\cdot}5037 & \\ \hline & & & -\ 2{\cdot}7737 & \end{array}$$

$$0 = -2{\cdot}7737-(2)+(4)-(12)+(14)-(23)+(24)$$

III.

$$\begin{array}{rrrrl} & {}^\circ & ' & '' & \\ B = & 106 & 22 & 37{\cdot}85 & -\ (8) + (11) \\ E = & 59 & 51 & 10{\cdot}19 & - (22) + (24) \\ C = & 13 & 46 & 11{\cdot}11 & - (12) + (13) \\ \hline & 179 & 59 & 59{\cdot}15 & \\ 180 + \epsilon = & 180 & 0 & 0{\cdot}1095 & \\ \hline & & & -\ 0{\cdot}9595 & \end{array}$$

$$0 = -0{\cdot}9595-(8)+(11)-(12)+(13)-(22)+(24)$$

IV.

$$\begin{array}{rrrrl} & {}^\circ & ' & '' & \\ D = & 63 & 17 & 22{\cdot}87 & - (16) + (19) \\ A = & 42 & 42 & 50{\cdot}62 & -\ (3) + (4) \\ E = & 73 & 59 & 45{\cdot}78 & - (23) + (25) \\ \hline & 179 & 59 & 59{\cdot}27 & \\ 180 + \epsilon = & 180 & 0 & 0{\cdot}4857 & \\ \hline & & & -\ 1{\cdot}2157 & \end{array}$$

$$0 = -1{\cdot}2157-(3)+(4)-(16)+(19)-(23)+(25)$$

V.

$$\begin{array}{rrrrl} & {}^\circ & ' & '' & \\ D = & 46 & 39 & 48{\cdot}62 & - (18) + (19) \\ A = & 42 & 42 & 51{\cdot}54 & -\ (3) + (5) \\ B = & 90 & 37 & 19{\cdot}30 & -\ (7) + (10) \\ \hline & 179 & 59 & 59{\cdot}46 & \\ 180 + \epsilon = & 180 & 0 & 0{\cdot}3804 & \\ \hline & & & -\ 0{\cdot}9204 & \end{array}$$

$$0 = -0{\cdot}9204-(3)+(5)-(7)+(10)-(18)+(19)$$

VI.

$$\begin{array}{rrrrl} & {}^\circ & ' & '' & \\ F = & 35 & 49 & 5{\cdot}81 & - (30) + (35) \\ A = & 67 & 37 & 37{\cdot}31 & -\ (1) + (4) \\ E = & 76 & 33 & 17{\cdot}06 & - (23) + (26) \\ \hline & 180 & 0 & 0{\cdot}18 & \\ 180 + \epsilon = & 180 & 0 & 1{\cdot}0224 & \\ \hline & & & -\ 0{\cdot}8424 & \end{array}$$

$$0 = -0{\cdot}8424-(1)+(4)-(23)+(26)-(30)+(35)$$

VII.

$$\begin{array}{rrrrl} & {}^\circ & ' & '' & \\ F = & 27 & 57 & 7{\cdot}94 & - (32) + (35) \\ A = & 67 & 37 & 38{\cdot}23 & -\ (1) + (5) \\ B = & 84 & 25 & 14{\cdot}31 & -\ (7) + (9) \\ \hline & 180 & 0 & 0{\cdot}48 & \\ 180 + \epsilon = & 180 & 0 & 0{\cdot}8001 & \\ \hline & & & -\ 0{\cdot}3201 & \end{array}$$

$$0 = -0{\cdot}3201-(1)+(5)-(7)+(9)-(32)+(35)$$

VIII.

$$\begin{array}{rrrrl} & {}^\circ & ' & '' & \\ F = & 33 & 27 & 20{\cdot}87 & - (31) + (35) \\ A = & 24 & 54 & 46{\cdot}69 & -\ (1) + (3) \\ D = & 121 & 37 & 53{\cdot}94 & - (19) + (15) \\ \hline & 180 & 0 & 1{\cdot}50 & \\ 180 + \epsilon = & 180 & 0 & 0{\cdot}5010 & \\ \hline & & & +\ 0{\cdot}9990 & \end{array}$$

$$0 = +0{\cdot}9990-(1)+(3)+(15)-(19)-(31)+(35)$$

IX.

$$\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
G = & 33 & 39 & 42{\cdot}25 & -(38)+(40)\\
F = & 62 & 13 & 4{\cdot}34 & -(30)+(36)\\
E = & 84 & 7 & 15{\cdot}13 & -(21)+(26)\\
\hline
 & 180 & 0 & 1{\cdot}72 & \\
180+\varepsilon = & 180 & 0 & 2{\cdot}6370 & \\
\hline
 & & & -0{\cdot}9170 &
\end{array}$$

$$0=-0{\cdot}9170-(21)+(26)-(30)+(36)-(38)+(40)$$

X.

$$\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
G = & 32 & 57 & 3{\cdot}25 & -(38)+(39)\\
F = & 54 & 21 & 6{\cdot}47 & -(32)+(36)\\
B = & 92 & 41 & 48{\cdot}59 & -(6)+(9)\\
\hline
 & 179 & 59 & 58{\cdot}31 & \\
180+\varepsilon = & 180 & 0 & 2{\cdot}3667 & \\
\hline
 & & & -4{\cdot}0567 &
\end{array}$$

$$0=-4{\cdot}0567-(6)+(9)-(32)+(36)-(38)+(39)$$

XI.

$$\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
G = & 76 & 42 & 30{\cdot}42 & -(38)+(41)\\
F = & 62 & 56 & 20{\cdot}44 & -(29)+(36)\\
H = & 40 & 21 & 10{\cdot}86 & -(46)+(48)\\
\hline
 & 180 & 0 & 1{\cdot}72 & \\
180+\varepsilon = & 180 & 0 & 7{\cdot}1598 & \\
\hline
 & & & -5{\cdot}4398 &
\end{array}$$

$$0=-5{\cdot}4398-(29)+(36)-(38)+(41)-(46)+(48)$$

XII.

$$\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
G = & 43 & 2 & 48{\cdot}17 & -(40)+(41)\\
E = & 97 & 1 & 37{\cdot}13 & -(27)+(21)\\
H = & 39 & 55 & 35{\cdot}46 & -(46)+(47)\\
\hline
 & 180 & 0 & 0{\cdot}76 & \\
180+\varepsilon = & 180 & 0 & 4{\cdot}4664 & \\
\hline
 & & & -3{\cdot}7064 &
\end{array}$$

$$0=-3{\cdot}7064+(21)-(27)-(40)+(41)-(46)+(47)$$

XIII.

$$\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
G = & 13 & 5 & 8{\cdot}49 & -(41)+(42)\\
H = & 78 & 31 & 48{\cdot}56 & -(49)+(46)\\
K = & 88 & 23 & 6{\cdot}94 & -(51)+(53)\\
\hline
 & 180 & 0 & 3{\cdot}99 & \\
180+\varepsilon = & 180 & 0 & 2{\cdot}2461 & \\
\hline
 & & & +1{\cdot}7439 &
\end{array}$$

$$0=+1{\cdot}7439-(41)+(42)+(46)-(49)-(51)+(53)$$

XIV.

$$\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
G = & 89 & 47 & 38{\cdot}91 & -(38)+(42)\\
F = & 53 & 34 & 19{\cdot}33 & -(33)+(36)\\
K = & 36 & 38 & 7{\cdot}62 & -(51)+(52)\\
\hline
 & 180 & 0 & 5{\cdot}86 & \\
180+\varepsilon = & 180 & 0 & 7{\cdot}2129 & \\
\hline
 & & & -1{\cdot}3529 &
\end{array}$$

$$0=-1{\cdot}3529-(33)+(36)-(38)+(42)-(51)+(52)$$

XV.

$$\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
L = & 59 & 54 & 7{\cdot}11 & -(57)+(60)\\
G = & 43 & 18 & 21{\cdot}28 & -(42)+(43)\\
K = & 76 & 47 & 38{\cdot}86 & -(54)+(51)\\
\hline
 & 180 & 0 & 7{\cdot}25 & \\
180+\varepsilon = & 180 & 0 & 7{\cdot}5069 & \\
\hline
 & & & -0{\cdot}2569 &
\end{array}$$

$$0=-0{\cdot}2569-(42)+(43)+(51)-(54)-(57)+(60)$$

XVI.

$$\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
L = & 56 & 34 & 23{\cdot}13 & -(57)+(59)\\
G = & 56 & 23 & 29{\cdot}77 & -(41)+(43)\\
H = & 67 & 2 & 13{\cdot}82 & -(50)+(46)\\
\hline
 & 180 & 0 & 6{\cdot}72 & \\
180+\varepsilon = & 180 & 0 & 9{\cdot}2973 & \\
\hline
 & & & -2{\cdot}5773 &
\end{array}$$

$$0=-2{\cdot}5773-(41)+(43)+(46)-(50)-(57)+(59)$$

XVII.

$$\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
L = & 18 & 21 & 21{\cdot}43 & -(57)+(58)\\
G = & 133 & 6 & 0{\cdot}19 & -(38)+(43)\\
F = & 28 & 32 & 34{\cdot}99 & -(34)+(36)\\
\hline
 & 179 & 59 & 56{\cdot}61 & \\
180+\varepsilon = & 180 & 0 & 5{\cdot}9265 & \\
\hline
 & & & -9{\cdot}3165 &
\end{array}$$

$$0=-9{\cdot}3165-(34)+(36)-(38)+(43)-(57)+(58)$$

XVIII.

$$\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
N = & 37 & 35 & 4{\cdot}21 & -(68)+(70)\\
G = & 25 & 1 & 47{\cdot}15 & -(43)+(44)\\
L = & 117 & 23 & 18{\cdot}71 & -(61)+(57)\\
\hline
 & 180 & 0 & 10{\cdot}07 & \\
180+\varepsilon = & 180 & 0 & 7{\cdot}5864 & \\
\hline
 & & & +2{\cdot}4836 &
\end{array}$$

$$0=+2{\cdot}4836-(43)+(44)+(57)-(61)-(68)+(70)$$

XIX.

$$\begin{array}{rrrrl}
 & {}^\circ & {}' & {}'' & \\
N = & 36 & 13 & 7{\cdot}76 & -(68)+(69) \\
G = & 68 & 20 & 8{\cdot}43 & -(42)+(44) \\
K = & 75 & 26 & 57{\cdot}90 & -(55)+(51) \\
\hline
 & 180 & 0 & 14{\cdot}09 & \\
180+\varepsilon = & 180 & 0 & 14{\cdot}8086 & \\
\hline
 & & & -0{\cdot}7186 &
\end{array}$$

$$0 = -0{\cdot}7186-(42)+(44)+(51)-(55)-(68)+(69)$$

XX.

$$\begin{array}{rrrrl}
 & {}^\circ & {}' & {}'' & \\
G = & 19 & 31 & 24{\cdot}63 & -(44)+(45) \\
N = & 28 & 3 & 39{\cdot}41 & -(67)+(68) \\
M = & 132 & 24 & 58{\cdot}51 & -(63)+(65) \\
\hline
 & 180 & 0 & 2{\cdot}55 & \\
180+\varepsilon = & 180 & 0 & 5{\cdot}5590 & \\
\hline
 & & & -3{\cdot}0090 &
\end{array}$$

$$0 = -3{\cdot}0090-(44)+(45)-(63)+(65)-(67)+(68)$$

XXI.

$$\begin{array}{rrrrl}
 & {}^\circ & {}' & {}'' & \\
G = & 44 & 33 & 11{\cdot}78 & -(43)+(45) \\
L = & 62 & 29 & 7{\cdot}77 & -(56)+(57) \\
M = & 72 & 57 & 47{\cdot}15 & -(63)+(64) \\
\hline
 & 180 & 0 & 6{\cdot}70 & \\
180+\varepsilon = & 180 & 0 & 8{\cdot}0157 & \\
\hline
 & & & -1{\cdot}3157 &
\end{array}$$

$$0 = -1{\cdot}3157-(43)+(45)-(56)+(57)-(63)+(64)$$

XXII.

$$\begin{array}{rrrrl}
 & {}^\circ & {}' & {}'' & \\
O = & 20 & 20 & 28{\cdot}35 & -(71)+(72) \\
F = & 40 & 2 & 57{\cdot}09 & -(36)+(28) \\
G = & 119 & 36 & 44{\cdot}25 & -(37)+(38) \\
\hline
 & 180 & 0 & 9{\cdot}69 & \\
180+\varepsilon = & 180 & 0 & 8{\cdot}6094 & \\
\hline
 & & & +1{\cdot}0806 &
\end{array}$$

$$0 = +1{\cdot}0806+(28)-(36)-(37)+(38)-(71)+(72)$$

XXIII.

$$\begin{array}{rrrrl}
 & {}^\circ & {}' & {}'' & \\
O = & 54 & 38 & 22{\cdot}53 & -(72)+(74) \\
G = & 82 & 15 & 28{\cdot}41 & -(44)+(37) \\
N = & 43 & 6 & 31{\cdot}81 & -(66)+(68) \\
\hline
 & 180 & 0 & 22{\cdot}75 & \\
180+\varepsilon = & 180 & 0 & 21{\cdot}6768 & \\
\hline
 & & & +1{\cdot}0732 &
\end{array}$$

$$0 = +1{\cdot}0732+(37)-(44)-(66)+(68)-(72)+(74)$$

XXIV.

$$\begin{array}{rrrrl}
 & {}^\circ & {}' & {}'' & \\
O = & 46 & 2 & 49{\cdot}69 & -(72)+(73) \\
G = & 62 & 44 & 3{\cdot}78 & -(45)+(37) \\
M = & 71 & 13 & 17{\cdot}27 & -(62)+(63) \\
\hline
 & 180 & 0 & 10{\cdot}74 & \\
180+\varepsilon = & 180 & 0 & 12{\cdot}3897 & \\
\hline
 & & & -1{\cdot}6497 &
\end{array}$$

$$0 = -1{\cdot}6497+(37)-(45)-(62)+(63)-(72)+(73)$$

In these equations the co-efficients of the unknown quantities (1), (2), (3) · · · are all units, positive or negative; in the side equations each has a co-efficient to be calculated, on this principle: if A be any given angle, and x a very small unknown angle whose square may be neglected,

$$\begin{aligned} \log \sin (A + x) &= \log (\sin A + x \cos A) \\ &= \log \sin A + M x \cot A \end{aligned}$$

Where M is the modulus of the common system of logarithms. If x be expressed in seconds, not in arc, then

$$\log \sin (A + x) = \log \sin A + \mu \cot A\, x$$

Where $\mu = M \sin 1''$. The quantity $\mu \cot A\, x$ has therefore to be added to log sin A; and as it is convenient to take out from the logarithmic tables those figures which are to be added to the last decimals of 7-figure logarithms, we must add 7 to the logarithm of μ, so that

$$\log \mu = \log \text{M} \sin 7'' = 1 \cdot 3233592$$

The calculation of the side equations will be as follows:—

XXV.

$$\frac{\sin \text{CEB} \sin \text{CAE} \sin \text{CBA}}{\sin \text{CBE} \sin \text{CEA} \sin \text{CAB}} = 1$$

$$\begin{array}{ll}
\text{CEB} = 59^\circ\ 51'\ 10''{\cdot}19 - 0''{\cdot}0365 - (22) + (24) & \text{CBE} = 106^\circ\ 22'\ 37''{\cdot}85 - 0''{\cdot}0365 - (8) + (11) \\
\text{CAE} = 53\ 30\ 51{\cdot}59 - 0{\cdot}1679 - (2) + (4) & \text{CEA} = 59\ 51\ 6{\cdot}75 - 0{\cdot}1679 - (23) + (24) \\
\text{CBA} = 73\ 37\ 18{\cdot}20 - 0{\cdot}1314 - (7) + (8) & \text{CAB} = 53\ 30\ 52{\cdot}51 - 0{\cdot}1314 - (2) + (5) \\
9{\cdot}936\ 8846\ 263 + 12{\cdot}2284\{(24) - (22)\} & 9{\cdot}982\ 0116\ 636 + 6{\cdot}1878\{(8) - (11)\} \\
9{\cdot}905\ 2588\ 175 + 15{\cdot}5719\{(4) - (2)\} & 9{\cdot}936\ 8802\ 589 + 12{\cdot}2289\{(24) - (23)\} \\
\underline{9{\cdot}982\ 0091\ 154} + 6{\cdot}1882\{(8) - (7)\} & \underline{9{\cdot}905\ 2603\ 069} + 15{\cdot}5717\{(5) - (2)\} \\
9{\cdot}824\ 1525\ 592 & 9{\cdot}824\ 1522\ 294 \\
\qquad\quad \underline{2\ 294} & \\
\qquad + 3\ 298 &
\end{array}$$

$$\begin{aligned}
0 = {} & + 3{\cdot}298 - 0{\cdot}0002\,(2) + 15{\cdot}5719\,(4) - 15{\cdot}5717\,(5) - 6{\cdot}1882\,(7) + 0{\cdot}0004\,(8) + 6{\cdot}1878\,(11) \\
& - 12{\cdot}2284\,(22) + 12{\cdot}2289\,(23) - 0{\cdot}0005\,(24).
\end{aligned}$$

XXVI.

$$\frac{\sin \text{ACB} \sin \text{AEC} \sin \text{ADE} \sin \text{ABD}}{\sin \text{ABC} \sin \text{ACE} \sin \text{AED} \sin \text{ADB}} = 1$$

$$\begin{array}{ll}
\text{ACB} = 52^\circ\ 51'\ 48''{\cdot}28 - 0''{\cdot}1314 - (13) + (14) & \text{ABC} = 73^\circ\ 37'\ 18''{\cdot}20 - 0''{\cdot}1314 - (7) + (8) \\
\text{AEC} = 59\ 51\ 6{\cdot}75 - 0{\cdot}1679 - (23) + (24) & \text{ACE} = 66\ 37\ 59{\cdot}39 - 0{\cdot}1679 - (12) + (14) \\
\text{ADE} = 63\ 17\ 22{\cdot}87 - 0{\cdot}1619 - (16) + (19) & \text{AED} = 73\ 59\ 45{\cdot}78 - 0{\cdot}1619 - (23) + (25) \\
\text{ABD} = 90\ 37\ 19{\cdot}30 - 0{\cdot}1268 - (7) + (10) & \text{ADB} = 46\ 39\ 48{\cdot}62 - 0{\cdot}1268 - (18) + (19) \\
9{\cdot}901\ 5662\ 958 + 15{\cdot}9451\{(14) - (13)\} & 9{\cdot}982\ 0091\ 154 + 6{\cdot}1882\{(8) - (7)\} \\
9{\cdot}936\ 8802\ 589 + 12{\cdot}2289\{(24) - (23)\} & 9{\cdot}962\ 8351\ 240 + 9{\cdot}0970\{(14) - (12)\} \\
9{\cdot}950\ 9925\ 396 + 10{\cdot}5944\{(19) - (16)\} & 9{\cdot}982\ 8329\ 529 + 6{\cdot}0391\{(25) - (23)\} \\
\underline{9{\cdot}999\ 9744\ 089} + 0{\cdot}2286\{(7) - (10)\} & \underline{9{\cdot}861\ 7347\ 001} + 19{\cdot}8668\{(19) - (18)\} \\
9{\cdot}789\ 4135\ 032 & 9{\cdot}789\ 4118\ 924 \\
\qquad\quad \underline{18\ 924} & \\
\qquad + 16\ 108 &
\end{array}$$

$$\begin{aligned}
0 = {} & + 16{\cdot}108 + 6{\cdot}4168\,(7) - 6{\cdot}1882\,(8) - 0{\cdot}2286\,(10) + 9{\cdot}0970\,(12) - 15{\cdot}9451\,(13) + 6{\cdot}8481\,(14) \\
& - 10{\cdot}5944\,(16) + 19{\cdot}8668\,(18) - 9{\cdot}2724\,(19) - 6{\cdot}1898\,(23) + 12{\cdot}2289\,(24) \\
& - 6{\cdot}0391\,(25).
\end{aligned}$$

XXVII.

$$\frac{\sin \mathrm{AFB} \sin \mathrm{ADF} \sin \mathrm{ABD}}{\sin \mathrm{ABF} \sin \mathrm{AFD} \sin \mathrm{ADB}} = 1$$

$$\begin{array}{lrrrl}
\mathrm{AFB} = & 27^{\circ} & 57' & 7''{\cdot}94 & -0''{\cdot}2667-(32)+(35) \\
\mathrm{ADF} = & 121 & 37 & 53{\cdot}94 & -0{\cdot}1670-(19)+(15) \\
\mathrm{ABD} = & 90 & 37 & 19{\cdot}30 & -0{\cdot}1268-\ (7)+(10)
\end{array}$$

$$\begin{array}{rl}
9{\cdot}670\ 9262\ 051 & +39{\cdot}6790\{(35)-(32)\} \\
9{\cdot}930\ 1529\ 171 & +12{\cdot}9692\{(19)-(15)\} \\
9{\cdot}999\ 9744\ 089 & +\ 0{\cdot}2286\{\ (7)-)10)\} \\
\hline
9{\cdot}601\ 0535\ 311 & \\
46\ 155 & \\
\hline
-\ 10\ 844 &
\end{array}$$

$$\begin{array}{lrrrl}
\mathrm{ABF} = & 84^{\circ} & 25' & 14''{\cdot}31 & -0''{\cdot}2667-\ (7)+\ (9) \\
\mathrm{AFD} = & 33 & 27 & 20{\cdot}87 & -0{\cdot}1670-(31)+(35) \\
\mathrm{ADB} = & 46 & 39 & 48{\cdot}62 & -0{\cdot}1268-(18)+(19)
\end{array}$$

$$\begin{array}{rl}
9{\cdot}997\ 9375\ 820 & +\ 2{\cdot}0568\{\ (9)-\ (7)\} \\
9{\cdot}741\ 3823\ 334 & +31{\cdot}8644\{(35)-(31)\} \\
9{\cdot}861\ 7347\ 001 & +19{\cdot}8668\{(19)-(18)\} \\
\hline
9{\cdot}601\ 0546\ 155 &
\end{array}$$

$$\begin{aligned}
0 = & -10{\cdot}844 + 2{\cdot}2854\,(7) - 2{\cdot}0568\,(9) - 0{\cdot}2286\,(10) - 12{\cdot}9692\,(15) + 19{\cdot}8668\,(18) - 6{\cdot}8976\,(19) \\
& + 31{\cdot}8644\,(31) - 39{\cdot}6790\,(32) + 7{\cdot}8146\,(35).
\end{aligned}$$

XXVIII.

$$\frac{\sin \mathrm{AFD} \sin \mathrm{AEF} \sin \mathrm{ADE}}{\sin \mathrm{ADF} \sin \mathrm{AFE} \sin \mathrm{AED}} = 1$$

$$\begin{array}{lrrrl}
\mathrm{AFD} = & 33^{\circ} & 27' & 20''{\cdot}87 & -0''{\cdot}1670-(31)+(35) \\
\mathrm{AEF} = & 76 & 33 & 17{\cdot}06 & -0{\cdot}3408-(23)+(26) \\
\mathrm{ADE} = & 63 & 17 & 22{\cdot}87 & -0{\cdot}1619-(16)+(19)
\end{array}$$

$$\begin{array}{rl}
9{\cdot}741\ 3823\ 334 & +31{\cdot}8644\{(35)-(31)\} \\
9{\cdot}987\ 9307\ 466 & +\ 5{\cdot}0337\{(26)-(23)\} \\
9{\cdot}950\ 9925\ 396 & +10{\cdot}5944\{(19)-(16)\} \\
\hline
9{\cdot}680\ 3056\ 196 & \\
13\ 787 & \\
\hline
+\ 42\ 409 &
\end{array}$$

$$\begin{array}{lrrrl}
\mathrm{ADF} = & 121^{\circ} & 37' & 53''{\cdot}94 & -0''{\cdot}1670-(19)+(15) \\
\mathrm{AFE} = & 35 & 49 & 5{\cdot}81 & -0{\cdot}3408-(30)+(35) \\
\mathrm{AED} = & 73 & 59 & 45{\cdot}78 & -0{\cdot}1619-(23)+(25)
\end{array}$$

$$\begin{array}{rl}
9{\cdot}930\ 1529\ 171 & +12{\cdot}9692\{(19)-(15)\} \\
9{\cdot}767\ 3155\ 087 & +29{\cdot}1742\{(35)-(30)\} \\
9{\cdot}982\ 8329\ 529 & +\ 6{\cdot}0391\{(25)-(23,\} \\
\hline
9{\cdot}680\ 3013\ 787 &
\end{array}$$

$$\begin{aligned}
0 = & +42{\cdot}409 + 12{\cdot}9692\,(15) - 10{\cdot}5944\,(16) - 2{\cdot}3748\,(19) + 1{\cdot}0054\,(23) - 6{\cdot}0391\,(25) + 5{\cdot}0337\,(26) \\
& + 29{\cdot}1742\,(30) - 31{\cdot}8644\,(31) + 2{\cdot}6902\,(35).
\end{aligned}$$

XXIX.

$$\frac{\sin \mathrm{FGB} \sin \mathrm{FEG} \sin \mathrm{FBE}}{\sin \mathrm{FBG} \sin \mathrm{FGE} \sin \mathrm{FEB}} = 1$$

FGB =	$32^{\circ}\ 57'\ 3''{\cdot}25 - 0''{\cdot}7889 - (38) + (39)$	FBG =	$92^{\circ}\ 41'\ 48''{\cdot}59 - 0''{\cdot}7889 - (6) + (9)$
FEG =	$84\ \ 7\ \ 15{\cdot}13 - 0{\cdot}8790 - (21) + (26)$	FGE =	$33\ \ 39\ \ 42{\cdot}25 - 0{\cdot}8790 - (38) + (40)$
FBE =	$95\ \ 34\ \ 41{\cdot}74 - 0{\cdot}0741 - (9) + (11)$	FEB =	$76\ \ 33\ \ 20{\cdot}50 - 0{\cdot}0741 - (22) + (26)$

$9{\cdot}735\ 5326\ 024 + 32{\cdot}4833\{(39) - (38)\}$	$9{\cdot}999\ 5188\ 215 + 0{\cdot}9917\{(6) - (9)\}$
$9{\cdot}997\ 7094\ 753 + 2{\cdot}1682\{(26) - (21)\}$	$9{\cdot}743\ 7332\ 453 + 31{\cdot}6169\{(40) - (38)\}$
$9{\cdot}997\ 9384\ 645 + 2{\cdot}0564\{(9) - (11)\}$	$9{\cdot}987\ 9326\ 124 + 5{\cdot}0333\{(26) - (22)\}$
$9{\cdot}731\ 1805\ 422$	$9{\cdot}731\ 1846\ 792$
$46\ 792$	
$-\ 41\ 370$	

$$0 = -41{\cdot}370 - 0{\cdot}9917\,(6) + 3{\cdot}0481\,(9) - 2{\cdot}0564\,(11) - 2{\cdot}1682\,(21) + 5{\cdot}0333\,(22) - 2{\cdot}8651\,(26)$$
$$- 0{\cdot}8664\,(38) + 32{\cdot}4833\,(39) - 31{\cdot}6169\,(40).$$

XXX.

$$\frac{\sin \mathrm{GED} \sin \mathrm{GFE} \sin \mathrm{GDF}}{\sin \mathrm{GDE} \sin \mathrm{GEF} \sin \mathrm{GFD}} = 1$$

GED =	$81^{\circ}\ 33'\ 43''{\cdot}85 - 0''{\cdot}4200 - (21) + (25)$	GDE =	$81^{\circ}\ 7'\ 53''{\cdot}31 - 0''{\cdot}4200 - (16) + (20)$
GFE =	$62\ \ 13\ \ 4{\cdot}34 - 0{\cdot}8790 - (30) + (36)$	GEF =	$84\ \ 7\ \ 15{\cdot}13 - 0{\cdot}8790 - (21) + (26)$
GDF =	$103\ \ 47\ \ 23{\cdot}50 - 0{\cdot}4471 - (20) + (15)$	GFD =	$59\ \ 51\ \ 19{\cdot}40 - 0{\cdot}4471 - (31) + (36)$

$9{\cdot}995\ 2733\ 042 + 3{\cdot}1234\{(25) - (21)\}$	$9{\cdot}994\ 7764\ 574 + 3{\cdot}28535\{(20) - (16)\}$
$9{\cdot}946\ 8080\ 174 + 11{\cdot}0929\{(36) - (30)\}$	$9{\cdot}997\ 7094\ 753 + 2{\cdot}1682\{(26) - (21)\}$
$9{\cdot}987\ 2983\ 848 + 5{\cdot}16765\{(20) - (15)\}$	$9{\cdot}936\ 8953\ 861 + 12{\cdot}2272\{(36) - (31)\}$
$9{\cdot}929\ 3797\ 064$	$9{\cdot}929\ 3813\ 188$
$813\ 188$	
$-\ 16\ 124$	

$$0 = -16{\cdot}124 - 5{\cdot}16765\,(15) + 3{\cdot}28535\,(16) + 1{\cdot}8823\,(20) - 0{\cdot}9552\,(21) + 3{\cdot}1234\,(25) - 2{\cdot}1682\,(26)$$
$$- 11{\cdot}0929\,(30) + 12{\cdot}2272\,(31) - 1{\cdot}1343\,(36).$$

XXXI.

$$\frac{\sin GFE \sin GHF \sin GEH}{\sin GEF \sin GFH \sin GHE} = 1$$

$$\begin{array}{ll}
GFE = & 62^\circ\ 13'\ \ 4''{\cdot}34 - 0''{\cdot}8790 - (30) + (36) \\
GHF = & 40\ \ 21\ \ 10{\cdot}86 - 2{\cdot}3866 - (46) + (48) \\
GEH = & 97\ \ \ 1\ \ 37{\cdot}13 - 1{\cdot}4888 - (27) + (21)
\end{array}$$

$$\begin{array}{ll}
GEF = & 84^\circ\ \ 7'\ 15''{\cdot}13 - 0''{\cdot}8790 - (21) + (26) \\
GFH = & 62\ \ 56\ \ 20{\cdot}44 - 2{\cdot}3866 - (29) + (36) \\
GHE = & 39\ \ 55\ \ 35{\cdot}46 - 1{\cdot}4888 - (46) + (47)
\end{array}$$

$$\begin{array}{rl}
9{\cdot}946\ 8080\ 174 & + 11{\cdot}0929\{(36)-(30)\} \\
9{\cdot}811\ 2306\ 465 & + 24{\cdot}7815\{(48)-(46)\} \\
\underline{9{\cdot}996\ 7259\ 368} & +\ \ 2{\cdot}5952\{(27)-(21)\} \\
9{\cdot}754\ 7646\ 007 & \\
\underline{510\ 209} & \\
+\ 135\ 798 &
\end{array}
\qquad
\begin{array}{rl}
9{\cdot}997\ 7094\ 753 & +\ \ 2{\cdot}1682\{(26)-(21)\} \\
9{\cdot}949\ 6424\ 594 & + 10{\cdot}7567\{(36)-(29)\} \\
\underline{9{\cdot}807\ 3990\ 862} & + 25{\cdot}1584\{(47)-(46)\} \\
9{\cdot}754\ 7510\ 209 &
\end{array}$$

$$\begin{aligned}
0 = {} & +135{\cdot}798 - 0{\cdot}4270\,(21) - 2{\cdot}1682\,(26) + 2{\cdot}5952\,(27) + 10{\cdot}7567\,(29) - 11{\cdot}0929\,(30) + 0{\cdot}3362\,(36) \\
& + 0{\cdot}3769\,(46) - 25{\cdot}1584\,(47) + 24{\cdot}7815\,(48).
\end{aligned}$$

XXXII.

$$\frac{\sin GHE \sin GDH \sin GED}{\sin GEH \sin GHD \sin GDE} = 1$$

$$\begin{array}{ll}
GHE = & 39^\circ\ 55'\ 35''{\cdot}46 - 1'{\cdot}4888 - (46) + (47) \\
GDH = & 80\ \ \ 1\ \ 54{\cdot}82 - 1{\cdot}8977 - (17) + (20) \\
GED = & 81\ \ 33\ \ 43{\cdot}85 - 0{\cdot}4200 - (21) + (25)
\end{array}$$

$$\begin{array}{ll}
GEH = & 97^\circ\ \ 1'\ 37''{\cdot}13\ \ - 1''{\cdot}4888\ - (27) + (21) \\
GHD = & 39\ \ 36\ \ 53{\cdot}4546 - 1{\cdot}8977 \begin{cases} -(15)+(17) \\ -(29)+(31) \\ -(46)+(48) \end{cases} \\
GDE = & 81\ \ \ 7\ \ 53{\cdot}31\ \ - 0{\cdot}4200\ - (16) + (20)
\end{array}$$

$$\begin{array}{rl}
9{\cdot}807\ 3990\ 862 & + 25{\cdot}1584\{(47)-(46)\} \\
9{\cdot}993\ 3933\ 153 & +\ \ 3{\cdot}7007\{(20)-(17)\} \\
\underline{9{\cdot}995\ 2733\ 042} & +\ \ 3{\cdot}1234\{(25)-(21)\} \\
9{\cdot}796\ 0657\ 057 & \\
\underline{19\ 844} & \\
+\ 37\ 213 &
\end{array}
\qquad
\begin{array}{rl}
9{\cdot}996\ 7259\ 368 & +\ \ 2{\cdot}5952\{(27)-(21)\} \\
9{\cdot}804\ 5595\ 902 & + 25{\cdot}4384 \begin{cases} (17)-(15) \\ (31)-(29) \\ (48)-(46) \end{cases} \\
\underline{9{\cdot}994\ 7764\ 574} & +\ \ 3{\cdot}28535\{(20)-(16)\} \\
9{\cdot}796\ 0619\ 844 &
\end{array}$$

$$\begin{aligned}
0 = {} & +37{\cdot}213 + 25{\cdot}4384(15) + 3{\cdot}28535\,(16) - 29{\cdot}1391\,(17) + 0{\cdot}41535(20) - 0{\cdot}5282\,(21) + 3{\cdot}1234(25) \\
& - 2{\cdot}5952\,(27) + 25{\cdot}4384\,(29) - 25{\cdot}4384(31) + 0{\cdot}2800\,(46) + 25{\cdot}1584\,(47) - 25{\cdot}4384(48).
\end{aligned}$$

XXXIII.

$$\frac{\sin \mathrm{GFH}\ \sin \mathrm{GKF}\ \sin \mathrm{GHK}}{\sin \mathrm{GHF}\ \sin \mathrm{GFK}\ \sin \mathrm{GKH}} = 1$$

$$\begin{array}{ll}
\mathrm{GFH} = 62^\circ\ 56'\ 20''\!\cdot\!44 - 2''\!\cdot\!3866 - (29) + (36) & \mathrm{GHF} = 40^\circ\ 21'\ 10''\!\cdot\!86 - 2''\!\cdot\!3866 - (46) + (48) \\
\mathrm{GKF} = 36\ \ 38\ \ \ 7\cdot62 - 2\cdot4043 - (51) + (52) & \mathrm{GFK} = 53\ \ 34\ \ 19\cdot33 - 2\cdot4043 - (33) + (36) \\
\mathrm{GHK} = 78\ \ 31\ \ 48\cdot56 - 0\cdot7487 - (49) + (46) & \mathrm{GKH} = 88\ \ 23\ \ \ 6\cdot94 - 0\cdot7487 - (51) + (53)
\end{array}$$

$$\begin{array}{ll}
9\cdot949\ 6424\ 594 + 10\cdot7567\{(36) - (29)\} & 9\cdot811\ 2306\ 465 + 24\cdot7815\{(48) - (46)\} \\
9\cdot775\ 7648\ 632 + 28\cdot3149\{(52) - (51)\} & 9\cdot905\ 5784\ 924 + 15\cdot5395\{(36) - (33)\} \\
\underline{9\cdot991\ 2388\ 284} + \ 4\cdot2723\{(46) - (49)\} & \underline{9\cdot999\ 8274\ 627} + \ 0\cdot5936\{(53) - (51)\} \\
9\cdot716\ 6461\ 510 & 9\cdot716\ 6366\ 016 \\
\underline{366\ 016} & \\
+\ 95\ 494 &
\end{array}$$

$$\begin{aligned}
0 = {} & +95\cdot494 - 10\cdot7567\,(29) + 15\cdot5395\,(33) - 4\cdot7828\,(36) + 29\cdot0538\,(46) - 24\cdot7815\,(48) - 4\cdot2723\,(49) \\
& - 27\cdot7213\,(51) + 28\cdot3149\,(52) - 0\cdot5936\,(53)
\end{aligned}$$

XXXIV.

$$\frac{\sin \mathrm{GFK}\ \sin \mathrm{GLF}\ \sin \mathrm{GKL}}{\sin \mathrm{GKF}\ \sin \mathrm{GFL}\ \sin \mathrm{GLK}} = 1$$

$$\begin{array}{ll}
\mathrm{GFK} = 53^\circ\ 34'\ 19''\!\cdot\!33 - 2''\!\cdot\!4043 - (33) + (36) & \mathrm{GKF} = 36^\circ\ 38'\ \ 7''\!\cdot\!62 - 2''\!\cdot\!4043 - (51) + (52) \\
\mathrm{GLF} = 18\ \ 21\ \ 21\cdot43 - 1\cdot9755 - (57) + (58) & \mathrm{GFL} = 28\ \ 32\ \ 34\cdot99 - 1\cdot9755 - (34) + (36) \\
\mathrm{GKL} = 76\ \ 47\ \ 38\cdot86 - 2\cdot5023 - (54) + (51) & \mathrm{GLK} = 59\ \ 54\ \ \ 7\cdot11 - 2\cdot5023 - (57) + (60)
\end{array}$$

$$\begin{array}{ll}
9\cdot905\ 5784\ 924 + 15\cdot5395\{(36) - (33)\} & 9\cdot775\ 7648\ 632 + 28\cdot3149\{(52) - (51)\} \\
9\cdot498\ 1869\ 857 + 63\cdot4591\{(58) - (57)\} & 9\cdot679\ 2557\ 496 + 38\cdot7103\{(36) - (34)\} \\
\underline{9\cdot988\ 3594\ 756} + \ 4\cdot9410\{(51) - (54)\} & \underline{9\cdot937\ 0977\ 489} + 12\cdot2046\{(60) - (57)\} \\
9\cdot392\ 1249\ 537 & 9\cdot392\ 1183\ 617 \\
\underline{183\ 617} & \\
+\ 65\ 920 &
\end{array}$$

$$\begin{aligned}
0 = {} & +65\cdot920 - 15\cdot5395\,(33) + 38\cdot7103\,(34) - 23\cdot1708\,(36) + 33\cdot2559\,(51) - 28\cdot3149\,(52) - 4\cdot9410\,(54) \\
& - 51\cdot2545\,(57) + 63\cdot4591\,(58) - 12\cdot2046\,(60)
\end{aligned}$$

XXXV.

$$\frac{\sin GLK \ \sin GHL \ \sin GKH}{\sin GKL \ \sin GLH \ \sin GHK} = 1$$

GLK = 59° 54′ 7″·11 − 2″·5023 − (57) + (60)
GHL = 67 2 13·82 − 3·0991 − (50) + (46)
GKH = 88 23 6·94 − 0·7487 − (51) + (53)

9·937 0977 489 + 12·2046 {(60) − (57)}
9·964 1428 102 + 8·92165 {(46) − (50)}
9·999 8274 627 + 0·5936 {(53) − (51)}
9·901 0680 218
67 802
+ 12 416

GKL = 76° 47′ 38″·86 − 2″·5023 − (54) + (51)
GLH = 56 34 23·13 − 3·0991 − (57) + (59)
GHK = 78 31 48·56 − 0·7487 − (49) + (46)

9·988 3594 756 + 4·9410 {(51) − (54)}
9·921 4684 762 + 13·8980 {(59) − (57)}
9·991 2388 284 + 4·2723 {(46) − (49)}
9·901 0667 802

$$0 = +\ 12{\cdot}416 + 4{\cdot}64935\ (46) + 4{\cdot}2723\ (49) - 8{\cdot}92165\ (50) - 5{\cdot}5346\ (51) + 0{\cdot}5936\ (53) + 4{\cdot}9410\ (54) + 1{\cdot}6934\ (57) - 13{\cdot}8980\ (59) + 12{\cdot}2046\ (60)$$

XXXVI.

$$\frac{\sin GNL \ \sin GKN \ \sin GLK}{\sin GLN \ \sin GNK \ \sin GKL} = 1$$

GNL = 37° 35′ 4″·21 − 2″·5288 − (68) + (70)
GKN = 75 26 57·90 − 4·9362 − (55) + (51)
GLK = 59 54 7·11 − 2·5023 − (57) + (60)

9·785 2736 788 + 27·3567 {(70) − (68)}
9·985 8395 551 + 5·4656 {(51) − (55)}
9·937 0977 489 + 12·2046 {(60) − (57)}
9·708 2109 828
082 392
+ 27 436

GLN = 117° 23′ 18″·71 − 2″·5288 − (61) + (57)
GNK = 36 13 7·76 − 4·9362 − (68) + (69)
GKL = 76 47 38·86 − 2·5023 − (54) + (51)

9·948 3704 779 + 10·9083 {(61) − (57)}
9·771 4782 857 + 28·7499 {(69) − (68)}
9·988 3594 756 + 4·9410 {(51) − (54)}
9·708 2082 392

$$0 = +\ 27{\cdot}436 + 0{\cdot}5246\ (51) + 4{\cdot}9410\ (54) - 5{\cdot}4656\ (55) - 1{\cdot}2963\ (57) + 12{\cdot}2046\ (60) - 10{\cdot}9083\ (61) + 1{\cdot}3932\ (68) - 28{\cdot}7499\ (69) + 27{\cdot}3567\ (70)$$

XXXVII.

$$\frac{\sin \mathrm{GMN} \sin \mathrm{GLM} \sin \mathrm{GNL}}{\sin \mathrm{GNM} \sin \mathrm{GML} \sin \mathrm{GLN}} = 1$$

$$\begin{aligned}
\mathrm{GMN} &= 132^{\circ}\ 24'\ 58''{\cdot}51 - 1''{\cdot}8530 - (63) + (65)\\
\mathrm{GLM} &= 62\ \ 29\ \ 7{\cdot}77 - 2{\cdot}6719 - (56) + (57)\\
\mathrm{GNL} &= 37\ \ 35\ \ 4{\cdot}21 - 2{\cdot}5288 - (68) + (70)
\end{aligned}$$

$$\begin{aligned}
&9{\cdot}868\ 2152\ 767 + 19{\cdot}2366\{(63) - (65)\}\\
&9{\cdot}947\ 8687\ 283 + 10{\cdot}9678\{(57) - (56)\}\\
&\underline{9{\cdot}785\ 2736\ 788} + 27{\cdot}3567\{(70) - (68)\}\\
&9{\cdot}601\ 3576\ 838\\
&\qquad\ \ \underline{491\ 539}\\
&\qquad + \ 85\ 299
\end{aligned}$$

$$\begin{aligned}
\mathrm{GNM} &= 28^{\circ}\ 3'\ 39''{\cdot}41 - 1''{\cdot}8530 - (67) + (68)\\
\mathrm{GML} &= 72\ \ 57\ \ 47{\cdot}15 - 2{\cdot}6719 - (63) + (64)\\
\mathrm{GLN} &= 117\ \ 23\ \ 18{\cdot}71 - 2{\cdot}5288 - (61) + (57)
\end{aligned}$$

$$\begin{aligned}
&9{\cdot}672\ 4697\ 014 + 39{\cdot}4985\{(68) - (67)\}\\
&9{\cdot}980\ 5089\ 746 + \ 6{\cdot}4523\{(64) - (63)\}\\
&\underline{9{\cdot}948\ 3704\ 779} + 10{\cdot}9083\{(61) - (57)\}\\
&9{\cdot}601\ 3491\ 539
\end{aligned}$$

$$\begin{aligned}
0 = {} & +85{\cdot}299 - 10{\cdot}9678\,(56) + 21{\cdot}8761\,(57) - 10{\cdot}9083\,(61) + 25{\cdot}6889\,(63) - 6{\cdot}4523\,(64) - 19{\cdot}2366\,(65)\\
& +39{\cdot}4985\,(67) - 66{\cdot}8552\,(68) + 27{\cdot}3567\,(70)
\end{aligned}$$

XXXVIII.

$$\frac{\sin \mathrm{GNM} \sin \mathrm{GON} \sin \mathrm{GMO}}{\sin \mathrm{GMN} \sin \mathrm{GNO} \sin \mathrm{GOM}} = 1$$

$$\begin{aligned}
\mathrm{GNM} &= 28^{\circ}\ 3'\ 39''{\cdot}41 - 1''{\cdot}8530 - (67) + (68)\\
\mathrm{GON} &= 54\ \ 38\ \ 22{\cdot}53 - 7{\cdot}2256 - (72) + (74)\\
\mathrm{GOM} &= 71\ \ 13\ \ 17{\cdot}27 - 4{\cdot}1299 - (62) + (63)
\end{aligned}$$

$$\begin{aligned}
&9{\cdot}672\ 4697\ 014 + 39{\cdot}4985\ \{(68) - (67)\}\\
&9{\cdot}911\ 4280\ 201 + 14{\cdot}9424\ \{(74) - (72)\}\\
&\underline{9{\cdot}976\ 2414\ 945} + \ 7{\cdot}15945\{(63) - (62)\}\\
&9{\cdot}560\ 1392\ 160\\
&\qquad\ \ \ \underline{57\ 859}\\
&\qquad + 34\ 301
\end{aligned}$$

$$\begin{aligned}
\mathrm{GMN} &= 132^{\circ}\ 24'\ 58''{\cdot}51 - 1''{\cdot}8530 - (63) + (65)\\
\mathrm{GNO} &= 43\ \ 6\ \ 31{\cdot}81 - 7{\cdot}2256 - (66) + (68)\\
\mathrm{GOM} &= 46\ \ 2\ \ 49{\cdot}69 - 4{\cdot}1299 - (72) + (73)
\end{aligned}$$

$$\begin{aligned}
&9{\cdot}868\ 2152\ 767 + 19{\cdot}2366\{(63) - (65)\}\\
&9{\cdot}834\ 6500\ 599 + 22{\cdot}4947\{(68) - (66)\}\\
&\underline{9{\cdot}857\ 2704\ 493} + 20{\cdot}3001\{(73) - (72)\}\\
&9{\cdot}560\ 1357\ 859
\end{aligned}$$

$$\begin{aligned}
0 = {} & +34{\cdot}301 - 7{\cdot}15945(62) - 12{\cdot}07725(63) + 19{\cdot}2366(65) + 22{\cdot}4947(66) - 39{\cdot}4985(67) + 17{\cdot}0038\,(68)\\
& +5{\cdot}3577\ (72) - 20{\cdot}3001\ (73) + 14{\cdot}9424\,(74).
\end{aligned}$$

XXXIX.

$$\frac{\sin GKF \sin GNK \sin GON \sin GFO}{\sin GFK \sin GKN \sin GNO \sin GOF} = 1$$

GKF = 36° 38′ 7″·62 − 2″·4043 − (51) + (52)
GNK = 36 13 7·76 − 4·9362 − (68) + (69)
GON = 54 38 22·53 − 7·2256 − (72) + (74)
GFO = 40 2 57·09 − 2·8698 − (36) + (28)

GFK = 53° 34′ 19″·33 − 2″·4043 − (33) + (36)
GKN = 75 26 57·90 − 4·9362 − (55) + (51)
GNO = 43 6 31·81 − 7·2256 − (66) + (68)
GOF = 20 20 28·35 − 2·8698 − (71) + (72)

9·775 7648 632 + 28·3149{(52) − (51)}
9·771 4782 857 + 28·7499{(69) − (68)}
9·911 4280 201 + 14·9424{(74) − (72)}
9·808 5042 860 + 25·0496{(28) − (36)}
9·267 1754 550
442 089
+ 312 461

9·905 5784 924 + 15·5395{(36) − (33)}
9·985 8395 551 + 5·4656{(51) − (55)}
9·834 6500 599 + 22·4947{(68) − (66)}
9·541 0761 015 + 56·7964{(72) − (71)}
9·267 1442 089

$$0 = +312·461 + 25·0496\,(28) + 15·5395\,(33) - 40·5891\,(36) - 33·7805\,(51) + 28·3149\,(52) + 5·4656\,(55) + 22·4947\,(66) - 51·2446\,(68) + 28·7499\,(69) + 56·7964\,(71) - 71·7388\,(72) + 14·9424\,(74)$$

The 39 geometrical equations of condition, when expressed algebraically, are then as follows:—

Equations A.

I. 0 = − 1·4042 − (2) + (5) − (7) + (8) − (13) + (14)
II. 0 = − 2·7737 − (2) + (4) − (12) + (14) − (23) + (24)
III. 0 = − 0·9595 − (8) + (11) − (12) + (13) − (22) + (24)
IV. 0 = − 1·2157 − (3) + (4) − (16) + (19) − (23) + (25)
V. 0 = − 0·9204 − (3) + (5) − (7) + (10) − (18) + (19)
VI. 0 = − 0·8424 − (1) + (4) − (23) + (26) − (30) + (35)
VII. 0 = − 0·3201 − (1) + (5) − (7) + (9) − (32) + (35)
VIII. 0 = + 0·9990 − (1) + (3) + (15) − (19) − (31) + (35)
IX. 0 = − 0·9170 − (21) + (26) − (30) + (36) − (38) + (40)
X. 0 = − 4·0567 − (6) + (9) − (32) + (36) − (38) + (39)
XI. 0 = − 5·4398 − (29) + (36) − (38) + (41) − (46) + (48)
XII. 0 = − 3·7064 + (21) − (27) − (40) + (41) − (46) + (47)
XIII. 0 = + 1·7439 − (41) + (42) + (46) − (49) + (51) + (53)
XIV. 0 = − 1·3529 − (33) + (36) − (38) + (42) − (51) + (52)
XV. 0 = − 0·2569 − (42) + (43) + (51) − (54) − (57) + (60)
XVI. 0 = − 2·5773 − (41) + (43) + (46) − (50) − (57) + (59)

XVII. $0 = -\ 9{\cdot}3165 - (34) + (36) - (38) + (43) - (57) + (58)$

XVIII. $0 = +\ 2{\cdot}4836 - (43) + (44) + (57) - (61) - (68) + (70)$

XIX. $0 = -\ 0{\cdot}7186 - (42) + (44) + (51) - (55) - (68) + (69)$

XX. $0 = -\ 3{\cdot}0090 - (44) + (45) - (63) + (65) - (67) + (68)$

XXI. $0 = -\ 1{\cdot}3157 - (43) + (45) - (56) + (57) - (63) + (64)$

XXII. $0 = +\ 1{\cdot}0806 + (28) - (36) - (37) + (38) - (71) + (72)$

XXIII. $0 = +\ 1{\cdot}0732 + (37) - (44) - (66) + (68) - (72) + (74)$

XXIV. $0 = -\ 1{\cdot}6497 + (37) - (45) - (62) + (63) - (72) + (73)$

XXV. $0 = +\ 3{\cdot}2980 - 0{\cdot}00015\,(2) + 15{\cdot}5719\,(4) - 15{\cdot}57175\,(5) - 6{\cdot}1882\,(7) + 0{\cdot}0004577\,(8)$
$+ 6{\cdot}1878\,(11) - 12{\cdot}2284\,(22) + 12{\cdot}2289\,(23) - 0{\cdot}0005\,(24)$

XXVI. $0 = +\ 16{\cdot}1080 + 6{\cdot}4168\,(7) - 6{\cdot}1882\,(8) - 0{\cdot}2286\,(10) + 9{\cdot}0970\,(12) - 15{\cdot}9451\,(13)$
$+ 6{\cdot}8481\,(14) - 10{\cdot}5944\,(16) + 19{\cdot}8668\,(18) - 9{\cdot}2724\,(19) - 6{\cdot}1898\,(23)$
$+ 12{\cdot}2289\,(24) - 6{\cdot}0391\,(25)$

XXVII. $0 = -\ 10{\cdot}8440 + 2{\cdot}2854\,(7) - 2{\cdot}0568\,(9) - 0{\cdot}2286\,(10) - 12{\cdot}9692\,(15) + 19{\cdot}8668\,(18)$
$- 6{\cdot}8970\,(19) + 31{\cdot}8644\,(31) - 39{\cdot}6790\,(32) + 7{\cdot}8146\,(35)$

XXVIII. $0 = +\ 42{\cdot}4090 + 12{\cdot}9692\,(15) - 10{\cdot}5944\,(16) - 2{\cdot}3748\,(19) + 1{\cdot}0054\,(23) - 6{\cdot}0391\,(25)$
$+ 5{\cdot}0337\,(26) + 29{\cdot}1742\,(30) - 31{\cdot}8644\,(31) + 2{\cdot}6902\,(35)$

XXIX. $0 = -\ 41{\cdot}3700 - 0{\cdot}9917\,(6) + 3{\cdot}0481\,(9) - 2{\cdot}0564\,(11) - 2{\cdot}1682\,(21) + 5{\cdot}0333\,(22)$
$- 2{\cdot}8651\,(26) - 0{\cdot}8664\,(38) + 32{\cdot}4833\,(39) - 31{\cdot}6169\,(40)$

XXX. $0 = -\ 16{\cdot}1240 - 5{\cdot}16765\,(15) + 3{\cdot}28535\,(16) + 1{\cdot}8823\,(20) - 0{\cdot}9552\,(21) + 3{\cdot}1234\,(25)$
$- 2{\cdot}1682\,(26) - 11{\cdot}0929\,(30) + 12{\cdot}2272\,(31) - 1{\cdot}1343\,(36)$

XXXI. $0 = +135{\cdot}7980 - 0{\cdot}4270\,(21) - 2{\cdot}1682\,(26) + 2{\cdot}5952\,(27) + 10{\cdot}7567\,(29) - 11{\cdot}0929\,(30)$
$+ 0{\cdot}3362\,(36) + 0{\cdot}3769\,(46) - 25{\cdot}1584\,(47) + 24{\cdot}7815\,(48)$

XXXII. $0 = +\ 37{\cdot}2130 + 25{\cdot}4384\,(15) + 3{\cdot}28535\,(16) - 29{\cdot}1391\,(17) + 0{\cdot}41535\,(20) - 0{\cdot}5282\,(21)$
$+ 3{\cdot}1234\,(25) - 2{\cdot}5952\,(27) + 25{\cdot}4384\,(29) - 25{\cdot}4384\,(31) + 0{\cdot}2800\,(46)$
$+ 25{\cdot}1584\,(47) - 25{\cdot}4384\,(48)$

XXXIII. $0 = +\ 95{\cdot}4940 - 10{\cdot}7567\,(29) + 15{\cdot}5395\,(33) - 4{\cdot}7828\,(36) + 29{\cdot}0538\,(46) - 24{\cdot}7815\,(48)$
$- 4{\cdot}2723\,(49) - 27{\cdot}7213\,(51) + 28{\cdot}3149\,(52) - 0{\cdot}5936\,(53)$

XXXIV. $0 = +\ 65{\cdot}9200 - 15{\cdot}5395\,(33) + 38{\cdot}7103\,(34) - 23{\cdot}1708\,(36) + 33{\cdot}2559\,(51) - 28{\cdot}3149\,(52)$
$- 4{\cdot}9410\,(54) - 51{\cdot}2545\,(57) + 63{\cdot}4591\,(58) - 12{\cdot}2046\,(60)$

XXXV. $0 = +\ 12{\cdot}4160 + 4{\cdot}64935\,(46) + 4{\cdot}2723\,(49) - 8{\cdot}92165\,(50) - 5{\cdot}5346\,(51) + 0{\cdot}5936\,(53)$
$+ 4{\cdot}9410\,(54) + 1{\cdot}6934\,(57) - 13{\cdot}8980\,(59) + 12{\cdot}2046\,(60)$

XXXVI. $0 = +\ 27{\cdot}4360 + 0{\cdot}5246\,(51) + 4{\cdot}9410\,(54) - 5{\cdot}4656\,(55) - 1{\cdot}2963\,(57) + 12{\cdot}2046\,(60)$
$- 10{\cdot}9083\,(61) + 1{\cdot}3932\,(68) - 28{\cdot}7499\,(69) + 27{\cdot}3567\,(70)$

XXXVII. $0 = +\ 85{\cdot}2990 - 10{\cdot}9678\,(56) + 21{\cdot}8761\,(57) - 10{\cdot}9083\,(61) + 25{\cdot}6889\,(63) - 6{\cdot}4523\,(64)$
$- 19{\cdot}2366\,(65) + 39{\cdot}4985\,(67) - 66{\cdot}8552\,(68) + 27{\cdot}3567\,(70)$

XXXVIII. $0 = +\ 34{\cdot}3010 - 7{\cdot}15945\,(62) - 12{\cdot}07725\,(63) + 19{\cdot}2366\,(65) + 22{\cdot}4947\,(66) - 39{\cdot}4988\,(67)$
$+ 17{\cdot}0038\,(68) + 5{\cdot}3577\,(72) - 20{\cdot}3001\,(73) + 14{\cdot}9424\,(74)$

XXXIX. $0 = +312{\cdot}4610 + 25{\cdot}0496\,(28) + 15{\cdot}5395\,(33) - 40{\cdot}5891\,(36) - 33{\cdot}7805\,(51) + 28{\cdot}3149\,(52)$
$+\ 5{\cdot}4656\,(55) + 22{\cdot}4947\,(66) - 51{\cdot}2446\,(68) + 28{\cdot}7499\,(69) + 56{\cdot}7964\,(71)$
$- 71{\cdot}7388\,(72) + 14{\cdot}9424\,(74)$

These equations correspond to the equations of condition (1)

$$\begin{aligned}
0 &= a + a_1 x_1 + a_2 x_2 + a_3 x_3 + \cdots \\
0 &= b + b_1 x_1 + b_2 x_2 + b_3 x_3 + \cdots \\
0 &= c + c_1 x_1 + c_2 x_2 + c_3 x_3 + \cdots \\
&\vdots \\
0 &= q + q_1 x_1 + q_2 x_2 + q_3 x_3 + \cdots
\end{aligned}$$

at page 262, and we have now to form the next set of equations; namely equations (5) page 263

$$\begin{aligned}
w_1 x_1 &= a_1 \lambda_1 + b_1 \lambda_2 + c_1 \lambda_3 + \cdots q_1 \lambda_n \\
w_2 x_2 &= a_2 \lambda_1 + b_2 \lambda_2 + c_2 \lambda_3 + \cdots q_2 \lambda_n \\
w_3 x_3 &= a_3 \lambda_1 + b_3 \lambda_2 + c_3 \lambda_3 + \cdots q_3 \lambda_n \\
&\vdots \\
w_m x_m &= a_m \lambda_1 + b_m \lambda_2 + c_m \lambda_3 + \cdots q_m \lambda_n
\end{aligned}$$

$\lambda_1\ \lambda_2\ \lambda_3 \cdots$ being a new system of unknown quantities. These quantities are generally designated by the Roman numerals

I, II, III, IV, V, ...

The method of formation of this second set of equations is obvious, the symbols $x_1\ x_2\ x_3 \cdots x_n$ corresponding with the symbols (1), (2), (3) ... (n). They are as follow:—

Equations B.

$\frac{1}{0{\cdot}080}(1) = -\text{VI} - \text{VII} - \text{VIII}$

$\frac{1}{0{\cdot}206}(2) = -\text{I} - \text{II} - 0{\cdot}0002\ \text{XXV}$

$\frac{1}{0{\cdot}158}(3) = -\text{IV} - \text{V} + \text{VIII}$

$\frac{1}{0{\cdot}338}(4) = +\text{II} + \text{IV} + \text{VI} + 15{\cdot}5719\ \text{XXV}$

$\frac{1}{0{\cdot}226}(5) = +\text{I} + \text{V} + \text{VII} - 15{\cdot}5717\ \text{XXV}$

$\frac{1}{3{\cdot}4355}(6) = -\text{X} - 0{\cdot}9917\ \text{XXIX}$

$\frac{1}{1{\cdot}211}(7) = -\text{I} - \text{V} - \text{VII} - 6{\cdot}1882\ \text{XXV} + 6{\cdot}4168\ \text{XXVI} + 2{\cdot}2854\ \text{XXVII}$

$\frac{1}{0{\cdot}292}(8) = +\text{I} - \text{III} + 0{\cdot}0004\ \text{XXV} - 6{\cdot}1882\ \text{XXVI}$

$\frac{1}{0{\cdot}886}(9) = +\text{VII} + \text{X} - 2{\cdot}0568\ \text{XXVII} + 3{\cdot}0481\ \text{XXIX}$

$\frac{1}{1{\cdot}023}(10) = +\text{V} - 0{\cdot}2286\ \text{XXVI} - 0{\cdot}2286\ \text{XXVII}$

$\frac{1}{0{\cdot}141}(11) = +\text{III} + 6{\cdot}1878\ \text{XXV} - 2{\cdot}0564\ \text{XXIX}$

$\frac{1}{0{\cdot}251}(12) = -\text{II} - \text{III} + 9{\cdot}0970\ \text{XXVI}$

$\frac{1}{0{\cdot}285}(13) = -\text{I} + \text{III} - 15{\cdot}9451\ \text{XXVI}$

$\frac{1}{0{\cdot}314}(14) = +\text{I} + \text{II} + 6{\cdot}8481\ \text{XXVI}$

$\frac{1}{1{\cdot}714}(15) = +\text{VIII} - 12{\cdot}9692\ \text{XXVII} + 12{\cdot}9692\ \text{XXVIII} - 5{\cdot}16765\ \text{XXX} + 25{\cdot}4384\ \text{XXXII}$

$\frac{1}{0{\cdot}359}(16) = -\text{IV} - 10{\cdot}5944\ \text{XXVI} - 10{\cdot}5944\ \text{XXVIII} + 3{\cdot}28535\ \text{XXX} + 3{\cdot}28535\ \text{XXXII}$

$$\frac{1}{1\cdot694}(17) = -\ 29\cdot1391\ \text{XXXII}$$

$$\frac{1}{0\cdot091}(18) = -\ \text{V} + 19\cdot8668\ \text{XXVI} + 19\cdot8668\ \text{XXVII}$$

$$\frac{1}{0\cdot291}(19) = +\ \text{IV} + \text{V} - \text{VIII} - 9\cdot2724\ \text{XXVI} - 6\cdot8970\ \text{XXVII} - 2\cdot3748\ \text{XXVIII}$$

$$\frac{1}{1\cdot695}(20) = +\ 1\cdot8823\ \text{XXX} + 0\cdot41535\ \text{XXXII}$$

$$\frac{1}{0\cdot638}(21) = -\ \text{IX} + \text{XII} - 2\cdot1682\ \text{XXIX} - 0\cdot9552\ \text{XXX} - 0\cdot4270\ \text{XXXI} - 0\cdot5282\ \text{XXXII}$$

$$\frac{1}{0\cdot064}(22) = -\ \text{III} - 12\cdot2284\ \text{XXV} + 5\cdot0333\ \text{XXIX}$$

$$\frac{1}{0\cdot206}(23) = -\ \text{II} - \text{IV} - \text{VI} + 12\cdot2289\ \text{XXV} - 6\cdot1898\ \text{XXVI} + 1\cdot0054\ \text{XXVIII}$$

$$\frac{1}{0\cdot218}(24) = +\ \text{II} + \text{III} - 0\cdot0005\ \text{XXV} + 12\cdot2289\ \text{XXVI}$$

$$\frac{1}{0\cdot193}(25) = +\ \text{IV} - 6\cdot0391\ \text{XXVI} - 6\cdot0391\ \text{XXVIII} + 3\cdot1234\ \text{XXX} + 3\cdot1234\ \text{XXXII}$$

$$\frac{1}{0\cdot383}(26) = +\ \text{VI} + \text{IX} + 5\cdot0337\ \text{XXVIII} - 2\cdot8651\ \text{XXIX} - 2\cdot1682\ \text{XXX} - 2\cdot1682\ \text{XXXI}$$

$$\frac{1}{0\cdot371}(27) = -\ \text{XII} + 2\cdot5952\ \text{XXXI} - 2\cdot5952\ \text{XXXII}$$

$$\frac{1}{2\cdot9005}(28) = +\ \text{XXII} + 25\cdot0496\ \text{XXXIX}$$

$$\frac{1}{1\cdot0861}(29) = -\ \text{XI} + 10\cdot7567\ \text{XXXI} + 25\cdot4384\ \text{XXXII} - 10\cdot7567\ \text{XXXIII}$$

$$\frac{1}{0\cdot2808}(30) = -\ \text{VI} - \text{IX} + 29\cdot1742\ \text{XXVIII} - 11\cdot0929\ \text{XXX} - 11\cdot0929\ \text{XXXI}$$

$$\frac{1}{0\cdot4105}(31) = -\ \text{VIII} + 31\cdot8644\ \text{XXVII} - 31\cdot8644\ \text{XXVIII} + 12\cdot2272\ \text{XXX} - 25\cdot4384\ \text{XXXII}$$

$$\frac{1}{0\cdot3671}(32) = -\ \text{VII} - \text{X} - 39\cdot6790\ \text{XXVII}$$

$$\frac{1}{1\cdot8257}(33) = -\ \text{XIV} + 15\cdot5395\ \text{XXXIII} - 15\cdot5395\ \text{XXXIV} + 15\cdot5395\ \text{XXXIX}$$

$$\frac{1}{5\cdot4849}(34) = -\ \text{XVII} + 38\cdot7103\ \text{XXXIV}$$

$$\frac{1}{1\cdot7072}(35) = +\ \text{VI} + \text{VII} + \text{VIII} + 7\cdot8146\ \text{XXVII} + 2\cdot6902\ \text{XXVIII}$$

$$\begin{aligned}\frac{1}{0\cdot3139}(36) = {} & +\ \text{IX} + \text{X} + \text{XI} + \text{XIV} + \text{XVII} - \text{XXII} - 1\cdot1343\ \text{XXX} + 0\cdot3362\ \text{XXXI} - 4\cdot7828\ \text{XXXIII} \\ & - 23\cdot1708\ \text{XXXIV} - 40\cdot5891\ \text{XXXIX}\end{aligned}$$

$$\frac{1}{4\cdot1919}(37) = -\ \text{XXII} + \text{XXIII} + \text{XXIV}$$

$$\frac{1}{0\cdot3935}(38) = -\ \text{IX} - \text{X} - \text{XI} - \text{XIV} - \text{XVII} + \text{XXII} - 0\cdot8664\ \text{XXIX}$$

$$\frac{1}{0\cdot1972}(39) = +\ \text{X} + 32\cdot4833\ \text{XXIX}$$

$$\frac{1}{0\cdot1944}(40) = +\ \text{IX} - \text{XII} - 31\cdot6169\ \text{XXIX}$$

$$\frac{1}{0\cdot6304}(41) = +\ \text{XI} + \text{XII} - \text{XIII} - \text{XVI}$$

$$\frac{1}{0\cdot3952}(42) = +\ \text{XIII} + \text{XIV} - \text{XV} - \text{XIX}$$

$$\frac{1}{0\cdot5746}(43) = +\ \text{XV} + \text{XVI} + \text{XVII} - \text{XVIII} - \text{XXI}$$

$$\frac{1}{2\cdot4091}(44) = +\ \text{XVIII} + \text{XIX} - \text{XX} - \text{XXIII}$$

$$\frac{1}{4\cdot9953}(45) = +\ \text{XX} + \text{XXI} - \text{XXIV}$$

$$\begin{aligned}\frac{1}{0\cdot4652}(46) = {} & -\ \text{XI} - \text{XII} + \text{XIII} + 0\cdot3769\ \text{XXXI} + 0\cdot2800\ \text{XXXII} + 29\cdot0538\ \text{XXXIII} + 4\cdot64935\ \text{XXXV} \\ & + \text{XVI}\end{aligned}$$

$$\frac{1}{0\cdot1752}(47) = +\ \text{XII} - 25\cdot1584\ \text{XXXI} + 25\cdot1584\ \text{XXXII}$$

$$\frac{1}{0\cdot2178}(48) = +\ \text{XI} + 24\cdot7815\ \text{XXXI} - 25\cdot4384\ \text{XXXII} - 24\cdot7815\ \text{XXXIII}$$

$$\frac{1}{0\cdot3475}(49) = -\ \text{XIII} - 4\cdot2723\ \text{XXXIII} + 4\cdot2723\ \text{XXXV}$$

$$\frac{1}{0\cdot4856}(50) = -\ \text{XVI} - 8\cdot92165\ \text{XXXV}$$

$$\begin{aligned}\frac{1}{2\cdot6068}(51) = {} & -\ \text{XIII} - \text{XIV} + \text{XV} + \text{XIX} - 27\cdot7213\ \text{XXXIII} + 33\cdot2559\ \text{XXXIV} - 5\cdot5346\ \text{XXXV} \\ & + 0\cdot5246\ \text{XXXVI} - 33\cdot7805\ \text{XXXIX}\end{aligned}$$

$$\frac{1}{3\cdot4087}(52) = +\ \text{XIV} + 28\cdot3149\ \text{XXXIII} - 28\cdot3149\ \text{XXXIV} + 28\cdot3149\ \text{XXXIX}$$

$$\frac{1}{0\cdot2530}(53) = +\ \text{XIII} - 0\cdot5936\ \text{XXXIII} + 0\cdot5936\ \text{XXXV}$$

$$\frac{1}{0\cdot5222}(54) = -\ \text{XV} - 4\cdot9410\ \text{XXXIV} + 4\cdot9410\ \text{XXXV} + 4\cdot9410\ \text{XXXVI}$$

$\frac{1}{2\cdot2144}(55) = -\text{XIX} - 5\cdot4656\ \text{XXXVI} + 5\cdot4656\ \text{XXXIX}$

$\frac{1}{0\cdot2775}(56) = -\text{XXI} - 10\cdot9678\ \text{XXXVII}$

$\frac{1}{0\cdot5482}(57) = -\text{XV} - \text{XVI} - \text{XVII} + \text{XXI} - 51\cdot2545\ \text{XXXIV} + 1\cdot6934\ \text{XXXV} - 1\cdot2963\ \text{XXXVI} + 21\cdot8761\ \text{XXXVII} + \text{XVIII}$

$\frac{1}{0\cdot2427}(58) = +\text{XVII} + 63\cdot4591\ \text{XXXIV}$

$\frac{1}{0\cdot5023}(59) = +\text{XVI} - 13\cdot8980\ \text{XXXV}$

$\frac{1}{0\cdot9509}(60) = +\text{XV} - 12\cdot2046\ \text{XXXIV} + 12\cdot2046\ \text{XXXV} + 12\cdot2046\ \text{XXXVI}$

$\frac{1}{0\cdot7301}(61) = -\text{XVIII} - 10\cdot9083\ \text{XXXVI} - 10\cdot9083\ \text{XXXVII}$

$\frac{1}{15\cdot5191}(62) = -\text{XXIV} - 7\cdot15945\ \text{XXXVIII}$

$\frac{1}{7\cdot7595}(63) = -\text{XX} + \text{XXIV} + 25\cdot6889\ \text{XXXVII} - 12\cdot07725\ \text{XXXVIII} - \text{XXI}$

$\frac{1}{0\cdot4175}(64) = +\text{XXI} - 6\cdot4523\ \text{XXXVII}$

$\frac{1}{2\cdot1358}(65) = +\text{XX} - 19\cdot2366\ \text{XXXVII} + 19\cdot2366\ \text{XXXVIII}$

$\frac{1}{0\cdot2432}(66) = -\text{XXIII} + 22\cdot4947\ \text{XXXVIII} + 22\cdot4947\ \text{XXXIX}$

$\frac{1}{2\cdot6985}(67) = -\text{XX} + 39\cdot4985\ \text{XXXVII} - 39\cdot4985\ \text{XXXVIII}$

$\frac{1}{0\cdot1815}(68) = -\text{XVIII} - \text{XIX} + \text{XX} + \text{XXIII} + 1\cdot3932\ \text{XXXVI} - 66\cdot8552\ \text{XXXVII} + 17\cdot0038\ \text{XXXVIII} - 51\cdot2446\ \text{XXXIX}$

$\frac{1}{0\cdot2107}(69) = +\text{XIX} - 28\cdot7499\ \text{XXXVI} + 28\cdot7499\ \text{XXXIX}$

$\frac{1}{0\cdot2886}(70) = +\text{XVIII} + 27\cdot3567\ \text{XXXVI} + 27\cdot3567\ \text{XXXVII}$

$\frac{1}{1\cdot3201}(71) = -\text{XXII} + 56\cdot7964\ \text{XXXIX}$

$\frac{1}{0\cdot4924}(72) = +\text{XXII} - \text{XXIII} - \text{XXIV} + 5\cdot3577\ \text{XXXVIII} - 71\cdot7388\ \text{XXXIX}$

$\frac{1}{0\cdot5455}(73) = +\text{XXIV} - 20\cdot3001\ \text{XXXVIII}$

$\frac{1}{0\cdot8247}(74) = +\text{XXIII} + 14\cdot9424\ \text{XXXVIII} + 14\cdot9424\ \text{XXXIX}$

From these equations we immediately derive the values of (1), (2), (3) . . . in terms of I, II, III . . . .

Equations C.

(1) = − 0·0800 VI − 0·0800 VII − 0·0800 VIII
(2) = − 0·2060 I − 0·2060 II − 0·0000309 XXV
(3) = − 0·1580 IV − 0·1580 V + 0·1580 VIII
(4) = + 0·3380 II + 0·3380 IV + 0·3380 VI + 5·263302 XXV
(5) = + 0·2260 I + 0·2260 V + 0·2260 VII − 3·519215 XXV
(6) = − 3·4355 X − 3·406950 XXIX
(7) = − 1·2110 I − 1·2110 V − 1·2110 VII − 7·493936 XXV + 7·770745 XXVI + 2·767619 XXVII
(8) = + 0·2920 I − 0·2920 III + 0·000133648 XXV − 1·806961 XXVI
(9) = + 0·8860 VII + 0·8860 X − 1·822325 XXVII + 2·700616 XXIX
(10) = + 1·0230 V − 0·2338578 XXVI − 0·2338578 XXVII
(11) = + 0·1410 III + 0·872475 XXV − 0·2899533 XXIX
(12) = − 0·2510 II − 0·2510 III + 2·283347 XXVI
(13) = − 0·2850 I + 0·2850 III − 4·544353 XXVI

(14) = + 0·3140 I + 0·3140 II + 2·150303 XXVI

(15) = + 1·7140 VIII − 22·229306 XXVII + 22·229306 XXVIII − 8·857355 XXX + 43·60142 XXXII

(16) = − 0·3590 IV − 3·803400 XXVI − 3·803400 XXVIII + 1·179440 XXX + 1·179440 XXXII

(17) = − 49·361636 XXXII

(18) = − 0·0910 V + 1·807879 XXVI + 1·807879 XXVII

(19) = + 0·2910 IV + 0·2910 V − 0·2910 VIII − 2·698269 XXVI − 2·007202 XXVII − 0·6910668 XXVIII

(20) = + 3·190499 XXX + 0·704182 XXXII

(21) = − 0·6380 IX + 0·6380 XII − 1·383300 XXIX − 0·6094176 XXX − 0·272426 XXXI − 0·3369916 XXXII

(22) = − 0·0640 III − 0·7826204 XXV + 0·3221293 XXIX

(23) = − 0·2060 II − 0·2060 IV − 0·2060 VI + 2·519159 XXV − 1·275099 XXVI + 0·2071124 XXVIII

(24) = + 0·2180 II + 0·2180 III − 0·000109 XXV + 2·66590 XXVI

(25) = + 0·1930 IV − 1·165537 XXVI − 1·165540 XXVIII + 0·602817 XXX + 0·602817 XXXII

(26) = + 0·3830 VI + 0·3830 IX + 1·927896 XXVIII − 1·097333 XXIX − 0·830414 XXX − 0·830414 XXXI

(27) = − 0·3710 XII + 0·9628056 XXXI − 0·9628056 XXXII

(28) = + 2·9005 XXII + 72·65638 XXXIX

(29) = − 1·0861 XI + 11·68286 XXXI + 27·62865 XXXII − 11·68286 XXXIII

(30) = − 0·2808 VI − 0·2808 IX + 8·192121 XXVIII − 3·114876 XXX − 3·114876 XXXI

(31) = − 0·4105 VIII + 13·08033 XXVII − 13·08033 XXVIII + 5·019283 XXX − 10·44247 XXXII

(32) = − 0·3671 VII − 0·3671 X − 14·56616 XXVII

(33) = − 1·8257 XIV + 28·37042 XXXIII − 28·37042 XXXIV + 28·37042 XXXIX

(34) = − 5·4849 XVII + 212·3223 XXXIV

(35) = + 1·7072 VI + 1·7072 VII + 1·7072 VIII + 13·34109 XXVII + 4·592710 XXVIII

(36) = + 0·3139 IX + 0·3139 X + 0·3139 XI + 0·3139 XIV + 0·3139 XVII − 0·3139 XXII − 0·3560567 XXX + 0·1055332 XXXI − 1·501321 XXXIII − 7·273313 XXXIV − 12·74092 XXXIX

(37) = − 4·1919 XXII + 4·1919 XXIII + 4·1919 XXIV

(38) = − 0·3935 IX − 0·3935 X − 0·3935 XI − 0·3935 XIV − 0·3935 XVII + 0·3935 XXII − 0·3409283 XXIX

(39) = + 0·1972 X + 6·405713 XXIX

(40) = + 0·1944 IX − 0·1944 XII − 6·146330 XXIX

(41) = + 0·6304 XI + 0·6304 XII − 0·6304 XIII − 0·6304 XVI

(42) = + 0·3952 XIII + 0·3952 XIV − 0·3952 XV − 0·3952 XIX

(43) = + 0·5746 XV + 0·5746 XVI + 0·5746 XVII − 0·5746 XVIII − 0·5746 XXI

(44) = + 2·4091 XVIII + 2·4091 XIX − 2·4091 XX − 2·4091 XXXIII

(45) = + 4·9953 XX + 4·9953 XXI − 4·9953 XXIV

(46) = − 0·4652 XI − 0·4652 XII + 0·4652 XIII + 0·4652 XVI + 0·1753339 XXXI + 0·130256 XXXII + 13·51583 XXXIII + 2·162878 XXXV

(47) = + 0·1752 XII − 4·407756 XXXI + 4·407756 XXXII

(48) = + 0·2178 XI + 5·397409 XXXI − 5·540485 XXXII − 5·397409 XXXIII

(49) = − 0·3475 XIII − 1·484613 XXXIII + 1·484613 XXXV

(50) = − 0·4856 XVI − 4·332356 XXXV

(51) = − 2·6068 XIII − 2·6068 XIV + 2·6068 XV + 2·6068 XIX − 72·26388 XXXIII + 86·69148 XXXIV − 14·42759 XXXV + 1·367527 XXXVI − 88·05902 XXXIX

(52) = + 3·4087 XIV + 96·51704 XXXIII − 96·51704 XXXIV + 96·51704 XXXIX

(53) = + 0·2530 XIII − 0·1501863 XXXIII + 0·1501863 XXXV

(54) = − 0·5222 XV − 2·580189 XXXIV + 2·580189 XXXV + 2·580189 XXXVI

(55) = − 2·2144 XIX − 12·10306 XXXVI + 12·10306 XXXIX

(56) = − 0·2775 XXI − 3·043555 XXXVII

(57) = − 0·5482 XV − 0·5482 XVI − 0·5482 XVII + 0·5482 XVIII + 0·5482 XXI − 28·09771 XXXIV + 0·928322 XXXV − 0·7106316 XXXVI + 11·99248 XXXVII

(58) = + 0·2427 XVII + 15·40153 XXXIV

(59) = + 0·5023 XVI − 6·980960 XXXV

(60) = + 0·9509 XV − 11·60539 XXXIV + 11·60539 XXXV + 11·60539 XXXVI

(61) = − 0·7301 XVIII − 7·96415 XXXVI − 7·96415 XXXVII

(62) = − 15·5191 XXIV − 111·10815 XXXVIII

(63) = − 7·7595 XX − 7·7595 XXI + 7·7595 XXIV + 199·3330 XXXVII − 93·71343 XXXVIII

(64) = + 0·4175 XXI − 2·693856 XXXVII

(65) = + 2·1358 XX − 41·08561 XXXVII + 41·08561 XXXVIII

(66) = − 0·2432 XXIII + 5·470710 XXXVIII + 5·470710 XXXIX

(67) = − 2·6985 XX + 106·5867 XXXVII − 106·5867 XXXVIII

(68) = − 0·1815 XVIII − 0·1815 XIX + 0·1815 XX + 0·1815 XXIII + 0·252866 XXXVI − 12·134218 XXXVII + 3·086189 XXXVIII − 9·300894 XXXIX

(69) = + 0·2167 XIX − 6·230103 XXXVI + 6·230103 XXXIX

(70) = + 0·2886 XVIII + 7·89515 XXXVI + 7·89515 XXXVII

(71) = − 1·3281 XXII + 75·43135 XXXIX

(72) = + 0·4924 XXII − 0·4924 XXIII − 0·4924 XXIV + 2·638131 XXXVIII − 35·32418 XXXIX

(73) = + 0·5455 XXIV − 11·07372 XXXVIII

(74) = + 0·8247 XXIII + 12·32297 XXXVIII + 12·32297 XXXIX

The next step is the substitution of the quantities (1), (2), (3) thus expressed, in the 39 equations of condition A., and the following is the result of the substitution :—

Equations D.

0 = − 1·4042 + 2·5340 I + 0·5200 II − 0·5770 III + 1·4370 V + 1·4370 VII + 3·974886 XXV − 2·883050 XXVI − 2·767619 XXVII

0 = − 2·7737 + 0·5200 I + 1·5330 II + 0·4690 III + 0·5440 IV + 0·5440 VI + 2·744065 XXV + 3·807955 XXVI − 0·2071124 XXVIII

0 = − 0·9595 − 0·5770 I + 0·4690 II + 1·2510 III + 1·6548528 XXV − 2·354839 XXVI − 0·6120826 XXIX

0 = − 1·2157 + 0·5440 II + 1·5450 IV + 0·4490 V + 0·5440 VI − 0·4490 VIII + 2·744143 XXV + 1·214693 XXVI − 2·007202 XXVII + 1·739681 XXVIII − 0·5766226 XXX − 0·5766226 XXXII

0 = − 0·9204 + 1·4370 I + 0·4490 IV + 3·0000 V + 1·4370 VII − 0·4490 VIII + 3·974721 XXV − 12·510751 XXVI − 6·816558 XXVII − 0·6910668 XXVIII

$$\begin{aligned}0 = &-0{\cdot}8424 + 0{\cdot}5440\ \mathrm{II} + 0{\cdot}5440\ \mathrm{IV} + 2{\cdot}9950\ \mathrm{VI} + 1{\cdot}7872\ \mathrm{VII} + 1{\cdot}7872\ \mathrm{VIII} + 0{\cdot}6638\ \mathrm{IX}\\ &+ 2{\cdot}744143\ \mathrm{XXV} + 1{\cdot}275099\ \mathrm{XXVI} + 13{\cdot}34109\ \mathrm{XXVII} - 1{\cdot}878627\ \mathrm{XXVIII}\\ &- 1{\cdot}097333\ \mathrm{XXIX} + 2{\cdot}284462\ \mathrm{XXX} + 2{\cdot}284462\ \mathrm{XXXI}\end{aligned}$$

$$\begin{aligned}0 = &-0{\cdot}3201 + 1{\cdot}4370\ \mathrm{I} + 1{\cdot}4370\ \mathrm{V} + 1{\cdot}7872\ \mathrm{VI} + 4{\cdot}4773\ \mathrm{VII} + 1{\cdot}7872\ \mathrm{VIII} + 1{\cdot}2531\ \mathrm{X}\\ &+ 3{\cdot}974721\ \mathrm{XXV} - 7{\cdot}770745\ \mathrm{XXVI} + 23{\cdot}31731\ \mathrm{XXVII} + 4{\cdot}592710\ \mathrm{XXVIII}\\ &+ 2{\cdot}700616\ \mathrm{XXIX}\end{aligned}$$

$$\begin{aligned}0 = &+0{\cdot}9990 - 0{\cdot}4490\ \mathrm{IV} - 0{\cdot}4490\ \mathrm{V} + 1{\cdot}7872\ \mathrm{VI} + 1{\cdot}7872\ \mathrm{VII} + 4{\cdot}3607\ \mathrm{VIII} + 2{\cdot}698269\ \mathrm{XXVI}\\ &- 19{\cdot}96134\ \mathrm{XXVII} + 40{\cdot}593413\ \mathrm{XXVIII} - 13{\cdot}876638\ \mathrm{XXX} + 54{\cdot}04389\ \mathrm{XXXII}\end{aligned}$$

$$\begin{aligned}0 = &-0{\cdot}9170 + 0{\cdot}6638\ \mathrm{VI} + 2{\cdot}2036\ \mathrm{IX} + 0{\cdot}7074\ \mathrm{X} + 0{\cdot}7074\ \mathrm{XI} - 0{\cdot}8324\ \mathrm{XII} + 0{\cdot}7074\ \mathrm{XIV}\\ &+ 0{\cdot}7074\ \mathrm{XVII} - 0{\cdot}7074\ \mathrm{XXII} - 6{\cdot}264225\ \mathrm{XXVIII} - 5{\cdot}519438\ \mathrm{XXIX} + 2{\cdot}537823\ \mathrm{XXX}\\ &+ 2{\cdot}662421\ \mathrm{XXXI} + 0{\cdot}3369916\ \mathrm{XXXII} - 1{\cdot}501321\ \mathrm{XXXIII} - 7{\cdot}273313\ \mathrm{XXXIV}\\ &- 12{\cdot}74092\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = &-4{\cdot}0567 + 1{\cdot}2531\ \mathrm{VII} + 0{\cdot}7074\ \mathrm{IX} + 5{\cdot}5932\ \mathrm{X} + 0{\cdot}7074\ \mathrm{XI} + 0{\cdot}7074\ \mathrm{XIV} + 0{\cdot}7074\ \mathrm{XVII}\\ &- 0{\cdot}7074\ \mathrm{XXII} + 12{\cdot}743835\ \mathrm{XXVII} + 12{\cdot}854207\ \mathrm{XXIX} - 0{\cdot}3560567\ \mathrm{XXX}\\ &+ 0{\cdot}1055332\ \mathrm{XXXI} - 1{\cdot}501321\ \mathrm{XXXIII} - 7{\cdot}273313\ \mathrm{XXXIV} - 12{\cdot}74092\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = &-5{\cdot}4398 + 0{\cdot}7074\ \mathrm{IX} + 0{\cdot}7074\ \mathrm{X} + 3{\cdot}1069\ \mathrm{XI} + 1{\cdot}0956\ \mathrm{XII} - 1{\cdot}0956\ \mathrm{XIII} + 0{\cdot}7074\ \mathrm{XIV}\\ &- 1{\cdot}0956\ \mathrm{XVI} + 0{\cdot}7074\ \mathrm{XVII} - 0{\cdot}7074\ \mathrm{XXII} + 0{\cdot}3409283\ \mathrm{XXIX} - 0{\cdot}3560567\ \mathrm{XXX}\\ &- 6{\cdot}355252\ \mathrm{XXXI} - 33{\cdot}299391\ \mathrm{XXXII} - 8{\cdot}73170\ \mathrm{XXXIII} - 7{\cdot}273313\ \mathrm{XXXIV}\\ &- 2{\cdot}162878\ \mathrm{XXXV} - 12{\cdot}74092\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = &-3{\cdot}7064 - 0{\cdot}8324\ \mathrm{IX} + 1{\cdot}0956\ \mathrm{XI} + 2{\cdot}4742\ \mathrm{XII} - 1{\cdot}0956\ \mathrm{XIII} - 1{\cdot}0956\ \mathrm{XVI} + 4{\cdot}763033\ \mathrm{XXIX}\\ &- 0{\cdot}6094176\ \mathrm{XXX} - 5{\cdot}8183215\ \mathrm{XXXI} + 4{\cdot}903314\ \mathrm{XXXII} - 13{\cdot}51583\ \mathrm{XXXIII}\\ &- 2{\cdot}162878\ \mathrm{XXXV}\end{aligned}$$

$$\begin{aligned}0 = &+1{\cdot}7439 - 1{\cdot}0956\ \mathrm{XI} - 1{\cdot}0956\ \mathrm{XII} + 4{\cdot}6981\ \mathrm{XIII} + 3{\cdot}0020\ \mathrm{XIV} - 3{\cdot}0020\ \mathrm{XV} + 1{\cdot}0956\ \mathrm{XVI}\\ &- 3{\cdot}0020\ \mathrm{XIX} + 0{\cdot}1753339\ \mathrm{XXXI} + 0{\cdot}130256\ \mathrm{XXXII} + 87{\cdot}11414\ \mathrm{XXXIII}\\ &- 86{\cdot}69148\ \mathrm{XXXIV} + 15{\cdot}256041\ \mathrm{XXXV} - 1{\cdot}367527\ \mathrm{XXXVI} + 88{\cdot}05902\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = &-1{\cdot}3529 + 0{\cdot}7074\ \mathrm{IX} + 0{\cdot}7074\ \mathrm{X} + 0{\cdot}7074\ \mathrm{XI} + 3{\cdot}0020\ \mathrm{XIII} + 8{\cdot}9438\ \mathrm{XIV} - 3{\cdot}0020\ \mathrm{XV}\\ &+ 0{\cdot}7074\ \mathrm{XVII} - 3{\cdot}0020\ \mathrm{XIX} - 0{\cdot}7074\ \mathrm{XXII} + 0{\cdot}3409283\ \mathrm{XXIX} - 0{\cdot}3560567\ \mathrm{XXX}\\ &+ 0{\cdot}1055332\ \mathrm{XXXI} + 138{\cdot}90918\ \mathrm{XXXIII} - 162{\cdot}11141\ \mathrm{XXXIV} + 14{\cdot}42759\ \mathrm{XXXV}\\ &- 1{\cdot}367527\ \mathrm{XXXVI} + 143{\cdot}46472\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = &-0{\cdot}2569 - 3{\cdot}0020\ \mathrm{XIII} - 3{\cdot}0020\ \mathrm{XIV} + 5{\cdot}5979\ \mathrm{XV} + 1{\cdot}1228\ \mathrm{XVI} + 1{\cdot}1228\ \mathrm{XVII}\\ &- 1{\cdot}1228\ \mathrm{XVIII} + 3{\cdot}0020\ \mathrm{XIX} - 1{\cdot}1228\ \mathrm{XXI} - 72{\cdot}26388\ \mathrm{XXXIII} + 105{\cdot}76399\ \mathrm{XXXIV}\\ &- 6{\cdot}330711\ \mathrm{XXXV} + 11{\cdot}10336\ \mathrm{XXXVI} - 11{\cdot}99248\ \mathrm{XXXVII} - 88{\cdot}05902\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = &-2{\cdot}5773 - 1{\cdot}0956\ \mathrm{XI} - 1{\cdot}0956\ \mathrm{XII} + 1{\cdot}0956\ \mathrm{XIII} + 1{\cdot}1228\ \mathrm{XV} + 3{\cdot}2063\ \mathrm{XVI} + 1{\cdot}1228\ \mathrm{XVII}\\ &- 1{\cdot}1228\ \mathrm{XVIII} - 1{\cdot}1228\ \mathrm{XXI} + 0{\cdot}1753339\ \mathrm{XXXI} + 0{\cdot}130256\ \mathrm{XXXII} + 13{\cdot}51583\ \mathrm{XXXIII}\\ &+ 28{\cdot}09771\ \mathrm{XXXIV} - 1{\cdot}414048\ \mathrm{XXXV} + 0{\cdot}7106316\ \mathrm{XXXVI} - 11{\cdot}99248\ \mathrm{XXXVII}\end{aligned}$$

$$\begin{aligned}0 = &-9{\cdot}3165 + 0{\cdot}7074\ \mathrm{IX} + 0{\cdot}7074\ \mathrm{X} + 0{\cdot}7074\ \mathrm{XI} + 0{\cdot}7074\ \mathrm{XIV} + 1{\cdot}1228\ \mathrm{XV} + 1{\cdot}1228\ \mathrm{XVI}\\ &+ 7{\cdot}5578\ \mathrm{XVII} - 1{\cdot}1228\ \mathrm{XVIII} - 1{\cdot}1228\ \mathrm{XXI} - 0{\cdot}7074\ \mathrm{XXII} + 0{\cdot}3409283\ \mathrm{XXIX}\\ &- 0{\cdot}3560567\ \mathrm{XXX} + 0{\cdot}1055332\ \mathrm{XXXI} - 1{\cdot}501321\ \mathrm{XXXIII} - 176{\cdot}09637\ \mathrm{XXXIV}\\ &- 0{\cdot}928322\ \mathrm{XXXV} + 0{\cdot}7106316\ \mathrm{XXXVI} - 11{\cdot}99248\ \mathrm{XXXVII} - 12{\cdot}74092\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = &+2{\cdot}4836 - 1{\cdot}1228\ \mathrm{XV} - 1{\cdot}1228\ \mathrm{XVI} - 1{\cdot}1228\ \mathrm{XVII} + 4{\cdot}7321\ \mathrm{XVIII} + 2{\cdot}5906\ \mathrm{XIX}\\ &- 2{\cdot}5906\ \mathrm{XX} + 1{\cdot}1228\ \mathrm{XXI} - 2{\cdot}5906\ \mathrm{XXIII} - 28{\cdot}09771\ \mathrm{XXXIV} + 0{\cdot}928322\ \mathrm{XXXV}\\ &+ 14{\cdot}895802\ \mathrm{XXXVI} + 39{\cdot}98600\ \mathrm{XXXVII} - 3{\cdot}086189\ \mathrm{XXXVIII} + 9{\cdot}300894\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = &-0{\cdot}7186 - 3{\cdot}0020\ \mathrm{XIII} - 3{\cdot}0020\ \mathrm{XIV} + 3{\cdot}0020\ \mathrm{XV} + 2{\cdot}5906\ \mathrm{XVIII} + 8{\cdot}0237\ \mathrm{XIX}\\ &- 2{\cdot}5906\ \mathrm{XX} - 2{\cdot}5906\ \mathrm{XXIII} - 72{\cdot}26388\ \mathrm{XXXIII} + 86{\cdot}69148\ \mathrm{XXXIV} - 14{\cdot}42759\ \mathrm{XXXV}\\ &+ 6{\cdot}987618\ \mathrm{XXXVI} + 12{\cdot}134218\ \mathrm{XXXVII} - 3{\cdot}086189\ \mathrm{XXXVIII} + 84{\cdot}63108\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = {} & - 3\cdot0090 - 2\cdot5906\ \mathrm{XVIII} - 2\cdot5906\ \mathrm{XIX} + 20\cdot1797\ \mathrm{XX} + 12\cdot7548\ \mathrm{XXI} + 2\cdot5906\ \mathrm{XXIII}\\ & - 12\cdot7548\ \mathrm{XXIV} + 0\cdot252866\ \mathrm{XXXVI} - 359\cdot1395\ \mathrm{XXXVII} + 244\cdot47193\ \mathrm{XXXVIII}\\ & - 9\cdot300894\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = {} & - 1\cdot3157 - 1\cdot1228\ \mathrm{XV} - 1\cdot1228\ \mathrm{XVI} - 1\cdot1228\ \mathrm{XVII} + 1\cdot1228\ \mathrm{XVIII} + 12\cdot7548\ \mathrm{XX}\\ & + 14\cdot5726\ \mathrm{XXI} - 12\cdot7548\ \mathrm{XXIV} - 28\cdot09771\ \mathrm{XXXIV} + 0\cdot928322\ \mathrm{XXXV}\\ & - 0\cdot7106316\ \mathrm{XXXVI} - 186\cdot99082\ \mathrm{XXXVII} + 93\cdot71343\ \mathrm{XXXVIII}\end{aligned}$$

$$\begin{aligned}0 = {} & + 1\cdot0806 - 0\cdot7074\ \mathrm{IX} - 0\cdot7074\ \mathrm{X} - 0\cdot7074\ \mathrm{XI} - 0\cdot7074\ \mathrm{XIV} - 0\cdot7074\ \mathrm{XVII} + 9\cdot6203\ \mathrm{XXII}\\ & - 4\cdot6843\ \mathrm{XXIII} - 4\cdot6843\ \mathrm{XXIV} - 0\cdot3409283\ \mathrm{XXIX} + 0\cdot3560567\ \mathrm{XXX} - 0\cdot1055332\ \mathrm{XXXI}\\ & + 1\cdot501321\ \mathrm{XXXIII} + 7\cdot273313\ \mathrm{XXXIV} + 2\cdot638131\ \mathrm{XXXVIII} - 25\cdot35823\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = {} & + 1\cdot0732 - 2\cdot5906\ \mathrm{XVIII} - 2\cdot5906\ \mathrm{XIX} + 2\cdot5906\ \mathrm{XX} - 4\cdot6843\ \mathrm{XXII} + 8\cdot3428\ \mathrm{XXIII}\\ & + 4\cdot6843\ \mathrm{XXIV} + 0\cdot252866\ \mathrm{XXXVI} - 12\cdot134218\ \mathrm{XXXVII} + 7\cdot30032\ \mathrm{XXXVIII}\\ & + 32\cdot87555\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = {} & - 1\cdot6497 - 12\cdot7548\ \mathrm{XX} - 12\cdot7548\ \mathrm{XXI} - 4\cdot6843\ \mathrm{XXII} + 4\cdot6843\ \mathrm{XXIII} + 33\cdot5037\ \mathrm{XXIV}\\ & + 199\cdot3330\ \mathrm{XXXVII} + 3\cdot68287\ \mathrm{XXXVIII} + 35\cdot32418\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = {} & + 3\cdot2980 + 3\cdot974886\ \mathrm{I} + 2\cdot744065\ \mathrm{II} + 1\cdot6548528\ \mathrm{III} + 2\cdot744143\ \mathrm{IV} + 3\cdot974721\ \mathrm{V}\\ & + 2\cdot744143\ \mathrm{VI} + 3\cdot97421\ \mathrm{VII} + 228\cdot90961\ \mathrm{XXV} - 63\cdot68234\ \mathrm{XXVI} - 17\cdot12664\ \mathrm{XXVII}\\ & + 2\cdot532763\ \mathrm{XXVIII} - 5\cdot733303\ \mathrm{XXIX}\end{aligned}$$

$$\begin{aligned}0 = {} & + 16\cdot1080 - 2\cdot883050\ \mathrm{I} + 3\cdot807955\ \mathrm{II} - 2\cdot354839\ \mathrm{III} + 1\cdot214693\ \mathrm{IV} - 12\cdot510751\ \mathrm{V}\\ & + 1\cdot275099\ \mathrm{VI} - 7\cdot770745\ \mathrm{VII} + 2\cdot698269\ \mathrm{VIII} - 63\cdot68234\ \mathrm{XXV} + 317\cdot81935\ \mathrm{XXVI}\\ & + 72\cdot34106\ \mathrm{XXVII} + 52\cdot45947\ \mathrm{XXVIII} - 16\cdot135952\ \mathrm{XXX} - 16\cdot135952\ \mathrm{XXXII}\end{aligned}$$

$$\begin{aligned}0 = {} & - 10\cdot8440 - 2\cdot76719\ \mathrm{I} - 2\cdot007202\ \mathrm{IV} - 6\cdot816558\ \mathrm{V} + 13\cdot34109\ \mathrm{VI} + 23\cdot31731\ \mathrm{VII}\\ & - 19\cdot96134\ \mathrm{VIII} + 12\cdot743835\ \mathrm{X} - 17\cdot12664\ \mathrm{XXV} + 72\cdot34106\ \mathrm{XXVI} + 1447\cdot2082\ \mathrm{XXVII}\\ & - 664\cdot4370\ \mathrm{XXVIII} - 5\cdot554628\ \mathrm{XXIX} + 274\cdot8096\ \mathrm{XXX} - 898\cdot2208\ \mathrm{XXXII}\end{aligned}$$

$$\begin{aligned}0 = {} & + 42\cdot4090 - 0\cdot2071124\ \mathrm{II} + 1\cdot739681\ \mathrm{IV} - 0\cdot6910668\ \mathrm{V} - 1\cdot878627\ \mathrm{VI} + 4\cdot59271\ \mathrm{VII}\\ & + 40\cdot593413\ \mathrm{VIII} - 6\cdot264225\ \mathrm{IX} + 2\cdot532763\ \mathrm{XXV} + 52\cdot45947\ \mathrm{XXVI} - 664\cdot4370\ \mathrm{XXVII}\\ & + 1015\cdot3352\ \mathrm{XXVIII} - 5\cdot523615\ \mathrm{XXIX} - 385\cdot9996\ \mathrm{XXX} - 95\cdot05411\ \mathrm{XXXI}\\ & + 882\cdot0848\ \mathrm{XXXII}\end{aligned}$$

$$\begin{aligned}0 = {} & - 41\cdot3700 - 0\cdot6120826\ \mathrm{III} - 1\cdot097333\ \mathrm{VI} + 2\cdot700616\ \mathrm{VII} - 5\cdot519438\ \mathrm{IX} + 12\cdot854207\ \mathrm{X}\\ & + 0\cdot3409283\ \mathrm{XI} + 4\cdot763033\ \mathrm{XII} + 0\cdot3409283\ \mathrm{XIV} + 0\cdot3409283\ \mathrm{XVII} - 0\cdot3409823\ \mathrm{XXII}\\ & - 5\cdot733303\ \mathrm{XXV} - 5\cdot554628\ \mathrm{XXVII} - 5\cdot523615\ \mathrm{XXVIII} + 422\cdot6737\ \mathrm{XXIX}\\ & + 3\cdot700548\ \mathrm{XXX} + 2\cdot969888\ \mathrm{XXXI} + 0\cdot7306593\ \mathrm{XXXII}\end{aligned}$$

$$\begin{aligned}0 = {} & - 16\cdot1240 - 0\cdot5766226\ \mathrm{IV} + 2\cdot284462\ \mathrm{VI} - 13\cdot876632\ \mathrm{VIII} + 2\cdot537823\ \mathrm{IX} - 0\cdot3560567\ \mathrm{X}\\ & - 0\cdot3560567\ \mathrm{XI} - 0\cdot6094176\ \mathrm{XII} - 0\cdot3560567\ \mathrm{XIV} - 0\cdot3560567\ \mathrm{XVII} + 0\cdot3560567\ \mathrm{XXII}\\ & - 16\cdot135952\ \mathrm{XXVI} + 274\cdot8096\ \mathrm{XXVII} - 385\cdot9996\ \mathrm{XXVIII} + 3\cdot700548\ \mathrm{XXIX}\\ & + 156\cdot24630\ \mathrm{XXX} + 36\cdot49391\ \mathrm{XXXI} - 345\cdot5948\ \mathrm{XXXII} + 1\cdot702948\ \mathrm{XXXIII}\\ & + 8\cdot250119\ \mathrm{XXXIV} + 14\cdot45202\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = {} & + 135\cdot7980 + 2\cdot284462\ \mathrm{VI} + 2\cdot662421\ \mathrm{IX} + 0\cdot1055332\ \mathrm{X} - 6\cdot355252\ \mathrm{XI} - 5\cdot8183215\ \mathrm{XII}\\ & + 0\cdot1753339\ \mathrm{XIII} + 0\cdot1055332\ \mathrm{XIV} + 0\cdot1753339\ \mathrm{XVI} + 0\cdot1055332\ \mathrm{XVII}\\ & - 0\cdot1055332\ \mathrm{XXII} - 95\cdot05411\ \mathrm{XXVIII} + 2\cdot969888\ \mathrm{XXIX} + 36\cdot49391\ \mathrm{XXX}\\ & + 409\cdot3870\ \mathrm{XXXI} + 46\cdot6940\ \mathrm{XXXII} - 254\cdot8355\ \mathrm{XXXIII} - 2\cdot445288\ \mathrm{XXXIV}\\ & + 0\cdot815189\ \mathrm{XXXV} - 4\cdot283497\ \mathrm{XXXIX}\end{aligned}$$

$$\begin{aligned}0 = {} & + 37\cdot2130 - 0\cdot5766226\ \mathrm{IV} + 54\cdot04389\ \mathrm{VIII} + 0\cdot3369916\ \mathrm{IX} - 33\cdot299391\ \mathrm{XI} + 4\cdot903314\ \mathrm{XII}\\ & + 0\cdot1302560\ \mathrm{XIII} + 0\cdot130256\ \mathrm{XVI} - 16\cdot135952\ \mathrm{XXVI} - 898\cdot2208\ \mathrm{XXVII}\\ & + 882\cdot0848\ \mathrm{XXVIII} + 0\cdot7306593\ \mathrm{XXIX} - 345\cdot5948\ \mathrm{XXX} + 46\cdot6940\ \mathrm{XXXI}\\ & + 3776\cdot5692\ \mathrm{XXXII} - 156\cdot10743\ \mathrm{XXXIII} + 0\cdot605606\ \mathrm{XXXV}\end{aligned}$$

$$
\begin{aligned}
0 = +\ 95{\cdot}4940 &- 1{\cdot}501321\ \textsc{ix} - 1{\cdot}501321\ \textsc{x} - 8{\cdot}73170\ \textsc{xi} - 13{\cdot}51583\ \textsc{xii} + 87{\cdot}11414\ \textsc{xiii} \\
&+ 138{\cdot}90918\ \textsc{xiv} - 72{\cdot}26388\ \textsc{xv} + 13{\cdot}51583\ \textsc{xvi} - 1{\cdot}501321\ \textsc{xvii} - 72{\cdot}26388\ \textsc{xix} \\
&+ 1{\cdot}501321\ \textsc{xxii} + 1{\cdot}702948\ \textsc{xxx} - 254{\cdot}8355\ \textsc{xxxi} - 156{\cdot}10743\ \textsc{xxxii} \\
&+ 5842{\cdot}7053\ \textsc{xxxiii} - 5542{\cdot}1468\ \textsc{xxxiv} + 456{\cdot}35964\ \textsc{xxxv} - 37{\cdot}90963\ \textsc{xxxvi} \\
&+ 5675{\cdot}7803\ \textsc{xxxix}
\end{aligned}
$$

$$
\begin{aligned}
0 = +\ 65{\cdot}9200 &- 7{\cdot}273313\ \textsc{ix} - 7{\cdot}273313\ \textsc{x} - 7{\cdot}273313\ \textsc{xi} - 86{\cdot}69148\ \textsc{xiii} - 162{\cdot}11141\ \textsc{xiv} \\
&+ 105{\cdot}76399\ \textsc{xv} + 28{\cdot}09771\ \textsc{xvi} - 176{\cdot}0937\ \textsc{xvii} - 28{\cdot}09771\ \textsc{xviii} + 86{\cdot}69148\ \textsc{xix} \\
&- 28{\cdot}09771\ \textsc{xxi} + 7{\cdot}273313\ \textsc{xxii} + 8{\cdot}250119\ \textsc{xxx} - 2{\cdot}445288\ \textsc{xxxi} \\
&- 5542{\cdot}1468\ \textsc{xxxiii} + 17016{\cdot}2169\ \textsc{xxxiv} - 681{\cdot}7716\ \textsc{xxxv} - 72{\cdot}4869\ \textsc{xxxvi} \\
&- 614{\cdot}6684\ \textsc{xxxvii} - 5806{\cdot}9977\ \textsc{xxxix}
\end{aligned}
$$

$$
\begin{aligned}
0 = +\ 12{\cdot}4160 &- 2{\cdot}162878\ \textsc{xi} - 2{\cdot}162878\ \textsc{xii} + 15{\cdot}256041\ \textsc{xiii} + 14{\cdot}42759\ \textsc{xiv} - 6{\cdot}330711\ \textsc{xv} \\
&- 1{\cdot}414048\ \textsc{xvi} - 0{\cdot}928322\ \textsc{xvii} + 0{\cdot}928322\ \textsc{xviii} - 14{\cdot}42759\ \textsc{xix} + 0{\cdot}928322\ \textsc{xxi} \\
&+ 0{\cdot}815189\ \textsc{xxxi} + 0{\cdot}605606\ \textsc{xxxii} + 456{\cdot}35964\ \textsc{xxxiii} - 681{\cdot}7716\ \textsc{xxxiv} \\
&+ 387{\cdot}9721\ \textsc{xxxv} + 145{\cdot}61618\ \textsc{xxxvi} + 20{\cdot}30806\ \textsc{xxxvii} + 487{\cdot}3713\ \textsc{xxxix}
\end{aligned}
$$

$$
\begin{aligned}
0 = +\ 27{\cdot}4360 &- 1{\cdot}367527\ \textsc{xiii} - 1{\cdot}367527\ \textsc{xiv} + 11{\cdot}10336\ \textsc{xv} + 0{\cdot}7106316\ \textsc{xvi} + 0{\cdot}7106316\ \textsc{xvii} \\
&+ 14{\cdot}895802\ \textsc{xviii} + 6{\cdot}987618\ \textsc{xix} + 0{\cdot}252866\ \textsc{xx} - 0{\cdot}7106316\ \textsc{xxi} + 0{\cdot}252866\ \textsc{xxiii} \\
&- 37{\cdot}90963\ \textsc{xxxiii} - 72{\cdot}4869\ \textsc{xxxiv} + 145{\cdot}61618\ \textsc{xxxv} + 704{\cdot}50542\ \textsc{xxxvi} \\
&+ 270{\cdot}4095\ \textsc{xxxvii} + 4{\cdot}299679\ \textsc{xxxviii} - 304{\cdot}41928\ \textsc{xxxix}
\end{aligned}
$$

$$
\begin{aligned}
0 = +\ 85{\cdot}2990 &- 11{\cdot}99248\ \textsc{xv} - 11{\cdot}99248\ \textsc{xvi} - 11{\cdot}99248\ \textsc{xvii} + 39{\cdot}98600\ \textsc{xviii} + 12{\cdot}134218\ \textsc{xix} \\
&- 359{\cdot}1395\ \textsc{xx} - 186{\cdot}99082\ \textsc{xxi} - 12{\cdot}134218\ \textsc{xxiii} + 199{\cdot}3330\ \textsc{xxiv} \\
&- 614{\cdot}6684\ \textsc{xxxiv} + 20{\cdot}30806\ \textsc{xxxv} + 270{\cdot}4095\ \textsc{xxxvi} + 11548{\cdot}2165\ \textsc{xxxvii} \\
&- 7614{\cdot}0860\ \textsc{xxxviii} + 621{\cdot}8131\ \textsc{xxxix}
\end{aligned}
$$

$$
\begin{aligned}
0 = +\ 34{\cdot}3010 &- 3{\cdot}086189\ \textsc{xviii} - 3{\cdot}086189\ \textsc{xix} + 244{\cdot}47193\ \textsc{xx} + 93{\cdot}71343\ \textsc{xxi} + 2{\cdot}638131\ \textsc{xxii} \\
&+ 7{\cdot}30032\ \textsc{xxiii} + 3{\cdot}68287\ \textsc{xxiv} + 4{\cdot}299679\ \textsc{xxxvi} - 7614{\cdot}0860\ \textsc{xxxvii} \\
&+ 7526{\cdot}2424\ \textsc{xxxviii} - 40{\cdot}21064\ \textsc{xxxix}
\end{aligned}
$$

$$
\begin{aligned}
0 = +\ 312{\cdot}4610 &- 12{\cdot}74092\ \textsc{ix} - 12{\cdot}74092\ \textsc{x} - 12{\cdot}74092\ \textsc{xi} + 88{\cdot}05902\ \textsc{xiii} + 143{\cdot}46472\ \textsc{xiv} \\
&- 88{\cdot}05902\ \textsc{xv} - 12{\cdot}74092\ \textsc{xvii} + 9{\cdot}300894\ \textsc{xviii} - 84{\cdot}631083\ \textsc{xix} - 9{\cdot}300894\ \textsc{xx} \\
&- 25{\cdot}35823\ \textsc{xxii} + 32{\cdot}87555\ \textsc{xxiii} + 35{\cdot}32418\ \textsc{xxiv} + 14{\cdot}45202\ \textsc{xxx} \\
&- 4{\cdot}283497\ \textsc{xxxi} + 5675{\cdot}7803\ \textsc{xxxiii} - 5806{\cdot}9977\ \textsc{xxxiv} + 487{\cdot}3713\ \textsc{xxxv} \\
&- 304{\cdot}41928\ \textsc{xxxvi} + 621{\cdot}8131\ \textsc{xxxvii} - 40{\cdot}21064\ \textsc{xxxviii} + 16332{\cdot}9962\ \textsc{xxxix}
\end{aligned}
$$

From these 39 equations, which correspond to equations (6) page 263, the values of the 39 unknown quantities I, II, III . . . . XXXIX have to be extricated. Their logarithmic values are found to be as follows*

Log I = 9·9196637 +
II = 0·1117152 +
III = 9·8197849 +
IV = 0·0899605 +
V = 8·5525879 +
VI = 9·5502414 −
VII = 9·8652624 −

Log VIII = 0·2081743 +
IX = 8·8848523 −
X = 9·6905475 +
XI = 9·9213808 +
XII = 9·5092750 +
XIII = 9·8820947 −
XIV = 9·8668881 +

Log XV = 0·0979064 −
XVI = 0·3118859 +
XVII = 9·7202759 +
XVIII = 9·6104961 +
XIX = 9·4443796 +
XX = 0·1043927 +
XXI = 9·4412122 −

---

* The sign placed after the logarithm indicates the sign of the corresponding natural number; thus, VIII is a positive quantity, IX a negative quantity.

Log xxii = 9·2646586 +
xxiii = 9·7925531 −
xxiv = 9·9164167 +
xxv = 8·8178796 −
xxvi = 8·8435415 −
xxvii = 8·3985923 −
Log xxviii = 8·9635387 −
xxix = 8·9041661 +
xxx = 9·2665455 +
xxxi = 9·5932397 −
xxxii = 7·9263407 +
xxxiii = 8·8042253 −
Log xxxiv = 8·1902448 −
xxxv = 8·7135943 +
xxxvi = 8·5559922 −
xxxvii = 8·6481505 −
xxxviii = 8·9397225 −
xxxix = 7·9797450 −

By the substitution of these quantities in the equations C. we obtain finally the required corrections (1), (2), (3) . . . Their values are:—

(1) = − 0·0421
(2) = − 0·4376
(3) = + 0·0552
(4) = + 0·3869
(5) = + 0·2616
(6) = − 1·9580
(7) = − 0·2803
(8) = + 0·1759
(9) = + 0·0470
(10) = + 0·0587
(11) = + 0·0125
(12) = − 0·6496
(13) = + 0·2683
(14) = + 0·5171
(15) = + 0·0125
(16) = + 0·4012
(17) = − 0·4166
(18) = − 0·1746
(19) = + 0·2004
(20) = + 0·5953
(21) = + 0·1355
(22) = + 0·0350
(23) = + 0·5424
(24) = + 0·2400
(25) = + 0·5423
(26) = − 0·2585
(27) = − 0·5054
(28) = − 0·1600
(29) = − 4·5079
(30) = + 0·0135
(31) = + 1·0513
(32) = + 0·4539
(33) = − 2·9824
(34) = − 6·1707
(35) = + 0·1429
(36) = + 0·9528
(37) = + 0·0871
(38) = − 0·9424
(39) = + 0·6104
(40) = − 0·5706
(41) = − 0·0825
(42) = + 0·3748
(43) = + 0·6845
(44) = + 0·0833
(45) = + 0·8522
(46) = − 0·7560
(47) = + 1·8215
(48) = − 1·6367
(49) = + 0·4362
(50) = − 1·2198
(51) = + 0·8335
(52) = − 3·0660
(53) = − 0·1755
(54) = + 0·7348
(55) = − 0·2962
(56) = + 0·2120
(57) = − 0·6775
(58) = − 0·1112
(59) = + 0·6690
(60) = − 0·8289
(61) = + 0·3430
(62) = − 3·1312
(63) = − 2·0329
(64) = + 0·0045
(65) = + 0·9675
(66) = − 0·3776
(67) = + 1·1048
(68) = + 0·3445
(69) = + 0·2250
(70) = − 0·5175
(71) = − 0·9642
(72) = + 0·0973
(73) = + 1·4139
(74) = − 1·7017

The application of these corrections to the observed bearings will give those which must be considered the *most probable* bearings, and with these the triangulation may be worked out in the usual manner.

The process is precisely the same for each of the figures. The geometrical and corresponding algebraical equations of condition are given in the following pages.

GEOMETRICAL EQUATIONS OF CONDITION.

FIGURE 2.

Angle Equations.

I. $0 = O' + N' + B - 180^\circ - \varepsilon_1$

II. $0 = O' + N' + A - 180^\circ - \varepsilon_2$

III. $0 = O' + A + B - 180^\circ - \varepsilon_3$

IV. $0 = O' + B + C - 180^\circ - \varepsilon_4$

V. $0 = O' + A + C - 180^\circ - \varepsilon_5$

VI. $0 = C + B + D - 180^\circ - \varepsilon_6$

VII. $0 = C + D + E - 180^\circ - \varepsilon_7$

VIII. $0 = C + E + H - 180^\circ - \varepsilon_8$

IX. $0 = C + H + I - 180^\circ - \varepsilon_9$

X. $0 = C + O' + I - 180^\circ - \varepsilon_{10}$

XI. $0 = O' + I + K - 180^\circ - \varepsilon_{11}$

XII. $0 = O' + K + F' - 180^\circ - \varepsilon_{12}$

Side Equations.

XIII. $\frac{O'N}{O'A} \cdot \frac{O'A}{O'B} \cdot \frac{O'B}{O'N'} = 1$

XIV. $\frac{O'B}{O'C} \cdot \frac{O'C}{O'A} \cdot \frac{O'A}{O'B} = 1$

XV. $\frac{CO'}{CB} \cdot \frac{CB}{CD} \cdot \frac{CD}{CE} \cdot \frac{CE}{CH} \cdot \frac{CH}{CI} \cdot \frac{CI}{CO'} = 1$

XVI. $\frac{O'N'}{O'B} \cdot \frac{O'B}{O'C} \cdot \frac{O'C}{O'I} \cdot \frac{O'I}{O'K} \cdot \frac{O'K}{O'F'} \cdot \frac{O'F'}{O'G'} \cdot \frac{O'G'}{O'N'} = 1$

FIGURE 3.

Angle Equations.

I. $0 = E' + F + C' - 180^\circ - \varepsilon_1$

II. $0 = F + C' + D' - 180^\circ - \varepsilon_2$

III. $0 = K + E' + F - 180^\circ - \varepsilon_3$

IV. $0 = K + E' + C' - 180^\circ - \varepsilon_4$

V. $0 = G + C' + F - 180^\circ - \varepsilon_5$

VI. $0 = G + C' + E' - 180^\circ - \varepsilon_6$

VII. $0 = G + C' + K - 180^\circ - \varepsilon_7$

VIII. $0 = K + H + G - 180^\circ - \varepsilon_8$

IX. $0 = D' + H + G + C' - 360^\circ - \varepsilon_9$

X. $0 = K + I + H - 180^\circ - \varepsilon_{10}$

XI. $0 = E' + K + L - 180^\circ - \varepsilon_{11}$

XII. $0 = L + K + I - 180^\circ - \varepsilon_{12}$

Side Equations.

XIII. $\frac{C'F}{C'E'} \cdot \frac{C'E'}{C'D'} \cdot \frac{C'D'}{C'F} = 1$

XIV. $\frac{E'F}{E'C'} \cdot \frac{E'C'}{E'K} \cdot \frac{E'K}{E'F} = 1$

XV. $\frac{C'K}{C'G} \cdot \frac{C'G}{C'E'} \cdot \frac{C'E'}{C'K} = 1$

XVI. $\frac{GF}{GK} \cdot \frac{GK}{GC'} \cdot \frac{GC'}{GF} = 1$

XVII. $\frac{C'M}{C'D'} \cdot \frac{C'D'}{C'F} \cdot \frac{C'F}{C'G} \cdot \frac{C'G}{C'M} = 1$

XVIII. $\frac{MF}{MC'} \cdot \frac{MC'}{MG} \cdot \frac{MG}{MF} = 1$

XIX. $\frac{MF}{MC'} \cdot \frac{MC'}{MK} \cdot \frac{MK}{MF} = 1$

XX. $\frac{ME'}{MC'} \cdot \frac{MC'}{MK} \cdot \frac{MK}{ME'} = 1$

XXI. $\frac{MH}{MG} \cdot \frac{MG}{MC'} \cdot \frac{MC'}{MD'} \cdot \frac{MD'}{MH} = 1$

XXII. $\frac{MK}{MH} \cdot \frac{MH}{MG} \cdot \frac{MG}{MF} \cdot \frac{MF}{MK} = 1$

XXIII. $\frac{IG}{IK} \cdot \frac{IK}{IH} \cdot \frac{IH}{IG} = 1$

XXIV. $\frac{KI}{KL} \cdot \frac{KL}{KE'} \cdot \frac{KE'}{KG} \cdot \frac{KG}{KH} \cdot \frac{KH}{KI} = 1$

XXV. $\frac{AB'}{AC'} \cdot \frac{AC'}{AD'} \cdot \frac{AD'}{AB'} = 1$

XXVI. $\frac{AE'}{AK} \cdot \frac{AK}{AC'} \cdot \frac{AC'}{AE'} = 1$

XXVII. $\frac{AF}{AK} \cdot \frac{AK}{AC'} \cdot \frac{AC'}{AF} = 1$

XXVIII. $\frac{AK}{AG} \cdot \frac{AG}{AF} \cdot \frac{AF}{AK} = 1$

XXIX. $\frac{AG}{AH} \cdot \frac{AH}{AK} \cdot \frac{AK}{AG} = 1$

XXX. $\frac{AD'}{AC'} \cdot \frac{AC'}{AK} \cdot \frac{AK}{AH} \cdot \frac{AH}{AD'} = 1$

FIGURE 4.

Angle Equations.

I. $0 = H' + L' + D - 180° - \varepsilon_1$

II. $0 = K' + L' + D - 180° - \varepsilon_2$

III. $0 = N' + K' + D - 180° - \varepsilon_3$

IV. $0 = A + L' + D - 180° - \varepsilon_4$

V. $0 = A + N' + D - 180° - \varepsilon_5$

VI. $0 = A + N' + B' - 180° - \varepsilon_6$

VII. $0 = N' + A + C - 180° - \varepsilon_7$

VIII. $0 = N' + B' + C - 180° - \varepsilon_8$

IX. $0 = B' + V + N' - 180° - \varepsilon_9$

X. $0 = A + D + E - 180° - \varepsilon_{10}$

XI. $0 = A + C + F - 180° - \varepsilon_{11}$

XII. $0 = A + D + F - 180° - \varepsilon_{12}$

XIII. $0 = A + E + F - 180° - \varepsilon_{13}$

XIV. $0 = C + A + M - 180° - \varepsilon_{14}$

XV. $0 = A + D + M - 180° - \varepsilon_{15}$

XVI. $0 = A + E + M - 180° - \varepsilon_{16}$

XVII. $0 = A + F + M - 180° - \varepsilon_{17}$

XVIII. $0 = P + M + F - 180° - \varepsilon_{18}$

XIX. $0 = R + C + M - 180° - \varepsilon_{19}$

XX. $0 = R + C + F - 180° - \varepsilon_{20}$

XXI. $0 = R + M + P - 180° - \varepsilon_{21}$

XXII. $0 = Q + P + F - 180° - \varepsilon_{22}$

XXIII. $0 = Q + R + F - 180° - \varepsilon_{23}$

XXIV. $0 = R + S + Q - 180° - \varepsilon_{24}$

XXV. $0 = R + P + S - 180° - \varepsilon_{25}$

XXVI. $0 = T + C + R - 180° - \varepsilon_{26}$

XXVII. $0 = T + R + S - 180° - \varepsilon_{27}$

XXVIII. $0 = T + R + P - 180° - \varepsilon_{28}$

XXIX. $0 = U + T + R - 180° - \varepsilon_{29}$

Side Equations.

XXX. $\frac{H'K'}{H'D} \cdot \frac{H'D}{H'L'} \cdot \frac{H'L'}{H'G'} \cdot \frac{H'G'}{H'K'} = 1$

XXXI. $\frac{L'H'}{L'D} \cdot \frac{L'D}{L'N'} \cdot \frac{L'N'}{L'G'} \cdot \frac{L'G'}{L'H'} = 1$

XXXII. $\frac{L'A}{L'N'} \cdot \frac{L'N'}{L'D} \cdot \frac{L'D}{L'A} = 1$

XXXIII. $\frac{K'A}{K'D} \cdot \frac{K'D}{K'L} \cdot \frac{K'L}{K'A} = 1$

XXXIV. $\frac{N'A}{N'B'} \cdot \frac{N'B'}{N'O'} \cdot \frac{N'O'}{N'G'} \cdot \frac{N'G'}{N'K'} \cdot \frac{N'K'}{N'D} \cdot \frac{N'D}{N'A} = 1$

XXXV. $\frac{N'C}{N'B'} \cdot \frac{N'B'}{N'A} \cdot \frac{N'A}{N'C} = 1$

XXXVI. $\frac{VA}{VN'} \cdot \frac{VN'}{VC} \cdot \frac{VC}{VA} = 1$

XXXVII. $\frac{VN'}{VC} \cdot \frac{VC}{VB'} \cdot \frac{VB'}{VN'} = 1$

XXXVIII. $\frac{EN'}{EA} \cdot \frac{EA}{ED} \cdot \frac{ED}{EN'} = 1$

XXXIX. $\frac{AE}{AD} \cdot \frac{AD}{AF} \cdot \frac{AF}{AE} = 1$

XL. $\frac{AF}{AC} \cdot \frac{AC}{AN'} \cdot \frac{AN'}{AD} \cdot \frac{AD}{AF} = 1$

XLI. $\frac{AD}{AF} \cdot \frac{AF}{AN'} \cdot \frac{AN'}{AD} = 1$

XLII. $\frac{AM}{AF} \cdot \frac{AF}{AC} \cdot \frac{AC}{AM} = 1$

XLIII. $\frac{AE}{AD} \cdot \frac{AD}{AM} \cdot \frac{AM}{AE} = 1$

XLIV. $\frac{FA}{FE} \cdot \frac{FE}{FM} \cdot \frac{FM}{FA} = 1$

XLV. $\frac{FC}{FI} \cdot \frac{FI}{FA} \cdot \frac{FA}{FC} = 1$

XLVI. $\frac{IM}{IF} \cdot \frac{IF}{IA} \cdot \frac{IA}{IM} = 1$

XLVII. $\frac{FC}{FP} \cdot \frac{FP}{FM} \cdot \frac{FM}{FA} \cdot \frac{FA}{FC} = 1$

XLVIII. $\frac{IM}{IF} \cdot \frac{IF}{IP} \cdot \frac{IP}{IM} = 1$

XLIX. $\frac{FC}{FR} \cdot \frac{FR}{FM} \cdot \frac{FM}{FA} \cdot \frac{FA}{FC} = 1$

L. $\frac{RF}{RM} \cdot \frac{RM}{RP} \cdot \frac{RP}{RF} = 1$

LI. $\frac{IC}{IM} \cdot \frac{IM}{IR} \cdot \frac{IR}{IC} = 1$

LII. $\frac{RP}{RF} \cdot \frac{RF}{RQ} \cdot \frac{RQ}{RP} = 1$

LIII. $\frac{IQ}{IP} \cdot \frac{IP}{IF} \cdot \frac{IF}{IQ} = 1$

LIV. $\frac{PR}{PS} \cdot \frac{PS}{PQ} \cdot \frac{PQ}{PR} = 1$

LV. $\frac{RT}{RC} \cdot \frac{RC}{RF} \cdot \frac{RF}{RQ} \cdot \frac{RQ}{RS} \cdot \frac{RS}{RT} = 1$

LVI. $\frac{RS}{RP} \cdot \frac{RP}{RT} \cdot \frac{RT}{RS} = 1$

LVII. $\frac{IR}{IC} \cdot \frac{IC}{IT} \cdot \frac{IT}{IR} = 1$

LVIII. $\frac{TC}{TR} \cdot \frac{TR}{TU} \cdot \frac{TU}{TC} = 1$

LIX. $\frac{SU}{SR} \cdot \frac{SR}{ST} \cdot \frac{ST}{SU} = 1$

FIGURE 5.

Angle Equations.

I. $0 = K + F + Q - 180^\circ - \epsilon_1$
II. $0 = K + F + E - 180^\circ - \epsilon_2$
III. $0 = K + F + D - 180^\circ - \epsilon_3$
IV. $0 = V + D + K - 180^\circ - \epsilon_4$
V. $0 = V + E + K - 180^\circ - \epsilon_5$
VI. $0 = V + F + K - 180^\circ - \epsilon_6$
VII. $0 = B + V + K - 180^\circ - \epsilon_7$
VIII. $0 = B + E + K - 180^\circ - \epsilon_8$
IX. $0 = J + V + B - 180^\circ - \epsilon_9$
X. $0 = J + V + F - 180^\circ - \epsilon_{10}$
XI. $0 = J + V + K - 180^\circ - \epsilon_{11}$
XII. $0 = W + V + E - 180^\circ - \epsilon_{12}$
XIII. $0 = W + B + V - 180^\circ - \epsilon_{13}$
XIV. $0 = W + V + K - 180^\circ - \epsilon_{14}$
XV. $0 = W + V + J - 180^\circ - \epsilon_{15}$
XVI. $0 = X + W + V - 180^\circ - \epsilon_{16}$
XVII. $0 = X + W + B - 180^\circ - \epsilon_{17}$
XVIII. $0 = X + W + J - 180^\circ - \epsilon_{18}$
XIX. $0 = Z + X + J - 180^\circ - \epsilon_{19}$
XX. $0 = Z + J + W - 180^\circ - \epsilon_{20}$
XXI. $0 = X + J + Y - 180^\circ - \epsilon_{21}$
XXII. $0 = X + Z + Y - 180^\circ - \epsilon_{22}$
XXIII. $0 = K + Q + H - 180^\circ - \epsilon_{23}$
XXIV. $0 = Q + F + H - 180^\circ - \epsilon_{24}$
XXV. $0 = H + K + L - 180^\circ - \epsilon_{25}$
XXVI. $0 = J + K + L - 180^\circ - \epsilon_{26}$
XXVII. $0 = N + H + L - 180^\circ - \epsilon_{27}$
XXVIII. $0 = N + L + J - 180^\circ - \epsilon_{28}$
XXIX. $0 = G + Q + H - 180^\circ - \epsilon_{29}$
XXX. $0 = G + Q + K - 180^\circ - \epsilon_{30}$
XXXI. $0 = G + L + H - 180^\circ - \epsilon_{31}$
XXXII. $0 = G + L + N - 180^\circ - \epsilon_{32}$
XXXIII. $0 = O + N + J - 180^\circ - \epsilon_{33}$
XXXIV. $0 = O + N + L - 180^\circ - \epsilon_{34}$
XXXV. $0 = O + N + H - 180^\circ - \epsilon_{35}$
XXXVI. $0 = O + N + G - 180^\circ - \epsilon_{36}$

Side Equations.

XXXVII. $\frac{FE}{FK} \cdot \frac{FK}{FD} \cdot \frac{FD}{FE} = 1$

XXXVIII. $\frac{FE}{FK} \cdot \frac{FK}{FQ} \cdot \frac{FQ}{FR} \cdot \frac{FR}{FM} \cdot \frac{FM}{FE} = 1$

XXXIX. $\frac{IK}{IQ} \cdot \frac{IQ}{IF} \cdot \frac{IF}{IK} = 1$

XL. $\frac{ED}{EV} \cdot \frac{EV}{EF} \cdot \frac{EF}{ED} = 1$

XLI. $\frac{FV}{FK} \cdot \frac{FK}{FD} \cdot \frac{FD}{FV} = 1$

XLII. $\frac{BE}{BK} \cdot \frac{BK}{BV} \cdot \frac{BV}{BE} = 1$

XLIII. $\frac{VE}{VD} \cdot \frac{VD}{VB} \cdot \frac{VB}{VE} = 1$

XLIV. $\frac{BF}{BE} \cdot \frac{BE}{BK} \cdot \frac{BK}{BF} = 1$

XLV. $\frac{KV}{KF} \cdot \frac{KF}{KJ} \cdot \frac{KJ}{KV} = 1$

XLVI. $\frac{BK}{BV} \cdot \frac{BV}{BJ} \cdot \frac{BJ}{BK} = 1$

XLVII. $\frac{VB}{VE} \cdot \frac{VE}{VW} \cdot \frac{VW}{VB} = 1$

XLVIII. $\frac{VJ}{VK} \cdot \frac{VK}{VW} \cdot \frac{VW}{VJ} = 1$

XLIX. $\frac{WB}{WV} \cdot \frac{WV}{WJ} \cdot \frac{WJ}{WB} = 1$

L. $\frac{JW}{JB} \cdot \frac{JB}{JX} \cdot \frac{JX}{JW} = 1$

LI. $\frac{JV}{JB} \cdot \frac{JB}{JX} \cdot \frac{JX}{JV} = 1$

LII. $\frac{JX}{JW} \cdot \frac{JW}{JZ} \cdot \frac{JZ}{JX} = 1$

LIII. $\frac{JY}{JX} \cdot \frac{JX}{JZ} \cdot \frac{JZ}{JY} = 1$

LIV. $\frac{HF}{HK} \cdot \frac{HK}{HQ} \cdot \frac{HQ}{HF} = 1$

LV. $\frac{KJ}{KL} \cdot \frac{KL}{KH} \cdot \frac{KH}{KF} \cdot \frac{KF}{KJ} = 1$

LVI. $\frac{LK}{LJ} \cdot \frac{LJ}{LN} \cdot \frac{LN}{LH} \cdot \frac{LH}{LK} = 1$

LVII. $\frac{HQ}{HK} \cdot \frac{HK}{HL} \cdot \frac{HL}{HN} \cdot \frac{HN}{HQ} = 1$

LVIII. $\frac{LG}{LH} \cdot \frac{LH}{LN} \cdot \frac{LN}{LG} = 1$

LIX. $\frac{KG}{KH} \cdot \frac{KH}{KL} \cdot \frac{KL}{KG} = 1$

LX. $\frac{HQ}{HK} \cdot \frac{HK}{HG} \cdot \frac{HG}{HQ} = 1$

LXI. $\frac{LK}{LJ} \cdot \frac{LJ}{LG} \cdot \frac{LG}{LK} = 1$

LXII. $\frac{GL}{GH} \cdot \frac{GH}{GO} \cdot \frac{GO}{GL} = 1$

LXIII. $\frac{GN}{GH} \cdot \frac{GH}{GO} \cdot \frac{GO}{GN} = 1$

LXIV. $\frac{NJ}{NO} \cdot \frac{NO}{NL} \cdot \frac{NL}{NJ} = 1$

FIGURE 6.

Angle Equations.

I. $0 = A + B + C - 180^\circ - \varepsilon_1$

II. $0 = B + E + C - 180^\circ - \varepsilon_2$

III. $0 = C + B + D - 180^\circ - \varepsilon_3$

IV. $0 = C + E + D - 180^\circ - \varepsilon_4$

V. $0 = C + E + F - 180^\circ - \varepsilon_5$

VI. $0 = C + D + F - 180^\circ - \varepsilon_6$

VII. $0 = C + E + H - 180^\circ - \varepsilon_7$

VIII. $0 = F + E + H - 180^\circ - \varepsilon_8$

IX. $0 = E + D + H - 180^\circ - \varepsilon_9$

X. $0 = G + H + F - 180^\circ - \varepsilon_{10}$

XI. $0 = G + H + D - 180^\circ - \varepsilon_{11}$

Side Equations.

XII. $\frac{EA}{EB}\cdot\frac{EB}{EC}\cdot\frac{EC}{EA}=1$

XIII. $\frac{CB}{CD}\cdot\frac{CD}{CE}\cdot\frac{CE}{CB}=1$

XIV. $\frac{EF}{EC}\cdot\frac{EC}{ED}\cdot\frac{ED}{EF}=1$

XV. $\frac{EH}{EC}\cdot\frac{EC}{EF}\cdot\frac{EF}{EH}=1$

XVI. $\frac{EH}{ED}\cdot\frac{ED}{EF}\cdot\frac{EF}{EH}=1$

XVII. $\frac{HD}{HG}\cdot\frac{HG}{HF}\cdot\frac{HF}{HD}=1$

FIGURE 7.

Angle Equations.

I. $0=C+G+D-180^\circ-\varepsilon_1$

II. $0=C+D+E-180^\circ-\varepsilon_2$

III. $0=C+G+E-180^\circ-\varepsilon_3$

IV. $0=C+G+F-180^\circ-\varepsilon_4$

V. $0=H+C+F-180^\circ-\varepsilon_5$

VI. $0=H+C+G-180^\circ-\varepsilon_6$

VII. $0=H+D+G-180^\circ-\varepsilon_7$

VIII. $0=K+C+F-180^\circ-\varepsilon_8$

IX. $0=K+H+F-180^\circ-\varepsilon_9$

X. $0=K+D+C-180^\circ-\varepsilon_{10}$

XI. $0=K+E+C-180^\circ-\varepsilon_{11}$

XII. $0=I+E+K-180^\circ-\varepsilon_{12}$

XIII. $0=I+K+H-180^\circ-\varepsilon_{13}$

XIV. $0=I+E+G-180^\circ-\varepsilon_{14}$

XV. $0=I+E+C-180^\circ-\varepsilon_{15}$

XVI. $0=I+E+D-180^\circ-\varepsilon_{16}$

XVII. $0=K+I+L-180^\circ-\varepsilon_{17}$

XVIII. $0=K+L+P-180^\circ-\varepsilon_{18}$

XIX. $0=P+K+C+G-360^\circ-\varepsilon_{19}$

XX. $0=L+P+M-180^\circ-\varepsilon_{20}$

XXI. $0=K+P+M-180^\circ-\varepsilon_{21}$

XXII. $0=P+A+M-180^\circ-\varepsilon_{22}$

XXIII. $0=P+B+A-180^\circ-\varepsilon_{23}$

Side Equations.

XXIV. $\frac{GC}{GE}\cdot\frac{GE}{GD}\cdot\frac{GD}{GC}=1$

XXV. $\frac{CF}{CH}\cdot\frac{CH}{CG}\cdot\frac{CG}{CF}=1$

XXVI. $\frac{CG}{CD}\cdot\frac{CD}{CH}\cdot\frac{CH}{CG}=1$

XXVII. $\frac{EC}{EG}\cdot\frac{EG}{EH}\cdot\frac{EH}{EC}=1$

XXVIII. $\frac{KH}{KF}\cdot\frac{KF}{KC}\cdot\frac{KC}{KH}=1$

XXIX. $\frac{CK}{CD}\cdot\frac{CD}{CH}\cdot\frac{CH}{CK}=1$

XXX. $\frac{KC}{KE}\cdot\frac{KE}{KD}\cdot\frac{KD}{KC}=1$

XXXI. $\frac{KG}{KC}\cdot\frac{KC}{KF}\cdot\frac{KF}{KC}=1$

XXXII. $\frac{KH}{KI}\cdot\frac{KI}{KG}\cdot\frac{KG}{KH}=1$

XXXIII. $\frac{EG}{EI}\cdot\frac{EI}{EH}\cdot\frac{EH}{EG}=1$

XXXIV. $\frac{EC}{EI}\cdot\frac{EI}{EH}\cdot\frac{EH}{EC}=1$

XXXV. $\frac{KD}{KI}\cdot\frac{KI}{KE}\cdot\frac{KE}{KD}=1$

XXXVI. $\frac{LK}{LI}\cdot\frac{LI}{LH}\cdot\frac{LH}{LK}=1$

XXXVII. $\frac{KG}{KP}\cdot\frac{KP}{KL}\cdot\frac{KL}{KI}\cdot\frac{KI}{KG}=1$

XXXVIII. $\frac{CG}{CP}\cdot\frac{CP}{CK}\cdot\frac{CK}{CG}=1$

XXXIX. $\frac{PK}{PM}\cdot\frac{PM}{PL}\cdot\frac{PL}{PK}=1$

XL. $\frac{OK}{OP}\cdot\frac{OP}{OM}\cdot\frac{OM}{OK}=1$

XLI. $\frac{OP}{OB}\cdot\frac{OB}{OA}\cdot\frac{OA}{OP}=1$

XLII. $\frac{OP}{OA} \cdot \frac{OA}{OM} \cdot \frac{OM}{OP} = 1$

XLIII. $\frac{NK}{NP} \cdot \frac{NP}{NM} \cdot \frac{NM}{NK} = 1$

XLIV. $\frac{MN}{MP} \cdot \frac{MP}{MA} \cdot \frac{MA}{MN} = 1$

XLV. $\frac{BN}{BA} \cdot \frac{BA}{BP} \cdot \frac{BP}{BN} = 1$

FIGURE 8.

Angle Equations.

I. $0 = F + E' + A' - 180° - \varepsilon_1$

II. $0 = F + E' + H - 180° - \varepsilon_2$

III. $0 = E' + H + C' - 180° - \varepsilon_3$

IV. $0 = K + H + E' - 180° - \varepsilon_4$

V. $0 = K + H + M - 180° - \varepsilon_5$

VI. $0 = M + H + C' - 180° - \varepsilon_6$

VII. $0 = K + M + L - 180° - \varepsilon_7$

VIII. $0 = M + H + L - 180° - \varepsilon_8$

IX. $0 = C' + O + H - 180° - \varepsilon_9$

X. $0 = O + K + H - 180° - \varepsilon_{10}$

XI. $0 = K + O + P - 180° - \varepsilon_{11}$

XII. $0 = K + M + P - 180° - \varepsilon_{12}$

XIII. $0 = K + O + N - 180° - \varepsilon_{13}$

XIV. $0 = K + P + N - 180° - \varepsilon_{14}$

XV. $0 = P + M + N - 180° - \varepsilon_{15}$

XVI. $0 = Q + N + M - 180° - \varepsilon_{16}$

XVII. $0 = Q + N + P - 180° - \varepsilon_{17}$

XVIII. $0 = Q + N + O - 180° - \varepsilon_{18}$

XIX. $0 = S + N + Q - 180° - \varepsilon_{19}$

XX. $0 = S + M + Q - 180° - \varepsilon_{20}$

XXI. $0 = R + Q + S - 180° - \varepsilon_{21}$

XXII. $0 = R + Q + N - 180° - \varepsilon_{22}$

XXIII. $0 = R + Q + M - 180° - \varepsilon_{23}$

XXIV. $0 = T + Q + N - 180° - \varepsilon_{24}$

XXV. $0 = T + Q + R - 180° - \varepsilon_{25}$

XXVI. $0 = T + Q + S - 180° - \varepsilon_{26}$

XXVII. $0 = U + S + T - 180° - \varepsilon_{27}$

XXVIII. $0 = U + S + R - 180° - \varepsilon_{28}$

Side Equations.

XXIX. $\frac{E'F}{E'A'} \cdot \frac{E'A'}{E'C} \cdot \frac{E'C'}{E'H} \cdot \frac{E'H}{E'F} = 1$

XXX. $\frac{A'F}{A'G} \cdot \frac{A'G}{A'E'} \cdot \frac{A\,E'}{A'F} = 1$

XXXI. $\frac{HF}{HE'} \cdot \frac{HE'}{HG} \cdot \frac{HG}{HF} = 1$

XXXII. $\frac{KH}{KG} \cdot \frac{KG}{KE'} \cdot \frac{KE'}{KH} = 1$

XXXIII. $\frac{HE'}{HF} \cdot \frac{HF}{HK} \cdot \frac{HK}{HE'} = 1$

XXXIV. $\frac{FK}{FH} \cdot \frac{FH}{FM} \cdot \frac{FM}{FK} = 1$

XXXV. $\frac{HE'}{HK} \cdot \frac{HK}{HM} \cdot \frac{HM}{HC'} \cdot \frac{HC'}{HE'} = 1$

XXXVI. $\frac{KL}{KH} \cdot \frac{KH}{KM} \cdot \frac{KM}{KL} = 1$

XXXVII. $\frac{FK}{FL} \cdot \frac{FL}{FH} \cdot \frac{FH}{FK} = 1$

XXXVIII. $\frac{HK}{HO} \cdot \frac{HO}{HC'} \cdot \frac{HC'}{HE'} \cdot \frac{HE'}{HK} = 1$

XXXIX. $\frac{KM}{KP} \cdot \frac{KP}{KO} \cdot \frac{KO}{KH} \cdot \frac{KH}{KM} = 1$

XL. $\frac{KP}{KO} \cdot \frac{KO}{KN} \cdot \frac{KN}{KP} = 1$

XLI. $\frac{KM}{KP} \cdot \frac{KP}{KN} \cdot \frac{KN}{KM} = 1$

XLII. $\frac{QM}{QN} \cdot \frac{QN}{QP} \cdot \frac{QP}{QM} = 1$

XLIII. $\frac{QP}{QN} \cdot \frac{QN}{QO} \cdot \frac{QO}{QP} = 1$

XLIV. $\frac{MS}{MN} \cdot \frac{MN}{MQ} \cdot \frac{MQ}{MS} = 1$

XLV. $\frac{RN}{RQ} \cdot \frac{RQ}{RS} \cdot \frac{RS}{RN} = 1$

XLVI. $\frac{RM}{RQ} \cdot \frac{RQ}{RN} \cdot \frac{RN}{RM} = 1$

XLVII. $\frac{RP}{RQ} \cdot \frac{RQ}{RM} \cdot \frac{RM}{RP} = 1$

XLVIII. $\frac{QR}{QN} \cdot \frac{QN}{QT} \cdot \frac{QT}{QR} = 1$

XLIX. $\frac{QS}{QN} \cdot \frac{QN}{QT} \cdot \frac{QT}{QS} = 1$

L. $\frac{RU}{RS} \cdot \frac{RS}{RT} \cdot \frac{RT}{RU} = 1$

FIGURE 9.

Angle Equations.

I. $0 = W + P + Q - 180^\circ - \varepsilon_1$
II. $0 = W + O + Q - 180^\circ - \varepsilon_2$
III. $0 = W + O + C' - 180^\circ - \varepsilon_3$
IV. $0 = W + A' + D' - 180^\circ - \varepsilon_4$
V. $0 = W + C' + D' - 180^\circ - \varepsilon_5$
VI. $0 = V + Q + P - 180^\circ - \varepsilon_6$
VII. $0 = V + Q + R - 180^\circ - \varepsilon_7$
VIII. $0 = V + Q + T - 180^\circ - \varepsilon_8$
IX. $0 = X + Q + V - 180^\circ - \varepsilon_9$
X. $0 = X + P + V - 180^\circ - \varepsilon_{10}$
XI. $0 = X + O + Q - 180^\circ - \varepsilon_{11}$
XII. $0 = X + W + Q - 180^\circ - \varepsilon_{12}$
XIII. $0 = X + W + C' - 180^\circ - \varepsilon_{13}$
XIV. $0 = X + W + D' - 180^\circ - \varepsilon_{14}$
XV. $0 = D + W + D' - 180^\circ - \varepsilon_{15}$
XVI. $0 = D + C' + B' - 180^\circ - \varepsilon_{16}$
XVII. $0 = D + W + C' - 180^\circ - \varepsilon_{17}$
XVIII. $0 = D + W + X - 180^\circ - \varepsilon_{18}$
XIX. $0 = D + W + Q - 180^\circ - \varepsilon_{19}$
XX. $0 = D + W + P - 180^\circ - \varepsilon_{20}$
XXI. $0 = D + W + O - 180^\circ - \varepsilon_{21}$
XXII. $0 = C + X + V - 180^\circ - \varepsilon_{22}$
XXIII. $0 = C + X + D - 180^\circ - \varepsilon_{23}$
XXIV. $0 = B + X + D - 180^\circ - \varepsilon_{24}$
XXV. $0 = B + C + D - 180^\circ - \varepsilon_{25}$
XXVI. $0 = A + D + B - 180^\circ - \varepsilon_{26}$
XXVII. $0 = A + D + D' - 180^\circ - \varepsilon_{27}$
XXVIII. $0 = B + Y + A - 180^\circ - \varepsilon_{28}$
XXIX. $0 = D + B + Y + B' - 360^\circ - \varepsilon_{29}$
XXX. $0 = Z + D + D' - 180^\circ - \varepsilon_{30}$
XXXI. $0 = Z + D + B + Y - 360^\circ - \varepsilon_{31}$

Side Equations.

XXXII. $\frac{QO}{QP} \cdot \frac{QP}{QW} \cdot \frac{QW}{QO} = 1$

XXXIII. $\frac{OC}{OW} \cdot \frac{OW}{OQ} \cdot \frac{OQ}{ON} \cdot \frac{ON}{OK} \cdot \frac{OK}{OH} \cdot \frac{OH}{OC'} = 1$

XXXIV. $\frac{D'C'}{D'A'} \cdot \frac{D'A'}{D'W} \cdot \frac{D'W}{D'C'} = 1$

XXXV. $\frac{C'O}{C'H} \cdot \frac{C'H}{C'E'} \cdot \frac{C'E'}{C'A'} \cdot \frac{C'A'}{C'D'} \cdot \frac{C'D}{C'W} \cdot \frac{C'W}{C'O} = 1$

XXXVI. $\frac{RQ}{RV} \cdot \frac{RV}{RP} \cdot \frac{RP}{RQ} = 1$

XXXVII. $\frac{QT}{QR} \cdot \frac{QR}{QV} \cdot \frac{QV}{QT} = 1$

XXXVIII. $\frac{XQ}{XP} \cdot \frac{XP}{XV} \cdot \frac{XV}{XQ} = 1$

XXXIX. $\frac{QO}{QP} \cdot \frac{QP}{QX} \cdot \frac{QX}{QO} = 1$

XL. $\frac{QW}{QO} \cdot \frac{QO}{QX} \cdot \frac{QX}{QW} = 1$

XLI. $\frac{XW}{XO} \cdot \frac{XO}{XC'} \cdot \frac{XC'}{XW} = 1$

XLII. $\frac{WD'}{WC'} \cdot \frac{WC'}{WX} \cdot \frac{WX}{WD'} = 1$

XLIII. $\frac{C'D'}{C'B'} \cdot \frac{C'B'}{C'X} \cdot \frac{C'X}{C'D'} = 1$

XLIV. $\frac{WX}{WQ} \cdot \frac{WQ}{WD} \cdot \frac{WD}{WX} = 1$

XLV. $\frac{WX}{WP} \cdot \frac{WP}{WD} \cdot \frac{WD}{WX} = 1$

XLVI. $\frac{WX}{WO} \cdot \frac{WO}{WD} \cdot \frac{WD}{WX} = 1$

XLVII. $\frac{C'X}{C'O} \cdot \frac{C'O}{C'D} \cdot \frac{C'D}{C'X} = 1$

XLVIII. $\frac{XD'}{XC'} \cdot \frac{XC'}{XD} \cdot \frac{XD}{XD'} = 1$

XLIX. $\frac{C'D'}{C'B'} \cdot \frac{C'B'}{C'D} \cdot \frac{C'D}{C'D'} = 1$

L. $\frac{XC}{XV} \cdot \frac{XV}{XP} \cdot \frac{XP}{XD} \cdot \frac{XD}{XC} = 1$

LI. $\frac{VX}{VC} \cdot \frac{VC}{VP} \cdot \frac{VP}{VX} = 1$

LII. $\frac{BX}{BD} \cdot \frac{BD}{BC} \cdot \frac{BC}{BX} = 1$

LIII. $\frac{DA}{DB} \cdot \frac{DB}{DX} \cdot \frac{DX}{DD'} \cdot \frac{DD'}{DA} = 1$

LIV. $\frac{B'D}{B'D'} \cdot \frac{B'D'}{B'A} \cdot \frac{B'A}{B'D} = 1$

LV. $\frac{YB}{YA} \cdot \frac{YA}{YD} \cdot \frac{YD}{YB} = 1$

LVI. $\frac{DY}{DB} \cdot \frac{DB}{DX} \cdot \frac{DX}{DW} \cdot \frac{DW}{DY} = 1$

LVII. $\frac{YW}{YD'} \cdot \frac{YD'}{YD} \cdot \frac{YD}{YW} = 1$

LVIII. $\frac{YD}{YB'} \cdot \frac{YB'}{YD} \cdot \frac{YD}{YD'} = 1$

LIX. $\frac{D'Y}{D'Z} \cdot \frac{D'Z}{D'D} \cdot \frac{D'D}{D'Y} = 1$

LX. $\frac{B'D}{B'D'} \cdot \frac{B'D'}{B'Z} \cdot \frac{B'Z}{B'D} = 1$

FIGURE 10.

Angle Equations.

I. $0 = F + C' + E' - 180^\circ - \varepsilon_1$

II. $0 = F + A' + E' - 180^\circ - \varepsilon_2$

III. $0 = F + B' + E' - 180^\circ - \varepsilon_3$

IV. $0 = H + F + E' - 180^\circ - \varepsilon_4$

V. $0 = H + B' + E' - 180^\circ - \varepsilon_5$

VI. $0 = B' + F + G - 180^\circ - \varepsilon_6$

VII. $0 = H + F + G - 180^\circ - \varepsilon_7$

Side Equations.

VIII. $\frac{B'E'}{B'F} \cdot \frac{B'F}{B'A'} \cdot \frac{B'A'}{B'E'} = 1$

IX. $\frac{B'E'}{B'F} \cdot \frac{B'F}{B'C'} \cdot \frac{B'C'}{B'E'} = 1$

X. $\frac{FD'}{FC'} \cdot \frac{FC'}{FE'} \cdot \frac{FE'}{FD'} = 1$

XI. $\frac{E'F}{E'B'} \cdot \frac{E'B'}{E'H} \cdot \frac{E'H}{E'F} = 1$

XII. $\frac{FG}{FH} \cdot \frac{FH}{FE'} \cdot \frac{FE'}{FB'} \cdot \frac{FB'}{FG} = 1$

FIGURE 11.

Angle Equations.

I. $0 = O' + Q' + H' - 180^\circ - \varepsilon_1$

II. $0 = O' + R' + H' - 180^\circ - \varepsilon_2$

III. $0 = O' + G' + Q' - 180^\circ - \varepsilon_3$

IV $0 = O' + G' + R' - 180^\circ - \varepsilon_4$

V. $0 = H' + G' + Q' - 180^\circ - \varepsilon_5$

VI. $0 = G' + Q' + V' - 180^\circ - \varepsilon_6$

VII. $0 = V' + G' + E' - 180^\circ - \varepsilon_7$

VIII. $0 = N' + V' + E' - 180^\circ - \varepsilon_8$

IX. $0 = N' + K' + E' - 180^\circ - \varepsilon_9$

X. $0 = K' + E' + C' + B' - 360^\circ - \varepsilon_{10}$

XI. $0 = I' + K' + B' - 180^\circ - \varepsilon_{11}$

XII. $0 = A + V' + T' - 180^\circ - \varepsilon_{12}$

XIII. $0 = A + V' + R' - 180^\circ - \varepsilon_{13}$

XIV. $0 = A + V' + Q' - 180^\circ - \varepsilon_{14}$

XV. $0 = A + G' + Q' - 180^\circ - \varepsilon_{15}$

XVI. $0 = A + G' + F' - 180^\circ - \varepsilon_{16}$

XVII. $0 = A + G' + E' - 180^\circ - \varepsilon_{17}$

XVIII. $0 = A + V' + X' - 180^\circ - \varepsilon_{18}$

XIX. $0 = A + V' + N' - 180^\circ - \varepsilon_{19}$

XX. $0 = B + A + F' - 180^\circ - \varepsilon_{20}$

XXI. $0 = B + A + G' - 180^\circ - \varepsilon_{21}$

XXII. $0 = B + A + Q' - 180^\circ - \varepsilon_{22}$

XXIII. $0 = B + A + N' - 180^\circ - \varepsilon_{23}$

XXIV. $0 = B + N' + E' - 180^\circ - \varepsilon_{24}$

XXV. $0 = B + E' + D' - 180^\circ - \varepsilon_{25}$

XXVI. $0 = B + N' + L' - 180^\circ - \varepsilon_{26}$

XXVII. $0 = C + N' + K' - 180^\circ - \varepsilon_{27}$

XXVIII. $0 = C + N' + L' - 180^\circ - \varepsilon_{28}$

XXIX. $0 = C + N' + V' - 180^\circ - \varepsilon_{29}$

XXX. $0 = C + N' + A - 180^\circ - \varepsilon_{30}$

XXXI. $0 = C + N' + B - 180^\circ - \varepsilon_{31}$

XXXII. $0 = C + E' + D' - 180^\circ - \varepsilon_{32}$

XXXIII. $0 = C + E' + B - 180^\circ - \varepsilon_{33}$

XXXIV. $0 = C + E' + A' - 180^\circ - \varepsilon_{34}$

XXXV. $0 = C + E' + C' - 180^\circ - \varepsilon_{35}$

XXXVI. $0 = C + B' + K' - 180^\circ - \varepsilon_{36}$

Side Equations.

XXXVII. $$\frac{O'Q'}{O'R'} \cdot \frac{O'R'}{O'H'} \cdot \frac{O'H'}{O'Q'} = 1$$

XXXVIII. $$\frac{Q'H'}{Q'G'} \cdot \frac{Q'G'}{Q'O'} \cdot \frac{Q'O'}{Q'H'} = 1$$

XXXIX. $$\frac{Q'R'}{Q'O'} \cdot \frac{Q'O'}{Q'G'} \cdot \frac{Q'G'}{Q'V'} \cdot \frac{Q'V'}{Q'R'} = 1$$

XL. $$\frac{Q'V'}{Q'R'} \cdot \frac{Q'R'}{Q'G'} \cdot \frac{Q'G'}{Q'V'} = 1$$

XLI. $$\frac{P'Q'}{P'S'} \cdot \frac{P'S'}{P'O'} \cdot \frac{P'O'}{P'G'} \cdot \frac{P'G'}{P'Q'} = 1$$

XLII. $$\frac{G'H'}{G'F'} \cdot \frac{G'F'}{G'E'} \cdot \frac{G'E'}{G'V'} \cdot \frac{G'V'}{G'Q'} \cdot \frac{G'Q'}{G'H} = 1$$

XLIII. $$\frac{F'V'}{F'G'} \cdot \frac{F'G'}{F'E'} \cdot \frac{F'E'}{F'V'} = 1$$

XLIV. $$\frac{V'N'}{V'Y'} \cdot \frac{V'Y'}{V'W'} \cdot \frac{V'W'}{V'T'} \cdot \frac{V'T'}{V'Q'} \cdot \frac{V'Q'}{V'G'} \cdot \frac{V'G'}{V'E'} \cdot \frac{V'E'}{V'N'} = 1$$

XLV. $$\frac{N'K'}{N'M'} \cdot \frac{N'M'}{N'Y'} \cdot \frac{N'Y'}{N'V'} \cdot \frac{N'V'}{N'E'} \cdot \frac{N'E'}{N'K'} = 1$$

XLVI. $$\frac{E'G'}{E'D'} \cdot \frac{E'D'}{E'A'} \cdot \frac{E'A'}{E'K'} \cdot \frac{E'K'}{E'N'} \cdot \frac{E'N'}{E'V'} \cdot \frac{E'V'}{E'G'} = 1$$

XLVII. $$\frac{C'E'}{C'A'} \cdot \frac{C'A'}{C'K'} \cdot \frac{C'K'}{C'E'} = 1$$

XLVIII. $$\frac{C'E'}{C'A'} \cdot \frac{C'A'}{C'B'} \cdot \frac{C'B'}{C'K'} \cdot \frac{C'K'}{C'E'} = 1$$

XLIX. $$\frac{K'I'}{K'M'} \cdot \frac{K'M'}{K'N'} \cdot \frac{K'N'}{K'E'} \cdot \frac{K'E'}{K'A'} \cdot \frac{K'A'}{K'B'} \cdot \frac{K'B'}{K'I'} = 1$$

L. $$\frac{C'I'}{C'K'} \cdot \frac{C'K'}{C'B'} \cdot \frac{C'B'}{C'I'} = 1$$

LI. $$\frac{R'V'}{R'T'} \cdot \frac{R'T'}{R'A} \cdot \frac{R'A}{R'V} = 1$$

LII. $$\frac{Q'V'}{Q'R'} \cdot \frac{Q'R'}{Q'A} \cdot \frac{Q'A}{Q'V'} = 1$$

LIII. $$\frac{AQ'}{AG'} \cdot \frac{AG'}{AV'} \cdot \frac{AV'}{AQ'} = 1$$

LIV. $$\frac{G'A}{G'Q'} \cdot \frac{G'Q'}{G'H'} \cdot \frac{G'H'}{G'F'} \cdot \frac{G'F'}{G'E'} \cdot \frac{G'E'}{G'A} = 1$$

LV. $\frac{AN'}{AV'} \cdot \frac{AV'}{AE'} \cdot \frac{AE'}{AN'} = 1$

LVI. $\frac{AF'}{AG'} \cdot \frac{AG'}{AE'} \cdot \frac{AE'}{AF'} = 1$

LVII. $\frac{V'A}{V'N'} \cdot \frac{V'N'}{V'X'} \cdot \frac{V'X'}{V'A} = 1$

LVIII. $\frac{O'R'}{O'T'} \cdot \frac{O'T'}{O'A} \cdot \frac{O'A}{O'R'} = 1$

LIX. $\frac{U'A}{U'T'} \cdot \frac{U'T'}{U'V'} \cdot \frac{U'V'}{U'A} = 1$

LX. $\frac{W'A}{W'T'} \cdot \frac{W'T'}{W'V'} \cdot \frac{W'V'}{W'A} = 1$

LXI. $\frac{AG'}{AB} \cdot \frac{AB}{AQ'} \cdot \frac{AQ'}{AG'} = 1$

LXII. $\frac{AF'}{AB} \cdot \frac{AB}{AG'} \cdot \frac{AG'}{AF'} = 1$

LXIII. $\frac{G'D'}{G'B} \cdot \frac{G'B}{G'F'} \cdot \frac{G'F'}{G'D'} = 1$

LXIV. $\frac{BD'}{BG'} \cdot \frac{BG'}{BE'} \cdot \frac{BE'}{BD'} = 1$

LXV. $\frac{BG'}{BE'} \cdot \frac{BE'}{BN'} \cdot \frac{BN'}{BA} \cdot \frac{BA}{BG'} = 1$

LXVI. $\frac{BK'}{BE'} \cdot \frac{BE'}{BN'} \cdot \frac{BN'}{BK'} = 1$

LXVII. $\frac{BV'}{BA} \cdot \frac{BA}{BN'} \cdot \frac{BN'}{BV'} = 1$

LXVIII. $\frac{N'L'}{N'B} \cdot \frac{N'B}{N'K'} \cdot \frac{N'K'}{N'M'} \cdot \frac{N'M'}{N'L'} = 1$

LXIX. $\frac{N'L'}{N'C} \cdot \frac{N'C}{N'K'} \cdot \frac{N'K'}{N'M'} \cdot \frac{N'M'}{N'L'} = 1$

LXX. $\frac{N'K'}{N'M'} \cdot \frac{N'M'}{N'Y'} \cdot \frac{N'Y'}{N'V'} \cdot \frac{N'V'}{N'C} \cdot \frac{N'C}{N'K'} = 1$

LXXI. $\frac{N'A}{N'V'} \cdot \frac{N'V'}{N'C} \cdot \frac{N'C}{N'A} = 1$

LXXII. $\frac{CA}{CN'} \cdot \frac{CN'}{CB} \cdot \frac{CB}{CA} = 1$

LXXIII. $\frac{CB}{CN'} \cdot \frac{CN'}{CE'} \cdot \frac{CE'}{CB} = 1$

LXXIV. $\frac{E'B}{E'C} \cdot \frac{E'C}{E'D'} \cdot \frac{E'D'}{E'B} = 1$

LXXV. $\frac{E'C}{E'D'} \cdot \frac{E'D'}{E'A'} \cdot \frac{E'A'}{E'C} = 1$

LXXVI. $\frac{C'E'}{C'A'} \cdot \frac{C'A'}{C'C} \cdot \frac{C'C}{C'E'} = 1$

LXXVII. $\frac{CE'}{CA'} \cdot \frac{CA'}{CB'} \cdot \frac{CB'}{CK'} \cdot \frac{CK'}{CE'} = 1$

FIGURE 12.

Angle Equations.

I. $0 = A + E + B - 180^\circ - \epsilon_1$
II. $0 = A + D + B - 180^\circ - \epsilon_2$
III. $0 = A + D + E - 180^\circ - \epsilon_3$
IV. $0 = F + A + E - 180^\circ - \epsilon_4$
V. $0 = F + K + E - 180^\circ - \epsilon_5$
VI. $0 = F + K + A - 180^\circ - \epsilon_6$
VII. $0 = A + D + K - 180^\circ - \epsilon_7$
VIII. $0 = B + D + C - 180^\circ - \epsilon_8$
IX. $0 = D + C + K - 180^\circ - \epsilon_9$
X. $0 = D + C + L - 180^\circ - \epsilon_{10}$
XI. $0 = D + K + L - 180^\circ - \epsilon_{11}$
XII. $0 = F + H + K - 180^\circ - \epsilon_{12}$
XIII. $0 = F + H + E - 180^\circ - \epsilon_{13}$
XIV. $0 = K + M + H - 180^\circ - \epsilon_{14}$
XV. $0 = F + K + N - 180^\circ - \epsilon_{15}$
XVI. $0 = F + H + N - 180^\circ - \epsilon_{16}$
XVII. $0 = F + G + H - 180^\circ - \epsilon_{17}$
XVIII. $0 = G + N + H - 180^\circ - \epsilon_{18}$
XIX. $0 = O + K + M - 180^\circ - \epsilon_{19}$
XX. $0 = O + H + M - 180^\circ - \epsilon_{20}$
XXI. $0 = O + N + K - 180^\circ - \epsilon_{21}$
XXII. $0 = Q + N + K + M - 360^\circ - \epsilon_{22}$
XXIII. $0 = Q + M + K + L - 360^\circ - \epsilon_{23}$
XXIV. $0 = N + P + Q - 180^\circ - \epsilon_{24}$

Side Equations.

XXV. $\frac{AD}{AE}\cdot\frac{AE}{AB}\cdot\frac{AB}{AD}=1$

XXVI. $\frac{FE}{FK}\cdot\frac{FK}{FA}\cdot\frac{FA}{FE}=1$

XXVII. $\frac{DE}{DK}\cdot\frac{DK}{DA}\cdot\frac{DA}{DE}=1$

XXVIII. $\frac{DK}{DA}\cdot\frac{DA}{DB}\cdot\frac{DB}{DC}\cdot\frac{DC}{DK}=1$

XXIX. $\frac{DL}{DK}\cdot\frac{DK}{DC}\cdot\frac{DC}{DL}=1$

XXX. $\frac{FK}{FH}\cdot\frac{FH}{FE}\cdot\frac{FE}{FK}=1$

XXXI. $\frac{DH}{DK}\cdot\frac{DK}{DE}\cdot\frac{DE}{DH}=1$

XXXII. $\frac{KM}{KH}\cdot\frac{KH}{KE}\cdot\frac{KE}{KD}\cdot\frac{KD}{KL}\cdot\frac{KL}{KM}=1$

XXXIII. $\frac{FH}{FN}\cdot\frac{FN}{FK}\cdot\frac{FK}{FH}=1$

XXXIV. $\frac{MH}{MK}\cdot\frac{MK}{MN}\cdot\frac{MN}{MH}=1$

XXXV. $\frac{HG}{HN}\cdot\frac{HN}{HF}\cdot\frac{HF}{HG}=1$

XXXVI. $\frac{MH}{MK}\cdot\frac{MK}{MO}\cdot\frac{MO}{MH}=1$

XXXVII. $\frac{KN}{KH}\cdot\frac{KH}{KO}\cdot\frac{KO}{KN}=1$

XXXVIII. $\frac{MQ}{MN}\cdot\frac{MN}{MK}\cdot\frac{MK}{ML}\cdot\frac{ML}{MQ}=1$

XXXIX. $\frac{NP}{NK}\cdot\frac{NK}{NO}\cdot\frac{NO}{NP}=1$

XL. $\frac{NP}{NQ}\cdot\frac{NQ}{NM}\cdot\frac{NM}{NK}\cdot\frac{NK}{NP}=1$

FIGURE 13.

Angle Equations.

I. $0 = A' + D' + K - 180^\circ - \varepsilon_1$

II. $0 = A' + D' + F - 180^\circ - \varepsilon_2$

III. $0 = A' + F + K - 180^\circ - \varepsilon_3$

IV. $0 = G + F + K - 180^\circ - \varepsilon_4$

V. $0 = G + K + D' - 180^\circ - \varepsilon_5$

VI. $0 = G + E' + D' - 180^\circ - \varepsilon_6$

VII. $0 = E' + F + D' - 180^\circ - \varepsilon_7$

VIII. $0 = L + A' + K - 180^\circ - \varepsilon_8$

IX. $0 = D' + E' + H - 180^\circ - \varepsilon_9$

X. $0 = H + E' + G + K + L - 540^\circ - \varepsilon_{10}$

XI. $0 = M + H + L - 180^\circ - \varepsilon_{11}$

XII. $0 = M + H + D' - 180^\circ - \varepsilon_{12}$

XIII. $0 = M + D' + K - 180^\circ - \varepsilon_{13}$

XIV. $0 = O + M + L - 180^\circ - \varepsilon_{14}$

XV. $0 = O + M + K - 180^\circ - \varepsilon_{15}$

XVI. $0 = M + R + O - 180^\circ - \varepsilon_{16}$

XVII. $0 = R + O + L - 180^\circ - \varepsilon_{17}$

XVIII. $0 = S + O + L - 180^\circ - \varepsilon_{18}$

XIX. $0 = S + O + M - 180^\circ - \varepsilon_{19}$

XX. $0 = S + O + R - 180^\circ - \varepsilon_{20}$

XXI. $0 = O + R + P - 180^\circ - \varepsilon_{21}$

XXII. $0 = O + S + P - 180^\circ - \varepsilon_{22}$

XXIII. $0 = P + L + M - 180^\circ - \varepsilon_{23}$

XXIV. $0 = P + M + R - 180^\circ - \varepsilon_{24}$

XXV. $0 = B' + A' + K - 180^\circ - \varepsilon_{25}$

Side Equations.

XXVI. $\frac{A'F}{A'D'}\cdot\frac{A'D'}{A'K}\cdot\frac{A'K}{A'F}=1$

XXVII. $\frac{FG}{FK}\cdot\frac{FK}{FD'}\cdot\frac{FD'}{FG}=1$

XXVIII. $\frac{E'F}{E'D'}\cdot\frac{E'D'}{E'G}\cdot\frac{E'G}{E'F}=1$

XXIX. $\frac{A'L}{A'K}\cdot\frac{A'K}{A'F}\cdot\frac{A'F}{A'D'}\cdot\frac{A'D'}{A'L}=1$

XXX. $\frac{KL}{KA'}\cdot\frac{KA'}{KF}\cdot\frac{KF}{KG}\cdot\frac{KG}{KL}=1$

XXXI. $\frac{GE'}{GD'}\cdot\frac{GD'}{GH}\cdot\frac{GH}{GE'}=1$

XXXII. $\frac{GH}{GE'} \cdot \frac{GE'}{GF} \cdot \frac{GF}{GK} \cdot \frac{GK}{GL} \cdot \frac{GL}{GH} = 1$

XXXIII. $\frac{GM}{GH} \cdot \frac{GH}{GL} \cdot \frac{GL}{GM} = 1$

XXXIV. $\frac{HL}{HD'} \cdot \frac{HD'}{HM} \cdot \frac{HM}{HL} = 1$

XXXV. $\frac{GL}{GK} \cdot \frac{GK}{GM} \cdot \frac{GM}{GL} = 1$

XXXVI. $\frac{ML}{MK} \cdot \frac{MK}{MO} \cdot \frac{MO}{ML} = 1$

XXXVII. $\frac{ML}{MH} \cdot \frac{MH}{MN} \cdot \frac{MN}{MO} \cdot \frac{MO}{ML} = 1$

XXXVIII. $\frac{OM}{OL} \cdot \frac{OL}{OR} \cdot \frac{OR}{OM} = 1$

XXXIX. $\frac{NO}{NM} \cdot \frac{NM}{NR} \cdot \frac{NR}{NO} = 1$

XL. $\frac{OM}{OS} \cdot \frac{OS}{OL} \cdot \frac{OL}{OM} = 1$

XLI. $\frac{OR}{OS} \cdot \frac{OS}{OM} \cdot \frac{OM}{OR} = 1$

XLII. $\frac{NS}{NO} \cdot \frac{NO}{NR} \cdot \frac{NR}{NS} = 1$

XLIII. $\frac{RO}{RP} \cdot \frac{RP}{RM} \cdot \frac{RM}{RO} = 1$

XLIV. $\frac{RO}{RP} \cdot \frac{RP}{RL} \cdot \frac{RL}{RO} = 1$

XLV. $\frac{OS}{OP} \cdot \frac{OP}{OR} \cdot \frac{OR}{OS} = 1$

XLVI. $\frac{PL}{PO} \cdot \frac{PO}{PK} \cdot \frac{PK}{PL} = 1$

XLVII. $\frac{QS}{QP} \cdot \frac{QP}{QR} \cdot \frac{QR}{QS} = 1$

XLVIII. $\frac{PR}{PQ} \cdot \frac{PQ}{PL} \cdot \frac{PL}{PR} = 1$

XLIX. $\frac{PR}{PQ} \cdot \frac{PQ}{PM} \cdot \frac{PM}{PR} = 1$

L. $\frac{A'D'}{A'K} \cdot \frac{A'K}{A'B'} \cdot \frac{A'B'}{A'D'} = 1$

LI. $\frac{A'D'}{A'B'} \cdot \frac{A'B'}{A'F} \cdot \frac{A'F}{A'D'} = 1$

LII. $\frac{E'F}{E'G} \cdot \frac{E'G}{E'B'} \cdot \frac{E'B'}{E'F} = 1$

LIII. $\frac{D'E'}{D'C'} \cdot \frac{D'C'}{D'H} \cdot \frac{D'H}{D'E'} = 1$

LIV. $\frac{FD'}{FB'} \cdot \frac{FB'}{FE'} \cdot \frac{FE'}{FD'} = 1$

FIGURE 14.

Angle Equations.

I. $0 = F + H + K - 180^\circ - \varepsilon_1$

II. $0 = E + H + F - 180^\circ - \varepsilon_2$

III. $0 = D + K + F - 180^\circ - \varepsilon_3$

IV. $0 = D + K + H - 180^\circ - \varepsilon_4$

V. $0 = D + E + F - 180^\circ - \varepsilon_5$

VI. $0 = C + F + E - 180^\circ - \varepsilon_6$

VII. $0 = C + F + D - 180^\circ - \varepsilon_7$

VIII. $0 = C + H + D - 180^\circ - \varepsilon_8$

IX. $0 = B + H + D - 180^\circ - \varepsilon_9$

X. $0 = B + D + F - 180^\circ - \varepsilon_{10}$

XI. $0 = B + K + D - 180^\circ - \varepsilon_{11}$

XII. $0 = B + D + A - 180^\circ - \varepsilon_{12}$

XIII. $0 = D + C + A - 180^\circ - \varepsilon_{13}$

Side Equations.

XIV. $\frac{KF}{KE} \cdot \frac{KE}{KH} \cdot \frac{KH}{KF} = 1$

XV. $\frac{KE}{KD} \cdot \frac{KD}{KF} \cdot \frac{KF}{KE} = 1$

XVI. $\frac{KF}{KD} \cdot \frac{KD}{KH} \cdot \frac{KH}{KF} = 1$

XVII. $\frac{KC}{KD} \cdot \frac{KD}{KF} \cdot \frac{KF}{KC} = 1$

XVIII. $\frac{DF}{DC} \cdot \frac{DC}{DH} \cdot \frac{DH}{DF} = 1$

XIX. $\frac{DC}{DE} \cdot \frac{DE}{DF} \cdot \frac{DF}{DC} = 1$

XX. $\frac{DH}{DB} \cdot \frac{DB}{DK} \cdot \frac{DK}{DH} = 1$

XXI. $\frac{DF}{DB} \cdot \frac{DB}{DK} \cdot \frac{DK}{DF} = 1$

XXII. $\frac{BF}{BE} \cdot \frac{BE}{BH} \cdot \frac{BH}{BF} = 1$

XXIII. $\frac{BE}{BF} \cdot \frac{BF}{BC} \cdot \frac{BC}{BE} = 1$

XXIV. $\frac{BC}{BA} \cdot \frac{BA}{BD} \cdot \frac{BD}{BC} = 1$

XXV. $\frac{EA}{ED} \cdot \frac{ED}{EC} \cdot \frac{EC}{EA} = 1$

FIGURE 15.

Angle Equations.

I. $0 = A' + N' + H' - 180^\circ - \varepsilon_1$
II. $0 = A' + N' + K' - 180^\circ - \varepsilon_2$
III. $0 = A' + N' + L' - 180^\circ - \varepsilon_3$
IV. $0 = A + H' + N' - 180^\circ - \varepsilon_4$
V. $0 = A + H' + L' - 180^\circ - \varepsilon_5$
VI. $0 = A + H' + M' - 180^\circ - \varepsilon_6$
VII. $0 = B + N' + A - 180^\circ - \varepsilon_7$
VIII. $0 = B + M' + A - 180^\circ - \varepsilon_8$
IX. $0 = B + L' + A - 180^\circ - \varepsilon_9$
X. $0 = B + H' + A - 180^\circ - \varepsilon_{10}$
XI. $0 = C + N' + B - 180^\circ - \varepsilon_{11}$
XII. $0 = C + M' + B - 180^\circ - \varepsilon_{12}$
XIII. $0 = C + L' + B - 180^\circ - \varepsilon_{13}$
XIV. $0 = C + A + B - 180^\circ - \varepsilon_{14}$
XV. $0 = D + N' + B - 180^\circ - \varepsilon_{15}$
XVI. $0 = D + L' + B - 180^\circ - \varepsilon_{16}$
XVII. $0 = D + M' + B - 180^\circ - \varepsilon_{17}$
XVIII. $0 = D + N' + C - 180^\circ - \varepsilon_{18}$
XIX. $0 = E + B + D - 180^\circ - \varepsilon_{19}$
XX. $0 = E + D + C - 180^\circ - \varepsilon_{20}$
XXI. $0 = E' + A' + N' - 180^\circ - \varepsilon_{21}$
XXII. $0 = E' + N' + C - 180^\circ - \varepsilon_{22}$
XXIII. $0 = E' + N' + D - 180^\circ - \varepsilon_{23}$
XXIV. $0 = E' + E + C - 180^\circ - \varepsilon_{24}$
XXV. $0 = B' + E' + E - 180^\circ - \varepsilon_{25}$
XXVI. $0 = B' + E' + D - 180^\circ - \varepsilon_{26}$
XXVII. $0 = C' + B' + E - 180^\circ - \varepsilon_{27}$
XXVIII. $0 = D' + C' + E - 180^\circ - \varepsilon_{28}$

Side Equations.

XXIX. $\frac{N'K'}{N'A'} \cdot \frac{N'A'}{N'H'} \cdot \frac{N'H'}{N'K'} = 1$

XXX. $\frac{N'K'}{N'A'} \cdot \frac{N'A'}{N'L'} \cdot \frac{N'L'}{N'K'} = 1$

XXXI. $\frac{N'G'}{N'A'} \cdot \frac{N'A'}{N'H'} \cdot \frac{N'H'}{N'G'} = 1$

XXXII. $\frac{F'G'}{F'H'} \cdot \frac{F'H'}{F'A'} \cdot \frac{F'A'}{F'G'} = 1$

XXXIII. $\frac{H'M'}{H'N'} \cdot \frac{H'N'}{H'A} \cdot \frac{H'A}{H'M'} = 1$

XXXIV. $\frac{H'M'}{H'L'} \cdot \frac{H'L'}{H'A} \cdot \frac{H'A}{H'M'} = 1$

XXXV. $\frac{AL'}{AH'} \cdot \frac{AH'}{AB} \cdot \frac{AB}{AL'} = 1$

XXXVI. $\frac{AM'}{AL'} \cdot \frac{AL'}{AB} \cdot \frac{AB}{AM'} = 1$

XXXVII. $\frac{AN'}{AL'} \cdot \frac{AL'}{AB} \cdot \frac{AB}{AN'} = 1$

XXXVIII. $\frac{BM'}{BA} \cdot \frac{BA}{BC} \cdot \frac{BC}{BM'} = 1$

XXXIX. $\frac{BL'}{BA} \cdot \frac{BA}{BC} \cdot \frac{BC}{BL'} = 1$

XL. $\frac{BN'}{BM'} \cdot \frac{BM}{BC} \cdot \frac{BC}{BN'} = 1$

XLI. $\frac{N'C}{N'D} \cdot \frac{N'D}{N'B} \cdot \frac{N'B}{N'C} = 1$

XLII. $\frac{BN'}{BM'} \cdot \frac{BM'}{BD} \cdot \frac{BD}{BN'} = 1$

XLIII. $\frac{BN'}{BL'} \cdot \frac{BL'}{BD} \cdot \frac{BD}{BN'} = 1$

XLIV. $\frac{FC}{FD} \cdot \frac{FD}{FB} \cdot \frac{FB}{FC} = 1$

XLV. $\frac{FB}{FC} \cdot \frac{FC}{FM'} \cdot \frac{FM'}{FB} = 1$

XLVI. $\frac{FD}{FM'} \cdot \frac{FM'}{FN'} \cdot \frac{FN'}{FD} = 1$

XLVII. $\frac{FC}{FE'} \cdot \frac{FE'}{FB} \cdot \frac{FB}{FC} = 1$

XLVIII. $\frac{FD}{FE} \cdot \frac{FE}{FB} \cdot \frac{FB}{FD} = 1$

XLIX. $\frac{EC}{EE'} \cdot \frac{EE'}{ED} \cdot \frac{ED}{EC} = 1$

L. $\frac{N'E'}{N'A'} \cdot \frac{N'A'}{N'L'} \cdot \frac{N'L'}{N'C} \cdot \frac{N'C}{N'E'} = 1$

LI. $\frac{N'E'}{N'A'} \cdot \frac{N'A'}{N'L'} \cdot \frac{N'L'}{N'D} \cdot \frac{N'D}{N'E'} = 1$

LII. $\frac{B'E}{B'E'} \cdot \frac{B'E'}{B'D} \cdot \frac{B'D}{B'E} = 1$

LIII. $\frac{E'B'}{E'A'} \cdot \frac{E'A'}{E'N'} \cdot \frac{E'N'}{E'D} \cdot \frac{E'D}{E'B'} = 1$

LIV. $\frac{E'C'}{E'B'} \cdot \frac{E'B'}{E'E} \cdot \frac{E'E}{E'C'} = 1$

LV. $\frac{E'C'}{E'B} \cdot \frac{E'B}{E'D} \cdot \frac{E'D}{E'C'} = 1$

LVI. $\frac{E'D'}{E'C'} \cdot \frac{E'C'}{E'E} \cdot \frac{E'E}{E'D'} = 1$

FIGURE 16.

Angle Equations.

I. $0 = H' + F' + L - 180° - \varepsilon_1$
II. $0 = A' + H' + L - 180° - \varepsilon_2$
III. $0 = U + A' + L - 180° - \varepsilon_3$
IV. $0 = U + L + G - 180° - \varepsilon_4$
V. $0 = L + A' + G - 180° - \varepsilon_5$
VI. $0 = L + U + K - 180° - \varepsilon_6$
VII. $0 = L + G + K - 180° - \varepsilon_7$
VIII. $0 = G + K + H - 180° - \varepsilon_8$
IX. $0 = H + I + L + K - 360° - \varepsilon_9$
X. $0 = H' + A + M - 180° - \varepsilon_{10}$
XI. $0 = H' + L + M - 180° - \varepsilon_{11}$
XII. $0 = I + L + M - 180° - \varepsilon_{12}$
XIII. $0 = B + A + N - 180° - \varepsilon_{13}$
XIV. $0 = A + M + N - 180° - \varepsilon_{14}$
XV. $0 = M + W + N - 180° - \varepsilon_{15}$
XVI. $0 = M + I + W - 180° - \varepsilon_{16}$
XVII. $0 = M + I + R - 180° - \varepsilon_{17}$
XVIII. $0 = M + R + W - 180° - \varepsilon_{18}$
XIX. $0 = M + I + S - 180° - \varepsilon_{19}$
XX. $0 = M + S + W - 180° - \varepsilon_{20}$
XXI. $0 = M + T + W - 180° - \varepsilon_{21}$
XXII. $0 = M + S + T - 180° - \varepsilon_{22}$
XXIII. $0 = M + T + R - 180° - \varepsilon_{23}$
XXIV. $0 = U + T + W - 180° - \varepsilon_{24}$
XXV. $0 = U + T + R - 180° - \varepsilon_{25}$
XXVI. $0 = M + Q + W - 180° - \varepsilon_{26}$
XXVII. $0 = P + N + W - 180° - \varepsilon_{27}$
XXVIII. $0 = P + Q + W - 180° - \varepsilon_{28}$
XXIX. $0 = B + N + O - 180° - \varepsilon_{29}$
XXX. $0 = M + N + O - 180° - \varepsilon_{30}$
XXXI. $0 = N + O + P - 180° - \varepsilon_{31}$

Side Equations.

XXXII. $\frac{H'F'}{H'L} \cdot \frac{H'L}{H'A'} \cdot \frac{H'A'}{H'F'} = 1$

XXXIII. $\frac{LG'}{LH'} \cdot \frac{LH'}{LF'} \cdot \frac{LF'}{LG'} = 1$

XXXIV. $\frac{A'N'}{A'E'} \cdot \frac{A'E'}{A'Q'} \cdot \frac{A'Q'}{A'P'} \cdot \frac{A'P'}{A'L} \cdot \frac{A'L}{A'H'} \cdot \frac{A'H'}{A'N'} = 1$

XXXV. $\frac{LP'}{LA'} \cdot \frac{LA'}{LG} \cdot \frac{LG}{LP'} = 1$

XXXVI. $\frac{LG}{LP'} \cdot \frac{LP'}{LK} \cdot \frac{LK}{LG} = 1$

XXXVII. $$\frac{GK}{GL} \cdot \frac{GL}{GH} \cdot \frac{GH}{GK} = 1$$

XXXVIII. $$\frac{LH'}{LA'} \cdot \frac{LA'}{LG} \cdot \frac{LG}{LH} \cdot \frac{LH}{LI} \cdot \frac{LI}{LM} \cdot \frac{LM}{LH'} = 1$$

XXXIX. $$\frac{H'L}{H'M} \cdot \frac{H'M}{H'A} \cdot \frac{H'A}{H'N'} \cdot \frac{H'N'}{H'A'} \cdot \frac{H'A'}{H'L} = 1$$

XL. $$\frac{AH'}{AM} \cdot \frac{AM}{AN} \cdot \frac{AN}{AB} \cdot \frac{AB}{AH'} = 1$$

XLI. $$\frac{MI}{MW} \cdot \frac{MW}{MN} \cdot \frac{MN}{MA} \cdot \frac{MA}{MH'} \cdot \frac{MH'}{ML} \cdot \frac{ML}{MI} = 1$$

XLII. $$\frac{MR}{MI} \cdot \frac{MI}{MW} \cdot \frac{MW}{MR} = 1$$

XLIII. $$\frac{MS}{MI} \cdot \frac{MI}{MW} \cdot \frac{MW}{MS} = 1$$

XLIV. $$\frac{MS}{MR} \cdot \frac{MR}{MW} \cdot \frac{MW}{MS} = 1$$

XLV. $$\frac{MT}{MS} \cdot \frac{MS}{MW} \cdot \frac{MW}{MT} = 1$$

XLVI. $$\frac{RT}{RW} \cdot \frac{RW}{RM} \cdot \frac{RM}{RT} = 1$$

XLVII. $$\frac{MT}{MW} \cdot \frac{MW}{MU} \cdot \frac{MU}{MT} = 1$$

XLVIII. $$\frac{RU}{RT} \cdot \frac{RT}{RW} \cdot \frac{RW}{RM} \cdot \frac{RM}{RU} = 1$$

XLIX. $$\frac{MQ}{MN} \cdot \frac{MN}{MW} \cdot \frac{MW}{MQ} = 1$$

L. $$\frac{QN}{QW} \cdot \frac{QW}{QP} \cdot \frac{QP}{QN} = 1$$

LI. $$\frac{NM}{NO} \cdot \frac{NO}{NB} \cdot \frac{NB}{NA} \cdot \frac{NA}{NM} = 1$$

LII. $$\frac{NM}{NW} \cdot \frac{NW}{NP} \cdot \frac{NP}{NO} \cdot \frac{NO}{NM} = 1$$

LIII. $$\frac{TM}{TW} \cdot \frac{TW}{TV} \cdot \frac{TV}{TM} = 1$$

LIV. $$\frac{VT}{VR} \cdot \frac{VR}{VM} \cdot \frac{VM}{VT} = 1$$

LV. $$\frac{TR}{TU} \cdot \frac{TU}{TV} \cdot \frac{TV}{TR} = 1$$

LVI. $$\frac{VW}{VS} \cdot \frac{VS}{VM} \cdot \frac{VM}{VW} = 1$$

FIGURE 17.

Angle Equations.

I. $0 = B + C' + D' - 180^\circ - \varepsilon_1$

II. $0 = D + C' + B - 180^\circ - \varepsilon_2$

III. $0 = D + B + C - 180^\circ - \varepsilon_3$

IV. $0 = C + B + D' - 180^\circ - \varepsilon_4$

V. $0 = C + D' + E' - 180^\circ - \varepsilon_5$

VI. $0 = L' + C + D' - 180^\circ - \varepsilon_6$

VII. $0 = L' + C + E' - 180^\circ - \varepsilon_7$

VIII. $0 = L' + C + D - 180^\circ - \varepsilon_8$

IX. $0 = G + M' + N' - 180^\circ - \varepsilon_9$

X. $0 = G + M' + L' + E' - 360^\circ - \varepsilon_{10}$

XI. $0 = G + E' + H' - 180^\circ - \varepsilon_{11}$

XII. $0 = G + H' + K' - 180^\circ - \varepsilon_{12}$

XIII. $0 = K + H' + K' - 180^\circ - \varepsilon_{13}$

XIV. $0 = K + H' + G - 180^\circ - \varepsilon_{14}$

XV. $0 = K + G + N' - 180^\circ - \varepsilon_{15}$

XVI. $0 = K + Q' + N' - 180^\circ - \varepsilon_{16}$

XVII. $0 = K + Q' + P' - 180^\circ - \varepsilon_{17}$

XVIII. $0 = H + G + E' - 180^\circ - \varepsilon_{18}$

XIX. $0 = H + G + H' - 180^\circ - \varepsilon_{19}$

XX. $0 = H + G + K' - 180^\circ - \varepsilon_{20}$

Side Equations.

XXI. $\frac{BB'}{BD'} \cdot \frac{BD'}{BC'} \cdot \frac{BC'}{BB'} = 1$

XXII. $\frac{D'B}{D'B'} \cdot \frac{D'E'}{D'B} = \frac{D'E'}{D'B'}$

XXIII. $\frac{AB'}{AB} \cdot \frac{AB}{AC'} \cdot \frac{AC'}{AB'} = 1$

XXIV. $\frac{B'A}{B'A'} \cdot \frac{B'A'}{B'D'} \cdot \frac{B'D'}{B'B} \cdot \frac{B'B}{B'A} = 1$

XXV. $\frac{AC'}{AB} \cdot \frac{AB}{AD} \cdot \frac{AD}{AC'} = 1$

XXVI. $\frac{BE'}{BD'} \cdot \frac{BD'}{BC} \cdot \frac{BC}{BE'} = 1$

XXVII. $\frac{BC}{BD} \cdot \frac{BD}{BC'} \cdot \frac{BC'}{BD'} \cdot \frac{BD'}{BC} = 1$

XXVIII. $\frac{AC}{AB} \cdot \frac{AB}{AD} \cdot \frac{AD}{AC} = 1$

XXIX. $\frac{CE'}{CD'} \cdot \frac{CD'}{CL'} \cdot \frac{CL'}{CE'} = 1$

XXX. $\frac{CL'}{CD} \cdot \frac{CD}{CB} \cdot \frac{CB}{CD'} \cdot \frac{CD'}{CL'} = 1$

XXXI. $\frac{CD'}{CB} \cdot \frac{CB}{CL'} \cdot \frac{CL'}{CD'} = 1$

XXXII. $\frac{AL'}{AC} \cdot \frac{AC}{AD} \cdot \frac{AD}{AL'} = 1$

XXXIII. $\frac{FM'}{FL'} \cdot \frac{FL'}{FE'} \cdot \frac{FE'}{FM'} = 1$

XXXIV. $\frac{EM'}{EL'} \cdot \frac{EL'}{EE'} \cdot \frac{EE'}{EM'} = 1$

XXXV. $\frac{EG}{EE'} \cdot \frac{EE'}{EM'} \cdot \frac{EM'}{EG} = 1$

XXXVI. $\frac{FG}{FE'} \cdot \frac{FE'}{FM'} \cdot \frac{FM'}{FG} = 1$

XXXVII. $\frac{M'L'}{M'E'} \cdot \frac{M'E'}{M'G} \cdot \frac{M'G}{M'N'} = \frac{M'L'}{M'N'}$

XXXVIII. $\frac{E'H'}{E'G} \cdot \frac{E'G}{E'M'} \cdot \frac{E'M'}{E'L'} \cdot \frac{E'L'}{E'D'} = \frac{E'H'}{E'D'}$

XXXIX. $\frac{H'K'}{H'G} \cdot \frac{H'G}{H'E'} \cdot \frac{H'E'}{H'K'} = 1$

XL. $\frac{Q'N'}{Q'K} \cdot \frac{Q'K}{Q'P'} \cdot \frac{Q'P'}{Q'N'} = 1$

XLI. $\frac{N'M'}{N'G} \cdot \frac{N'G}{N'K} \cdot \frac{N'K}{N'Q'} = \frac{N'M'}{N'Q'}$

XLII. $\frac{GK}{GN'} \cdot \frac{GN'}{GM'} \cdot \frac{GM'}{GE'} \cdot \frac{GE'}{GH'} \cdot \frac{GH'}{GK} = 1$

XLIII. $\frac{GH'}{GK'} \cdot \frac{GK'}{GK} \cdot \frac{GK}{GH'} = 1$

XLIV. $\frac{E'H}{E'H'} \cdot \frac{E'H'}{E'G} \cdot \frac{E'G}{E'H} = 1$

XLV. $\frac{H'K'}{H'H} \cdot \frac{H'H}{H'G} \cdot \frac{H'G}{H'K'} = 1$

XLVI. $\frac{HF'}{HE'} \cdot \frac{HE'}{HH'} \cdot \frac{HH'}{HF'} = 1$

XLVII. $\frac{HG'}{HF'} \cdot \frac{HF'}{HH'} \cdot \frac{HH'}{HG'} = 1$

FIGURE 18.

Angle Equations.

I. $0 = B + E' + D' - 180° - \epsilon_1$

II. $0 = B + D' + I' - 180° - \epsilon_2$

III. $0 = B + E' + H' - 180° - \epsilon_3$

IV. $0 = A + B + I' - 180° - \epsilon_4$

V. $0 = D + A' + D' - 180° - \epsilon_5$

VI. $0 = D + B + D' - 180° - \epsilon_6$

VII. $0 = D + A' + C - 180° - \epsilon_7$

VIII. $0 = A' + C' + C - 180° - \epsilon_8$

IX. $0 = B + E + D' - 180° - \epsilon_9$

X. $0 = E + D + D' - 180° - \epsilon_{10}$

XI. $0 = E + D + C - 180° - \epsilon_{11}$

XII. $0 = F + B + D - 180° - \epsilon_{12}$

XIII. $0 = F + B + E - 180° - \epsilon_{13}$

XIV. $0 = A + F + B - 180° - \epsilon_{14}$

XV. $0 = D + G + F - 180° - \epsilon_{15}$

Side Equations.

XVI. $\frac{E'H'}{E'G'} \cdot \frac{E'G'}{E'D'} \cdot \frac{E'D'}{E'B} \cdot \frac{E'B}{E'H'} = 1$

XVII. $\frac{BH'}{BE'} \cdot \frac{BE'}{BD'} \cdot \frac{BD'}{BI'} \cdot \frac{BI'}{BH'} = 1$

XVIII. $\frac{F'B}{F'I'} \cdot \frac{F'I'}{F'D'} \cdot \frac{F'D'}{F'B} = 1$

XIX. $\frac{F'A}{F'B} \cdot \frac{F'B}{F'I'} \cdot \frac{F'I'}{F'A} = 1$

XX. $\frac{C'D}{C'A'} \cdot \frac{C'A'}{C'D'} \cdot \frac{C'D'}{C'D} = 1$

XXI. $\frac{D'E'}{D'A'} \cdot \frac{D'A'}{D'D} \cdot \frac{D'D}{D'B} \cdot \frac{D'B}{D'E'} = 1$

XXII. $\frac{C'D}{C'A'} \cdot \frac{C'A'}{C'C} \cdot \frac{C'C}{C'D} = 1$

XXIII. $\frac{CA'}{CD} \cdot \frac{CD}{CD'} \cdot \frac{CD'}{CA'} = 1$

XXIV. $\frac{B'C}{B'A'} \cdot \frac{B'A'}{B'C'} \cdot \frac{B'C'}{B'C} = 1$

XXV. $\frac{D'E}{D'B} \cdot \frac{D'B}{D'D} \cdot \frac{DD}{D'E} = 1$

XXVI. $\frac{CD'}{CD} \cdot \frac{CD}{CE} \cdot \frac{CE}{CD'} = 1$

XXVII. $\frac{EB}{ED} \cdot \frac{ED}{EF} \cdot \frac{EF}{EB} = 1$

XXVIII. $\frac{BA}{BI'} \cdot \frac{BI'}{BD'} \cdot \frac{BD'}{BE} \cdot \frac{BE}{BF} \cdot \frac{BF}{BA} = 1$

XXIX. $\frac{GE}{GD} \cdot \frac{GD}{GF} \cdot \frac{GF}{GE} = 1$

XXX. $\frac{GB}{GD} \cdot \frac{GD}{GF} \cdot \frac{GF}{GB} = 1$

XXXI. $\frac{GA}{GB} \cdot \frac{GB}{GF} \cdot \frac{GF}{GA} = 1$

FIGURE 19.

Angle Equations.

I. $0 = C' + A' + A - 180° - \varepsilon_1$

II. $0 = D' + A' + B - 180° - \varepsilon_2$

III. $0 = C' + B' + B + A - 360° - \varepsilon_3$

IV. $0 = A + B + A' - 180° - \varepsilon_4$

V. $0 = C + D' + E' - 180° - \varepsilon_5$

VI. $0 = C + F + E' - 180° - \varepsilon_6$

VII. $0 = F + E' + D' - 180° - \varepsilon_7$

VIII. $0 = C + E + F - 180° - \varepsilon_8$

IX. $0 = C + E + B + D' - 360° - \varepsilon_9$

X. $0 = D + A + B - 180° - \varepsilon_{10}$

XI. $0 = D + B + E - 180° - \varepsilon_{11}$

XII. $0 = C + G + E' - 180° - \varepsilon_{12}$

XIII. $0 = F + G + E' - 180° - \varepsilon_{13}$

XIV. $0 = F + G + H - 180° - \varepsilon_{14}$

XV. $0 = E + H + F - 180° - \varepsilon_{15}$

XVI. $0 = I + E + H - 180° - \varepsilon_{16}$

XVII. $0 = L + D + A - 180° - \varepsilon_{17}$

XVIII. $0 = M + I + E - 180° - \varepsilon_{18}$

XIX. $0 = M + E + D + L - 360° - \varepsilon_{19}$

XX. $0 = K + E + I - 180° - \varepsilon_{20}$

XXI. $0 = K + M + I - 180° - \varepsilon_{21}$

XXII. $0 = K + M + L - 180° - \varepsilon_{22}$

XXIII. $0 = K + L + A - 180° - \varepsilon_{23}$

Side Equations.

XXIV. $\frac{B'C'}{B'A} \cdot \frac{B'A}{B'A'} = \frac{B'C'}{B'A'}$

XXV. $\frac{A'C'}{A'A} \cdot \frac{A'A}{A'B} \cdot \frac{A'B}{A'D'} = \frac{A'C'}{A'D'}$

XXVI. $\frac{B'B}{B'A'} \cdot \frac{B'A'}{B'A} \cdot \frac{B'A}{B'B} = 1$

XXVII. $\frac{D'A'}{D'B} \cdot \frac{D'B}{D'C} \cdot \frac{D'C}{D'E'} = \frac{D'A'}{D'E'}$

XXVIII. $\frac{E'C}{E'F} \cdot \frac{E'F}{E'D'} \cdot \frac{E'D'}{E'C} = 1$

XXIX. $\frac{CE'}{CD'} \cdot \frac{CD'}{CB} \cdot \frac{CB}{CF} \cdot \frac{CF}{CE'} = 1$

XXX. $\frac{CE}{CF} \cdot \frac{CF}{CB} \cdot \frac{CB}{CE} = 1$

XXXI. $\frac{ED'}{EF} \cdot \frac{EF}{EB} \cdot \frac{EB}{ED'} = 1$

XXXII. $\frac{BD}{BE} \cdot \frac{BE}{BD'} \cdot \frac{BD'}{BA'} \cdot \frac{BA'}{BA} \cdot \frac{BA}{BD} = 1$

XXXIII. $\frac{FE'}{FC} \cdot \frac{FC}{FG} \cdot \frac{FG}{FE'} = 1$

XXXIV. $\frac{FG}{FC} \cdot \frac{FC}{FE} \cdot \frac{FE}{FH} \cdot \frac{FH}{FG} = 1$

XXXV. $\frac{IF}{IE} \cdot \frac{IE}{IH} \cdot \frac{IH}{IF} = 1$

XXXVI. $\frac{DE}{DB} \cdot \frac{DB}{DA} \cdot \frac{DA}{DL} \cdot \frac{DL}{DM} \cdot \frac{DM}{DE} = 1$

XXXVII. $\frac{EI}{EH} \cdot \frac{EH}{EF} \cdot \frac{EF}{EB} \cdot \frac{EB}{ED} \cdot \frac{ED}{EM} \cdot \frac{EM}{EI} = 1$

XXXVIII. $\frac{KI}{KM} \cdot \frac{KM}{KE} \cdot \frac{KE}{KI} = 1$

XXXIX. $\frac{MI}{ME} \cdot \frac{ME}{MD} \cdot \frac{MD}{ML} \cdot \frac{ML}{MK} \cdot \frac{MK}{MI} = 1$

XL. $\frac{LK}{LM} \cdot \frac{LM}{LD} \cdot \frac{LD}{LA} \cdot \frac{LA}{LK} = 1$

FIGURE 20.

Angle Equations.

I. $0 = A + F' + E' - 180^\circ - \varepsilon_1$

II. $0 = A + F' + B - 180^\circ - \varepsilon_2$

III. $0 = F' + G' + B - 180^\circ - \varepsilon_3$

IV. $0 = G' + H' + B - 180^\circ - \varepsilon_4$

V. $0 = E + H' + B - 180^\circ - \varepsilon_5$

VI. $0 = E + H' + G' - 180^\circ - \varepsilon_6$

VII. $0 = E + H' + B' - 180^\circ - \varepsilon_7$

VIII. $0 = F + H' + E - 180^\circ - \varepsilon_8$

IX. $0 = F + H' + B' - 180^\circ - \varepsilon_9$

X. $0 = F + H' + A' - 180^\circ - \varepsilon_{10}$

XI. $0 = C + A + B - 180^\circ - \varepsilon_{11}$

XII. $0 = D + C + B - 180^\circ - \varepsilon_{12}$

XIII. $0 = D + E + B - 180^\circ - \varepsilon_{13}$

XIV. $0 = I + E + F - 180^\circ - \varepsilon_{14}$

XV. $0 = I + E + B - 180^\circ - \varepsilon_{15}$

XVI. $0 = I + E + D - 180^\circ - \varepsilon_{16}$

XVII. $0 = K + A' + F - 180^\circ - \varepsilon_{17}$

XVIII. $0 = K + I + F - 180^\circ - \varepsilon_{18}$

XIX. $0 = G + D + C - 180^\circ - \varepsilon_{19}$

XX. $0 = G + D + I - 180^\circ - \varepsilon_{20}$

XXI. $0 = H + I + D - 180^\circ - \varepsilon_{21}$

XXII. $0 = H + I + G - 180^\circ - \varepsilon_{22}$

XXIII. $0 = H + I + K - 180^\circ - \varepsilon_{23}$

Side Equations.

XXIV. $\frac{F'G'}{F'B} \cdot \frac{F'B}{F'A} \cdot \frac{F'A}{F'E'} \cdot \frac{F'E'}{F'D'} \cdot \frac{F'D'}{F'G'} = 1$

XXV. $\frac{G'F'}{G'D'} \cdot \frac{G'D'}{G'H'} \cdot \frac{G'H'}{G'B} \cdot \frac{G'B}{G'F'} = 1$

XXVI. $\frac{H'B}{H'G'} \cdot \frac{H'G'}{H'E} \cdot \frac{H'E}{H'B} = 1$

XXVII. $\frac{H'G'}{H'C'} \cdot \frac{H'C'}{H'B'} \cdot \frac{H'B'}{H'E} \cdot \frac{H'E}{H'G'} = 1$

XXVIII. $\frac{A'H'}{A'B'} \cdot \frac{A'B'}{A'E} \cdot \frac{A'E}{A'H'} = 1$

XXIX. $\frac{H'F}{H'E} \cdot \frac{H'E}{H'B'} \cdot \frac{H'B'}{H'F} = 1$

XXX. $\frac{H'A'}{H'F} \cdot \frac{H'F}{H'B'} \cdot \frac{H'B'}{H'A'} = 1$

XXXI. $\frac{FC'}{FH'} \cdot \frac{FH'}{FB'} \cdot \frac{FB'}{FC'} = 1$

XXXII. $\frac{BA}{BF'} \cdot \frac{BF'}{BG'} \cdot \frac{BG'}{BE} \cdot \frac{BE}{BD} \cdot \frac{BD}{BC} \cdot \frac{BC}{BA} = 1$

XXXIII. $\frac{ED}{EI} \cdot \frac{EI}{EB} \cdot \frac{EB}{ED} = 1$

XXXIV. $\frac{EI}{EB} \cdot \frac{EB}{EH'} \cdot \frac{EH'}{EF} \cdot \frac{EF}{EI} = 1$

XXXV. $\frac{FI}{FE} \cdot \frac{FE}{FH'} \cdot \frac{FH'}{FA'} \cdot \frac{FA'}{FK} \cdot \frac{FK}{FI} = 1$

XXXVI. $\frac{KE}{KF} \cdot \frac{KF}{KI} \cdot \frac{KI}{KE} = 1$

XXXVII. $\frac{DC}{DB} \cdot \frac{DB}{DI} \cdot \frac{DI}{DG} \cdot \frac{DG}{DC} = 1$

XXXVIII. $\frac{IG}{IH} \cdot \frac{IH}{ID} \cdot \frac{ID}{IG} = 1$

XXXIX. $\frac{ID}{IE} \cdot \frac{IE}{IF} \cdot \frac{IF}{IK} \cdot \frac{IK}{IH} \cdot \frac{IH}{ID} = 1$

FIGURE 21.

Angle Equations.

I. $0 = G' + B + F' - 180° - \epsilon_1$
II. $0 = E + E' + G' + B - 360° - \epsilon_2$
III. $0 = A + F' + B - 180° - \epsilon_3$
IV. $0 = A + B + E - 180° - \epsilon_4$
V. $0 = D + E' + G' + B - 360° - \epsilon_5$
VI. $0 = D + A + B - 180° - \epsilon_6$
VII. $0 = C + A + D - 180° - \epsilon_7$
VIII. $0 = C + A + E - 180° - \epsilon_8$
IX. $0 = B' + C' + F - 180° - \epsilon_9$
X. $0 = C + E + F - 180° - \epsilon_{10}$
XI. $0 = E + F + C' + E' - 360° - \epsilon_{11}$
XII. $0 = A' + Q + B' - 180° - \epsilon_{12}$
XIII. $0 = S + H' + K' - 180° - \epsilon_{13}$
XIV. $0 = S + H' + D' - 180° - \epsilon_{14}$
XV. $0 = S + D' + A' + Q - 360° - \epsilon_{15}$
XVI. $0 = P + Q + S - 180° - \epsilon_{16}$
XVII. $0 = P + Q + B' - 180° - \epsilon_{17}$
XVIII. $0 = O + S + K' - 180° - \epsilon_{18}$
XIX. $0 = O + S + Q - 180° - \epsilon_{19}$
XX. $0 = O + S + P - 180° - \epsilon_{20}$
XXI. $0 = M + L' + K' - 180° - \epsilon_{21}$
XXII. $0 = M + O + K' - 180° - \epsilon_{22}$
XXIII. $0 = N + L' + K' - 180° - \epsilon_{23}$
XXIV. $0 = N + M + K' - 180° - \epsilon_{24}$
XXV. $0 = K + M + L' - 180° - \epsilon_{25}$
XXVI. $0 = K + M + N - 180° - \epsilon_{26}$
XXVII. $0 = K + M + O - 180° - \epsilon_{27}$
XXVIII. $0 = K + O + P + B' + F - 540° - \epsilon_{28}$
XXIX. $0 = K + L + M - 180° - \epsilon_{29}$
XXX. $0 = K + L + N - 180° - \epsilon_{30}$
XXXI. $0 = M' + L' + N + L - 360° - \epsilon_{31}$

Side Equations.

XXXII. $\frac{G'F'}{G'B} \cdot \frac{G'B}{G'E} \cdot \frac{G'E}{G'E'} = \frac{G'F'}{G'E'}$

XXXIII. $\frac{G'A}{G'B} \cdot \frac{G'B}{G'F'} \cdot \frac{G'F'}{G'A} = 1$

XXXIV. $\frac{G'B}{G'A} \cdot \frac{G'A}{G'E} \cdot \frac{G'E}{G'B} = 1$

XXXV. $\frac{G'D}{G'E'} \cdot \frac{G'E'}{G'E} \cdot \frac{G'E}{G'B} \cdot \frac{G'B}{G'D} = 1$

XXXVI. $\frac{BD}{BA} \cdot \frac{BA}{BG'} \cdot \frac{BG'}{BD} = 1$

XXXVII. $\frac{AE}{AC} \cdot \frac{AC}{AD} \cdot \frac{AD}{AB} \cdot \frac{AB}{AE} = 1$

XXXVIII. $\frac{C'E'}{C'G} \cdot \frac{C'G}{C'B'} = \frac{C'E'}{C'B'}$

XXXIX. $\frac{E'G'}{E'E} \cdot \frac{E'E}{E'G} \cdot \frac{E'G}{E'C'} = \frac{E'G'}{E'C'}$

XL. $\frac{EE'}{EG'}\cdot\frac{EG'}{EA}\cdot\frac{EA}{EC}\cdot\frac{EC}{EG}\cdot\frac{EG}{EE'}=1$

XLI. $\frac{GE}{GC}\cdot\frac{GC}{GF}\cdot\frac{GF}{GE}=1$

XLII. $\frac{GE}{GF}\cdot\frac{GF}{GC'}\cdot\frac{GC'}{GE'}\cdot\frac{GE'}{GE}=1$

XLIII. $\frac{GB'}{GF}\cdot\frac{GF}{GC'}\cdot\frac{GC'}{GB'}=1$

XLIV. $\frac{A'B'}{A'Q}\cdot\frac{A'Q}{A'D'}=\frac{A'B'}{A'D'}$

XLV. $\frac{H'D'}{H'S}\cdot\frac{H'S}{H'K'}=\frac{H'D'}{H'K'}$

XLVI. $\frac{D'A'}{D'Q}\cdot\frac{D'Q}{D'S}\cdot\frac{D'S}{D'H'}=\frac{D'A'}{D'H'}$

XLVII. $\frac{SN'}{SK'}\cdot\frac{SK'}{SH'}\cdot\frac{SH'}{SN'}=1$

XLVIII. $\frac{RD'}{RA'}\cdot\frac{RA'}{RQ}\cdot\frac{RQ}{RS}\cdot\frac{RS}{RD'}=1$

XLIX. $\frac{RA'}{RD'}\cdot\frac{RD'}{RQ}\cdot\frac{RQ}{RA'}=1$

L. $\frac{RH'}{RS}\cdot\frac{RS}{RD'}\cdot\frac{RD'}{RH'}=1$

LI. $\frac{QA'}{QB'}\cdot\frac{QB'}{QP}\cdot\frac{QP}{QS}\cdot\frac{QS}{QD'}\cdot\frac{QD'}{QA'}=1$

LII. $\frac{B'C'}{B'F}\cdot\frac{B'F}{B'H}\cdot\frac{B'H}{B'P}\cdot\frac{B'P}{B'Q}\cdot\frac{B'Q}{B'A'}=\frac{B'C'}{B'A'}$

LIII. $\frac{SO}{SK'}\cdot\frac{SK'}{SH'}\cdot\frac{SH'}{SD'}\cdot\frac{SD'}{SQ}\cdot\frac{SQ}{SO}=1$

LIV. $\frac{SP}{SO}\cdot\frac{SO}{SQ}\cdot\frac{SQ}{SP}=1$

LV. $\frac{PH}{PO}\cdot\frac{PO}{PQ}\cdot\frac{PQ}{PB'}\cdot\frac{PB'}{PH}=1$

LVI. $\frac{K'H'}{K'S}\cdot\frac{K'S}{K'O}\cdot\frac{K'O}{K'M}\cdot\frac{K'M}{K'L'}=\frac{K'H'}{K'L'}$

LVII. $\frac{ML'}{MN}\cdot\frac{MN}{MK'}\cdot\frac{MK'}{ML'}=1$

LVIII. $\frac{MN}{MK}\cdot\frac{MK}{ML'}\cdot\frac{ML'}{MN}=1$

LIX. $\frac{ML'}{MK'}\cdot\frac{MK'}{MO}\cdot\frac{MO}{MK}\cdot\frac{MK}{ML'}=1$

LX. $\frac{OK}{OM}\cdot\frac{OM}{OK'}\cdot\frac{OK'}{OS}\cdot\frac{OS}{OP}\cdot\frac{OP}{OH}\cdot\frac{OH}{OK}=1$

LXI. $\frac{HK}{HO}\cdot\frac{HO}{HP}\cdot\frac{HP}{HB'}\cdot\frac{HB'}{HF}\cdot\frac{HF}{HK}=1$

LXII. $\frac{NK}{NL}\cdot\frac{NL}{NM}\cdot\frac{NM}{NK}=1$

LXIII. $\frac{L'K'}{L'M}\cdot\frac{L'M}{L'L}\cdot\frac{L'L}{L'M'}=\frac{L'K'}{L'M'}$

RESULTING

ALGEBRAICAL

EQUATIONS OF CONDITION.

FIGURE 2.

Notation.

(1)	*O'C*	(2)	*O'I*	(3)	*O'K*	(4)	*O'B*	(5)	*O'A*	(6)	*N'B*	(7)	*N'A*
(8)	*BD*	(9)	*BC*	(10)	*BA*	(11)	*BO'*	(12)	*BN'*	(13)	*AC*	(14)	*AO'*
(15)	*AN'*	(16)	*AB*	(17)	*CE*	(18)	*CH*	(19)	*CI*	(20)	*CO'*	(21)	*CA*
(22)	*CB*	(23)	*CD*	(24)	*DE*	(25)	*DC*	(26)	*DB*	(27)	*EH*	(28)	*EC*
(29)	*ED*	(30)	*HE*	(31)	*HI*	(32)	*HC*	(33)	*IH*	(34)	*IK*	(35)	*IO'*
(36)	*IC*	(37)	*KI*	(38)	*KF*	(39)	*KO*	(40)	*FK*				

Equations.

I. $0 = +\ 0{\cdot}0060 + (4) - (6) - (11) + (12)$

II. $0 = +\ 5{\cdot}8930 + (5) - (7) - (14) + (15)$

III. $0 = +\ 7{\cdot}7866 - (4) + (5) - (10) + (11) - (14) + (16)$

IV. $0 = -\ 0{\cdot}5211 + (1) - (4) - (9) + (11) - (20) + (22)$

V. $0 = -\ 0{\cdot}8381 + (1) - (5) - (13) + (14) - (20) + (21)$

VI. $0 = +\ 7{\cdot}1555 - (8) + (9) - (22) + (23) - (25) + (26)$

VII. $0 = -\ 2{\cdot}5064 + (17) - (23) - (24) + (25) - (28) + (29)$

VIII. $0 = -\ 1{\cdot}6834 - (17) + (18) - (27) + (28) + (30) - (32)$

IX. $0 = -\ 3{\cdot}7607 - (18) + (19) - (31) + (32) + (33) - (36)$

X. $0 = +\ 1{\cdot}8104 - (1) + (2) - (19) + (20) - (35) + (36)$

XI. $0 = +\ 2{\cdot}8791 - (2) + (3) - (34) + (35) + (37) - (39)$

XII. $0 = +\ 1{\cdot}6281 - (3) - (38) + (39) + (40)$

XIII. $$\begin{aligned} 0 = +\ 423{\cdot}696 &- 4{\cdot}4833\,(6) + 19{\cdot}4147\,(7) - 32{\cdot}3375\,(10) + 47{\cdot}8757\,(11) - 15{\cdot}5382\,(12) \\ &- 25{\cdot}9066\,(14) + 10{\cdot}5081\,(15) + 15{\cdot}3985\,(16) \end{aligned}$$

XIV. $$\begin{aligned} 0 = +\ 245{\cdot}639 &+ 7{\cdot}9611\,(9) - 32{\cdot}3375\,(10) + 24{\cdot}3764\,(11) + 9{\cdot}6672\,(13) - 25{\cdot}0657\,(14) \\ &+ 15{\cdot}3985\,(16) + 18{\cdot}2567\,(20) - 31{\cdot}0101\,(21) + 12{\cdot}7534\,(22) \end{aligned}$$

XV. $$\begin{aligned} 0 = -\ 187{\cdot}742 &- 24{\cdot}8522\,(1) + 8{\cdot}3520\,(2) + 16{\cdot}5002\,(4) + 36{\cdot}9493\,(8) - 44{\cdot}9104\,(9) \\ &+ 7{\cdot}9611\,(11) + 17{\cdot}1010\,(24) - 20{\cdot}2208\,(25) + 3{\cdot}1198\,(26) + 12{\cdot}7692\,(27) \\ &- 56{\cdot}0291\,(28) + 43{\cdot}2599\,(29) + 16{\cdot}2913\,(30) - 4{\cdot}6934\,(31) - 11{\cdot}5979\,(32) \\ &+ 14{\cdot}9415\,(33) + 5{\cdot}9998\,(35) - 20{\cdot}9413\,(36) \end{aligned}$$

XVI. $$\begin{aligned} 0 = -\ 114{\cdot}888 &+ 4{\cdot}4833\,(6) + 7{\cdot}9611\,(9) - 23{\cdot}4993\,(11) + 15{\cdot}5382\,(12) + 27{\cdot}3973\,(19) \\ &- 40{\cdot}1507\,(20) + 12{\cdot}7534\,(22) + 19{\cdot}7460\,(34) - 25{\cdot}7458\,(35) + 5{\cdot}9998\,(36) \\ &+ 3{\cdot}3104\,(37) + 5{\cdot}4228\,(38) - 8{\cdot}7332\,(39) + 24{\cdot}8605\,(40) \end{aligned}$$

The logarithmic values of the multipliers are as follow:

Log I = 9·9870812 −
II = 9·1924604 +
III = 7·6641943 −
IV = 9·8382516 −
V = 9·5983072 −

Log VI = 9·9866076 −
VII = 9·4361077 +
VIII = 9·5294442 +
IX = 9·0333518 +
X = 9·4680706 −
XI = 9·5412088 −

Log XII = 9·2510890 −
XIII = 8·4851144 −
XIV = 8·0311923 −
XV = 7·8577019 +
XVI = 7·8316477 +

FIGURE 3.

Notation.

(1)	*B′A*	(2)	*D′H*	(3)	*D′M*	(4)	*D′F*	(5)	*D′A*	(6)	*C′G*	(7)	*C′K*
(8)	*C′F*	(9)	*C′M*	(10)	*C′A*	(11)	*E′A*	(12)	*E′F*	(13)	*E′M*	(14)	*E′G*
(15)	*E′K*	(16)	*E′L*	(17)	*FK*	(18)	*FE′*	(19)	*FC′*	(20)	*FA*	(21)	*FD′*
(22)	*FM*	(23)	*FG*	(24)	*GH*	(25)	*GI*	(26)	*GK*	(27)	*GE′*	(28)	*GF*
(29)	*GC′*	(30)	*GA*	(31)	*GM*	(32)	*HI*	(33)	*HK*	(34)	*HG*	(35)	*HA*
(36)	*HM*	(37)	*HD′*	(38)	*KL*	(39)	*KE′*	(40)	*KF*	(41)	*KC*	(42)	*KA*
(43)	*KM*	(44)	*KG*	(45)	*KH*	(46)	*KI*	(47)	*LE′*	(48)	*LK*	(49)	*LI*
(50)	*IL*	(51)	*IK*	(52)	*IH*								

Equations.

I. $0 = -\,0·1311 - (8) + (12) - (18) + (19)$

II. $0 = +\,6·3997 - (4) + (8) - (19) + (21)$

III. $0 = -\,1·1280 - (12) + (15) - (17) + (18) - (39) + (40)$

IV. $0 = -\,4·6627 - (7) + (15) - (39) + (41)$

V. $0 = +\,1·9887 - (6) + (8) - (19) + (23) - (28) + (29)$

VI. $0 = +\,4·2596 - (6) + (14) - (27) + (29)$

VII. $0 = +\,4·7523 - (6) + (7) - (26) + (29) - (41) + (44)$

VIII. $0 = +\,3·0055 - (24) + (26) - (33) + (34) - (44) + (45)$

IX. $0 = -\,8·3978 - (2) + (6) + (24) - (29) - (34) + (37)$

X. $0 = -\,0·9747 - (32) + (33) - (45) + (46) - (51) + (52)$

XI. $0 = +\,0·1682 - (15) + (16) - (38) + (39) - (47) + (48)$

XII. $0 = +\,1·9619 + (38) - (46) - (48) + (49) - (50) + (51)$

XIII. $0 = +\,31·621 + 17·1342\,(4) + 43·4918\,(12) - 24·4848\,(18) + 0·1525\,(19) + 24·3323\,(21)$

XIV. $0 = +\,123·635 + 50·2675\,(7) - 79·3501\,(8) - 4·7567\,(17) - 19·7281\,(18) + 24·4848\,(19)$
$+\,10·7787\,(39) - 5·6930\,(40) - 5·0857\,(41)$

XV. $0 = +\,13·244 + 28·6329\,(14) - 15·4709\,(15) + 6·9748\,(26) - 0·7423\,(27) - 6·2325\,(29)$
$+\,5·0857\,(39) + 17·6937\,(41) - 22·7794\,(44)$

XVI. $0 = -\,16·473 - 9·9482\,(6) + 38·1009\,(7) - 28·1527\,(8) - 2·9490\,(17) - 11·4822\,(19)$
$+\,14·4312\,(23) - 7·0620\,(40) + 22·7794\,(41) - 15·7174\,(44)$

XVII. $0 = +\,148·978 - 8·0016\,(3) + 17·1342\,(4) - 0·4581\,(6) - 1·4045\,(9) - 12·8501\,(19)$
$+\,24·3323\,(21) - 11·4822\,(23) - 3·0291\,(28) + 15·7331\,(29) - 12·7040\,(31)$

XVIII. $0 = -\,212·749 - 34·4545\,(6) + 8·4121\,(8) + 26·0424\,(9) + 21·1520\,(19) - 92·1172\,(22)$
$+\,70·9652\,(23) - 26·5953\,(28) - 12·2459\,(29) + 38·8412\,(31)$

XIX. $0 = -25\cdot087 - 11\cdot9829\,(7) + 8\cdot4121\,(8) + 3\cdot5708\,(9) - 3\cdot1665\,(17) + 21\cdot1520\,(19)$
$- 17\cdot9855\,(22) + 9\cdot1763\,(40) - 27\cdot2028\,(41) + 18\cdot0265\,(43)$

XX. $0 = -89\cdot482 - 11\cdot9829\,(7) + 9\cdot4285\,(9) - 83\cdot5292\,(13) + 44\cdot7022\,(15) - 26\cdot3012\,(39)$
$- 27\cdot2028\,(41) + 53\cdot5040\,(43)$

XXI. $0 = -961\cdot998 + 43\cdot3659\,(2) - 53\cdot2301\,(3) + 34\cdot4545\,(6) - 92\cdot1551\,(9) - 9\cdot2639\,(24)$
$+ 12\cdot2459\,(29) - 2\cdot9820\,(31) + 30\cdot7283\,(34) - 153\cdot3575\,(36) + 122\cdot6292\,(37)$

XXII. $0 = -312\cdot602 + 3\cdot1665\,(17) - 74\cdot1317\,(22) + 70\cdot9652\,(23) - 9\cdot2639\,(24) - 26\cdot5953\,(28)$
$+ 35\cdot8592\,(31) + 3\cdot1235\,(33) + 30\cdot7283\,(34) - 33\cdot8518\,(36) - 9\cdot1763\,(40)$
$+ 31\cdot8469\,(43) - 22\cdot6706\,(45)$

XXIII. $0 = -20\cdot997 - 26\cdot5235\,(24) + 51\cdot9210\,(25) - 25\cdot3975\,(26) - 27\cdot0900\,(32) + 17\cdot7993\,(33)$
$+ 9\cdot2907\,(34) + 0\cdot8181\,(44) + 15\cdot1035\,(45) - 15\cdot9216\,(46)$

XXIV. $0 = +215\cdot243 + 67\cdot3411\,(14) - 77\cdot6050\,(15) + 10\cdot2639\,(16) + 4\cdot4356\,(24) - 76\cdot4021\,(26)$
$+ 71\cdot9665\,(27) + 17\cdot7993\,(32) - 28\cdot0599\,(33) + 10\cdot2606\,(34) + 17\cdot5921\,(47)$
$- 35\cdot9789\,(48) + 18\cdot3868\,(49) + 5\cdot4243\,(50) - 10\cdot7276\,(51) + 5\cdot3033\,(52)$

XXV. $0 = +960\cdot464 + 159\cdot7293\,(1) - 39\cdot8062\,(5) - 35\cdot4587\,(10)$

XXVI. $0 = +311\cdot824 + 20\cdot5315\,(7) - 12\cdot2161\,(10) + 88\cdot2097\,(11) - 32\cdot3917\,(15) + 15\cdot8832\,(39)$
$+ 48\cdot5417\,(41) - 64\cdot4249\,(42)$

XXVII. $0 = -47\cdot704 + 20\cdot5315\,(7) - 15\cdot5298\,(8) - 5\cdot0017\,(10) + 8\cdot3614\,(17) - 34\cdot0078\,(19)$
$+ 25\cdot6464\,(20) - 16\cdot3558\,(40) + 48\cdot5417\,(41) - 32\cdot1859\,(42)$

XXVIII. $0 = -84\cdot027 + 8\cdot3614\,(17) - 45\cdot3776\,(20) + 37\cdot0162\,(23) + 17\cdot7536\,(26) - 5\cdot1025\,(28)$
$- 12\cdot6511\,(30) - 16\cdot3558\,(40) + 76\cdot4861\,(42) - 60\cdot1303\,(44)$

XXIX. $0 = -296\cdot540 + 39\cdot1985\,(24) + 17\cdot7536\,(26) - 56\cdot9521\,(30) + 4\cdot5596\,(33) - 85\cdot9662\,(34)$
$+ 81\cdot4066\,(35) + 46\cdot5486\,(42) - 60\cdot1303\,(44) + 13\cdot5817\,(45)$

XXX. $0 = +87\cdot800 - 15\cdot0140\,(2) + 43\cdot6914\,(5) - 20\cdot5315\,(7) + 50\cdot9603\,(10) - 4\cdot5596\,(33)$
$+ 40\cdot4485\,(35) - 35\cdot8889\,(37) - 48\cdot5417\,(41) + 62\cdot1234\,(42) - 13\cdot5817\,(45)$

The logarithmic values of the multipliers are as follow:

Log I = 0·5901368 −
II = 0·2568569 −
III = 9·8976666 −
IV = 0·2428871 +
V = 9·2694805 +
VI = 9·6271300 −
VII = 9·9611520 −
VIII = 9·9761600 −
IX = 0·1091774 −
X = 9·9086995 −

Log XI = 9·1789756 −
XII = 9·8732519 −
XIII = 9·1734659 +
XIV = 7·6475800 −
XV = 7·8060222 +
XVI = 9·0972027 +
XVII = 9·0940712 −
XVIII = 7·4939438 −
XIX = 7·8792245 +
XX = 7·0116055 −

Log XXI = 8·2469789 +
XXII = 8·0343120 +
XXIII = 7·0275058 +
XXIV = 7·7875585 −
XXV = 8·1839522 −
XXVI = 7·6206600 −
XXVII = 8·8460658 −
XXVIII = 8·7100859 −
XXIX = 8·1513407 +
XXX = 8·2339243 +

FIGURE 4.

Notation.

(1)	*H'D*	(2)	*L'D*	(3)	*L'A*	(4)	*K'D*	(5)	*N'D*	(6)	*N'E*	(7)	*N'F*
(8)	*N'A*	(9)	*N'C*	(10)	*N'V'*	(11)	*B'V'*	(12)	*B'A*	(13)	*B'C*	(14)	*V'B'*
(15)	*V'N'*	(16)	*DA*	(17)	*DN'*	(18)	*DL'*	(19)	*DK'*	(20)	*DH'*	(21)	*DE*
(22)	*DF*	(23)	*DM*	(24)	*AB'*	(25)	*AV'*	(26)	*AN'*	(27)	*AL'*	(28)	*AK'*
(29)	*AD*	(30)	*AE*	(31)	*AF*	(32)	*AM*	(33)	*AI*	(34)	*AC*	(35)	*EF*
(36)	*EM*	(37)	*EA*	(38)	*ED*	(39)	*FM*	(40)	*FC*	(41)	*FA*	(42)	*FD*
(43)	*FE*	(44)	*FQ*	(45)	*FI*	(46)	*FP*	(47)	*FR*	(48)	*MC*	(49)	*MA*
(50)	*MD*	(51)	*ME*	(52)	*MF*	(53)	*MI*	(54)	*MP*	(55)	*MR*	(56)	*CB'*
(57)	*CV'*	(58)	*CN'*	(59)	*CA*	(60)	*CF*	(61)	*CM*	(62)	*CI*	(63)	*CR*
(64)	*CT*	(65)	*QR*	(66)	*QP*	(67)	*QI*	(68)	*QF*	(69)	*QS*	(70)	*PC*
(71)	*PM*	(72)	*PF*	(73)	*PI*	(74)	*PQ*	(75)	*PS*	(76)	*PT*	(77)	*PR*
(78)	*RT*	(79)	*RU*	(80)	*RC*	(81)	*RM*	(82)	*RF*	(83)	*RI*	(84)	*RP*
(85)	*RQ*	(86)	*RS*	(87)	*UC*	(88)	*UR*	(89)	*US*	(90)	*UT*	(91)	*TU*
(92)	*TC*	(93)	*TR*	(94)	*TP*	(95)	*TI*	(96)	*TS*	(97)	*ST*	(98)	*SR*
(99)	*SP*	(100)	*SQ*										

Equations.

I. $0 = -10{\cdot}6748 - (1) + (2) - (18) + (20)$

II. $0 = +6{\cdot}3417 + (2) - (4) - (18) + (19)$

III. $0 = +4{\cdot}6191 - (4) + (5) - (17) + (19)$

IV. $0 = +1{\cdot}8441 - (2) + (3) - (16) + (18) - (27) + (29)$

V. $0 = +0{\cdot}8679 - (5) + (8) - (16) + (17) - (26) + (29)$

VI. $0 = +1{\cdot}9983 - (8) + (12) - (24) + (26)$

VII. $0 = +0{\cdot}7171 - (8) + (9) + (26) - (34) - (58) + (59)$

VIII. $0 = -4{\cdot}8379 - (9) + (13) - (56) + (58)$

IX. $0 = -1{\cdot}3201 - (10) + (11) - (14) + (15)$

X. $0 = -2{\cdot}0621 + (16) - (21) - (29) + (30) - (37) + (38)$

XI. $0 = +5{\cdot}7016 - (31) + (34) - (40) + (41) - (59) + (60)$

XII. $0 = -2{\cdot}2003 + (16) - (22) - (29) + (31) - (41) + (42)$

XIII. $0 = +4{\cdot}1603 - (30) + (31) - (35) + (37) - (41) + (43)$

XIV. $0 = +0{\cdot}8399 - (32) + (34) - (48) + (49) - (59) + (61)$

XV. $0 = -1{\cdot}3524 + (16) - (23) - (29) + (32) - (49) + (50)$

XVI. $0 = +7{\cdot}9437 - (30) + (32) - (36) + (37) - (49) + (51)$

XVII. $0 = +2{\cdot}4265 - (31) + (32) - (39) + (41) - (49) + (52)$

XVIII. $0 = +0{\cdot}4656 + (39) - (46) - (52) + (54) - (71) + (72)$

XIX. $0 = +0{\cdot}8404 + (48) - (55) - (61) + (63) - (80) + (81)$

XX. $0 = -1{\cdot}8471 + (40) - (47) - (60) + (63) - (80) + (82)$

XXI. $0 = +0{\cdot}5895 - (54) + (55) + (71) - (77) - (81) + (84)$

XXII. $0 = +0{\cdot}9234 - (44) + (46) - (66) + (68) - (72) + (74)$

XXIII. $0 = +1{\cdot}6096 - (44) + (47) - (65) + (68) - (82) + (85)$

XXIV. $0 = +4{\cdot}3269 + (65) - (69) - (85) + (86) - (98) + (100)$

XXV. $0 = +1{\cdot}8929 - (75) + (77) - (84) + (86) - (98) + (99)$

XXVI. $0 = -5{\cdot}1172 - (63) + (64) - (78) + (80) - (92) + (93)$

XXVII. $0 = -0{\cdot}2006 + (78) - (86) - (93) + (96) - (97) + (98)$

XXVIII. $0 = +0{\cdot}7338 - (76) + (77) + (78) - (84) - (93) + (94)$

XXIX. $0 = -8{\cdot}2069 - (78) + (79) - (88) + (90) - (91) + (93)$

XXX. $0 = -130{\cdot}299 + 6{\cdot}6857\,(2) + 7{\cdot}2406\,(4) + 33{\cdot}7636\,(18) - 175{\cdot}4396\,(19) + 141{\cdot}6760\,(20)$

XXXI. $0 = +76{\cdot}177 + 5{\cdot}3804\,(1) + 18{\cdot}8489\,(5) + 64{\cdot}1035\,(17) - 97{\cdot}8671\,(18) + 33{\cdot}7636\,(20)$

XXXII. $0 = +174{\cdot}626 - 18{\cdot}8489\,(5) - 3{\cdot}4697\,(8) + 15{\cdot}7134\,(16) - 64{\cdot}1035\,(17)$
$+ 48{\cdot}3901\,(18) + 39{\cdot}5491\,(26) - 49{\cdot}0576\,(27) + 9{\cdot}5085\,(29)$

XXXIII. $0 = +47{\cdot}860 + 8{\cdot}0589\,(2) + 17{\cdot}7279\,(3) - 4{\cdot}3330\,(16) + 44{\cdot}9391\,(18)$
$- 40{\cdot}6061\,(19) - 74{\cdot}6251\,(27) + 92{\cdot}3309\,(28) - 17{\cdot}7058\,(29)$

XXXIV. $0 = +219{\cdot}000 + 1{\cdot}0353\,(4) + 34{\cdot}6509\,(12) + 29{\cdot}9784\,(16) - 52{\cdot}3315\,(17)$
$+ 22{\cdot}3531\,(19) + 19{\cdot}8048\,(24) - 18{\cdot}4331\,(26) - 1{\cdot}3717\,(29)$

XXXV. $0 = -26{\cdot}721 - 34{\cdot}6509\,(12) + 3{\cdot}6396\,(13) - 19{\cdot}8048\,(24) + 27{\cdot}9759\,(26)$
$- 8{\cdot}1711\,(34) + 24{\cdot}5649\,(56) - 67{\cdot}6659\,(58) + 43{\cdot}1010\,(59)$

XXXVI. $0 = -144{\cdot}384 + 1{\cdot}6139\,(8) + 16{\cdot}6727\,(9) - 18{\cdot}2866\,(10) + 30{\cdot}2138\,(25)$
$- 20{\cdot}7409\,(26) - 9{\cdot}4729\,(34) - 21{\cdot}3600\,(57) + 34{\cdot}9968\,(58) - 13{\cdot}6368\,(59)$

XXXVII. $0 = +221{\cdot}013 + 16{\cdot}6727\,(9) - 174{\cdot}8898\,(10) - 59{\cdot}3751\,(11) + 14{\cdot}7882\,(13)$
$+ 124{\cdot}9016\,(56) - 159{\cdot}8984\,(57) + 34{\cdot}9968\,(58)$

XXXVIII. $0 = +99{\cdot}158 - 62{\cdot}3830\,(5) + 95{\cdot}3834\,(6) - 33{\cdot}0004\,(8) + 5{\cdot}7351\,(16)$
$+ 7{\cdot}5998\,(17) - 13{\cdot}3349\,(21) + 14{\cdot}2402\,(26) + 35{\cdot}9704\,(29) - 50{\cdot}2106\,(30)$

XXXIX. $0 = -21{\cdot}548 - 9{\cdot}5744\,(16) - 5{\cdot}7351\,(21) + 15{\cdot}3095\,(22) - 10{\cdot}0855\,(35) + 15{\cdot}7690\,(37)$
$- 5{\cdot}6835\,(38) - 8{\cdot}1199\,(41) + 10{\cdot}7672\,(42) - 2{\cdot}6473\,(43)$

XL. $0 = -124{\cdot}706 + 16{\cdot}9344\,(5) - 39{\cdot}7086\,(8) + 22{\cdot}7742\,(9) - 45{\cdot}2879\,(16) + 29{\cdot}9784\,(17)$
$+ 15{\cdot}3095\,(22) + 17{\cdot}4917\,(40) - 28{\cdot}2589\,(41) + 10{\cdot}7672\,(42) + 43{\cdot}1010\,(58)$
$- 70{\cdot}2723\,(59) + 27{\cdot}1713\,(60)$

XLI. $0 = +527{\cdot}853 + 16{\cdot}9344\,(5) - 207{\cdot}8092\,(7) + 190{\cdot}8748\,(8) - 45{\cdot}2879\,(16)$
$+ 29{\cdot}9784\,(17) + 15{\cdot}3095\,(22) - 115{\cdot}1430\,(26) + 115{\cdot}1430\,(31) - 10{\cdot}7672\,(41)$
$+ 10{\cdot}7672\,(42)$

XLII. $0 = -155{\cdot}630 - 16{\cdot}6412\,(39) + 17{\cdot}4917\,(40) - 0{\cdot}8505\,(41) - 11{\cdot}2309\,(48)$
$+ 0{\cdot}7569\,(49) + 10{\cdot}4740\,(52) - 0{\cdot}2240\,(59) + 27{\cdot}1713\,(60) - 26{\cdot}9473\,(61)$

XLIII. $0 = +57{\cdot}239 - 16{\cdot}0023\,(16) - 5{\cdot}7351\,(21) + 21{\cdot}7374\,(23) - 16{\cdot}2994\,(36)$
$+ 21{\cdot}9829\,(37) - 5{\cdot}6835\,(38) - 9{\cdot}1590\,(49) + 11{\cdot}7031\,(50) - 2{\cdot}5441\,(51)$

XLIV. $0 = +367{\cdot}323 + 32{\cdot}7188\,(30) - 132{\cdot}8590\,(31) + 100{\cdot}1402\,(32) + 87{\cdot}7171\,(35)$
$- 97{\cdot}8026\,(36) + 10{\cdot}0855\,(37) - 10{\cdot}4740\,(49) - 32{\cdot}0090\,(51) + 42{\cdot}4830\,(52)$

XLV. $0 = -380{\cdot}963 - 97{\cdot}8691\,(31) + 97{\cdot}1531\,(33) + 0{\cdot}7160\,(34) - 4{\cdot}2651\,(40)$
$+ 34{\cdot}6238\,(41) - 30{\cdot}3587\,(45) - 27{\cdot}1713\,(59) + 84{\cdot}4766\,(60) - 57{\cdot}3053\,(62)$

XLVI. $0 = +443{\cdot}185 - 62{\cdot}5293\,(31) + 178{\cdot}2645\,(32) - 115{\cdot}7352\,(33) + 4{\cdot}3190\,(39)$
$+ 17{\cdot}7213\,(41) - 22{\cdot}0403\,(45) - 71{\cdot}4367\,(49) + 3{\cdot}7229\,(52) + 67{\cdot}7138\,(53)$

XLVII. $0 = -262{\cdot}714 - 100{\cdot}8562\,(31) + 100{\cdot}1402\,(32) + 0{\cdot}7160\,(34) + 33{\cdot}2846\,(40)$
$- 33{\cdot}2846\,(46) - 10{\cdot}4740\,(49) + 15{\cdot}3898\,(52) - 4{\cdot}9158\,(54) - 27{\cdot}1713\,(59)$
$+ 27{\cdot}1713\,(60) - 35{\cdot}4147\,(70) + 79{\cdot}7477\,(71) - 44{\cdot}3330\,(72)$

XLVIII. $0 = -166{\cdot}271 + 4{\cdot}3190\,(39) + 67{\cdot}5042\,(45) - 71{\cdot}8232\,(46) + 3{\cdot}7229\,(52)$
$- 52{\cdot}9236\,(53) + 49{\cdot}2007\,(54) + 20{\cdot}4654\,(71) - 35{\cdot}0091\,(72) + 14{\cdot}5437\,(73)$

XLIX. $0 = -11{\cdot}258 - 100{\cdot}8562\,(31) + 100{\cdot}1402\,(32) + 0{\cdot}7160\,(34) - 10{\cdot}4740\,(49)$
$+ 19{\cdot}7404\,(52) - 9{\cdot}2664\,(55) - 27{\cdot}1713\,(59) + 51{\cdot}9068\,(60) - 24{\cdot}7355\,(63)$
$- 2{\cdot}4543\,(80) + 104{\cdot}4825\,(81) - 102{\cdot}0282\,(82)$

L. $0 = +53{\cdot}087 - 14{\cdot}8242\,(39) - 166{\cdot}2857\,(46) + 181{\cdot}1099\,(47) + 9{\cdot}2664\,(52)$
$+ 112{\cdot}3710\,(54) - 121{\cdot}6374\,(55) - 16{\cdot}7730\,(71) + 28{\cdot}2805\,(72) - 11{\cdot}5075\,(77)$

LI. 0 = − 288·424 − 4·3408 (48) + 35·8154 (53) − 31·4746 (55) + 53·6650 (61)
− 115·6815 (62) + 62·0165 (63) − 7·4236 (80) − 27·4776 (81) + 34·9012 (83)

LII. 0 = + 438·457 + 33·1464 (44) − 166·2857 (46) + 133·1393 (47) + 90·0994 (65)
− 88·2215 (66) − 1·8779 (68) + 28·2805 (72) + 28·5049 (74) − 56·7854 (77)

LIII. 0 = + 524·479 + 134·9228 (44) − 206·7460 (45) + 71·8232 (46) + 12·0494 (66)
− 65·6230 (67) + 53·5736 (68) + 35·0091 (72) − 58·3154 (73) + 23·3063 (74)

LIV. 0 = + 158·662 + 88·2215 (65) − 60·8365 (66) − 27·3850 (69) − 64·0408 (84)
+ 49·5355 (85) + 14·5053 (86) − 47·4335 (98) + 83·3237 (99) − 35·8902 (100)

LV. 0 = − 360·030 + 14·0722 (40) + 33·1464 (44) − 47·2186 (47) + 24·7355 (60)
− 78·2265 (63) + 53·4910 (64) − 45·1216 (65) − 1·8779 (68) + 46·9995 (69)
+ 6·4337 (92) − 41·8863 (93) + 35·4526 (96) − 8·5800 (97) − 6·5306 (98)
+ 15·1106 (100)

LVI. 0 = − 624·395 − 34·3576 (75) + 92·3015 (76) − 57·9439 (77) − 184·6974 (93)
+ 220·1500 (94) − 35·4526 (96) + 8·5800 (97) + 38·8535 (98) − 47·4335 (99)

LVII. 0 = + 226·915 − 37·1348 (62) + 62·0165 (63) − 24·8817 (64) − 78·9687 (78)
− 7·4236 (80) + 86·3923 (83) − 3·4413 (92) + 155·5163 (93) − 152·0750 (95)

LVIII. 0 = + 239·841 + 53·4910 (63) − 53·4910 (64) + 55·0429 (78) − 56·6985 (79)
+ 1·6556 (80) + 208·3694 (87) − 13·8783 (88) − 194·4911 (90) + 238·3808 (91)
− 238·3808 (92)

LIX. 0 = + 177·170 − 27·8166 (78) + 13·4157 (79) + 14·4009 (86) + 39·4210 (88)
− 78·1964 (89) + 38·7754 (90) − 20·1362 (91) − 35·4526 (93) + 55·5888 (96)

The logarithmic values of the multipliers are as follow:

Log I = 9·9386663 +
II = 0·3787853 −
III = 0·1144088 +
IV = 9·7435278 −
V = 9·2522873 −
VI = 0·5247987 +
VII = 0·5528706 −
VIII = 9·8436849 −
IX = 9·7674572 +
X = 8·3200632 +
XI = 0·2555262 −
XII = 9·4065574 +
XIII = 0·2221406 −
XIV = 8·3738972 −
XV = 9·4759462 −
XVI = 9·6251369 −
XVII = 8·6711349 −
XVIII = 0·0007198 +
XIX = 9·2837939 +
XX = 0·1358889 −

Log XXI = 9·9589597 +
XXII = 9·2809485 +
XXIII = 9·8453975 −
XXIV = 9·8077968 −
XXV = 8·9927779 −
XXVI = 9·8048858 −
XXVII = 8·8561226 +
XXVIII = 9·6575788 +
XXIX = 0·5587721 +
XXX = 8·0144600 +
XXXI = 8·4969474 −
XXXII = 8·9203735 −
XXXIII = 7·8900858 +
XXXIV = 8·9216569 −
XXXV = 8·6040323 +
XXXVI = 8·9948085 +
XXXVII = 8·1856824 −
XXXVIII = 7·9752209 −
XXXIX = 8·9521893 +
XL = 8·4700998 −

Log XLI = 8·2993604 −
XLII = 8·6640524 +
XLIII = 8·5129342 +
XLIV = 7·6965464 −
XLV = 7·8363203 −
XLVI = 8·0569139 −
XLVII = 7·9882062 +
XLVIII = 7·3126416 −
XLIX = 7·9843685 +
L = 6·9512656 +
LI = 8·3957517 +
LII = 8·1389918 −
LIII = 6·8526288 −
LIV = 7·8741256 +
LV = 8·3249710 +
LVI = 7·9991345 +
LVII = 7·9967856 −
LVIII = 7·8508316 +
LIX = 8·9251873 −

FIGURE 5.

Notation.

(1)	*DV*	(2)	*DB*	(3)	*DK*	(4)	*EV*	(5)	*EW*	(6)	*EB*	(7)	*EK*
(8)	*FV*	(9)	*FB*	(10)	*FJ*	(11)	*FK*	(12)	*FH*	(13)	*QK*	(14)	*QH*
(15)	*QG*	(16)	*VK*	(17)	*VD*	(18)	*VW*	(19)	*VX*	(20)	*VJ*	(21)	*VB*
(22)	*VK*	(23)	*VF*	(24)	*BE*	(25)	*BV*	(26)	*BW*	(27)	*BX*	(28)	*BJ*
(29)	*BK*	(30)	*KF*	(31)	*KE*	(32)	*KD*	(33)	*KV*	(34)	*KB*	(35)	*KW*
(36)	*KJ*	(37)	*KL*	(38)	*KG*	(39)	*KH*	(40)	*KQ*	(41)	*KI*	(42)	*HF*
(43)	*HK*	(44)	*HL*	(45)	*HN*	(46)	*HO*	(47)	*HG*	(48)	*HQ*	(49)	*WE*
(50)	*WV*	(51)	*WX*	(52)	*WZ*	(53)	*WJ*	(54)	*WK*	(55)	*WB*	(56)	*XB*
(57)	*XV*	(58)	*XW*	(59)	*XY*	(60)	*XZ*	(61)	*XJ*	(62)	*JK*	(63)	*JF*
(64)	*JB*	(65)	*JV*	(66)	*JW*	(67)	*JX*	(68)	*JY*	(69)	*JZ*	(70)	*JO*
(71)	*JN*	(72)	*JG*	(73)	*JL*	(74)	*ZJ*	(75)	*ZW*	(76)	*ZX*	(77)	*ZY*
(78)	*YX*	(79)	*YZ*	(80)	*YJ*	(81)	*LH*	(82)	*LK*	(83)	*LJ*	(84)	*LN*
(85)	*LO*	(86)	*LG*	(87)	*NQ*	(88)	*NH*	(89)	*NL*	(90)	*NJ*	(91)	*NO*
(92)	*NG*	(93)	*GQ*	(94)	*GH*	(95)	*GK*	(96)	*GL*	(97)	*GN*	(98)	*GO*
(99)	*OG*	(100)	*OH*	(101)	*OL*	(102)	*ON*	(103)	*OJ*				

Equations.

I. $0 = - 1{\cdot}0396 - (11) + (13) + (30) - (40)$

II. $0 = + 0{\cdot}5809 - (7) + (11) - (30) + (31)$

III. $0 = + 1{\cdot}5982 - (3) + (11) - (30) + (32)$

IV. $0 = - 4{\cdot}7687 - (1) + (3) + (17) - (22) - (32) + (33)$

V. $0 = - 4{\cdot}4335 - (4) + (7) + (16) - (22) - (31) + (33)$

VI. $0 = + 3{\cdot}4296 - (8) + (11) - (22) + (23) - (30) + (33)$

VII. $0 = + 1{\cdot}4635 - (21) + (22) + (25) - (29) - (33) + (34)$

VIII. $0 = - 2{\cdot}8090 - (6) + (7) + (24) - (29) - (31) + (34)$

IX. $0 = + 0{\cdot}7453 - (20) + (21) - (25) + (28) - (64) + (65)$

X. $0 = - 0{\cdot}2764 - (8) + (10) - (20) + (23) - (63) + (65)$

XI. $0 = - 1{\cdot}6250 - (20) + (22) - (33) + (36) - (62) + (65)$

XII. $0 = + 0{\cdot}5595 - (4) + (5) + (16) - (18) - (49) + (50)$

XIII. $0 = + 1{\cdot}0919 - (18) + (21) - (25) + (26) + (50) - (55)$

XIV. $0 = - 1{\cdot}8643 - (18) + (22) - (33) + (35) + (50) - (54)$

XV. $0 = + 1{\cdot}5999 - (18) + (20) + (50) - (53) - (65) + (66)$

XVI. $0 = + 3{\cdot}7658 - (18) + (19) + (50) - (51) - (57) + (58)$

XVII. $0 = + 0{\cdot}2042 - (26) + (27) - (51) + (55) - (56) + (58)$

XVIII. $0 = + 0{\cdot}4907 - (51) + (53) + (58) - (61) - (66) + (67)$

XIX. $0 = + 2{\cdot}8169 - (60) + (61) - (67) + (69) - (74) + (76)$

XX. $0 = + 3{\cdot}3622 - (52) + (53) - (66) + (69) - (74) + (75)$

XXI. $0 = + 0{\cdot}0328 - (59) + (61) - (67) + (68) + (78) - (80)$

XXII. $0 = - 0{\cdot}0221 - (59) + (60) - (76) + (77) + (78) - (79)$

XXIII. $0 = - 0{\cdot}5017 - (13) + (14) - (39) + (40) + (43) - (48)$

XXIV. $0 = - 4{\cdot}2903 - (12) + (14) + (42) - (48)$

XXV. $0 = - 1{\cdot}3934 - (37) + (39) - (43) + (44) - (81) + (82)$

XXVI. $0 = + 4{\cdot}5389 - (36) + (37) + (62) - (73) - (82) + (83)$

XXVII. $0 = - 0{\cdot}2953 - (44) + (45) + (81) - (84) - (88) + (89)$

XXVIII. $0 = - 1{\cdot}3885 - (71) + (73) - (83) + (84) - (89) + (90)$

XXIX. $0 = + 1{\cdot}5123 - (14) + (15) - (47) + (48) - (93) + (94)$

XXX. $0 = -2{\cdot}1516 - (13) + (15) - (38) + (40) - (93) + (95)$

XXXI. $0 = +0{\cdot}0297 - (44) + (47) + (81) - (86) - (94) + (96)$

XXXII. $0 = -2{\cdot}3421 - (84) + (86) + (89) - (92) - (96) + (97)$

XXXIII. $0 = -1{\cdot}1521 - (70) + (71) - (90) + (91) - (102) + (103)$

XXXIV. $0 = -1{\cdot}5781 - (84) + (85) + (89) - (91) - (101) + (102)$

XXXV. $0 = +0{\cdot}6762 - (45) + (46) + (88) - (91) - (100) + (102)$

XXXVI. $0 = +3{\cdot}0522 - (91) + (92) - (97) + (98) - (99) + (102)$

XXXVII. $0 = +84{\cdot}936 - 50{\cdot}3070\,(3) + 21{\cdot}7576\,(7) - 0{\cdot}8653\,(30) + 17{\cdot}3026\,(31) - 16{\cdot}4373\,(32)$

XXXVIII. $0 = +251{\cdot}917 + 21{\cdot}7576\,(7) + 25{\cdot}5147\,(13) - 18{\cdot}1796\,(30) + 17{\cdot}3026\,(31) + 0{\cdot}8770\,(40)$

XXXIX. $0 = -51{\cdot}879 + 11{\cdot}3254\,(11) + 11{\cdot}6777\,(13) + 9{\cdot}6987\,(30) + 51{\cdot}2182\,(40) - 60{\cdot}9169\,(41)$

XL. $0 = -65{\cdot}337 + 20{\cdot}1358\,(1) + 58{\cdot}5535\,(8) - 68{\cdot}4859\,(16) + 17{\cdot}0643\,(17) + 51{\cdot}4216\,(23)$

XLI. $0 = -7{\cdot}037 + 8{\cdot}9109\,(1) - 50{\cdot}3070\,(3) + 6{\cdot}3395\,(17) + 35{\cdot}8446\,(22) - 42{\cdot}1841\,(23) + 14{\cdot}2236\,(30) - 16{\cdot}4373\,(32) + 2{\cdot}2137\,(33)$

XLII. $0 = -134{\cdot}866 - 33{\cdot}6100\,(4) + 44{\cdot}8879\,(6) - 11{\cdot}2779\,(7) - 3{\cdot}9942\,(16) - 38{\cdot}1266\,(21) + 42{\cdot}1208\,(22) - 21{\cdot}4385\,(31) + 107{\cdot}5900\,(33) - 86{\cdot}1515\,(34)$

XLIII. $0 = +231{\cdot}007 + 93{\cdot}3791\,(1) - 113{\cdot}5149\,(2) - 36{\cdot}2905\,(4) + 33{\cdot}6100\,(6) - 29{\cdot}2666\,(17) + 29{\cdot}2666\,(21) + 8{\cdot}2192\,(24) - 8{\cdot}2192\,(25)$

XLIV. $0 = -4{\cdot}676 + 17{\cdot}2700\,(6) - 11{\cdot}2779\,(7) - 47{\cdot}1615\,(9) + 17{\cdot}9259\,(11) - 1{\cdot}8686\,(30) - 21{\cdot}4385\,(31) + 23{\cdot}3071\,(34)$

XLV. $0 = -198{\cdot}069 - 9{\cdot}5631\,(8) + 38{\cdot}5668\,(10) - 29{\cdot}0037\,(11) - 12{\cdot}6021\,(20) + 48{\cdot}4467\,(22) - 35{\cdot}8446\,(23) - 61{\cdot}2925\,(62) + 68{\cdot}7208\,(63) - 7{\cdot}4283\,(65)$

XLVI. $0 = -222{\cdot}819 + 32{\cdot}9993\,(20) - 75{\cdot}1201\,(21) + 42{\cdot}1208\,(22) + 107{\cdot}5900\,(33) - 133{\cdot}3496\,(34) + 25{\cdot}7596\,(36) + 15{\cdot}5787\,(62) - 84{\cdot}1681\,(64) + 68{\cdot}5894\,(65)$

XLVII. $0 = -201{\cdot}799 + 159{\cdot}6947\,(4) - 193{\cdot}3047\,(5) + 33{\cdot}6100\,(6) + 8{\cdot}2192\,(24) - 33{\cdot}2448\,(25) + 25{\cdot}0256\,(26) - 79{\cdot}4389\,(49) + 67{\cdot}4023\,(50) + 12{\cdot}0366\,(55)$

XLVIII. $0 = +428{\cdot}380 + 82{\cdot}7026\,(33) - 100{\cdot}1618\,(35) + 17{\cdot}4592\,(36) + 22{\cdot}5786\,(50) - 11{\cdot}3308\,(53) - 11{\cdot}2478\,(54) + 7{\cdot}4283\,(62) - 88{\cdot}4120\,(65) + 80{\cdot}9837\,(66)$

XLIX. $0 = +47{\cdot}612 + 15{\cdot}7046\,(18) - 19{\cdot}5385\,(20) + 3{\cdot}8339\,(21) + 25{\cdot}0256\,(25) - 24{\cdot}9083\,(26) - 0{\cdot}1173\,(28) + 34{\cdot}1724\,(64) - 80{\cdot}9837\,(65) + 46{\cdot}8113\,(66)$

L. $0 = +7{\cdot}552 + 0{\cdot}1173\,(26) + 17{\cdot}2924\,(27) - 17{\cdot}4097\,(28) - 16{\cdot}3228\,(51) + 29{\cdot}4582\,(53) - 13{\cdot}1354\,(55) + 14{\cdot}2885\,(56) - 1{\cdot}5218\,(58) - 12{\cdot}7667\,(61)$

LI. $0 = +19{\cdot}039 - 24{\cdot}3433\,(19) + 57{\cdot}3426\,(20) - 32{\cdot}9993\,(21) + 17{\cdot}9163\,(25) + 17{\cdot}2924\,(27) - 35{\cdot}2087\,(28) + 14{\cdot}2885\,(56) - 2{\cdot}7160\,(57) - 11{\cdot}5725\,(61)$

LII. $0 = +63{\cdot}558 - 16{\cdot}3228\,(51) + 81{\cdot}6791\,(52) - 65{\cdot}3563\,(53) - 1{\cdot}5218\,(58) - 45{\cdot}6573\,(60) + 47{\cdot}1791\,(61) - 15{\cdot}1994\,(74) + 19{\cdot}1882\,(75) - 3{\cdot}9888\,(76)$

LIII. $0 = +85{\cdot}230 - 10{\cdot}1834\,(59) + 45{\cdot}6573\,(60) - 35{\cdot}4739\,(61) - 12{\cdot}6799\,(74) + 3{\cdot}9888\,(76) + 8{\cdot}6911\,(77) - 3{\cdot}0543\,(78) - 43{\cdot}8174\,(79) + 46{\cdot}8717\,(80)$

LIV. $0 = -724{\cdot}579 + 33{\cdot}7109\,(11) - 88{\cdot}9464\,(12) - 83{\cdot}5088\,(13) + 68{\cdot}0318\,(14) - 3{\cdot}4232\,(30) + 105{\cdot}8220\,(39) - 102{\cdot}3988\,(40)$

LV. $0 = -180{\cdot}390 + 38{\cdot}5668\,(10) - 72{\cdot}2777\,(11) + 33{\cdot}7109\,(12) + 18{\cdot}4466\,(42) - 34{\cdot}7009\,(43) + 16{\cdot}2543\,(44) - 120{\cdot}3432\,(62) + 68{\cdot}7208\,(63) + 51{\cdot}6224\,(73) + 10{\cdot}2552\,(81) - 8{\cdot}4059\,(82) - 1{\cdot}8493\,(83)$

LVI. $$\begin{aligned} 0 = {} & + 234{\cdot}327 + 10{\cdot}8243\ (36) - 21{\cdot}2596\ (37) + 10{\cdot}4353\ (39) + 16{\cdot}2543\ (43) \\ & - 51{\cdot}8789\ (44) + 35{\cdot}6246\ (45) + 51{\cdot}6224\ (62) + 43{\cdot}8608\ (71) - 95{\cdot}4832\ (73) \\ & + 33{\cdot}4197\ (88) - 40{\cdot}8293\ (89) + 7{\cdot}4096\ (90) \end{aligned}$$

LVII. $$\begin{aligned} 0 = {} & + 146{\cdot}218 + 83{\cdot}5088\ (13) - 83{\cdot}5088\ (14) + 10{\cdot}4353\ (37) - 112{\cdot}8341\ (39) \\ & + 102{\cdot}3988\ (40) + 26{\cdot}0133\ (45) - 26{\cdot}0133\ (48) + 0{\cdot}5674\ (81) + 10{\cdot}2552\ (82) \\ & - 10{\cdot}8226\ (84) + 90{\cdot}2019\ (87) - 123{\cdot}6216\ (88) + 33{\cdot}4197\ (89) \end{aligned}$$

LVIII. $$\begin{aligned} 0 = {} & - 283{\cdot}660 + 31{\cdot}6854\ (44) - 35{\cdot}6246\ (45) + 3{\cdot}9392\ (47) - 33{\cdot}4197\ (88) + 37{\cdot}3551\ (89) \\ & - 3{\cdot}9354\ (92) + 33{\cdot}4122\ (94) - 69{\cdot}0306\ (96) + 35{\cdot}6184\ (97) \end{aligned}$$

LIX. $$\begin{aligned} 0 = {} & - 55{\cdot}719 + 35{\cdot}0374\ (43) - 16{\cdot}2543\ (44) - 18{\cdot}7831\ (47) - 10{\cdot}2552\ (81) \\ & + 29{\cdot}4884\ (82) - 19{\cdot}2332\ (86) + 67{\cdot}8170\ (94) - 146{\cdot}5643\ (95) + 78{\cdot}7473\ (96) \end{aligned}$$

LX. $$\begin{aligned} 0 = {} & + 671{\cdot}869 + 83{\cdot}5088\ (13) - 87{\cdot}8649\ (14) + 4{\cdot}3561\ (15) + 35{\cdot}0207\ (38) \\ & - 137{\cdot}4195\ (39) + 102{\cdot}3988\ (40) + 40{\cdot}1795\ (93) - 107{\cdot}9965\ (94) + 67{\cdot}8170\ (95) \end{aligned}$$

LXI. $$\begin{aligned} 0 = {} & + 177{\cdot}586 + 10{\cdot}8243\ (36) - 43{\cdot}7215\ (37) + 32{\cdot}8972\ (38) + 51{\cdot}6224\ (62) \\ & + 96{\cdot}2750\ (72) - 147{\cdot}8974\ (73) + 44{\cdot}6749\ (83) - 44{\cdot}6749\ (86) + 78{\cdot}7473\ (95) \\ & - 78{\cdot}7473\ (96) \end{aligned}$$

LXII. $$\begin{aligned} 0 = {} & + 4{\cdot}155 - 3{\cdot}9392\ (44) + 92{\cdot}2212\ (46) - 88{\cdot}2820\ (47) - 8{\cdot}3452\ (81) \\ & - 65{\cdot}2049\ (85) + 73{\cdot}5501\ (86) - 16{\cdot}8532\ (99) + 26{\cdot}2765\ (100) - 9{\cdot}4233\ (101) \end{aligned}$$

LXIII. $$\begin{aligned} 0 = {} & - 96{\cdot}851 - 18{\cdot}4206\ (45) + 92{\cdot}2212\ (46) - 73{\cdot}8006\ (47) - 8{\cdot}3472\ (88) \\ & - 43{\cdot}0367\ (91) + 51{\cdot}3839\ (92) - 25{\cdot}6532\ (99) + 26{\cdot}2765\ (100) - 0{\cdot}6233\ (102) \end{aligned}$$

LXIV. $$\begin{aligned} 0 = {} & - 346{\cdot}404 + 188{\cdot}5746\ (70) - 232{\cdot}4354\ (71) + 43{\cdot}8608\ (73) + 2{\cdot}3081\ (83) \\ & - 37{\cdot}4501\ (84) + 35{\cdot}1420\ (85) + 51{\cdot}0461\ (101) - 160{\cdot}3904\ (102) + 109{\cdot}3443\ (103) \end{aligned}$$

The logarithmic values of the multipliers are as follow :

Log I = 0·5293965 +
II = 0·3975379 +
III = 0·7958668 +
IV = 0·7921918 +
V = 0·3036479 +
VI. = 0·6884514 −
VII = 9·9751943 −
VIII = 0·0043347 +
IX = 0·6717515 −
X = 0·6715208 −
XI = 0·5242174 +
XII = 0·1932298 −
XIII = 0·3503383 −
XIV = 9·6264556 +
XV = 0·5704234 +
XVI = 0·1565700 −
XVII = 9·5989273 +
XVIII = 9·0004148 −
XIX = 9·5596608 +
XX = 0·6387752 −
XXI = 0·1818181 −

Log XXII = 0·3916240 +
XXIII = 0·5731195 −
XXIV = 0·8155366 +
XXV = 0·5168457 +
XXVI = 9·9144170 +
XXVII = 9·2693061 +
XXVIII = 0·5913993 +
XXIX = 9·9462862 +
XXX = 0·4163191 −
XXXI = 0·6672112 +
XXXII = 9·9960731 −
XXXIII = 0·7658271 +
XXXIV = 0·7011470 +
XXXV = 9·6687342 +
XXXVI = 0·0256032 −
XXXVII = 9·8460294 +
XXXVIII = 9·8753184 −
XXXIX = 8·4556441 +
XL = 8·8153312 −
XLI = 9·8320881 −
XLII = 8·3789766 +
XLIII = 7·7388931 −

Log XLIV = 8·1403302 −
XLV = 9·6208954 +
XLVI = 8·1044492 −
XLVII = 7·1647008 −
XLVIII = 6·9762179 −
XLIX = 9·2481761 −
L = 9·0767849 +
LI = 9·1691290 −
LII = 8·8525309 −
LIII = 9·0753716 −
LIV = 9·2335784 −
LV = 9·4477501 −
LVI = 8·8866621 −
LVII = 8·0337204 −
LVIII = 8·1593081 +
LIX = 8·8585001 −
LX = 8·9322040 −
LXI = 8·3961392 −
LXII = 8·5850545 +
LXIII = 7·8265083 +
LXIV = 8·4594747 +

FIGURE 6.

Notation.

(1)	*AB*	(2)	*AE*	(3)	*AC*	(4)	*BE*	(5)	*BD*	(6)	*BC*	(7)	*BA*
(8)	*CA*	(9)	*CB*	(10)	*CE*	(11)	*CD*	(12)	*CH*	(13)	*CF*	(14)	*DC*
(15)	*DB*	(16)	*DE*	(17)	*DH*	(18)	*DG*	(19)	*DF*	(20)	*EB*	(21)	*EH*
(22)	*EF*	(23)	*ED*	(24)	*EC*	(25)	*FC*	(26)	*FD*	(27)	*FE*	(28)	*FH*
(29)	*FG*	(30)	*GF*	(31)	*GD*	(32)	*GH*	(33)	*HF*	(34)	*HC*	(35)	*HD*
(36)	*HE*	(37)	*HG*										

Equations.

I. $0 = +3{\cdot}5387 - (1) + (3) - (6) + (7) - (8) + (9)$

II. $0 = +2{\cdot}1970 - (4) + (6) - (9) + (10) + (20) - (24)$

III $0 = -0{\cdot}3500 - (5) + (6) - (9) + (11) - (14) + (15)$

IV. $0 = -0{\cdot}0953 - (10) + (11) - (14) + (16) - (23) + (24)$

V. $0 = -2{\cdot}9738 - (10) + (13) - (22) + (24) - (25) + (27)$

VI. $0 = -0{\cdot}7019 - (11) + (13) + (14) - (19) - (25) + (26)$

VII. $0 = -3{\cdot}7769 - (10) + (12) - (21) + (24) - (34) + (36)$

VIII. $0 = +0{\cdot}6564 - (21) + (22) - (27) + (28) - (33) + (36)$

IX. $0 = -7{\cdot}2969 - (16) + (17) - (21) + (23) - (35) + (36)$

X. $0 = -2{\cdot}8910 - (28) + (29) - (30) + (32) + (33) - (37)$

XI. $0 = +3{\cdot}4623 - (17) + (18) - (31) + (32) + (35) - (37)$

XII. $0 = -48{\cdot}099 + 42{\cdot}1448\,(1) - 102{\cdot}5105\,(2) + 60{\cdot}3657\,(3) + 28{\cdot}4691\,(4) - 16{\cdot}6282\,(6) - 11{\cdot}8409\,(7) - 18{\cdot}9141\,(8) - 9{\cdot}6582\,(9) + 28{\cdot}5723\,(10)$

XIII. $0 = -17{\cdot}897 - 16{\cdot}6282\,(4) + 24{\cdot}8198\,(5) - 8{\cdot}1916\,(6) - 15{\cdot}5474\,(14) + 17{\cdot}2021\,(15) - 1{\cdot}6547\,(16) - 10{\cdot}7552\,(20) - 7{\cdot}2860\,(23) + 18{\cdot}0412\,(24)$

XIV. $0 = +83{\cdot}338 + 19{\cdot}4190\,(10) - 48{\cdot}2364\,(11) + 28{\cdot}8174\,(13) - 1{\cdot}6547\,(14) - 34{\cdot}9142\,(16) + 36{\cdot}5689\,(19) + 12{\cdot}0992\,(25) - 68{\cdot}3803\,(26) + 56{\cdot}2811\,(27)$

XV. $0 = +59{\cdot}415 - 3{\cdot}3787\,(10) + 32{\cdot}1961\,(12) - 28{\cdot}8174\,(13) - 12{\cdot}0992\,(25) + 4{\cdot}5617\,(27) + 7{\cdot}5375\,(28) - 23{\cdot}0799\,(33) + 29{\cdot}8961\,(34) - 6{\cdot}8162\,(36)$

XVI. $0 = +321{\cdot}247 - 25{\cdot}7495\,(16) - 10{\cdot}8194\,(17) + 36{\cdot}5689\,(19) - 68{\cdot}3803\,(26) + 60{\cdot}8428\,(27) + 7{\cdot}5375\,(28) - 23{\cdot}0799\,(33) + 51{\cdot}9724\,(35) - 28{\cdot}8925\,(36)$

XVII. $0 = -124{\cdot}859 + 62{\cdot}1656\,(17) - 94{\cdot}7478\,(18) + 32{\cdot}5822\,(19) - 15{\cdot}7576\,(26) - 74{\cdot}9706\,(28) + 90{\cdot}7282\,(29) - 29{\cdot}5206\,(30) + 14{\cdot}5907\,(31) + 14{\cdot}9299\,(32)$

The logarithmic values of the multipliers are as follow :

Log I = 0·0305141 −
II = 9·9414277 −
III = 9·0169674 +
IV = 0·5720127 −
V = 0·5472919 +
VI = 0·4991391 −

Log VII = 8·8373977 +
VIII = 9·5507719 +
IX = 9·4909562 −
X = 9·9179106 +
XI = 9·5362399 −
XII = 7·8354559 +

Log XIII = 8·5483305 +
XIV = 8·3918345 −
XV = 8·1234760 −
XVI = 8·9044266 −
XVII = 6·1283993 +

FIGURE 7.

Notation.

(1)	*CF*	(2)	*CG*	(3)	*CP*	(4)	*CK*	(5)	*CH*	(6)	*CI*	(7)	*CE*
(8)	*CD*	(9)	*FK*	(10)	*FH*	(11)	*FG*	(12)	*FC*	(13)	*GF*	(14)	*GP*
(15)	*GK*	(16)	*GI*	(17)	*GH*	(18)	*GE*	(19)	*GD*	(20)	*GC*	(21)	*DC*
(22)	*DG*	(23)	*DK*	(24)	*DH*	(25)	*DI*	(26)	*DE*	(27)	*ED*	(28)	*EC*
(29)	*EG*	(30)	*EK*	(31)	*EI*	(32)	*HC*	(33)	*HD*	(34)	*HC*	(35)	*HG*
(36)	*HF*	(37)	*HK*	(38)	*HL*	(39)	*HI*	(40)	*KE*	(41)	*KH*	(42)	*KD*
(43)	*KC*	(44)	*KF*	(45)	*KP*	(46)	*KO*	(47)	*KN*	(48)	*KM*	(49)	*KL*
(50)	*KI*	(51)	*IE*	(52)	*ID*	(53)	*IC*	(54)	*IH*	(55)	*IG*	(56)	*IK*
(57)	*IL*	(58)	*LI*	(59)	*LK*	(60)	*LP*	(61)	*LM*	(62)	*ML*	(63)	*MK*
(64)	*MP*	(65)	*MO*	(66)	*MN*	(67)	*MA*	(68)	*PK*	(69)	*PG*	(70)	*PB*
(71)	*PA*	(72)	*PO*	(73)	*PN*	(74)	*PM*	(75)	*PL*	(76)	*AM*	(77)	*AN*
(78)	*AO*	(79)	*AP*	(80)	*AB*	(81)	*BN*	(82)	*BO*	(83)	*BP*	(84)	*BA*

Equations.

I. $0 = -1{\cdot}7545 - (2) + (8) - (19) + (20) - (21) + (22)$

II. $0 = -0{\cdot}7343 - (18) + (19) - (22) + (26) - (27) + (29)$

III. $0 = +0{\cdot}4095 - (2) + (7) - (18) + (20) - (28) + (29)$

IV. $0 = +4{\cdot}9734 - (1) + (2) - (11) + (12) + (13) - (20)$

V. $0 = +0{\cdot}1711 - (1) + (5) - (10) + (12) - (34) + (36)$

VI. $0 = +1{\cdot}5737 - (2) + (5) - (17) + (20) - (34) + (35)$

VII. $0 = +3{\cdot}8800 - (17) + (19) - (22) + (24) - (33) + (35)$

VIII. $0 = -0{\cdot}6179 - (1) + (4) - (9) + (12) - (43) + (44)$

IX. $0 = -3{\cdot}3731 - (9) + (10) - (36) + (37) - (41) + (44)$

X. $0 = +0{\cdot}9324 - (4) + (8) - (21) + (23) - (42) + (43)$

XI. $0 = +3{\cdot}1264 - (4) + (7) - (28) + (30) - (40) + (43)$

XII. $0 = -0{\cdot}3108 - (30) + (31) + (40) - (50) - (51) + (56)$

XIII. $0 = +3{\cdot}4130 - (37) + (39) + (41) - (50) - (54) + (56)$

XIV. $0 = +3{\cdot}5314 - (16) + (18) - (29) + (31) - (51) + (55)$

XV. $0 = +7{\cdot}6777 - (6) + (7) - (28) + (31) - (51) + (53)$

XVI. $0 = +4{\cdot}8698 - (25) + (26) - (27) + (31) - (51) + (52)$

XVII. $0 = -2{\cdot}7555 - (49) + (50) - (56) + (57) - (58) + (59)$

XVIII. $0 = -0{\cdot}3474 - (45) + (49) - (59) + (60) + (68) - (75)$

XIX. $0 = +0{\cdot}8732 - (2) + (4) - (14) + (20) - (43) + (45) - (68) + (69)$

XX. $0 = -0{\cdot}7505 - (60) + (61) - (62) + (64) - (74) + (75)$

XXI. $0 = -3{\cdot}3194 - (45) + (48) - (63) + (64) + (68) - (74)$

XXII. $0 = +3{\cdot}6755 - (64) + (67) - (71) + (74) - (76) + (79)$

XXIII. $0 = -2{\cdot}2697 - (70) + (71) - (79) + (80) + (83) - (84)$

XXIV. $0 = +34{\cdot}415 + 4{\cdot}1042\,(2) + 0{\cdot}0571\,(7) - 4{\cdot}1613\,(8) + 33{\cdot}1458\,(21) - 28{\cdot}2034\,(22)$
$- 4{\cdot}9424\,(26) + 10{\cdot}4970\,(27) - 36{\cdot}2319\,(28) + 25{\cdot}7349\,(29)$

XXV. $0 = -59{\cdot}621 + 12{\cdot}2004\,(10) - 18{\cdot}5005\,(11) + 6{\cdot}3001\,(12) + 4{\cdot}8239\,(13) + 0{\cdot}7489\,(17)$
$- 5{\cdot}5728\,(20) + 5{\cdot}7622\,(34) - 40{\cdot}6764\,(35) + 34{\cdot}9142\,(36)$

XXVI. $0 = +92{\cdot}753 + 074{\cdot}89\,(17) + 20{\cdot}0543\,(19) - 20{\cdot}8032\,(20) - 37{\cdot}6207\,(21) + 33{\cdot}1458\,(22)$
$+ 4{\cdot}4749\,(24) - 27{\cdot}5208\,(33) + 68{\cdot}1972\,(34) - 40{\cdot}6764\,(35)$

XXVII. $0 = -52{\cdot}422 - 0{\cdot}0571\,(2) - 37{\cdot}1263\,(5) + 37{\cdot}1834\,(7) + 33{\cdot}2372\,(17) - 45{\cdot}5493\,(18)$
$+ 12{\cdot}3121\,(20) + 12{\cdot}6532\,(32) - 16{\cdot}8803\,(34) + 4{\cdot}2271\,(35)$

$$\begin{aligned}\text{XXVIII.}\quad 0 = &- 201{\cdot}136 - 4{\cdot}6427\,(1) + 108{\cdot}7641\,(4) - 104{\cdot}1214\,(5) - 72{\cdot}7759\,(9) + 78{\cdot}4432\,(10)\\ &- 5{\cdot}6673\,(12) + 19{\cdot}0165\,(34) - 4{\cdot}0905\,(36) - 14{\cdot}9260\,(37)\end{aligned}$$

$$\begin{aligned}\text{XXIX.}\quad 0 = &- 120{\cdot}804 - 1{\cdot}3286\,(21) - 3{\cdot}1463\,(23) + 4{\cdot}4749\,(24) - 27{\cdot}5208\,(33) + 8{\cdot}5043\,(34)\\ &+ 19{\cdot}0165\,(37) - 28{\cdot}4756\,(41) + 37{\cdot}2358\,(42) - 8{\cdot}7602\,(43)\end{aligned}$$

$$\begin{aligned}\text{XXX.}\quad 0 = &- 6{\cdot}202 + 7{\cdot}7881\,(4) - 24{\cdot}2292\,(7) + 16{\cdot}4411\,(8) - 3{\cdot}1463\,(21) - 24{\cdot}6575\,(23)\\ &+ 27{\cdot}8038\,(26) - 20{\cdot}2258\,(27) + 3{\cdot}9057\,(28) + 16{\cdot}3201\,(30)\end{aligned}$$

$$\begin{aligned}\text{XXXI.}\quad 0 = &+ 176{\cdot}488 + 4{\cdot}6427\,(1) - 18{\cdot}1977\,(2) + 13{\cdot}5550\,(4) + 37{\cdot}0464\,(9) - 42{\cdot}7137\,(11)\\ &+ 5{\cdot}6673\,(12) + 31{\cdot}7242\,(13) - 39{\cdot}6663\,(15) + 7{\cdot}9421\,(20)\end{aligned}$$

$$\begin{aligned}\text{XXXII.}\quad 0 = &+ 80{\cdot}936 - 22{\cdot}7400\,(15) + 85{\cdot}1985\,(16) - 62{\cdot}4585\,(17) + 5{\cdot}5314\,(35) + 7{\cdot}1595\,(37)\\ &- 12{\cdot}6909\,(39) - 13{\cdot}7473\,(54) + 20{\cdot}8744\,(55) - 7{\cdot}1271\,(56)\end{aligned}$$

$$\begin{aligned}\text{XXXIII.}\quad 0 = &+ 24{\cdot}747 + 27{\cdot}8389\,(16) - 33{\cdot}2372\,(17) + 5{\cdot}3983\,(18) - 6{\cdot}8318\,(32) - 4{\cdot}2271\,(35)\\ &+ 11{\cdot}0589\,(39) + 9{\cdot}4500\,(51) - 28{\cdot}1213\,(54) + 18{\cdot}6713\,(55)\end{aligned}$$

$$\begin{aligned}\text{XXXIV.}\quad 0 = &- 82{\cdot}191 - 37{\cdot}1263\,(5) + 41{\cdot}9355\,(6) - 4{\cdot}8092\,(7) + 5{\cdot}8214\,(32) - 16{\cdot}8803\,(34)\\ &+ 11{\cdot}0589\,(39) - 10{\cdot}3158\,(51) + 38{\cdot}4371\,(53) - 28{\cdot}1213\,(54)\end{aligned}$$

$$\begin{aligned}\text{XXXV.}\quad 0 = &+ 149{\cdot}706 + 29{\cdot}9170\,(23) - 57{\cdot}7208\,(25) + 27{\cdot}8038\,(26) - 20{\cdot}2258\,(27) - 26{\cdot}8020\,(30)\\ &+ 47{\cdot}0278\,(31) - 1{\cdot}3550\,(51) - 0{\cdot}4618\,(52) + 1{\cdot}8168\,(56)\end{aligned}$$

$$\begin{aligned}\text{XXXVI.}\quad 0 = &- 299{\cdot}782 + 21{\cdot}4380\,(37) - 103{\cdot}2242\,(38) + 81{\cdot}7862\,(39) - 5{\cdot}1560\,(41) + 30{\cdot}6632\,(49)\\ &- 25{\cdot}5072\,(50) - 32{\cdot}5693\,(54) + 0{\cdot}0953\,(56) + 32{\cdot}4740\,(57)\end{aligned}$$

$$\begin{aligned}\text{XXXVII.}\quad 0 = &+ 252{\cdot}453 + 87{\cdot}0842\,(14) - 172{\cdot}2827\,(15) + 85{\cdot}1985\,(16) + 20{\cdot}8744\,(55) - 20{\cdot}7791\,(56)\\ &- 0{\cdot}0953\,(57) + 17{\cdot}5415\,(58) - 21{\cdot}9355\,(59) + 4{\cdot}3940\,(60) - 70{\cdot}2600\,(68)\\ &+ 49{\cdot}6645\,(69) + 20{\cdot}5955\,(75)\end{aligned}$$

$$\begin{aligned}\text{XXXVIII.}\quad 0 = &+ 257{\cdot}239 + 46{\cdot}0857\,(2) - 121{\cdot}6034\,(3) + 75{\cdot}5177\,(4) + 71{\cdot}2701\,(14) - 28{\cdot}9345\,(15)\\ &- 42{\cdot}3356\,(20) + 21{\cdot}2790\,(68) - 21{\cdot}2790\,(69)\end{aligned}$$

$$\begin{aligned}\text{XXXIX.}\quad 0 = &+ 31{\cdot}935 + 10{\cdot}6707\,(45) - 24{\cdot}7897\,(48) + 14{\cdot}1190\,(49) + 4{\cdot}3940\,(59) - 17{\cdot}7166\,(60)\\ &+ 13{\cdot}3226\,(61) - 0{\cdot}9964\,(62) - 10{\cdot}0834\,(63) + 11{\cdot}0798\,(64)\end{aligned}$$

$$\begin{aligned}\text{XL.}\quad 0 = &- 181{\cdot}931 + 39{\cdot}2217\,(45) - 137{\cdot}3120\,(47) + 98{\cdot}0903\,(48) - 25{\cdot}0260\,(63)\\ &- 5{\cdot}4395\,(64) + 30{\cdot}4655\,(66) - 8{\cdot}9854\,(68) + 36{\cdot}0668\,(73) - 27{\cdot}0814\,(74)\end{aligned}$$

$$\begin{aligned}\text{XLI.}\quad 0 = &+ 269{\cdot}689 + 10{\cdot}5151\,(70) - 51{\cdot}4287\,(71) + 40{\cdot}9136\,(73) + 82{\cdot}9103\,(77) - 77{\cdot}7681\,(79)\\ &- 5{\cdot}1422\,(80) - 95{\cdot}5760\,(81) + 60{\cdot}6485\,(83) + 34{\cdot}9275\,(84)\end{aligned}$$

$$\begin{aligned}\text{XLII.}\quad 0 = &- 157{\cdot}664 + 5{\cdot}4395\,(64) - 61{\cdot}9665\,(66) + 56{\cdot}5270\,(67) + 51{\cdot}4287\,(71) - 78{\cdot}5101\,(73)\\ &+ 27{\cdot}0814\,(74) + 136{\cdot}3295\,(76) - 214{\cdot}0976\,(77) + 77{\cdot}7681\,(79)\end{aligned}$$

$$\begin{aligned}\text{XLIII.}\quad 0 = &- 55{\cdot}803 + 53{\cdot}9713\,(45) - 115{\cdot}0126\,(46) + 61{\cdot}0413\,(48) - 18{\cdot}8367\,(63) - 8{\cdot}7616\,(64)\\ &+ 27{\cdot}5983\,(65) - 18{\cdot}6553\,(68) + 32{\cdot}6999\,(72) - 14{\cdot}0446\,(74)\end{aligned}$$

$$\begin{aligned}\text{XLIV.}\quad 0 = &- 6{\cdot}682 - 14{\cdot}0430\,(64) - 4{\cdot}0615\,(65) + 18{\cdot}1045\,(67) + 12{\cdot}0933\,(71) - 28{\cdot}0876\,(72)\\ &+ 15{\cdot}9943\,(74) - 10{\cdot}0435\,(76) - 37{\cdot}4061\,(78) + 47{\cdot}4496\,(79)\end{aligned}$$

$$\begin{aligned}\text{XLV.}\quad 0 = &- 56{\cdot}796 - 13{\cdot}6990\,(70) + 24{\cdot}0538\,(71) - 10{\cdot}3548\,(72) - 13{\cdot}5021\,(78) + 0{\cdot}5241\,(79)\\ &+ 12{\cdot}9780\,(80) + 3{\cdot}4614\,(82) + 10{\cdot}6774\,(83) - 14{\cdot}1388\,(84)\end{aligned}$$

The logarithmic values of the multipliers are as follow:

Log I = 0·6886588 −
II = 0·2222725 −
III = 0·8859485 +
IV = 9·8524366 −
V = 0·3953496 −
VI = 0·5667297 −

Log VII = 0·0029665 −
VIII = 0·5218201 +
IX = 0·4905806 −
X = 0·3714597 +
XI = 0·4282211 +
XII = 0·6933756 +

Log XIII = 0·1601638 −
XIV = 9·2698267 −
XV = 0·5070954 −
XVI = 0·3996050 −
XVII = 0·6390890 +
XVIII = 0·4311153 +

Log XIX = 9·0972469 −
XX = 0·0358242 +
XXI = 9·6121636 −
XXII = 0·1457979 −
XXIII = 0·3465679 −
XXIV = 9·3295629 −
XXV = 8·1454820 +
XXVI = 9·1074823 −
XXVII = 9·1854048 −

Log XXVIII = 7·7234285 +
XXIX = 8·9598740 +
XXX = 8·9665120 +
XXXI = 8·0221754 +
XXXII = 7·9319356 −
XXXIII = 7·6570157 +
XXXIV = 8·6962025 +
XXXV = 8·0279055 +
XXXVI = 7·9125576 +

Log XXXVII = 7·4796287 −
XXXVIII = 6·9322992 −
XXXIX = 7·5478818 −
XL = 6·9774635 +
XLI = 7·6008689 −
XLII = 7·6172851 +
XLIII = 8·4265598 +
XLIV = 8·6292516 −
XLV = 9·3017605 +

FIGURE 8.

Notation.

(1)	*US*	(2)	*UR*	(3)	*UT*	(4)	*TU*	(5)	*TS*	(6)	*TN*	(7)	*TR*
(8)	*TQ*	(9)	*RU*	(10)	*RS*	(11)	*RN*	(12)	*RM*	(13)	*RQ*	(14)	*RT*
(15)	*SN*	(16)	*SM*	(17)	*SQ*	(18)	*SR*	(19)	*ST*	(20)	*SU*	(21)	*QT*
(22)	*QR*	(23)	*QS*	(24)	*QN*	(25)	*QM*	(26)	*QP*	(27)	*QO*	(28)	*PR*
(29)	*PS*	(30)	*PM*	(31)	*PK*	(32)	*PO*	(33)	*PQ*	(34)	*OP*	(35)	*ON*
(36)	*OK*	(37)	*OH*	(38)	*OC*	(39)	*OQ*	(40)	*NK*	(41)	*NM*	(42)	*NO*
(43)	*NP*	(44)	*NQ*	(45)	*NR*	(46)	*NT*	(47)	*NS*	(48)	*MS*	(49)	*MN*
(50)	*MK*	(51)	*MH*	(52)	*MF*	(53)	*ML*	(54)	*MC*	(55)	*MP*	(56)	*MQ*
(57)	*MR*	(58)	*LM*	(59)	*LK*	(60)	*LH*	(61)	*LF*	(62)	*KN*	(63)	*KG*
(64)	*KH*	(65)	*KF*	(66)	*KE'*	(67)	*KL*	(68)	*KO*	(69)	*KP*	(70)	*KM*
(71)	*HK*	(72)	*HG*	(73)	*HF*	(74)	*HE'*	(75)	*HC*	(76)	*HO*	(77)	*HL*
(78)	*HM*	(79)	*CO*	(80)	*CM*	(81)	*CH*	(82)	*E'K*	(83)	*EH*	(84)	*EF*
(85)	*EG*	(86)	*A'F*	(87)	*A'G*	(88)	*FH*	(89)	*FG*	(90)	*FA'*	(91)	*FE'*

Equations.

I. 0 = − 6·3841 − (84) + (86) − (90) + (91)
II. 0 = + 6·3356 − (73) + (74) − (83) + (84) + (88) − (91)
III. 0 = + 1·4183 − (74) + (75) − (81) + (83)
IV. 0 = + 4·8551 − (64) + (66) + (71) − (74) − (82) + (83)
V. 0 = + 3·1818 − (50) + (51) − (64) + (70) + (71) − (78)
VI. 0 = − 2·7757 − (51) + (54) − (75) + (78) − (80) + (81)
VII. 0 = − 0·0156 − (50) + (53) − (58) + (59) − (67) + (70)
VIII. 0 = − 3·8351 − (51) + (53) − (58) + (60) − (77) + (78)
IX. 0 = − 1·3457 − (37) + (38) − (75) + (76) − (79) + (81)
X. 0 = − 5·2424 − (36) + (37) − (64) + (68) + (71) − (76)
XI. 0 = − 3·5667 − (31) + (32) − (34) + (36) − (68) + (69)
XII. 0 = + 4·2114 − (30) + (31) − (50) + (55) − (69) + (70)
XIII. 0 = − 0·1076 − (35) + (36) − (40) + (42) + (62) − (68)
XIV. 0 = + 0·9718 − (29) + (31) − (40) + (43) + (62) − (69)
XV. 0 = + 0·9711 − (29) + (30) − (41) + (43) + (49) − (55)
XVI. 0 = + 1·9638 − (24) + (25) − (41) + (44) + (49) − (56)
XVII. 0 = + 2·1715 − (24) + (26) + (29) − (33) − (43) + (44)
XVIII. 0 = + 2·2244 − (24) + (27) + (35) − (39) − (42) + (44)

XIX. $0 = -4.3345 - (15) + (17) - (23) + (24) - (44) + (47)$

XX. $0 = -2.3785 - (16) + (17) - (23) + (25) + (48) - (56)$

XXI. $0 = +0.2735 - (10) + (13) - (17) + (18) - (22) + (23)$

XXII. $0 = -3.7085 - (11) + (13) - (22) + (24) - (44) + (45)$

XXIII. $0 = +0.9382 - (12) + (13) - (22) + (25) - (56) + (57)$

XXIV. $0 = -0.4239 - (6) + (8) - (21) + (24) - (44) + (46)$

XXV. $0 = +0.9412 - (7) + (8) - (13) + (14) - (21) + (22)$

XXVI. $0 = -0.6181 - (5) + (8) - (17) + (19) - (21) + (23)$

XXVII. $0 = +4.6412 - (1) + (3) - (4) + (5) - (19) + (20)$

XXVIII. $0 = -0.7339 - (1) + (2) - (9) + (10) - (18) + (20)$

XXIX. $0 = -1067.450 + 465.4882\,(73) - 516.9380\,(74) + 51.4498\,(75) + 21.5606\,(81) + 256.5430\,(86) - 21.1027\,(88) + 58.5673\,(90) - 37.4646\,(91)$

XXX. $0 = -351.669 - 46.2719\,(84) - 103.7048\,(85) + 105.6050\,(86) + 17.6031\,(87) + 83.8605\,(89) - 142.4278\,(90) + 58.5673\,(91)$

XXXI. $0 = +117.029 - 0.7943\,(72) + 13.2034\,(73) - 12.4091\,(74) - 0.9048\,(83) + 23.1010\,(84) - 22.1962\,(85) - 0.0832\,(88) + 21.1859\,(89) - 21.1027\,(91)$

XXXII. $0 = -1153.888 + 195.1558\,(63) - 204.8161\,(64) + 9.6603\,(66) + 105.4196\,(71) - 130.9236\,(72) + 25.5040\,(74) + 65.6416\,(82) - 79.0185\,(83) + 13.3769\,(85)$

XXXIII. $0 = +285.824 - 86.7917\,(64) + 132.7318\,(65) + 45.9401\,(66) + 82.0482\,(71) - 82.0482\,(73) - 79.0185\,(82) + 102.1195\,(83) - 23.1010\,(84) - 21.1027\,(88) + 21.1027\,(91)$

XXXIV. $0 = +62.979 + 9.2984\,(50) - 30.5948\,(51) + 21.2964\,(52) + 50.6836\,(64) - 51.8226\,(65) + 1.1390\,(70) - 27.9901\,(71) + 22.6832\,(73) + 5.3069\,(78)$

XXXV. $0 = -640.637 - 34.1758\,(50) + 48.3181\,(51) - 14.1423\,(54) - 53.3815\,(64) + 45.9401\,(66) + 7.4414\,(70) - 21.3596\,(80) + 42.9202\,(81) + 79.0185\,(82) - 88.1402\,(83)$

XXXVI. $0 = +59.677 - 26.5720\,(50) + 34.1758\,(51) - 7.6038\,(53) - 8.0675\,(58) + 27.0957\,(59) - 19.0282\,(60) - 15.5793\,(71) - 10.5150\,(77) + 26.0943\,(78)$

XXXVII. $0 = -136.909 + 5.7012\,(59) + 13.5404\,(60) - 19.2416\,(61) - 50.6836\,(64) + 70.9582\,(65) - 20.2746\,(67) + 27.9901\,(71) - 31.8603\,(73) + 3.8702\,(77)$

XXXVIII. $0 = -717.203 - 48.5849\,(36) + 63.3146\,(37) - 14.7297\,(38) - 45.1017\,(64) + 45.9401\,(66) - 0.8384\,(68) - 5.3958\,(79) + 26.9564\,(81) + 79.0185\,(82) - 88.1402\,(83)$

XXXIX. $0 = -44.447 - 75.1583\,(30) + 81.7779\,(31) - 6.6196\,(32) + 4.7699\,(34) + 43.8150\,(36) - 48.5849\,(37) - 66.9666\,(50) + 34.1758\,(51) + 32.7908\,(55) - 17.9480\,(71) - 8.1463\,(76) + 26.0943\,(78)$

XL. $0 = -47.010 - 27.1218\,(29) + 33.7414\,(31) - 6.6196\,(32) + 4.7699\,(34) + 27.5878\,(35) - 32.3577\,(36) - 1.8584\,(40) + 10.3287\,(42) - 8.4703\,(43)$

XLI. $0 = -89.124 + 27.1218\,(29) - 75.1583\,(30) + 48.0365\,(31) + 11.0985\,(40) - 19.5688\,(41) + 8.4703\,(43) - 5.3016\,(49) - 27.4892\,(50) + 32.7908\,(55)$

XLII. $0 = +48.880 - 2.7642\,(29) + 11.9877\,(30) - 9.2235\,(33) + 23.1725\,(41) + 54.0939\,(43) - 30.9214\,(44) + 7.8978\,(49) - 42.6529\,(55) + 34.7551\,(56)$

XLIII. $0 = -122.032 + 2.7642\,(29) + 39.8441\,(32) - 42.6083\,(33) - 48.6486\,(34) + 0.4365\,(35) + 48.2121\,(39) + 44.1761\,(42) - 54.0939\,(43) + 9.9178\,(44)$

XLIV. $0 = -82.232 + 30.3236\,(15) - 47.6210\,(16) + 17.2974\,(17) + 9.5929\,(23) - 41.0035\,(24) + 31.4106\,(25) + 26.3437\,(41) - 23.1725\,(44) - 3.1712\,(47)$

XLV. 0 = + 196·795 − 6·4951 (15) − 52·7217 (17) + 59·2168 (18) + 57·3804 (22) − 72·3734 (23) + 14·9930 (24) + 36·4019 (44) − 79·0842 (45) + 42·6823 (47)

XLVI. 0 = − 37·887 + 11·9320 (22) − 14·9930 (24) + 3·0610 (25) + 6·7170 (41) − 36·4019 (44) + 29·6849 (45) + 6·4606 (49) + 27·3238 (56) − 33·7844 (57)

XLVII. 0 = + 21·740 + 13·2407 (22) − 3·0610 (25) − 10·1797 (26) − 30·1878 (28) + 2·9960 (30) + 27·1918 (33) + 10·3190 (55) − 27·3238 (56) + 17·0048 (57)

XLVIII. 0 = − 43·110 − 12·8515 (6) + 33·6551 (7) − 20·8036 (8) − 1·9931 (11) + 15·9507 (13) − 13·9576 (14) − 13·3637 (44) + 36·4019 (45) − 23·0382 (46)

XLIX. 0 = − 19·349 + 6·0816 (5) − 12·8515 (6) + 6·7699 (8) + 1·7045 (15) − 11·3169 (17) + 9·6124 (19) + 8·9978 (44) − 23·0382 (46) + 14·0404 (47)

L. 0 = + 43·770 + 26·2108 (1) − 55·7143 (2) + 29·5035 (3) − 14·5864 (4) − 23·5003 (5) + 38·0867 (7) + 12·9062 (18) − 22·0387 (19) + 9·1325 (20)

The logarithmic values of the multipliers are as follow:

Log I = 0·4283263 −
II = 9·2907821 +
III = 9·3647971 +
IV = 9·1288847 −
V = 0·6918108 −
VI = 9·8123425 −
VII = 0·5130646 +
VIII = 9·7523412 +
IX = 0·0803963 +
X = 0·2924153 +
XI = 0·3597306 +
XII = 0·0414945 −
XIII = 9·9617855 −
XIV = 0·4727129 +
XV = 0·4883180 +
XVI = 9·9880129 −
XVII = 8·7966825 −

Log XVIII = 9·5706921 −
XIX = 0·6358501 +
XX = 0·1484740 −
XXI = 0·3449634 +
XXII = 9·5830747 +
XXIII = 0·0979125 −
XXIV = 0·3270535 −
XXV = 9·5382508 +
XXVI = 9·6299985 +
XXVII = 0·5933206 −
XXVIII = 0·4159666 +
XXIX = 7·7727637 +
XXX = 8·3243628 +
XXXI = 8·9641150 −
XXXII = 7·3732306 +
XXXIII = 7·9726905 +
XXXIV = 8·6064233 −

Log XXXV = 8·5296897 +
XXXVI = 8·8213057 −
XXXVII = 8·0291041 −
XXXVIII = 8·3178675 −
XXXIX = 8·6176608 −
XL = 8·0537098 −
XLI = 9·1080847 +
XLII = 9·0457163 −
XLIII = 8·5327901 +
XLIV = 8·6082128 +
XLV = 8·6011666 −
XLVI = 8·3134690 +
XLVII = 7·7165618 +
XLVIII = 8·6793920 −
XLIX = 9·3563810 +
L = 8·6128918 +

FIGURE 9.

Notation.

(1)	*A′W*	(2)	*CW*	(3)	*CD*	(4)	*CX*	(5)	*D′X*	(6)	*D′W*	(7)	*D′H*
(8)	*D′Z*	(9)	*D′Y*	(10)	*D′D*	(11)	*B′Y*	(12)	*B′D*	(13)	*WQ*	(14)	*WP*
(15)	*WO*	(16)	*WA′*	(17)	*WC*	(18)	*WD′*	(19)	*WY*	(20)	*WD*	(21)	*WX*
(22)	*OW*	(23)	*OD*	(24)	*OX*	(25)	*PW*	(26)	*PD*	(27)	*PX*	(28)	*PC*
(29)	*PV*	(30)	*QW*	(31)	*QD*	(32)	*QX*	(33)	*QV*	(34)	*XV*	(35)	*XQ*
(36)	*XP*	(37)	*XO*	(38)	*XW*	(39)	*XC*	(40)	*XD′*	(41)	*XB′*	(42)	*XD*
(43)	*XB*	(44)	*XC*	(45)	*DC*	(46)	*DX*	(47)	*DQ*	(48)	*DP*	(49)	*DO*
(50)	*DW*	(51)	*DC*	(52)	*DD′*	(53)	*DB′*	(54)	*DZ*	(55)	*DY*	(56)	*DA*
(57)	*DB*	(58)	*YB′*	(59)	*YZ*	(60)	*YA*	(61)	*YB*	(62)	*ZY*	(63)	*ZD*
(64)	*ZD′*	(65)	*ZB′*	(66)	*AB*	(67)	*AD*	(68)	*AY*	(69)	*AD′*	(70)	*AB′*
(71)	*VT*	(72)	*VR*	(73)	*VQ*	(74)	*VP*	(75)	*VX*	(76)	*VC*	(77)	*CV*
(78)	*CX*	(79)	*CD*	(80)	*CB*	(81)	*BC*	(82)	*BX*	(83)	*BD*	(84)	*BY*
(85)	*BA*	(86)	*XV*	(87)	*TV*								

Equations.

I. $0 = -0{\cdot}8266 - (13) + (14) - (25) + (30)$

II. $0 = -2{\cdot}2188 - (13) + (15) - (22) + (30)$

III. $0 = -0{\cdot}2931 - (2) - (15) + (17) + (22)$

IV. $0 = +0{\cdot}8944 + (1) - (6) - (16) + (18)$

V. $0 = +2{\cdot}9716 + (2) - (6) - (17) + (18)$

VI. $0 = +0{\cdot}8860 - (29) + (33) - (73) + (74)$

VII. $0 = +2{\cdot}7166 - (33) - (72) + (73) + (86)$

VIII. $0 = +0{\cdot}8381 - (33) - (71) + (73) + (87)$

IX. $0 = +0{\cdot}6801 - (32) + (33) - (34) + (35) - (73) + (75)$

X. $0 = -0{\cdot}6700 - (27) + (29) - (34) + (36) - (74) + (75)$

XI. $0 = +1{\cdot}8604 - (24) + (32) - (35) + (37)$

XII. $0 = +0{\cdot}8109 + (13) - (21) - (30) + (32) - (35) + (38)$

XIII. $0 = +0{\cdot}9078 + (2) - (4) - (17) + (21) - (38) + (39)$

XIV. $0 = +1{\cdot}4709 - (5) + (6) - (18) + (21) - (38) + (40)$

XV. $0 = -0{\cdot}8129 + (6) - (10) - (18) + (20) - (50) + (52)$

XVI. $0 = +0{\cdot}6301 + (3) - (12) - (51) + (53)$

XVII. $0 = +3{\cdot}3889 + (2) - (3) - (17) + (20) - (50) + (51)$

XVIII. $0 = -0{\cdot}1595 - (20) + (21) - (38) + (42) - (46) + (50)$

XIX. $0 = -0{\cdot}6965 + (13) - (20) - (30) + (31) - (47) + (50)$

XX. $0 = -1{\cdot}5510 + (14) - (20) - (25) + (26) - (48) + (50)$

XXI. $0 = -3{\cdot}9183 + (15) - (20) - (22) + (23) - (49) + (50)$

XXII. $0 = -1{\cdot}6297 + (34) - (44) - (75) + (76) - (77) + (78)$

XXIII. $0 = +1{\cdot}1807 - (42) + (44) - (45) + (46) - (78) + (79)$

XXIV. $0 = -0{\cdot}5549 - (42) + (43) + (46) - (57) - (82) + (83)$

XXV. $0 = +1{\cdot}0435 + (45) - (57) - (79) + (80) - (81) + (83)$

XXVI. $0 = -0{\cdot}0792 - (56) + (57) - (66) + (67) - (83) + (85)$

XXVII. $0 = +1{\cdot}7336 - (7) + (10) - (52) + (56) - (67) + (69)$

XXVIII. $0 = -1{\cdot}8601 - (60) + (61) - (66) + (68) - (84) + (85)$

XXIX. $0 = +3{\cdot}6860 - (11) + (12) - (53) + (57) + (58) - (61) - (83) + (84)$

XXX. $0 = +2{\cdot}9014 - (8) + (10) - (52) + (54) - (63) + (64)$

XXXI. $0 = +2{\cdot}1660 - (54) + (57) + (59) - (61) - (62) + (63) - (83) + (84)$

XXXII. $0 = +86{\cdot}514 - 8{\cdot}5491\,(13) + 53{\cdot}1363\,(14) - 44{\cdot}5872\,(15) + 22{\cdot}7623\,(22) - 22{\cdot}1428\,(25)$

XXXIII. $0 = +256{\cdot}266 + 92{\cdot}0671\,(2) + 44{\cdot}5872\,(13) - 9{\cdot}5767\,(15) - 35{\cdot}0105\,(17) + 66{\cdot}8183\,(30)$

XXXIV. $0 = -37{\cdot}816 - 12{\cdot}9640\,(1) - 2{\cdot}8037\,(2) - 23{\cdot}8550\,(16) + 30{\cdot}4620\,(17) - 6{\cdot}6070\,(18)$

XXXV. $0 = -172{\cdot}284 + 19{\cdot}1158\,(6) - 35{\cdot}0105\,(15) + 4{\cdot}5485\,(17) + 30{\cdot}4620\,(18) + 64{\cdot}2618\,(22)$

XXXVI. $0 = -17{\cdot}091 - 10{\cdot}1133\,(29) - 10{\cdot}5926\,(33) - 17{\cdot}2894\,(72) + 28{\cdot}1519\,(73) - 10{\cdot}8625\,(74)$

XXXVII. $0 = -125{\cdot}794 + 10{\cdot}8263\,(71) - 28{\cdot}1519\,(72) + 17{\cdot}3256\,(73) - 42{\cdot}2273\,(86) + 45{\cdot}2854\,(87)$

XXXVIII. $0 = +12{\cdot}880 + 15{\cdot}3195\,(27) - 23{\cdot}7731\,(29) - 22{\cdot}0890\,(32) + 16{\cdot}9920\,(33) + 0{\cdot}3411\,(73)$
$- 10{\cdot}6576\,(74) + 10{\cdot}3165\,(75)$

XXXIX. $0 = -14{\cdot}424 - 9{\cdot}4295\,(24) + 8{\cdot}4536\,(27) - 6{\cdot}3036\,(35) + 29{\cdot}5363\,(36) - 23{\cdot}2327\,(37)$

XL. $0 = +178{\cdot}688 + 59{\cdot}8198\,(13) - 44{\cdot}5872\,(15) - 15{\cdot}2326\,(21) + 22{\cdot}7623\,(22) + 9{\cdot}4295\,(24)$
$- 16{\cdot}1613\,(35) + 23{\cdot}2327\,(37) - 7{\cdot}0714\,(38)$

XLI. $0 = -40{\cdot}862 - 38{\cdot}9962\,(2) + 14{\cdot}9845\,(4) - 3{\cdot}9423\,(15) + 18{\cdot}7104\,(17) - 14{\cdot}7681\,(21)$
$- 7{\cdot}1045\,(22) + 6{\cdot}9188\,(24)$

$$\begin{aligned}\text{XLII.}\ 0 &= -\ 161{\cdot}422 - 41{\cdot}7999\,(2) + 38{\cdot}9962\,(4) - 23{\cdot}0286\,(5) + 42{\cdot}1444\,(6) - 33{\cdot}2480\,(38)\\ &\quad + 57{\cdot}8209\,(39) - 24{\cdot}5729\,(40)\end{aligned}$$

$$\text{XLIII.}\ 0 = -\ 30{\cdot}933 + 5{\cdot}9339\,(4) + 0{\cdot}0742\,(5) + 5{\cdot}9887\,(39) - 56{\cdot}0701\,(40) + 50{\cdot}0814\,(41)$$

$$\begin{aligned}\text{XLIV.}\ 0 &= +\ 10{\cdot}289 + 5{\cdot}4954\,(30) - 20{\cdot}5422\,(31) + 15{\cdot}0468\,(32) + 7{\cdot}0714\,(35) - 10{\cdot}2402\,(38)\\ &\quad + 3{\cdot}1688\,(42) + 7{\cdot}0508\,(46) - 15{\cdot}8063\,(47) + 8{\cdot}7555\,(50)\end{aligned}$$

$$\begin{aligned}\text{XLV.}\ 0 &= -\ 36{\cdot}424 + 9{\cdot}4959\,(25) - 17{\cdot}8680\,(26) + 8{\cdot}3721\,(27) + 29{\cdot}0317\,(36) - 32{\cdot}2005\,(38)\\ &\quad + 3{\cdot}1688\,(42) + 7{\cdot}0508\,(46) - 40{\cdot}4259\,(48) + 33{\cdot}3751\,(50)\end{aligned}$$

$$\begin{aligned}\text{XLVI.}\ 0 &= +\ 84{\cdot}091 + 9{\cdot}9777\,(22) - 17{\cdot}0822\,(23) + 7{\cdot}1045\,(24) + 37{\cdot}5971\,(37) - 40{\cdot}7659\,(38)\\ &\quad + 3{\cdot}1688\,(42) + 7{\cdot}0508\,(46) - 50{\cdot}6242\,(49) + 43{\cdot}5734\,(50)\end{aligned}$$

$$\begin{aligned}\text{XLVII.}\ 0 &= +\ 72{\cdot}940 - 8{\cdot}0455\,(23) + 0{\cdot}1857\,(24) + 18{\cdot}1370\,(37) - 29{\cdot}6018\,(39) + 11{\cdot}4648\,(42)\\ &\quad - 4{\cdot}1515\,(46) - 16{\cdot}3027\,(49) + 20{\cdot}4542\,(51)\end{aligned}$$

$$\begin{aligned}\text{XLVIII.}\ 0 &= -\ 224{\cdot}196 + 67{\cdot}1238\,(3) - 59{\cdot}1329\,(4) + 74{\cdot}2389\,(5) - 74{\cdot}3131\,(10) - 9{\cdot}6961\,(46)\\ &\quad - 4{\cdot}1515\,(51) + 13{\cdot}8476\,(52)\end{aligned}$$

$$\text{XLIX.}\ 0 = -\ 31{\cdot}349 + 6{\cdot}0463\,(10) + 1{\cdot}0156\,(12) + 15{\cdot}0924\,(51) - 51{\cdot}6544\,(52) + 36{\cdot}5620\,(53)$$

$$\begin{aligned}\text{L.}\ 0 &= +\ 45{\cdot}838 + 62{\cdot}4419\,(26) - 86{\cdot}2150\,(27) + 23{\cdot}7731\,(29) + 33{\cdot}7036\,(45) - 55{\cdot}5277\,(46)\\ &\quad + 21{\cdot}8241\,(48) + 10{\cdot}6576\,(74) - 55{\cdot}9238\,(75) + 45{\cdot}2662\,(76) + 1{\cdot}8529\,(77)\\ &\quad - 19{\cdot}0228\,(78) + 17{\cdot}1699\,(79)\end{aligned}$$

$$\begin{aligned}\text{LI.}\ 0 &= +\ 4{\cdot}037 + 23{\cdot}7731\,(27) - 44{\cdot}7953\,(28) + 21{\cdot}0222\,(29) - 13{\cdot}1459\,(34) + 5{\cdot}5172\,(36)\\ &\quad + 7{\cdot}6287\,(44) - 12{\cdot}9289\,(74) + 12{\cdot}9289\,(76) - 1{\cdot}8529\,(77) + 1{\cdot}8529\,(78)\end{aligned}$$

$$\begin{aligned}\text{LII.}\ 0 &= +\ 33{\cdot}268 + 12{\cdot}0632\,(42) - 39{\cdot}9921\,(43) + 27{\cdot}9289\,(44) - 22{\cdot}2020\,(45) + 5{\cdot}4549\,(46)\\ &\quad + 16{\cdot}7471\,(57) - 10{\cdot}6667\,(78) - 9{\cdot}3467\,(79) + 20{\cdot}0134\,(80)\end{aligned}$$

$$\begin{aligned}\text{LIII.}\ 0 &= -\ 262{\cdot}090 + 74{\cdot}3131\,(5) + 24{\cdot}8557\,(7) - 99{\cdot}1688\,(10) + 24{\cdot}3513\,(40) - 36{\cdot}4145\,(42)\\ &\quad + 12{\cdot}0632\,(43) + 27{\cdot}5623\,(66) - 40{\cdot}1062\,(67) + 12{\cdot}5439\,(69) + 21{\cdot}5503\,(82)\\ &\quad - 32{\cdot}7681\,(83) + 11{\cdot}2178\,(85)\end{aligned}$$

$$\begin{aligned}\text{LIV.}\ 0 &= +\ 38{\cdot}297 - 4{\cdot}4481\,(7) - 11{\cdot}3564\,(10) + 154{\cdot}5060\,(52) - 161{\cdot}0410\,(53) + 6{\cdot}5350\,(56)\\ &\quad + 8{\cdot}6918\,(67) - 143{\cdot}3958\,(69) + 134{\cdot}7040\,(70)\end{aligned}$$

$$\begin{aligned}\text{LV.}\ 0 &= +\ 35{\cdot}030 - 53{\cdot}0170\,(55) + 47{\cdot}5499\,(56) + 5{\cdot}4671\,(57) - 13{\cdot}7141\,(66) + 59{\cdot}3097\,(67)\\ &\quad - 45{\cdot}5956\,(68) - 36{\cdot}2134\,(83) + 70{\cdot}2019\,(84) - 33{\cdot}9885\,(85)\end{aligned}$$

$$\begin{aligned}\text{LVI.}\ 0 &= -\ 180{\cdot}073 + 77{\cdot}9572\,(19) - 119{\cdot}1508\,(20) + 41{\cdot}1936\,(21) + 3{\cdot}1688\,(38) - 15{\cdot}2320\,(42)\\ &\quad + 12{\cdot}0632\,(43) + 16{\cdot}0130\,(50) - 36{\cdot}8709\,(55) + 20{\cdot}8579\,(57) + 21{\cdot}5503\,(82)\\ &\quad - 78{\cdot}6216\,(83) + 57{\cdot}0713\,(84)\end{aligned}$$

$$\begin{aligned}\text{LVII.}\ 0 &= +\ 118{\cdot}648 + 4{\cdot}0131\,(6) + 50{\cdot}9774\,(9) - 54{\cdot}9905\,(10) + 16{\cdot}9746\,(18) - 78{\cdot}9188\,(19)\\ &\quad + 61{\cdot}9442\,(20) - 7{\cdot}0371\,(50) - 13{\cdot}8652\,(52) + 20{\cdot}9023\,(55)\end{aligned}$$

$$\begin{aligned}\text{LVIII.}\ 0 &= -\ 165{\cdot}330 - 52{\cdot}2601\,(9) + 54{\cdot}9905\,(10) + 57{\cdot}8949\,(11) - 64{\cdot}7737\,(12) + 13{\cdot}8652\,(52)\\ &\quad + 18{\cdot}3845\,(53) + 4{\cdot}5193\,(55)\end{aligned}$$

$$\begin{aligned}\text{LIX.}\ 0 &= +\ 34{\cdot}104 - 9{\cdot}4498\,(8) + 4{\cdot}8150\,(9) + 4{\cdot}6348\,(10) - 6{\cdot}3098\,(52) + 15{\cdot}5402\,(54)\\ &\quad - 9{\cdot}2304\,(55) - 3{\cdot}4958\,(62) - 2{\cdot}6219\,(63) + 6{\cdot}1177\,(64)\end{aligned}$$

$$\begin{aligned}\text{LX.}\ 0 &= -\ 50{\cdot}400 - 0{\cdot}3648\,(8) - 11{\cdot}3564\,(10) + 154{\cdot}5060\,(52) - 174{\cdot}9744\,(53) + 20{\cdot}4684\,(54)\\ &\quad - 7{\cdot}1610\,(63) - 101{\cdot}8058\,(64) + 108{\cdot}9668\,(65)\end{aligned}$$

The logarithmic values of the multipliers are as follow :

Log I = 0·9429133 +
II = 1·3020450 −
III = 1·3153901 −
IV = 7·8832411 −
V = 9·8965607 +
VI = 9·6691458 +
VII = 0·3039591 −
VIII = 0·3841409 −
IX = 0·6632037 −
X = 0·2514039 +
XI = 9·8082755 +
XII = 0·7031322 −
XIII = 1·2403970 +
XIV = 1·2211784 −
XV = 1·4311404 +
XVI = 0·7971839 −
XVII = 1·4319170 −
XVIII = 0·3322506 −
XIX = 9·1765198 −
XX = 9·6358077 −

Log XXI = 0·8926260 +
XXII = 9·9126585 −
XXIII = 0·2166834 −
XXIV = 9·9026820 +
XXV = 0·2349931 −
XXVI = 9·4912015 −
XXVII = 9·1438476 −
XXVIII = 9·2045400 −
XXIX = 9·6552321 +
XXX = 9·7901434 −
XXXI = 9·9644769 −
XXXII = 9·2006317 −
XXXIII = 8·4854758 −
XXXIV = 7·8070612 +
XXXV = 9·5333186 +
XXXVI = 9·1271456 +
XXXVII = 7·9197421 +
XXXVIII = 7·8694077 −
XXXIX = 9·1187768 −
XL = 9·0232369 −

Log XLI = 9·7416345 +
XLII = 9·4200602 −
XLIII = 8·4375524 −
XLIV = 8·6603569 −
XLV = 8·8582670 +
XLVI = 9·4032380 −
XLVII = 8·7921528 +
XLVIII = 9·5123951 −
XLIX = 9·8066049 +
L = 8·0624617 −
LI = 7·5163747 +
LII = 8·3073570 +
LIII = 8·3522597 +
LIV = 8·3890863 +
LV = 8·3813678 −
LVI = 8·4407863 +
LVII = 8·3422667 +
LVIII = 8·7023642 +
LIX = 8·5239753 +
LX = 8·4356676 +

FIGURE 10.

Notation.

(1)	*E′F*	(2)	*E′H*	(3)	*D′F*	(4)	*CF*	(5)	*A′F*	(6)	*B′F*	(7)	*B′H*
(8)	*B′G*	(9)	*FH*	(10)	*FE′*	(11)	*FC*	(12)	*FA′*	(13)	*FB′*	(14)	*FG*
(15)	*HE′*	(16)	*HB′*	(17)	*HF*	(18)	*HG*	(19)	*GH*	(20)	*GF*	(21)	*GB′*

Equations.

I $0 = - 1{\cdot}9732 + (1) - (4) - (10) + (11)$

II $0 = - 0{\cdot}1137 + (1) - (5) - (10) + (12)$

III $0 = - 4{\cdot}7979 + (1) - (6) - (10) + (13)$

IV $0 = + 4{\cdot}1475 - (1) + (2) - (9) + (10) - (15) + (17)$

V $0 = - 2{\cdot}3080 + (2) - (7) - (15) + (16)$

VI $0 = - 0{\cdot}2322 + (6) - (8) - (13) + (14) - (20) + (21)$

VII $0 = + 1{\cdot}4964 + (9) - (14) - (17) + (18) - (19) + (20)$

VIII $0 = + 178{\cdot}619 - 148{\cdot}8676\,(1) + 47{\cdot}8071\,(5) + 26{\cdot}8616\,(10) + 7{\cdot}9700\,(12) - 34{\cdot}8316\,(13)$

IX $0 = + 171{\cdot}349 - 148{\cdot}8676\,(1) + 81{\cdot}0836\,(4) + 26{\cdot}8616\,(10) - 2{\cdot}9689\,(11) - 23{\cdot}8927\,(13)$

X $0 = + 9{\cdot}372 + 17{\cdot}1856\,(1) - 75{\cdot}7595\,(3) + 35{\cdot}8801\,(4)$

XI $0 = + 30{\cdot}256 - 36{\cdot}4113\,(6) + 45{\cdot}1760\,(7) - 35{\cdot}7680\,(9) + 8{\cdot}9064\,(10) + 26{\cdot}8616\,(13)$
$- 1{\cdot}2565\,(15) - 11{\cdot}5963\,(16) + 12{\cdot}8528\,(17)$

XII $0 = + 114{\cdot}060 - 188{\cdot}2737\,(1) + 39{\cdot}4061\,(2) - 61{\cdot}1368\,(6) + 24{\cdot}7255\,(8) - 12{\cdot}8528\,(15)$
$- 57{\cdot}2041\,(17) + 70{\cdot}0569\,(18) + 16{\cdot}6399\,(19) - 27{\cdot}1060\,(20) + 10{\cdot}4661\,(21)$

The logarithmic values of the multipliers are as follow:

Log I = 8·5450020 +
II = 0·6249841 −
III = 0·5226363 +
IV = 0·2564729 −

Log V = 9·9616757 +
VI = 9·9413961 +
VII = 9·8026935 −
VIII = 8·4493316 −

Log IX = 8·3590965 +
X = 8·3705973 −
XI = 7·3318361 +
XII = 8·1731029 −

FIGURE 11.

Notation.

(1)	$O'H'$	(2)	$O'G'$	(3)	$Q'H'$	(4)	$Q'G'$	(5)	QB	(6)	QA	(7)	$R'H'$
(8)	$R'G'$	(9)	$R'A$	(10)	TA	(11)	$X'A$	(12)	$V'G'$	(13)	$V'A$	(14)	VF'
(15)	VB	(16)	VE'	(17)	$V'C$	(18)	$N'A$	(19)	$N'B$	(20)	$N'E'$	(21)	$N'C$
(22)	$L'B$	(23)	$L'C$	(24)	$K'B$	(25)	$K'C$	(26)	$K'E'$	(27)	$K'C'$	(28)	$K'B'$
(29)	$I'C$	(30)	$I'B'$	(31)	AU'	(32)	AT'	(33)	AR'	(34)	AQ'	(35)	AO'
(36)	AG'	(37)	AF'	(38)	AB	(39)	AE'	(40)	AC	(41)	AN'	(42)	AX'
(43)	AV'	(44)	AW'	(45)	HO'	(46)	HR'	(47)	HQ'	(48)	$G'R'$	(49)	$G'Q'$
(50)	$G'P'$	(51)	$G'O'$	(52)	$G'B$	(53)	$G'A$	(54)	$G'V'$	(55)	FB	(56)	$F'A$
(57)	$D'C$	(58)	$D'B$	(59)	BA	(60)	BQ'	(61)	BG'	(62)	BF'	(63)	BD'
(64)	BE'	(65)	BC	(66)	BL'	(67)	BN'	(68)	CV'	(69)	CA	(70)	CB
(71)	CD'	(72)	CE'	(73)	CA'	(74)	CC'	(75)	CB'	(76)	CK'	(77)	CL'
(78)	CN'	(79)	$E'V'$	(80)	$E'A$	(81)	$E'B$	(82)	$E'K'$	(83)	$E'C$	(84)	EN'
(85)	$A'C$	(86)	$A'K'$	(87)	$C'C$	(88)	$B'C$	(89)	$B'I'$	(90)	$B'K'$		

Equations.

I. $0 = -3{\cdot}7889 - (1) + (3) + (45) - (47)$
II. $0 = -2{\cdot}3402 - (1) + (7) + (45) - (46)$
III. $0 = -4{\cdot}7466 - (2) + (4) - (49) + (51)$
IV. $0 = -7{\cdot}5260 - (2) + (8) - (48) + (51)$
V. $0 = -1{\cdot}6697 - (3) + (4) + (47) - (49)$
VI. $0 = +2{\cdot}2707 - (4) + (12) + (49) - (54)$
VII. $0 = -0{\cdot}3159 - (12) + (16) + (54) - (79)$
VIII. $0 = -2{\cdot}3205 - (16) + (20) + (79) - (84)$
IX. $0 = -1{\cdot}0618 - (20) + (26) - (82) + (84)$
X. $0 = -0{\cdot}5765 - (26) + (28) + (82) - (90)$
XI. $0 = +0{\cdot}2944 - (28) + (30) - (89) + (90)$
XII. $0 = -6{\cdot}2161 - (10) + (13) + (32) - (43)$
XIII. $0 = -0{\cdot}8422 - (9) + (13) + (33) - (43)$
XIV. $0 = -0{\cdot}6529 - (6) + (13) + (34) - (43)$
XV. $0 = +0{\cdot}8115 - (4) + (6) - (34) + (36) + (49) - (53)$
XVI. $0 = -0{\cdot}2317 - (36) + (37) + (53) - (56)$
XVII. $0 = +2{\cdot}8952 - (36) - (39) + (53) - (80)$
XVIII. $0 = -3{\cdot}4764 + (11) - (13) - (42) + (43)$
XIX. $0 = -1{\cdot}1644 - (13) + (18) - (41) + (43)$
XX. $0 = +4{\cdot}3220 - (37) + (38) - (55) + (56) - (59) + (62)$
XXI. $0 = +3{\cdot}5748 - (36) + (38) - (52) + (53) - (59) + (61)$
XXII. $0 = +7{\cdot}2494 - (5) + (6) - (34) + (38) - (59) + (60)$

XXIII. $0 = -6{\cdot}1170 - (18) + (19) - (38) + (41) + (59) - (67)$

XXIV. $0 = +1{\cdot}7139 - (19) + (20) - (64) + (67) + (81) - (84)$

XXV. $0 = -0{\cdot}2186 + (58) - (63) + (64) - (81)$

XXVI. $0 = +3{\cdot}3798 - (19) + (22) - (66) + (67)$

XXVII. $0 = -0{\cdot}0256 - (21) + (25) - (76) + (78)$

XXVIII. $0 = +1{\cdot}5007 - (21) + (23) - (77) + (78)$

XXIX. $0 = -1{\cdot}1339 - (17) + (21) + (68) - (78)$

XXX. $0 = -0{\cdot}5902 - (18) + (21) - (40) + (41) + (69) - (78)$

XXXI. $0 = +3{\cdot}0432 - (19) + (21) - (65) + (67) + (70) - (78)$

XXXII. $0 = +0{\cdot}5341 + (57) - (71) + (72) - (83)$

XXXIII. $0 = -3{\cdot}3487 - (64) + (65) - (70) + (72) + (81) - (83)$

XXXIV. $0 = +0{\cdot}9520 - (72) + (73) - (83) - (85)$

XXXV. $0 = +2{\cdot}4789 - (72) + (74) + (83) - (87)$

XXXVI. $0 = +0{\cdot}7263 - (25) + (28) - (75) + (76) + (88) - (90)$

XXXVII. $0 = -39{\cdot}268 + 5{\cdot}4670\,(3) - 10{\cdot}5117\,(7) - 1{\cdot}3799\,(45) - 13{\cdot}2726\,(46) - 14{\cdot}6525\,(47)$

XXXVIII. $0 = -93{\cdot}744 + 18{\cdot}0535\,(1) - 20{\cdot}0643\,(2) + 14{\cdot}6525\,(45) - 5{\cdot}0240\,(47) + 6{\cdot}4281\,(49) - 28{\cdot}3275\,(51)$

XXXIX. $0 = +214{\cdot}431 + 20{\cdot}0643\,(2) + 25{\cdot}2446\,(12) - 40{\cdot}3708\,(49) + 28{\cdot}3275\,(51) + 12{\cdot}0433\,(54)$

XL. $0 = -283{\cdot}414 - 35{\cdot}2440\,(8) + 25{\cdot}2446\,(12) - 239{\cdot}4933\,(48) + 227{\cdot}4500\,(49) + 12{\cdot}0433\,(54)$

XLI. $0 = +208{\cdot}252 + 19{\cdot}5631\,(2) - 2{\cdot}7427\,(4) + 43{\cdot}6886\,(49) - 153{\cdot}1148\,(50) + 109{\cdot}4262\,(51)$

XLII. $0 = +6{\cdot}053 + 53{\cdot}3125\,(3) - 57{\cdot}0478\,(4) - 40{\cdot}7074\,(12) + 15{\cdot}4628\,(16) - 9{\cdot}6285\,(47) + 16{\cdot}3990\,(79)$

XLIII. $0 = +58{\cdot}713 + 80{\cdot}4248\,(12) - 106{\cdot}3938\,(14) + 25{\cdot}9690\,(16) - 18{\cdot}4182\,(54) + 4{\cdot}4440\,(79)$

XLIV. $0 = +22{\cdot}503 + 3{\cdot}7353\,(4) - 0{\cdot}6253\,(20) + 12{\cdot}0433\,(49) - 17{\cdot}9987\,(54) - 55{\cdot}8222\,(79) + 39{\cdot}4232\,(84)$

XLV. $0 = +126{\cdot}022 + 12{\cdot}0617\,(16) + 1{\cdot}4871\,(26) + 39{\cdot}4232\,(79) + 37{\cdot}3966\,(82) - 76{\cdot}8198\,(84)$

XLVI. $0 = -150{\cdot}357 + 15{\cdot}4628\,(12) - 27{\cdot}5245\,(16) - 9{\cdot}3455\,(20) - 1{\cdot}4871\,(26) + 5{\cdot}9554\,(54) - 42{\cdot}1716\,(82) + 56{\cdot}9619\,(86)$

XLVII. $0 = +1224{\cdot}885 + 36{\cdot}4029\,(26) + 306{\cdot}5869\,(27) - 342{\cdot}9898\,(28) + 6{\cdot}1932\,(82) - 420{\cdot}6563\,(86) + 342{\cdot}9898\,(90)$

XLVIII. $0 = -198{\cdot}377 + 36{\cdot}4029\,(26) - 99{\cdot}8323\,(27) + 63{\cdot}4294\,(28) + 6{\cdot}1932\,(82) + 3{\cdot}7795\,(90)$

XLIX. $0 = -89{\cdot}269 + 9{\cdot}9708\,(20) + 1{\cdot}0373\,(30) - 34{\cdot}2294\,(82) + 37{\cdot}3966\,(84) - 31{\cdot}1347\,(86) + 84{\cdot}2319\,(89) - 78{\cdot}2163\,(90)$

L. $0 = +301{\cdot}189 + 66{\cdot}0709\,(27) - 63{\cdot}4294\,(28) - 74{\cdot}3538\,(29) + 66{\cdot}5306\,(30) - 1{\cdot}4202\,(89) - 3{\cdot}7795\,(90)$

LI. $0 = -340{\cdot}518 - 30{\cdot}1218\,(10) + 14{\cdot}8115\,(13) - 63{\cdot}0710\,(32) + 60{\cdot}9992\,(33) + 2{\cdot}0718\,(43)$

LII. $0 = -56{\cdot}476 - 8{\cdot}4076\,(9) + 18{\cdot}9374\,(13) - 132{\cdot}3470\,(33) + 133{\cdot}6053\,(34) - 1{\cdot}2583\,(43)$

LIII. $0 = +31.550 + 23.8899\,(4) - 50.3138\,(6) - 146.1049\,(12) + 127.1675\,(13) + 7.6569\,(49) + 114.4511\,(53) - 122.1080\,(54)$

LIV. $0 = -124.360 + 53.3125\,(3) - 77.2024\,(4) + 23.8899\,(6) + 8.2542\,(34) - 8.7331\,(36) + 0.4789\,(39) - 9.6285\,(47) + 41.5403\,(80)$

LV. $0 = -81.995 + 26.6051\,(13) - 20.6851\,(16) - 43.2377\,(18) + 14.4733\,(20) - 44.7280\,(79) + 29.0426\,(80) + 15.6854\,(84)$

LVI. $0 = +98.855 - 22.9262\,(53) + 45.7226\,(56) - 25.6010\,(80)$

LVII. $0 = +457.670 - 76.2467\,(11) + 28.7644\,(18) + 26.8599\,(41) - 94.3695\,(42) + 67.5096\,(43)$

LVIII. $0 = -93.256 - 12.1474\,(9) - 5.0560\,(10) - 20.2588\,(32) + 40.1988\,(33) - 19.9400\,(35)$

LIX. $0 = +164.571 + 33.3764\,(10) + 3.0206\,(13) + 148.6583\,(31) - 135.0126\,(32) - 13.6457\,(43)$

LX. $0 = +14.845 + 1.8940\,(10) + 19.8063\,(13) - 27.8251\,(32) - 38.2975\,(43) + 66.1226\,(44)$

LXI. $0 = -205.654 + 23.8899\,(4) - 108.1228\,(5) + 84.2329\,(6) + 7.6569\,(49) + 16.4962\,(52) - 24.1531\,(53) + 66.6717\,(59) - 91.1524\,(60) + 24.4807\,(61)$

LXII. $0 = -114.293 - 16.4962\,(52) + 29.3458\,(53) + 22.3507\,(55) - 59.2887\,(56) + 20.1020\,(59) - 24.4807\,(61) + 4.3787\,(62)$

LXIII. $0 = +16.183 - 6.4472\,(55) + 16.9957\,(58) + 27.9276\,(61) - 27.3867\,(62) - 0.5409\,(63)$

LXIV. $0 = +106.861 - 67.4498\,(52) + 41.0588\,(58) - 44.5715\,(81)$

LXV. $0 = +279.746 + 28.8707\,(18) - 88.6853\,(19) + 59.8146\,(20) + 0.9631\,(36) - 7.5991\,(38) + 6.6360\,(41) - 111.4459\,(52) + 16.4962\,(53) - 59.9957\,(81) + 14.5615\,(84)$

LXVI. $0 = +346.091 - 57.6216\,(19) + 59.8146\,(20) + 63.7353\,(24) - 53.7174\,(26) - 12.6139\,(81) - 1.9476\,(82) + 14.5615\,(84)$

LXVII. $0 = -108.721 + 30.1110\,(13) - 37.4683\,(15) - 28.8707\,(18) + 22.1543\,(19) + 14.5505\,(38) - 6.6360\,(41) - 7.9145\,(43)$

LXVIII. $0 = +162.044 - 34.5062\,(19) + 0.3389\,(22) + 44.5241\,(24) - 105.3177\,(66) + 105.3177\,(67)$

LXIX. $0 = -28.069 + 9.9079\,(23) + 5.3751\,(25) + 19.9345\,(76) - 88.8925\,(77) + 68.9580\,(78)$

LXX. $0 = +124.242 + 20.8045\,(17) + 5.3751\,(25) + 28.9265\,(68) + 19.9345\,(76) - 48.8610\,(78)$

LXXI. $0 = +83.891 + 5.9200\,(13) + 20.8045\,(17) - 19.6645\,(40) + 46.5244\,(41) - 26.8599\,(43) + 28.9265\,(68) - 7.3973\,(69) - 21.5292\,(78)$

LXXII. $0 = -131.676 - 11.0067\,(18) + 42.6061\,(19) - 31.5994\,(21) - 44.0454\,(38) + 63.7099\,(40) - 19.6645\,(41) + 11.7302\,(59) - 30.8674\,(65) + 19.1372\,(67)$

LXXIII. $0 = +190.952 - 42.6061\,(19) + 173.8684\,(20) - 131.2623\,(21) - 13.3915\,(64) + 32.5287\,(65) - 19.1372\,(67) - 8.4685\,(81) - 84.5370\,(83) + 93.0055\,(84)$

LXXIV. $0 = -142.805 + 105.1237\,(57) - 24.0631\,(58) - 16.9529\,(63) + 30.3444\,(64) - 13.3915\,(65) - 15.0927\,(70) + 91.7121\,(71) - 76.6194\,(72)$

LXXV. $0 = -88.162 + 105.1237\,(57) + 91.7121\,(71) - 118.4205\,(72) + 26.7084\,(73) + 42.1043\,(85)$

LXXVI. $0 = + 23{\cdot}320 + 16{\cdot}2036\,(72) - 83{\cdot}4013\,(73) + 67{\cdot}1977\,(74) - 0{\cdot}0869\,(83) - 22{\cdot}1762\,(85)$

LXXVII. $0 = - 146{\cdot}964 + 103{\cdot}2693\,(25) - 116{\cdot}0757\,(26) + 12{\cdot}8064\,(28) - 70{\cdot}5122\,(82) + 80{\cdot}4119\,(83) - 45{\cdot}6179\,(85) - 34{\cdot}9047\,(88) + 25{\cdot}8911\,(90)$

The logarithmic values of the multipliers are as follow:

Log I = 0·4049316 +
II = 0·1230085 −
III = 0·7751602 −
IV = 0·0175138 +
V = 0·9169162 +
VI = 0·9854319 −
VII = 0·5941234 +
VIII = 0·3620531 +
IX = 0·6474205 +
X = 1·0664646 +
XI = 0·8815449 +
XII = 0·4780612 −
XIII = 9·5334272 +
XIV = 9·7412979 +
XV = 9·8301911 −
XVI = 0·2486421 −
XVII = 0·3949979 −
XVIII = 9·8336016 −
XIX = 1·0236208 +
XX = 0·3205971 +
XXI = 9·8532763 +
XXII = 9·8235858 −
XXIII = 9·7175043 +
XXIV = 0·6345447 −
XXV = 0·7420813 +
XXVI = 0·6439164 −

Log XXVII = 0·0786685 −
XXVIII = 0·1292990 −
XXIX = 9·6347616 +
XXX = 0·5144541 +
XXXI = 0·4857572 +
XXXII = 0·3307567 −
XXXIII = 0·8838135 +
XXXIV = 9·9800827 +
XXXV = 0·2288640 −
XXXVI = 9·7452712 −
XXXVII = 8·6789671 +
XXXVIII = 8·7526070 −
XXXIX = 9·5502138 −
XL = 7·8519911 +
XLI = 7·7049428 +
XLII = 9·3548562 −
XLIII = 8·4316460 +
XLIV = 9·2486009 −
XLV = 8·6272466 −
XLVI = 9·5963568 +
XLVII = 8·6144258 +
XLVIII = 9·1275357 +
XLIX = 9·1037383 +
L = 8·6651646 −
LI = 8·9619154 +
LII = 8·0261834 −

Log LIII = 7·9770694 −
LIV = 9·5038224 +
LV = 8·9827049 −
LVI = 8·5158238 +
LVII = 8·7782869 −
LVIII = 8·3986277 −
LIX = 8·3922868 −
LX = 8·4648323 −
LXI = 7·7636915 −
LXII = 9·0999248 +
LXIII = 8·9979099 +
LXIV = 9·5268476 −
LXV = 9·2421950 +
LXVI = 8·6896373 −
LXVII = 9·6365472 +
LXVIII = 8·6536908 +
LXIX = 8·2379522 +
LXX = 9·5105440 −
LXXI = 9·4006849 +
LXXII = 9·0855183 +
LXXIII = 8·3155085 +
LXXIV = 9·0441124 −
LXXV = 9·1010576 +
LXXVI = 8·5529661 +
LXXVII = 8·2204089 +

FIGURE 12.

Notation.

(1)	*BD*	(2)	*BE*	(3)	*BA*	(4)	*BC*	(5)	*AB*	(6)	*AD*	(7)	*AE*
(8)	*AK*	(9)	*AF*	(10)	*FN*	(11)	*FG*	(12)	*FA*	(13)	*FE*	(14)	*FK*
(15)	*FH*	(16)	*EK*	(17)	*EH*	(18)	*EF*	(19)	*EA*	(20)	*EB*	(21)	*ED*
(22)	*DK*	(23)	*DH*	(24)	*DE*	(25)	*DA*	(26)	*DB*	(27)	*DC*	(28)	*DL*
(29)	*CL*	(30)	*CK*	(31)	*CD*	(32)	*CB*	(33)	*LQ*	(34)	*LK*	(35)	*LD*
(36)	*LC*	(37)	*KO*	(38)	*KN*	(39)	*KH*	(40)	*KF*	(41)	*KA*	(42)	*KE*
(43)	*KD*	(44)	*KC*	(45)	*KL*	(46)	*KM*	(47)	*MO*	(48)	*MN*	(49)	*MH*
(50)	*MK*	(51)	*ML*	(52)	*MQ*	(53)	*QN*	(54)	*QP*	(55)	*QM*	(56)	*QL*
(57)	*PO*	(58)	*PN*	(59)	*PK*	(60)	*PQ*	(61)	*HO*	(62)	*HN*	(63)	*HG*
(64)	*HF*	(65)	*HE*	(66)	*HK*	(67)	*HM*	(68)	*GF*	(69)	*GH*	(70)	*GN*
(71)	*NG*	(72)	*NF*	(73)	*MH*	(74)	*NK*	(75)	*NP*	(76)	*NQ*	(77)	*NO*
(78)	*ON*	(79)	*OH*	(80)	*OK*	(81)	*OM*						

Equations.

I. $0 = -2.3951 - (2) + (3) - (5) + (7) - (19) + (20)$

II. $0 = +0.0074 - (1) + (3) - (5) + (6) - (25) + (26)$

III. $0 = -0.2203 - (6) + (7) - (19) + (21) - (24) + (25)$

IV. $0 = +1.7918 - (7) + (9) - (12) + (13) - (18) + (19)$

V. $0 = -2.9055 - (13) + (14) - (16) + (18) - (40) + (42)$

VI. $0 = +0.1559 - (8) + (9) - (12) + (14) - (40) + (41)$

VII. $0 = -0.9176 - (6) + (8) - (22) + (25) - (41) + (43)$

VIII. $0 = +0.1209 + (1) - (4) - (26) + (27) - (31) + (32)$

IX. $0 = +1.9227 + (22) - (27) - (30) + (31) - (43) + (44)$

X. $0 = -0.2522 - (27) + (28) - (29) + (31) - (35) + (36)$

XI. $0 = +0.5640 + (22) - (28) - (34) + (35) - (43) + (45)$

XII. $0 = -5.3237 - (14) + (15) - (39) + (40) - (64) + (66)$

XIII. $0 = -1.7055 - (13) + (15) - (17) + (18) - (64) + (65)$

XIV. $0 = +3.7027 + (39) - (46) - (49) + (50) - (66) + (67)$

XV. $0 = +0.7918 + (10) - (14) - (38) + (40) - (72) + (74)$

XVI. $0 = +5.7245 + (10) - (15) - (62) + (64) - (72) + (73)$

XVII. $0 = +7.8303 + (11) - (15) - (63) + (64) - (68) + (69)$

XVIII. $0 = +2.7628 - (62) + (63) - (69) + (70) - (71) + (73)$

XIX. $0 = +1.0890 + (37) - (46) - (47) + (50) - (79) + (80)$

XX. $0 = -6.6946 - (47) + (49) + (61) - (67) - (78) + (80)$

XXI. $0 = -4.5944 - (37) + (38) - (74) + (76) - (77) + (79)$

XXII. $0 = -2.8493 + (38) - (46) + (50) - (52) - (53) + (55) - (74) + (81)$

XXIII. $0 = -0.6668 - (33) + (34) - (45) + (46) - (50) + (52) - (55) + (56)$

XXIV. $0 = -4.4765 - (53) + (54) + (58) - (60) - (75) + (81)$

XXV. $0 = +20.290 + 7.6602\,(1) - 12.4659\,(2) + 4.8057\,(3) + 27.0097\,(19) - 9.1647\,(20)$
$- 17.8450\,(21) + 22.8259\,(24) - 33.6137\,(25) + 10.7878\,(26)$

XXVI. $0 = +1.315 + 33.5344\,(7) - 71.6484\,(8) + 38.1140\,(9) + 4.3369\,(16) - 19.3414\,(18)$
$+ 15.0045\,(19) + 40.5079\,(40) - 57.2132\,(41) + 16.7053\,(42)$

XXVII. $0 = +168.503 + 120.6336\,(6) - 170.7849\,(7) + 50.1513\,(8) + 2.5213\,(16) + 17.8450\,(19)$
$- 20.3663\,(21) + 17.8978\,(41) - 63.7826\,(42) + 45.8848\,(43)$

XXVIII. $0 = -81.862 - 27.5133\,(1) + 7.6602\,(3) + 19.8531\,(4) + 19.5514\,(5) - 69.7027\,(6)$
$+ 50.1513\,(8) + 45.3601\,(30) - 53.6350\,(31) + 8.2749\,(32) + 17.8978\,(41)$
$- 53.8688\,(43) + 35.9710\,(44)$

XXIX. $0 = +39.841 + 11.4089\,(29) - 45.3601\,(30) + 33.9512\,(31) + 20.4001\,(34) - 25.3972\,(35)$
$+ 4.9971\,(36) + 19.0055\,(43) - 35.9710\,(44) + 16.9655\,(45)$

XXX. $0 = -111.986 + 4.3369\,(16) - 17.4493\,(17) + 13.1124\,(18) + 10.6348\,(39) - 27.3401\,(40)$
$+ 16.7053\,(42) + 19.0282\,(64) - 22.7336\,(65) + 3.7054\,(66)$

XXXI. $0 = -406.622 - 2.5213\,(16) - 111.2679\,(17) + 113.7892\,(21) - 40.0664\,(22)$
$+ 165.9896\,(23) - 125.9232\,(24) - 20.4024\,(39) + 63.7826\,(42) - 43.3802\,(43)$

XXXII. $0 = +272.770 - 37.0602\,(16) + 39.5815\,(17) - 2.5213\,(21) - 12.4635\,(22) + 9.8616\,(24)$
$+ 2.6019\,(28) - 20.4001\,(34) + 20.4001\,(35) - 79.0931\,(45) + 79.0931\,(46)$
$+ 14.1217\,(49) - 93.7734\,(50) + 79.6517\,(51) + 27.7259\,(65) - 73.6156\,(66)$
$+ 45.8897\,(67)$

XXXIII. 0 = + 163·181 − 3·9556 (38) + 10·6348 (39) − 6·6792 (40) − 26·8039 (62) + 23·0985 (64)
+ 3·7054 (66) − 25·3861 (72) + 41·4660 (73) − 16·0799 (74)

XXXIV. 0 = − 145·920 − 64·9881 (38) − 3·4117 (39) + 68·3998 (46) + 40·8835 (48) + 21·5364 (49)
− 62·4199 (50) + 12·3885 (62) + 45·8897 (66) − 58·2782 (67)

XXXV. 0 = − 268·120 + 106·0244 (10) − 45·5978 (11) − 60·4266 (15) − 11·6663 (68) + 24·2082 (69)
− 12·5419 (70) − 6·3813 (71) + 41·4660 (72) − 35·0847 (73)

XXXVI. 0 = − 79·942 − 7·3240 (37) − 3·4117 (39) + 10·7357 (46) − 3·2511 (61) + 45·8897 (66)
− 42·6386 (67) + 21·2888 (78) − 71·1121 (79) + 49·8233 (80)

XXXVII. 0 = − 1·173 + 13·8963 (61) − 23·4916 (62) + 9·5953 (66) + 43·7269 (73) − 33·1353 (74)
− 10·5916 (76) + 17·0083 (77) − 39·2835 (78) + 22·2752 (79)

XXXVIII. 0 = + 26·365 + 64·9881 (38) + 84·1048 (45) − 149·0929 (46) − 110·1809 (48)
+ 141·5130 (50) − 129·9698 (51) + 98·6377 (52) + 63·5602 (53) − 123·1973 (55)
+ 59·6371 (56)

XXXIX. 0 = + 0·270 + 97·1730 (37) − 97·1730 (38) − 55·3449 (57) + 74·2092 (58) − 18·8643 (59)
+ 23·6289 (74) − 28·8578 (75) + 5·2289 (76) − 17·0083 (77) + 17·0083 (79)

XL. 0 = − 27·426 + 2·5682 (38) − 2·5682 (46) − 0·5423 (48) − 4·3549 (50) + 4·8972 (52)
− 127·6730 (53) + 143·4722 (54) − 15·7992 (55) + 58·4513 (58) + 18·8643 (59)
− 77·3156 (60) − 23·6289 (74) + 23·6289 (75)

The logarithmic values of the multipliers are as follow:

Log I = 0·2248932 +
II = 8·9688680 −
III = 0·6215364 −
IV = 9·5618712 −
V = 9·6224763 −
VI = 9·9239776 +
VII = 9·7742029 +
VIII = 9·7319780 −
IX = 9·9275444 −
X = 9·5821022 +
XI = 9·7385247 +
XII = 0·7721137 +
XIII = 9·2221213 −

Log XIV = 0·7390302 +
XV = 0·3849114 −
XVI = 0·3282291 +
XVII = 0·3715612 −
XVIII = 9·9056930 −
XIX = 0·3922171 −
XX = 0·6385232 +
XXI = 9·8617436 +
XXII = 0·3500430 +
XXIII = 9·5855898 +
XXIV = 9·9624003 +
XXV = 9·0530143 −
XXVI = 7·1573356 +
XXVII = 8·2002517 −

Log XXVIII = 8·4348489 +
XXIX = 8·5408173 +
XXX = 8·8396196 +
XXXI = 8·0678869 +
XXXII = 8·6175089 +
XXXIII = 8·6965003 +
XXXIV = 8·8072679 +
XXXV = 8·8324203 +
XXXVI = 5·1503729 −
XXXVII = 8·6767298 −
XXXVIII = 8·2983058 +
XXXIX = 8·1713874 +
XL = 7·9164417 −

FIGURE 13.

(1)	$D'H$	(2)	$D'G$	(3)	$D'M$	(4)	$D'K$	(5)	$D'F$	(6)	$E'H$	(7)	$E'G$
(8)	$E'F$	(9)	$A'F$	(10)	$A'K$	(11)	$A'L$	(12)	HN	(13)	HM	(14)	HL
(15)	HG	(16)	HD'	(17)	HE'	(18)	GK	(19)	GF	(20)	GD'	(21)	GE'
(22)	FG	(23)	FK	(24)	FA'	(25)	FD'	(26)	FE'	(27)	KM	(28)	KL
(29)	KO	(30)	KA'	(31)	KB'	(32)	KF	(33)	KD'	(34)	KG	(35)	LM
(36)	LQ	(37)	LR	(38)	LS	(39)	LO	(40)	LP	(41)	LA'	(42)	LK
(43)	LD'	(44)	LG	(45)	LH	(46)	MN	(47)	MQ	(48)	MR	(49)	MS
(50)	MO	(51)	MP	(52)	ML	(53)	MK	(54)	MD'	(55)	MG	(56)	MH
(57)	RQ	(58)	RP	(59)	RS	(60)	RO	(61)	RL	(62)	RM	(63)	RN
(64)	SR	(65)	SQ	(66)	SP	(67)	SO	(68)	SL	(69)	SM	(70)	SN
(71)	OR	(72)	OS	(73)	OP	(74)	OK	(75)	OL	(76)	OM	(77)	ON
(78)	PQ	(79)	PK	(80)	PL	(81)	PM	(82)	PO	(83)	PS	(84)	PR
(85)	$B'G$	(86)	$B'Q$	(87)	$B'K$	(88)	CH						

Equations.

I. $0 = -1{\cdot}9343 - (4) + (10) - (30) + (33)$

II. $0 = +4{\cdot}9821 - (5) + (9) - (24) + (25)$

III. $0 = -7{\cdot}0598 - (9) + (10) - (23) + (24) - (30) + (32)$

IV. $0 = +6{\cdot}9041 - (18) + (19) - (22) + (23) - (32) + (34)$

V. $0 = +2{\cdot}9969 - (2) + (4) - (18) + (20) - (33) + (34)$

VI. $0 = -3{\cdot}2119 + (2) - (7) - (20) + (21)$

VII. $0 = +0{\cdot}7633 + (5) - (8) - (25) + (26)$

VIII. $0 = +0{\cdot}4588 - (10) + (11) - (28) + (30) - (41) + (42)$

IX. $0 = -1{\cdot}9885 + (1) - (6) - (16) + (17)$

X. $0 = +2{\cdot}0244 - (6) + (7) - (14) + (17) + (18) - (21) + (28) - (34) - (42) + (45)$

XI. $0 = -3{\cdot}0395 - (13) + (14) + (35) - (45) - (52) + (56)$

XII. $0 = +0{\cdot}8635 - (1) + (3) - (13) + (16) - (54) + (56)$

XIII. $0 = -1{\cdot}6807 - (3) + (4) + (27) - (33) - (53) + (54)$

XIV. $0 = +4{\cdot}7459 - (35) + (39) - (50) + (52) - (75) + (76)$

XV. $0 = +5{\cdot}5356 - (27) + (29) - (50) + (53) - (74) + (76)$

XVI. $0 = -5{\cdot}3301 - (48) + (50) - (60) + (62) + (71) - (76)$

XVII. $0 = -7{\cdot}7811 - (37) + (39) - (60) + (61) + (71) - (75)$

XVIII. $0 = -2{\cdot}6250 - (38) + (39) - (67) + (68) + (72) - (75)$

XIX. $0 = -5{\cdot}0651 - (49) + (50) - (67) - (69) + (72) - (76)$

XX. $0 = +0{\cdot}3756 - (59) + (60) + (64) - (67) - (71) + (72)$

XXI. $0 = +7{\cdot}0660 - (58) + (60) - (71) + (73) - (82) + (84)$

XXII. $0 = +9{\cdot}4772 - (66) + (67) - (72) + (73) - (82) + (83)$

XXIII. $0 = +3{\cdot}9404 - (35) + (40) - (51) + (52) - (80) + (81)$

XXIV. $0 = +2{\cdot}1228 - (48) + (51) - (58) + (62) - (81) + (84)$

XXV. $0 = -3{\cdot}4642 + (10) - (30) + (31) - (87)$

XXVI. $0 = +48{\cdot}039 + 6{\cdot}5930\,(4) - 8{\cdot}8931\,(5) + 29{\cdot}7044\,(23) - 10{\cdot}1845\,(24) - 19{\cdot}5199\,(25)$
$+ 16{\cdot}6427\,(30) - 61{\cdot}1104\,(32) + 44{\cdot}4677\,(33)$

XXVII. $0 = +95{\cdot}259 + 55{\cdot}7925\,(2) - 218{\cdot}2243\,(4) + 162{\cdot}4318\,(5) + 0{\cdot}1009\,(18) - 18{\cdot}4440\,(19)$
$+ 18{\cdot}3431\,(20) + 156{\cdot}8638\,(32) - 189{\cdot}9146\,(33) + 33{\cdot}0508\,(34)$

XXVIII. $0 = +90{\cdot}473 - 13{\cdot}8485\,(2) + 4{\cdot}7288\,(5) + 4{\cdot}3687\,(19) - 37{\cdot}4585\,(20) + 33{\cdot}0898\,(21)$
$+ 5{\cdot}3031\,(22) + 30{\cdot}6102\,(25) - 35{\cdot}9133\,(26)$

XXIX.
$$\begin{aligned}0 = {}&+ 463{\cdot}519 - 8{\cdot}8931\,(5) + 6{\cdot}2784\,(11) + 29{\cdot}7044\,(23) - 10{\cdot}1845\,(24) - 19{\cdot}5199\,(25)\\&+ 65{\cdot}0388\,(28) - 3{\cdot}9284\,(30) - 61{\cdot}1104\,(32) + 16{\cdot}6302\,(41) - 73{\cdot}4728\,(42)\\&+ 56{\cdot}8426\,(43)\end{aligned}$$

XXX.
$$\begin{aligned}0 = {}&+ 1211{\cdot}817 + 71{\cdot}9244\,(9) - 691{\cdot}0395\,(10) + 619{\cdot}1151\,(11) - 0{\cdot}1009\,(18)\\&+ 0{\cdot}1009\,(19) + 13{\cdot}2709\,(22) + 16{\cdot}4335\,(23) - 29{\cdot}7044\,(24) + 158{\cdot}5989\,(28)\\&- 158{\cdot}5989\,(34) + 73{\cdot}4728\,(41) - 275{\cdot}2221\,(42) + 201{\cdot}7493\,(44)\end{aligned}$$

XXXI.
$$\begin{aligned}0 = {}&+ 235{\cdot}339 + 64{\cdot}0824\,(1) - 50{\cdot}2339\,(2) - 40{\cdot}0147\,(6) + 38{\cdot}5450\,(7) - 10{\cdot}4369\,(15)\\&+ 19{\cdot}1855\,(16) - 8{\cdot}7486\,(17)\end{aligned}$$

XXXII.
$$\begin{aligned}0 = {}&- 62{\cdot}051 - 40{\cdot}0147\,(6) + 83{\cdot}4598\,(7) - 43{\cdot}4451\,(8) - 15{\cdot}6295\,(14) + 24{\cdot}3781\,(15)\\&- 8{\cdot}7486\,(17) + 18{\cdot}5740\,(22) - 13{\cdot}2709\,(23) - 5{\cdot}3031\,(26) + 31{\cdot}7240\,(28)\\&- 33{\cdot}0508\,(32) + 1{\cdot}3268\,(34) - 43{\cdot}1504\,(42) + 84{\cdot}5374\,(44) - 41{\cdot}3870\,(45)\end{aligned}$$

XXXIII.
$$\begin{aligned}0 = {}&- 91{\cdot}531 - 9{\cdot}7866\,(13) + 15{\cdot}6295\,(14) - 5{\cdot}8429\,(15) + 9{\cdot}2067\,(35) - 50{\cdot}5937\,(44)\\&+ 41{\cdot}3870\,(45) - 14{\cdot}9694\,(52) + 56{\cdot}1167\,(55) - 41{\cdot}1473\,(56)\end{aligned}$$

XXXIV.
$$\begin{aligned}0 = {}&+ 215{\cdot}523 + 39{\cdot}2393\,(1) - 39{\cdot}2393\,(3) + 32{\cdot}9617\,(14) - 32{\cdot}9617\,(16) + 1{\cdot}2311\,(35)\\&+ 53{\cdot}0422\,(43) - 54{\cdot}2733\,(45) + 3{\cdot}0762\,(52) - 25{\cdot}9526\,(54) + 22{\cdot}8764\,(56)\end{aligned}$$

XXXV.
$$\begin{aligned}0 = {}&- 126{\cdot}550 + 15{\cdot}1254\,(27) - 31{\cdot}7240\,(28) + 16{\cdot}5986\,(34) - 9{\cdot}2067\,(35)\\&+ 43{\cdot}1504\,(42) - 33{\cdot}9437\,(44) + 14{\cdot}9694\,(52) - 30{\cdot}1239\,(53) + 15{\cdot}1545\,(55)\end{aligned}$$

XXXVI.
$$\begin{aligned}0 = {}&- 173{\cdot}874 - 10{\cdot}9789\,(27) - 55{\cdot}6171\,(28) - 44{\cdot}6382\,(29) + 4{\cdot}0539\,(35)\\&+ 20{\cdot}7105\,(39) - 24{\cdot}7644\,(42) - 183{\cdot}0077\,(74) + 200{\cdot}6476\,(75) - 17{\cdot}6399\,(76)\end{aligned}$$

XXXVII.
$$\begin{aligned}0 = {}&- 438{\cdot}043 + 67{\cdot}5249\,(12) - 169{\cdot}5768\,(13) + 102{\cdot}0519\,(14) - 21{\cdot}9416\,(35)\\&+ 20{\cdot}7105\,(39) + 1{\cdot}2311\,(45) - 11{\cdot}2458\,(46) - 11{\cdot}5990\,(50) + 22{\cdot}8448\,(56)\\&+ 200{\cdot}6476\,(75) - 237{\cdot}5208\,(76) + 36{\cdot}8732\,(77)\end{aligned}$$

XXXVIII.
$$\begin{aligned}0 = {}&- 123{\cdot}443 - 20{\cdot}7105\,(35) + 257{\cdot}3350\,(37) - 236.6245\,(39) - 210{\cdot}4532\,(48)\\&+ 193{\cdot}6823\,(50) + 16{\cdot}7709\,(52) - 4{\cdot}6032\,(60) + 28{\cdot}3740\,(61) - 23{\cdot}7708\,(62)\end{aligned}$$

XXXIX.
$$\begin{aligned}0 = {}&+ 210{\cdot}553 + 6{\cdot}4799\,(46) - 33{\cdot}7504\,(48) + 27{\cdot}2705\,(50) + 12{\cdot}1622\,(60)\\&- 63{\cdot}0936\,(62) + 50{\cdot}9314\,(63) - 7{\cdot}3623\,(71) + 48{\cdot}4722\,(76) - 41{\cdot}1099\,(77)\end{aligned}$$

XL.
$$\begin{aligned}0 = {}&- 912{\cdot}923 + 20{\cdot}7105\,(35) - 447{\cdot}6529\,(38) + 426{\cdot}9424\,(39) + 351{\cdot}5191\,(49)\\&- 334{\cdot}7482\,(50) - 16{\cdot}7709\,(52) + 6{\cdot}9854\,(67) - 37{\cdot}6689\,(68) - 30{\cdot}6835\,(69)\end{aligned}$$

XLI.
$$\begin{aligned}0 = {}&- 150{\cdot}432 + 210{\cdot}4532\,(48) - 351{\cdot}5191\,(49) + 141{\cdot}0659\,(50) + 56{\cdot}1897\,(59)\\&- 79{\cdot}9605\,(60) + 23{\cdot}7708\,(62) - 36{\cdot}6129\,(64) + 67{\cdot}2964\,(67) - 30{\cdot}6835\,(69)\end{aligned}$$

XLII.
$$\begin{aligned}0 = {}&+ 35{\cdot}973 + 3{\cdot}5122\,(59) - 12{\cdot}1622\,(60) + 8{\cdot}6500\,(63) - 2{\cdot}1378\,(64) + 15{\cdot}1296\,(67)\\&- 12{\cdot}9918\,(70) + 7{\cdot}3623\,(71) - 11{\cdot}4956\,(72) + 4{\cdot}1333\,(77)\end{aligned}$$

XLIII.
$$\begin{aligned}0 = {}&+ 684{\cdot}435 + 39{\cdot}4800\,(48) - 210{\cdot}4532\,(50) + 170{\cdot}9732\,(51) - 7{\cdot}3093\,(71)\\&- 12{\cdot}1554\,(73) + 19{\cdot}4647\,(76) + 2{\cdot}0536\,(81) - 6{\cdot}4442\,(82) + 4{\cdot}3906\,(84)\end{aligned}$$

XLIV.
$$\begin{aligned}0 = {}&+ 437{\cdot}447 + 70{\cdot}5432\,(37) - 257{\cdot}3350\,(39) + 186{\cdot}7918\,(40) - 11{\cdot}8472\,(71)\\&- 12{\cdot}1554\,(73) + 24{\cdot}0026\,(75) + 0{\cdot}0099\,(80) - 6{\cdot}4442\,(82) + 6{\cdot}4343\,(84)\end{aligned}$$

XLV.
$$\begin{aligned}0 = {}&+ 86{\cdot}160 - 19{\cdot}6222\,(58) + 56{\cdot}1897\,(59) - 36{\cdot}5675\,(60) - 36{\cdot}6129\,(64)\\&+ 7{\cdot}7836\,(66) + 28{\cdot}8293\,(67) - 5{\cdot}8843\,(82) + 12{\cdot}3285\,(83) - 6{\cdot}4442\,(84)\end{aligned}$$

XLVI.
$$\begin{aligned}0 = {}&+ 1339{\cdot}052 + 687{\cdot}6802\,(39) - 707{\cdot}9681\,(40) + 20{\cdot}2879\,(42) + 746{\cdot}1758\,(73)\\&- 684{\cdot}1290\,(74) - 62{\cdot}0468\,(75) + 548{\cdot}7606\,(79) + 195{\cdot}4357\,(80) - 744{\cdot}1963\,(82)\end{aligned}$$

XLVII.
$$\begin{aligned}0 = {}&- 10{\cdot}332 - 11{\cdot}1342\,(57) - 2{\cdot}5757\,(58) + 13{\cdot}9099\,(59) - 31{\cdot}1624\,(64)\\&+ 25{\cdot}0673\,(65) + 6{\cdot}0951\,(66) - 17{\cdot}3025\,(78) - 25{\cdot}5347\,(83) + 42{\cdot}8372\,(84)\end{aligned}$$

XLVIII.
$$\begin{aligned}0 = {}&+ 34{\cdot}979 - 199{\cdot}3356\,(36) + 186{\cdot}7918\,(37) + 12{\cdot}5438\,(40) + 11{\cdot}1758\,(57)\\&- 13{\cdot}5393\,(58) + 2{\cdot}3635\,(61) - 0{\cdot}2804\,(78) - 13{\cdot}4711\,(80) + 13{\cdot}7515\,(84)\end{aligned}$$

XLIX.
$$\begin{aligned}0 = {}&- 26{\cdot}546 - 174{\cdot}2768\,(47) + 170{\cdot}9732\,(48) + 3{\cdot}3036\,(51) + 11{\cdot}1758\,(57)\\&- 11{\cdot}7085\,(58) + 0{\cdot}5327\,(62) - 2{\cdot}5751\,(78) - 11{\cdot}1764\,(81) + 13{\cdot}7515\,(84)\end{aligned}$$

L. $0 = -112{\cdot}664 + 6{\cdot}5930\,(4) + 39{\cdot}7285\,(30) - 84{\cdot}1962\,(31) + 44{\cdot}4677\,(33) - 45{\cdot}7964\,(87)$

LI. $0 = +542{\cdot}459 - 8{\cdot}8931\,(5) - 39{\cdot}6297\,(9) + 19{\cdot}5199\,(24) - 19{\cdot}5199\,(25) + 80{\cdot}8336\,(86)$

LII. $0 = -81{\cdot}473 - 27{\cdot}4199\,(7) + 15{\cdot}4634\,(8) - 4{\cdot}3687\,(19) + 4{\cdot}3687\,(21) - 5{\cdot}3031\,(22)$
$+ 5{\cdot}3031\,(26) + 71{\cdot}3477\,(85) - 45{\cdot}2226\,(86)$

LIII. $0 = +33{\cdot}544 - 29{\cdot}8555\,(6) + 15{\cdot}6761\,(16) - 15{\cdot}6761\,(17) + 110{\cdot}4537\,(88)$

LIV. $0 = +244{\cdot}094 - 20{\cdot}5791\,(5) + 8{\cdot}8215\,(8) + 26{\cdot}3305\,(86)$

The logarithmic values of the multipliers are as follow:

Log I = 0·3357402 +
II = 0·6680586 −
III = 0·9028661 −
IV = 9·7571802 −
V = 0·5552083 −
VI = 0·3087398 +
VII = 0·8361258 −
VIII = 0·0491172 +
IX = 0·6456229 +
X = 0·5568856 −
XI = 0·1802842 +
XII = 0·1082335 −
XIII = 0·0125374 +
XIV = 0·4279482 +
XV = 8·9395886 −
XVI = 9·8742507 +
XVII = 0·2832146 +
XVIII = 0·1413915 −

Log XIX = 0·3942867 +
XX = 0·5510484 −
XXI = 0·4911574 +
XXII = 0·2419524 −
XXIII = 0·6225395 −
XXIV = 0·2623455 −
XXV = 0·7412896 +
XXVI = 9·5784364 −
XXVII = 8·7449783 −
XXVIII = 9·2606399 −
XXIX = 8·9430367 +
XXX = 7·6568901 −
XXXI = 8·4468962 −
XXXII = 8·6932485 +
XXXIII = 7·5813175 −
XXXIV = 9·0902871 −
XXXV = 9·0048053 +
XXXVI = 8·1400947 +

Log XXXVII = 7·7019375 +
XXXVIII = 7·7195917 +
XXXIX = 7·8504249 −
XL = 7·8368841 +
XLI = 6·6197910 +
XLII = 8·6629859 −
XLIII = 8·2237804 −
XLIV = 8·0013494 +
XLV = 8·7777939 −
XLVI = 7·4686291 −
XLVII = 8·4151247 −
XLVIII = 6·6266218 −
XLIX = 7·1903675 +
L = 8·8289200 +
LI = 9·0049788 +
LII = 8·5614607 −
LIII = 7·8838826 −
LIV = 9·8558511 −

FIGURE 14.

Notation.

(1)	*FK*	(2)	*FD*	(3)	*FE*	(4)	*FC*	(5)	*FB*	(6)	*FH*	(7)	*HK*
(8)	*HD*	(9)	*HE*	(10)	*HC*	(11)	*HF*	(12)	*HB*	(13)	*KD*	(14)	*KB*
(15)	*KF*	(16)	*KH*	(17)	*DA*	(18)	*DC*	(19)	*DE*	(20)	*DB*	(21)	*DF*
(22)	*DH*	(23)	*DK*	(24)	*EK*	(25)	*ED*	(26)	*EC*	(27)	*EB*	(28)	*EF*
(29)	*EH*	(30)	*CK*	(31)	*CE*	(32)	*CD*	(33)	*CA*	(34)	*CB*	(35)	*CF*
(36)	*CH*	(37)	*BH*	(38)	*BF*	(39)	*BK*	(40)	*BD*	(41)	*BA*	(42)	*AC*
(43)	*AE*	(44)	*AD*	(45)	*AB*								

Equations.

I. $0 = -4{\cdot}9488 + (1) - (6) - (7) + (11) - (15) + (16)$

II. $0 = -0{\cdot}8530 + (3) - (6) - (9) + (11) - (28) + (29)$

III. $0 = +2{\cdot}9816 - (1) + (2) - (13) + (15) - (21) + (23)$

IV. $0 = -2{\cdot}6356 - (7) + (8) - (13) + (16) - (22) + (23)$

V. $0 = +0{\cdot}2722 - (2) + (3) - (19) + (21) + (25) - (28)$

VI. 0 = − 3·6662 − (3) + (4) − (26) + (28) + (31) − (35)
VII. 0 = − 2·1007 − (2) + (4) − (18) + (21) + (32) − (35)
VIII. 0 = + 2·3401 − (8) + (10) − (18) + (22) + (32) − (36)
IX. 0 = − 2·3464 − (8) + (12) − (20) + (22) − (37) + (40)
X. 0 = + 0·3093 − (2) + (5) − (20) + (21) − (38) + (40)
XI. 0 = − 0·0672 − (13) + (14) − (20) + (23) − (39) + (40)
XII. 0 = + 0·9802 − (17) + (20) − (40) + (41) + (44) − (45)
XIII. 0 = + 0·7477 − (17) + (18) − (32) + (33) − (42) + (44)
XIV. 0 = − 48·952 + 20·3640 (1) − 11·3552 (3) − 9·0088 (6) − 13·7879 (7) + 16·3964 (9)
− 2·6085 (11) − 9·7400 (24) − 6·2830 (28) + 16·0230 (29)
XV. 0 = − 15·489 − 3·1691 (1) + 14·5243 (2) − 11·3552 (3) + 3·1382 (19) + 7·9194 (21)
− 11·0576 (23) + 13·4241 (24) − 7·1411 (25) − 6·2830 (28)
XVI. 0 = − 81·658 + 23·5331 (1) − 14·5243 (2) − 9·0088 (6) − 17·9620 (7) + 20·5705 (8)
− 2·6085 (11) − 7·9194 (21) + 18·3712 (22) − 10·4518 (23)
XVII. 0 = − 95·186 − 20·7284 (1) + 14·5243 (2) + 6·2041 (4) + 11·1171 (18) + 7·9194 (21)
− 19·0365 (23) + 50·6538 (30) − 29·8373 (32) − 20·8165 (35)
XVIII. 0 = + 52·118 + 3·7618 (2) − 17·0400 (4) + 13·2782 (6) − 4·2129 (8) + 31·8814 (10)
− 27·6685 (11) − 1·1119 (32) − 3·5098 (35) + 4·6217 (36)
XIX. 0 = − 96·004 − 174·8918 (2) + 191·9318 (3) − 17·0400 (4) + 74·4236 (25)
− 44·7413 (26) − 29·6823 (28) + 205·6716 (31) − 202·1618 (32) − 3·5098 (35)
XX. 0 = − 123·427 − 20·5705 (7) + 26·9188 (8) − 6·3483 (12) − 14·7138 (13)
+ 16·3940 (14) − 1·6802 (16) − 13·2630 (37) + 36·4321 (39) − 23·1691 (40)
XXI. 0 = − 9·395 − 14·5243 (1) + 3·6595 (2) + 10·8648 (5) − 1·7669 (13) + 16·3940 (14)
− 14·6271 (15) − 30·9666 (38) + 36·4321 (39) − 5·4655 (40)
XXII. 0 = − 20·983 − 8·0967 (3) + 20·4834 (5) − 12·3867 (6) − 9·0252 (9) + 29·0319 (11)
− 20·0067 (12) − 15·8750 (27) + 30·4394 (28) − 14·5644 (29)
XXIII. 0 = + 11·953 + 8·0967 (3) + 9·2524 (4) − 17·3491 (5) − 43·4009 (26) + 73·8403 (27)
− 30·4394 (28) + 22·1434 (31) − 33·4559 (34) + 11·3125 (35)
XXIV. 0 = + 28·815 − 31·5037 (17) + 57·9946 (18) − 26·4909 (20) − 27·2306 (32)
+ 3·4649 (33) + 23·7657 (34) + 5·8788 (42) + 5·0707 (44) − 10·9495 (45)
XXV. 0 = + 446·366 − 32·2252 (17) + 59·9337 (18) − 27·7085 (19) − 233·5183 (31)
+ 205·6716 (32) + 27·8417 (33) − 49·8114 (42) + 242·7768 (43) − 192·9654 (44)

The logarithmic values of the multipliers are as follow:

Log I = 0·0867411 +
II = 0·1807674 +
III = 9·5609206 +
IV = 8·6847529 −
V = 0·2487918 +
VI = 0·5948216 +
VII = 0·1916991 +
VIII = 0·4752867 −

Log IX = 9·6790084 +
X = 0·1500379 −
XI = 0·0552831 +
XII = 8·7176344 −
XIII = 9·9465396 −
XIV = 8·6162617 +
XV = 8·3932264 +
XVI = 9·0646936 +
XVII = 8·7692891 +

Log XVIII = 8·6450689 +
XIX = 8·1572618 −
XX = 7·5556467 −
XXI = 8·5349471 +
XXII = 8·2305158 −
XXIII − 8·6931705 −
XXIV = 8·6876129 −
XXV = 7·9365944 −

FIGURE 15.

Notation.

(1)	$G'A'$	(2)	$F'A'$	(3)	$H'A'$	(4)	$H'A$	(5)	$H'B$	(6)	$L'C$	(7)	$L'D$
(8)	$L'A'$	(9)	$L'A$	(10)	$L'B$	(11)	$M'C$	(12)	$M'D$	(13)	$M'F$	(14)	$M'A$
(15)	$M'B$	(16)	$N'E'$	(17)	$N'A'$	(18)	$N'A'$	(19)	$N'B$	(20)	$N'F$	(21)	$N'C$
(22)	$N'D$	(23)	$K'A'$	(24)	AB	(25)	AC	(26)	AN'	(27)	AM'	(28)	AL'
(29)	AH'	(30)	BD	(31)	BC	(32)	BE	(33)	BF	(34)	BN'	(35)	BM'
(36)	BL'	(37)	BH'	(38)	BA	(39)	CD	(40)	CE	(41)	CE'	(42)	CN'
(43)	CF	(44)	CL'	(45)	CM'	(46)	CA	(47)	CB	(48)	DB'	(49)	DE
(50)	DE'	(51)	DN'	(52)	DF	(53)	DL'	(54)	DN'	(55)	DC	(56)	DB
(57)	$E'D'$	(58)	EC	(59)	EB'	(60)	EE'	(61)	EF	(62)	EB	(63)	EC
(64)	ED	(65)	$A'H'$	(66)	$A'K'$	(67)	$A'L'$	(68)	$A'N'$	(69)	$E'N'$	(70)	$E'C$
(71)	$E'D$	(72)	$E'E$	(73)	$L'E$	(74)	CE	(75)	CD	(76)	$B'E$	(77)	$B'D$

Equations.

I. $0 = +\,2{\cdot}5810 + (3) - (17) - (65) + (68)$

II. $0 = -\,0{\cdot}3932 - (17) + (23) - (66) + (68)$

III. $0 = +\,2{\cdot}6510 + (8) - (17) - (67) + (68)$

IV. $0 = -\,2{\cdot}3160 - (4) + (18) - (26) + (29)$

V. $0 = +\,2{\cdot}7920 - (4) + (9) - (28) + (29)$

VI. $0 = +\,1{\cdot}4589 - (4) + (14) - (27) + (29)$

VII. $0 = -\,0{\cdot}4427 - (18) + (19) - (24) + (26) - (34) + (38)$

VIII. $0 = -\,1{\cdot}3457 - (14) + (15) - (24) + (27) - (35) + (38)$

IX. $0 = -\,3{\cdot}2584 - (9) + (10) - (24) + (28) - (36) + (38)$

X. $0 = -\,0{\cdot}5341 - (4) + (5) - (24) + (29) - (37) + (38)$

XI. $0 = +\,3{\cdot}5566 - (19) + (21) - (31) + (34) - (42) + (47)$

XII. $0 = +\,0{\cdot}8705 + (11) - (15) - (31) + (35) - (45) + (47)$

XIII. $0 = +\,4{\cdot}1626 + (6) - (10) - (31) + (36) - (44) + (47)$

XIV. $0 = +\,3{\cdot}7955 - (24) + (25) - (31) + (38) - (46) + (47)$

XV. $0 = -\,0{\cdot}8394 - (19) + (22) - (30) + (34) - (51) + (56)$

XVI. $0 = -\,2{\cdot}3226 + (7) - (10) - (30) + (36) - (53) + (56)$

XVII. $0 = +\,0{\cdot}6990 + (12) - (15) - (30) + (35) - (54) + (56)$

XVIII. $0 = -\,0{\cdot}2398 - (21) + (22) - (39) + (42) - (51) + (55)$

XIX. $0 = +\,0{\cdot}1744 - (30) + (32) - (49) + (56) - (62) + (64)$

XX. $0 = +\,2{\cdot}8361 - (39) + (40) - (49) + (55) - (63) + (64)$

XXI. $0 = -\,1{\cdot}9838 - (16) + (17) - (68) + (69)$

XXII. $0 = +\,0{\cdot}5434 + (16) - (21) - (41) + (42) - (69) + (70)$

XXIII. $0 = +\,3{\cdot}3166 + (16) - (22) - (50) + (51) - (69) + (71)$

XXIV. $0 = -\,2{\cdot}5223 - (40) + (41) - (60) + (63) - (70) + (72)$

XXV. $0 = +\,0{\cdot}7754 - (59) + (60) - (72) + (76)$

XXVI. $0 = -\,2{\cdot}2156 - (48) + (50) - (71) + (77)$

XXVII. $0 = +\,1{\cdot}7682 - (58) + (59) + (74) - (76)$

XXVIII. $0 = +\,1{\cdot}0479 - (57) + (58) + (73) - (74)$

XXIX. $0 = +\,31{\cdot}734 + 34{\cdot}7236\,(3) - 1{\cdot}7012\,(23) + 16{\cdot}5873\,(65) - 17{\cdot}8476\,(66) + 1{\cdot}2603\,(68)$

XXX. $0 = -\,58{\cdot}677 + 22{\cdot}3687\,(8) - 1{\cdot}7012\,(23) - 17{\cdot}8476\,(66) + 28{\cdot}3341\,(67) - 10{\cdot}4865\,(68)$

XXXI. $0 = +\,57{\cdot}832 - 31{\cdot}0871\,(1) + 34{\cdot}7236\,(3) + 11{\cdot}9778\,(8) + 16{\cdot}5873\,(65) - 11{\cdot}9778\,(67) - 4{\cdot}6095\,(68)$

XXXII. $0 = +\,103{\cdot}732 - 111{\cdot}2059\,(1) + 61{\cdot}2787\,(2) + 88{\cdot}1951\,(3)$

XXXIII. $0 = -29{\cdot}695 + 4{\cdot}9640\,(14) - 22{\cdot}8544\,(18) - 17{\cdot}6513\,(26) + 22{\cdot}0219\,(27) - 4{\cdot}3706\,(29)$

XXXIV. $0 = -21{\cdot}728 - 7{\cdot}7204\,(9) + 4{\cdot}9640\,(14) + 22{\cdot}0219\,(27) - 41{\cdot}3147\,(28) + 19{\cdot}2928\,(29)$

XXXV. $0 = -49{\cdot}089 + 40{\cdot}0335\,(4) - 42{\cdot}5693\,(5) - 36{\cdot}7408\,(9) + 29{\cdot}0204\,(10) + 16{\cdot}9736\,(36) - 31{\cdot}4921\,(37) + 14{\cdot}5185\,(38)$

XXXVI. $0 = +27{\cdot}673 + 11{\cdot}7283\,(9) - 29{\cdot}0204\,(10) - 10{\cdot}1019\,(14) + 18{\cdot}9033\,(15) + 14{\cdot}3544\,(35) - 16{\cdot}9736\,(36) + 2{\cdot}6192\,(38)$

XXXVII. $0 = +29{\cdot}898 + 39{\cdot}9530\,(9) - 29{\cdot}0204\,(10) - 59{\cdot}2408\,(18) + 33{\cdot}7324\,(19) + 4{\cdot}4222\,(34) - 16{\cdot}9736\,(36) + 12{\cdot}5514\,(38)$

XXXVIII. $0 = +3{\cdot}135 + 10{\cdot}6087\,(11) + 18{\cdot}9033\,(14) - 29{\cdot}5120\,(15) + 19{\cdot}2764\,(24) - 24{\cdot}4475\,(25) + 5{\cdot}1711\,(27) + 10{\cdot}6503\,(45) - 36{\cdot}5053\,(46) + 25{\cdot}8550\,(47)$

XXXIX. $0 = +135{\cdot}639 + 19{\cdot}1949\,(6) + 29{\cdot}0204\,(9) - 48{\cdot}2153\,(10) + 25{\cdot}5182\,(24) - 24{\cdot}4475\,(25) - 1{\cdot}0707\,(28) + 5{\cdot}9611\,(44) - 36{\cdot}5053\,(46) + 30{\cdot}5442\,(47)$

XL. $0 = +142{\cdot}689 - 10{\cdot}6087\,(11) + 21{\cdot}0116\,(15) - 44{\cdot}4092\,(19) + 20{\cdot}4291\,(21) - 4{\cdot}8557\,(42) - 10{\cdot}6503\,(45) + 15{\cdot}5060\,(47)$

XLI. $0 = -27{\cdot}051 - 31{\cdot}5250\,(30) + 34{\cdot}8347\,(31) - 3{\cdot}3097\,(34) - 3{\cdot}0512\,(39) + 7{\cdot}9069\,(42) - 4{\cdot}8557\,(47) - 7{\cdot}1154\,(51) + 7{\cdot}2217\,(55) - 0{\cdot}1063\,(56)$

XLII. $0 = +54{\cdot}901 - 8{\cdot}2425\,(12) + 18{\cdot}6454\,(15) - 37{\cdot}8896\,(19) + 13{\cdot}9095\,(22) + 0{\cdot}1063\,(51) - 14{\cdot}6243\,(54) + 14{\cdot}5180\,(56)$

XLIII. $0 = +48{\cdot}364 - 15{\cdot}9446\,(7) + 12{\cdot}7893\,(10) - 20{\cdot}9507\,(19) + 13{\cdot}9095\,(22) + 0{\cdot}1063\,(51) - 9{\cdot}2058\,(53) + 9{\cdot}0995\,(56)$

XLIV. $0 = -67{\cdot}399 - 39{\cdot}8413\,(30) + 44{\cdot}6093\,(31) - 4{\cdot}7680\,(33) - 9{\cdot}8661\,(39) + 8{\cdot}4400\,(43) + 1{\cdot}4261\,(47) - 9{\cdot}2324\,(52) + 17{\cdot}0337\,(55) - 7{\cdot}8013\,(56)$

XLV. $0 = -58{\cdot}556 - 56{\cdot}3929\,(11) + 54{\cdot}0806\,(13) + 2{\cdot}3123\,(15) + 44{\cdot}6093\,(31) - 83{\cdot}7027\,(33) + 39{\cdot}0934\,(35) + 48{\cdot}2819\,(43) - 49{\cdot}7080\,(45) + 1{\cdot}4261\,(47)$

XLVI. $0 = +403{\cdot}235 - 77{\cdot}9730\,(12) + 110{\cdot}9468\,(13) + 40{\cdot}2962\,(20) - 29{\cdot}0575\,(22) - 57{\cdot}7187\,(51) + 139{\cdot}4160\,(52) - 81{\cdot}6973\,(54)$

XLVII. $0 = -23{\cdot}603 + 44{\cdot}6093\,(31) - 52{\cdot}4669\,(32) + 7{\cdot}8576\,(33) + 1{\cdot}5758\,(40) - 3{\cdot}0019\,(43) + 1{\cdot}4261\,(47) + 6{\cdot}4421\,(61) - 37{\cdot}8317\,(62) + 31{\cdot}3896\,(63)$

XLVIII. $0 = -58{\cdot}288 + 39{\cdot}8413\,(30) - 52{\cdot}4669\,(32) + 12{\cdot}6256\,(33) - 1{\cdot}0717\,(49) - 6{\cdot}7296\,(52) + 7{\cdot}8013\,(56) + 13{\cdot}2012\,(61) - 37{\cdot}8317\,(62) + 24{\cdot}6305\,(64)$

XLIX. $0 = +573{\cdot}179 - 37{\cdot}3870\,(39) + 165{\cdot}5312\,(40) - 128{\cdot}1442\,(41) - 185{\cdot}5757\,(49) + 156{\cdot}6582\,(50) + 28{\cdot}9175\,(55) - 75{\cdot}2742\,(70) + 112{\cdot}1791\,(71) - 36{\cdot}9049\,(72)$

L. $0 = +90{\cdot}094 + 31{\cdot}4157\,(6) + 22{\cdot}3687\,(8) + 12{\cdot}3889\,(41) - 50{\cdot}6985\,(42) + 38{\cdot}3096\,(44) + 28{\cdot}3341\,(67) - 34{\cdot}4392\,(68) - 46{\cdot}8789\,(69) + 25{\cdot}6105\,(70)$

LI. $0 = -49{\cdot}214 + 38{\cdot}5980\,(7) + 22{\cdot}3687\,(8) + 9{\cdot}7212\,(50) - 58{\cdot}5492\,(51) + 48{\cdot}8280\,(53) + 28{\cdot}3341\,(67) - 34{\cdot}4392\,(68) - 42{\cdot}7532\,(69) + 21{\cdot}4848\,(71)$

LII. $0 = +117{\cdot}284 + 43{\cdot}7850\,(48) - 104{\cdot}9318\,(49) + 61{\cdot}1468\,(50) - 46{\cdot}5335\,(59) - 32{\cdot}0790\,(60) + 78{\cdot}6125\,(64) - 34{\cdot}1043\,(71) + 23{\cdot}1229\,(72)$

LIII. $0 = +235{\cdot}239 - 18{\cdot}9651\,(16) + 11{\cdot}4521\,(17) + 7{\cdot}5130\,(22) + 61{\cdot}1468\,(48) - 70{\cdot}8680\,(50) + 9{\cdot}7212\,(51) + 6{\cdot}1051\,(68) + 93{\cdot}5124\,(77)$

LIV. $0 = +255{\cdot}567 + 20{\cdot}9242\,(58) - 32{\cdot}0790\,(59) + 11{\cdot}1548\,(60) + 98{\cdot}6375\,(74) - 132{\cdot}3168\,(76)$

LV. $0 = -7{\cdot}536 - 21{\cdot}1423\,(48) + 21{\cdot}1423\,(50) + 101{\cdot}7188\,(75) - 133{\cdot}5169\,(77)$

LVI. $0 = +112{\cdot}843 + 9{\cdot}6370\,(57) - 20{\cdot}9242\,(58) + 11{\cdot}2872\,(60) + 82{\cdot}5722\,(73) - 98{\cdot}6375\,(74)$

The logarithmic values of the multipliers are as follow:

Log I = 0·7900400 −
II = 0·6848780 +
III = 0·0787725 −
IV = 1·4159362 +
V = 1·2840132 −
VI = 0·9317037 −
VII = 1·1293676 +
VIII = 0·4055921 −
IX = 0·9833578 −
X = 9·8097320 −
XI = 0·9976422 −
XII = 8·9932290 +
XIII = 0·5908574 −
XIV = 0·3729639 −
XV = 1·2701573 +
XVI = 0·3162786 −
XVII = 0·4170129 −
XVIII = 0·4013442 −
XIX = 0·2918009 −

Log XX = 9·8315825 +
XXI = 9·2205369 +
XXII = 0·0558102 +
XXIII = 0·0784902 +
XXIV = 9·3693788 −
XXV = 9·1714078 −
XXVI = 9·9214317 −
XXVII = 9·4640362 +
XXVIII = 9·6731183 +
XXIX = 9·5644411 −
XXX = 9·3905093 +
XXXI = 8·4030635 +
XXXII = 7·4033591 −
XXXIII = 9·5961369 −
XXXIV = 9·0637900 +
XXXV = 8·3819704 −
XXXVI = 9·0888938 −
XXXVII = 9·5276980 +
XXXVIII = 9·5481392 +

Log XXXIX = 9·5415141 −
XL = 9·7145425 +
XLI = 0·0246147 −
XLII = 8·3776649 −
XLIII = 9·7766917 −
XLIV = 9·7179255 +
XLV = 8·6288226 −
XLVI = 8·4114867 +
XLVII = 7·8654356 +
XLVIII = 8·2987047 −
XLIX = 8·1864935 −
L = 8·6655906 −
LI = 9·1639869 −
LII = 8·4280849 +
LIII = 8·8982671 −
LIV = 8·6673619 −
LV = 8·7373943 −
LVI = 8·5603551 −

FIGURE 16.

Notation.

(1)	*PG*	(2)	*P'K*	(3)	*PL*	(4)	*A'G*	(5)	*A'L*	(6)	*G'L*	(7)	*FL*
(8)	*H'L*	(9)	*H'M*	(10)	*AM*	(11)	*AN*	(12)	*BN*	(13)	*BO*	(14)	*GA'*
(15)	*GP'*	(16)	*GH*	(17)	*GK*	(18)	*GL*	(19)	*LA'*	(20)	*LP'*	(21)	*LG*
(22)	*LK*	(23)	*LH*	(24)	*LI*	(25)	*LM*	(26)	*LH'*	(27)	*LF*	(28)	*KP'*
(29)	*KG*	(30)	*KH*	(31)	*KL*	(32)	*HK*	(33)	*HG*	(34)	*HI*	(35)	*IL*
(36)	*IH*	(37)	*IS*	(38)	*IR*	(39)	*IW*	(40)	*IM*	(41)	*RM*	(42)	*RI*
(43)	*RU*	(44)	*RW*	(45)	*RT*	(46)	*RV*	(47)	*ST*	(48)	*SM*	(49)	*SV*
(50)	*SR*	(51)	*SI*	(52)	*SW*	(53)	*UV*	(54)	*UM*	(55)	*UR*	(56)	*UW*
(57)	*UT*	(58)	*TM*	(59)	*TV*	(60)	*TR*	(61)	*TU*	(62)	*TS*	(63)	*TW*
(64)	*WQ*	(65)	*WN*	(66)	*WM*	(67)	*WV*	(68)	*WT*	(69)	*WF*	(70)	*WR*
(71)	*WU*	(72)	*WS*	(73)	*WP*	(74)	*MA*	(75)	*MH'*	(76)	*ML*	(77)	*MI*
(78)	*MR*	(79)	*MV*	(80)	*MS*	(81)	*MT*	(82)	*MW*	(83)	*MQ*	(84)	*MO*
(85)	*MN*	(86)	*QM*	(87)	*QW*	(88)	*QP*	(89)	*PO*	(90)	*PN*	(91)	*PQ*
(92)	*PW*	(93)	*NB*	(94)	*NA*	(95)	*NM*	(96)	*NW*	(97)	*NQ*	(98)	*NP*
(99)	*NO*	(100)	*OB*	(101)	*ON*	(102)	*OM*	(103)	*OP*				

Equations.

I. 0 = − 3·0694 − (7) + (8) − (26) + (27)
II. 0 = + 4·6972 − (5) + (8) + (19) − (26)
III. 0 = − 8·6161 − (3) + (5) − (19) + (20)
IV. 0 = + 6·9100 − (1) + (3) + (15) − (18) − (20) + (21)
V. 0 = − 1·1984 − (4) + (5) + (14) − (18) − (19) + (21)
VI. 0 = + 0·6412 − (2) + (3) − (20) + (22) + (28) − (31)
VII. 0 = − 5·5176 − (17) + (18) − (21) + (22) + (29) − (31)

VIII. $0 = + 0{\cdot}3758 - (16) + (17) - (29) + (30) - (32) + (33)$

IX. $0 = + 4{\cdot}6322 - (22) + (24) - (30) + (31) + (32) - (34) - (35) + (36)$

X. $0 = + 0{\cdot}5638 - (9) + (10) - (74) + (75)$

XI. $0 = - 2{\cdot}5037 - (8) + (9) - (25) + (26) - (75) + (76)$

XII. $0 = - 1{\cdot}6502 - (24) + (25) + (35) - (40) - (76) + (77)$

XIII. $0 = - 3{\cdot}1353 - (11) + (12) - (93) + (94)$

XIV. $0 = + 3{\cdot}0856 - (10) + (11) + (74) - (85) - (94) + (95)$

XV. $0 = - 1{\cdot}1086 - (65) + (66) - (82) + (85) - (95) + (96)$

XVI. $0 = + 3{\cdot}6593 - (39) + (40) - (67) + (69) - (77) + (82)$

XVII. $0 = + 0{\cdot}2575 - (38) + (40) - (41) + (42) - (77) + (78)$

XVIII. $0 = + 2{\cdot}8605 + (41) - (44) - (66) + (70) - (78) + (82)$

XIX. $0 = + 0{\cdot}8043 - (37) + (40) - (48) + (51) - (77) + (80)$

XX. $0 = - 3{\cdot}7892 + (48) - (52) - (66) + (72) - (80) + (82)$

XXI. $0 = - 0{\cdot}0791 + (58) - (63) - (66) + (68) - (81) + (82)$

XXII. $0 = + 0{\cdot}5438 - (47) + (48) - (58) + (62) - (80) + (81)$

XXIII. $0 = + 2{\cdot}7721 + (41) - (45) - (58) + (60) - (78) + (81)$

XXIV. $0 = - 1{\cdot}6997 - (56) + (57) - (61) + (63) - (68) + (71)$

XXV. $0 = - 1{\cdot}8928 - (43) + (45) + (55) - (57) - (60) + (61)$

XXVI. $0 = - 0{\cdot}6992 - (64) + (66) - (82) + (83) - (86) + (87)$

XXVII. $0 = - 1{\cdot}1087 + (65) - (73) - (90) + (92) - (96) + (98)$

XXVIII. $0 = + 2{\cdot}5577 + (64) - (73) - (87) + (88) - (91) + (92)$

XXIX. $0 = + 2{\cdot}8919 - (12) + (13) + (93) - (99) - (100) + (101)$

XXX. $0 = - 2{\cdot}0934 - (84) + (85) - (95) + (99) - (101) + (102)$

XXXI. $0 = - 0{\cdot}8624 - (89) + (90) - (98) + (99) - (101) + (103)$

XXXII. $0 = + 29{\cdot}386 - 61{\cdot}0081\,(5) + 10{\cdot}5815\,(7) - 5{\cdot}2001\,(19) - 1{\cdot}0990\,(26) + 6{\cdot}2991\,(27)$

XXXIII. $0 = - 124{\cdot}193 + 57{\cdot}1619\,(6) - 33{\cdot}7352\,(7) - 13{\cdot}9279\,(8)$

XXXIV. $0 = - 111{\cdot}429 + 22{\cdot}6348\,(3) + 1{\cdot}9038\,(8) - 21{\cdot}7651\,(19) + 16{\cdot}5650\,(20) + 5{\cdot}2001\,(26)$

XXXV. $0 = + 57{\cdot}519 - 26{\cdot}4182\,(1) + 49{\cdot}0530\,(3) + 16{\cdot}9247\,(4) - 15{\cdot}1795\,(5) + 17{\cdot}2430\,(14)$
$+ 9{\cdot}8838\,(15) - 27{\cdot}1268\,(18)$

XXXVI. $0 = - 15{\cdot}709 - 26{\cdot}4182\,(1) + 42{\cdot}4106\,(2) - 15{\cdot}9924\,(3) + 9{\cdot}8838\,(15) - 32{\cdot}1189\,(17)$
$+ 22{\cdot}2351\,(18) - 5{\cdot}9438\,(28) + 14{\cdot}8637\,(29) - 8{\cdot}9199\,(31)$

XXXVII. $0 = - 60{\cdot}469 + 11{\cdot}3090\,(16) - 11{\cdot}3090\,(18) - 30{\cdot}0818\,(21) + 53{\cdot}3635\,(22)$
$- 23{\cdot}2817\,(23) + 37{\cdot}7995\,(29) - 22{\cdot}9358\,(30) - 14{\cdot}8637\,(31) + 65{\cdot}0596\,(32)$
$- 65{\cdot}0596\,(33)$

XXXVIII. $0 = + 277{\cdot}957 + 16{\cdot}9247\,(4) - 77{\cdot}9328\,(5) + 10{\cdot}2047\,(8) - 12{\cdot}1085\,(9) + 17{\cdot}2430\,(14)$
$+ 24{\cdot}5349\,(16) - 41{\cdot}7779\,(18) + 11{\cdot}3090\,(21) - 19{\cdot}7099\,(23) + 8{\cdot}4009\,(24)$
$- 23{\cdot}9618\,(35) + 18{\cdot}2046\,(36) + 5{\cdot}7572\,(40) + 86{\cdot}3055\,(75) - 102{\cdot}1626\,(76)$
$+ 15{\cdot}8571\,(77)$

XXXIX. $0 = - 255{\cdot}699 + 61{\cdot}0081\,(5) - 4{\cdot}2260\,(10) + 5{\cdot}2001\,(19) + 20{\cdot}0599\,(25)$
$- 25{\cdot}2600\,(26) + 23{\cdot}4327\,(74) - 109{\cdot}7382\,(75) + 86{\cdot}3055\,(76)$

XL. $0 = + 23{\cdot}415 + 28{\cdot}2373\,(9) + 20{\cdot}0946\,(12) - 21{\cdot}8990\,(74) + 23{\cdot}4327\,(75)$
$- 1{\cdot}5337\,(85) + 36{\cdot}3995\,(93) - 53{\cdot}6225\,(94) + 17{\cdot}2230\,(95)$

XLI. $0 = + 69{\cdot}123 - 12{\cdot}1085\,(8) - 16{\cdot}1288\,(9) - 25{\cdot}7130\,(10) + 29{\cdot}9390\,(11)$
$+ 16{\cdot}2870\,(24) - 30{\cdot}3469\,(25) + 20{\cdot}0599\,(26) + 5{\cdot}7572\,(35) + 30{\cdot}2933\,(39)$
$- 36{\cdot}0505\,(40) + 24{\cdot}6464\,(65) - 34{\cdot}0888\,(66) + 9{\cdot}4424\,(69) + 17{\cdot}2230\,(94)$
$- 28{\cdot}5676\,(95) + 11{\cdot}3446\,(96)$

XLII. 0 = + 65·767 − 12·8017 (38) + 30·2933 (39) − 17·4916 (40) + 17·8608 (41) + 3·0018 (42) − 20·8626 (44) − 5·1855 (66) + 9·4424 (69) − 4·2569 (70)

XLIII. 0 = − 2·569 − 11·9875 (37) + 30·2933 (39) − 18·3058 (40) + 24·2278 (48) − 3·3031 (51) − 20·9247 (52) − 10·9285 (66) + 9·4424 (69) + 1·4861 (72)

XLIV. 0 = − 105·246 − 20·8626 (41) + 20·8626 (44) + 26·6409 (48) − 5·7162 (50) − 20·9247 (52) − 5·7430 (66) + 4·2569 (70) + 1·4861 (72) + 1·3506 (78) − 1·3506 (80)

XLV. 0 = + 67·015 − 54·4884 (47) + 33·5637 (48) + 20·9247 (52) − 30·1458 (58) + 31·4698 (62) − 1·3240 (63) + 11·0938 (66) − 9·6077 (68) − 1·4861 (72)

XLVI. 0 = − 4·458 + 9·9425 (58) − 62·9067 (60) + 52·9642 (63) + 4·2569 (66) − 90·4918 (68) + 86·2349 (70) + 25·4172 (78) − 39·5297 (81) + 14·1125 (82)

XLVII. 0 = − 17·469 − 11·3456 (54) − 14·1015 (56) + 25·4471 (57) + 12·9641 (58) − 14·2881 (61) + 1·3240 (63) − 5·6867 (66) + 9·6077 (68) − 3·9210 (71)

XLVIII. 0 = − 86·845 + 100·7245 (41) − 100·7245 (43) − 88·2373 (54) + 146·9642 (55) − 58·7269 (57) − 187·6722 (60) + 134·7080 (61) + 52·9642 (63) + 4·2569 (66) − 90·4918 (68) + 86·2349 (70) − 14·1125 (78) + 14·1125 (82)

XLIX. 0 = + 0·576 + 15·3323 (64) − 24·6464 (65) + 9·3141 (66) − 10·6818 (83) + 10·6818 (85) − 1·0653 (86) + 1·0653 (87) − 3·7936 (95) − 11·3446 (96) + 15·1382 (97)

L. 0 = − 146·077 − 110·6033 (64) + 88·1705 (65) + 22·4328 (73) + 39·3772 (90) − 71·6652 (91) + 32·2880 (92) + 71·7001 (96) − 92·1922 (97) + 20·4921 (98)

LI. 0 = − 77·055 + 29·9390 (10) − 24·8394 (11) − 100·8645 (12) + 80·7699 (13) − 1·5337 (74) + 107·8566 (84) − 106·3229 (85) + 23·0536 (100) − 116·4577 (101) + 93·4041 (102)

LII. 0 = + 86·329 − 38·5213 (65) + 24·6464 (66) + 13·8749 (73) + 4·5490 (82) − 107·8566 (84) + 103·3076 (85) + 31·4541 (89) − 43·0091 (90) + 11·5550 (92) + 102·6810 (101) − 93·4041 (102) − 9·2769 (103)

LIII. 0 = + 346·015 + 0·5666 (58) − 86·0403 (59) + 85·4737 (63) − 9·6077 (66) + 249·3621 (67) − 239·7544 (68) − 72·7641 (79) + 112·1542 (81) − 39·3901 (82)

LIV. 0 = − 189·796 − 61·5715 (41) − 65·5663 (45) + 127·1378 (46) − 6·6636 (58) + 29·3703 (59) − 22·7067 (60) − 98·8738 (78) + 172·2045 (79) − 73·3307 (81)

LV. 0 = + 118·968 + 109·9578 (43) − 163·6805 (45) + 53·7227 (46) − 26·2609 (53) − 58·7269 (55) + 84·9878 (57) + 12·1127 (59) − 11·8436 (60) − 0·2691 (61)

LVI. 0 = + 735·308 + 222·3683 (48) − 205·0657 (49) − 17·3026 (52) − 13·0796 (66) + 42·1811 (67) − 29·1015 (72) − 336·6793 (79) + 358·3716 (80) − 21·6923 (82)

The logarithmic values of the multipliers are as follow:

Log I = 9·3256339 +
II = 0·6845849 +
III = 1·0246388 +
IV = 9·6581463 −
V = 9·9336864 −
VI = 0·3399802 +

Log VII = 0·3608894 +
VIII = 0·6390802 −
IX = 0·0476890 −
X = 0·6083462 +
XI = 0·6917649 +
XII = 0·5259586 +

Log XIII = 0·7352728 +
XIV = 0·4020549 +
XV = 0·1722565 +
XVI = 0·4884429 −
XVII = 0·4404791 −
XVIII = 0·2099442 +

Log XIX = 0·7913370 +
XX = 0·7944788 +
XXI = 9·8760980 −
XXII = 0·2406153 −
XXIII = 0·5464470 −
XXIV = 0·7173478 +
XXV = 9·4038878 +
XXVI = 8·4093937 +
XXVII = 0·4476657 +
XXVIII = 0·7146212 −
XXIX = 9·7251423 +
XXX = 9·7505042 −
XXXI = 0·0345124 +

Log XXXII = 8·5634107 +
XXXIII = 7·9934192 +
XXXIV = 9·0246809 −
XXXV = 8·0030519 −
XXXVI = 8·3944275 −
XXXVII = 8·8339300 −
XXXVIII = 8·8292242 +
XXXIX = 8·9160489 +
XL = 8·7511509 +
XLI = 8·5832032 +
XLII = 9·4819076 +
XLIII = 9·6709376 −
XLIV = 9·6597939 +

Log XLV = 8·5405967 +
XLVI = 8·7553162 −
XLVII = 8·6517256 −
XLVIII = 8·3312160 +
XLIX = 9·4645718 +
L = 8·7451251 +
LI = 8·3040449 +
LII = 8·1746653 +
LIII = 7·8961683 −
LIV = 9·9717976 −
LV = 8·3779216 +
LVI = 7·8079933 −

FIGURE 17.

Notation.

(1)	*D'B*	(2)	*D'L'*	(3)	*D'C*	(4)	*A'A*	(5)	*B'A*	(6)	*CB*	(7)	*CA*
(8)	*CD*	(9)	*E'L'*	(10)	*E'C*	(11)	*E'H*	(12)	*E'G*	(13)	*E'F*	(14)	*E'E*
(15)	*DC'*	(16)	*DA*	(17)	*DB*	(18)	*DC*	(19)	*DL'*	(20)	*CD*	(21)	*CA*
(22)	*CB*	(23)	*CD'*	(24)	*CE'*	(25)	*CL'*	(26)	*BD*	(27)	*BA*	(28)	*BC*
(29)	*BB'*	(30)	*BD'*	(31)	*BE'*	(32)	*BL'*	(33)	*BC*	(34)	*L'D*	(35)	*L'A*
(36)	*L'C*	(37)	*L'D'*	(38)	*L'E'*	(39)	*L'E*	(40)	*L'F*	(41)	*M'F*	(42)	*M'E*
(43)	*M'E'*	(44)	*M'G*	(45)	*N'G*	(46)	*N'K*	(47)	*P'K*	(48)	*Q'K*	(49)	*GM'*
(50)	*GF*	(51)	*GE*	(52)	*GE'*	(53)	*GH*	(54)	*GK'*	(55)	*GH'*	(56)	*GK*
(57)	*GN'*	(58)	*KP'*	(59)	*KN'*	(60)	*KG*	(61)	*KK'*	(62)	*KH'*	(63)	*KQ'*
(64)	*HE'*	(65)	*HH'*	(66)	*HK'*	(67)	*HG*	(68)	*KG*	(69)	*KH*	(70)	*KK*
(71)	*H'G*	(72)	*H'H*	(73)	*HK*	(74)	*G'H*	(75)	*F'H*				

Equations.

I. 0 = + 2·3877 − (1) + (6) − (28) + (30)
II. 0 = + 1·5569 − (6) + (8) − (15) + (17) − (26) + (28)
III. 0 = − 8·1617 − (17) + (18) − (20) + (22) + (26) − (33)
IV. 0 = − 0·0287 + (1) − (3) − (22) + (23) − (30) + (33)
V. 0 = + 9·7167 + (3) − (10) − (23) + (24)
VI. 0 = + 5·5764 − (2) + (3) − (23) + (25) − (36) + (37)
VII. 0 = − 4·8792 − (9) + (10) − (24) + (25) − (36) + (38)
VIII. 0 = + 0·2026 − (18) + (19) + (20) − (25) − (34) + (36)
IX. 0 = − 1·6392 − (44) + (45) + (49) − (57)
X. 0 = + 1·4024 + (9) − (12) − (38) + (44) − (49) + (52)
XI. 0 = + 1·4227 + (12) − (52) + (55) − (71)
XII. 0 = + 5·4146 − (54) + (55) + (68) − (71)
XIII. 0 = − 4·2153 − (61) + (62) + (70) − (73)
XIV. 0 = − 7·0631 − (55) + (56) − (60) + (62) + (71) − (73)
XV. 0 = + 3·3134 − (45) + (46) − (56) + (57) − (59) + (60)

XVI. $0 = + 2{\cdot}4934 - (46) + (48) + (59) - (63)$

XVII. $0 = + 3{\cdot}4954 - (47) + (48) + (58) - (63)$

XVIII. $0 = + 2{\cdot}1383 - (11) + (12) - (52) + (53) + (64) - (67)$

XIX. $0 = + 6{\cdot}6650 - (53) + (55) - (65) + (67) - (71) + (72)$

XX. $0 = + 0{\cdot}1715 - (53) + (54) - (66) + (67) - (68) + (69)$

XXI. $0 = + 353{\cdot}977 - 81{\cdot}0997\ (1) + 117{\cdot}1668\ (6) + 13{\cdot}7259\ (28) - 114{\cdot}0522\ (29) + 100{\cdot}3263\ (30)$

XXII. $0 = - 747{\cdot}061 + 210{\cdot}6435\ (1) + 95{\cdot}8186\ (29) - 210{\cdot}4861\ (30) + 114{\cdot}6675\ (31)$

XXIII. $0 = + 48{\cdot}659 + 69{\cdot}4454\ (1) - 94{\cdot}5083\ (5) - 180{\cdot}7518\ (6) + 183{\cdot}4765\ (7) + 95{\cdot}7861\ (27) - 118{\cdot}8276\ (28) + 92{\cdot}4869\ (29) - 69{\cdot}4454\ (30)$

XXIV. $0 = - 52{\cdot}401 - 3{\cdot}1614\ (1) - 21{\cdot}3233\ (4) + 25{\cdot}2849\ (5) + 10{\cdot}5337\ (27) - 18{\cdot}5338\ (29) + 8{\cdot}0001\ (30)$

XXV. $0 = - 71{\cdot}696 + 180{\cdot}7518\ (6) - 215{\cdot}3438\ (7) + 34{\cdot}5920\ (8) + 8{\cdot}3434\ (15) - 38{\cdot}6207\ (16) + 30{\cdot}2773\ (17) + 38{\cdot}3264\ (26) - 157{\cdot}1540\ (27) + 118{\cdot}8276\ (28)$

XXVI. $0 = - 1350{\cdot}582 + 17{\cdot}5884\ (1) + 92{\cdot}4097\ (3) + 36{\cdot}5403\ (22) + 83{\cdot}6396\ (23) - 120{\cdot}1799\ (24) - 110{\cdot}3172\ (30) + 232{\cdot}5314\ (31) - 122{\cdot}2142\ (33)$

XXVII. $0 = + 311{\cdot}144 - 120{\cdot}9827\ (1) + 92{\cdot}4097\ (3) - 157{\cdot}6019\ (6) + 26{\cdot}9769\ (8) - 4{\cdot}9379\ (15) - 61{\cdot}7554\ (17) + 66{\cdot}6933\ (18) - 16{\cdot}9297\ (20) - 66{\cdot}7099\ (22) + 83{\cdot}6396\ (23)$

XXVIII. $0 = + 43{\cdot}248 - 14{\cdot}0254\ (16) + 30{\cdot}2773\ (17) - 16{\cdot}2519\ (18) - 18{\cdot}9531\ (20) + 22{\cdot}3658\ (21) - 3{\cdot}4127\ (22) + 38{\cdot}3264\ (26) - 27{\cdot}3464\ (27) - 10{\cdot}9800\ (33)$

XXIX. $0 = - 15{\cdot}719 + 36{\cdot}4925\ (2) - 32{\cdot}0293\ (3) - 15{\cdot}3723\ (9) + 71{\cdot}4379\ (10) - 90{\cdot}9876\ (36) + 98{\cdot}8469\ (37) - 7{\cdot}8593\ (38)$

XXX. $0 = - 315{\cdot}060 + 92{\cdot}4097\ (1) + 36{\cdot}4925\ (2) - 128{\cdot}9022\ (3) + 66{\cdot}6933\ (17) - 70{\cdot}6952\ (18) + 4{\cdot}0019\ (19) + 31{\cdot}6004\ (26) - 41{\cdot}3849\ (30) + 9{\cdot}7845\ (33) + 52{\cdot}9391\ (34) - 151{\cdot}7860\ (36) + 98{\cdot}8469\ (37)$

XXXI. $0 = - 1254{\cdot}334 - 92{\cdot}4097\ (1) - 36{\cdot}4925\ (2) + 128{\cdot}9022\ (3) + 260{\cdot}3625\ (22) - 260{\cdot}3625\ (25) + 41{\cdot}3849\ (30) + 309{\cdot}3628\ (32) - 350{\cdot}7477\ (33) + 98{\cdot}8469\ (36) - 98{\cdot}8469\ (37)$

XXXII. $0 = - 1626{\cdot}991 - 34{\cdot}9296\ (16) + 16{\cdot}2519\ (18) + 18{\cdot}6777\ (19) + 18{\cdot}9531\ (20) - 34{\cdot}8662\ (21) + 15{\cdot}9131\ (25) - 117{\cdot}6780\ (34) + 220{\cdot}7584\ (35) - 103{\cdot}0804\ (36)$

XXXIII. $0 = + 399{\cdot}063 + 27{\cdot}2897\ (9) + 105{\cdot}2039\ (12) - 132{\cdot}4936\ (13) + 38{\cdot}6732\ (38) - 63{\cdot}8167\ (40) - 126{\cdot}7512\ (41) + 215{\cdot}3279\ (43) - 105{\cdot}2039\ (44) + 105{\cdot}2039\ (49) - 105{\cdot}2039\ (52)$

XXXIV. $0 = + 306{\cdot}215 + 28{\cdot}4679\ (9) + 95{\cdot}1081\ (12) - 123{\cdot}5760\ (14) + 82{\cdot}6460\ (38) - 97{\cdot}8229\ (39) - 239{\cdot}0389\ (42) + 320{\cdot}5728\ (43) - 95{\cdot}1081\ (44) + 95{\cdot}1081\ (49) - 95{\cdot}1081\ (52)$

XXXV. $0 = + 674{\cdot}012 + 69{\cdot}6360\ (12) - 69{\cdot}6360\ (14) - 206{\cdot}0916\ (42) + 320{\cdot}5728\ (43) - 114{\cdot}4812\ (44) + 94{\cdot}7410\ (49) + 52{\cdot}8528\ (51) - 147{\cdot}5938\ (52)$

XXXVI. $0 = + 580{\cdot}243 + 78{\cdot}6498\ (12) - 78{\cdot}6498\ (13) - 94{\cdot}1838\ (41) + 215{\cdot}3279\ (43) - 121{\cdot}1441\ (44) + 96{\cdot}4270\ (49) + 30{\cdot}9732\ (50) - 127{\cdot}4002\ (52)$

XXXVII. $0 = - 282{\cdot}655 + 26{\cdot}6146\ (38) - 59{\cdot}4819\ (43) + 41{\cdot}1578\ (44) + 7{\cdot}1393\ (45) - 49{\cdot}8894\ (49) + 33{\cdot}1343\ (52) + 16{\cdot}7551\ (57)$

XXXVIII. $0 = - 73{\cdot}192 + 18{\cdot}9288\ (2) + 13{\cdot}4115\ (37) - 21{\cdot}7020\ (38) - 34{\cdot}2949\ (43) + 23{\cdot}3458\ (44) - 8{\cdot}0235\ (49) + 8{\cdot}9347\ (52) - 0{\cdot}9112\ (55) + 21{\cdot}5303\ (71)$

XXXIX. $0 = + 681{\cdot}838 + 22{\cdot}4513\ (12) - 0{\cdot}9112\ (52) - 297{\cdot}1907\ (54) + 298{\cdot}1019\ (55) + 63{\cdot}2619\ (68)$

XL. $0 = -108{\cdot}640 + 30{\cdot}5880\,(46) - 27{\cdot}0816\,(47) - 32{\cdot}9671\,(58) + 15{\cdot}3980\,(59) + 17{\cdot}5691\,(63)$

XLI. $0 = -124{\cdot}044 + 13{\cdot}5475\,(44) - 0{\cdot}6019\,(48) + 16{\cdot}7551\,(49) + 10{\cdot}0517\,(56) - 26{\cdot}8068\,(57)$
$- 46{\cdot}2081\,(59) + 30{\cdot}8101\,(60) + 15{\cdot}3980\,(63)$

XLII. $0 = +396{\cdot}681 - 22{\cdot}4513\,(12) + 64{\cdot}5036\,(43) - 78{\cdot}0511\,(44) - 10{\cdot}4092\,(45) + 3{\cdot}2699\,(46)$
$+ 41{\cdot}1578\,(49) - 41{\cdot}1578\,(52) + 30{\cdot}8101\,(59) - 41{\cdot}2277\,(60) + 10{\cdot}4176\,(62)$
$- 26{\cdot}9216\,(71) + 5{\cdot}3913\,(73)$

XLIII. $0 = -87{\cdot}452 - 9{\cdot}5087\,(60) + 19{\cdot}9263\,(61) - 10{\cdot}4176\,(62) - 63{\cdot}7454\,(68) + 0{\cdot}4835\,(70)$
$+ 87{\cdot}6473\,(71) - 5{\cdot}3913\,(73)$

XLIV. $0 = -471{\cdot}692 + 32{\cdot}9743\,(52) - 32{\cdot}0631\,(53) - 0{\cdot}9112\,(55) - 71{\cdot}4693\,(64) + 61{\cdot}8643\,(65)$
$+ 9{\cdot}6050\,(67) + 21{\cdot}5303\,(71) - 153{\cdot}3506\,(72)$

XLV. $0 = -534{\cdot}581 - 12{\cdot}5582\,(53) + 297{\cdot}1907\,(54) - 284{\cdot}6325\,(55) - 161{\cdot}3413\,(65)$
$+ 163{\cdot}4534\,(66) - 2{\cdot}1121\,(67) - 63{\cdot}2619\,(68) - 37{\cdot}1746\,(69)$

XLVI. $0 = +298{\cdot}277 - 75{\cdot}4616\,(11) - 149{\cdot}9509\,(72) + 65{\cdot}8909\,(75)$

XLVII. $0 = +33{\cdot}891 - 11{\cdot}7082\,(72) + 28{\cdot}7347\,(74) - 14{\cdot}3470\,(75)$

The logarithmic values of the multipliers are as follow:

Log I = 0·7001492 +
II = 9·9057507 −
III = 8·9590199 −
IV = 1·2961872 −
V = 1·2402109 −
VI = 9·6507133 +
VII = 0·8211164 −
VIII = 8·8466519 −
IX = 0·1134610 −
X = 0·4891544 −
XI = 9·8353075 −
XII = 1·8623616 −
XIII = 0·8035058 −
XIV = 0·5816320 +
XV = 0·5939799 −
XVI = 0·2916685 +

Log XVII = 0·2378173 +
XVIII = 0·1679205 −
XIX = 0·1889868 −
XX = 8·4455108 +
XXI = 8·8330113 +
XXII = 9·2309470 +
XXIII = 8·3120561 −
XXIV = 7·7648878 +
XXV = 7·5461266 −
XXVI = 8·5621584 −
XXVII = 9·0376488 +
XXVIII = 8·2080272 −
XXIX = 9·1520939 −
XXX = 9·0710051 +
XXXI = 8·2070257 −
XXXII = 7·8366746 +

Log XXXIII = 7·9870447 −
XXXIV = 8·0596783 −
XXXV = 8·1944041 +
XXXVI = 7·7868391 +
XXXVII = 9·5493557 +
XXXVIII = 7·9435623 −
XXXIX = 9·3942400 +
XL = 9·2091238 +
XLI = 9·4905609 +
XLII = 9·3080893 +
XLIII = 9·9093460 −
XLIV = 7·8564933 −
XLV = 7·4793262 +
XLVI = 8·0836415 −
XLVII = 7·9248022 −

FIGURE 18.

Notation.

(1)	*A'D*	(2)	*A'C*	(3)	*C'C*	(4)	*D'D*	(5)	*D'C*	(6)	*D'E*	(7)	*D'B*
(8)	*E'B*	(9)	*H'B*	(10)	*I'B*	(11)	*I'A*	(12)	*AI'*	(13)	*AF'*	(14)	*AB*
(15)	*AG*	(16)	*AF*	(17)	*BI'*	(18)	*BH'*	(19)	*BF'*	(20)	*BE'*	(21)	*BD'*
(22)	*BD*	(23)	*BE*	(24)	*BG*	(25)	*BF*	(26)	*BA*	(27)	*CC'*	(28)	*CB'*
(29)	*CA'*	(30)	*CD*	(31)	*CE*	(32)	*ED'*	(33)	*EC*	(34)	*ED*	(35)	*EG*
(36)	*EF*	(37)	*EB*	(38)	*DC*	(39)	*DD'*	(40)	*DC'*	(41)	*DA'*	(42)	*DG*
(43)	*DF*	(44)	*DE*	(45)	*DB*	(46)	*FA*	(47)	*FB*	(48)	*FE*	(49)	*FD*
(50)	*FG*	(51)	*GD*	(52)	*GF*								

Equations.

I. $0 = -2.8722 - (7) + (8) - (20) + (21)$

II. $0 = -0.6059 - (7) + (10) - (17) + (21)$

III. $0 = -2.2503 - (8) + (9) - (18) + (20)$

IV. $0 = -2.6353 - (10) + (11) - (12) + (14) + (17) - (26)$

V. $0 = -3.1593 - (1) + (4) - (39) + (41)$

VI. $0 = +1.9694 - (4) + (7) - (21) + (22) + (39) - (45)$

VII. $0 = -2.0604 - (1) + (2) - (29) + (30) - (38) + (41)$

VIII. $0 = +1.8252 - (2) + (3) - (27) + (29)$

IX. $0 = -1.4803 - (6) + (7) - (21) + (23) + (32) - (37)$

X. $0 = -2.5461 - (4) + (6) - (32) + (34) + (39) - (44)$

XI. $0 = +1.0512 - (30) + (31) - (33) + (34) + (38) - (44)$

XII. $0 = +5.5268 - (22) + (25) - (43) + (45) - (47) + (49)$

XIII. $0 = +4.1169 - (23) + (25) - (36) + (37) - (47) + (48)$

XIV. $0 = +0.2976 - (14) + (16) - (25) + (26) - (46) + (47)$

XV. $0 = -8.5792 - (42) + (43) - (49) + (50) + (51) - (52)$

XVI. $0 = -5.596 - 28.7221\,(7) + 3.7491\,(9) + 17.8248\,(18) - 76.2202\,(20) + 58.3954\,(21)$

XVII. $0 = +233.224 + 48.6108\,(7) - 88.9196\,(8) - 58.1163\,(9) + 46.9010\,(10)$

XVIII. $0 = -20.745 + 0.0669\,(7) + 3.3376\,(10) + 18.7311\,(17) - 46.5300\,(19) + 27.7989\,(21)$

XIX. $0 = +21.666 + 3.3376\,(10) + 8.7475\,(11) - 35.5933\,(12) + 70.9164\,(13) - 35.3231\,(14)$
$+ 18.7311\,(17) - 40.0599\,(19) + 21.3288\,(26)$

XX. $0 = +244.018 + 4.4811\,(1) - 8.8085\,(4) - 39.9016\,(39) + 69.4362\,(40) - 29.5346\,(41)$

XXI. $0 = -137.761 + 4.4895\,(1) + 71.4678\,(8) + 58.3954\,(20) - 73.5017\,(21) + 15.1063\,(22)$
$- 33.8196\,(39) + 10.5875\,(41) + 23.2321\,(45)$

XXII. $0 = +169.432 + 4.4811\,(1) + 20.5244\,(2) - 23.6729\,(27) + 23.3305\,(29) + 0.3424\,(30)$
$- 25.6645\,(38) + 55.1991\,(40) - 29.5346\,(41)$

XXIII. $0 = +179.194 + 14.0512\,(1) - 67.0136\,(2) - 45.8222\,(4) + 35.2837\,(5) + 97.3671\,(38)$
$- 103.0677\,(39) + 5.7006\,(41)$

XXIV. $0 = -189.895 - 6.7325\,(2) + 84.3241\,(3) - 288.8981\,(27) + 315.9476\,(28) - 27.0495\,(29)$

XXV. $0 = +63.950 + 8.1408\,(21) - 15.1063\,(22) + 6.9655\,(23) - 10.4322\,(32) + 4.6343\,(34)$
$+ 5.7979\,(37) + 7.1577\,(39) + 16.0744\,(44) - 23.2321\,(45)$

XXVI. $0 = -76.050 + 45.8222\,(4) - 90.7836\,(5) + 44.9614\,(6) + 74.3379\,(32) - 85.6404\,(33)$
$+ 11.3025\,(34) - 127.2083\,(38) + 103.0677\,(39) + 24.1406\,(44)$

XXVII. $0 = -269.977 + 67.3836\,(22) - 76.5746\,(23) + 9.1910\,(25) + 38.9499\,(43)$
$- 153.0616\,(44) + 114.1117\,(45) + 34.7915\,(47) - 77.5774\,(48) + 42.7859\,(49)$

XXVIII. $0 = +14.023 + 31.5701\,(6) - 51.4588\,(7) - 53.3034\,(10) + 34.2676\,(11) + 11.4774\,(12)$
$- 8.1275\,(14) - 3.3499\,(16) + 5.7979\,(32) + 2.8086\,(36) - 8.6065\,(37)$
$+ 34.2463\,(46) - 69.0378\,(47) + 34.7915\,(48)$

XXIX. $0 = -65.615 + 12.1620\,(34) - 21.7944\,(35) + 9.6324\,(36) + 12.8611\,(42)$
$- 16.3782\,(43) + 3.5171\,(44) + 6.4480\,(48) - 19.7925\,(49) + 13.3445\,(50)$

XXX. $0 = -50.887 + 24.5500\,(22) - 47.0098\,(24) + 22.4598\,(25) + 16.7350\,(42)$
$- 16.3782\,(43) - 0.3568\,(45) - 5.3101\,(47) - 19.7925\,(49) + 25.1026\,(50)$

XXXI. $0 = -34.701 + 7.1557\,(14) - 47.0731\,(15) + 39.9174\,(16) + 23.3892\,(24)$
$- 22.4598\,(25) - 0.9294\,(26) - 21.6055\,(46) + 5.3101\,(47) + 16.2954\,(50)$

The logarithmic values of the multipliers are as follow:

Log I = 0·8157103 +
II = 9·1267212 −
III = 0·3289483 +
IV = 9·8845124 +
V = 0·8016538 +
VI = 9·8885949 −
VII = 0·7593692 −
VIII = 0·7323468 −
IX = 0·2900560 +
X = 0·4917551 +

Log XI = 0·4021730 −
XII = 9·8579187 −
XIII = 9·3607321 +
XIV = 9·7667154 −
XV = 9·7241568 +
XVI = 8·7966108 −
XVII = 8·2738882 +
XVIII = 8·3464482 −
XIX = 7·8230366 +
XX = 8·9900697 −
XXI = 7·9599091 −

Log XXII = 8·8972770 +
XXIII = 8·2777369 +
XXIV = 7·4915465 +
XXV = 8·9900848 −
XXVI = 8·3841822 +
XXVII = 7·6891100 −
XXVIII = 8·3349969 −
XXIX = 8·6074617 +
XXX = 7·7253128 +
XXXI = 8·0188482 +

FIGURE 19.

Notation.

(1)	*CA*	(2)	*B′B*	(3)	*A′A*	(4)	*A′B*	(5)	*D′B*	(6)	*D′E*	(7)	*D′F*
(8)	*D′C*	(9)	*E′C*	(10)	*E′F*	(11)	*E′G*	(12)	*AB′*	(13)	*AC′*	(14)	*AL*
(15)	*AK*	(16)	*AD*	(17)	*AB*	(18)	*AA′*	(19)	*BA′*	(20)	*BB′*	(21)	*BA*
(22)	*BD*	(23)	*BE*	(24)	*BC*	(25)	*BD′*	(26)	*CD′*	(27)	*CE*	(28)	*CF*
(29)	*CG*	(30)	*CE′*	(31)	*FC*	(32)	*FD′*	(33)	*FB*	(34)	*FE*	(35)	*FI*
(36)	*FH*	(37)	*FG*	(38)	*FE′*	(39)	*GC*	(40)	*GF*	(41)	*GH*	(42)	*GE′*
(43)	*HF*	(44)	*HE*	(45)	*HI*	(46)	*HG*	(47)	*EB*	(48)	*ED*	(49)	*EK*
(50)	*EM*	(51)	*EI*	(52)	*EH*	(53)	*EF*	(54)	*EC*	(55)	*DA*	(56)	*DL*
(57)	*DM*	(58)	*DE*	(59)	*DB*	(60)	*LA*	(61)	*LM*	(62)	*LK*	(63)	*LD*
(64)	*MK*	(65)	*ML*	(66)	*MI*	(67)	*ME*	(68)	*IE*	(69)	*IK*	(70)	*IM*
(71)	*IH*	(72)	*KA*	(73)	*KL*	(74)	*KM*	(75)	*KI*	(76)	*KE*		

Equations.

I. $0 = +\,2{\cdot}8486 - (1) + (3) + (13) - (18)$
II. $0 = -\,3{\cdot}2191 - (4) + (5) + (19) - (25)$
III. $0 = -\,0{\cdot}3284 - (1) + (2) + (13) - (17) - (20) + (21)$
IV. $0 = -\,0{\cdot}6592 - (3) + (4) - (17) + (18) - (19) + (21)$
V. $0 = +\,2{\cdot}6535 - (8) + (9) + (26) - (30)$
VI. $0 = -\,0{\cdot}1104 - (9) + (10) - (28) + (30) + (31) - (38)$
VII. $0 = -\,1{\cdot}8525 - (7) + (10) + (32) - (38)$
VIII. $0 = -\,4{\cdot}3456 - (27) + (28) - (31) + (34) - (53) + (54)$
IX. $0 = -\,1{\cdot}0715 - (5) + (8) - (23) + (25) - (26) + (27) + (47) - (54)$
X. $0 = +\,0{\cdot}6456 - (16) + (17) - (21) + (22) + (55) - (59)$
XI. $0 = +\,0{\cdot}3109 - (22) + (23) - (47) + (48) - (58) + (59)$
XII. $0 = +\,2{\cdot}0204 - (9) + (11) - (29) + (30) + (39) - (42)$
XIII. $0 = -\,1{\cdot}2054 - (10) + (11) - (37) + (38) + (40) - (42)$
XIV. $0 = +\,6{\cdot}1630 - (36) + (37) - (40) + (41) + (43) - (46)$
XV. $0 = +\,0{\cdot}0364 - (34) + (36) - (43) + (44) - (52) + (53)$

XVI. $0 = -4{\cdot}0510 - (44) + (45) - (51) + (52) + (68) - (71)$

XVII. $0 = -4{\cdot}5065 - (14) + (16) - (55) + (56) + (60) - (63)$

XVIII. $0 = -4{\cdot}6427 - (50) + (51) - (66) + (67) - (68) + (70)$

XIX. $0 = +1{\cdot}4200 - (48) + (50) - (56) + (58) - (61) + (63) + (65) - (67)$

XX. $0 = +0{\cdot}3535 - (49) + (51) - (68) + (69) - (75) + (76)$

XXI. $0 = +0{\cdot}1835 + (64) - (66) - (69) + (70) - (74) + (75)$

XXII. $0 = -2{\cdot}8148 - (61) + (62) - (64) + (65) - (73) + (74)$

XXIII. $0 = -4{\cdot}5110 - (14) + (15) + (60) - (62) - (72) + (73)$

XXIV. $0 = +175{\cdot}550 + 18{\cdot}1582\,(1) + 15{\cdot}3327\,(3) - 53{\cdot}7646\,(12) + 16{\cdot}5975\,(13) + 37{\cdot}1671\,(18)$

XXV. $0 = +69{\cdot}422 + 19{\cdot}2402\,(1) + 11{\cdot}5887\,(5) + 3{\cdot}2281\,(13) + 13{\cdot}3899\,(17) - 16{\cdot}6180\,(18) - 17{\cdot}1656\,(19) + 4{\cdot}9513\,(21) + 12{\cdot}2143\,(25)$

XXVI. $0 = -21{\cdot}278 - 15{\cdot}3327\,(3) - 3{\cdot}5704\,(4) + 36{\cdot}0923\,(12) + 1{\cdot}0748\,(17) - 37{\cdot}1671\,(18) + 40{\cdot}8844\,(19) - 58{\cdot}8554\,(20) + 17{\cdot}9710\,(21)$

XXVII. $0 = -123{\cdot}826 + 12{\cdot}6781\,(4) + 18{\cdot}6609\,(5) - 18{\cdot}6609\,(8) + 20{\cdot}8212\,(9) + 12{\cdot}2143\,(19) + 28{\cdot}5519\,(24) - 40{\cdot}7662\,(25) + 3{\cdot}3456\,(26) - 3{\cdot}3456\,(30)$

XXVIII. $0 = -17{\cdot}343 - 24{\cdot}9215\,(7) + 29{\cdot}3532\,(8) - 3{\cdot}3456\,(26) - 3{\cdot}1492\,(28) + 6{\cdot}4948\,(30) + 12{\cdot}2953\,(31) - 6{\cdot}5540\,(32) - 5{\cdot}7413\,(38)$

XXIX. $0 = +601{\cdot}437 - 66{\cdot}7537\,(5) + 75{\cdot}8161\,(7) - 38{\cdot}4156\,(8) - 73{\cdot}5260\,(9) + 52{\cdot}7048\,(10) - 85{\cdot}7071\,(24) + 85{\cdot}7071\,(25) - 29{\cdot}6741\,(31) - 75{\cdot}8161\,(32) + 93{\cdot}1949\,(33) + 12{\cdot}2953\,(38)$

XXX. $0 = -262{\cdot}888 + 75{\cdot}8161\,(5) - 75{\cdot}8161\,(7) + 30{\cdot}6548\,(23) + 45{\cdot}1613\,(24) - 75{\cdot}8161\,(25) + 18{\cdot}3308\,(31) + 75{\cdot}8161\,(32) - 93{\cdot}1949\,(33) - 0{\cdot}9520\,(34) + 3{\cdot}0974\,(47) + 27{\cdot}6579\,(53) - 30{\cdot}7553\,(54)$

XXXI. $0 = +474{\cdot}001 - 22{\cdot}6101\,(5) + 83{\cdot}0367\,(6) - 60{\cdot}4266\,(7) - 3{\cdot}4558\,(23) + 3{\cdot}4558\,(25) - 3{\cdot}8660\,(32) + 84{\cdot}5603\,(33) - 80{\cdot}6943\,(34) - 61{\cdot}2783\,(47) + 61{\cdot}2783\,(53)$

XXXII. $0 = +98{\cdot}620 + 20{\cdot}5563\,(3) - 33{\cdot}2344\,(4) - 61{\cdot}4228\,(5) + 49{\cdot}8341\,(6) + 22{\cdot}4158\,(16) - 35{\cdot}8057\,(17) + 13{\cdot}3899\,(18) - 27{\cdot}2240\,(23) + 27{\cdot}2240\,(25) - 29{\cdot}5708\,(47) + 29{\cdot}5708\,(48) + 9{\cdot}0430\,(55) - 0{\cdot}2037\,(58) - 8{\cdot}8393\,(59)$

XXXIII. $0 = -282{\cdot}914 + 52{\cdot}7048\,(9) - 85{\cdot}5042\,(10) + 32{\cdot}7994\,(11) + 35{\cdot}1018\,(28) - 31{\cdot}9526\,(29) - 3{\cdot}1492\,(30) - 51{\cdot}4221\,(39) + 48{\cdot}7136\,(40) + 2{\cdot}7085\,(42)$

XXXIV. $0 = +84{\cdot}840 + 17{\cdot}5868\,(27) - 49{\cdot}5394\,(28) + 31{\cdot}9526\,(29) + 51{\cdot}4221\,(39) - 62{\cdot}2616\,(40) + 10{\cdot}8395\,(41) - 33{\cdot}2372\,(43) + 32{\cdot}0766\,(44) + 1{\cdot}1606\,(46) + 31{\cdot}1522\,(52) - 58{\cdot}8101\,(53) + 27{\cdot}6579\,(54)$

XXXV. $0 = -21{\cdot}614 + 5{\cdot}8861\,(34) - 32{\cdot}5624\,(35) + 26{\cdot}6763\,(36) - 9{\cdot}0709\,(43) - 3{\cdot}7028\,(44) + 12{\cdot}7737\,(45) + 18{\cdot}1378\,(51) - 23{\cdot}3501\,(52) + 5{\cdot}2123\,(53)$

XXXVI. $0 = +100{\cdot}145 + 28{\cdot}6030\,(14) - 51{\cdot}0188\,(16) + 22{\cdot}4158\,(17) + 7{\cdot}6485\,(21) - 22{\cdot}9494\,(22) + 15{\cdot}3009\,(23) + 29{\cdot}5708\,(47) - 71{\cdot}1065\,(48) + 41{\cdot}5357\,(50) + 50{\cdot}9410\,(56) - 93{\cdot}3817\,(57) + 42{\cdot}4407\,(58) + 32{\cdot}3281\,(60) + 35{\cdot}3192\,(61) - 67{\cdot}6473\,(63)$

XXXVII. $0 = +267{\cdot}989 + 15{\cdot}3009\,(22) - 15{\cdot}3009\,(23) + 84{\cdot}5603\,(33) - 75{\cdot}7679\,(34) - 8{\cdot}7924\,(36) + 32{\cdot}0766\,(43) - 35{\cdot}7794\,(44) + 3{\cdot}7028\,(45) - 61{\cdot}2783\,(47) + 42{\cdot}4407\,(48) - 42{\cdot}4407\,(50) + 61{\cdot}2783\,(53) + 54{\cdot}0387\,(57) - 53{\cdot}8350\,(58) - 0{\cdot}2037\,(59) + 30{\cdot}9499\,(66) - 30{\cdot}9499\,(67) - 6{\cdot}0675\,(68) - 7{\cdot}1242\,(70) + 13{\cdot}1917\,(71)$

XXXVIII. $0 = + 84{\cdot}283 - 83{\cdot}3234\,(49) + 101{\cdot}7779\,(50) - 18{\cdot}4545\,(51) - 76{\cdot}5788\,(64)$
$- 19{\cdot}9588\,(66) + 96{\cdot}5376\,(67) - 13{\cdot}2687\,(68) + 30{\cdot}3724\,(69) - 17{\cdot}1037\,(70)$

XXXIX. $0 = - 194{\cdot}224 - 0{\cdot}9050\,(48) - 26{\cdot}9574\,(50) + 27{\cdot}8624\,(51) + 35{\cdot}0832\,(56)$
$- 46{\cdot}6812\,(57) + 11{\cdot}5980\,(58) + 39{\cdot}0841\,(61) - 23{\cdot}4623\,(62) - 15{\cdot}6218\,(63)$
$- 7{\cdot}1242\,(68) - 17{\cdot}1037\,(69) + 24{\cdot}2279\,(70) + 4{\cdot}3045\,(73) - 1{\cdot}5537\,(74)$
$- 2{\cdot}7508\,(75)$

XL. $0 = + 45{\cdot}915 + 28{\cdot}3694\,(14) - 56{\cdot}9724\,(15) + 28{\cdot}6030\,(16) - 7{\cdot}8995\,(55)$
$- 78{\cdot}1247\,(56) + 86{\cdot}0242\,(57) - 50{\cdot}9410\,(61) + 50{\cdot}9410\,(63) - 28{\cdot}4130\,(64)$
$+ 28{\cdot}4130\,(65) - 23{\cdot}4075\,(72) + 19{\cdot}1030\,(73) + 4{\cdot}3045\,(74)$

The logarithmic values of the multipliers are as follow:

Log I = 0·3724948 −
II = 0·4069450 +
III = 9·8081068 +
IV = 0·2792521 +
V = 0·3158436 +
VI = 0·6715666 +
VII = 0·0389693 −
VIII = 0·6617245 +
IX = 0·5803044 +
X = 0·3527149 +
XI = 0·4752469 +
XII = 9·7910152 −
XIII = 9·5007925 +

Log XIV = 9·4429038 +
XV = 0·2728804 +
XVI = 0·3358004 +
XVII = 0·2912955 +
XVIII = 0·2884929 +
XIX = 0·1764783 +
XX = 9·8486244 −
XXI = 0·1479960 −
XXII = 9·6123788 −
XXIII = 9·5316819 +
XXIV = 8·9438312 −
XXV = 8·5986290 −
XXVI = 8·3535254 −
XXVII = 6·3889178 −

Log XXVIII = 8·6550029 −
XXIX = 7·4252114 −
XXX = 8·2008387 +
XXXI = 7·6064244 +
XXXII = 7·9424673 −
XXXIII = 8·6396017 +
XXXIV = 7·9293535 −
XXXV = 7·8089573 −
XXXVI = 8·2273099 −
XXXVII = 7·8067849 +
XXXVIII = 7·5162246 −
XXXIX = 8·9361567 +
XL = 6·7471709 +

FIGURE 20.

Notation.

(1)	*E'A*	(2)	*C'F*	(3)	*B'F*	(4)	*BE*	(5)	*F'B*	(6)	*F'A*	(7)	*G'E*
(8)	*G'B*	(9)	*H'F*	(10)	*H'E*	(11)	*H'B*	(12)	*A'K*	(13)	*A'F*	(14)	*AE'*
(15)	*AF'*	(16)	*AB*	(17)	*AC*	(18)	*BF'*	(19)	*BG'*	(20)	*BH'*	(21)	*BE*
(22)	*BI*	(23)	*BD*	(24)	*BC*	(25)	*BA*	(26)	*EG'*	(27)	*EH'*	(28)	*EB'*
(29)	*EA'*	(30)	*EF*	(31)	*EK*	(32)	*EI*	(33)	*ED*	(34)	*EB*	(35)	*FB'*
(36)	*FA'*	(37)	*FK*	(38)	*FI*	(39)	*FE*	(40)	*FH'*	(41)	*CA*	(42)	*CB*
(43)	*CD*	(44)	*CG*	(45)	*DB*	(46)	*DE*	(47)	*DI*	(48)	*DH*	(49)	*DG*
(50)	*DC*	(51)	*IE*	(52)	*IF*	(53)	*IK*	(54)	*IH*	(55)	*IG*	(56)	*ID*
(57)	*IB*	(58)	*KF*	(59)	*KA'*	(60)	*KH*	(61)	*KI*	(62)	*GC*	(63)	*GD*
(64)	*GI*	(65)	*GH*	(66)	*HI*	(67)	*HK*	(68)	*HG*	(69)	*HD*		

Equations.

I. $0 = + 2{\cdot}3360 + (1) - (6) - (14) + (15)$

II. $0 = - 1{\cdot}6022 - (5) + (6) - (15) + (16) + (18) - (25)$

III. $0 = - 3{\cdot}6030 + (5) - (8) - (18) + (19)$

IV. $0 = + 4{\cdot}3216 + (8) - (11) - (19) + (20)$

V. $0 = +\,0{\cdot}1829 - (10) + (11) - (20) + (21) + (27) - (34)$

VI. $0 = +\,3{\cdot}3466 + (7) - (10) - (26) + (27)$

VII. $0 = -\,4{\cdot}1496 - (4) + (10) - (27) + (28)$

VIII. $0 = +\,1{\cdot}1571 - (9) + (10) - (27) + (30) - (39) + (40)$

IX. $0 = -\,1{\cdot}0621 - (3) + (9) + (35) - (40)$

X. $0 = -\,4{\cdot}4407 + (9) - (13) + (36) - (40)$

XI. $0 = +\,3{\cdot}4901 - (16) + (17) - (24) + (25) - (41) + (42)$

XII. $0 = -\,4{\cdot}6005 - (23) + (24) - (42) + (43) + (45) - (50)$

XIII. $0 = -\,0{\cdot}0751 - (21) + (23) - (33) + (34) - (45) + (46)$

XIV. $0 = +\,0{\cdot}2560 - (30) + (32) - (38) + (39) - (51) + (52)$

XV. $0 = -\,1{\cdot}3209 - (21) + (22) - (32) + (34) + (51) - (57)$

XVI. $0 = -\,0{\cdot}3795 - (32) + (33) - (46) + (47) + (51) - (56)$

XVII. $0 = +\,3{\cdot}4800 - (12) + (13) - (36) + (37) - (58) + (59)$

XVIII. $0 = +\,3{\cdot}8353 - (37) + (38) - (52) + (53) + (58) - (61)$

XIX. $0 = -\,1{\cdot}5761 - (43) + (44) - (49) + (50) - (62) + (63)$

XX. $0 = +\,0{\cdot}3891 - (47) + (49) - (55) + (56) - (63) + (64)$

XXI. $0 = -\,7{\cdot}3847 - (47) + (48) - (54) + (56) + (66) - (69)$

XXII. $0 = -\,8{\cdot}3419 - (54) + (55) - (64) + (65) + (66) - (68)$

XXIII. $0 = +\,3{\cdot}9513 - (53) + (54) - (60) + (61) - (66) + (67)$

XXIV. $0 = +\,1{\cdot}107 + 23{\cdot}8794\,(1) + 4{\cdot}4659\,(8) + 6{\cdot}5068\,(14) - 25{\cdot}4127\,(15) + 18{\cdot}9059\,(16)$
$-\,52{\cdot}4989\,(18) + 38{\cdot}5322\,(19) + 13{\cdot}9667\,(25)$

XXV. $0 = -\,186{\cdot}879 + 6{\cdot}3085\,(5) + 21{\cdot}3384\,(11) + 38{\cdot}5322\,(18) - 54{\cdot}6772\,(19) + 15{\cdot}5450\,(20)$

XXVI. $0 = -\,20{\cdot}416 - 47{\cdot}9777\,(7) + 3{\cdot}0254\,(8) + 15{\cdot}5450\,(19) - 51{\cdot}9639\,(20)$
$+\,36{\cdot}4189\,(21) - 25{\cdot}4049\,(26) + 20{\cdot}9017\,(27) + 4{\cdot}5032\,(34)$

XXVII. $0 = +\,380{\cdot}304 + 64{\cdot}9480\,(4) + 47{\cdot}9777\,(7) + 25{\cdot}4049\,(26) - 56{\cdot}1484\,(27) + 30{\cdot}7435\,(28)$

XXVIII. $0 = -\,21{\cdot}303 - 10{\cdot}0002\,(4) + 4{\cdot}9700\,(10) + 3{\cdot}6719\,(27) - 20{\cdot}5459\,(28) + 16{\cdot}8740\,(29)$

XXIX. $0 = -\,189{\cdot}853 + 24{\cdot}4760\,(3) - 64{\cdot}9480\,(4) + 32{\cdot}2462\,(27) - 30{\cdot}7435\,(28)$
$-\,1{\cdot}5027\,(30) + 14{\cdot}9285\,(35) + 22{\cdot}6416\,(39) - 37{\cdot}5701\,(40)$

XXX. $0 = +\,28{\cdot}698 - 24{\cdot}4760\,(3) + 33{\cdot}2739\,(13) - 14{\cdot}9285\,(35) - 9{\cdot}3537\,(36) + 24{\cdot}2822\,(40)$

XXXI. $0 = +\,105{\cdot}982 + 113{\cdot}1141\,(2) - 33{\cdot}9192\,(3) - 29{\cdot}3336\,(9)$

XXXII. $0 = -\,45{\cdot}999 - 11{\cdot}7621\,(5) + 5{\cdot}4536\,(6) + 13{\cdot}0911\,(7) - 17{\cdot}5570\,(8) + 18{\cdot}9059\,(15)$
$-\,38{\cdot}5307\,(16) + 19{\cdot}6248\,(17) + 26{\cdot}6833\,(26) + 14{\cdot}8818\,(33) - 41{\cdot}5651\,(34)$
$+\,11{\cdot}6338\,(41) - 16{\cdot}3694\,(42) + 4{\cdot}7356\,(43) - 30{\cdot}3233\,(45) + 14{\cdot}4320\,(46)$
$+\,15{\cdot}8913\,(50)$

XXXIII. $0 = +\,88{\cdot}132 - 27{\cdot}9426\,(21) + 35{\cdot}7392\,(22) - 7{\cdot}7966\,(23) - 14{\cdot}4320\,(45)$
$+\,39{\cdot}2756\,(46) - 24{\cdot}8436\,(47) - 21{\cdot}5618\,(51) - 0{\cdot}5050\,(56) + 22{\cdot}0668\,(57)$

XXXIV. $0 = -\,30{\cdot}762 + 22{\cdot}5811\,(9) - 29{\cdot}4067\,(10) + 6{\cdot}8256\,(11) + 36{\cdot}4189\,(20)$
$-\,72{\cdot}1581\,(21) + 35{\cdot}7392\,(22) + 12{\cdot}3277\,(38) - 34{\cdot}9693\,(39) + 22{\cdot}6416\,(40)$
$-\,48{\cdot}8242\,(51) + 26{\cdot}7574\,(52) + 22{\cdot}0668\,(57)$

XXXV. $0 = +\,2{\cdot}233 - 54{\cdot}1265\,(9) + 22{\cdot}5811\,(10) + 11{\cdot}6060\,(12) - 44{\cdot}8799\,(13)$
$-\,1{\cdot}5027\,(27) - 1{\cdot}4003\,(30) + 2{\cdot}9030\,(32) + 26{\cdot}7574\,(51) - 28{\cdot}0204\,(52)$
$+\,1{\cdot}2630\,(53) - 62{\cdot}2413\,(58) + 35{\cdot}3065\,(59) + 26{\cdot}9348\,(61)$

XXXVI. $0 = +\,19{\cdot}077 + 18{\cdot}8022\,(30) - 50{\cdot}1186\,(31) + 31{\cdot}3164\,(32) + 24{\cdot}3645\,(37)$
$-\,14{\cdot}5148\,(38) - 9{\cdot}8497\,(39) - 14{\cdot}6153\,(51) - 1{\cdot}2630\,(52) + 15{\cdot}8783\,(53)$

XXXVII. 0 = + 230·199 + 25·8376 (22) − 43·6823 (23) + 17·8447 (24) + 4·7356 (42)
− 46·5505 (43) + 41·8149 (44) + 41·9225 (55) − 62·9999 (56) + 21·0774 (57)
+ 22·1324 (62) − 38·6747 (63) + 16·5423 (64)

XXXVIII. 0 = + 25·347 − 23·8572 (47) + 19·5781 (48) + 4·2791 (49) − 16·5423 (63)
+ 32·5621 (64) − 16·0198 (65) − 13·2532 (66) − 11·4556 (68) + 24·7088 (69)

XXXIX. 0 = + 69·198 + 2·9030 (30) − 19·8971 (32) + 16·9941 (33) + 14·5148 (37)
− 26·8425 (38) + 12·3277 (39) + 24·8436 (46) − 44·4217 (47) + 19·5781 (48)
+ 26·9348 (58) + 14·1166 (60) − 41·0514 (61) − 32·4913 (66) + 7·7825 (67)
+ 24·7088 (69)

The logarithmic values of the multipliers are as follow:

Log I = 9·9504678 −
II = 9·6325438 +
III = 0·0056570 +
IV = 0·7835832 −
V = 0·7866830 −
VI = 9·3045681 +
VII = 0·0748563 +
VIII = 0·5874408 −
IX = 8·2550553 −
X = 0·1799876 −
XI = 9·8644668 −
XII = 0·0862690 +
XIII = 0·0551282 −

Log XIV = 0·1458103 −
XV = 0·0098870 −
XVI = 9·2038683 +
XVII = 9·8621625 −
XVIII = 0·2231397 −
XIX = 9·0969240 −
XX = 9·3506273 −
XXI = 9·2090051 −
XXII = 9·6827376 +
XXIII = 9·1783546 −
XXIV = 9·1173024 +
XXV = 9·3795405 +
XXVI = 8·3307980 −

Log XXVII = 9·2731265 −
XXVIII = 8·8061610 −
XXIX = 9·1688924 −
XXX = 9·0604901 −
XXXI = 8·2076842 −
XXXII = 8·9040258 +
XXXIII = 7·1870128 +
XXXIV = 8·5444322 −
XXXV = 8·7179146 −
XXXVI = 7·8700567 +
XXXVII = 8·1212923 −
XXXVIII = 8·4427748 +
XXXIX = 8·5068234 −

FIGURE 21.

Notation.

(1)	*B'G*	(2)	*B'F*	(3)	*B'H*	(4)	*B'P*	(5)	*B'Q*	(6)	*CG*	(7)	*CF*
(8)	*E'D*	(9)	*E'E*	(10)	*E'G*	(11)	*G'B*	(12)	*FA*	(13)	*FB*	(14)	*AC*
(15)	*AE*	(16)	*AD*	(17)	*AB*	(18)	*AG'*	(19)	*AF*	(20)	*BE*	(21)	*BD*
(22)	*BG'*	(23)	*BF'*	(24)	*BA*	(25)	*DE'*	(26)	*DG'*	(27)	*DB*	(28)	*DA*
(29)	*DC*	(30)	*CE*	(31)	*CG*	(32)	*CD*	(33)	*CA*	(34)	*CF*	(35)	*EG*
(36)	*EE'*	(37)	*EG'*	(38)	*EB*	(39)	*EA*	(40)	*EC*	(41)	*EF*	(42)	*FB'*
(43)	*FC'*	(44)	*FG*	(45)	*FE*	(46)	*FC*	(47)	*FK*	(48)	*FH*	(49)	*A'Q*
(50)	*A'R*	(51)	*D'R*	(52)	*D'Q*	(53)	*D'S*	(54)	*QA'*	(55)	*QB'*	(56)	*QP*
(57)	*QO*	(58)	*QS*	(59)	*QR*	(60)	*PQ*	(61)	*PB'*	(62)	*PH*	(63)	*PO*
(64)	*PS*	(65)	*OQ*	(66)	*OP*	(67)	*OH*	(68)	*OK*	(69)	*OM*	(70)	*OK'*
(71)	*OS*	(72)	*SD'*	(73)	*SR*	(74)	*SQ*	(75)	*SP*	(76)	*SO*	(77)	*SK'*
(78)	*SH'*	(79)	*H'R*	(80)	*HS*	(81)	*N'S*	(82)	*K'S*	(83)	*K'O*	(84)	*K'M*
(85)	*K'N*	(86)	*MO*	(87)	*MK*	(88)	*ML*	(89)	*MN*	(90)	*ML'*	(91)	*MK'*
(92)	*NK'*	(93)	*NM*	(94)	*NK*	(95)	*NL*	(96)	*NL'*	(97)	*KO*	(98)	*KH*
(99)	*KF*	(100)	*KL*	(101)	*KL'*	(102)	*KN*	(103)	*KM*	(104)	*LN*	(105)	*LM*
(106)	*LK*	(107)	*LM'*.	(108)	*L'M*	(109)	*L'N*	(110)	*L'K*	(111)	*M'L*		

Equations.

I. $0 = -1.9605 + (11) - (13) - (22) + (23)$

II. $0 = +0.7348 + (9) - (11) - (20) + (22) - (36) + (38)$

III. $0 = +5.9907 - (12) + (13) - (17) + (19) - (23) + (24)$

IV. $0 = -1.7529 - (15) + (17) + (20) - (24) - (38) + (39)$

V. $0 = +1.4182 + (8) - (11) - (21) + (22) - (25) + (27)$

VI. $0 = -3.7588 - (16) + (17) + (21) - (24) - (27) + (28)$

VII. $0 = -4.5374 - (14) + (16) - (28) + (29) - (32) + (33)$

VIII. $0 = -4.9481 - (14) + (15) - (30) + (33) - (39) + (40)$

IX. $0 = +2.0758 + (2) - (7) - (42) + (43)$

X. $0 = +0.2797 + (30) - (34) - (40) + (41) - (45) + (46)$

XI. $0 = -5.0707 + (7) - (9) + (36) - (41) - (43) + (45)$

XII. $0 = -4.1145 - (5) + (49) - (54) + (55)$

XIII. $0 = +3.0329 - (77) + (78) - (80) + (82)$

XIV. $0 = +2.6955 - (53) + (72) - (78) + (80)$

XV. $0 = +4.7781 - (49) + (53) + (54) - (58) - (72) + (74)$

XVI. $0 = -4.9495 - (56) + (58) + (60) - (64) - (74) + (75)$

XVII. $0 = +2.1921 - (4) + (5) - (55) + (56) - (60) + (61)$

XVIII. $0 = -11.1443 - (70) + (71) - (76) + (77) - (82) + (83)$

XIX. $0 = -4.2297 - (57) + (58) + (65) - (71) - (74) + (76)$

XX. $0 = +3.6534 - (63) + (64) + (66) - (71) - (75) + (76)$

XXI. $0 = -1.6949 - (84) - (90) + (91) + (108)$

XXII. $0 = +2.9605 - (69) + (70) - (83) + (84) + (86) - (91)$

XXIII. $0 = -1.2312 - (85) + (92) - (96) + (109)$

XXIV. $0 = -2.6938 - (84) + (85) - (89) + (91) - (92) + (93)$

XXV. $0 = +1.4675 - (87) + (90) - (101) + (103) - (108) + (110)$

XXVI. $0 = +2.1241 - (87) + (89) - (93) + (94) - (102) + (103)$

XXVII. $0 = +3.0067 - (68) + (69) - (86) + (87) + (97) - (103)$

XXVIII. $0 = -5.6584 - (2) + (42) - (47) + (4) - (61) + (63) - (66) + (68) - (97) + (99)$

XXIX. $0 = -5.8508 - (87) + (88) - (100) + (103) - (105) + (106)$

XXX. $0 = -1.4723 - (94) + (95) - (100) + (102) - (104) + (106)$

XXXI. $0 = -0.2615 - (95) + (96) + (104) - (107) - (109) + (111)$

XXXII. $0 = -27.999 - 6.6342\,(9) + 9.3845\,(13) - 4.2127\,(20) - 3.8258\,(22) + 8.0385\,(23) + 20.8691\,(36) - 44.1826\,(37) + 23.3135\,(38)$

XXXIII. $0 = -261.221 - 0.7961\,(12) - 9.3845\,(13) + 28.9920\,(17) - 44.0215\,(18) + 15.0295\,(19) + 27.2974\,(22) - 8.0385\,(23) - 19.2589\,(24)$

XXXIV. $0 = +204.647 + 5.4070\,(15) - 28.9920\,(17) + 23.5850\,(18) + 4.2127\,(20) - 23.4716\,(22) + 19.2589\,(24) + 9.2397\,(37) - 23.3135\,(38) + 14.0738\,(39)$

XXXV. $0 = +202.550 + 13.4865\,(8) + 6.6342\,(9) + 4.2127\,(20) + 8.9110\,(21) - 13.1237\,(22) + 2.0958\,(25) - 3.0490\,(26) + 0.9532\,(27) - 20.8691\,(36) + 44.1826\,(37) - 23.3135\,(38)$

XXXVI. $0 = -918.817 - 109.2821\,(16) + 241.1887\,(17) - 131.9066\,(18) - 44.0815\,(21) + 146.9961\,(22) - 102.9146\,(24) - 45.0347\,(26) + 184.1906\,(27) - 139.1559\,(28)$

XXXVII. $0 = + 455\cdot221 + 15\cdot4301\ (20) - 59\cdot4236\ (21) + 43\cdot9935\ (24) + 139\cdot1559\ (27)$
$- 148\cdot5888\ (28) + 9\cdot4329\ (29) + 15\cdot3329\ (30) + 9\cdot8467\ (32) - 25\cdot1796\ (33)$
$- 83\cdot4889\ (38) + 115\cdot3664\ (39) - 31\cdot8775\ (40)$

XXXVIII. $0 = - 367\cdot623 + 78\cdot8604\ (1) - 9\cdot7285\ (6) + 14\cdot9171\ (10)$

XXXIX. $0 = + 643\cdot243 + 30\cdot7023\ (6) + 27\cdot5493\ (9) + 17\cdot1498\ (10) + 44\cdot5404\ (35)$
$- 106\cdot2786\ (36) + 61\cdot7382\ (37)$

XL. $0 = - 244\cdot061 - 48\cdot1713\ (9) + 13\cdot9364\ (10) + 56\cdot3423\ (14) - 80\cdot5999\ (15)$
$+ 24\cdot2576\ (18) + 204\cdot8841\ (30) - 189\cdot5512\ (31) - 15\cdot3329\ (33) + 110\cdot1816\ (35)$
$+ 27\cdot5493\ (36) - 59\cdot7197\ (37) + 18\cdot8506\ (39) - 96\cdot8618\ (40)$

XLI. $0 = + 635\cdot050 - 92\cdot6894\ (30) + 94\cdot4381\ (31) - 1\cdot7487\ (34) - 73\cdot4662\ (35)$
$+ 45\cdot0260\ (40) + 28\cdot4402\ (41) + 67\cdot5498\ (44) - 84\cdot9732\ (45) + 17\cdot4234\ (46)$

XLII. $0 = - 1664\cdot735 - 51\cdot7756\ (6) + 17\cdot2433\ (7) + 27\cdot2562\ (9) - 46\cdot0033\ (10)$
$- 29\cdot4200\ (35) + 57\cdot8602\ (36) - 28\cdot4402\ (41) + 81\cdot1823\ (43) - 166\cdot1555\ (44)$
$+ 84\cdot9732\ (45)$

XLIII. $0 = - 876\cdot779 + 77\cdot4080\ (1) - 12\cdot1061\ (2) - 22\cdot8491\ (6) + 17\cdot2433\ (7) - 11\cdot0333\ (42)$
$+ 81\cdot1823\ (43) - 70\cdot1490\ (44)$

XLIV. $0 = - 194\cdot682 + 45\cdot5884\ (5) - 9\cdot2847\ (49) + 63\cdot0069\ (52) + 7\cdot3537\ (54) - 7\cdot3537\ (55)$

XLV. $0 = - 98\cdot790 + 15\cdot4324\ (53) + 14\cdot1283\ (72) + 10\cdot5889\ (77) - 24\cdot7172\ (78)$
$+ 20\cdot9104\ (82)$

XLVI. $0 = - 26\cdot220 + 8\cdot4042\ (49) - 35\cdot0218\ (52) + 25\cdot7371\ (53) - 34\cdot7932\ (72)$
$+ 20\cdot6649\ (74) + 14\cdot1283\ (78) + 7\cdot6209\ (80)$

XLVII. $0 = - 3\cdot570 - 13\cdot8288\ (80) + 31\cdot4766\ (81) - 16\cdot5402\ (82)$

XLVIII. $0 = + 177\cdot265 + 4\cdot8761\ (49) - 81\cdot1416\ (50) - 111\cdot3147\ (51) + 20\cdot6977\ (53)$
$+ 13\cdot3035\ (54) + 19\cdot1064\ (58) + 16\cdot9701\ (72) - 33\cdot1774\ (73) + 16\cdot2073\ (74)$
$- 32\cdot4099\ (59)$

XLIX. $0 = - 604\cdot267 + 136\cdot1700\ (49) + 81\cdot1416\ (50) + 234\cdot5767\ (51) - 285\cdot0058\ (52)$
$- 154\cdot3496\ (54)$

L. $0 = + 125\cdot303 - 24\cdot1273\ (51) + 20\cdot6977\ (53) + 16\cdot9701\ (72) - 23\cdot5160\ (73) + 6\cdot5459\ (78)$
$+ 60\cdot3536\ (79) - 32\cdot8902\ (80)$

LI. $0 = + 131\cdot036 + 22\cdot1689\ (4) - 67\cdot7573\ (5) - 19\cdot4831\ (49) - 81\cdot4922\ (52) + 27\cdot7700\ (53)$
$- 17\cdot9481\ (60) + 0\cdot9062\ (61) + 17\cdot0419\ (64) - 5\cdot0722\ (72) - 27\cdot7608\ (74)$
$+ 32\cdot8330\ (75)$

LII. $0 = - 154\cdot720 - 9\cdot7023\ (2) + 15\cdot1719\ (7) - 45\cdot8543\ (42) + 19\cdot0886\ (43) + 26\cdot7657\ (48)$
$- 12\cdot3358\ (49) + 22\cdot0381\ (4) + 20\cdot3636\ (49) - 7\cdot3537\ (54) - 10\cdot9874\ (55)$
$+ 18\cdot3411\ (56) + 0\cdot9062\ (60) - 15\cdot4042\ (61) + 14\cdot4980\ (62)$

LIII. $0 = + 137\cdot623 + 53\cdot5071\ (52) - 68\cdot9395\ (53) + 11\cdot3588\ (57) - 11\cdot3588\ (58) + 12\cdot6884\ (65)$
$+ 13\cdot8077\ (70) - 26\cdot4961\ (71) + 25\cdot7371\ (72) - 25\cdot7371\ (74) - 14\cdot6652\ (80)$
$- 41\cdot4873\ (82) + 20\cdot5769\ (83)$

LIV. $0 = - 121\cdot885 - 2\cdot3297\ (56) - 11\cdot3588\ (57) + 13\cdot6885\ (58) + 17\cdot0419\ (60) + 16\cdot4394\ (63)$
$- 33\cdot4813\ (64) - 12\cdot6884\ (65) - 4\cdot1825\ (66) + 16\cdot8709\ (71)$

LV. $0 = + 73\cdot463 + 64\cdot0779\ (3) - 86\cdot2468\ (4) + 22\cdot1689\ (5) + 18\cdot3411\ (55) - 48\cdot7946\ (56)$
$+ 30\cdot4535\ (57) + 22\cdot0381\ (61) - 30\cdot2310\ (62) + 8\cdot1929\ (63) + 23\cdot1319\ (65)$
$- 48\cdot0532\ (66) + 24\cdot9213\ (67)$

LVI. $0 = - 192\cdot800 + 26\cdot2888\ (69) - 40\cdot0965\ (70) + 13\cdot8077\ (71) + 4\cdot6312\ (76) - 15\cdot2201\ (77)$
$+ 10\cdot5889\ (78) + 7\cdot0443\ (80) + 2\cdot9215\ (86) + 7\cdot6174\ (90) - 10\cdot5389\ (91)$
$+ 12\cdot3506\ (108)$

LVII. 0 = − 206·950 − 49·6832 (84) + 67·1735 (85) + 9·9179 (92) − 33·2196 (93) + 23·3017 (96)
+ 94·3990 (108) − 82·0484 (109)

LVIII. 0 = + 76·912 − 7·7497 (93) − 15·5520 (94) + 23·3017 (96) + 52·5564 (101) − 66·5390 (102)
+ 13·9826 (103) + 28·7844 (108) − 82·0484 (109) + 53·2640 (110)

LIX. 0 = + 357·962 + 27·5031 (68) − 53·7919 (69) + 26·2888 (70) + 12·5471 (83) − 30·0374 (84)
+ 7·0955 (97) + 52·5564 (101) − 59·6519 (103) − 65·6146 (108) + 53·2640 (110)

LX. 0 = + 2·000 + 20·4823 (62) − 36·9217 (63) + 16·4394 (64) + 8·1929 (66) − 12·0777 (67) −
+ 3·8848 (68) + 41·7771 (75) − 46·4083 (76) + 4·6312 (77) + 20·5769 (82)
− 33·1240 (83) + 12·5471 (84) − 10·0941 (86) + 7·1726 (87) + 2·9215 (91)
− 53·9679 (97) + 46·8724 (98) + 7·0955 (103)

LXI. 0 = − 32·121 + 29·7822 (2) + 36·4680 (42) − 47·1654 (48) + 10·6974 (47) − 71·8220 (3)
+ 42·0398 (4) − 7·5401 (61) − 4·7493 (62) + 12·2894 (63) + 16·7284 (66)
− 22·6236 (67) + 5·8952 (68) + 42·9876 (97) − 53·4156 (98) + 10·4280 (99)

LXII. 0 = + 184·809 − 7·2056 (87) − 48·3309 (88) + 55·5365 (89) + 35·9013 (100)
− 102·4403 (102) + 66·5390 (103) + 82·5310 (104) − 95·0913 (105) + 12·5603 (106)

LXIII. 0 = − 101·784 + 17·4903 (84) + 24·0221 (88) − 31·6395 (90) + 7·6174 (91) + 13·7080 (100)
− 37·9349 (101) + 24·2269 (103) − 18·5705 (105) + 14·3230 (106) + 4·2475 (107)
+ 38·4457 (108) − 38·4457 (110) + 23·0296 (111)

The logarithmic values of the multipliers are as follow:

Log I = 0·0889695 −
II = 0·7414385 −
III = 9·1598588 +
IV = 0·3144831 −
V = 0·5532633 +
VI = 0·1835627 +
VII = 0·0316456 +
VIII = 9·7149179 −
IX = 0·2311781 −
X = 9·7116141 −
XI = 0·6504931 −
XII = 1·0061771 +
XIII = 1·5279569 +
XIV = 1·4947610 +
XV = 0·0023475 −
XVI = 1·2535760 −
XVII = 9·2336458 +
XVIII = 1·2985920 +
XIX = 0·3488471 +
XX = 1·4649921 +
XXI = 0·2983023 +

Log XXII = 1·0044255 +
XXIII = 0·4474535 +
XXIV = 0·8086689 +
XXV = 9·7022154 +
XXVI = 0·6388290 +
XXVII = 0·7439605 +
XXVIII = 0·2655749 +
XXIX = 9·8500314 +
XXX = 9·7550390 +
XXXI = 0·5150703 +
XXXII = 9·4543977 −
XXXIII = 8·3100326 −
XXXIV = 9·0484461 −
XXXV = 9·4766255 −
XXXVI = 7·8149841 −
XXXVII = 7·3617543 −
XXXVIII = 9·2565835 +
XXXIX = 8·5993786 +
XL = 8·0282628 +
XLI = 8·2707860 +
XLII = 8·8713732 +

Log XLIII = 9·1856657 −
XLIV = 0·2679188 +
XLV = 0·2221317 +
XLVI = 0·3703722 +
XLVII = 8·4438869 +
XLVIII = 7·8415347 +
XLIX = 7·8332963 +
L = 8·1007712 −
LI = 0·0533081 +
LII = 9·5408247 +
LIII = 0·0663444 +
LIV = 0·3207788 +
LV = 9·6277542 +
LVI = 9·4188744 +
LVII = 8·3802181 +
LVIII = 8·7413799 −
LIX = 8·7259402 +
LX = 9·5105849 +
LXI = 9·4364831 +
LXII = 7·1515108 +
LXIII = 7·7187731 −

CORRECTIONS

RESULTING FROM THE PRECEDING EQUATIONS.

Figure I.

From	To	Correction to observed Bearing.
SOUTH END OF BASE	Slieve Snaght .	− 0″·0421
	Drung Point . .	− 0·4376
	Cundtham . .	+ 0·0552
	Mount Sandy .	+ 0·3869
	North End of Base	+ 0·2616
NORTH END OF BASE	Sawel	− 1·9580
	South End of Base	− 0·2803
	Drung Point . .	+ 0·1759
	Slieve Snaght .	+ 0·0470
	Cundtham . .	+ 0·0587
	Mount Sandy .	+ 0·0125
DRUNG POINT . .	Mount Sandy .	− 0·6496
	North End of Base	+ 0·2683
	South End of Base	+ 0·5171
CUNDTHAM . . .	Slieve Snaght .	+ 0·0125
	Mount Sandy .	+ 0·4012
	Knocklayd . .	− 0·4166
	North End of Base	− 0·1746
	South End of Base	+ 0·2004
	Sawel	+ 0·5953
MOUNT SANDY . .	Sawel	+ 0·1355
	North End of Base	+ 0·0350
	South End of Base	− 0·5424
	Drung Point . .	+ 0·2400
	Cundtham . .	+ 0·5423
	Slieve Snaght .	− 0·2585
	Knocklayd . .	− 0·5054
SLIEVE SNAGHT . .	Cuilcagh . . .	− 0·1600
	Knocklayd . .	− 4·5079
	Mount Sandy .	+ 0·0135
	Cundtham . .	+ 1·0513
	North End of Base	+ 0·4539
	Trostan . . .	− 2·9824
	Divis	− 6·1707
	South End of Base	+ 0·1429
	Sawel	+ 0·9528
SAWEL	Cuilcagh . . .	+ 0″·0871
	Slieve Snaght .	− 0·9424
	North End of Base	+ 0·6104
	Mount Sandy .	− 0·5706
	Knocklayd . .	− 0·0825
	Trostan . . .	+ 0·3748
	Divis	+ 0·6845
	Slieve Donard .	+ 0·0833
	Vicars Carn . .	+ 0·8522
KNOCKLAYD . . .	Sawel	− 0·7560
	Mount Sandy .	+ 1·8215
	Slieve Snaght .	− 1·6367
	Trostan . . .	+ 0·4362
	Divis	− 1·2198
TROSTAN	Sawel	+ 0·8335
	Slieve Snaght .	− 3·0660
	Knocklayd . .	− 0·1755
	Divis	+ 0·7348
	Slieve Donard .	− 0·2962
DIVIS	Vicars Carn . .	+ 0·2120
	Sawel	− 0·6775
	Slieve Snaght .	− 0·1112
	Knocklayd . .	+ 0·6690
	Trostan . . .	− 0·8289
	Slieve Donard .	+ 0·3430
VICARS CARN . .	Cuilcagh . . .	− 3·1312
	Sawel	− 2·0329
	Divis	+ 0·0045
	Slieve Donard .	+ 0·9675
SLIEVE DONARD .	Cuilcagh . . .	− 0·3776
	Vicars Carn . .	+ 1·1048
	Sawel	+ 0·3445
	Trostan . .	+ 0·2250
	Divis	− 0·5175
CUILCAGH . . .	Slieve Snaght .	− 0·9642
	Sawel	+ 0·0973
	Vicars Carn . .	+ 1·4139
	Slieve Donard .	− 1·7017

FIGURE 2.

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
		″			″
CUILCAGH	Keeper	− 0·4289	*KEEPER—continued*	Kippure	+ 2·0217
	Nephin	+ 0·0786		Knockanaffrin	− 0·5632
	Slieve League	− 0·0647			
	Kippure	− 3·0128	*KNOCKANAFFRIN*	Baurtregaum	− 0·1896
	Croghan	+ 1·4682		Keeper	+ 0·2159
				Kippure	− 2·2030
SLIEVE DONARD	Kippure	+ 0·5816			
	Croghan	− 0·2957	*BAURTREGAUM*	Bencorr	− 0·7474
				Keeper	− 1·4331
KIPPURE	Knockanaffrin	+ 1·2093		Knockanaffrin	+ 0·4120
	Keeper	− 0·9425			
	Croghan	+ 1·4031	*BENCORR*	Baurtregaum	+ 1·0107
	Cuilcagh	− 3·9239		Nephin	− 0·1221
	Slieve Donard	− 0·3403		Keeper	− 0·2646
CROGHAN	Keeper	+ 1·5402	*NEPHIN*	Bencorr	+ 0·8503
	Cuilcagh	+ 5·4228		Slieve League	+ 1·6855
	Slieve Donard	− 2·2389		Cuilcagh	− 0·4185
	Kippure	− 1·5178		Keeper	− 3·1501
KEEPER	Baurtregaum	− 0·3073	*SLIEVE LEAGUE*	Nephin	− 0·4587
	Bencorr	+ 0·7865		Slieve Snaght	+ 1·8829
	Nephin	+ 0·6893		Cuilcagh	+ 0·1731
	Cuilcagh	+ 1·1030			
	Croghan	− 0·0444	*SLIEVE SNAGHT*	Slieve League	− 0·0147

FIGURE 3.

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
		″			″
KIPPURE	Galtymore	− 5·7867	*BAURTREGAUM*	Caherbarnagh	+ 0·5254
			(continued.)	Hungry Hill	− 0·1040
KNOCKANAFFRIN	Doolieve	+ 0·3653			
	Knockmealdown	+ 0·0124	*TAUR*	Caherbarnagh	− 0·1267
	Taur	+ 2·5485		Baurtregaum	− 0·0785
	Galtymore	+ 0·4027		Keeper	+ 0·4345
				Galtymore	+ 0·5238
KEEPER	Knocknaskagh	− 0·5079		Knockanaffrin	− 4·5896
	Caherbarnagh	− 0·3244		Knockmealdown	− 0·8680
	Taur	+ 1·1729		Knocknaskagh	+ 0·3830
	Knockmealdown	− 0·8584			
	Galtymore	+ 0·5676	*KNOCKNASKAGH*	Doolieve	− 0·2074
				Carrigfadda	+ 0·2088
BAURTREGAUM	Galtymore	− 3·3516		Caherbarnagh	− 0·4778
	Taur	+ 0·7909		Baurtregaum	− 0·1816
	Knockmealdown	+ 0·7808		Taur	+ 0·6513
	Knocknaskagh	− 1·9823		Keeper	− 2·9668

FIGURE 3—*continued.*

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
		″			″
KNOCKNASKAGH	Galtymore	− 0·6327	*CAHERBARNAGH*	Galtymore	+ 1·5694
(*continued.*)	Knockmealdown	+ 1·8638	(*continued.*)	Knockmealdown	+ 3·3900
				Knocknaskagh	+ 0·2863
DOOLIEVE	Carrigfadda	+ 0·1323		Doolieve	− 1·4789
	Caherbarnagh	+ 0·9473		Carrigfadda	− 0·0976
	Knocknaskagh	− 0·0227			
	Galtymore	+ 2·8879	*HUNGRY HILL*	Baurtregaum	+ 0·0696
	Knockmealdown	− 0·8486		Caherbarnagh	+ 0·7932
	Knockanaffrin	+ 6·4890		Carrigfadda	− 0·2775
CAHERBARNAGH	Hungry Hill	− 0·8170	*CARRIGFADDA*	Hungry Hill	+ 0·3346
	Baurtregaum	− 1·0795		Caherbarnagh	+ 0·1628
	Taur	+ 0·2658		Doolieve	− 1·0588
	Keeper	+ 2·7334			

FIGURE 4.

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
		″			″
KNOCKLAYD	Merrick	−12·9183	*MERRICK*—*continued*	Sea Fell	+ 0·0468
				Black Comb	+ 0·1302
DIVIS	Merrick	− 1·6858			
	South Berule	− 0·5780	*SOUTH BERULE*	Kippure	− 1·6019
				Howth	+ 3·0094
TROSTAN	Merrick	+ 3·5449		Slieve Donard	− 0·3433
				Divis	+ 2·4595
SLIEVE DONARD	Merrick	+ 0·1750		Trostan	+ 0·8993
	Criffel	− 0·3329		Merrick	− 1·5201
	Sea Fell	+ 1·9194		Criffel	+ 0·5292
	South Berule	− 0·6820		Sea Fell	− 0·8793
	Snowdon	− 0·4843		Black Comb	− 0·6319
	Howth	+ 0·1671		Ingleborough	+ 2·4597
				Snowdon	+ 0·3038
KIPPURE	Howth	+ 0·3121			
	South Berule	− 3·9389	*CRIFFEL*	Sea Fell	+ 0·3022
	Snowdon	− 0·3871		Black Comb	+ 4·4464
				South Berule	− 0·4194
HOWTH	Kippure	− 0·2239		Merrick	− 0·2780
	Slieve Donard	+ 0·9512			
			SCA FELL	Black Comb	+ 0·0482
MERRICK	South Berule	− 0·2676		Snowdon	+ 4·3755
	Slieve Donard	+ 0·8982		South Berule	− 0·5789
	Divis	+ 0·7600		Merrick	+ 1·2950
	Trostan	− 0·3509		Criffel	− 2·6092
	Knocklayd	+ 0·2024		Great Whernside	+ 0·4044
	Criffel	− 0·1390		Ingleborough	+ 2·9812

FIGURE 4—*continued.*

From	To	Correction to observed Bearing
SCA FELL—continued	Pendle	″ + 1·0079
	Whittle	− 1·7864
BLACK COMB	Snowdon	− 1·8094
	South Berule	+ 0·9525
	Merrick	+ 1·8145
	Criffel	− 0·9644
	Sca Fell	− 1·0943
	Ingleborough	+ 0·6939
	Pendle	+ 0·9586
	Whittle	− 2·9114
SNOWDON	Kippure	− 0·8200
	Howth	+ 1·2149
	Slieve Donard	+ 3·9207
	South Berule	+ 3·6531
	Sca Fell	+ 1·7229
	Black Comb	− 0·8844
	Ingleborough	− 2·4194
	Whittle	− 1·1380
	Axedge	+ 1·7656
GREAT WHERNSIDE	Whittle	− 0·6678
	Pendle	+ 0·3813
	Ingleborough	+ 0·0328
	Sca Fell	− 0·4964
	Holme Moss	+ 1·5275
PENDLE	Snowdon	− 2·2990
	Black Comb	+ 1·6505
	Sca Fell	+ 0·0917
	Ingleborough	+ 0·0194

From	To	Correction to observed Bearing.
PENDLE—continued	Great Whernside	″ − 0·5576
	Holme Moss	− 0·8033
	Axedge	+ 1·3799
	Whittle	− 1·0515
WHITTLE	Axedge	+ 0·6523
	Mowcopt	+ 6·4766
	Snowdon	+ 1·2829
	Black Comb	− 0·4058
	Sca Fell	− 0·1711
	Ingleborough	+ 0·0166
	Pendle	+ 0·1726
	Great Whernside	+ 0·2387
	Holme Moss	− 0·5839
MOWCOPT	Snowdon	+ 0·9823
	Whittle	− 0·9457
	Holme Moss	+ 0·5225
	Axedge	− 0·1818
AXEDGE	Mowcopt	− 0·2621
	Snowdon	− 0·2264
	Whittle	+ 1·3566
	Pendle	+ 2·5745
	Ingleborough	+ 2·3305
	Holme Moss	− 1·9697
HOLME MOSS	Axedge	− 1·2792
	Whittle	+ 1·0116
	Pendle	+ 0·1235
	Great Whernside	− 0·2974

FIGURE 5.

From	To	Correction to observed Bearing
MERRICK	Hart Fell	″ − 2·0541
	Wisp	+ 0·0477
	Cross Fell	− 0·6502
CRIFFEL	Hart Fell	− 0·7285
	Dunrich	− 2·4089
	Wisp	− 1·3396
	Cross Fell	− 3·6593

From	To	Correction to observed Bearing.
SCA FELL	Hart Fell	″ + 4·3461
	Wisp	+ 1·6895
	Cheviot	+ 5·7597
	Cross Fell	+ 1·8711
	Water Crag	− 2·4748
GREAT WHERNSIDE	Cross Fell	− 2·4252
	Water Crag	− 0·3423
	Botton Head	− 1·4004

FIGURE 5—*continued.*

From	To	Correction to observed Bearing. ″
HART FELL	Criffel	+ 2·5777
	Merrick	+ 0·5700
	Dunrich	− 0·6895
	Sayrs Law	+ 0·8013
	Cheviot	− 0·5531
	Wisp	− 0·6347
	Cross Fell	− 1·5806
	Sea Fell	+ 0·3700
WISP	Criffel	+ 1·8705
	Hart Fell	− 0·5697
	Dunrich	+ 0·3515
	Sayrs Law	− 0·6244
	Cheviot	− 1·5334
	Cross Fell	− 1·3660
CROSS FELL	Sea Fell	+ 3·1672
	Criffel	− 2·9441
	Merrick	− 0·9523
	Hart Fell	+ 0·2619
	Wisp	− 1·0519
	Dunrich	+ 4·4155
	Cheviot	+ 2·7016
	Collier Law	+ 0·4182
	Botton Head	− 2·0497
	Water Crag	+ 0·4810
	Great Whernside	− 2·1688
	Ingleborough	− 2·2879
WATER CRAG	Sea Fell	+ 0·7176
	Cross Fell	− 0·3718
	Collier Law	− 0·3719
	Wordeslow	− 2·7750
	Easington	+ 0·7810
	Botton Head	+ 1·0310
	Great Whernside	− 1·4403
DUNRICH	Criffel	+ 0·6800
	Hart Fell	− 1·4663
	Sayrs Law	+ 0·1997
	Blackheddon	− 0·3334
	Cheviot	+ 0·0120
	Cross Fell	− 0·0681
	Wisp	+ 0·6017
SAYRS LAW	Wisp	− 0·9644
	Hart Fell	+ 2·9960
	Dunrich	− 0·5946
	Lumsden	+ 0·0825
	Blackheddon	− 0·1185

From	To	Correction to observed Bearing. ″
SAYRS LAW—cont.	Cheviot	− 0·0151
CHEVIOT	Cross Fell	+ 0·0149
	Sea Fell	+ 2·2880
	Wisp	− 0·0723
	Hart Fell	+ 0·2277
	Dunrich	− 0·0303
	Sayrs Law	+ 0·2463
	Lumsden	− 0·8838
	Blackheddon	− 1·5242
	Easington	− 0·0687
	Wordeslow	− 0·8625
	Botton Head	− 0·2200
	Collier Law	+ 0·2176
BLACKHEDDON	Cheviot	+ 0·8279
	Dunrich	− 1·3858
	Sayrs Law	− 0·3220
	Lumsden	+ 0·2105
LUMSDEN	Sayrs Law	+ 0·5038
	Blackheddon	+ 0·8132
	Cheviot	− 0·6911
COLLIER LAW	Water Crag	− 0·7879
	Cross Fell	+ 0·5428
	Cheviot	− 1·5101
	Wordeslow	− 0·2246
	Easington	+ 0·4158
	Botton Head	− 0·0723
WORDESLOW	Great Whernside	− 0·1809
	Water Crag	− 2·5964
	Collier Law	+ 0·6654
	Cheviot	− 0·3117
	Easington	+ 0·2181
	Botton Head	+ 0·0835
BOTTON HEAD	Great Whernside	− 0·6276
	Water Crag	+ 1·3896
	Cross Fell	+ 0·6183
	Collier Law	+ 0·6727
	Wordeslow	+ 2·2806
	Easington	− 0·4799
EASINGTON	Botton Head	+ 0·0722
	Water Crag	+ 1·3327
	Collier Law	− 0·5754
	Wordeslow	− 0·0850
	Cheviot	+ 1·3311

FIGURE 6.

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
		″			″
FAIR ISLE	Foula	+ 0·4936	*RONAS*	Foula	− 1·0084
	Ronas	− 1·3974		Saxaford	− 0·0793
	Brass	− 1·5170		Fetlar	− 2·2350
				Yell	+ 1·0015
FOULA	Ronas	+ 0·4069		Brassa	+ 0·3377
	Yell	+ 0·4151			
	Brassa	+ 0·1584	*FETLAR*	Brassa	− 0·1983
	Fair Isle	− 0·3182		Yell	+ 0·3324
				Ronas	+ 0·5157
BRASSA	Fair Isle	+ 0·9467		Saxaford	− 0·3647
	Foula	− 0·1047		Balta	+ 0·2332
	Ronas	− 0·7071			
	Yell	+ 0·1628	*BALTA*	Fetlar	− 0·6720
	Saxaford	− 0·1704		Yell	+ 0·1098
	Fetlar	+ 0·0114		Saxaford	+ 0·2481
YELL	Brassa	− 0·0243	*SAXAFORD*	Fetlar	+ 1·2001
	Foula	+ 0·3148		Brassa	− 0·2750
	Ronas	− 0·1351		Yell	− 2·8100
	Saxaford	+ 0·7227		Ronas	+ 2·5483
	Balta	− 0·2410		Balta	− 0·1731
	Fetlar	− 0·3469			

FIGURE 7.

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
		″			″
BEN CLIBRIG	Fashven	− 0·0599	*BEN HUTIG—cont.*	Ben Cheilt	+ 0·3237
	Ben Hutig	− 0·4570		Scarabin	− 0·4692
	Fitty Hill	+ 0·6511		Ben Clibrig	+ 0·9195
	Wart Hill Hoy	− 1·1637			
	Dunnet Head	− 2·7410	*SCARABIN*	Ben Clibrig	− 0·0811
	South Ronaldshay	+ 1·1713		Ben Hutig	− 0·1115
	Ben Cheilt	− 0·9785		Wart Hill Hoy	+ 0·0499
	Scarabin	− 0·0606		Dunnet Head	− 0·5682
				South Ronaldshay	+ 1·8724
FASHVEN	Wart Hill Hoy	− 0·9556		Ben Cheilt	− 0·1453
	Dunnet Head	− 0·2212			
	Ben Hutig	+ 0·0026	*BEN CHEILT*	Scarabin	− 0·1122
	Ben Clibrig	+ 0·2783		Ben Clibrig	+ 1·9325
				Ben Hutig	+ 1·4488
BEN HUTIG	Fashven	− 3·9326		Wart Hill Hoy	− 0·7504
	Fitty Hill	− 0·3810		South Ronaldshay	− 1·3038
	Wart Hill Hoy	+ 0·4377			
	South Ronaldshay	− 0·6483	*DUNNET HEAD*	Ben Cheilt	− 0·1868
	Dunnet Head	− 0·1505		Scarabin	+ 2·7982

FIGURE 7—*continued.*

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
DUNNET HEAD—cont.	Ben Clibrig	+ 0″·0532	*STRONSAY*	Deerness	− 0″·2004
	Ben Hutig	− 0·3065		Wart Hill Hoy	− 0·0633
	Fashven	+ 2·0636		Fitty Hill	+ 1·2130
	Wart Hill Hoy	+ 0·0770		N. Ronaldshay Lighthouse	+ 0·1753
	Deerness	− 2·7116		Start Lighthouse	− 0·2688
	South Ronaldshay	− 0·0443		Fair Isle	− 0·4129
WART HILL HOY	Ben Cheilt	+ 1·4740	*FITTY HILL*	Wart Hill Hoy	+ 0·4390
	Dunnet Head	− 3·2921		Ben Hutig	− 0·0131
	Scarabin	+ 3·0118		Foula	− 0·1916
	Ben Clibrig	+ 0·8454		Fair Isle	+ 1·1037
	Fashven	+ 1·3331		N. Ronaldshay Lighthouse	− 0·0090
	Fitty Hill	− 0·1696		Start Lighthouse	− 0·3208
	N. Ronaldshay Lighthouse	− 0·0197		Stronsay	− 0·2509
	Start Lighthouse	− 0·6875		Deerness	− 0·5027
	Stronsay	+ 1·1836	*FAIR ISLE*	Stronsay	+ 0·2941
	Deerness	− 0·6991		Start Lighthouse	− 0·3441
	South Ronaldshay	+ 0·1059		N. Ronaldshay Lighthouse	− 1·1841
SOUTH RONALDSHAY	Ben Cheilt	+ 0·2861		Fitty Hill	− 0·4009
	Scarabin	− 1·3744		Foula	+ 0·4936
	Ben Clibrig	− 2·0055	*FOULA*	Start Lighthouse	+ 1·8031
	Dunnet Head	− 0·3243		N. Ronaldshay Lighthouse	+ 0·9799
	Ben Hutig	− 1·4648		Fitty Hill	− 0·2382
	Wart Hill Hoy	− 0·2179		Fair Isle	− 0·3182
	Deerness	+ 0·5690			
DEERNESS	South Ronaldshay	− 0·5238			
	Wart Hill Hoy	+ 0·6399			
	Fitty Hill	+ 0·5751			
	Stronsay	+ 0·1641			

FIGURE 8.

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
BEN TARTEVIL	Ben Heynish	+ 0″·7156	*BEN MORE, MULL*	Ben Tartevil	− 0″·9705
	Ben More, Mull	+ 0·2592		Ben Heynish	+ 0·2428
	Jura	− 0·3762		Ben More, S. Uist	− 0·4327
JURA	Ben Tartevil	+ 1·5959		Storr	+ 0·2793
	Ben Heynish	− 0·7559		Ben Nevis	+ 0·3202
	Ben More, S. Uist	− 0·0571		Jura	+ 0·1994
	Ben More, Mull	− 0·1412	*BEN HEYNISH*	Ben More, S. Uist	− 1·1740
	Ben Nevis	+ 0·1862		Storr	− 3·1756

FIGURE 8—*continued.*

From	To	Correction to observed Bearing.
		″
BEN HEYNISH—cont.	Ben Nevis	+ 1·5497
	Ben More, Mull	− 0·3863
	Jura	+ 0·7883
	Ben Tartevil	− 0·4092
BEN NEVIS	Jura	+ 0·2429
	Ben More, Mull	− 0·9049
	Ben Heynish	+ 0·6802
	Ben More, S. Uist	+ 1·2782
	Storr	− 0·5965
	Mamsuil	− 0·0225
	Scournalapich	− 0·0383
MAMSUIL	Ben More, Mull	− 0·0796
	Ben More, S. Uist	− 0·7437
	Storr	− 0·6502
	Cleisham	+ 0·7449
	Scournalapich	+ 0·6832
	Ben Nevis	− 0·1039
SCOURNALAPICH	Mamsuil	− 1·2697
	Ben More, S. Uist	+ 0·0261
	Cleisham	− 0·2880
	Monach	+ 0·4968
	Ben Clibrig	+ 1·3066
	Ben Nevis	+ 0·4676
BEN MORE, S. UIST	Cleisham	− 0·0262
	Storr	− 0·0684
	Scournalapich	+ 0·1081
	Mamsuil	− 0·1274
	Ben Nevis	− 0·3583
	Ben More, Mull	+ 0·4142
	Jura	− 1·2131
	Ben Heynish	+ 0·6544
STORR	Ben Heynish	− 1·0133
	Ben More, S. Uist	+ 0·2576
	Cleisham	+ 0·5594
	Monach	− 1·3489
	Cnocghiubhais	− 0·7851
	Ru Rea	+ 0·7025
	Ben Clibrig	− 0·9119
	Mamsuil	+ 1·2632
		″
STORR—continued	Ben Nevis	+ 0·0568
	Ben More, Mull	− 1·2307
RU REA	Storr	− 0·5471
	Cleisham	+ 0·2953
	Monach	+ 0·8504
	Cnocghiubhais	+ 0·1552
CLEISHAM	Ben More, S. Uist	+ 2·7278
	North Rona	+ 2·7912
	Monach	− 0·3563
	Cnocghiubhais	+ 1·3845
	Fashven	+ 0·5908
	Ru Rea	− 0·2534
	Scournalapich	+ 2·4404
	Mamsuil	+ 5·0870
	Storr	− 1·2232
MONACH	Cleisham	+ 0·1895
	North Rona	− 0·5398
	Cnocghiubhais	+ 0·0268
	Fashven	− 1·5262
	Ben Clibrig	− 0·0784
	Scournalapich	− 1·4714
	Ru Rea	+ 0·2097
	Storr	+ 0·5960
BEN CLIBRIG	Scournalapich	− 0·3829
	Storr	− 0·1182
	Monach	+ 1·5460
FASHVEN	Cleisham	+ 6·1978
	Monach	− 1·3202
	Cnocghiubhais	− 4·5296
	North Rona	− 2·4222
BEN HUTIG	Cnocghiubhais	+ 0·9905
	North Rona	+ 0·4766
CNOCGHIUBHAIS	Monach	− 0·7031
	North Rona	− 3·2460
	Ben Hutig	+ 0·0060
	Fashven	+ 0·8700

FIGURE 9.

From	To	Correction to observed Bearing.
		″
BEN HUTIG	Ben Wyvis	− 1·2593
BEN CLIBRIG	Ben Wyvis	− 1·5917
	Corryhabbie	− 1·0054
	Ben Macdui	− 1·1723
SCARABIN	Ben Macdui	+ 2·4383
	Ben Wyvis	+ 5·1029
	Mormonth	+ 0·3803
	Cowhythe	+ 0·3836
	Knock	− 2·6693
	Corryhabbie	− 0·4787
BEN CHEILT	Knock	+ 4·7148
	Corryhabbie	+ 2·9611
BEN WYVIS	Ben Nevis	− 1·4351
	Mamsuil	− 1·1102
	Scournalapich	+ 1·5010
	Ben Hutig	− 1·7747
	Ben Clibrig	− 0·0298
	Scarabin	+ 3·6930
	Knock	+ 0·3759
	Corryhabbie	− 1·0018
	Ben Macdui	− 0·0238
SCOURNALAPICH	Ben Wyvis	+ 0·2323
	Corryhabbie	+ 1·7802
	Ben Macdui	+ 0·5976
MAMSUIL	Ben Wyvis	− 0·9867
	Corryhabbie	− 0·5169
	Ben Macdui	− 0·5327
	Glashmeal	− 0·0392
	Ben Lawers	− 0·0433
BEN NEVIS	Ben Wyvis	− 0·4851
	Corryhabbie	+ 0·6854
	Ben Macdui	+ 0·4361
	Ben Lawers	− 0·0007
BEN MACDUI	Ben Lawers	+ 0·2503
	Ben Nevis	+ 0·5884
	Mamsuil	+ 0·0837
	Scournalapich	− 1·1105
	Ben Wyvis	+ 0·2676
	Ben Clibrig	− 0·2268
BEN MACDUI—cont.	Scarabin	− 0·1509
	Ben Cheilt	+ 0·6111
	Corryhabbie	− 0·4664
	Mount Battock	+ 0·0233
	Glashmeal	− 0·0323
CORRYHABBIE	Glashmeal	+ 0·0263
	Ben Macdui	+ 0·2861
	Ben Nevis	+ 0·2423
	Mamsuil	− 0·9879
	Scournalapich	+ 0·3340
	Ben Wyvis	+ 0·2017
	Ben Clibrig	− 1·6290
	Scarabin	+ 0·1278
	Ben Cheilt	+ 1·7074
	Cowhythe	+ 0·0982
	Knock	+ 0·1360
	Mormonth	− 0·1237
	Mount Battock	+ 0·0702
KNOCK	Ben Cheilt	+ 0·8828
	Cowhythe	− 0·2218
	Mormonth	− 0·0868
	Mount Battock	+ 1·5499
COWHYTHE	Knock	+ 0·2634
	Corryhabbie	− 0·4749
	Scarabin	− 2·4843
	Ben Cheilt	+ 0·6037
MORMONTH	Mount Battock	+ 0·2567
	Corryhabbie	− 0·1617
	Knock	+ 0·5484
	Scarabin	− 0·7848
	Ben Cheilt	+ 0·7634
BEN LAWERS	Jura	+ 0·7287
	Ben More in Mull	− 0·0487
	Ben Nevis	+ 0·3307
	Mamsuil	− 0·5979
	Ben Macdui	− 0·2507
	Glashmeal	+ 0·0990
GLASHMEAL	Ben Lawers	+ 0·5009
	Ben Macdui	+ 1·4984
	Corryhabbie	− 0·3762
	Mount Battock	− 0·7742

FIGURE 9—*continued.*

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
		″			″
MOUNT BATTOCK	Glashmeal	+ 0·2950	*BEN MORE, MULL*	Ben Lawers	− 3·0967
	Ben Macdui	− 0·1560			
	Corryhabbie	− 0·3067	*JURA*	Ben Lawers	− 0·4408
	Knock	+ 0·0653			
	Mormonth	− 0·0029			
FIGURE 10.					
		″			″
MOUNT BATTOCK	Dudwick	+ 0·7456	*DUDWICK—cont.*	Mount Battock	− 1·5933
	Blue Hill	− 0·3830		Corryhabbie	− 0·0466
				Knock	− 1·2138
BEN MACDUI	Dudwick	+ 0·4439		Mormonth	+ 1·8233
				Little Stirling	+ 0·6318
CORRYHABBIE	Dudwick	+ 0·3190			
			BLUE HILL	Mount Battock	+ 0·5059
KNOCK	Dudwick	+ 1·0113		Mormonth	+ 2·4901
				Dudwick	− 0·2684
MORMONTH	Dudwick	− 0·6358		Little Stirling	− 0·4858
	Blue Hill	− 0·7068			
	Little Stirling	− 0·4390	*LITTLE STIRLING*	Blue Hill	+ 0·7589
				Dudwick	− 0·5397
DUDWICK	Blue Hill	+ 0·6514		Mormonth	+ 1·0807
FIGURE 11.					
		″			″
SLIEVE SNAGHT	Ben Tartevil	− 5·3480	*MERRICK*	Jura	− 1·0859
	Jura	− 6·9694		Goat Fell	− 0·1874
				Ben More, Mull	− 1·0276
KNOCKLAYD	Ben Tartevil	− 0·2057		Ben Lomond	− 1·4036
	Jura	− 0·0376		Ben Lawers	− 1·9774
	Ben Lomond	+ 1·8246		Ben Cleugh	− 1·0259
	Goat Fell	+ 0·8683			
			HART FELL	Goat Fell	+ 0·8327
TROSTAN	Ben Tartevil	− 2·5430		Ben Lomond	+ 3·6391
	Jura	+ 1·3123		Ben Lawers	+ 0·6308
	Goat Fell	+ 0·2272		Ben Cleugh	+ 2·0522
DIVIS	Goat Fell	− 7·1948			
			DUNRICH	Ben Lomond	− 0·8744
CRIFFEL	Goat Fell	+ 4·6193		Ben Cleugh	− 0·3255

FIGURE 11—*continued.*

From	To	Correction to observed Bearing.
		″
SAYRS LAW	Ben Lomond	− 3·4941
	Ben Cleugh	− 0·2071
	Ben Lawers	− 0·7081
	Glashmeal	− 1·2755
	Mount Battock	+ 1·6711
LUMSDEN	Glashmeal	+ 2·8880
	Mount Battock	+ 1·5803
GOAT FELL	Slieve Donard	− 0·3540
	Divis	− 0·9388
	Trostan	+ 1·1094
	Knocklayd	+ 1·5611
	Slieve Snaght	+ 0·1993
	Jura	+ 0·0838
	Ben More, Mull	− 0·6080
	Ben Lomond	+ 0·1438
	Ben Lawers	− 0·6798
	Ben Cleugh	− 0·0707
	Hart Fell	− 0·2918
	Criffel	+ 1·1828
	Merrick	− 0·1475
	South Berule	− 0·4428
BEN TARTEVIL	Slieve Snaght	+ 0·6429
	Trostan	+ 1·1078
	Knocklayd	+ 1·9964
JURA	Trostan	− 0·9348
	Knocklayd	+ 0·4948
	Mount Sandy	− 0·5967
	Slieve Snaght	− 1·6905
	Ben Lomond	+ 0·1754
	Goat Fell	+ 0·7348
	Merrick	+ 1·7171
BEN MORE, MULL	Ben Lomond	+ 0·1178
	Goat Fell	− 0·1886
BEN NEVIS	Ben Cleugh	− 0·2003
	Ben Lomond	− 1·9522

From	To	Correction to observed Bearing.
		″
BEN LOMOND	Goat Fell	+ 4·6334
	Knocklayd	− 0·2424
	Jura	+ 0·4391
	Ben More, Mull	− 0·1339
	Ben Nevis	− 3·3754
	Ben Lawers	− 0·8708
	Ben Cleugh	+ 0·5048
	Dunrich	− 0·2464
	Hart Fell	+ 0·8872
BEN CLEUGH	Merrick	− 1·1591
	Goat Fell	+ 0·3769
	Ben Lomond	− 1·0536
	Ben Nevis	+ 0·9195
	Ben Lawers	+ 0·0965
	Ben Macdui	+ 0·3035
	Glashmeal	+ 0·2267
	Mount Battock	+ 0·4346
	Sayrs Law	− 1·4998
	Dunrich	− 0·0919
	Hart Fell	+ 0·7851
BEN LAWERS	Merrick	+ 0·5098
	Goat Fell	+ 2·8665
	Ben Lomond	+ 0·3338
	Sayrs Law	− 1·6033
	Ben Cleugh	− 0·4892
	Hart Fell	+ 0·7974
BEN MACDUI	Ben Cleugh	+ 0·6698
	Sayrs Law	+ 0·6974
GLASHMEAL	Ben Cleugh	+ 2·1200
MOUNT BATTOCK	Ben Cleugh	− 0·4707
	Lumsden	+ 0·4030
	Sayrs Law	+ 0·1994

FIGURE 12.

From	To	Correction to observed Bearing.
		″
CRADLE	Dunkery	− 0·2584
	Paracombe	− 0·0839
	Precelly	+ 0·2732
	Mendip	+ 0·2416
PRECELLY	Cradle	− 0·2708
	Dunkery	− 0·0974
	Paracombe	+ 0·1510
	High Wilhays	+ 0·0920
	Lundy Island	+ 0·5585
LUNDY ISLAND	Hensbarrow	+ 0·3988
	Trevose Head	− 2·8670
	Precelly	− 0·0840
	Paracombe	+ 0·0804
	High Wilhays	− 0·7008
	Brown Willy	+ 0·3819
PARACOMBE	High Wilhays	− 0·6987
	Brown Willy	− 1·2374
	Lundy Island	+ 1·5597
	Precelly	− 0·8040
	Cradle	+ 0·8123
	Dunkery	− 0·5655
DUNKERY	High Wilhays	− 1·4389
	Brown Willy	+ 1·2060
	Paracombe	+ 0·4174
	Precelly	+ 0·1509
	Cradle	− 0·5615
	Mendip	− 0·0223
	Pillesdon	− 0·0275
MENDIP	Pillesdon	+ 0·0044
	High Wilhays	+ 0·7598
	Dunkery	− 0·0501
	Cradle	− 0·2102
PILLESDON	Barrow Hill	− 0·0589
	High Wilhays	− 0·1830
	Dunkery	+ 0·0508
	Mendip	+ 0·3628
HIGH WILHAYS	Deadman	− 0·7365
	Hensbarrow	+ 0·8342
	Brown Willy	+ 0·2441
	Lundy Island	+ 0·1236

From	To	Correction to observed Bearing.
		″
HIGH WILHAYS (*continued.*)	Precelly	+ 0·1179
	Paracombe	+ 1·5518
	Dunkery	− 0·7437
	Mendip	− 0·4399
	Pillesdon	− 0·1301
	Ryders Hill	− 0·0569
RYDERS HILL	Deadman	− 0·2738
	Hensbarrow	+ 0·1944
	Brown Willy	+ 0·4792
	High Wilhays	− 0·0849
	Pillesdon	+ 0·2387
	Barrow Hill	+ 0·0272
BARROW HILL	Hensbarrow	− 0·9035
	Maker Tower	− 0·3957
	Ryders Hill	− 0·0782
	Pillesdon	+ 0·5273
MAKER TOWER	Deadman	− 0·4270
	Hensbarrow	+ 0·2488
	High Wilhays	− 0·0991
	Barrow Hill	− 0·1062
BROWN WILLY	Deadman	+ 1·8665
	Hensbarrow	− 1·0742
	Trevose Head	+ 0·4432
	Lundy Island	− 3·8892
	Paracombe	− 5·2824
	High Wilhays	+ 0·4723
	Ryders Hill	− 2·9672
TREVOSE HEAD	Lundy Island	+ 0·2926
	Brown Willy	+ 0·0436
	Hensbarrow	− 1·3863
HENSBARROW	Trevose Head	+ 0·3798
	Lundy Island	+ 0·4559
	Brown Willy	− 2·4705
	High Wilhays	− 0·7249
	Maker Tower	− 3·0935
	Barrow Hill	+ 0·5201
	Deadman	+ 0·2423
DEADMAN	Hensbarrow	− 1·1227
	Brown Willy	− 0·7722
	High Wilhays	+ 0·9340
	Ryders Hill	+ 0·3356

FIGURE 13.

From	To	Correction to observed Bearing. ″
HENSBARROW	Goonhilly	− 0·2565
	Karnbonellis	+ 3·0118
	Pertinny	+ 0·8779
	Karnminnis	+ 1·1020
	St. Agnes' Beacon	+ 0·9432
DEADMAN	Goonhilly	− 0·4872
	Karnbonellis	− 1·4458
	St. Agnes' Beacon	− 2·3356
TREVOSE HEAD	St. Agnes' Beacon	− 0·6766
	Karnminnis	+ 0·3462
	Karn Galver	− 0·5612
GOONHILLY	Wolf Rock	+ 0·0482
	Pertinny	− 0·1105
	Karn Galver	+ 0·3275
	Karnbonellis	+ 0·3267
	Hensbarrow	− 1·2062
	Deadman	+ 0·5516
KARNBONELLIS	Karnminnis	+ 0·8271
	St. Agnes' Beacon	− 0·0423
	Hensbarrow	+ 0·6969
	Deadman	− 0·5487
ST. AGNES' BEACON	Karnbonellis	+ 5·1037
	Karnminnis	− 0·9814
	Trevose Head	+ 3·3729
	Hensbarrow	+ 0·0106
	Deadman	− 4·0314
KARNMINNIS	Pertinny	+ 0·6811
	Karn Galver	− 0·5726
	St. Martin's Head	− 0·9688
	Trevose Head	− 1·6787
	Brown Willy	− 0·4708
	St. Agnes' Beacon	+ 0·0039
	Hensbarrow	+ 1·0114
	Karnbonellis	+ 0·0545
KARN GALVER	Pertinny	+ 2·0139
	St. Agnes' Lighth.	+ 0·1823
	Peninnis Windmill	+ 0·0943
	Telegraph Tower	− 1·8798
	St. Martin's Head	+ 1·1657
	Beacon Hill, Trescow	− 0·8377
	Trevose Head	+ 0·0074
KARN GALVER (*continued.*)	Karnminnis	+ 1·5621
	Hensbarrow	− 4·2636
	Karnbonellis	+ 0·0052
	Goonhilly	− 0·4766
PERTINNY	Wolf Rock	− 0·2398
	St. Agnes' Lighthouse	− 0·1425
	Peninnis Windmill	− 0·0511
	Telegraph Tower	− 0·1681
	St. Martin's Head	+ 1·4326
	Beacon Hill, Trescow	− 2·0989
	Karn Galver	− 0·6195
	Karnminnis	− 1·3931
	Hensbarrow	+ 0·3938
	Karnbonellis	+ 1·0424
	Goonhilly	− 0·5084
PENINNIS WINDMILL	St. Agnes' Lighthouse	+ 0·1121
	Beacon Hill, Trescow	− 0·3504
	Telegraph Tower	− 3·2115
	St. Martin's Head	− 2·7183
	Karn Galver	+ 3·1379
	Pertinny	− 1·5710
	Wolf Rock	− 4·5311
TELEGRAPH TOWER	Peninnis W. Mill	− 0·7145
	St. Agnes' Lighthouse	− 0·9377
	Beacon Hill, Trescow	+ 3·5862
	St. Martin's Head	− 1·4489
	Karn Galver	− 1·1198
	Pertinny	+ 0·9196
	Wolf Rock	+ 0·9233
ST. MARTIN'S HEAD	Peninnis W. Mill	+ 1·5066
	Telegraph Tower	− 0·0965
	Beacon Hill	− 1·8862
	Karnminnis	− 0·1325
	Karn Galver	+ 0·6532
	Pertinny	− 1·1925
	Wolf Rock	+ 0·3936
BEACON HILL, TRESCOW.	St. Agnes' Lighthouse	+ 0·4495

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
FIGURE 13—*continued.*					
		″			″
BEACON HILL, TRESCOW—continued.	Karnminnis	− 0·8327	*BROWN WILLY*	Karnbonellis	− 3·1102
	Karn Galver	+ 1·8924		St. Agnes' Beacon	− 7·7507
	Pertinny	− 0·6758		Karnminnis	− 1·9101
	St. Martin's Head	+ 1·7751			
	Telegraph Tower	− 0·8773	*RYDERS HILL*	Goonhilly	− 0·1859
	Peninnis W. Mill	+ 0·4698			
FIGURE 14.					
		″			″
BEACON HILL	Wingreen	+ 0·8302	*STOKE HILL*	Wingreen	− 0·0253
	Westbury Down	+ 0·1483		Westbury Down	+ 0·0766
	Stoke Hill	− 0·4091		Milk Hill	− 0·3243
	Milk Hill	+ 0·9153		Inkpen	− 0·3977
	Inkpen	− 0·0614		Beacon Hill	+ 0·3521
	Dean Hill	− 0·5064		Dean Hill	+ 0·5223
DEAN HILL	Wingreen	− 1·8301	*MILK HILL*	Wingreen	+ 1·4204
	Westbury Down	+ 0·9520		Stoke Hill	+ 1·0297
	Stoke Hill	− 0·5273		Westbury Down	+ 0·0382
	Milk Hill	− 0·9475		Upcot Down	− 0·2398
	Beacon Hill	+ 0·0581		Inkpen	+ 0·2325
	Inkpen	+ 0·1605		Beacon Hill	− 0·6358
				Dean Hill	+ 0·7095
WINGREEN	Westbury Down	− 0·2921			
	Inkpen	+ 0·9814	*INKPEN*	Dean Hill	− 3·3260
	Beacon Hill	− 1·1287		Beacon Hill	+ 0·3406
	Dean Hill	+ 0·5954		Wingreen	− 0·0159
				Westbury Down	+ 0·0755
WESTBURY DOWN	Upcot Down	+ 0·2340		Upcot Down	− 0·6101
	Milk Hill	− 0·2766			
	Stoke Hill	− 0·1775			
	Inkpen	+ 0·2177	*UPCOT DOWN*	Milk Hill	+ 0·1418
	Beacon Hill	+ 0·3832		Stoke Hill	− 0·5996
	Dean Hill	− 0·0460		Westbury Down	+ 0·1827
	Wingreen	− 1·0800		Inkpen	+ 0·4611
FIGURE 15.					
		″			″
MILK HILL	Mendip	− 0·1189	*BEACON HILL*	Coringdon	− 2·5579
				Swyre Barrow	+ 0·9975
UPCOT DOWN	Mendip	− 0·0096		Mendip	+ 0·0566
				Butser Hill	− 1·8433
INKPEN	Mendip	− 1·3194		Dunnose	− 0·5640
	Butser Hill	+ 0·5919			
	Dunnose	+ 0·3389	*DEAN HILL*	Coringdon	+ 0·2718

FIGURE 15—*continued.*

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
DEAN HILL—cont.	Swyre Barrow	− 2″·7977	*CORINGDON—cont.*	Butser Hill	+ 1″·4588
	Horton's Gazebo	+ 1·0438		Dunnose	− 1·1838
	Butser Hill	+ 0·1166			
	Dunnose	− 1·2125	*SWYRE BARROW*	High Wilhays	− 2·2790
				Black Down	+ 1·1121
WINGREEN	Pillesdon	+ 0·6396		Pillesdon	+ 0·2062
	Mendip	+ 1·0604		Wingreen	− 0·4936
	Butser Hill	+ 1·0831		Horton's Gazebo	− 5·3401
	Dunnose	+ 2·2651		Beacon Hill	+ 0·3646
	Horton's Gazebo	+ 0·5043		Dean Hill	+ 0·1707
	Coringdon	+ 0·3107		Coringdon	− 0·7091
	Swyre Barrow	+ 0·9484		Dunnose	+ 1·3656
WESTBURY DOWN	Mendip	+ 0·7776	*BLACK DOWN*	Barrow Hill	− 0·2667
				Ryders Hill	− 0·0404
BUTSER HILL	Dunnose	− 0·0873		High Wilhays	+ 0·0761
	Coringdon	− 1·3128		Pillesdon	− 0·7582
	Wingreen	− 1·3068		Horton's Gazebo	− 0·3048
	Dean Hill	+ 1·5016		Dunnose	+ 0·2832
	Beacon Hill	+ 0·8748		Coringdon	− 0·1791
	Inkpen	+ 0·5180		Swyre Barrow	+ 0·0743
DUNNOSE	Swyre Barrow	− 0·2204	*MENDIP*	Inkpen	+ 0·3144
	Coringdon	+ 0·4842		Westbury Down	− 0·5628
	Black Down	− 0·4395		Beacon Hill	+ 1·7603
	Horton's Gazebo	+ 0·9125		Wingreen	+ 0·1132
	Wingreen	+ 0·0765			
	Dean Hill	− 0·5291	*PILLESDON*	Wingreen	+ 1·6762
	Beacon Hill	− 0·4603		Coringdon	+ 0·8006
	Inkpen	+ 0·3749		Swyre Barrow	− 0·6319
	Butser Hill	+ 0·5568		Black Down	+ 1·6611
CORINGDON	Swyre Barrow	+ 0·1932	*BARROW HILL*	Black Down	− 1·4390
	Black Down	− 1·0752			
	Pillesdon	+ 0·0075	*RYDERS HILL*	Black Down	− 0·1648
	Wingreen	+ 0·0109		Swyre Barrow	− 1·6258
	Horton's Gazebo	+ 2·1795			
	Beacon Hill	+ 0·0405	*HIGH WILHAYS*	Black Down	+ 1·7199
	Dean Hill	+ 0·1577		Swyre Barrow	− 0·9015

FIGURE 16.

CRADLE	Malvern	+ 0″·5230	*MILK HILL*	Whitehorse Hill	+ 2″·1644
	Broadway Tower	− 0·3887			
	Whitehorse Hill	− 0·8134	*UPCOT DOWN*	Whithorse Hill	− 0·0196
MENDIP	Malvern	+ 1·8119	*INKPEN*	Whitehorse Hill	+ 0·0134
	Whitehorse Hill	+ 1·0996		Leith Hill Tower	+ 0·2504

FIGURE 16—*continued.*

From	To	Correction to observed Bearing.
		″
BUTSER HILL	Leith Hill Tower	+ 0·4745
	Ditchling	− 0·4846
DUNNOSE	Ditchling	+ 1·5858
	Beachy Head	+ 1·1529
MALVERN	Mendip	+ 0·3738
	Cradle	− 0·4074
	Arbury Hill	+ 1·0108
	Broadway Tower	− 0·4655
	Whitehorse Hill	+ 0·7445
WHITEHORSE HILL	Mendip	− 4·2465
	Cradle	+ 2·4566
	Malvern	− 1·9652
	Broadway Tower	+ 1·7912
	Arbury Hill	+ 0·1022
	Dunstable	− 0·6184
	Leith Hill Tower	− 0·6796
	Inkpen	− 0·6354
	Upcot Down	+ 2·4010
BROADWAY TOWER	Cradle	+ 0·2594
	Malvern	+ 0·3619
	Arbury Hill	− 0·3849
	Whitehorse Hill	− 0·1894
ARBURY HILL	Broadway Tower	− 1·7692
	Malvern	+ 0·0782
	Dunstable	+ 0·2628
DUNSTABLE	Whitehorse Hill	+ 0·4790
	Arbury Hill	+ 0·0929
	Epping Cupola	− 0·2021
	Berkhampstead	− 1·5237
	Wrotham	− 0·7478
	Leith Hill Tower	− 0·4014
BERKHAMPSTEAD	Leith Hill Tower	− 0·8734
	Dunstable	− 0·3071
	Chingford	+ 0·0419
	Wrotham	+ 1·5519
	Severndroog	+ 0·9647
	St. Paul's	+ 0·2760
EPPING CUPOLA	Severndroog	− 0·1403
	Leith Hill Tower	− 0·3577
	St. Paul's	+ 1·6958
	Berkhampstead	− 2·9947
		″
EPPING CUPOLA (*continued.*)	Dunstable	+ 0·7579
	Wrotham	− 2·3769
CHINGFORD	St. Paul's	− 0·0793
	Leith Hill Tower	− 0·2700
	Berkhampstead	+ 0·2563
	Wrotham	− 0·5638
	Severndroog	+ 0·2241
SEVERNDROOG	Leith Hill Tower	+ 0·2108
	St. Paul's	+ 0·0945
	Berkhampstead	− 1·0344
	Chingford	− 0·0966
	Epping Cupola	− 0·2013
	Wrotham	+ 0·2072
WROTHAM	Crowborough	− 0·7070
	Ditchling	− 0·2315
	Leith Hill Tower	+ 0·0977
	St. Paul's	− 1·3096
	Severndroog	− 0·1222
	Dunstable	− 2·5686
	Berkhampstead	− 0·9441
	Chingford	+ 0·4858
	Epping Cupola	+ 1·4866
	Fairlight	+ 0·3110
LEITH HILL TOWER	Butser Hill	− 0·7884
	Inkpen	− 1·5763
	Whitehorse Hill	+ 0·6463
	Dunstable	+ 1·4773
	Berkhampstead	− 0·4687
	St. Paul's	+ 0·4729
	Epping Cupola	− 0·2433
	Severndroog	− 0·1575
	Wrotham	+ 0·1379
	Crowborough	− 0·3051
	Beachy Head	+ 0·5739
	Ditchling	+ 1·2192
CROWBOROUGH	Leith Hill Tower	− 0·0278
	Wrotham	+ 0·3097
	Fairlight	− 0·9823
FAIRLIGHT	Beachy Head	− 0·1651
	Ditchling	− 0·1422
	Crowborough	+ 0·2211
	Wrotham	− 0·0267

FIGURE 16—*continued.*

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
		″			″
DITCHLING . . .	Dunnose . . .	− 0·8250	*DITCHLING—cont.* .	Beachy Head . .	− 0·0034
	Butser Hill . .	+ 0·2400			
	Leith Hill Tower	+ 0·1211	*BEACHY HEAD* . .	Dunnose . . .	− 0·0392
	Wrotham . . .	− 0·1809		Ditchling . . .	− 1·6566
	Crowborough . .	− 0·9635		Leith Hill Tower	− 0·0841
	Fairlight . . .	+ 1·3548		Fairlight . . .	+ 0·5411

FIGURE 17.

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
		″			″
KIPPURE	Ballycreen . .	+ 1·4287	*BALLYCREEN—cont.*	Snowdon . . .	+ 2·1158
	Precelly . . .	− 0·8249		Precelly . . .	− 1·5099
	Tara	− 1·8999		Tara	− 0·3250
CROGHAN	Mount Leinster .	− 0·8402	*PRECELLY* . . .	Forth	+ 1·9766
				Mount Leinster .	+ 8·3726
KEEPER	Mount Leinster .	+ 2·2976		Tara	− 0·4281
				Kippure . . .	− 1·2920
KNOCKANAFFRIN .	Ballycreen . .	− 0·7703		Snowdon . . .	+ 1·5496
	Mount Leinster .	− 1·1894		Cader Idris . .	+ 0·3299
	Forth	+ 0·7923		Plynlimmon . .	+ 0·4179
SNOWDON	Precelly . . .	+ 1·2779	*CRADLE*	Plynlimmon . .	+ 0·4277
	Tara	+ 1·4776		Cader Idris . .	− 0·6161
	Cyrn-y-Brain .	+ 1·5386		Snowdon . . .	− 2·0787
	Longmount Pole	+ 4·3601		Longmount Pole	+ 0·1864
	Plynlimmon . .	+ 0·6365			
	Cader Idris . .	+ 0·2171	*MALVERN*	Longmount Pole	+ 1·6678
				Bardon . . .	− 0·3150
FORTH	Knockanaffrin .	+ 0·3253			
	Mount Leinster .	+ 0·0210	*BROADWAY TOWER* .	Bardon . . .	− 0·5079
	Ballycreen . .	− 1·9150			
	Tara	− 0·2142	*ARBURY HILL* . .	Bardon . . .	+ 0·8985
	Precelly . . .	+ 0·4435			
			LONGMOUNT POLE .	Cradle	− 0·7323
TARA	Forth	− 1·1821		Plynlimmon . .	+ 0·0767
	Mount Leinster .	− 0·8278		Cader Idris . .	+ 0·1641
	Ballycreen . .	+ 4·5583		Snowdon . . .	+ 2·3109
	Kippure . . .	+ 0·9110		Cyrn-y-Brain .	+ 0·0993
	Snowdon . . .	− 5·4284		Mowcopt . . .	+ 0·5026
	Precelly . . .	− 2·7265		Axedge . . .	− 1·8017
				Bardon . . .	+ 1·6388
BALLYCREEN . .	Forth	+ 0·3954		Malvern . . .	− 0·8901
	Mount Leinster .	− 0·1642			
	Knockanaffrin .	− 0·4838	*BARDON*	Broadway Tower	− 0·9428
	Keeper . . .	+ 0·6466		Malvern . . .	+ 0·2521
	Kippure . . .	− 0·6725		Longmount Pole	+ 1·4504

FIGURE 17—*continued.*

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
BARDON—continued	Mowcopt	− 2″·2963	*MOWCOPT—cont.*	Cyrn-y-Brain	− 0″·0047
	Axedge	+ 6·8833		Bardon	− 1·4838
	Arbury Hill	+ 3·9590			
			AXEDGE	Longmount Pole	+ 1·6702
CYRN-Y-BRAIN	Snowdon	− 3·7428		Cyrn-y-Brain	+ 0·2921
	Axedge	+ 2·3913		Bardon	+ 3·4806
	Mowcopt	+ 1·0156			
	Longmount Pole	− 0·9945	*HOLME MOSS*	Cyrn-y-Brain	− 2·1089
MOWCOPT	Longmount Pole	− 1·4401	*WHITTLE*	Cyrn-y-Brain	− 2·0999

FIGURE 18.

From	To	Correction to observed Bearing.	From	To	Correction to observed Bearing.
EPPING CUPOLA	Danbury	− 0″·4880	*FRITTENFIELD—cont.*	Paddlesworth	− 0″·7404
	Gads Hill	− 0·2037			
			GADS HILL	Severndroog	+ 0·0880
SEVERNDROOG	Gads Hill	− 2·2889		St. Paul's	+ 1·4300
				Epping Cupola	+ 0·3481
WROTHAM	Danbury	+ 0·9820		Danbury	− 0·6057
	Gads Hill	− 1·3974		Norwood	− 0·1118
	Norwood	+ 1·0908	*NORWOOD*	Wrotham	+ 0·0577
	Frittenfield	− 0·4749		Gads Hill	+ 2·9995
				Danbury	+ 1·0049
LEITH HILL TOWER	Frittenfield	+ 1·7540		Walton Tower	− 1·1899
BEACHY HEAD	Frittenfield	+ 1·1947		St. Peter's Church Tower	+ 0·1412
FAIRLIGHT	Frittenfield	+ 0·3254		Frittenfield	− 2·3975
	Paddlesworth	+ 0·0192	*DANBURY*	Gads Hill	− 0·6595
				Wrotham	+ 0·3810
PADDLESWORTH	Fairlight	− 0·3099		Severndroog	− 2·2587
	Crowborough	+ 0·2754		Epping Cupola	+ 2·0703
	Frittenfield	+ 1·1000		Walton Tower	+ 0·0553
	Walton Tower	− 0·1363		St. Peter's Church Tower	+ 2·0142
	St. Peter's Church Tower	− 0·0905		Norwood	− 1·1091
				Frittenfield	+ 1·4768
FRITTENFIELD	Fairlight	+ 0·7913			
	Beachy Head	− 2·8560	*ST. PETER'S CHURCH TOWER*	Paddlesworth	− 0·1468
	Crowborough	+ 0·2520		Frittenfield	+ 1·4266
	Leith Hill Tower	− 0·0462		Norwood	+ 1·0960
	Wrotham	+ 0·5969		Danbury	− 2·3227
	Danbury	+ 1·1803		Walton Tower	+ 0·6924
	Norwood	+ 1·1878			
	Walton Tower	− 0·0015	*WALTON TOWER*	Danbury	+ 0·4262
	St. Peter's Church Tower	− 0·0599		St. Peter's Church Tower	− 3·1790

FIGURE 19.

From	To	Correction to observed Bearing.
		″
DUNSTABLE	Tharfield	− 0·4073
BERKHAMPSTEAD	Thaxted	+ 3·1319
EPPING CUPOLA	Tharfield	− 2·1634
	Thaxted	− 0·2408
DANBURY	Thaxted	+ 0·0873
	Lawshall	− 1·7571
	Naughton	+ 1·5071
	Stoke Tower	+ 2·1301
WALTON TOWER	Stoke Tower	+ 0·3083
	Naughton	− 0·1244
	Otley	+ 0·4736
THARFIELD	Berkhampstead	+ 2·4306
	Dunstable	− 0·8353
	Ely Minster	− 2·1245
	Brandon Tower	+ 0·2073
	Balsham	+ 1·1040
	Thaxted	− 0·6971
	Epping Cupola	+ 0·2572
THAXTED	Epping Cupola	+ 0·4184
	Berkhampstead	+ 1·2733
	Tharfield	− 1·7992
	Balsham	− 0·4772
	Lawshall	− 1·4400
	Stoke Tower	+ 1·9423
	Danbury	− 2·4726
STOKE TOWER	Danbury	− 0·6301
	Lawshall	− 0·4242
	Naughton	− 0·8302
	Otley	− 0·0281
	Walton Tower	+ 0·2015
NAUGHTON	Stoke Tower	− 0·0504
	Danbury	+ 2·2618
	Thaxted	− 0·3801
	Lawshall	+ 0·6763
	South Lopham	+ 2·4023
	Mickfield	+ 2·3793
	Otley	− 0·0458
	Walton Tower	− 1·2221
		″
OTLEY	Stoke Tower	− 2·2213
	Naughton	+ 1·9779
	Mickfield	+ 0·4631
	Walton Tower	+ 0·1940
MICKFIELD	Naughton	− 2·4587
	Lawshall	− 1·2752
	South Lopham	+ 1·7766
	Otley	− 0·2355
LAWSHALL	Thaxted	− 0·0528
	Balsham	+ 0·1892
	Brandon Tower	+ 0·4964
	Swaffham	− 1·6863
	South Lopham	+ 0·5348
	Mickfield	+ 0·6503
	Naughton	− 2·2726
	Stoke Tower	+ 0·0918
BALSHAM	Tharfield	+ 0·0609
	Ely Minster	+ 0·4963
	Swaffham	− 0·8967
	Lawshall	− 0·1824
	Thaxted	+ 0·2275
ELY MINSTER	Tharfield	+ 0·5713
	Swaffham	+ 0·1238
	Brandon Tower	− 0·5259
	Balsham	− 0·2713
SWAFFHAM	Brandon Tower	− 2·4476
	Ely Minster	+ 1·3217
	South Lopham	− 1·3711
	Lawshall	− 0·2076
SOUTH LOPHAM	Lawshall	+ 0·1272
	Brandon Tower	− 1·0144
	Swaffham	+ 1·3853
	Mickfield	− 0·7567
BRANDON TOWER	Tharfield	− 0·3385
	Ely Minster	+ 0·7434
	Swaffham	+ 0·4387
	South Lopham	− 1·0680
	Lawshall	− 0·3183

FIGURE 20.

From	To	Correction to observed Bearing.
WALTON TOWER	Orford Castle	+ 1″·4656
LAWSHALL	Hingham	− 0·9536
BRANDON TOWER	Hingham	− 0·8044
	Bunwell	− 2·9173
OTLEY	Laxfield	+ 1·5357
	Orford Castle	+ 1·2519
MICKFIELD	Bunwell	− 2·9728
	Laxfield	− 3·5772
SOUTH LOPHAM	Hingham	+ 0·8659
	Bunwell	+ 1·7025
	Laxfield	+ 3·5404
SWAFFHAM	Baconsthorpe	+ 0·6942
	Hingham	− 1·0044
ORFORD CASTLE	Walton Tower	+ 1·2096
	Otley	− 1·3402
	Laxfield	+ 0·1140
	Southwold	+ 0·7665
LAXFIELD	Otley	+ 0·6672
	Mickfield	− 0·8427
	South Lopham	+ 1·9533
	Bunwell	− 0·7538
	Norwich Spire	− 1·1786
	Tofts Tower	− 0·1263
	Southwold	+ 0·3033
	Orford Castle	+ 0·2353
BUNWELL	Mickfield	− 0·4607
	South Lopham	+ 0·8679
	Brandon Tower	+ 0·3978
	Swaffham	− 0·6723
	Hingham	− 0·5723
	Baconsthorpe	− 0·1633
	Norwich Spire	+ 0·0532
	Tofts Tower	+ 0·2786
BUNWELL—cont.	Laxfield	+ 0″·1817
HINGHAM	Brandon Tower	− 1·7186
	Swaffham	+ 1·4600
	Baconsthorpe	+ 1·0773
	Norwich Spire	+ 0·0958
	Bunwell	− 0·5570
	South Lopham	− 1·1104
SOUTHWOLD	Orford Castle	+ 0·7582
	Laxfield	− 3·3163
	Tofts Tower	+ 1·0073
	Gorleston	− 0·0246
TOFTS TOWER	Laxfield	− 0·2299
	Bunwell	− 0·6854
	Norwich Spire	+ 0·3354
	Happisburgh	− 0·1369
	Gorleston	+ 0·0273
	Southwold	− 0·0770
NORWICH SPIRE	Bunwell	+ 1·7127
	Hingham	+ 1·4839
	Baconsthorpe	− 1·3605
	Happisburgh	− 4·2402
	Gorleston	+ 2·0093
	Tofts Tower	+ 2·5794
	Laxfield	+ 0·0955
BACONSTHORPE	Hingham	+ 0·3037
	Swaffham	− 1·0949
	Happisburgh	− 0·0787
	Norwich Spire	+ 0·3131
GORLESTON	Southwold	− 2·1010
	Tofts Tower	+ 0·6114
	Norwich Spire	− 0·0397
	Happisburgh	+ 0·0457
HAPPISBURGH	Norwich Spire	+ 1·2600
	Baconsthorpe	− 0·2034
	Gorleston	− 0·7470
	Tofts Tower	+ 0·2225

FIGURE 21.

From	To	Correction to observed Bearing. ″
BARDON	Back Tor	+ 4·4441
	Lincoln Minster	+ 1·1644
	Buckminster	+ 2·8175
	Tilton	+ 3·4813
	Naseby Church Tower	+ 1·3486
AXEDGE	Back Tor	− 0·4030
	Lincoln Minster	+ 6·8877
HOLME MOSS	Garforth Cliff	− 4·0704
	Clifton Beacon	+ 8·5350
	Back Tor	+ 0·8877
GREAT WHERNSIDE	York Minster	+ 2·4675
BOTTON HEAD	Acklam Wold	− 1·1278
	York Minster	− 3·8752
ACKLAM WOLD	Crowle	+ 0·5809
	Clifton Beacon	+ 1·5598
	Garforth Cliff	+ 1·3310
	York Minster	+ 4·0979
	Great Whernside	− 0·6772
	Botton Head	− 0·8116
YORK MINSTER	Clifton Beacon	+ 0·4066
	Garforth Cliff	− 1·1763
	Great Whernside	+ 2·1452
	Botton Head	− 2·2370
	Acklam Wold	− 0·5709
GARFORTH CLIFF	Holme Moss	− 1·6076
	Great Whernside	+ 5·0110
	York Minster	+ 0·1905
	Acklam Wold	+ 1·7878
	Crowle	+ 1·4939
CROWLE	Clifton Beacon	+ 0·6454
	Back Tor	− 2·9347
	Garforth Cliff	− 1·4626
	Acklam Wold	+ 2·6186
	Lincoln Minster	+ 0·5484
CLIFTON BEACON	Back Tor	− 0·6340
	Holme Moss	+ 8·2329
	Great Whernside	+ 0·3563
	York Minster	− 0·3081
	Acklam Wold	− 2·0708
CLIFTON BEACON (continued.)	Crowle	− 0·0747
	Lincoln Minster	+ 1·2414
LINCOLN MINSTER	Bardon	− 1·9848
	Axedge	+ 1·6626
	Back Tor	− 3·7351
	Clifton Beacon	+ 1·3892
	Crowle	− 0·3037
	Boston	+ 5·2840
	Buckminster	− 0·5650
ARBURY HILL	Naseby Church Tower	− 0·3277
	Hanslope Spire	− 0·0301
DUNSTABLE	Hanslope Spire	+ 1·9403
	Naseby Church Tower	+ 2·7417
	Keysoe Spire	+ 2·4500
NASEBY CHURCH TOWER	Arbury Hill	− 3·7734
	Bardon	+ 2·0172
	Tilton	− 2·1849
	Easton Tower	+ 1·5088
	Keysoe Spire	+ 6·4331
	Hanslope Spire	+ 2·5600
TILTON	Naseby Church Tower	− 1·9796
	Bardon	+ 2·1631
	Buckminster	− 0·8136
	Easton Tower	+11·7850
	Keysoe Spire	+ 0·7355
EASTON TOWER	Naseby Church Tower	+ 1·0558
	Tilton	+ 8·1934
	Buckminster	+ 1·0945
	Boston Tower	+ 1·0132
	Walpole St. Peter's	− 1·5687
	Ely Minster	− 5·8211
	Keysoe Spire	− 2·4798
KEYSOE SPIRE	Dunstable	− 0·9194
	Hanslope Spire	+ 0·4534
	Naseby Ch. Tower	+ 1·7314
	Tilton	+ 0·7781
	Easton Tower	− 2·4992
	Ely Minster	+ 3·5184
	Tharfield	− 0·5046

FIGURE 21—*continued.*

From	To	Correction to observed Bearing.
		″
THARFIELD	Hanslope Spire	− 1·5585
	Keysoe Spire	+ 0·1694
BALSHAM	Keysoe Spire	+ 0·7971
ELY MINSTER	Keysoe Spire	+ 1·1594
	Easton Tower	+ 2·9451
	Walpole St. Peter's	+ 1·0520
	Lynn Tower	+ 1·8218
WALPOLE, ST. PETER'S	Easton Tower	+ 0·3132
	Boston Tower	+ 2·0348
	Docking Tower	+ 0·7483
	Lynn Tower	− 1·5121
	Swaffham	+ 0·1791
	Ely Minster	− 2·8718
LYNN TOWER	Ely Minster	− 1·0513
	Walpole St. Peter's	+ 2·2323
	Boston Tower	+ 2·6426
LYNN TOWER—cont.	Docking Tower	− 1·1794
	Swaffham	− 0·1851
BOSTON TOWER	Easton Tower	− 3·6780
	Buckminster	+ 1·8109
	Lincoln Minster	+ 4·4909
	Docking Tower	− 7·9679
	Swaffham	− 3·9543
	Lynn Tower	− 2·5441
	Walpole St. Peter's	− 1·5316
DOCKING TOWER	Lynn Tower	+ 0·6245
	Walpole St. Peter's	− 0·2060
	Boston Tower	+ 0·4950
	Baconsthorpe	− 2·1580
SWAFFHAM	Walpole St. Peter's	+ 5·7977
	Lynn Tower	+ 3·9192
	Boston Tower	+ 3·7631
BACONSTHORPE	Docking Tower	+ 0·4040

CONNECTION OF SPECIAL POINTS

WITH THE GENERAL TRIANGULATION.

The corrections given in the preceding pages, 1,554 in number, enable us to determine the most probable relative positions and distances of 218 principal points; before proceeding, however, to the calculation of the triangles, there remain the extremities of five base-lines, and several sector stations and observatories, whose positions with reference to the above points must be accurately determined. We proceed to give the Equations of Condition necessary for these determinations, and the corrections to the observed bearings that have thence resulted.

Hounslow Heath Base.

The triangulation for the connection of this base-line is shown in Figure 22. The points in this figure that are already fixed, are Leith Hill, Wrotham, Severndroog, Epping, Chingford, and St. Paul's Cathedral. Those to be fixed are Banstead, St. Ann's, Hanger Hill Tower, and the extremities of the base, King's Arbour and Hampton Poorhouse. The station here called Leith Hill is not identical with that used by Captain Kater in 1822, nor with that of General Roy in 1795; their relative positions will be found at page 25. The station Wrotham is identical with that of Captain Kater, but at a considerable distance from General Roy's station of that name. The stations Severndroog and Chingford are identical with Captain Kater's stations of 1822; the former is a few inches distant from General Roy's station (page 35). The station Banstead is 6·2 feet south (more exactly, 357° 7′) of the old station of that name; and from the station on Hanger Hill Tower, the old station of Hanger Hill (1792) is distant 504·8 feet in the direction 269° 4′ 24″.

The remaining points, King's Arbour, Hampton Poorhouse, and St. Ann's Hill, are identical with the points so called in the first volume of the "Account of the Trigonometrical Survey."

Instead of writing the geometrical Equations of Condition at full length, it will be sufficient to indicate them thus: ABC prefixed to an algebraical equation indicates that that equation arises from the sum of the angles of the triangle ABC being necessarily equal to a known quantity, and A : BCD prefixed to an algebraical equation indicates that the equation arises from the necessary relation—

$$\frac{\mathrm{AB}}{\mathrm{AC}} \cdot \frac{\mathrm{AC}}{\mathrm{AD}} \cdot \frac{\mathrm{AD}}{\mathrm{AB}} = \frac{\sin \mathrm{ACB}}{\sin \mathrm{ABC}} \cdot \frac{\sin \mathrm{ADC}}{\sin \mathrm{ACD}} \cdot \frac{\sin \mathrm{ABD}}{\sin \mathrm{ADB}} = 1$$

An inspection of the table of calculated corrections following the equations, will show to which bearings the symbolical corrections (1) (2) (3) . . . . . are respectively attached.

Equations of Condition.

I. *MD′B′* $0 = +2{\cdot}9782 - (3) + (5) + (9) - (17)$
II. *MD′C* $0 = -2{\cdot}4845 - (5) + (8) - (15) + (17)$
III. *GFB′* $0 = -0{\cdot}6694 - (2) + (4) + (18) - (24)$
IV. *GMB′* $0 = -3{\cdot}8794 - (2) + (3) - (9) + (13) + (18) - (25)$
V. *GD′B′* $0 = -2{\cdot}4362 - (2) + (7) + (18) - (23)$
VI. *LGB′* $0 = +2{\cdot}7352 - (1) + (2) - (18) + (20) - (27) + (30)$
VII. *LMB′* $0 = +4{\cdot}3151 - (1) + (3) - (9) + (10) - (29) + (30)$
VIII. *LKM* $0 = -3{\cdot}2068 - (10) + (11) - (26) + (29) + (31) - (33)$
IX. *HKL* $0 = +0{\cdot}2864 - (26) + (28) + (31) - (32) - (34) + (35)$
X. *HLG* $0 = +2{\cdot}9060 - (19) + (20) - (27) + (28) - (34) + (36)$
XI. *B′ : MCD′* $0 = -308{\cdot}675 - 202{\cdot}0300\,(5) - 291{\cdot}5474\,(8) - 180{\cdot}6479\,(9) + 95{\cdot}2285\,(15) + 85{\cdot}4194\,(17)$

xii. $A' : MB'D'$ $\quad 0 = -117\cdot312 + 147\cdot5334\,(3) + 9\cdot1600\,(5) - 79\cdot8601\,(9) + 41\cdot2690\,(14) + 38\cdot5911\,(17)$

xiii. $E' : MA'D'$ $\quad 0 = -80\cdot636 - 101\cdot6736\,(5) + 70\cdot5458\,(14) - 203\cdot3259\,(16) + 132\cdot7801\,(17)$

xiv. $D' : B'GF$ $\quad 0 = +39\cdot243 + 29\cdot6466\,(2) - 68\cdot1250\,(4) + 2\cdot4314\,(18) + 74\cdot9657\,(23) - 77\cdot3971\,(24)$

xv. $M : B'GD'$ $\quad 0 = +184\cdot150 + 40\cdot4729\,(2) - 192\cdot2492\,(3) - 216\cdot6342\,(5) + 14\cdot6042\,(7) + 52\cdot4533\,(18) + 11\cdot4121\,(23) - 63\cdot8654\,(25)$

xvi. $A' : MGD'$ $\quad 0 = -269\cdot889 + 9\cdot1600\,(5) - 106\cdot0036\,(7) + 30\cdot5365\,(13) - 69\cdot1276\,(14) + 38\cdot5911\,(17) + 115\cdot0605\,(22) - 122\cdot1792\,(23) + 7\cdot1187\,(25)$

xvii. $M : LGB'$ $\quad 0 = +72\cdot835 + 13\cdot0042\,(1) - 40\cdot4729\,(2) + 27\cdot4687\,(3) - 52\cdot4533\,(18) + 10\cdot8366\,(20) + 41\cdot6167\,(25) + 10\cdot1682\,(27) - 31\cdot9196\,(29) + 21\cdot7514\,(30)$

xviii. $L : D'GB'$ $\quad 0 = +44\cdot579 - 26\cdot0123\,(1) + 35\cdot2999\,(2) + 74\cdot1309\,(6) - 45\cdot5851\,(7) + 24\cdot3109\,(18) - 38\cdot6782\,(20) + 14\cdot3673\,(23)$

xix. $K : MLG$ $\quad 0 = -135\cdot588 + 44\cdot6263\,(10) - 84\cdot6437\,(11) + 40\cdot0174\,(13) - 68\cdot9615\,(20) + 65\cdot1521\,(21) + 3\cdot8094\,(25) + 59\cdot4385\,(26) - 62\cdot0348\,(27) + 2\cdot5963\,(29)$

xx. $K : HGL$ $\quad 0 = +312\cdot936 - 23\cdot3480\,(19) + 68\cdot9615\,(20) - 45\cdot6135\,(21) - 40\cdot4696\,(26) + 62\cdot0348\,(27) - 21\cdot5652\,(28) - 11\cdot4592\,(34) + 19\cdot9980\,(35) - 8\cdot5388\,(36)$

xxi. $H : MLG$ $\quad 0 = +330\cdot607 + 42\cdot1516\,(10) - 84\cdot4427\,(12) + 42\cdot2911\,(13) - 72\cdot2598\,(19) + 45\cdot0181\,(20) + 27\cdot2417\,(25) + 44\cdot0111\,(27) - 70\cdot3338\,(28) + 26\cdot3227\,(29)$

The logarithmic values of the multipliers are as follow:

Log I = 9·6680121 −
II = 0·6784650 +
III = 0·3401836 −
IV = 0·7208430 +
V = 0·6893755 +
VI = 0·4823861 +
VII = 0·6411409 −

Log VIII = 0·2604080 −
IX = 0·2341499 +
X = 0·3096631 −
XI = 8·4608365 +
XII = 8·8418597 −
XIII = 7·9748855 −
XIV = 8·4566329 +

Log XV = 8·7385372 −
XVI = 8·2067635 −
XVII = 8·7072380 −
XVIII = 7·4672759 +
XIX = 8·7842041 −
XX = 9·0014371 −
XXI = 7·9454110 −

Resulting Corrections.

From	To	Symbol.	Calculated Correction.
Leith Hill Tower	St. Ann's Hill .	(1)	+1″·1774
	Hanger Hill Tr.	(2)	−0·5529
	Banstead . .	(3)	+0·1792
Wrotham . . .	Hanger Hill Tr.	(4)	−0·4329
Severndroog Tower	Banstead . .	(5)	+0·9881
	St. Ann's Hill .	(6)	+0·1951
	Hanger Hill Tr.	(7)	+0·6179
Chingford . . .	Banstead . .	(8)	−0·7888
Banstead . . .	Leith Hill Tower	(9)	−0·3736
	St. Ann's Hill .	(10)	−2·2721
	King's Arbour .	(11)	+1·3993
	Hampton Poorhouse . . .	(12)	+0·1720
	Hanger Hill Tr.	(13)	+1·3308
	St. Paul's . .	(14)	−1·9324
	Chingford . .	(15)	−0·8478
	Epping Cupola .	(16)	+0·6680
	Severndroog .	(17)	+3·4135
Hanger Hill Tower	Leith Hill Tr. .	(18)	+0″·5454
	Hampton P. Ho.	(19)	+2·3278
	St. Ann's Hill .	(20)	−0·4229
	King's Arbour .	(21)	+3·1007
	St. Paul's . .	(22)	−0·5320
	Severndroog .	(23)	−0·7201
	Wrotham . .	(24)	−0·0041
	Banstead . .	(25)	−0·8976
St. Ann's Hill .	King's Arbour .	(26)	+0·3759
	Hanger Hill Tr.	(27)	−0·8157
	Hampton P. Ho.	(28)	+0·6242
	Banstead . .	(29)	+0·5661
	Leith Hill Tower	(30)	−0·8523
King's Arbour .	St. Ann's Hill .	(31)	−0·0224
	Hampton P. Ho.	(32)	−0·6049
	Banstead . .	(33)	+0·6324
Hampton Poorhouse	St. Ann's Hill .	(34)	+0·7706
	King's Arbour .	(35)	−0·3466
	Hanger Hill Tr.	(36)	−0·8246

MISTERTON CARR BASE.

The connection of this base-line with three fixed points in the triangulation, Crowle, Clifton Beacon, and Lincoln Minster, is shown in Figure 23. The points to be fixed are Gringley and the extremities of the base-line. The station here named Crowle is not identical with the station of 1806, of which no trace could be found in 1842, the date of the selection of the present station.

Equations of Condition.

I. $CB'G$ $\quad 0 = -1{\cdot}8284 + (3) - (4) - (5) + (8)$
II. $GB'S$ $\quad 0 = +2{\cdot}8655 - (2) + (3) - (5) + (6) + (10) - (12)$
III. $NB'G$ $\quad 0 = +9{\cdot}2523 - (1) + (3) - (5) + (7) + (14) - (15)$
IV. $NB'S$ $\quad 0 = +5{\cdot}4855 - (1) + (2) - (10) + (11) - (13) + (14)$
V. $L' : GCB'$ $\quad 0 = -236{\cdot}220 + 80{\cdot}5504\,(3) + 30{\cdot}2328\,(4) - 43{\cdot}8473\,(5) - 11{\cdot}9960\,(8) + 55{\cdot}8433\,(9)$
VI. $B' : SNG$ $\quad 0 = -169{\cdot}185 + 9{\cdot}4706\,(5) - 16{\cdot}9378\,(6) + 7{\cdot}4672\,(7) - 13{\cdot}0613\,(10) + 3{\cdot}3060\,(11)$
$\qquad + 9{\cdot}7553\,(12) - 12{\cdot}0154\,(13) + 6{\cdot}2883\,(14) + 5{\cdot}7271\,(15)$

The logarithmic values of the multipliers are as follow:

Log I = 0·1583086 +
II = 0·1512869 +
Log III = 0·2579842 −
IV = 9·7826815 +
Log V = 8·7200391 +
VI = 9·1805777 +

Resulting Corrections.

From	To	Symbol.	Calculated Correction.
CLIFTON BEACON	North End of Base	(1)	$+3''{\cdot}4825$
	South End of Base	(2)	$-6{\cdot}3974$
	Gringley Beacon .	(3)	$+0{\cdot}8954$
CROWLE . .	Gringley Beacon .	(4)	$+0{\cdot}4861$
GRINGLEY BEACON	Clifton Beacon .	(5)	$-0{\cdot}7334$
	South End of Base	(6)	$-9{\cdot}0073$
	North End of Base	(7)	$-4{\cdot}1688$
	Crowle Beacon .	(8)	$+0{\cdot}6858$
GRINGLEY BEACON	Lincoln Minster .	(9)	$+2''{\cdot}2469$
SOUTH END OF BASE.	Clifton Beacon .	(10)	$-1{\cdot}7512$
	North End of Base	(11)	$+2{\cdot}2655$
	Gringley Beacon .	(12)	$+0{\cdot}1331$
NORTH END OF BASE.	South End of Base	(13)	$-0{\cdot}7918$
	Clifton Beacon .	(14)	$-0{\cdot}4141$
	Gringley Beacon .	(15)	$+2{\cdot}8156$

RHUDDLAN MARSH BASE.

The triangulation for connecting this base-line with the points Snowdon, Cyrn-y-Brain, and Delamere* is shown in Figure 24. Besides the extremities of the base, the points Great Ormes Head, Gwaunysgaer, Garreg, Moelfre Issa, Llanelian, and Arrenig are to be fixed. The relative positions of the old and new stations at Delamere are given at page 16. There is some little doubt resting upon the re-finding of the exact centres of the old stations at Arrenig and Llanelian, and this circumstance must necessarily diminish the weight of the calculated length of the base-line.

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* The fixation of this Sector Station will be found at page 405.

Equations of Condition.

I. *CD′GL* $0 = -\,7{\cdot}6136 - (2) + (3) + (6) - (9) - (14) + (16)$

II. *RMW* $0 = +\,6{\cdot}8669 - (19) + (20) - (24) + (26) - (32) + (33)$

III. *EWM* $0 = +\,6{\cdot}5616 - (19) + (21) - (25) + (26) - (27) + (28)$

IV. *EWR* $0 = +\,0{\cdot}0248 - (24) + (25) - (28) + (30) - (31) + (33)$

V. *C : D′S′G* $0 = -\,193{\cdot}947 + 60{\cdot}4897\,(3) + 41{\cdot}3003\,(5) - 27{\cdot}0415\,(9) - 14{\cdot}2588\,(10)$

VI. *A : GCL* $0 = -\,98{\cdot}936 + 17{\cdot}1817\,(1) - 16{\cdot}2414\,(2) - 0{\cdot}9403\,(3) - 36{\cdot}5312\,(4) + 20{\cdot}2082\,(6)$
$+\,0{\cdot}9403\,(9) + 15{\cdot}3827\,(10) + 16{\cdot}5508\,(11) - 3{\cdot}7613\,(14) - 12{\cdot}7895\,(16)$

VII. *L : MCG* $0 = -\,604{\cdot}153 + 124{\cdot}4269\,(6) - 102{\cdot}6215\,(7) - 21{\cdot}8054\,(10) - 34{\cdot}5855\,(12) - 50{\cdot}0714\,(14)$
$+\,210{\cdot}5186\,(15) - 125{\cdot}8617\,(16) - 233{\cdot}5353\,(17) + 112{\cdot}3243\,(18) + 121{\cdot}2110\,(22)$

VIII. *O : MLR* $0 = -\,173{\cdot}193 + 14{\cdot}1428\,(12) - 46{\cdot}1351\,(13) + 31{\cdot}9923\,(15) - 3{\cdot}4333\,(17) - 41{\cdot}6479\,(18)$
$+\,45{\cdot}0812\,(20) - 28{\cdot}7795\,(32) + 28{\cdot}7795\,(34)$

IX. *W : MOR* $0 = -\,1297{\cdot}611 + 100{\cdot}1486\,(18) - 111{\cdot}3413\,(19) + 11{\cdot}1927\,(20) - 193{\cdot}5255\,(23) + 107{\cdot}8851\,(24)$
$+\,85{\cdot}6404\,(26) + 52{\cdot}9804\,(32) - 234{\cdot}1192\,(33) + 181{\cdot}1388\,(34)$

X. *L : MRGC*, $0 = +\,995{\cdot}867 - 30{\cdot}8920\,(6) + 9{\cdot}0866\,(8) + 21{\cdot}8054\,(10) + 90{\cdot}1718\,(13) + 3{\cdot}1454\,(14)$
$-\,219{\cdot}1789\,(15) + 125{\cdot}8617\,(16) + 165{\cdot}1722\,(17) - 43{\cdot}9612\,(20) - 121{\cdot}2110\,(22)$

XI. *W : EMR* $0 = +\,27{\cdot}641 + 0{\cdot}9403\,(19) - 11{\cdot}1927\,(20) + 10{\cdot}2524\,(21) + 24{\cdot}9075\,(27) - 3{\cdot}7046\,(28)$
$-\,21{\cdot}2029\,(30) + 46{\cdot}7787\,(31) - 52{\cdot}9804\,(32) + 6{\cdot}2017\,(33)$

XII. *M : WEO* $0 = -\,102{\cdot}353 - 10{\cdot}6528\,(18) + 85{\cdot}6404\,(19) - 74{\cdot}9876\,(21) + 77{\cdot}6606\,(23) + 5{\cdot}3451\,(25)$
$-\,83{\cdot}0057\,(26) + 71{\cdot}2472\,(27) + 24{\cdot}9075\,(28) - 96{\cdot}1547\,(29)$

The logarithmic values of the multipliers are as follow :

Log I = 9·9186648 +
II = 9·8749473 −
III = 9·5291529 −
IV = 9·8522429 +

Log V = 7·7652387 +
VI = 8·0336344 +
VII = 8·0920561 −
VIII = 8·5043912 −

Log IX = 7·8016327 +
X = 8·3408568 −
XI = 8·2500968 +
XII = 6·5666154 +

Resulting Corrections.

From	To	Symbol.	Calculated Correction.
Cyrn-y-Brain . .	Arrenig . . .	(1)	+0″·2574
	Llanelian . .	(2)	−3·8456
Delamere . . .	Garreg Mountⁿ.	(3)	+2·0356
Garreg Mountain	Arrenig . . .	(4)	−0·3624
	Snowdon . .	(5)	+0·4564
	Llanelian . .	(6)	+0·1281
	Gt. Ormes Head	(7)	+2·2298
	Gwaunysgaer .	(8)	−0·0239
	Delamere . .	(9)	−1·8607
	Cyrn-y-Brain .	(10)	−0·1158
Llanelian . . .	Arrenig . . .	(11)	+0·6586
	Gt. Ormes Head	(12)	−0·0168
	Gwaunysgaer .	(13)	−5·2266
	Garreg Mountⁿ.	(14)	−0·4526
	Moelfre Issa .	(15)	+0·0784
	Cyrn-y-Brain .	(16)	−0·7091
Moelfre Issa . .	Llanelian . .	(17)	−1·6643
Moelfre Issa—cont.	Gt. Ormes Head	(18)	+1″·1798
	West End of Base	(19)	+0·3241
	Gwaunysgaer .	(20)	−0·6642
	East End of Base	(21)	−0·5256
	Cyrn-y-Brain .	(22)	+1·3557
West End of Base	Gt. Ormes Head	(23)	−2·3856
	Gwaunysgaer .	(24)	+0·9211
	East End of Base	(25)	+2·6876
	Moelfre Issa .	(26)	−1·6218
East End of Base	Moelfre Issa .	(27)	+1·1111
	West End of Base	(28)	−0·2915
	Gt. Ormes Head	(29)	−0·1262
	Gwaunysgaer .	(30)	+0·8037
Gwaunysgaer . .	East End of Base	(31)	+0·5068
	Moelfre Issa .	(32)	+0·9561
	West End of Base	(33)	−2·3796
	Gt. Ormes Head	(34)	+1·0658

BELHELVIE BASE.

The triangulation for the connection of this base-line with the already fixed points Knock, Mormonth, Little Stirling, Blue Hill, Mount Battock, and Dudwick is given in Figure 25. The points to be fixed with relation to these, are the extremities of the base, and Over Hill and Brimmond.

Equations of Condition.

I. $BQ'M'$ $\quad 0 = -2{\cdot}3354 - (21) + (23) + (29) - (30)$

II. $BQ'D'$ $\quad 0 = -2{\cdot}0203 - (21) + (24) + (29) - (33)$

III. OBD' $\quad 0 = -1{\cdot}5922 - (14) + (17) - (24) + (25) - (32) + (33)$

IV. TOB $\quad 0 = +0{\cdot}8103 - (3) + (4) + (14) - (20) - (25) + (27)$

V. TOD' $\quad 0 = +0{\cdot}2971 - (4) + (5) - (17) + (20) - (31) + (32)$

VI. LOT $\quad 0 = -0{\cdot}4056 - (4) + (6) - (9) + (11) - (19) + (20)$

VII. LBT $\quad 0 = +1{\cdot}1426 - (3) + (6) - (9) + (10) - (26) + (27)$

VIII. $LD'T$ $\quad 0 = +3{\cdot}5664 - (5) + (6) - (9) + (12) + (31) - (34)$

IX. $D' : Q'BM'$ $\quad 0 = -247{\cdot}283 + 39{\cdot}1804\,(21) + 141{\cdot}7549\,(23) - 180{\cdot}9353\,(24) - 95{\cdot}2818\,(29) + 85{\cdot}4037\,(30)$

X. $K' : BM'Q'$ $\quad 0 = +57{\cdot}085 + 1{\cdot}4653\,(21) + 16{\cdot}6345\,(22) - 18{\cdot}0998\,(23) - 16{\cdot}2795\,(29) - 6{\cdot}3921\,(30)$

XI. $R' : BD'Q'$ $\quad 0 = +99{\cdot}644 + 0{\cdot}3464\,(21) - 11{\cdot}7652\,(24) + 11{\cdot}4188\,(28) + 75{\cdot}1326\,(29) + 80{\cdot}3070\,(33)$

XII. $Q' : OD'B$ $\quad 0 = +81{\cdot}650 - 148{\cdot}5797\,(14) + 120{\cdot}0695\,(15) + 28{\cdot}5102\,(17) - 73{\cdot}8633\,(21) - 39{\cdot}1804\,(24) + 113{\cdot}0437\,(25) - 42{\cdot}2861\,(32) + 74{\cdot}4481\,(33)$

XIII. $R' : BOQ'$ $\quad 0 = -54{\cdot}401 - 6{\cdot}9738\,(13) + 92{\cdot}3495\,(14) - 85{\cdot}3757\,(15) + 64{\cdot}6122\,(21) - 68{\cdot}5470\,(25) + 3{\cdot}9348\,(28) + 10{\cdot}8668\,(29)$

XIV. $O : R'BD'$ $\quad 0 = -908{\cdot}360 - 77{\cdot}9768\,(13) + 23{\cdot}6762\,(14) + 171{\cdot}7322\,(17) - 117{\cdot}4316\,(18) + 65{\cdot}9659\,(24) - 85{\cdot}3609\,(25) + 19{\cdot}3950\,(28) - 273{\cdot}3092\,(32) + 111{\cdot}6674\,(33)$

XV. $M' : OBD'$ $\quad 0 = +126{\cdot}308 - 31{\cdot}1333\,(14) - 180{\cdot}2022\,(16) + 211{\cdot}3355\,(17) + 98{\cdot}8721\,(23) - 141{\cdot}7549\,(24) + 42{\cdot}8828\,(25) - 102{\cdot}3938\,(32) + 51{\cdot}3437\,(33)$

XVI. $O : TBD'$ $\quad 0 = -11{\cdot}897 + 0{\cdot}5633\,(3) - 37{\cdot}7397\,(4) + 37{\cdot}1764\,(5) + 65{\cdot}9659\,(24) - 109{\cdot}0459\,(25) + 43{\cdot}0800\,(27) + 158{\cdot}2223\,(31) - 269{\cdot}8897\,(32) + 111{\cdot}6674\,(33)$

XVII. $R' : OTB$ $\quad 0 = -151{\cdot}350 + 41{\cdot}9536\,(1) - 16{\cdot}4673\,(3) - 25{\cdot}4863\,(4) - 66{\cdot}6603\,(13) + 28{\cdot}0837\,(14) + 38{\cdot}5766\,(20) - 4{\cdot}2812\,(25) - 5{\cdot}4662\,(27) + 9{\cdot}7474\,(28)$

XVIII. $R' : BTQ'$ $\quad 0 = +450{\cdot}446 + 8{\cdot}7522\,(1) - 122{\cdot}0134\,(2) + 113{\cdot}2612\,(3) + 97{\cdot}1403\,(21) - 91{\cdot}3277\,(27) - 5{\cdot}8126\,(28) - 21{\cdot}6613\,(29)$

XIX. $S' : OTD'$ $\quad 0 = -45{\cdot}178 - 15{\cdot}1957\,(4) + 45{\cdot}8695\,(5) - 30{\cdot}6738\,(7) - 43{\cdot}7005\,(17) + 32{\cdot}8938\,(18) + 10{\cdot}8067\,(20) - 6{\cdot}8515\,(31) + 10{\cdot}0904\,(32)$

XX. $O : LD'T$ $\quad 0 = +99{\cdot}330 - 15{\cdot}2531\,(4) + 37{\cdot}1764\,(5) - 21{\cdot}9233\,(6) - 24{\cdot}0359\,(9) + 13{\cdot}0255\,(11) + 11{\cdot}0104\,(12) + 158{\cdot}2223\,(31) - 76{\cdot}4505\,(32) - 81{\cdot}7718\,(34)$

XXI. $O : TLB$ $\quad 0 = -418{\cdot}781 - 0{\cdot}5633\,(3) + 22{\cdot}4866\,(4) - 21{\cdot}9233\,(6) - 24{\cdot}0359\,(9) + 88{\cdot}2288\,(10) - 64{\cdot}1929\,(11) - 152{\cdot}2404\,(25) + 195{\cdot}3204\,(26) - 43{\cdot}0800\,(27)$

XXII. $R' : D'BL$ $\quad 0 = +133{\cdot}349 - 98{\cdot}7183\,(8) + 35{\cdot}7819\,(10) + 62{\cdot}9364\,(12) + 11{\cdot}7652\,(24) - 1{\cdot}9683\,(26) - 9{\cdot}7969\,(28) - 80{\cdot}3070\,(33) - 114{\cdot}4305\,(34)$

The logarithmic values of the multipliers are as follow:

Log I = 9·9921867 +
II = 8·8504011 +
III = 0·7343735 +
IV = 9·2550312 −
V = 0·0724357 −
VI = 9·4474726 +
VII = 9·9774388 +

Log VIII = 0·2037114 −
IX = 8·1880672 +
X = 8·5209934 −
XI = 7·6353852 +
XII = 8·9828544 −
XIII = 9·0822298 −
XIV = 8·2626347 +
XV = 8·4699649 −

Log XVI = 8·1952064 −
XVII = 8·5173020 +
XVIII = 7·6938474 −
XIX = 7·9115076 +
XX = 8·1528886 +
XXI = 7·9674738 +
XXII = 7·6327530 −

Resulting Corrections.

From	To	Symbol.	Calculated Correction.	From	To	Symbol.	Calculated Correction.
TARBATHY . . .	Blue Hill . .	(1)	+0″·6260	*OVER HILL—cont.* .	Layton . . .	(19)	−0″·0931
	Mount Battock .	(2)	+2·3953		Tarbathy . .	(20)	+0·7000
	Brimmond . .	(3)	−0·3156				
	Over Hill . .	(4)	+0·2145	*BRIMMOND* . . .	Mount Battock .	(21)	−1·3803
	Dudwick . .	(5)	+0·4518		Knock . . .	(22)	−1·7604
	Layton . . .	(6)	−0·3048		Mormonth . .	(23)	+1·5140
	Stirling . . .	(7)	−6·9001		Dudwick . .	(24)	−0·2031
					Over Hill . .	(25)	+0·1195
LAYTON . . .	Blue Hill . .	(8)	+0·1563		Layton . . .	(26)	+1·8779
	Tarbathy . .	(9)	−0·3382		Tarbathy . .	(27)	−0·0231
	Brimmond . .	(10)	+0·4094		Blue Hill . .	(28)	+0·1572
	Over Hill . .	(11)	−0·2064				
	Dudwick . .	(12)	−1·5416	*MOUNT BATTOCK* .	Brimmond . .	(29)	−0·2293
OVER HILL . .	Blue Hill . .	(13)	−1·6679	*MORMONTH* . . .	Brimmond . .	(30)	+0·3296
	Brimmond . .	(14)	−0·4978				
	Mount Battock .	(15)	−2·2019	*DUDWICK* . . .	Tarbathy . .	(31)	−0·6098
	Mormonth . .	(16)	+3·1098		Over Hill . .	(32)	−0·9153
	Dudwick . .	(17)	+0·9289		Brimmond . .	(33)	−1·0724
	Stirling . . .	(18)	−4·4340		Layton . . .	(34)	+0·9966

SALISBURY PLAIN BASE.

The triangulation for connecting this base-line with the work will be found in Figure 26. The points that have to be fixed are the south end of the base, Old Sarum Castle, Four-mile Stone, and Old Lodge: these are connected with eight points already fixed.

Equations of Condition.

I. *GFN* $0 = -4{\cdot}5659 - (1) + (3) - (6) + (7) - (33) + (35)$

II. *GLF* $0 = -4{\cdot}5034 + (1) - (4) - (7) + (8) - (19) + (23)$

III. *LFN* $0 = -2{\cdot}2453 + (3) - (4) - (20) + (23) - (33) + (34)$

IV. *MNF* $0 = -6{\cdot}0132 - (2) + (3) - (9) + (12) - (33) + (37)$

V. *MNL* $0 = -7{\cdot}0312 - (9) + (14) - (18) + (20) - (34) + (37)$

VI. *MNG* $0 = -2{\cdot}0960 - (5) + (6) - (9) + (13) - (35) + (37)$

VII. *HLN* $0 = -1{\cdot}3763 - (16) + (20) - (27) + (28) - (34) + (36)$

VIII. *HLM* $0 = -0{\cdot}9189 - (14) + (15) - (16) + (18) - (26) + (28)$

IX. *HFN* $0 = -0{\cdot}0590 + (3) - (27) - (33) + (36)$

X. *ENF* $0 = +1{\cdot}7207 - (3) - (31) + (33) + (45)$

XI. *EMF* $0 = -1{\cdot}5514 - (2) - (10) + (12) + (44)$

XII. *EHL* $0 = -1{\cdot}1934 - (16) + (22) + (28) - (43)$

XIII. *DNF* $0 = -0{\cdot}8053 - (3) - (30) + (33) + (42)$

XIV. *DNL* $0 = -0{\cdot}5029 - (20) + (21) - (30) + (34) - (41) + (42)$

XV. *CDL* $0 = +0{\cdot}7411 - (21) + (24) - (39) + (41)$

XVI. *CDN* $0 = -1{\cdot}1233 - (30) + (32) - (38) + (42)$

XVII. *CEM* $0 = +1{\cdot}5208 - (10) + (11) - (40) + (44)$

XVIII. *F : GLN* $0 = +19{\cdot}438 - 1{\cdot}4826\,(6) + 27{\cdot}3128\,(7) - 25{\cdot}8302\,(8) - 8{\cdot}2595\,(19) + 15{\cdot}8421\,(20)$
$- 7{\cdot}5826\,(23) - 27{\cdot}7686\,(33) + 35{\cdot}4188\,(34) - 7{\cdot}6502\,(35)$

XIX. *N : GMF* $0 = -11{\cdot}046 + 47{\cdot}3029\,(1) - 50{\cdot}8765\,(2) + 3{\cdot}5736\,(3) + 11{\cdot}9574\,(5) - 13{\cdot}4400\,(6)$
$+ 1{\cdot}4826\,(7) + 12{\cdot}3456\,(9) - 3{\cdot}1670\,(12) - 9{\cdot}1786\,(13)$

xx. *F* : *MNL* $0 = +27{\cdot}487 + 3{\cdot}1670\,(9) - 30{\cdot}6904\,(12) + 27{\cdot}5234\,(14) + 8{\cdot}1507\,(18) - 15{\cdot}8421\,(20) + 7{\cdot}6914\,(23) + 30{\cdot}1965\,(33) - 35{\cdot}4188\,(34) + 5{\cdot}2223\,(37)$

xxi. *N* : *LFH* $0 = +6{\cdot}047 - 10{\cdot}6200\,(3) + 2{\cdot}2976\,(4) - 3{\cdot}1852\,(16) + 19{\cdot}0273\,(20) - 15{\cdot}8421\,(23) - 12{\cdot}0130\,(27) - 13{\cdot}5438\,(28)$

xxii. *M* : *LFH* $0 = -1{\cdot}898 - 14{\cdot}2289\,(2) - 6{\cdot}1389\,(4) - 9{\cdot}5505\,(16) + 17{\cdot}7012\,(18) - 8{\cdot}1507\,(23) - 11{\cdot}4825\,(26) - 12{\cdot}8123\,(28)$

xxiii. *K* : *LFH* $0 = +45{\cdot}551 + 1{\cdot}7052\,(4) - 11{\cdot}6424\,(16) + 17{\cdot}9274\,(17) - 6{\cdot}2850\,(23) + 3{\cdot}9796\,(28)$

xxiv. *K* : *LNH* $0 = +162{\cdot}935 - 11{\cdot}6424\,(16) + 68{\cdot}4474\,(17) - 56{\cdot}8050\,(20) + 22{\cdot}2231\,(27) + 3{\cdot}9796\,(28) - 26{\cdot}4032\,(29) + 32{\cdot}2428\,(34) - 5{\cdot}8396\,(36)$

xxv. *F* : *NEM* $0 = +39{\cdot}392 + 3{\cdot}1670\,(9) - 6{\cdot}8276\,(10) + 3{\cdot}6606\,(12) - 0{\cdot}9544\,(31) - 4{\cdot}2679\,(33) + 5{\cdot}2223\,(37) - 44{\cdot}3032\,(44) + 40{\cdot}7227\,(45)$

xxvi. *F* : *MEH* $0 = +32{\cdot}349 + 6{\cdot}8276\,(10) - 5{\cdot}6744\,(12) - 1{\cdot}1532\,(15) + 24{\cdot}2948\,(26) + 44{\cdot}3032\,(44)$

xxvii. *L* : *NEH* $0 = +79{\cdot}797 + 13{\cdot}5438\,(27) + 5{\cdot}0112\,(28) - 13{\cdot}8440\,(31) - 10{\cdot}0777\,(34) + 23{\cdot}9217\,(36) + 24{\cdot}7208\,(43) + 52{\cdot}0624\,(45)$

xxviii. *K* : *END* $0 = +26{\cdot}492 + 4{\cdot}3268\,(29) - 4{\cdot}3931\,(30) + 0{\cdot}0663\,(31) - 19{\cdot}3664\,(42) + 20{\cdot}3015\,(45)$

xxix. *K* : *LDH* $0 = +65{\cdot}330 - 11{\cdot}6424\,(16) + 29{\cdot}8065\,(17) - 18{\cdot}1641\,(21) + 3{\cdot}9796\,(28) - 10{\cdot}4597\,(41)$

xxx. *E* : *FCN* $0 = +1{\cdot}922 - 12{\cdot}1246\,(3) - 25{\cdot}2167\,(31) + 24{\cdot}2623\,(32) + 0{\cdot}9544\,(33) + 16{\cdot}9555\,(38)$

xxxi. *E* : *FCM* $0 = +30{\cdot}170 - 2{\cdot}7545\,(2) - 21{\cdot}8287\,(10) + 28{\cdot}6563\,(11) - 6{\cdot}8276\,(12) + 13{\cdot}2990\,(40)$

xxxii. *D* : *LCH* $0 = +5{\cdot}205 + 7{\cdot}7785\,(16) + 17{\cdot}2367\,(21) - 25{\cdot}0152\,(24) - 14{\cdot}7235\,(28) - 1{\cdot}5972\,(39)$

xxxiii. *B* : *DLC* $0 = -13{\cdot}834 + 2{\cdot}0194\,(21) + 14{\cdot}5298\,(24) - 16{\cdot}5492\,(25) + 13{\cdot}8698\,(39) - 30{\cdot}8693\,(41)$

The logarithmic values of the multipliers are as follow :

Log I = 1·1487492 +
II = 0·7389172 +
III = 0·9214591 +
IV = 0·7388908 +
V = 0·3201681 −
VI = 0·4899136 +
VII = 0·7914212 +
VIII = 0·2554530 +
IX = 1·1197214 −
X = 0·1020325 −
XI = 0·8295361 +

Log XII = 0·5700743 +
XIII = 0·5811130 +
XIV = 9·7969399 +
XV = 9·4791740 −
XVI = 9·9653536 −
XVII = 0·7567621 −
XVIII = 8·5855937 −
XIX = 9·3038867 +
XX = 9·4042583 +
XXI = 0·0337402 +
XXII = 9·7912589 −

Log XXIII = 0·0573730 −
XXIV = 9·2401011 +
XXV = 9·4525308 −
XXVI = 9·4891452 −
XXVII = 9·0621382 +
XXVIII = 9·2432695 +
XXIX = 8·8926510 −
XXX = 8·0836523 +
XXXI = 8·9263399 −
XXXII = 8·3833611 −
XXXIII = 8·3293730 +

Resulting Corrections.

From	To	Symbol.	Calculated Correction.	From	To	Symbol.	Calculated Correction.
BEACON HILL . .	Old Sarum Gun	(1)	″ +0·0517	OLD SARUM CASTLE (*continued.*)	Beacon Hill .	(12)	″ +0·8947
	Old Sarum Castle	(2)	−0·8256		Sarum Gun .	(13)	+2·0173
	Four-mile Stone	(3)	+0·1558		Old Lodge . .	(14)	+0·8800
	Old Lodge . .	(4)	−2·4124		Dean Hill . .	(15)	+1·2727
OLD SARUM GUN .	Sarum Castle .	(5)	−0·0816	OLD LODGE . .	Dean Hill . .	(16)	+0·3796
	Four-mile Stone	(6)	−1·2689		Wingreen . .	(17)	−1·9884
	Beacon Hill .	(7)	+0·3218		Sarum Castle .	(18)	−0·9646
	Old Lodge . .	(8)	+1·1703		Sarum Gun .	(19)	−0·4642
OLD SARUM CASTLE	Four-mile Stone	(9)	−2·3624		Four-mile Stone	(20)	+1·1887
	Stoke Hill . .	(10)	+0·1487		Westbury Down	(21)	+0·3851
	Milk Hill . .	(11)	−0·9789		Stoke Hill . .	(22)	+0·3400
					Beacon Hill .	(23)	+0·7266

Resulting Corrections—continued.

From	To	Symbol.	Calculated Correction.	From	To	Symbol.	Calculated Correction.
OLD LODGE—cont.	Milk Hill . .	(24)	$+0''{\cdot}1026$	*FOUR-MILE STONE*. (*continued.*)	Dean Hill . .	(36)	$-1''{\cdot}0533$
	Inkpen . . .	(25)	$-0{\cdot}1026$		Sarum Castle .	(37)	$+0{\cdot}9956$
DEAN HILL . .	Sarum Castle .	(26)	$-1{\cdot}0672$	*MILK HILL* . .	Four-mile Stone	(38)	$+0{\cdot}2021$
	Four-mile Stone	(27)	$-0{\cdot}1774$		Old Lodge . .	(39)	$+0{\cdot}2524$
	Old Lodge . .	(28)	$+0{\cdot}8032$		Sarum Castle .	(40)	$+0{\cdot}3731$
FOUR-MILE STONE .	Wingreen . .	(29)	$-2{\cdot}2853$	*WESTBURY DOWN* .	Old Lodge . .	(41)	$-0{\cdot}2062$
	Westbury Down	(30)	$-1{\cdot}6408$		Four-mile Stone	(42)	$+0{\cdot}0994$
	Stoke Hill . .	(31)	$-0{\cdot}2913$				
	Milk Hill . .	(32)	$-0{\cdot}4147$	*STOKE HILL* . .	Old Lodge . .	(43)	$-0{\cdot}4298$
	Beacon Hill .	(33)	$-0{\cdot}7791$		Sarum Castle .	(44)	$-0{\cdot}0201$
	Old Lodge . .	(34)	$-0{\cdot}6400$		Four-mile Stone	(45)	$-1{\cdot}0772$
	Sarum Gun .	(35)	$+2{\cdot}0920$				

SOUTHAMPTON SECTOR STATION.

In Figure 27 will be found the triangulation for connecting the Station at Southampton and Nodes Beacon in the Isle of Wight with Dunnose, Blackdown, Coringdon, Swyre Barrow, and other points to the north already fixed. The trigonometrical station at Southampton is 108 feet distant from the point over which Airy's Zenith-Sector was placed, and bearing 105° 53′ from the south.

Equations of Condition.

I. $H'ND'$ $\quad 0 = -2{\cdot}8745 - (1) - (13) + (14) + (22)$

II. $H'NA'$ $\quad 0 = -1{\cdot}9338 - (1) - (11) + (14) + (21)$

III. $H'NB$ $\quad 0 = +1{\cdot}1817 - (1) - (10) + (14) + (20)$

IV. $H'NC'$ $\quad 0 = -4{\cdot}1867 - (1) - (9) + (14) + (18)$

V. $H'NE'$ $\quad 0 = +3{\cdot}5068 + (1) - (5) + (8) - (14)$

VI. $H'NG'$ $\quad 0 = +0{\cdot}3907 + (1) - (3) + (7) - (14)$

VII. $H'NF'$ $\quad 0 = +0{\cdot}2591 + (1) - (4) + (6) - (14)$

VIII. NOB' $\quad 0 = +5{\cdot}6692 - (10) + (12) - (15) + (16) - (19) + (20)$

IX. NOH' $\quad 0 = -4{\cdot}5398 - (1) + (2) - (12) + (14) + (15) - (17)$

X. $H' : B'ND'$ $\quad 0 = -95{\cdot}915 + 9{\cdot}0871\,(10) + 15{\cdot}4088\,(13) - 24{\cdot}4959\,(14) - 47{\cdot}1343\,(20) + 39{\cdot}7747\,(22)$

XI. $B' : H'NA'$ $\quad 0 = +142{\cdot}910 + 22{\cdot}9091\,(1) + 88{\cdot}0070\,(10) - 78{\cdot}9199\,(11) - 9{\cdot}0871\,(14) - 72{\cdot}5905\,(21)$

XII. $N : B'C'H'$ $\quad 0 = +170{\cdot}221 - 34{\cdot}4336\,(1) - 112{\cdot}1703\,(18) + 46{\cdot}3174\,(20)$

XIII. $N : H'G'C'$ $\quad 0 = +233{\cdot}628 - 165{\cdot}7842\,(1) - 138{\cdot}7887\,(3) - 157{\cdot}8349\,(18)$

XIV. $N : H'F'C'$ $\quad 0 = -111{\cdot}555 - 144{\cdot}5296\,(1) - 144{\cdot}9775\,(4) - 148{\cdot}9335\,(18)$

XV. $N : F'H'E'$ $\quad 0 = +153{\cdot}798 - 70{\cdot}8826\,(1) + 98{\cdot}3132\,(4) - 372{\cdot}7796\,(5)$

XVI. $O : NB'H'$ $\quad 0 = -48{\cdot}575 + 15{\cdot}4743\,(1) + 91{\cdot}8374\,(2) + 35{\cdot}8519\,(10) - 38{\cdot}4673\,(12) + 2{\cdot}6154\,(14) + 40{\cdot}1532\,(19) + 22{\cdot}9621\,(20)$

The logarithmic values of the multipliers are as follow:

Log I = 9·6655159 +
II = 9·0729042 −
III = 0·1871984 −
IV = 0·3119478 +
V = 0·1372274 −

Log VI = 0·3642156 +
VII = 9·7705224 −
VIII = 8·1755156 −
IX = 8·7059959 +
X = 7·8812705 +
XI = 7·8064977 −

Log XII = 8·4188703 +
XIII = 8·2726883 −
XIV = 7·9716727 +
XV = 7·6301458 +
XVI = 7·7409708 +

Resulting Corrections.

From	To	Symbol.	Calculated Correction.	From	To	Symbol.	Calculated Correction.
DUNNOSE	Nodes Beacon	(1)	−0″·1889	*NODES BEACON—cont.*	Butser Hill.	(13)	−0″·3129
	Southampton	(2)	+0·1755		Dunnose	(14)	+0·8155
CORINGDON	Nodes Beacon	(3)	+0·5975	*SOUTHAMPTON*	Nodes Beacon	(15)	+2·4890
					Dean Hill	(16)	−0·6091
SWYRE BARROW	Nodes Beacon	(4)	−1·7658		Dunnose	(17)	−0·6132
BLACK DOWN	Nodes Beacon	(5)	−0·0172	*WINGREEN*	Nodes Beacon	(18)	+1·1532
NODES BEACON	Swyre Barrow	(6)	−1·0206	*DEAN HILL*	Southampton	(19)	+0·1274
	Coringdon	(7)	+1·2112		Nodes Beacon	(20)	−1·0228
	Black Down	(8)	−2·5196				
	Wingreen	(9)	−2·0292	*INKPEN*	Nodes Beacon	(21)	+2·0256
	Dean Hill	(10)	+1·1633				
	Inkpen	(11)	+1·0962	*BUTSER HILL*	Nodes Beacon	(22)	+1·5572
	Southampton	(12)	−0·2577				

ISLE OF WIGHT SECTOR STATIONS.

The four Zenith-Sector Stations in the Isle of Wight are Dunnose, Week Down, Boniface Down, and Port Valley, which latter is a point 263 feet distant from High Port Cliff, in a line between it and Boniface Down South-east Station.

The points Shanklin Down, Wroxall, Littletown Down, and Boniface Down South-east Station, serve to establish the connection, which is shown in Figure 28.

Equations of Condition.

I. *DA′B′* $0 = +\ 0{\cdot}1107 + (1) - (4) - (9) + (11)$

II. *DFB′* $0 = -\ 0{\cdot}3205 - (2) + (4) - (11) + (12) - (21) + (25)$

III. *CB′F* $0 = +\ 6{\cdot}1973 - (2) + (5) - (7) + (8) - (24) + (25)$

IV. *CB′D* $0 = -\ 0{\cdot}3236 - (4) + (5) + (6) - (7) - (10) + (11)$

V. *EB′F* $0 = -\ 2{\cdot}3424 - (2) + (3) - (16) + (17) - (22) + (25)$

VI. *DGF* $0 = -\ 48{\cdot}3409 - (12) + (13) + (21) - (26) - (28) + (31)$

VII. *EGF* $0 = -\ 24{\cdot}6503 - (17) + (18) + (22) - (26) - (29) + (31)$

VIII. *HGD* $0 = +\ 0{\cdot}8488 - (13) + (14) - (27) + (28) - (33) + (35)$

IX. *HGF* $0 = -\ 42{\cdot}9800 + (20) - (26) - (27) + (31) - (34) + (35)$

X. *IHG* $0 = -\ 35{\cdot}6206 + (27) - (32) - (35) + (36) - (37) + (38)$

XI. *B′ : DFA′* $0 = -\ 1340{\cdot}567 - 172{\cdot}5700\,(1) + 319{\cdot}5749\,(2) + 11{\cdot}7741\,(9) + 28{\cdot}3884\,(11) - 40{\cdot}1625\,(12)$
$+\ 6{\cdot}5023\,(21) + 324{\cdot}3476\,(23) - 330{\cdot}8499\,(25)$

XII. *B′ : DFC* $0 = -\ 694{\cdot}875 + 18{\cdot}1673\,(6) - 24{\cdot}7280\,(7) + 6{\cdot}5607\,(8) - 152{\cdot}5544\,(10) + 192{\cdot}7169\,(11)$
$-\ 40{\cdot}1625\,(12) + 6{\cdot}5023\,(21) + 55{\cdot}4766\,(24) - 61{\cdot}9789\,(25)$

XIII. *C : FB′E* $0 = +\ 282{\cdot}900 - 1{\cdot}2785\,(2) + 7{\cdot}8032\,(3) - 6{\cdot}5247\,(5) - 56{\cdot}1162\,(15) + 61{\cdot}1924\,(16)$
$-\ 5{\cdot}0762\,(17) - 1{\cdot}8799\,(22) + 57{\cdot}3565\,(24) - 55{\cdot}4766\,(25)$

XIV. *D : FA′G* $0 = +\ 377{\cdot}806 - 28{\cdot}5552\,(9) - 378{\cdot}1601\,(12) + 406{\cdot}7153\,(13) + 406{\cdot}3232\,(21)$
$-\ 414{\cdot}7268\,(23) + 8{\cdot}4036\,(26) - 438{\cdot}3480\,(28) + 448{\cdot}1660\,(30) - 9{\cdot}8180\,(31)$

XV. *F : GECD* $0 = +\ 2151{\cdot}766 - 13{\cdot}1078\,(6) + 13{\cdot}1078\,(8) - 29{\cdot}4922\,(10) + 402{\cdot}8037\,(12) - 373{\cdot}3115\,(13)$
$+\ 67{\cdot}4359\,(15) - 159{\cdot}5044\,(17) + 92{\cdot}0685\,(18) + 62{\cdot}3597\,(22) - 62{\cdot}3597\,(24)$
$-\ 9{\cdot}8180\,(28) + 15{\cdot}3688\,(29) - 5{\cdot}5508\,(31)$

XVI. *F* : *HGD* $$0 = -1470{\cdot}573 - 224{\cdot}1672\,(12) + 373{\cdot}3115\,(13) - 149{\cdot}1443\,(14) + 2{\cdot}4680\,(27) + 9{\cdot}8180\,(28) - 12{\cdot}2860\,(31) + 12{\cdot}8278\,(33) + 38{\cdot}8799\,(34) - 51{\cdot}7077\,(35)$$

XVII. *F* : *HGE* $$0 = -250{\cdot}397 - 68{\cdot}7506\,(17) + 92{\cdot}0685\,(18) - 23{\cdot}3179\,(19) + 3{\cdot}0031\,(20) - 3{\cdot}0031\,(22) + 2{\cdot}4680\,(27) + 15{\cdot}3688\,(29) - 17{\cdot}8368\,(31) + 51{\cdot}7077\,(34) - 51{\cdot}7077\,(35)$$

The logarithmic values of the multipliers are as follow:

Log I = 9·7478919 +
II = 0·2611427 +
III = 0·0800907 −
IV = 9·8140012 +
V = 9·4175014 −
VI = 0·3644958 +

Log VII = 9·6302700 −
VIII = 0·0189828 +
IX = 9·6601518 +
X = 0·1142154 +
XI = 7·2841168 +
XII = 7·1206394 +

Log XIII = 8·1545226 −
XIV = 6·9855190 −
XV = 5·6010164 +
XVI = 7·5705988 +
XVII = 7·8327489 +

Resulting Corrections.

From	To	Symbol.	Calculated Correction.
NODES BEACON .	Week Down .	(1)	+1″·2033
DUNNOSE . . .	Boniface Down .	(2)	+0·9212
	Wroxall Down	(3)	−2·6523
	Week Down .	(4)	+1·6406
	Shanklin Down	(5)	−2·7956
SHANKLIN DOWN .	Week Down .	(6)	+1·2636
	Dunnose . .	(7)	+0·1612
	Boniface Down	(8)	−1·3242
WEEK DOWN . .	Nodes Beacon .	(9)	−1·4285
	Shanklin Down	(10)	−4·7593
	Dunnose . .	(11)	−1·1019
	Boniface Down	(12)	+3·1472
	Boniface S.E. .	(13)	+3·8295
	Littletown Down	(14)	+2·8044
WROXALL DOWN .	Shanklin Down	(15)	+3·4034
	Dunnose . .	(16)	−2·5913
	Boniface Down	(17)	−1·0009
	Boniface S.E. .	(18)	+2·6928
	Littletown Down	(19)	−0·6719
BONIFACE DOWN .	Littletown Down	(20)	+4″·6860
	Week Down .	(21)	+2·1265
	Wroxall Down	(22)	−0·5526
	Nodes Beacon .	(23)	+7·4302
	Shanklin Down	(24)	+4·7680
	Dunnose . .	(25)	+3·7729
	Boniface S.E. .	(26)	−14·3180
BONIFACE SOUTH-EAST STATION. .	Littletown Down	(27)	−1·0536
	Week Down .	(28)	−0·5361
	Wroxall Down	(29)	+17·1924
	Nodes Beacon .	(30)	−1·8388
	Boniface Down .	(31)	+24·3836
	High Port Cliff .	(32)	−12·7821
LITTLETOWN DOWN	Week Down .	(33)	−0·9876
	Boniface Down	(34)	+0·1324
	Boniface S.E. .	(35)	−1·3288
	High Port Cliff.	(36)	+13·9381
HIGH PORT CLIFF .	Littletown Down	(37)	−6·1837
	Boniface S.E. .	(38)	+2·4415

FEAGHMAAN SECTOR STATION.

The triangulation for the connection of this point with the general triangulation will be found in Figure 29. Besides the sector station, the points Brandon, Knocknadober, and Knocknagante, which establish the connection, are to be fixed with reference to Bencorr, Keeper, Taur, Caherbarnagh, and Hungry Hill.

Equations of Condition.

I. *CFK* $0 = -\ 1{\cdot}4391 - (4) + (8) - (17) + (19)$

II. *CHK* $0 = +\ 8{\cdot}1817 - (8) + (10) - (16) + (17) - (24) + (25)$

III. *LCF* $0 = -\ 7{\cdot}1742 - (6) + (7) - (13) + (14)$

IV. *LCK* $0 = -\ 4{\cdot}0860 - (7) + (8) - (12) + (13) - (17) + (18)$

V. *GCL* $0 = -\ 3{\cdot}8048 - (7) + (9) - (11) + (13) - (21) + (23)$

VI. *GKL* $0 = -\ 18{\cdot}4752 - (11) + (12) + (15) - (18) - (22) + (23)$

VII. *C : A′B′D′FH* $0 = +\ 96{\cdot}605 + 105{\cdot}0802\,(1) + 59{\cdot}5653\,(5) - 16{\cdot}3272\,(10)$

VIII. *C : FHD′* $0 = -\ 796{\cdot}762 - 298{\cdot}5307\,(2) + 59{\cdot}5653\,(5) + 97{\cdot}0640\,(10)$

IX. *C : HE′D′* $0 = +\ 736{\cdot}846 + 298{\cdot}5307\,(2) + 477{\cdot}9250\,(3) - 290{\cdot}8889\,(10)$

X. *H : F′KC* $0 = -\ 122{\cdot}857 + 116{\cdot}0633\,(4) - 161{\cdot}6100\,(5) - 16{\cdot}8556\,(8) + 20{\cdot}1323\,(10) + 56{\cdot}3207\,(16) - 14{\cdot}1594\,(17) - 42{\cdot}1613\,(19)$

XI. *L : CKF′* $0 = +\ 293{\cdot}867 - 82{\cdot}0215\,(4) + 137{\cdot}7556\,(6) + 114{\cdot}3530\,(7) - 32{\cdot}2475\,(8) - 4{\cdot}0585\,(17) + 67{\cdot}7502\,(18) - 63{\cdot}6917\,(19)$

XII. *H : KCG* $0 = -\ 386{\cdot}063 - 16{\cdot}8556\,(8) + 19{\cdot}0509\,(9) - 2{\cdot}1953\,(10) + 8{\cdot}6014\,(15) + 5{\cdot}5580\,(16) - 14{\cdot}1594\,(17) - 6{\cdot}8451\,(20) + 27{\cdot}1517\,(21) - 20{\cdot}3066\,(22)$

XIII. *K : LGHC′* $0 = +\ 1196{\cdot}363 + 32{\cdot}2475\,(7) - 49{\cdot}1031\,(8) + 16{\cdot}8556\,(10) + 40{\cdot}7761\,(11) - 49{\cdot}3825\,(12) + 8{\cdot}6064\,(13) + 52{\cdot}7960\,(15) - 52{\cdot}7960\,(16) + 73{\cdot}1026\,(20) - 98{\cdot}1280\,(22) + 25{\cdot}0254\,(23) + 6{\cdot}5977\,(24) - 6{\cdot}5977\,(25)$

The logarithmic values of the multipliers are as follow:

Log I = 9·6812744 −
II = 9·2907549 −
III = 9·4080146 +
IV = 0·1332116 +

Log V = 0·0078131 −
VI = 0·0792531 +
VII = 7·7063062 −
VIII = 7·9417825 +
IX = 5·8854839 +

Log X = 6·6449229 −
XI = 7·7249087 −
XII = 8·5130445 −
XIII = 7·7374582 −

Resulting Corrections.

From	To	Symbol.	Calculated Correction.
Bencorr . . .	Brandon . .	(1)	−1″·3125
Caherbarnagh .	Brandon . .	(2)	−2·4204
Taur	Brandon . .	(3)	+0·1466
Hungry Hill . .	Knocknadober .	(4)	+0·4769
	Brandon . .	(5)	+0·7731
	Knocknagante .	(6)	−0·2573
Baurtregaum . .	Knocknagante .	(7)	−1·5931
	Knocknadober .	(8)	+0·5360
	Feaghmaan . .	(9)	−1·7127
	Brandon . .	(10)	+0·2899
Knocknagante .	Feaghmaan . .	(11)	−11·8933
	Knocknadober .	(12)	+0·7434
Knocknagante, cont.	Baurtregaum .	(13)	+1″·6958
	Hungry Hill .	(14)	+10·2058
Knocknadober .	Feaghmaan . .	(15)	+2·3747
	Brandon . .	(16)	+0·7062
	Baurtregaum .	(17)	−3·8005
	Knocknagante .	(18)	−2·7960
	Hungry Hill .	(19)	−2·4205
Feaghmaan . .	Brandon . .	(20)	−10·7509
	Baurtregaum .	(21)	+9·7576
	Knocknadober .	(22)	−0·5748
	Knocknagante .	(23)	+0·0930
Brandon . . .	Baurtregaum .	(24)	+1·3482
	Knocknadober .	(25)	−2·0807

DELAMERE SECTOR STATION.

The point Delamere is fixed by its relations with Whittle Hill, Holme Moss, Mowcopt, and Cyrn-y-Brain, shown in Figure 30. The old station of Delamere, over which Ramsden's Zenith-Sector was placed, is calculated by the method given at page 69 to have been 3·7 feet distant, and bearing 201° from the present station.

Equations of Condition.

I. *C′DM′* 0 = + 0·9460 + (1) − (4) + (5) − (7)
II. *W′DM′* 0 = − 0·5723 − (2) + (4) − (5) + (6)
III. *C′ : DW′M′* 0 = + 26·140 − 54·2793 (1) + 37·9550 (2) + 16·3243 (4) − 39·6697 (5) − 101·9365 (6)
IV. *H′ : DW′M′* 0 = − 214·648 + 41·2902 (2) − 56·4688 (3) + 15·1786 (4) − 1·4771 (5) − 0·1330 (6)

The logarithmic values of the multipliers are as follow:

Log I = 9·0305823 − Log II = 9·4627176 − Log IV = 8·4532300 +
III = 7·5892119 −

Resulting Corrections.

From	To	Symbol.	Calculated Correction.	From	To	Symbol.	Calculated Correction.
DELAMERE . .	Cyrn-y-Brain .	(1)	″ +0·0334	*MOWCOPT* . . .	Delamere . .	(5)	″ +0·0623
	Whittle Hill .	(2)	+0·0936	*WHITTLE HILL* .	Delamere . .	(6)	+0·3154
	Holme Moss .	(3)	−3·6241	*CYRN-Y-BRAIN* .	Delamere . .	(7)	+0·6289
	Mowcopt . .	(4)	+0·4127				

TAWNAGHMORE SECTOR STATION.

The connection of this station, together with Slieve More in Achill and Knockalongy, is shown in Figure 31. The points already fixed are Slieve League, Cuilcagh, Nephin, and Bencorr.

Equations of Condition.

I. *LKI* 0 = + 7·2449 + (5) − (7) − (12) + (14)
II. *MLI* 0 = − 3·8552 − (6) + (7) − (14) − (15) + (16)
III. *MHI* 0 = − 8·4733 + (6) − (9) − (16) + (17)
IV. *ILP* 0 = + 0·3053 − (7) + (8) − (13) + (14) − (18) + (19)
V. *PLK* 0 = + 10·5268 − (3) + (5) − (12) + (13) − (19) + (20)
VI. *PKO′* 0 = − 6·1889 − (1) + (3) − (20) + (21)
VII. *L : O′KI* 0 = − 117·52 + 99·5073 (2) − 52·2846 (5) − 18·9328 (7)
VIII. *I : HMLKO′C* 0 = + 296·742 + 46·4689 (5) + 22·1679 (9) + 7·8985 (11) − 3·3945 (12) − 4·5040 (14) + 20·4250 (15) − 26·7772 (16) + 6·3522 (17)
IX. *M : KIL* 0 = − 457·424 − 333·0297 (4) + 275·1799 (5) + 22·6879 (6) − 9·9567 (7) + 112·2873 (11) − 104·3888 (12) − 7·8985 (14)
X. *I : PKO′* 0 = − 87·910 − 111·9432 (1) − 40·3273 (3) − 56·4753 (18) + 12·3876 (20) + 44·0877 (21)
XI. *P : LIK* 0 = + 189·480 − 23·8447 (3) − 16·4826 (5) − 0·5770 (7) − 33·1687 (8) − 20·2765 (12) + 36·0975 (13) − 15·8210 (14)
XII. *I : HMLP* 0 = + 390·145 − 53·7198 (8) + 22·1679 (9) + 132·4011 (10) + 7·8985 (11) + 15·8210 (13) − 23·7195 (14) + 20·4250 (15) − 26·7772 (16) + 6·3522 (17) − 26·4784 (18) + 26·4784 (19)

The logarithmic values of the multipliers are as follow:

Log I = 0·8166955 + Log V = 0·6667926 − Log IX = 7·2996482 +
II = 0·3720919 + VI = 0·6126070 − X = 8·9464007 +
III = 0·3425474 + VII = 7·2344059 − XI = 9·2677854 −
IV = 0·7001880 − VIII = 9·1318820 − XII = 7·3198985 −

Resulting Corrections.

From	To	Symbol.	Calculated Correction.	From	To	Symbol.	Calculated Correction.
CUILCAGH . . .	Knockalongy .	(1)	−3″·5282	*BENCORR—cont.* .	Knockalongy .	(10)	−1″·3630
	Tawnaghmore .	(2)	−1·8539	*TAWNAGHMORE* .	Slieve More in Achill . .	(11)	+3·0885
SLIEVE LEAGUE .	Knockalongy .	(3)	+5·8432		Slieve League .	(12)	+1·4294
	Slieve More in Achill . .	(4)	−5·8126		Knockalongy .	(13)	−1·9931
	Tawnaghmore .	(5)	−6·0155		Nephin . . .	(14)	+0·8613
NEPHIN . . .	Slieve More in Achill . .	(6)	−0·3175	*SLIEVE MORE IN ACHILL.*	Tawnaghmore .	(15)	−0·3724
	Tawnaghmore .	(7)	+0·6614		Nephin . . .	(16)	+0·2768
	Knockalongy .	(8)	+1·0361		Bencorr . .	(17)	+6·4220
BENCORR . . .	Slieve More in Achill . .	(9)	−2·6456	*KNOCKALONGY* . .	Nephin . . .	(18)	+0·4554
					Tawnaghmore .	(19)	−3·0790
					Slieve League .	(20)	+1·6754
					Cuilcagh . .	(21)	−1·5070

GREAT STIRLING SECTOR STATION.

The fixation of this point is shown in Figure 32. The Reform Monument, though not observed from, affords two Equations of Condition.

Equations of Condition.

I. *M′L′P* $0 = -14{\cdot}2612 - (1) + (3) - (9) + (10)$

II. *L′PS* $0 = -1{\cdot}6486 - (3) + (5) - (6) + (8) + (9) - (11)$

III. *L′ : PSRM′* $0 = +252{\cdot}311 + 77{\cdot}1848\,(1) - 119{\cdot}8638\,(2) + 39{\cdot}9464\,(4) - 51{\cdot}7746\,(5) + 54{\cdot}5913\,(6) - 47{\cdot}6097\,(7) - 6{\cdot}9816\,(8) - 80{\cdot}3874\,(9) - 9{\cdot}8101\,(10) + 90{\cdot}1975\,(11)$

IV. *R : PSL′M′* $0 = +282{\cdot}338 + 194{\cdot}6150\,(1) - 326{\cdot}3070\,(2) - 30{\cdot}5347\,(4) + 13{\cdot}8407\,(5) - 4{\cdot}1649\,(6) - 163{\cdot}5483\,(7) + 167{\cdot}7132\,(8) - 11{\cdot}2030\,(10) + 118{\cdot}1442\,(11) - 106{\cdot}9412\,(12)$

The logarithmic values of the multipliers are as follow:

Log I = 9·1060989 + Log II = 8·9688224 + Log IV = 6·3648512 −
III = 6·7896507 +

Resulting Corrections.

From	To	Symbol.	Calculated Correction.	From	To	Symbol.	Calculated Correction.
MORMONTH . .	Peter Head W. M.	(1)	−0″·0508	*SECTOR STATION—(continued.)*	Reform Monumᵗ	(7)	+0″·3675
	Reform Monumᵗ	(2)	+0·0013		Peter Head W. M.	(8)	+1·7444
LITTLE STIRLING .	Peter Head W. M.	(3)	+2·7253	*PETER HEAD WINDMILL.*	Little Stirling .	(9)	−2·2698
	Reform Monumᵗ	(4)	+0·9695		Mormonth . .	(10)	+9·2153
	Sector Station .	(5)	+0·4140		Sector Station .	(11)	−2·3326
SECTOR STATION .	Little Stirling .	(6)	−2·1527		Reform Monumᵗ	(12)	+1·0358

GERTH OF SCAW SECTOR STATION.

The connection of this point by means of Nive Hill to Saxavord and Balta is shown in Figure 33.

Equations of Condition.

I. *B′NS′* $0 = -\ 8\cdot3169 + (1) - (4) - (5) + (6)$

II. *S′B′G* $0 = +\ 21\cdot1177 + (2) - (3) - (8) + (10)$

III. *S′NG* $0 = +\ 19\cdot2024 - (3) + (4) - (6) + (7) - (9) + (10)$

IV. *S′ : NB′G* $0 = -\ 534\cdot705 + 56\cdot0520\,(1) - 43\cdot6463\,(2) - 25\cdot4248\,(5) + 9\cdot8216\,(6) + 15\cdot6032\,(7)$
$+\ 4\cdot3343\,(8) - 1\cdot0903\,(9) - 3\cdot2440\,(10)$

The logarithmic values of the multipliers are as follow:

Log I = 7·9917438 −

Log II = 8·8154413 −
III = 8·8318146 −

Log IV = 7·4955424 +

Resulting Corrections.

From	To	Symbol.	Calculated Correction.	From	To	Symbol.	Calculated Correction.
BALTA	Nive Hill . .	(1)	+2″·5051	*NIVE HILL—cont.*	Saxavord . .	(6)	+2″·7737
	Gerth of Scaw .	(2)	−7·2309		Gerth of Scaw .	(7)	−0·6265
SAXAVORD . . .	Gerth of Scaw .	(3)	+6·5262	*GERTH OF SCAW* .	Balta . . .	(8)	+1·6262
	Nive Hill . .	(4)	−1·5538		Nive Hill . .	(9)	+1·9877
NIVE HILL . .	Balta . . .	(5)	−1·4843		Saxavord . .	(10)	−5·7344

GREENWICH ROYAL OBSERVATORY.

The Transit is fixed by observations from Epping Cupola, Chingford, and Severndroog. From the latter point the Dome was taken, but the plan of the building affords the means of calculating the angle subtended by the Dome and Transit at Severndroog; thus the observation is reduced to the Transit.

Putting (1) (2) (3) for the corrections upon the three observed bearings, the Equation of Condition is—

$$0 = 44\cdot572 + 126\cdot1686\,(1) - 86\cdot7897\,(2) - 7\cdot0837\,(3)$$

Whence there results—

From	To	Calculated Correction.
EPPING	Greenwich . .	−0″·2257
CHINGFORD . . .	” . .	+0·1833
SEVERNDROOG . .	” . .	+0·0267

EDINBURGH ROYAL OBSERVATORY
AND
KELLIE LAW SECTOR STATION.

The connection of these points is shown in Figure 35. The trigonometrical station* on Calton Hill, selected and observed from by Major-General Colby, is 6·85 feet distant from the centre of the Altitude and Azimuth Instrument or of the Dome with the bearing 27° 27′. The Zenith-Sector Station on Kellie Law was 201·1 feet from the trigonometrical station, bearing 302° 53′. The fixation of these points involves that of East Lomond; and in order to diminish the labour, this latter point being well observed from eight points, and observing them reciprocally, was first fixed independently of its connection with Calton Hill and Kellie Law. The equations and resulting corrections for the station East Lomond are as follow:

Equations of Condition.

I. $DL'K'$ $\quad 0 = +\,7{\cdot}7463 - (7) + (8) + (15) - (16)$
II. DCK' $\quad 0 = +\,2{\cdot}9042 + (1) - (7) - (9) + (15)$
III. DBE' $\quad 0 = +\,1{\cdot}2148 - (2) + (3) + (10) - (11)$
IV. DCE' $\quad 0 = +\,4{\cdot}4970 - (1) + (3) + (9) - (11)$
V. DCC' $\quad 0 = +\,2{\cdot}9093 - (1) + (4) + (9) - (12)$
VI. $DC'B'$ $\quad 0 = +\,1{\cdot}3886 - (4) + (5) + (12) - (13)$
VII. $DI'B'$ $\quad 0 = -\,4{\cdot}0508 - (5) + (6) + (13) - (14)$
VIII. $B : K'DL'$ $\quad 0 = -\,146{\cdot}679 - 15{\cdot}2690\,(2) + 14{\cdot}7352\,(7) + 0{\cdot}5338\,(8) - 31{\cdot}0739\,(15) + 17{\cdot}3759\,(16)$
IX. $C : K'DL'$ $\quad 0 = -\,240{\cdot}876 - 14{\cdot}3095\,(1) + 11{\cdot}1506\,(7) + 3{\cdot}1589\,(8) - 49{\cdot}6088\,(15) + 42{\cdot}8057\,(16)$
X. $C : E'DB$ $\quad 0 = +\,649{\cdot}250 + 143{\cdot}7954\,(1) - 169{\cdot}4997\,(2) + 25{\cdot}7043\,(3) - 264{\cdot}1193\,(10) + 41{\cdot}8914\,(11)$
XI. $C : C'DE'$ $\quad 0 = -\,189{\cdot}727 + 26{\cdot}8082\,(1) - 25{\cdot}7043\,(3) - 1{\cdot}1039\,(4) - 41{\cdot}8914\,(11) + 44{\cdot}2920\,(12)$
XII. $D : K'CB'$ $\quad 0 = -\,60{\cdot}945 - 48{\cdot}4158\,(9) - 121{\cdot}9941\,(13) - 78{\cdot}9163\,(15)$
XIII. $D : I'K'B'$ $\quad 0 = +\,38{\cdot}907 + 21{\cdot}9845\,(13) - 44{\cdot}2380\,(14) + 38{\cdot}8947\,(15)$

The logarithmic values of the multipliers are as follow:

Log I = 0·2004674 +
II = 1·5853962 +
III = 1·5519484 −
IV = 1·5628928 +

Log V = 0·3639746 −
VI = 9·4995194 −
VII = 9·4461661 +
VIII = 0·5948687 +
IX = 0·1501222 −

Log X = 9·1666089 −
XI = 8·7652974 −
XII = 8·6250252 −
XIII = 8·3699261 +

Resulting Corrections.

From	To	Symbol.	Calculated Correction.	From	To	Symbol.	Calculated Correction.
EAST LOMOND . .	Ben Cleugh .	(1)	+0″·9711	*BEN LAWERS* . .	East Lomond .	(11)	−1″·6184
	Ben Lomond .	(2)	+0·4215				
	Ben Lawers .	(3)	−4·4394	*GLASHMEAL* . .	„ „ .	(12)	−0·4533
	Glashmeal . .	(4)	−1·6866				
	Mount Battock .	(5)	−0·9080	*MOUNT BATTOCK* .	„ „ .	(13)	+1·7139
	Lumsden . .	(6)	+0·5319				
	Sayrs Law . .	(7)	+1·5909	*LUMSDEN* . . .	„ „ .	(14)	−0·8970
	Dunrich. . .	(8)	−1·0180	*SAYRS LAW* . .	„ „ .	(15)	−2·9893
BEN CLEUGH . .	East Lomond .	(9)	−0·7049	*DUNRICH* . . .	„ „ .	(16)	+2·1481
BEN LOMOND . .	East Lomond .	(10)	+2·0276				

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* Astronomical Observations made at the Royal Observatory, Edinburgh, 1835.

East Lomond being thus fixed, we have for the fixation of Kellie Law and the Observatory on Calton Hill the following—

Equations of Condition.

I. *EDI′* $0 = +0{\cdot}0152 + (2) - (5) - (16) + (20)$

II. *EDK′* $0 = -0{\cdot}6466 + (2) - (6) - (16) + (21)$

III. *EDC* $0 = -4{\cdot}5696 - (2) + (3) + (16) - (18)$

IV. *EDB′* $0 = +0{\cdot}6085 - (2) + (4) + (16) - (19)$

V. *FCL′* $0 = -0{\cdot}1024 - (7) + (9) + (13) - (15)$

VI. *FCB* $0 = -5{\cdot}6433 - (8) + (9) + (14) - (15)$

VII. *FCD* $0 = -0{\cdot}0533 - (9) + (11) + (15) - (17)$

VIII. *FED* $0 = +4{\cdot}1042 - (1) + (2) - (11) + (12) - (16) + (17)$

IX. *D : EI′B′* $0 = -319{\cdot}111 + 20{\cdot}2577\,(2) - 1{\cdot}0687\,(4) - 19{\cdot}1890\,(5) + 61{\cdot}2850\,(19) + 73{\cdot}4896\,(20)$

X. *D : I′EK′* $0 = +166{\cdot}745 - 16{\cdot}3905\,(2) + 19{\cdot}1890\,(5) - 2{\cdot}7985\,(6) - 73{\cdot}4896\,(20) + 37{\cdot}8999\,(21)$

XI. *D : B′CE* $0 = +210{\cdot}528 - 11{\cdot}3470\,(2) + 10{\cdot}2783\,(3) + 1{\cdot}0687\,(4) + 56{\cdot}9186\,(18) - 61{\cdot}2850\,(19)$

XII. *C : L′FB* $0 = -55{\cdot}315 + 14{\cdot}8885\,(7) + 62{\cdot}5075\,(8) - 77{\cdot}3960\,(9) - 39{\cdot}5247\,(13) + 76{\cdot}4695\,(14)$

XIII. *C : FBE′D* $0 = -90{\cdot}676 + 62{\cdot}5075\,(8) - 79{\cdot}9131\,(9) + 17{\cdot}4056\,(11) + 76{\cdot}4695\,(14) + 2{\cdot}2390\,(17)$

XIV. *D : FCC′E* $0 = +269{\cdot}480 + 16{\cdot}9925\,(1) - 27{\cdot}2708\,(2) + 10{\cdot}2783\,(3) + 17{\cdot}4056\,(9) - 41{\cdot}1570\,(11)$
$+ 23{\cdot}7514\,(12) + 20{\cdot}5834\,(15) + 56{\cdot}9186\,(18)$

XV. *E′ : FDC* $0 = +745{\cdot}523 - 112{\cdot}0793\,(9) + 137{\cdot}3678\,(10) - 25{\cdot}2885\,(11) + 56{\cdot}7490\,(15) + 13{\cdot}8048\,(17)$

The logarithmic values of the multipliers are as follow :

Log I = 0·8270589 −
II = 0·6344166 +
III = 0·3203218 +
IV = 0·2005022 −
V = 0·6026239 −

Log VI = 0·4820265 +
VII = 9·8612972 −
VIII = 9·7420390 −
IX = 9·3239755 +
X = 8·7643532 −

Log XI = 8·9777515 +
XII = 9·0512482 −
XIII = 8·9888977 +
XIV = 8·8161620 −
XV = 7·7249571 −

Resulting Corrections.

From	To	Symbol.	Calculated Correction.
KELLIE LAW . .	Calton Hill . .	(1)	−0″·7783
	East Lomond .	(2)	+3·3533
	Glashmeal . .	(3)	+1·6226
	Mount Battock .	(4)	−0·3835
	Lumsden . .	(5)	+0·2844
	Sayrs Law . .	(6)	−1·7997
CALTON HILL . .	Dunrich . .	(7)	+1·8469
	Ben Lomond .	(8)	−2·1404
	Ben Cleugh .	(9)	+0·0233
	Ben Lawers .	(10)	−4·9592
	East Lomond .	(11)	+0·8638
	Kellie Law . .	(12)	−2·4353
DUNRICH . . .	Calton Hill . .	(13)	+1·2076
BEN LOMOND . .	Calton Hill . .	(14)	+2″·7611
BEN CLEUGH . .	„ „ . .	(15)	−0·7185
EAST LOMOND . .	Kellie Law . .	(16)	+5·0054
	Calton Hill . .	(17)	+0·0687
GLASHMEAL . .	Kellie Law . .	(18)	−1·2950
MOUNT BATTOCK .	„ „ . .	(19)	+1·8771
LUMSDEN . . .	„ „ . .	(20)	+1·9212
SAYRS LAW . .	„ „ . .	(21)	+0·4990

DUBLIN OBSERVATORY.

The Dome of this Observatory is observed from Kippure, Lyons Hill, and the Hill of Howth, which stations were also observed from the Dome. The connection of Lyons Hill and the Observatory with the fixed points Kippure, Croghan, and the Hill of Howth is shown in Figure 36. The centre of the Dome is 57·25 feet east and 17·3 feet south of the centre of the Transit.

Equations of Condition.

I. $DH'K'$ $\quad 0 = -3{\cdot}7082 - (2) + (4) - (6) + (7)$

II. $LK'H'$ $\quad 0 = -0{\cdot}6551 - (1) + (3) - (10) + (11)$

III. $LK'D$ $\quad 0 = -1{\cdot}6260 - (1) + (2) + (5) - (7) - (9) + (11)$

IV. $LK'C$ $\quad 0 = -0{\cdot}7292 + (1) + (8) - (11) - (12)$

V. $K' : DLH'$ $\quad 0 = +1{\cdot}370 + 30{\cdot}4833\,(3) - 14{\cdot}1657\,(4) - 16{\cdot}6022\,(5) - 2{\cdot}1920\,(6) + 18{\cdot}7942\,(7)$
$\qquad - 2{\cdot}9654\,(9) + 12{\cdot}5418\,(10) - 9{\cdot}5764\,(11)$

VI. $K' : CLH'$ $\quad 0 = -210{\cdot}226 + 30{\cdot}4833\,(3) - 30{\cdot}0277\,(8) + 12{\cdot}5418\,(10) + 17{\cdot}4859\,(11) + 129{\cdot}1318\,(12)$

The logarithmic values of the multipliers are as follow:

Log I = 0·4701389 +
II = 0·3619191 +
Log III = 0·2843241 +
IV = 0·7273617 +
Log V = 8·1286971 +
VI = 8·7623300 +

Resulting Corrections.

From	To	Symbol.	Calculated Correction.	From	To	Symbol.	Calculated Correction.
KIPPURE . . .	Lyons Hill . .	(1)	+1″·1173	*DUBLIN OBSERVATORY*	Kippure. . .	(7)	+0″·9114
	Dublin Obs^{y} .	(2)	−0·3707	*LYONS HILL* . .	Croghan . .	(8)	+0·8908
HILL OF HOWTH .	Lyons Hill . .	(3)	+1·1352		Dublin Obs^{y} .	(9)	−2·3457
	Dublin Obs^{y} .	(4)	+1·2577		Hill of Howth .	(10)	−1·0935
					Kippure . .	(11)	−0·4563
DUBLIN OBSERVATORY	Lyons Hill . .	(5)	+2·1359				
	Hill of Howth .	(6)	−1·1685	*CROGHAN* . . .	Lyons Hill . .	(12)	+1·7352

CAMBRIDGE OBSERVATORY.

The Dome of this Observatory is observed from Ely Minster and Balsham; a Heliostat near the Dome was also observed from Brandon: the reduction to the Dome is given with the observed bearing. If (1) (2) (3) represent the corrections to the observed bearings from the three stations in the order above named, the Equation of Condition will be found to be—

$$0 = -188{\cdot}892 + 36{\cdot}994\,(1) + 15{\cdot}924\,(2) - 74{\cdot}764\,(3)$$

Whence there results—

From	To	Calculated Correction.
		″
ELY MINSTER . .	Cambridge Observatory Dome .	+0·4313
BALSHAM	„ .	+0·2156
BRANDON	„ .	−2·2672

The Dome is 45·4 feet east of the Transit, and in the same parallel of latitude.

DURHAM OBSERVATORY
AND
BURLEIGH MOOR SECTOR STATION.

The triangulation for the connection of these two points with Collier Law, Wordeslow, Easington, and Water Crag is shown in Figure 37. The Dome of the Observatory, which is the point observed from Collier Law, Brandon, and Merrington Church, is 12·5 feet west of the Transit, and in the same parallel of latitude.

Equations of Condition.

I. $C'MA'$ $\quad 0 = +\ 8{\cdot}4899 + (2) - (6) - (10) + (11)$

II. $C'MB'$ $\quad 0 = -\ 2{\cdot}0251 + (6) - (7) - (11) + (14)$

III. $KA'M$ $\quad 0 = +\ 11{\cdot}8765 - (1) + (2) - (10) + (12) + (15) - (21)$

IV. $KC'M$ $\quad 0 = +\ 5{\cdot}9767 - (4) + (6) - (11) + (12) + (16) - (21)$

V. $KC'B'$ $\quad 0 = -\ 3{\cdot}7947 + (4) - (8) - (16) + (17)$

VI. $E'BB'$ $\quad 0 = +\ 1{\cdot}9697 + (9) - (22) - (23) + (24)$

VII. $M : C'A'B'$ $\quad 0 = -\ 218{\cdot}675 - 136{\cdot}0402\,(2) + 31{\cdot}7070\,(6) - 61{\cdot}9052\,(7)$

VIII. $M : C'KA'$ $\quad 0 = -\ 44{\cdot}080 - 91{\cdot}6903\,(1) + 64{\cdot}6847\,(2) + 66{\cdot}4055\,(4) - 69{\cdot}9643\,(6) - 7{\cdot}4661\,(15) - 16{\cdot}1379\,(16) + 23{\cdot}6040\,(21)$

IX. $K : MC'B'$ $\quad 0 = -\ 772{\cdot}303 - 126{\cdot}8517\,(4) + 66{\cdot}4055\,(6) + 40{\cdot}3874\,(7) - 101{\cdot}1351\,(8) + 30{\cdot}1361\,(11) - 41{\cdot}5154\,(12) + 11{\cdot}3793\,(14)$

X. $D : KMC'$ $\quad 0 = -\ 618{\cdot}628 + 207{\cdot}3126\,(3) - 258{\cdot}7950\,(4) + 51{\cdot}4824\,(6) + 7{\cdot}6329\,(11) - 29{\cdot}9223\,(12) + 22{\cdot}2894\,(13) + 59{\cdot}6767\,(16) - 52{\cdot}8214\,(18) - 6{\cdot}8553\,(21)$

XI. $E' : KC'B'$ $\quad 0 = +\ 399{\cdot}136 - 101{\cdot}4627\,(4) + 6{\cdot}6551\,(8) + 75{\cdot}4569\,(16) - 15{\cdot}3465\,(17) - 60{\cdot}1104\,(19)$

XII. $B : B'C'E'$ $\quad 0 = +\ 120{\cdot}862 - 382{\cdot}1067\,(5) + 95{\cdot}2763\,(9) - 52{\cdot}3924\,(22)$

XIII. $E' : KBB'$ $\quad 0 = -\ 1710{\cdot}260 + 54{\cdot}6117\,(8) + 57{\cdot}2923\,(9) - 63{\cdot}3031\,(17) + 250{\cdot}0208\,(19) - 186{\cdot}7177\,(20) + 29{\cdot}2867\,(23) - 29{\cdot}2867\,(24)$

The logarithmic values of the multipliers are as follow:

Log I = 0·0509327 −
II = 8·8127808 +
III = 9·7361491 +
IV = 9·5780705 −

Log V = 0·4824777 +
VI = 8·9146314 −
VII = 8·4028697 +
VIII = 8·7283164 +
IX = 8·5757811 +

Log X = 7·0329938 +
XI = 8·4917703 +
XII = 7·3867015 −
XIII = 8·2431481 +

Resulting Corrections.

From	To	Symbol.	Calculated Correction.
WATER CRAG . .	Brandon Down	(1)	−2″·6415
	Merrington Ch.	(2)	−1·2887
COLLIER LAW . .	Durham Obsy .	(3)	+0·2487
	Brandon Down	(4)	−1·7849
	Burleigh Moor .	(5)	+0·3654
	Merrington Ch.	(6)	+1·1020
WORDESLOW . .	Merrington Ch.	(7)	−0·1361
	Brandon Down	(8)	−2·6328
	Burleigh Moor .	(9)	+0·2398
MERRINGTON CHURCH	Water Crag .	(10)	+6·7779
	Collier Law .	(11)	+0·6785
	Brandon Down	(12)	−4·2109
MERRINGTON CHURCH (*continued.*)	Durham Obsy .	(13)	+0″·5255
	Wordeslow . .	(14)	+1·4656
BRANDON DOWN .	Water Crag .	(15)	+0·1188
	Collier Law .	(16)	−1·6148
	Wordeslow . .	(17)	+1·3321
	Durham Obsy .	(18)	−0·3421
	Easington . .	(19)	+6·9942
	Burleigh Moor .	(20)	−0·6079
	Merrington Ch.	(21)	+2 3594
EASINGTON . . .	Burleigh Moor .	(22)	+0·0782
BURLEIGH MOOR .	Wordeslow . .	(23)	+1·3673
	Easington . .	(24)	−0·7640

MORDINGTON AND BURNSWARK.

These points were originally omitted in the calculation of the most probable corrections for the parts of the triangulation where they would naturally fall, as they were not necessary for the connection of the principal triangulation. Subsequently, however, it was considered desirable to add them, on account of the azimuthal determinations made at those points, that at Burnswark having a probable error of only $0''\cdot33$.

The stations Wisp, Dunrich, Sayrs Law, Blackheddon, and Cheviot are observed from Mordington, while Mordington is observed from the last four of these stations: these are the data for the determination of the position of Mordington. First, from the observations at Sayrs Law and Cheviot determine an approximate position, M′, of Mordington; then, using the corrected bearings and relative positions of the five fixed points, and denoting them by their initials, we have by exact calculation the following angles:—

Angle.	Calculated Angle.	Corresponding Observed Angle.	Difference.
	° ′ ″	° ′ ″	″
$M'CS$	45 51 34·7837	45 51 34·7837	+0·0000
$M'SC$	44 34 23·2749	44 34 23·2749	+0·0000
SDM'	22 37 16·5216	22 37 19·6103	−3·0887
CBM'	117 11 38·8915	117 11 42·8922	−4·0007
$CM'S$	89 34 5·4769	89 34 4·4300	+1·0469
$WM'S$	52 20 42·0608	52 20 42·4100	−0·3492
$DM'S$	26 56 24·6539	26 56 25·5000	−0·8461
$SM'B$	122 15 48·2929	122 15 47·5500	+0·7429

From the differences shown in the last column the correct position of Mordington must be determined. Let it be defined by co-ordinates x and y originating at M', SM' being the axis of x and the positive direction, y being measured perpendicular to SM' and northwards, then we have the following equations for determining x and y:—

$$\begin{array}{rl}
1\cdot6628\,y \quad . \quad . \quad . = 0 & \text{Weight} = 2\cdot44 \\
+\,1\cdot7003\,x - 0\cdot0104\,y \quad . \quad . \quad . = 0 & \quad \text{,,} \quad = 5\cdot26 \\
+\,0\cdot3807\,x - 0\cdot7491\,y - 3\cdot0887 = 0 & \quad \text{,,} \quad = 0\cdot12 \\
+\,2\cdot5492\,x + 1\cdot6093\,y - 4\cdot0007 = 0 & \quad \text{,,} \quad = 11\cdot11 \\
-\,1\cdot7003\,x - 1\cdot6524\,y + 1\cdot0469 = 0 & \quad \text{,,} \quad = 2\cdot22 \\
-\,0\cdot6127\,x - 1\cdot1900\,y - 0\cdot3492 = 0 & \quad \text{,,} \quad = 1\cdot47 \\
-\,0\cdot3807\,x - 0\cdot9136\,y - 0\cdot8461 = 0 & \quad \text{,,} \quad = 1\cdot69 \\
-\,2\cdot5492\,x - 3\cdot2721\,y + 0\cdot7429 = 0 & \quad \text{,,} \quad = 1\cdot22
\end{array}$$

Whence by the method of least squares—

$$\begin{array}{r}
104\cdot610\,x + 64\cdot810\,y - 120\cdot445 = 0 \\
64\cdot810\,x + 58\cdot947\,y - 77\cdot137 = 0
\end{array}$$

$$\therefore x = +\,1\cdot068 : y = +\,0\cdot134$$

which determine the exact or most probable position of Mordington. The resulting corrections upon the observed angles are as in the adjoining table:—

Angle.	Correction.	Angle.	Correction.	Angle.	Correction.	Angle.	Correction.
MCS .	+1″·8152	*CMS* .	−0″·9910	*SDM* .	−2″·7822	*DMS* .	−1″·3752
MSC .	−0·2227	*WMS* .	−1·1632	*CBM* .	−1·0616	*SMB* .	−2·4189

If *z* be the correction upon the bearing of Sayrs Law as observed from Mordington, the corrections to the four other bearings will be, *z*+0·9910, *z*+1·1632, *z*+1·3752, and *z*+2·4189, whence the most probable value of *z* = − 1″·1008, and the corrections to the bearings will be as follow:—

Resulting Corrections.

From	To	Calculated Correction.	From	To	Calculated Correction.
Mordington . .	Wisp . . .	+0″·0624	*Dunrich* . . .	Mordington . .	−2″·7822
	Dunrich . . .	+0·2744			
	Sayrs Law . .	−1·1008	*Cheviot* . . .	„ . .	+1·8152
	Blackheddon .	+1·3181			
	Cheviot . . .	−0·1098	*Blackheddon* . .	„ . .	−1·0616
Sayrs Law . .	Mordington . .	−0·2227			

The station Burnswark is observed from Hart Fell, Cross Fell, and Criffel; and Criffel, Merrick, Hart Fell, Wisp, Cross Fell, and Sea Fell are observed from Burnswark: these are the data for the fixation of Burnswark.

Proceeding in the same manner as in the fixation of Mordington, the corrections contained in the annexed table were obtained.

Resulting Corrections.

From	To	Calculated Correction.	From	To	Calculated Correction.
Hart Fell . .	Burnswark . .	+0″·9424	*Burnswark, cont.*	Merrick . . .	−0″·8573
				Hart Fell . .	−0·0470
Cross Fell . .	„ . .	+0·6203		Wisp . . .	−0·8479
				Cross Fell . .	−1·4778
Criffel . . .	„ . .	−4·1294		Sea Fell . . .	+0·8350
Burnswark . .	Criffel . . .	+1·0005			

A TABLE SHEWING THE MEAN AMOUNTS FOR THE DIFFERENT STATIONS OF THE

CORRECTIONS TO THE BEARINGS

OBSERVED WITH THE LARGER INSTRUMENTS.

Names.	No. of Bearings.	Average Recip. of Weight.	Average No. of Obs. to a Bearing.	Sum of Squares of Corrections.	Mean Value of Corrections.
Arbury Hill	6	1·00	8	4·12	0″·26
Axedge	11	1·23	7	80·50	2·71
Baconsthorpe Tower	5	0·24	8	1·56	0·56
Ballycreen	9	0·15	31	8·15	0·95
Balsham Tower	8	0·32	20	1·77	0·47
Balta	3	0·54	10	0·53	0·42
Banstead	10	0·49	12	25·61	1·60
Bardon Hill	11	0·73	5	114·35	3·23
Barrow Hill	5	0·72	15	3·33	0·82
Baurtregaum	13	2·33	7	25·30	1·40
Beacon Hill	15	0·20	28	19·76	1·15
Ben Cheilt	7	1·45	6	39·11	2·36
Ben Cleugh	13	0·39	19	7·66	0·77
Ben Clibrig	14	1·40	15	19·60	1·24
Bencorr	6	1·96	8	11·69	1·40
Ben Heynish	6	1·21	12	14·81	1·57
Ben Hutig	11	2·69	7	20·21	1·36
Ben Lawers	13	0·14	26	15·71	1·10
Ben Lomond	11	0·98	7	46·72	2·06
Ben Macdui	14	0·25	24	3·52	0·50
Ben More, Mull	9	0·81	10	11·05	1·11
Ben More, South Uist	8	0·30	30	2·23	0·53
Ben Nevis	13	0·31	24	8·07	0·79
Ben Tartevil	6	0·86	16	6·35	1·03
Ben Wyvis	9	2·23	4	23·48	1·62
Berkhampstead	7	1·84	9	14·08	1·42
Black Comb	8	7·43	6	19·47	1·56
Black Down	9	0·43	15	0·86	0·32
Blackheddon	5	0·14	16	2·75	0·74
Blue Hill	4	0·69	7	6·76	1·30
Botton Head	9	2·29	6	24·87	1·66
Brandon (Suffolk)	8	1·11	11	11·26	1·19
Brandon Down (Durham)	9	1·33	7	59·36	2·57
Brassa	6	0·49	12	1·48	0·50
Brimmond	8	1·18	8	10·91	1·17
Broadway	5	0·11	30	0·64	0·36
Bunwell Church Tower	9	0·34	11	2·04	0·48
Burleigh Moor	2	1·79	7	2·45	1·11
Burnswark	6	0·20	15	3·84	0·80
Butser Hill	10	0·54	14	9·61	0·98
Caherbarnagh	11	2·26	6	31·46	1·69
Calton Observatory	6	1·61	18	39·26	2·56
Carrigfadda	3	0·99	10	1·26	0·65
Cheviot	13	0·23	17	9·30	0·85
Chingford	7	0·15	42	1·17	0·41
Cleisham	8	4·85	8	51·01	2·53
Clifton Beacon	10	2·01	7	128·10	3·58
Collier Law	10	0·75	17	8·02	0·89
Coringdon	10	0·72	11	9·86	1″·00
Corryhabbie	14	0·15	35	7·00	0·71
Cowhythe	4	0·33	27	6·83	1·31
Cradle	12	0·58	7	6·21	0·72
Criffel	10	3·88	6	62·96	2·51
Cnocghiubhais	5	5·44	8	11·79	1·54
Croghan	6	5·70	5	42·81	2·67
Cross Fell	13	2·77	6	62·11	2·19
Crowborough	3	0·11	20	1·06	0·59
Cuilcagh	11	3·44	13	33·15	1·01
Cundtham	6	0·97	11	0·76	0·36
Deadman	7	0·54	18	10·62	1·23
Dean Hill	16	0·48	14	18·81	1·09
Deerness	4	0·24	9	1·04	0·51
Delamere	5	0·92	5	13·38	1·64
Ditchling	7	0·58	13	3·55	0·71
Divis	9	2·32	14	56·71	2·51
Docking Tower	4	0·39	6	5·33	1·15
Doolieve	6	4·94	7	52·08	2·95
Drung Point	3	0·28	16	0·76	0·50
Dublin Observatory	3	0·78	10	6·76	1·50
Dudwick	9	0·83	7	11·51	1·13
Dunkery Beacon	6	0·59	11	4·04	0·82
Dunnet Head	8	1·08	4	19·58	1·57
Dunnose	17	1·42	11	24·56	1·20
Dunrich	12	1·20	16	10·07	0·92
Dunstable	10	0·80	8	20·77	1·44
Easington	6	0·47	11	3·90	0·81
East Lomond	10	1·27	8	53·40	2·31
Ely Minster	9	0·25	15	15·13	1·30
Epping Poorhouse	11	1·66	11	23·28	1·46
Fair Isle	7	0·62	10	6·26	0·95
Fairlight	6	0·31	24	0·20	0·17
Fashven	7	4·16	7	67·58	3·11
Fetler	5	0·24	4	1·09	0·47
Fitty Hill	8	0·46	14	1·87	0·48
Forth	7	1·95	9	4·02	0·76
Foula	7	1·17	11	4·73	0·83
Four-mile Stone	9	0·44	23	15·66	1·32
Frittenfield	10	0·72	12	12·56	1·12
Gads Hill	5	0·17	14	2·55	0·71
Garforth Cliff	6	3·74	9	33·16	2·35
Garreg	7	1·17	9	8·81	1·12
Glashmeal	7	0·88	8	9·61	1·18
Goat Fell	15	0·23	15	7·27	0·69
Goonhilly	5	0·40	14	1·99	0·65
Great Whernside	8	0·95	5	17·22	1·47
Gringley	5	3·19	20	104·56	4·57
Gwaunysgaer	4	1·96	6	16·40	2·03

Names.	No. of Bearings.	Average Recip. of Weight.	Average No. of Obs. to a Bearing.	Sum of Squares of Corrections.	Mean Value of Corrections.
Hampton Poorhouse	3	0·46	6	1·39	0″·68
Hanger Hill Tower	8	0·87	12	17·12	1·46
Happisburgh Tower	4	1·68	5	2·24	0·75
Hart Fell	13	0·46	17	29·98	1·52
Hensbarrow	12	1·38	10	28·88	1·55
High Wilhays	12	0·49	11	8·28	0·83
Howth Hill	4	0·68	12	3·83	0·98
Hungry Hill	6	1·08	6	1·60	0·52
Inkpen	11	2·25	10	17·27	1·25
Jura	13	0·35	17	11·22	0·93
Karnbonellis	4	1·65	5	1·47	0·61
Karnminnis	8	0·64	9	5·80	0·85
Keeper	11	2·13	11	14·89	1·16
Kellie Law	6	0·71	22	17·95	1·61
Kings Arbour	3	0·22	4	0·77	0·51
Kippure	14	0·96	11	76·80	2·34
Knock	4	1·96	9	3·24	0·90
Knockanaffrin	11	0·90	14	14·36	1·14
Knocklayd	9	1·80	21	179·25	4·46
Knocknaskagh	8	3·22	7	13·45	1·30
Lawshall Tower	9	1·07	8	9·92	1·05
Laxfield	8	0·38	15	7·09	0·94
Layton	5	0·96	6	2·73	0·74
Leith Hill	16	0·57	16	12·96	0·90
Lincoln Minster	7	3·24	5	50·91	2·70
Llanelian	6	2·89	6	28·47	2·18
Long Mount	9	1·02	9	12·90	1·20
Lo. F. Base { N. end	6	0·97	10	3·95	0·81
Lo. F. Base { S. end	5	0·20	16	0·42	0·28
Lumsden	7	0·40	11	16·72	1·55
Lundy Island	7	0·24	13	9·03	1·14
Lynn	5	0·66	8	14·50	1·70
Lyons Hill	4	1·05	9	7·70	1·39
Maker Church Tower	5	0·46	19	0·27	0·22
Malvern	7	0·68	6	4·98	0·84
Mamsuil	11	0·50	23	3·54	0·57
Mendip	10	0·61	11	8·64	0·93
Merrick	18	0·84	21	15·52	0·93
Milk Hill	12	0·59	22	9·04	0·87
M. Carr Base { N. end	3	1·00	5	8·73	1·71
M. Carr Base { S. end	3	1·89	8	8·21	1·66
Moelfre Issa	6	1·67	5	6·82	1·07
Monach	8	4·44	12	5·23	0·81
Mordington	5	0·27	21	3·03	0·72
Mormonth	11	0·39	19	4·12	0·61
Mount Battock	13	0·20	18	7·85	0·77
Mount Sandy	9	1·21	16	0·99	0·33
Mowcopt	7	0·34	21	6·44	0·96
Nephin	6	1·90	9	15·27	1·60
Old Lodge	10	0·24	28	7·47	0·87
Orford Castle	4	0·56	10	3·86	0·99
Over Hill	8	1·23	5	38·57	2·20
Paddlesworth	5	0·57	11	6·08	1·11
Paracombe	6	1·14	12	6·08	1·01
Pendle Hill	9	4·59	6	11·98	1·15
Pertinney	9	0·87	13	10·39	1″·07
Pillesdon	8	0·54	16	6·78	0·92
Precelly	14	0·77	6	78·98	2·38
Rhuddlan Base { E. end	4	1·01	6	7·97	1·41
Rhuddlan Base { W. end	4	2·16	5	1·98	0·71
Ronas	5	0·55	12	7·14	1·20
Rue Rea	4	0·41	19	1·13	0·53
Ryders Hill	9	0·34	13	3·12	0·59
St. Agnes Beacon	5	3·10	5	54·64	3·31
St. Anne's Hill	5	0·34	5	2·24	0·67
Sarum Castle	7	0·51	14	13·82	1·40
Sarum Gun	4	0·11	35	3·09	0·88
Sawel	9	1·36	10	2·94	0·57
Sayrs Law	14	0·86	8	36·63	1·62
Sca Fell	14	3·17	6	105·76	2·75
Scarabin	12	1·43	19	43·50	1·91
Scour-na-lapich	10	0·33	20	5·84	0·76
Severndroog	11	0·34	40	7·85	0·84
Slieve Donard	13	0·50	18	6·78	0·72
Slieve League	6	4·04	6	107·90	4·24
Slieve Snaght	12	2·01	8	146·73	3·50
Snowdon	17	2·26	5	70·53	2·04
South Barule	11	0·92	17	28·50	1·61
South Lopham Ch. Tr.	7	0·56	9	19·71	1·68
South Ronaldsha	7	1·20	9	8·62	1·11
Southwold Tower	4	0·48	10	12·59	1·78
Stoke Hill	9	0·27	25	2·01	0·47
Stoke Tower	5	0·28	12	1·31	0·51
Storr Hill	10	0·47	22	8·28	0·91
Stronsay	6	0·51	12	1·79	0·55
Swyre Barrow	10	1·77	10	40·88	2·02
Tara	7	0·86	9	60·59	2·94
Tarbathy	7	4·56	5	54·18	2·78
Taur	8	1·37	9	22·46	1·60
Tharfield	9	0·99	9	15·39	1·31
Tilton	5	1·86	6	148·69	5·45
Tofts Tower	6	1·12	12	0·66	0·33
Trevose Head	6	0·47	19	2·90	0·69
Trostan	9	1·78	13	31·56	1·87
Upcot Down	6	0·29	18	0·63	0·33
Vicars Cairn	4	5·71	4	14·87	1·93
Walpole, Saint Peter's Church Tower	6	0·70	11	15·36	1·60
Walton Tower	6	1·45	7	12·77	1·46
Wart Hill Hoy	11	1·62	8	26·98	1·57
Water Crag	9	0·70	10	20·87	1·52
Westbury Down	10	0·30	21	2·18	0·47
Whitehorse Hill	9	1·29	9	38·15	2·06
Wingreen	12	0·54	12	11·76	0·99
Wisp Hill	6	5·22	10	8·55	1·20
Whittle Hill	11	1·68	3	49·17	2·11
Wordeslow	9	0·53	11	14·37	1·27
Wrotham	15	0·38	22	16·85	1·06
Yell	6	0·55	13	0·82	0·37
Yorkminster	5	0·71	15	11·48	1·52

A TABLE SHEWING THE MEAN AMOUNTS AT THE DIFFERENT STATIONS OF THE

CORRECTIONS TO THE BEARINGS

OBSERVED WITH THE SMALLER INSTRUMENTS.

Names.	No. of Bearings.	Average Recip. of Weight.	Average No. of Obs. to a Bearing.	Sum of Squares of Corrections.	Mean Value of Corrections.	Names.	No. of Bearings.	Average Recip. of Weight.	Average No. of Obs. to a Bearing.	Sum of Squares of Corrections.	Mean Value of Corrections.
Acklam Wold . . .	6	4·20	4	22·45	1″·93	Little Stirling . .	3	1·30	5	2·04	0″·83
Beachy Head . . .	5	1·15	21	4·48	0·95	Littletown Down . .	5	29·65	8	197·03	6·28
Beacon Hill, Trescow	7	0·72	19	9·07	1·14	Merrington Church .	5	7·79	13	66·56	3·62
Boniface Down . .	7	9·11	7	323·97	6·80	Mickfield Tower . .	6	1·09	22	32·52	2·33
Boniface Down, S.E. .	6	10·70	6	1058·30	13·28	Naseby	6	5·45	7	73·29	3·50
Boston Tower . . .	7	2·01	9	124·92	4·23	Naughton Tower . .	8	2·48	11	18·65	1·53
Brandon Hill (Ireland)	2	10·76	6	6·15	1·76	Nive Hill	3	28·46	19	10·29	1·85
Brown Willy . . .	10	1·26	13	130·28	3·61	Nodes Beacon . .	10	1·67	12	17·81	1·33
Carn Galver . . .	12	1·83	15	30·54	1·60	Norwich Spire . .	7	6·35	4	35·66	2·26
Crowle	7	5·08	4	18·56	1·63	Norwood	6	0·91	10	17·19	1·69
Cyrn-y-Brain . . .	6	4·22	17	41·08	2·62	O. M. O. Southampton	3	30·18	12	6·94	1·52
Danbury Spire . .	12	3·83	12	27·34	1·51	Otley Church Tower .	6	1·07	17	13·02	1·47
Easton Tower . .	7	2·14	8	112·96	4·02	Peninnis Windmill .	6	4·79	22	50·68	2·91
Feaghmaan . . .	4	80·85	3	211·14	7·27	Peterhead Windmill .	4	44·73	9	96·59	4·91
Gerth of Scaw . .	3	30·46	30	39·48	3·63	St. Martin's Day Mark	6	0·82	17	7·86	1·14
Gorleston Tower . .	4	4·80	9	4·79	1·10	St. Peter's Church Tr.	5	2·46	9	9·13	1·35
Great Stirling . .	3	38·43	11	7·81	1·61	Saxavord	5	0·61	8	15·93	1·79
High Port Cliff . .	2	3·31	11	44·21	4·70	Shanklin	3	1·10	8	3·38	1·06
Hingham	6	3·29	8	7·80	1·14	Slieve More in Achill	3	1·66	9	41·56	3·72
Holme Moss . . .	8	3·07	5	97·45	3·49	Swaffham Tower . .	9	3·43	10	74·29	2·87
Keysoe Church Spire	7	1·77	8	23·53	1·83	Telegraph Tower . .	6	1·34	19	19·30	1·79
Knockalongy . . .	4	5·90	8	14·76	1·92	Thaxted Spire . .	7	2·41	15	17·21	1·57
Knocknadober . .	5	5·78	5	34·27	2·62	Week Down . . .	6	3·72	9	58·34	3·12
Knocknagante . .	4	30·16	7	249·03	7·89	Wroxall	2	4·59	5	27·00	3·67

SECTION VIII.

TRIANGLES AND DISTANCES.

CALCULATION OF DISTANCES.

In the preceding section we have given the results of the calculations for determining the most probable corrections to be applied to the observed bearings for the principal triangulation. These corrections being applied to their respective bearings, the process of calculating the distances is sufficiently simple: if ABC be the angles of any triangle, ε the spherical excess $= \mathrm{A} + \mathrm{B} + \mathrm{C} - 180°$, then any one side c being given, we have

$$a = c \sin\left(\mathrm{A} - \frac{\varepsilon}{3}\right) \operatorname{cosec}\left(\mathrm{C} - \frac{\varepsilon}{3}\right)$$

$$b = c \sin\left(\mathrm{B} - \frac{\varepsilon}{3}\right) \operatorname{cosec}\left(\mathrm{C} - \frac{\varepsilon}{3}\right)$$

It remains now to consider, what is to be taken as the absolute length of any one side of the triangulation? For this purpose, we have the measured lengths of six base-lines, and from any one of them we might calculate the whole of the distances; but as in general each base-line should have weight in the final distances, we may proceed as follows: Let $\mathrm{A}_1\ \mathrm{A}_2\ \mathrm{A}_3 \cdots\cdot$ be the measured lengths of the bases, and let $\varrho_1\ \varrho_2\ \varrho_3 \cdots\cdot$ be the ratios of their lengths to that of any side in the triangulation as determined by the calculation of the triangles, and let x be the most probable length of this side; then if $\omega_1\ \omega_2\ \omega_3 \cdots\cdot$ be the weights of the measurement of the bases, x must be such that

$$\omega_1(\varrho_1 x - \mathrm{A}_1)^2 + \omega_2(\varrho_2 x - \mathrm{A}_2)^3 + \omega_3(\varrho_3 x - \mathrm{A}_3)^2 + \cdots\cdot$$

shall be a minimum. The differential coefficient with respect to x being put $= 0$, we have

$$x = \frac{\omega_1\varrho_1\mathrm{A}_1 + \omega_2\varrho_2\mathrm{A}_2 + \omega_3\varrho_3\mathrm{A}_3 + \cdots}{\omega_1\varrho_1^2 + \omega_2\varrho_2^2 + \omega_3\varrho_3^2 + \cdots\cdot}$$

which gives the most probable length of the side required, and the scale of absolute distance.

To determine the weight of x, put the denominator of the fraction $= \mathrm{V}$, and let W be the weight of x; then

$$\frac{1}{\mathrm{W}} = \frac{1}{\omega_1}\left(\frac{\omega_1\varrho_1}{\mathrm{V}}\right)^2 + \frac{1}{\omega_2}\left(\frac{\omega_2\varrho_2}{\mathrm{V}}\right)^2 + \frac{1}{\omega_3}\left(\frac{\omega_3\varrho_3}{\mathrm{V}}\right)^2$$

$$= \frac{\omega_1\varrho_1^2 + \omega_2\varrho_2^2 + \omega_3\varrho_3^2 + \cdots\cdot}{\mathrm{V}^2} = \frac{1}{\mathrm{V}}$$

$$\therefore \mathrm{W} = \omega_1\varrho_1^2 + \omega_2\varrho_2^2 + \omega_3\varrho_3^2 + \cdots\cdots\cdots$$

If the side x coincide with one of the base-lines A_1, then $\varrho_1 = 1$, and

$$x = \frac{\omega_1 A_1 + \omega_2 \varrho_2 A_2 + \omega_3 \varrho_3 A_3 + \cdots\cdot}{\omega_1 + \omega_2 \varrho_2^2 + \omega_3 \varrho_3^2 + \cdots}$$

$$\therefore \quad x - A_1 = \frac{\omega_2 \varrho_2 (A_2 - \varrho_2 A_1) + \omega_3 \varrho_3 (A_3 - \varrho_3 A_1) + \cdots\cdot}{\omega_1 + \omega_2 \varrho_2^2 + \omega_3 \varrho_3^2 + \cdots\cdot}$$

which is the correction to be applied to the measured length of the base-line A_1. If $L_1\ L_2\ L_3 \cdots\cdot$ be the logarithms of the measured lengths, and $L_1\ L_2'\ L_3' \cdots\cdot$ be the corresponding logarithms given by the triangulation (using the measured length of A_1 as the standard), then it will follow that the correction to the logarithm of any side will be

$$\frac{\omega_2 A_2^2 (L_2 - L_2') + \omega_3 A_3^2 (L_3 - L_3') + \cdots\cdot}{\omega_1 A_1^2 + \omega_2 A_2^2 + \omega_3 A_3^2 + \cdots\cdot\cdot}$$

In the application of this principle to our triangulation, we are stopped in the outset by the impossibility of ascribing with any kind of accuracy the proper relative weights to the measurements of the base lines. Two different methods of measurement have been employed, namely by steel chains of 100 feet in length, and by the compensation bars. The relative value of the two methods might approximately be ascertained by repeated measurements of the distance between two points on the ground; such experimental measures have not however been made, nor would the relative weights thus obtained enable us to assign weights to measures made under different circumstances of ground, and with different degrees of carefulness and experience. If the relative precision of the compensation method be at all proportioned to the complication of the apparatus and tediousness of the process, the weight of a chain measurement will be comparatively small,—and that the former method is much more accurate there can be no doubt.

In the absence of any means of ascertaining the comparative value of the old bases, the absolute distances have been made to depend upon the Lough Foyle and Salisbury Plain compensation bases. The relative weights of the measurements of these two bases it would be difficult to assign. The ground in the former case was very much more favourable than in the latter, while on the other hand, the measurement of the line on Salisbury Plain was more perfect, from the circumstance of the compensation microscopes having been frequently and regularly examined and compared with the standard brass scale: in the measurement of the Lough Foyle base these comparisons were either very few or not generally recorded.

It follows from the theory of accidental errors of observation, that their influence to be apprehended in the measurement of a line is proportioned to the square root of its length; and hence the absolute length of any side in the triangulation has been taken, such, that the apparent errors of the measured lines at Lough Foyle and on Salisbury Plain as compared with their lengths in the consistent triangulation shall be proportioned to the square roots of their lengths.

The logarithm of the measured length in feet of the base line on Salisbury Plain is $4 \cdot 5632182708$, and the logarithm of the length as calculated from the measured length of the Lough Foyle base is $4 \cdot 5632136242$, the former exceeding the latter by $\cdot 0000046466$. Let A_1 and A_2 be the measured lengths of the bases, $A_1 + x\sqrt{A_1}$ and $A_2 - x\sqrt{A_2}$ their lengths adopted in the triangulation of which the ratio is given, then

$$\text{Log } A_2 - \log\left(A_1 \frac{A_2 - x\sqrt{A_2}}{A_1 + x\sqrt{A_1}}\right) = \cdot 0000046466$$

$$= M x \frac{\sqrt{A_1} + \sqrt{A_2}}{\sqrt{A_1 A_2}}$$

Also, if u be the correction to the logarithm of any distance as calculated from the measured length of the Lough Foyle base

$$u = \log \frac{A_1 + x\sqrt{A_1}}{A_1} = M x \frac{1}{\sqrt{A_1}}$$

$$= \cdot 0000046466 \frac{\sqrt{A_2}}{\sqrt{A_1} + \sqrt{A_2}} = \cdot 0000022480$$

since $\sqrt{A_1} = 204 \cdot 06$ and $\sqrt{A_2} = 191 \cdot 25$: also $x = \cdot 0010563$, and consequently the *apparent errors* of the measured lengths will be

	Feet.		Inches.
Lough Foyle Base	+ 0·2155	=	+ 2·586
Salisbury Plain Base	− 0·2020	=	− 2·424

The difference between the lengths of the Salisbury Plain base as measured (M), and as calculated (C) from the Lough Foyle base, is given by the equation

$$\text{Log } \frac{M}{C} = 2\,(\text{Modulus}) \frac{M - C}{M + C} = \cdot 0000046466$$

$$\therefore\ M - C = \frac{M \times \cdot 0000046466}{(\text{Modulus})} = \cdot 3913$$

$$M = C + 4 \cdot 6956 \text{ inches}$$

The probable error of this determination depends upon the probable errors of the measurements of the two lines, and also upon the probable error of the ratio of the two lines as derived from the triangulation.

The calculation of this last probable error is possible, but quite impracticable. It may, however, be interesting to ascertain the amount of error that might be expected to result from the errors of observation of the triangulation, supposing one single chain of triangles only to connect the two bases. For this purpose, take the triangles contained in the following table.

No.	Names.	Angles.	Error of Sum.	No.	Names.	Angles.	Error of Sum.
		° ′	″			° ′	″
1	South End . . North End . . Slieve Snaght .	67·37 84·25 27·57	−0·32	9	Precelly . . . Dunkery . . . Cradle . . .	47·7 62·52 70·0	+0·02
2	North End . . Sawel . . . . Slieve Snaght .	92·41 32·57 54·21	−4·06	10	Cradle . . . . Mendip . . . Dunkery . . .	46·41 68·32 64·46	+0·13
3	Slieve Snaght . Cuilcagh . . . Sawel	40·3 20·20. 119·36	+2·08	11	Dunkery . . . Pillesdon . . . Mendip . . .	41·48 76·39 61·32	−0·26
4	Sawel . . . . Slieve Donard . Cuilcagh . . .	82·15 43·6 54·38	+1·07	12	Pillesdon . . . Wingreen . . . Mendip . . .	44·42 61·27 73·50	−1·62
5	Cuilcagh . . . Kippure . . . Slieve Donard .	48·27 53·34 77·58	−1·31	13	Mendip . . . Beacon Hill . . Wingreen . . .	36·36 43·16 100·7	+4·61
6	Slieve Donard . Snowdon . . . Kippure . . .	59·12 40·36 80·11	−5·83	14	Wingreen . . . Dean Hill . . . Beacon Hill . .	30·13 82·56 66·50	−4·95
7	Kippure . . . Precelly . . . Snowdon . . .	48·2 57·30 74·27	+1·30	15	Dean Hill . . . Four-mile Stone . Beacon Hill . .	39·29 72·5 68·25	−0·62
8	Snowdon . . . Cradle . . . . Precelly . . .	(48·58) 68·30 62·31	Not observed	16	Four-mile Stone . Old Sarum Gun . Beacon Hill . .	70·2 85·58 23·59	−4.57

If α, β, represent the first and second angle of any triangle,

$$\Sigma\,(\cot^2\alpha + \cot^2\beta) = 30{\cdot}04$$

$$\Sigma\,(\cot^2\alpha + \cot\alpha\cot\beta + \cot^2\beta) = 39{\cdot}46$$

The ratio of the lengths of the base-lines, putting α_n β_n for the first and second angles of the n^{th} triangle, is expressed by the equation

$$\frac{A_2}{A_1} = \frac{\sin\alpha_1 \sin\alpha_2 \sin\alpha_3 \cdots\cdots \sin\alpha_{16}}{\sin\beta_1 \sin\beta_2 \sin\beta_3 \cdots\cdots \sin\beta_{16}}$$

Suppose $\alpha_n + x_n$, $\beta_n + y_n$, to be the true angles, $-x_n$ and $-y_n$ being the errors of observation corresponding to the observed angles α_n β_n; then

$$A_2 = A_1 \frac{\sin\alpha_1 \cdots\cdot \sin\alpha_{16}}{\sin\beta_1 \cdots\cdot \sin\beta_{16}} \left(1 + x_1\cot\alpha_1 - y_1\cot\beta_1 + x_2\cot\alpha_2 - y_2\cot\beta_2 + \cdots + x_{16}\cot\alpha_{16} - y_{16}\cot\beta_{16}\right)$$

If now $x_1\,y_1\,x_2\,y_2\cdots$ be all independent, that is, supposing two angles only of each triangle to be observed, the probable error of A_2 will be

$$\pm A_2 \sqrt{\left((x_1' \cot \alpha_1)^2 + (y_1' \cot \beta_1)^2 + (x_2' \cot \alpha_2)^2 + (y_2' \cot \alpha_2)^2 + \cdots\cdots \right)}$$

$x_n'\,y_n'$ being the probable values of $x_n\,y_n$. If we assume a mean quantity, θ, as the probable error of observation of a bearing, the probable error of observation of an angle will be $\theta\sqrt{2}$, and the probable error of A_2 will be

$$\pm A_2\,\theta\,\sqrt{2}\,\sqrt{\Sigma\,(\cot^2 \alpha_n + \cot^2 \beta_n)}$$

or, expressing θ in seconds and putting $A_2 = 36578$ feet, we have

Probable error of calculated length of Salisbury Base

$$= \pm\, 36578\,\sqrt{2}\,\sin 1''\cdot\theta\,\sqrt{\Sigma\,(\cot^2 \alpha_n + \cot^2 \beta_n)}$$
$$= 1{\cdot}37\theta \text{ feet,}$$

which depends entirely on the value we choose to assign to θ. If the angles were well observed, so that θ might be taken as $= 0''{\cdot}5$, the probable error would be $\pm\, 0{\cdot}68$, or 8 inches.

If the three angles of each triangle be observed, and $\varepsilon_1\,\varepsilon_2\cdots\cdots$ be the errors of the sums of the angles, then the probable error of A_2 will be*

$$\pm \frac{1}{3} A_2 \sqrt{\left(\cot^2 \alpha_1 + \cot \alpha_1 \cot \beta_1 + \cot^2 \beta_1\right) \varepsilon_1^2 + \left(\cot^2 \alpha_2 + \cot \alpha_2 \cot \beta_2 + \cot^2 \beta_2\right) \varepsilon_2^2 + \cdots}$$

or if ε be the mean value of the quantities $\varepsilon_1\,\varepsilon_2\cdots\cdots$

$$\pm \frac{\varepsilon}{3} A_2 \sqrt{\Sigma\left(\cot^2 \alpha_n + \cot \alpha_n \cot \beta_n + \cot^2 \beta_n\right)}$$

or, expressing the quantity ε in seconds,

$$\pm\, 12193 \sin 1''{\cdot}\varepsilon \sqrt{\Sigma\left(\cot^2 \alpha_n + \cot \alpha_n \cot \beta_n + \cot^2 \beta_n\right)}$$

The sum of the squares of the quantities ε for 15 triangles in the above table is 129·16, so that we may put $\varepsilon = 2{\cdot}93$: also the value of the quantity under the radical being $= 39{\cdot}462$, we have,

Probable error $= \pm\, 1{\cdot}09$ feet.

If instead of restricting ε to its value, as resulting from the particular triangles under consideration, we put $\varepsilon = \theta\,\sqrt{6}$, θ being still the probable error of observation of a bearing, should have had,

Probable error $= \pm\, 0{\cdot}91{\cdot}\theta$ feet.

which would be the probable error of the length of the Salisbury Plain base as calculated from the Lough Foyle base, on the supposition of there being no other means than the single chain of triangles contained in the table, and of the three angles of each triangle being observed in each case with an even probable error of $\theta\sqrt{2}$.

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* Laplace: "Théorie Analytique des Probabilités: Deuxième Supplément." Paris, 1847.

If $\pm \alpha_1 A_1$ and $\pm \alpha_2 A_2$ be the probable errors of the measurements of the Lough Foyle and Salisbury Plain bases, and $\pm \varepsilon R$ the probable error of the ratio, R, of the two bases given by the triangulation, the probable value of the discrepancy will be,—expressed in feet,

$$\pm 36578 \sqrt{\alpha_1^2 + \alpha_2^2 + \varepsilon^2}$$

If $\varrho_1\ \varrho_2$ be the ratios (as given by the triangulation) of the two bases to any side in the triangulation, the absolute length, λ, of that side is

$$\lambda = \frac{A_1\sqrt{A_2} + A_2\sqrt{A_1}}{\varrho_1\sqrt{A_2} + \varrho_2\sqrt{A_1}}$$

If instead of assigning to each base an error proportional to the square root of its length, the errors had been assigned as the lengths directly, the absolute length of any side would have exceeded the above length in the proportion of

$$1 + \frac{\varrho_1 A_2 - \varrho_2 A_1}{\varrho_1 A_2 + \varrho_2 A_1} \cdot \frac{\sqrt{A_1} - \sqrt{A_2}}{\sqrt{A_1} + \sqrt{A_2}} : 1$$

or, substituting the values of $\frac{\varrho_1}{\varrho_2}$, A_1 and A_2, in the proportion of 1·000000173 to unity, equivalent to an increase of one inch in a hundred miles.

COMPARISON OF BASES.

The annexed table contains the lengths of the bases as measured and reduced to the same standard, and their lengths as shown in the triangulation, in accordance with the adopted scale of linear measure. The last column of the table shows the discrepancies.

Date.	Bases.	Length in Terms of Ramsden's Scale.	Length in Terms of Ord. Standard O_1.	Length in Triangulation.	Difference.
1791	*Hounslow Heath* .	27404·24	27406·190	27406·363	+0·173
1794	*Salisbury Plain* .	36574·23	36576·830	36577·656	+0·826
1801	*Misterton Carr* . .	26342·19	26344·060	26343·869	−0·191
1806	*Rhuddlan Marsh* .	24514·26	24516·000	24517·596	+1·596
1817	*Belhelvie* . . .	26515·65	26517·530	26517·770	+0·240
1827	*Lough Foyle* . .	. .	41640·887	41641·103	+0·216
1849	*Salisbury Plain* . .	. .	36577·858	36577·656	−0·202

We may now estimate the difference that would have resulted in the absolute distances, if the first five bases measured with the 100-feet chain had been allowed to have a weight in the determination. Suppose these bases to have an equal weight represented by unity, while the weights of the compensation bases are represented by ω. Let $\delta_1\ \delta_2 \cdots$ represent the differences in the last column of the table, A_1 A_2 .... representing the lengths of the bases. Now, if $1 + x$ be the multiplier by which all the distances should be increased, the quantity

$$(A_1 x + \delta_1)^2 + (A_2 x + \delta_2)^2 + \cdots + \omega (A_6 x + \delta_6)^2 + \omega (A_7 x + \delta_7)^2$$

must be a minimum with respect to x; this gives

$$x = - \frac{A_1 \delta_1 + A_2 \delta_2 + \cdots + \omega A_6 \delta_6 + \omega A_7 \delta_7}{A_1^2 + A_2^2 + \cdots + \omega A_6^2 + \omega A_7^2}$$

$$x = - \frac{7{\cdot}5409 + 0{\cdot}1593\, \omega}{408451 + 307012\, \omega}$$

from which it follows, that if we give all the bases equal weight, the assumed absolute distances would have to be multiplied by

$$0{\cdot}99998924 \ : \ \log = 9{\cdot}99999533$$

By assigning double weight to the compensation bases, the absolute distances would require to be multiplied by

$$0{\cdot}99999231 \ : \ \log = 9{\cdot}99999666$$

The alteration in the former case would be at the rate of $- 5 \cdot 8$ feet in every hundred miles; in the latter case $- 4 \cdot 1$ feet in every hundred miles.

And in general, leaving ω indeterminate, the correction to any distance, as given in the triangulation, will be at the rate of

$$- {\cdot}27396 \frac{47{\cdot}34 + \omega}{1{\cdot}33 + \omega} \text{ feet}$$

to one hundred miles; or the correction to the eight-figure logarithm of the distance will be

$$- 22{\cdot}53 \frac{47{\cdot}34 + \omega}{1{\cdot}33 + \omega}$$

The following table shows the *apparent* errors, in inches, of the measurements on the suppositions of $\omega = 1$, $\omega = 2$, and as actually assumed.

Bases.	Giving equal Weight to all Bases.		Giving double Weight to first two Bases.		Assumed as most probable.	
	Error of Measure.	Error per Mile.	Error of Measure.	Error per Mile.	Error of Measure.	Error per Mile.
Salisbury Plain .	+ 7·15	+ 1·03	+ 5·80	+ 0·84	+ 2·42	+ 0·35
Lough Foyle . .	+ 2·80	+ 0·36	+ 1·24	+ 0·16	− 2·59	− 0·33
Hounslow Heath .	+ 1·46	+ 0·28	+ 0·45	+ 0·09	− 2·08	− 0·40
Misterton Carr . .	+ 5·72	+ 1·15	+ 4·71	+ 0·94	+ 2·29	+ 0·45
Belhelvie . . .	+ 0·55	+ 0·12	− 0·43	− 0·08	− 2·88	− 0·57
Salisbury (Old) .	− 5·18	− 0·74	− 6·54	− 0·94	− 9·91	− 1·43
Rhuddlan Marsh .	−15·99	− 3·43	−16·89	− 3·63	−19·15	− 4·12
Mean Amounts . .	± 7·29	± 1·45	± 7·39	± 1·49	± 8·40	± 1·69

From the last column of this table it would appear that the last five bases, measured with Ramsden's Steel Chain, are in defect by the *average* amount of 1·21 inches per mile. This might be attributed to an error in the comparison between the standard measure used in reference to these bases, and O_1; but the individual differences are not sufficiently regular to give much probability to this hypothesis, which would in reality rest almost entirely upon the comparatively large error shown in the Rhuddlan Marsh base. In examining the fixation of this base-line, page 396, it will be observed that for the determination of 34 corrections there are only given twelve equations between them. The general proportion of the number of equations to the number of corrections or observed bearings in any part of the triangulation is nearly as 6 : 10, consequently the number of Equations of Condition for the fixation of the Rhuddlan Marsh base should be 20. The connection of this base-line is therefore defective in this respect. It has also been stated that there was some uncertainty with respect to the refinding of the centres of the old stations at Llanelian and Arrenig at the time of the new observations at Cyrn-y-Brain in 1852; the centre stone of the latter station was then found in good preservation.

The measurement of the Salisbury base-line in 1849 exceeded the old measurement with the chains in 1794 by 12·33 inches. Part of this difference may be owing to a small displacement of the guns marking the extremities of this base-line during the fifty-five years that elapsed from their first being sunk and their being revisited in 1849.

The errors shown by the remaining bases are sufficiently small, and do not call for special notice.

TABLE SHEWING RELATIVE DISTANCES OF BASE-LINES,

IN MILES.

Names.	Hounslow.	Salisbury.	Misterton.	Rhuddlan.	Belhelvie.	Lough Foyle.
Hounslow . . .	. .	62·48	142·93	183·22	406·10	370·78
Salisbury . . .	62·48	. .	166·68	166·59	422·52	349·13
Misterton . . .	142·93	166·68	. .	108·97	262·26	269·15
Rhuddlan . . .	183·22	166·59	108·97	. .	279·78	187·58
Belhelvie . . .	406·10	422·52	262·26	279·78	. .	241·39
Lough Foyle . .	370·78	349·13	269·15	187·58	241·39	. .

PRINCIPAL TRIANGLES

OF THE

ORDNANCE SURVEY OF GREAT BRITAIN AND IRELAND.

TRIANGLES.

FIGURE I.

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
1	South End of Base	53 30 53·209	4·62321693	41996·870	8·0	−1·40
	North End of Base	73 37 18·656	4·69996493	50114·676	9·5	
	Drung Point	52 51 48·529	4·61952222	41641·103	7·9	
		180 0 0·394				
2	South End of Base	53 30 52·415	4·66834381	46595·482	8·8	−2·77
	Drung Point	66 37 60·557	4·72591989	53201·011	10·1	
	Mount Sandy	59 51 7·532	4·69996493	50114·676	9·5	
		180 0 0·504				
3	South End of Base	67 37 37·739	4·92461702	84065·35	15·9	−0·84
	Mount Sandy	76 33 17·344	4·94653490	88416·82	16·8	
	Slieve Snaght	35 49 5·939	4·72591989	53201·01	10·1	
		180 0 1·022				
4	South End of Base	67 37 38·534	4·91461101	82150·651	15·6	−0·32
	North End of Base	84 25 14·637	4·94653490	88416·822	16·8	
	Slieve Snaght	27 57 7·629	4·61952222	41641·103	7·9	
		180 0 0·800				
5	Slieve Snaght	54 21 6·969	5·08895548	122731·34	23·2	−4·06
	North End of Base	92 41 50·595	5·17859199	150866·21	28·6	
	Sawel	32 57 4·803	4·91461101	82150·65	15·6	
		180 0 2·367				
6	Mount Sandy	84 7 14·736	5·17859199	150866·21	28·6	−0·92
	Slieve Snaght	62 13 5·279	5·12769166	134181·20	25·4	
	Sawel	33 39 42·622	4·92461702	84065·35	15·9	
		180 0 2·637				
7	Sawel	76 42 31·280	5·35557056	226762·15	42·9	−5·44
	Slieve Snaght	62 56 25·901	5·31701186	207497·02	39·3	
	Knocklayd	40 21 9·979	5·17859199	150866·21	28·6	
		180 0 7·160				
8	Sawel	43 2 48·658	5·15444656	142707·42	27·0	−3·71
	Mount Sandy	97 1 37·771	5·31701186	207497·02	39·3	
	Knocklayd	39 55 38·037	5·12769166	134181·20	25·4	
		180 0 4·466				
9	Sawel	60 21 12·157	5·26268200	183097·33	34·7	Supplementary.
	Knocklayd	39 36 57·704	5·12818857	134334·81	25·4	
	Cundtham	80 1 55·832	5·31701186	207497·02	39·3	
		180 0 5·693				
10	Sawel	16 21 19·123	4·64091218	43743·365	8·3	”
	Slieve Snaght	59 51 19·301	5·12818857	134334·81	25·4	
	Cundtham	103 47 22·917	5·17859199	150866·21	28·6	
		180 0 1·341				

FIGURE 1—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
11	Mount Sandy	73 59 46·865	4·75776118	57248·113	10·8	−1·22
	South End of Base	42 42 50·952	4·60637539	40399·444	7·7	
	Cundtham	63 17 22·669	4·72591989	53201·011	10·1	
		180 0 0·486				
12	Mount Sandy	73 59 49·727	4·58923649	38836·179	7·4	+0·11
	Cundtham	16 37 33·674	4·06295444	11559·910	2·2	
	North End of Base	89 22 36·704	4·60637539	40399·444	7·7	
		180 0 0·105				
13	Sawel	13 5 8·947	4·67207372	46997·387	8·9	+1·74
	Knocklayd	78 31 47·368	5·30842277	203433·64	38·5	
	Trostan	88 23 5·931	5·31701186	207497·02	39·3	
		180 0 2·246				
14	Sawel	89 47 40·227	5·40283536	252833·93	47·9	−1·35
	Slieve Snaght	53 34 23·265	5·30842277	203433·64	38·5	
	Trostan	36 38 3·721	5·17859199	150866·21	28·6	
		180 0 7·213				
15	Sawel	68 20 8·138	5·50512565	319982·08	60·6	−0·72
	Trostan	75 26 59·030	5·52278501	333261·40	63·1	
	Slieve Donard	36 13 7·640	5·30842277	203433·64	38·5	
		180 0 14·808				
16	Sawel	82 15 28·414	5·60738062	404930·62	76·7	+1·07
	Slieve Donard	43 6 32·532	5·44601136	279261·72	52·9	
	Cuilcagh	54 38 20·731	5·52278501	333261·40	63·1	
		180 0 21·677				
17	Sawel	119 36 43·221	5·57672864	377336·34	71·5	+1·08
	Slieve Snaght	40 2 55·977	5·44601136	279261·72	52·9	
	Cuilcagh	20 20 29·411	5·17859199	150866·21	28·6	
		180 0 8·609				
18	Sawel	62 44 3·015	5·41861291	262188·06	49·7	−1·65
	Cuilcagh	46 2 51·007	5·32704220	212345·08	40·2	
	Vicars Carn	71 13 18·368	5·44601136	279261·72	52·9	
		180 0 12·390				
19	Sawel	44 33 11·948	5·22524141	167973·75	31·8	−1·32
	Vicars Carn	72 57 49·187	5·35968473	228920·52	43·4	
	Divis	62 29 6·881	5·32704220	212345·08	40·2	
		180 0 8·016				
20	Divis	54 54 10·809	5·17856719	150857·60	28·6	+0·79
	Vicars Carn	59 27 12·323	5·20083103	158792·88	30·1	
	Slieve Donard	65 38 41·998	5·22524141	167973·75	31·8	
		180 0 5·130				
21	Sawel	43 18 21·590	5·20757696	161278·68	30·5	−0·26
	Trostan	76 47 38·959	5·35968473	228920·52	43·4	
	Divis	59 54 6·958	5·30842277	203433·64	38·5	
		180 0 7·507				

FIGURE 1—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
	Sawel	56 23 30·537	5·31609989	207061·75	39·2	
22	Knocklayd	67 2 14·284	5·35968473	228920·52	43·4	−2·58
	Divis	56 34 24·476	5·31701186	207497·02	39·3	
		180 0 9·297				
	Knocklayd	107 23 24·263	5·54382111	349801·06	66·2	
23	Divis	38 13 2·480	5·35557056	226762·15	42·9	+1·30
	Slieve Snaght	34 23 43·787	5·31609989	207061·75	39·2	
		180 0 10·530				

FIGURE 2.

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Feet.	Miles.	Error of Δ.
	Cuilcagh	48 26 57·849	5·57587896	376598·83	71·3	
1	Slieve Donard	77 58 57·091	5·69215523	492215·44	93·2	−1·31
	Kippure	53 34 40·123	5·60738062	404930·62	76·7	
		180 0 35·063				
	Cuilcagh	51 55 14·884	5·65605572	452955·69	85·8	
2	Kippure	69 17 30·429	5·73100149	538271·62	101·9	−0·52
	Keeper	58 48 3·718	5·69215523	492215·44	93·2	
		180 0 49·031				
	Cuilcagh	31 9 50·633	5·48641948	306492·21	58·0	
3	Keeper	34 10 38·462	5·52206794	332711·60	63·0	−0·84
	Croghan	114 39 52·693	5·73100149	538271·62	101·9	
		180 0 21·788				
	Cuilcagh	20 45 24·251	5·33464833	216096·80	40·9	
4	Kippure	33 4 5·583	5·52206794	332711·60	63·0	+7·79
	Croghan	126 10 43·809	5·69215523	492215·44	93·2	
		180 0 13·643				
	Cuilcagh	69 12 22·100	5·62642300	423080·49	80·1	
5	Slieve Donard	47 19 26·318	5·52206794	332711·60	63·0	+4·57
	Croghan	63 28 41·188	5·60738062	404930·62	76·7	
		180 0 29·606				
	Kippure	86 38 45·707	5·62642300	423080·49	80·1	
6	Slieve Donard	30 39 30·773	5·33464833	216096·80	40·9	+1·90
	Croghan	62 42 2·621	5·57587896	376598·83	71·3	
		180 0 19·100				
	Kippure	36 13 24·846	5·48641948	306492·21	58·0	
7	Keeper	24 37 25·256	5·33464833	216096·80	40·9	−7·47
	Croghan	119 9 23·498	5·65605572	452955·69	85·8	
		180 0 13·600				
	Cuilcagh	68 22 0·878	5·71622432	520264·65	98·5	
8	Keeper	37 32 47·704	5·53282660	341056·71	64·6	+1·81
	Nephin	74 5 51·538	5·73100149	538271·62	101·9	
		180 0 40·120				

FIGURE 2—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		″ ′ °				″
9	Cuilcagh	52 5 58·287	5·43524110	272421·32	51·6	+2·88
	Nephin	46 50 19·546	5·40110340	251827·64	47·7	
	Slieve League	81 3 58·098	5·53282660	341056·71	64·6	
		180 0 15·931				
10	Cuilcagh	64 10 57·961	5·54500251	350753·90	66·4	+2·46
	Slieve Snaght	40 15 51·475	5·40110340	251827·64	47·7	
	Slieve League	75 33 30·670	5·57672864	377336·34	71·5	
		180 0 20·106				
11	Keeper	22 47 43·713	5·31492750	206503·54	39·1	−3·76
	Nephin	54 38 31·770	5·63819669	434707·06	82·3	
	Bencorr	102 34 5·118	5·71622432	520264·65	98·5	
		180 0 20·601				
12	Keeper	68 58 9·994	5·67626133	474527·44	89·9	−1·68
	Bencorr	52 16 23·595	5·60433034	402096·55	76·2	
	Baurtregaum	58 46 4·774	5·63819669	434707·06	82·3	
		180 0 38·363				
13	Keeper	103 7 55·176	5·70283550	504470·19	95·5	−2·51
	Baurtregaum	25 57 18·935	5·35545620	226702·44	42·9	
	Knockanaffrin	50 55 6·765	5·60433034	402096·55	76·2	
		180 0 20·876				
14	Keeper	68 45 19·695	5·63020198	426777·96	80·8	+7·16
	Kippure	29 40 39·868	5·35545620	226702·44	42·9	
	Knockanaffrin	81 34 22·941	5·65605572	452955·69	85·8	
		180 0 22·504				
15	Nephin	121 9 37·343	5·57018541	371693·88	70·4	Supplementary.
	Slieve League	19 59 55·616	5·17185971	148545·57	28·1	
	Slieve More	38 50 35·182	5·43524110	272421·32	51·6	
		180 0 8·141				
16	Nephin	63 15 39·803	5·28472087	192628·64	36·5	−7·50
	Bencorr	43 31 35·853	5·17185971	148545·57	28·1	
	Slieve More	73 12 50·785	5·31492750	206503·54	39·1	
		180 0 6·441				
17	Nephin	143 36 58·584	5·52591909	335675·07	63·6	Supplementary.
	Bencorr	14 58 53·019	5·16517958	146278·19	27·7	
	Knockalongy	21 24 12·610	5·31492750	206503·54	39·1	
		180 0 4·213				
18	Slieve League	27 34 5·638	5·16517958	146278·19	27·7	+5·73
	Nephin	31 57 44·271	5·22353502	167315·05	31·7	
	Knockalongy	120 28 15·050	5·43524110	272421·32	51·6	
		180 0 4·959				

FIGURE 2—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
19	Slieve League	53 29 52·460	5·30788538	203182·07	38·5	−5·95
	Cuilcagh........................	41 26 52·863	5·22353502	167315·05	31·7	
	Knockalongy	85 3 22·638	5·40110340	251827·64	47·7	
		180 0 7·961				
20	Nephin	64 41 30·619	5·15662611	143425·41	27·2	−3·86
	Slieve More	45 52 15·459	5·05643344	113876·32	21·6	
	Tawnaghmore	69 26 17·517	5·17185971	148545·57	28·1	
		180 0 3·595				
21	Slieve League	24 22 27·433	5·05643344	113876·32	21·6	+5·10
	Nephin	56 28 6·724	5·36175872	230016·36	43·6	
	Tawnaghmore	99 9 31·922	5·43524110	272421·32	51·6	
		180 0 6·079				
22	Slieve League	51 56 33·071	5·26221507	182900·57	34·6	+10·53
	Knockalongy..................	81 58 49·314	5·36175872	230016·36	43·6	
	Tawnaghmore	46 4 44·738	5·22353502	167315·05	31·7	
		180 0 7·123				
23	Slieve League	105 26 25·531	5·58390302	383621·57	72·7	Supplementary.
	Cuilcagh........................	35 18 28·889	5·36175872	230016·36	43·6	
	Tawnaghmore	39 15 18·705	5·40110340	251827·64	47·7	
		180 0 13·125				
24	Croghan........................	9 15 37·207	4·78225756	60570·00	11·5	+2·19
	Kippure........................	25 46 38·584	5·21401486	163687·25	31·0	
	Lyon's Hill	144 57 45·547	5·33464833	216096·80	40·9	
		180 0 1·338				
25	Kippure........................	86 8 49·445	5·02667635	106335·03	20·1	−1·19
	Howth	34 38 1·609	4·78225756	60570·00	11·5	
	Lyon's Hill	59 13 10·247	4·96171958	91562·91	17·3	
		180 0 1·301				
26	Kippure........................	46 16 19·602	4·74616279	55739·46	10·6	−1·63
	Lyon's Hill	81 59 1·559	4·88298225	76380·46	14·5	
	Dublin Observatory	51 44 39·625	4·78225756	60570·00	11·5	
		180 0 0·786				
27	Kippure........................	39 52 29·843	4·77099459	59019·37	11·2	−4·24
	Howth	56 4 5·691	4·88298225	76380·46	14·5	
	Dublin Observatory	84 3 25·520	4·96171958	91562·91	17·3	
		180 0 1·054				

FIGURE 3.

No. of Δ.	Names of Stations.	Corrected Angles.			Log. Distances.	Distances in		Error of Δ.
		°	′	″		Feet.	Miles.	″
1	Keeper	51	39	17·491	5·49906554	315548·08	59·8	+3·13
	Baurtregaum	36	19	48·031	5·37722386	238354·77	45·1	
	Knocknaskagh	92	1	12·154	5·60433034	402096·55	76·2	
		180	0	17·676				
2	Keeper	36	47	37·871	5·21113297	162604·65	30·8	+1·99
	Knocknaskagh	81	48	47·332	5·42931026	268726·35	50·9	
	Taur	61	23	43·818	5·37722386	238354·77	45·1	
		180	0	9·021				
3	Keeper	22	43	41·547	5·20361750	159814·99	30·3	−5·79
	Baurtregaum	53	41	37·208	5·52290041	333349·96	63·1	
	Caherbarnagh	103	34	53·423	5·60433034	402096·55	76·2	
		180	0	12·178				
4	Caherbarnagh	74	52	13·875	5·19910588	158163·36	30·0	−1·13
	Baurtregaum	27	51	37·325	4·88403598	76566·00	14·5	
	Taur	77	16	11·578	5·20361750	159814·99	30·3	
		180	0	2·778				
5	Caherbarnagh	65	52	5·807	5·27887967	190055·16	36·0	+0·17
	Baurtregaum	64	0	43·921	5·27230051	187197·70	35·5	
	Hungry Hill	50	7	16·694	5·20361750	159814·99	30·3	
		180	0	6·422				
6	Caherbarnagh	55	34	37·441	5·20264660	159458·11	30·2	+1·96
	Hungry Hill	48	52	14·009	5·16317630	145605·00	27·6	
	Carrigfadda	75	33	13·838	5·27230051	187197·70	35·5	
		180	0	5·288				
7	Caherbarnagh	92	13	32·116	5·35787605	227969·13	43·2	Supplementary.
	Carrigfadda	48	6	57·683	5·23006366	169849·26	32·2	
	Knocknaskagh	39	39	36·013	5·16317630	145605·00	27·6	
		180	0	5·812				
8	Caherbarnagh	37	52	39·805	5·06448245	116006·53	22·0	+3·01
	Knocknaskagh	78	6	15·029	5·26690429	184886·11	35·0	
	Doolieve...........................	64	1	9·700	5·23006366	169849·26	32·2	
		180	0	4·534				
9	Carrigfadda	27	44	48·646	5·06448245	116006·53	22·0	Supplementary.
	Knocknaskagh	38	26	39·016	5·19012015	154924·52	29·3	
	Doolieve	113	48	36·205	5·35787605	227969·13	43·2	
		180	0	3·867				
10	Caherbarnagh	57	10	33·062	5·34154065	219553·64	41·6	„
	Doolieve...........................	77	46	57·011	5·40714005	255352·46	48·4	
	Galtymore	45	2	39·257	5·26690429	184886·11	35·0	
		180	0	9·330				
11	Doolieve...........................	30	24	3·031	5·13499247	136455·95	25·8	„
	Galtymore	95	5	32·260	5·42909338	268592·19	50·9	
	Knockanaffrin	54	30	31·727	5·34154065	219553·64	41·6	
		180	0	7·018				

FIGURE 3—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
12	Keeper	68 26 60·645	5·57290763	374031·02	70·8	Supplementary.
	Baurtregaum	20 39 62·232	5·15206064	141925·57	26·9	
	Galtymore	90 53 9·605	5·60433034	402096·55	76·2	
		180 0 12·482				
13	Keeper	103 26 14·226	5·70342878	505159·79	95·7	”
	Kippure	15 51 35·444	5·15206064	141925·57	26·9	
	Galtymore	60 42 25·033	5·65605572	452955·69	85·8	
		180 0 14·703				
14	Knocknaskagh	103 37 21·166	5·33641560	216977·95	41·1	”
	Taur	29 37 55·249	5·04290510	110383·74	20·9	
	Galtymore	46 44 47·687	5·21113297	162604·65	30·8	
		180 0 4·102				
15	Doolieve	98 26 18·724	5·46632376	292633·31	55·4	”
	Caherbarnagh	42 52 61·041	5·30387912	201316·38	38·1	
	Knockmealdown	38 40 48·894	5·26690429	184886·11	35·0	
		180 0 8·659				
16	Taur	16 31 34·011	5·09453556	124318·44	23·6	”
	Knockmealdown	21 50 32·286	5·21113297	162604·65	30·8	
	Knockaskagh	141 37 56·653	5·43342053	271281·72	51·4	
		180 0 2·950				
17	Keeper	20 2 50·405	4·89217803	78014·98	14·8	”
	Knockanaffrin	64 53 51·783	5·31408139	206101·61	39·0	
	Knockmealdown	95 3 21·578	5·35545620	226702·44	42·9	
		180 0 3·766				
18	Taur	40 52 12·526	5·35545620	226702·44	42·9	+5·62
	Keeper	88 16 15·556	5·53946170	346307·34	65·6	
	Knockanaffrin	50 51 46·237	5·42931026	268726·35	50·9	
		180 0 14·319				
19	Caherbarnagh	141 19 23·399	5·63263909	429179·62	81·3	Supplementary.
	Baurtregaum	25 13 17·305	5·46632376	292633·31	55·4	
	Knockmealdown	13 27 26·170	5·20361750	159814·99	30·3	
		180 0 6·874				
20	Baurtregaum	84 40 47·623	5·67698034	475313·71	90·0	”
	Bencorr	11 34 43·677	4·98140024	95807·66	18·1	
	Brandon	83 44 39·345	5·67626133	474527·44	89·9	
		180 0 10·645				
21	Baurtregaum	98 50 46·474	5·35335987	225610·79	42·7	”
	Hungry Hill	24 48 36·836	4·98140024	95807·66	18·1	
	Brandon	56 20 40·922	5·27887967	190055·16	36·0	
		180 0 4·232				

FIGURE 3—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
22	Baurtregaum	51 19 16·454	4·95488778	90133·82	17·1	+8·18
	Brandon	72 36 2·591	5·04208481	110175·44	20·9	
	Knocknadober	56 4 42·893	4·98140024	95807·66	18·1	
		180 0 1·938				
23	Baurtregaum	47 31 30·020	5·15029738	141350·51	26·8	−1·61
	Hungry Hill	35 5 33·773	5·04208481	110175·44	20·9	
	Knocknadober	97 22 59·840	5·27887967	190055·16	36·0	
		180 0 3·633				
24	Baurtregaum	33 8 32·259	4·81339779	65072·54	12·3	−4·09
	Knocknadober	79 5 25·045	5·06771328	116872 76	22·1	
	Knocknagante	67 46 4·352	5·04208481	110175·44	20·9	
		180 0 1·656				
25	Baurtregaum	14 22 57·761	4·91458716	82146·14	15·6	−7·35
	Hungry Hill	20 41 44·547	5·06771328	116872·76	22·1	
	Knocknagante	144 55 18·990	5·27887967	190055·16	36·0	
		180 0 1·298				
26	Baurtregaum	36 36 6·460	4·96991340	93306·82	17·7	−3·80
	Knocknagante	95 4 53·559	5·19277728	155875·29	29·5	
	Feaghmaan	48 19 2·536	5·06771328	116872·76	22·1	
		180 0 2·555				
27	Baurtregaum	47 51 42·253	5·06415052	115917·90	22·0	Supplementary.
	Brandon	94 20 26·494	5·19277728	155875·29	29·5	
	Feaghmaan	37 47 53·858	4·98140024	95807·66	18·1	
		180 0 2·605				
28	Knocknadober	112 36 38·190	4·96991340	93306·82	17·7	−18·48
	Knocknagante	27 18 49·207	4·66632685	46379·58	8·8	
	Feaghmaan	40 4 33·258	4·81339779	65072·54	12·3	
		180 0 0·655				
29	Brandon	45 36 45·116	5·27230051	187197·70	35·5	Supplementary.
	Hungry Hill	74 55 53·530	5·40303105	252947·89	47·9	
	Caherbarnagh	59 27 30·947	5·35335987	225610·79	42·7	
		180 0 9·593				
30	Brandon	17 24 40·133	4·88403598	76566·00	14·5	"
	Caherbarnagh..................	81 16 48·736	5·40299659	252927·82	47·9	
	Taur	81 18 35·633	5·40303105	252947·89	47·9	
		180 0 4·502				

FIGURE 4.

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Feet.	Miles.	Error of Δ.
1	Slieve Donard	101 57 56·384	5·70396709	505786·34	95·8	+1·08
	Kippure	31 17 7·841	5·42889800	268471·48	50·8	
	South Berule..................	46 45 19·029	5·57587896	376598·83	71·3	
		180 0 23·254				

FIGURE 4—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
	Slieve Donard	100 43 26·353	5·65738687	454346·17	86·0	
2	Trostan	35 29 35·366	5·42889800	268471·48	50·8	Supplementary.
	South Berule	43 47 18·123	5·50512565	319982·08	60·6	
		180 0 19·842				
	Slieve Donard	94 23 6·459	5·61787735	414836·87	78·6	
3	South Berule	45 25 53·857	5·47186881	296393·59	56·1	„
	Howth	40 11 18·335	5·42889800	268471·48	50·8	
		180 0 18·651				
	Slieve Donard	59 12 33·636	5·69642725	497081·10	94·1	
4	Kippure	80 11 47·813	5·75603663	570212·36	108·0	−5·76
	Snowdon	40 36 21·921	5·57587896	376598·83	71·3	
		180 0 43·370				
	Kippure	54 55 5·491	5·65391288	450726·28	85·4	
5	Snowdon	9 34 12·035	4·96171958	91562·90	17·3	Supplementary.
	Howth	115 30 51·230	5·69642725	497081·10	94·1	
		180 0 8·756				
	Slieve Donard	55 33 27·006	5·73199581	539505·42	102·2	
6	Snowdon	63 48 46·282	5·76867715	587052·78	111·2	„
	Sca Fell	60 38 51·612	5·75603663	570212·36	108·0	
		180 0 64·900				
	Slieve Donard	38 23 29·264	5·56181329	364597·17	69·1	
7	Sca Fell	52 33 15·339	5·66849586	466117·98	88·3	„
	Merrick	89 3 55·341	5·76867715	587052·78	111·2	
		180 0 39·944				
	Slieve Donard	49 31 52·830	5·55025292	355020·08	67·2	
8	Trostan	87 11 11·499	5·66849586	466117·98	88·3	+5·14
	Merrick	43 17 22·341	5·50512565	319982·08	60·6	
		180 0 26·670				
	Knocklayd	75 40 14·648	5·57879591	379136·77	71·8	
9	Divis	72 22 65·235	5·57166410	372961·58	70·6	−8·79
	Merrick	31 56 57·702	5·31609989	207061·75	39·2	
		180 0 17·585				
	Merrick	53 15 67·768	5·52296532	333399·79	63·1	
10	Divis	61 2 27·428	5·56108339	363984·92	68·9	+1·84
	South Berule	65 41 50·800	5·57879591	379136·77	71·8	
		180 0 25·996				
	Snowdon	37 46 29·140	5·51936549	330647·69	62·6	
11	Sca Fell	50 16 60·076	5·61827517	415217·05	78·6	+5·70
	South Berule	91 56 63·043	5·73199581	539505·42	102·2	
		180 0 32·259				
	Snowdon	38 0 12·582	5·46199301	289729·69	54·9	
12	South Berule	80 4 35·246	5·66609198	463545·08	87·8	+0·84
	Black Comb	61 55 40·032	5·61827517	415217·05	78·6	
		180 0 27·860				

FIGURE 4—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
13	South Berule	74 58 55·338	5·60444615	402203·78	76·2	−1·35
	Merrick	44 5 19·572	5·46199301	289729·69	54·9	
	Black Comb	60 56 9·032	5·56108339	363984·92	68·9	
		180 0 23·942				
14	Slieve Donard	18 39 7·082	5·27979903	190457·92	36·1	Supplementary.
	Merrick	109 50 56·667	5·74834132	560197·70	106·1	
	Criffel	51 30 15·880	5·66849586	466117·98	88·3	
		180 0 19·629				
15	Slieve Donard	19 44 22·182	5·29753558	198397·22	37·6	„
	Sca Fell	72 28 12·844	5·74834132	560197·70	106·1	
	Criffel	87 47 51·081	5·76867715	587052·78	111·2	
		180 0 26·107				
16	South Berule	30 20 40·289	5·27979903	190457·92	36·1	−2·06
	Merrick	74 45 52·142	5·56081440	363759·55	68·9	
	Criffel	74 53 43·291	5·56108339	363984·92	68·9	
		180 0 15·722				
17	South Berule	44 38 15·049	5·41066981	257436·31	48·8	+7·94
	Criffel	52 15 20·994	5·46199301	289729·69	54·9	
	Black Comb	83 6 41·363	5·56081440	363759·55	68·9	
		180 0 17·406				
18	Snowdon	40 24 27·109	5·54661849	352061·47	66·7	+1·85
	Sca Fell	56 14 54·032	5·65474726	451593·05	85·5	
	Whittle	83 21 15·986	5·73199581	539505·42	102·2	
		180 0 37·127				
19	Snowdon	40 10 43·666	5·49764190	314515·39	59·6	+0·84
	Black Comb	67 52 10·962	5·65474726	451593·05	85·5	
	Whittle	71 57 37·121	5·66609198	463545·08	87·8	
		180 0 31·749				
20	Snowdon	32 5 21·632	5·41995099	262997·12	49·8	Supplementary.
	Black Comb	78 28 53·572	5·68585105	485122·09	91·9	
	Pendle Hill	69 26 12·879	5·66609198	463545·08	87·8	
		180 0 28·083				
21	Snowdon	32 18 65·075	5·46221444	289877·46	54·9	„
	Sca Fell	63 27 51·357	5·68585105	485122·09	91·9	
	Pendle Hill	84 13 36·461	5·73199581	539505·42	102·2	
		180 0 32·893				
22	Snowdon	29 34 39·388	5·38804552	244368·67	46·3	„
	Pendle Hill	71 57 18·571	5·67279104	470750·77	89·2	
	Axedge	78 28 28·541	5·68585105	485122·09	91·9	
		180 0 26·500				
23	Snowdon	21 29 17·353	5·23794415	172959·39	32·8	−5·12
	Whittle	85 30 21·331	5·67279104	470750·77	89·2	
	Axedge	73 0 39·623	5·65474726	451593·05	85·5	
		180 0 18·307				

FIGURE 4—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
24	Snowdon	26 32 11·174	5·30498411	201829·25	38·2	Supplementary.
	Whittle	65 7 52·996	5·61266289	409885·81	77·6	
	Mowcopt	88 20 15·272	5·65474726	451593·05	85·5	
		180 0 19·442				
25	Mowcopt	56 36 35·974	5·23794415	172959·39	32·8	−8·22
	Whittle	20 22 28·334	4·85805603	72120·05	13·7	
	Axedge	103 0 58·549	5·30498411	201829·25	38·2	
		180 0 2·857				
26	South Berule	73 20 20·844	5·73380915	541762·76	102·6	Supplementary.
	Snowdon	59 25 38·538	5·68742159	486879·61	92·2	
	Ingleborough	47 14 46·153	5·61827517	415217·05	78·6	
		180 0 45·535				
27	Snowdon	40 14 35·065	5·54975463	354612·98	67·2	”
	Axedge	80 43 18·447	5·73380915	541762·76	102·6	
	Ingleborough	59 2 45·221	5·67279104	470750·77	89·2	
		180 0 38·733				
28	Axedge	30 42 18·904	4·97940219	95367·89	18·1	−0·20
	Whittle	37 7 25·366	5·05200814	112721·86	21·3	
	Holme Moss	112 10 18·071	5·23794415	172959·39	32·8	
		180 0 2·341				
29	Holme Moss	54 20 3·441	5·27764803	189516·94	35·9	+4·33
	Whittle	101 32 7·157	5·35900153	228560·68	43·3	
	Great Whernside	24 7 53·565	4·97940219	95367·89	18·1	
		180 0 4·163				
30	Sea Fell	25 12 32·633	5·09613514	124777·17	23·6	+0·92
	Pendle Hill	73 7 8·441	5·44768611	280340·67	53·1	
	Great Whernside	81 40 27·062	5·46221444	289877·46	54·9	
		180 0 8·136				
31	Great Whernside	37 33 18·824	5·17696826	150303·21	28·5	+1·81
	Holme Moss	30 23 54·619	5·09613514	124777·17	23·6	
	Pendle Hill	112 2 50·644	5·35900153	228560·68	43·3	
		180 0 4·087				
32	Pendle Hill	31 30 3·952	4·97940219	95367·89	18·1	+1·89
	Holme Moss	23 56 8·822	4·86952130	74049·35	14·0	
	Whittle	124 33 48·593	5·17696826	150303·21	28·5	
		180 0 1·367				
33	Whittle	57 29 53·701	5·23228182	170718·99	32·3	Supplementary.
	Mowcopt	28 6 28·548	4·97940219	95367·89	18·1	
	Holme Moss	94 23 41·567	5·30498411	201829·25	38·2	
		180 0 3·816				
34	Great Whernside	60 13 6·942	5·04472446	110847·13	21·0	”
	Pendle Hill	42 5 42·333	4·93255067	85615·16	16·2	
	Ingleborough	77 41 12·905	5·09613514	124777·17	23·6	
		180 0 2·180				

FIGURE 4—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
35	Sca Fell	23 33 15·862	5·26642265	184681·18	35·0	Supplementary.
	Whittle	26 4 6·868	5·30768300	203087·41	38·5	
	Ingleborough	130 22 43·987	5·54661849	352061·47	66·7	
		180 0 6·717				
36	Sca Fell	78 24 26·517	5·30541592	202030·03	38·3	„
	Ingleborough	21 37 11·698	4·88073675	75986·55	14·4	
	Black Comb	79 58 25·338	5·30768300	203087·41	38·5	
		180 0 3·553				

FIGURE 5.

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
1	Sca Fell	39 41 47·141	5·38595101	243192·97	46·1	−5·35
	Merrick	67 3 48·821	5·54488140	350656·10	66·4	
	Hart Fell	73 14 43·230	5·56181329	364597·17	69·1	
		180 0 19·192				
2	Sca Fell	36 56 29·804	5·41838451	262050·21	49·6	−0·28
	Hart Fell	89 31 53·433	5·63951494	436028·56	82·6	
	Cheviot	53 31 58·359	5·54488140	350656·10	66·4	
		180 0 21·596				
3	Sca Fell	65 34 23·775	5·50654047	321026·19	60·8	+3·43
	Hart Fell	30 25 54·700	5·25184402	178584·61	33·8	
	Cross Fell	83 59 54·925	5·54488140	350656·10	66·4	
		180 0 13·400				
4	Cross Fell	50 20 10·790	5·41838451	262050·21	49·6	−1·62
	Hart Fell	59 5 58·732	5·46552539	292095·85	55·3	
	Cheviot	70 34 7·443	5·50654047	321026·19	60·8	
		180 0 16·965				
5	Cheviot	22 11 22·577	5·04429778	110738·28	21·0	+4·54
	Cross Fell	62 47 33·977	5·41627822	260782·37	49·4	
	Collier Law	95 1 10·207	5·46552539	292095·85	55·3	
		180 0 6·761				
6	Cheviot	25 38 38·770	5·07787782	119640·39	22·7	−1·39
	Collier Law	83 44 42·245	5·43902784	274807·03	52·0	
	Wordeslow	70 36 46·273	5·41627822	260782·37	49·4	
		180 0 7·288				
7	Cross Fell	95 24 46·768	5·65369439	450499·58	85·4	Supplementary.
	Cheviot	44 23 18·165	5·50041761	316531·99	59·9	
	Botton Head	40 12 16·701	5·46552539	292095·85	55·3	
		180 0 21·634				
8	Cheviot	9 48 58·809	4·89099059	77801·97	14·7	„
	Botton Head	70 58 52·141	5·63494110	431460·56	81·7	
	Easington	99 12 16·839	5·65369439	450499·58	85·3	
		180 0 7·789				

FIGURE 5—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
	Botton Head	44 54 18·496	5·34957552	223653·41	42·4	
9	Cross Fell	42 38 5·731	5·33160691	214588·73	40·6	−2·15
	Great Whernside	92 27 47·045	5·50041761	316531·99	59·9	
		180 0 11·272				
	Sca Fell	52 51 18·143	5·34957552	223653·41	42·4	
10	Great Whernside	39 31 49·451	5·25184402	178584·61	33·8	−1·94
	Cross Fell	87 36 61·786	5·44768610	280340·67	53·1	
		180 0 9·380				
	Botton Head	90 27 52·478	5·44598710	279246·09	52·9	
11	Great Whernside	39 19 25·542	5·24787858	176961·41	33·5	Supplementary.
	Wordeslow	50 12 50·906	5·33160691	214588·73	40·6	
		180 0 8·926				
	Botton Head	62 48 27·791	5·32042514	209134·24	39·6	
12	Wordeslow	68 22 27·960	5·33959240	218570·93	41·4	−2·02
	Water Crag	48 49 12·336	5·24787858	176961·41	33·5	
		180 0 8·087				
	Botton Head	27 39 24·687	5·01536908	103602·22	19·6	
13	Great Whernside	78 18 40·502	5·33959240	218570·93	41·4	+1·51
	Water Crag	74 1 59·929	5·33160691	214588·73	40·6	
		180 0 5·118				
	Wordeslow	94 26 39·466	5·43743696	273802·22	51·9	
14	Water Crag	35 57 31·926	5·20752595	161259·74	30·5	+0·68
	Easington	49 35 56·512	5·32042514	209134·24	39·6	
		180 0 7·904				
	Water Crag	66 32 34·273	5·40089214	251705·17	47·7	
15	Easington	27 10 61·262	5·09811393	125347·00	23·7	+1·96
	Collier Law	86 16 31·866	5·43743696	273802·22	51·9	
		180 0 7·401				
	Water Crag	52 19 57·450	5·04429778	110738·28	21·0	
16	Collier Law	64 1 53·810	5·09958514	125772·34	23·8	−1·39
	Cross Fell	63 38 11·673	5·09811393	125347·00	23·7	
		180 0 2·933				
	Cross Fell	99 14 8·636	5·36986766	234351·46	44·4	
17	Sca Fell	31 59 14·714	5·09958514	125772·34	23·8	+2·75
	Water Crag	48 46 41·861	5·25184402	178584·61	33·8	
		180 0 5·211				
	Cross Fell	22 20 50·261	4·93255067	85615·16	16·2	
18	Great Whernside	60 59 9·572	5·29428158	196916·26	37·3	Supplementary.
	Ingleborough	96 40 4·103	5·34957552	223653·41	42·4	
		180 0 3·936				
	Cross Fell	65 16 11·525	5·30768300	203087·41	38·5	
19	Sca Fell	61 43 32·240	5·29428158	196916·26	37·3	”
	Ingleborough	53 0 23·743	5·25184402	178584·61	33·8	
		180 0 7·508				

FIGURE 5—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
20	Sca Fell	55 40 44·474	5·50229433	317902·78	60·2	Supplementary.
	Merrick	53 1 50·609	5·48789407	307534·66	58·2	
	Wisp Hill	71 17 46·681	5·56181329	364597·17	69·1	
		180 0 21·764				
21	Wisp Hill	35 20 26·746	5·25184402	178584·61	33·8	”
	Sca Fell	49 35 26·442	5·37122258	235083·73	44·5	
	Cross Fell	95 4 16·641	5·48789407	307534·66	58·2	
		180 0 9·829				
22	Cross Fell	39 15 49·073	5·26735043	185076·14	35·1	−3·83
	Cheviot	53 30 11·483	5·37122258	235083·73	44·5	
	Wisp Hill	87 14 9·658	5·46552539	292095·85	55·3	
		180 0 10·214				
23	Wisp Hill	50 36 16·471	5·23766058	172846·50	32·7	+1·54
	Cheviot	73 33 31·168	5·33147340	214522·77	40·6	
	Sayrs Law	55 50 19·571	5·26735043	185076·14	35·1	
		180 0 7·210				
24	Cheviot	56 29 35·209	5·34303506	220310·43	41·7	−1·66
	Hart Fell	40 51 28·395	5·23766058	172846·50	32·7	
	Sayrs Law	82 39 5·271	5·41838451	262050·21	49·6	
		180 0 8·875				
25	Cheviot	85 8 34·105	5·54337483	349441·78	66·2	+1·84
	Cross Fell	38 27 47·776	5·33872431	218134·48	41·3	
	Dunrich	56 23 53·040	5·46552539	292095·85	55·3	
		180 0 14·921				
26	Cheviot	41 55 8·547	5·16467961	146109·89	27·7	+0·49
	Sayrs Law	85 51 58·790	5·33872431	218134·48	41·3	
	Dunrich	52 12 58·582	5·23766058	172846·50	32·7	
		180 0 5·919				
27	Cheviot	31 38 22·622	5·05853271	114428·11	21·7	+1·25
	Dunrich	58 2 32·970	5·26735043	185076·14	35·1	
	Wisp Hill	90 19 9·385	5·33872431	218134·48	41·3	
		180 0 4·977				
28	Wisp Hill	40 4 33·311	4·87436661	74880·13	14·2	+1·09
	Dunrich	60 14 39·212	5·00420915	100973·90	19·1	
	Hart Fell	79 40 49·225	5·05853271	114428·11	21·7	
		180 0 1·748				
29	Wisp Hill	73 41 34·357	5·40816348	255954·92	48·5	−2·81
	Cross Fell	44 29 4·562	5·27153407	186867·63	35·4	
	Criffel	61 49 30·990	5·37122258	235083·73	44·5	
		180 0 9·909				
30	Cross Fell	50 35 12·079	5·29753558	198397·22	37·6	−2·33
	Criffel	44 3 42·811	5·25184402	178584·61	33·8	
	Sca Fell	85 21 13·410	5·40816348	255954·92	48·5	
		180 0 8·300				

FIGURE 5—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
31	Wisp Hill	68 40 33·290	5·24840969	177177·96	33·6	−0·16
	Criffel	32 3 55·949	5·00420915	100973·90	19·1	
	Hart Fell	79 15 34·892	5·27153407	186867·63	35·4	
		180 0 4·131				
32	Hart Fell	103 40 37·931	5·64955355	446224·64	84·5	−4·77
	Cross Fell	31 58 36·254	5·38595101	243192·97	46·1	
	Merrick	44 20 63·644	5·50654047	321026·19	60·8	
		180 0 17·829				
33	Hart Fell	50 58 37·082	5·27979904	190457·92	36·1	+0·54
	Merrick	46 16 47·495	5·24840969	177177·96	33·6	
	Criffel	82 44 43·290	5·38595101	243192·97	46·1	
		180 0 7·867				
34	Criffel	87 40 29·630	5·54337483	349441·78	66·2	−6·86
	Cross Fell	45 17 5·859	5·39535683	248517·42	47·1	
	Dunrich	47 2 39·448	5·40816348	255954·92	48·5	
		180 0 14·937				
35	Cheviot	75 58 14·130	5·23216557	170673·29	32·3	+2·82
	Sayrs Law	24 45 26·793	4·86729673	73671·03	14·0	
	Blackheddon	79 16 21·980	5·23766058	172846·50	32·7	
		180 0 2·903				
36	Sayrs Law	110 37 25·584	5·41639562	260852·87	49·4	−0·06
	Blackheddon	31 36 60·734	5·16467961	146109·89	27·7	
	Dunrich	37 45 39·167	5·23216557	170673·29	32·3	
		180 0 548·5				
37	Sayrs Law	39 25 54·889	5·05541698	113610·11	21·5	−0·02
	Blackheddon	33 9 24·962	4·99046774	97829·03	18·5	
	Lumsden	107 24 42·641	5·23216557	170673·29	32·3	
		180 0 2·492				
38	Sayrs Law	64 11 21·6 2	5·19653829	157231·04	29·8	+0·03
	Cheviot	34 3 53·370	4·99046774	97829·03	18·5	
	Lumsden	81 44 48·525	5·23766058	172846·50	32·7	
		180 0 3·577				
39	Botton Head	32 13 5·133	5·09811393	125347·00	23·7	+0·03
	Water Crag	79 24 14·683	5·36380680	231103·64	43·8	
	Collier Law	68 22 46·514	5·33959240	218570·93	41·4	
		180 0 6·330				
40	Wordeslow	11 18 46·521	4·62611735	422782·84	8·0	+2·27
	Easington	37 7 26·607	5·11420130	130077·24	24·6	
	Burleigh Moor	131 33 47·839	5·20752595	161259·74	30·5	
		180 0 0·967				

FIGURE 5—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
41	Wordeslow	115 20 39·996	5·32445841	211085·50	40·0	Supplementary.
	Collier Law	33 50 36·080	5·11420130	130077·24	24·6	
	Burleigh Moor	30 48 47·230	5·07787783	119640·39	22·7	
		180 0 3·306				
42	Wordeslow	96 13 38·257	5·17838689	150794·98	28·6	„
	Burleigh Moor	24 44 1·612	4·80254840	63467·06	12·0	
	Brandon Down	59 2 22·060	5·11420130	130077·24	24·6	
		180 0 1·929				
43	Wordeslow	107 32 24·779	5·27936422	190267·33	36·0	„
	Easington	18 32 44·903	4·80254840	63467·06	12·0	
	Brandon Down	53 54 52·612	5·20752595	161259·74	30·5	
		180 0 2·294				
44	Wordeslow	46 39 3·372	4·94236764	87572·48	16·6	−2·92
	Collier Law	36 47 50·636	4·85813977	72133·96	13·7	
	Merrington Church	96 33 7·467	5·07787783	119640·39	22·7	
		180 0 1·475				
45	Collier Law	80 24 23·100	5·14747460	140434·75	26·6	+8·91
	Water Crag	37 56 31·103	4·94236764	87572·48	16·6	
	Merrington Church	61 39 8·341	5·09811393	125347·00	23·7	
		180 0 2·544				
46	Collier Law	97 59 58·267	5·17031457	148018·01	28·0	+3·01
	Water Crag	25 0 31·170	4·80064730	63189·85	12·0	
	Brandon Down	56 59 32·407	5·09811393	125347·00	23·7	
		180 0 1·844				
47	Water Crag	12 55 59·933	4·52308453	33349·131	6·3	+11·88
	Brandon Down	70 28 30·509	5·14747460	140434·754	26·6	
	Merrington Church	96 35 30·651	5·17031457	148018·013	28·0	
		180 0 1·093				
48	Brandon Down	71 57 57·872	4·52084335	33177·477	6·3	Supplementary.
	Merrington Church	35 8 2·256	4·30275839	20079·754	3·8	
	Durham Observatory	72 53 60·022	4·52308453	33349·131	6·3	
		180 0 ·150				
49	Merrington Church	70 4 24·567	4·91591090	82396·905	15·6	„
	Collier Law	22 14 37·693	4·52084335	33177·477	6·3	
	Durham Observatory	87 40 58·382	4·94236764	87572·478	16·6	
		180 0 0·642				
50	Cheviot	45 51 36·599	5·09357859	124044·81	23·5	−0·79
	Sayrs Law	44 34 23·498	5·08389610	121309·86	23·0	
	Mordington	89 34 3·439	5·23766058	172846·50	32·7	
		180 0 3·536				

FIGURE 5—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
51	Dunrich	22 37 16·828	5·09357859	124044·81	23·5	+4·73
	Sayrs Law	130 26 22·288	5·38996909	245453·42	46·5	
	Mordington	26 56 24·125	5·16467961	146109·89	27·7	
		180 0 3·241				
52	Wisp	27 14 41·834	5·09357859	124044·81	23·5	Supplementary.
	Sayrs Law	100 24 43·068	5·42570541	266505·03	50·5	
	Mordington	52 20 41·247	5·33147340	214522·77	40·6	
		180 0 6·149				
53	Cheviot	30 6 37·531	4·83518610	68420·49	13·0	+6·66
	Mordington	32 41 41·692	4·86729673	73671·03	14·0	
	Blackheddon	117 11 41·830	5·08389610	121309·86	23·0	
		180 0 1·053				
54	Merrick	27 27 45·4420	5·06871642	117143·02	22·2	Supplementary.
	Hart Fell	79 19 32·1676	5·39728255	249621·82	47·3	
	Burnswark	73 12 48·9703	5·38595101	243192·97	46·1	
		180 0 6·5799				
55	Merrick	39 35 63·3789	5·37014383	234500·53	44·4	”
	Sca Fell	42 43 46·3578	5·39728255	249621·82	47·3	
	Burnswark	97 40 23·8977	5·56181329	364597·17	69·1	
		180 0 13·6344				
56	Merrick	16 53 18·2019	5·34179186	219680·68	41·6	”
	Cross Fell	19 16 29·6726	5·39728255	249621·82	47·3	
	Burnswark	143 50 19·7305	5·64955355	446224·64	84·5	
		180 0 7·6050				
57	Merrick	13 25 47·2301	4·97716804	94878·55	18·0	”
	Wisp	37 39 52·5122	5·39728255	249621·82	47·3	
	Burnswark	128 54 24·5894	5·50229433	317902·78	60·2	
		180 0 4·3317				
58	Merrick	18 49 2·0531	4·96680087	92640·49	17·5	”
	Burnswark	41 32 17·3722	5·27979904	190457·92	36·1	
	Criffel	119 38 44·1786	5·39728255	249621·82	47·3	
		180 0 3·6039				

FIGURE 6.

1	Fair Isle	45 46 32·409	5·26057665	182211·86	34·5	+3·54
	Foula	67 39 7·723	5·37138595	235172·18	44·5	
	Brassa	66 34 29·099	5·36793827	233312·64	44·2	
		180 0 9·231				

FIGURE 6—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.			Log. Distances.	Distances in		Error of Δ.
		°	′	″		Feet.	Miles.	″
	Brassa	65	21	32·598	5·26945531	185975·32	35·2	
2	Foula	51	42	2·991	5·20567031	160572·18	30·4	+2·20
	Ronas	62	56	30·654	5·26057665	182211·86	34·5	
		180	0	6·243				
	Foula	119	21	10·715	5·55956106	362711·28	68·7	
3	Fair Isle	26	32	47·869	5·26945531	185975·32	35·2	Supplementary.
	Ronas	34	6	10·295	5·36793827	233312·64	44·2	
		180	0	8·879				
	Ronas	70	54	44·116	5·18244724	152211·42	28·8	
4	Brassa	23	34	54·210	4·80912521	64435·50	12·2	−0·10
	Yell	85	30	23·969	5·20567031	160572·18	30·4	
		180	0	2·295				
	Ronas	133	51	14·770	5·37153330	235251·99	44·6	
5	Foula	11	23	30·008	4·80912521	64435·50	12·2	+2·45
	Yell	34	45	17·250	5·26945531	185975·32	35·2	
		180	0	2·028				
	Brassa	36	9	14·478	5·03844808	109256·70	20·7	
6	Ronas	83	43	48·803	5·26502522	184087·89	34·9	−2·97
	Fetlar	60	6	60·813	5·20567031	160572·18	30·4	
		180	0	4·094				
	Fetlar	43	0	10·101	5·18244724	152211·420	28·8	
7	Brassa	12	34	20·269	4·68643895	48577·924	9·2	−0·70
	Yell	124	25	31·062	5·26502522	184087·889	34·9	
		180	0	1·432				
	Brassa	33	10	62·367	5·18362746	152625·63	28·9	
8	Ronas	111	39	36·007	5·41358413	259169·64	49·1	−3·78
	Saxavord	35	9	26·973	5·20567031	160572·18	30·4	
		180	0	5·347				
	Fetlar	109	41	48·661	5·18362746	152625·63	28·9	
9	Ronas	27	55	47·204	4·88041605	75930·46	14·4	+0·66
	Saxavord	42	22	25·969	5·03844808	109256·70	20·7	
		180	0	1·834				
	Saxavord	22	3	20·248	4·80912521	64435·50	12·2	
10	Ronas	40	44	51·891	5·04924262	112006·34	21·2	−7·30
	Yell	117	11	49·368	5·18362746	152625·63	28·9	
		180	0	1·507				
	Saxavord	42	45	1·693	4·96615055	92501·878	17·5	
11	Yell	12	31	43·186	4·47071871	29560·972	5·6	+3·46
	Balta	124	43	15·649	5·04924262	112006·347	21·2	
		180	0	0·528				
	Saxavord	22	25	55·973	4·69808366	49898·060	9·5	
12	Fetlar	13	3	55·748	4·47071871	29560·972	5·6	−2·89
	Balta	144	30	8·480	4·88041605	75930·463	14·4	
		180	0	0·201				

FIGURE 6—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
13	Saxavord	52 37 8·644	4·37988539	23981·999	4·5	+21·12
	Balta	25 45 3·199	4·11767665	13112·233	2·5	
	Gerth of Scaw	101 37 48·229	4·47071871	29560·972	5·6	
		180 0 0·072				
14	Saxavord	33 34 46·310	3·95553067	9026·734	1·7	+19·20
	Gerth of Scaw	92 57 43·698	4·21215114	16298·632	3·1	
	Nive Hill	53 27 30·020	4·11767665	13112·233	2·5	
		180 0 0·028				
15	Saxavord	19 2 22·334	4·17954288	15119·690	2·9	−8·32
	Balta	20 35 19·365	4·21215114	16298·632	3·1	
	Nive Hill	140 22 18·338	4·47071871	29560·972	5·6	
		180 0 0·037				

FIGURE 7.

1	Ben Clibrig	90 9 20·248	5·36738588	233016·07	44·1	+0·41
	Ben Cheilt.....................	30 9 44·166	5·06847423	117077·71	22·2	
	Ben Hutig......................	59 40 61·116	5·30352254	201151·16	38·1	
		180 0 5·530				
2	Ben Cheilt.....................	68 59 55·733	5·34611286	221877·30	42·0	Supplementary.
	Ben Hutig......................	32 21 15·504	5·10443731	127185·41	24·1	
	Dunnet Head	78 38 55·260	5·36738588	233016·07	44·1	
		180 0 6·497				
3	Dunnet Head	64 47 9·415	5·33083134	214205·85	40·6	+3·88
	Ben Hutig......................	45 38 32·831	5·22861358	169283·09	32·1	
	Scarabin........................	69 34 25·733	5·34611286	221877·30	42·0	
		180 0 7·979				
4	Dunnet Head	27 22 3·970	5·06847423	117077·71	22·2	+1·57
	Ben Hutig......................	92 2 16·620	5·40573309	254526·55	48·2	
	Ben Clibrig	60 35 45·506	5·34611286	221877·30	42·0	
		180 0 6·096				
5	Ben Clibrig	88 59 52·329	5·46853012	294123·77	55·7	+0·17
	Dunnet Head	31 5 37·420	5·18160562	151916·74	28·8	
	Fashven.........................	59 54 39·330	5·40573309	254526·55	48·2	
		180 0 9·079				
6	Dunnet Head..................	104 43 12·514	5·40975785	256896·30	48·7	Supplementary.
	Ben Hutig......................	18 37 46·422	4·92864206	84848·09	16·1	
	Wart Hill Hoy..............	56 39 5·340	5·34611286	221877·30	42·0	
		180 0 4·276				
7	Dunnet Head	100 59 39·063	5·50689488	321288·28	60·9	−3·37
	Fashven.........................	15 1 31·954	4·92864206	84848·09	16·1	
	Wart Hill Hoy..............	63 58 54·735	5·46853012	294123·77	55·7	
		180 0 5·752				

FIGURE 7—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
8	Wart Hill Hoy	27 30 2·548	5·18160562	151916·74	28·8	−0·62
	Fashven	74 56 11·284	5·50201710	317699·88	60·2	
	Ben Clibrig	77 33 57·236	5·50689488	321288·28	60·9	
		180 0 11·068				
9	Wart Hill Hoy	29 29 11·284	5·19895844	158109·76	29·9	+0·93
	Ben Clibrig	52 0 58·883	5·40343638	253184·07	48·0	
	Scarabin	98 29 59·131	5·50201701	317699·88	60·2	
		180 0 9·298				
10	Wart Hill Hoy	38 30 7·281	5·30352254	201151·16	38·1	+2·05
	Ben Clibrig	40 59 29·835	5·32622427	211945·53	40·1	
	Ben Cheilt	100 30 32·727	5·50201701	317699·88	60·2	
		180 0 10·923				
11	Wart Hill Hoy	64 13 13·742	4·96021802	91246·88	17·3	+3·41
	Dunnet Head	58 55 15·349	4·93845147	86786·36	16·4	
	Ronaldshay, South	56 51 32·466	4·92864206	84848·09	16·1	
		180 0 1·557				
12	Wart Hill Hoy	71 12 54·646	5·37976783	239755·08	45·4	−5·88
	Scarabin	20 2 30·293	4·93845147	86786·36	16·4	
	Ronaldshay, South	88 44 39·946	5·40343638	253184·07	48·0	
		180 0 4·885				
13	Ben Cheilt	94 27 57·437	5·49205537	310495·54	58·8	+3·53
	Ben Hutig	37 6 8·372	5·27385631	187869·51	35·6	
	Ronaldshay, South	48 26 4·439	5·36738588	233016·07	44·1	
		180 0 10·248				
14	Ronaldshay, South	19 43 16·941	5·06847423	117077·71	22·2	−3·74
	Ben Hutig	96 47 9·488	5·53723354	344535·16	65·3	
	Ben Clibrig	63 29 42·048	5·49205537	310495·54	58·8	
		180 0 8·477				
15	Ronaldshay, South	90 15 34·817	5·05291349	112957·09	21·4	−2·76
	Wart Hill Hoy	39 32 19·315	4·85678282	71908·93	13·6	
	Deerness	50 12 7·334	4·93845147	86786·36	16·4	
		180 0 1·466				
16	Deerness	31 45 29·358	4·92864206	84848·09	16·1	Supplementary.
	Wart Hill Hoy	103 45 33·057	5·19473787	156580·56	29·7	
	Dunnet Head	44 28 59·771	5·05291349	112957·09	21·4	
		180 0 2·186				
17	Deerness	78 12 45·105	5·18942889	154678·12	29·3	−0·36
	Wart Hill Hoy	56 9 19·650	5·11805079	131235·34	24·9	
	Fitty Hill	45 37 58·652	5·05291349	112957·09	21·4	
		180 0 3·407				
18	Fitty Hill	22 58 29·368	5·40975785	256896·30	48·7	Supplementary.
	Wart Hill Hoy	143 26 1·952	5·59340827	392110·33	74·3	
	Ben Hutig	13 35 34·239	5·18942889	154678·12	29·3	
		180 0 5·559				

FIGURE 7—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
	Fitty Hill	11 55 48·461	5·06847423	117077·71	22·2	
19	Ben Hutig	124 15 37·280	5·67036490	468128·30	88·7	Supplementary.
	Ben Clibrig	43 48 43·168	5·59340827	392110·33	74·3	
		180 0 8·909				
	Fitty Hill	29 36 49·318	4·81239323	64922·20	12·3	
20	Deerness	57 40 36·069	5·04541548	111023·65	21·0	−0·75
	Stronsay	92 42 36·303	5·11805079	131235·34	24·9	
		180 0 1·690				
	Stronsay	64 24 38·636	5·18942889	154678·12	29·3	
21	Fitty Hill	75 14 47·970	5·21970505	165846·02	31·4	−3·32
	Wart Hill Hoy	40 20 37·293	5·04541548	111023·65	21·0	
		180 0 3·899				
	Stronsay	75 30 51·888	5·06858792	117108·36	22·2	
22	Fitty Hill	37 51 51·980	4·87064032	74240·40	14·1	Supplementary.
	Start Lighthouse	66 37 18·006	5·04541548	111023·65	21·0	
		180 0 1·874				
	Start Lighthouse	38 39 42·341	5·18942889	154678·12	29·3	
23	Fitty Hill	113 6 39·950	5·35741459	227727·03	43·1	„
	Wart Hill Hoy	28 13 41·622	5·06858792	117108·36	22·2	
		180 0 3·913				
	Fitty Hill	60 7 43·545	5·37541190	237362·39	45·0	
24	Stronsay	95 56 38·004	5·43498100	272258·22	51·6	+3·68
	Fair Isle	23 55 44·605	5·04541548	111023·65	21·0	
		180 0 6·154				
	Fair Isle	15 8 58·233	5·06858792	117108·36	22·2	
25	Fitty Hill	22 15 51·566	5·22988839	169780·73	32·2	Supplementary.
	Start Lighthouse	142 35 13·037	5·43498100	272258·22	51·6	
		180 0 2·836				
	Fair Isle	88 34 33·084	5·54914673	354116·96	67·1	
26	Fitty Hill	41 11 55·155	5·36793827	233312·64	44·2	−2·27
	Foula	50 13 46·670	5·43498100	272258·22	51·6	
		180 0 14·909				
	Foula	31 5 2·611	5·22988839	169780·73	32·2	
27	Fair Isle	103 43 31·318	5·50442033	319462·82	60·5	Supplementary.
	Start Lighthouse	45 11 35·105	5·36793827	233312·64	44·2	
		180 0 9·034				
	Foula	32 9 3·228	5·17479109	149551·61	28·3	
28	Fair Isle	91 43 59·798	5·44856687	280909·79	53·2	„
	North Ronaldshay Lt.-house	56 7 5·163	5·36793827	233312·64	44·2	
		180 0 8·189				
	Stronsay	67 24 23·412	5·09064342	123209·28	23·3	
29	Fitty Hill	56 17 43·338	5·04539781	111019·13	21·0	„
	North Ronaldshay Lt.-house	56 17 55·922	5·04541548	111023·65	21·0	
		180 0 2·672				

FIGURE 7—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.			Log. Distances.	Distances in		Error of Δ.
		°	′	″		Feet.	Miles.	″
30	North Ronaldshay Lt.-house	27	8	48·752	5·18942889	154678·12	29·3	Supplementary.
	Fitty Hill	131	32	31·308	5·40438480	253737·60	48·1	
	Wart Hill Hoy	21	18	43·290	5·09064342	123209·28	23·3	
		180	0	3·350				

FIGURE 8.

No. of Δ.	Names of Stations.	Corrected Angles.			Log. Distances.	Distances in		Error of Δ.
		°	′	″		Feet.	Miles.	″
1	Scournalapich	55	1	37·730	5·56589632	368041·10	69·7	−1·35
	Ben Clibrig	75	37	45·849	5·63859075	435101·67	82·4	
	Monach	49	21	4·957	5·53246544	340773·21	64·5	
		180	0	28·536				
2	Ben Clibrig	44	19	15·314	5·44751575	280230·72	53·1	+1·76
	Monach	22	15	26·978	5·18160563	151916·74	28·8	
	Fashven	113	25	26·881	5·56589632	368041·10	69·7	
		180	0	9·173				
3	Monach	55	26	42·484	5·42795235	267887·44	50·7	Supplementary.
	Fashven	65	4	14·218	5·46977217	294966·15	55·9	
	North Rona	59	29	19·285	5·44751575	280230·72	53·1	
		180	0	15·987				
4	Monach	52	51	20·257	5·40045073	251449·48	47·6	”
	North Rona	57	54	38·462	5·42692701	267255·72	50·6	
	Cnoc-Ghiubhais	69	14	16·037	5·46977217	294966·15	55·8	
		180	0	14·756				
5	Fashven	79	59	22·250	5·66644394	463920·90	87·9	”
	North Rona	65	21	34·149	5·63163719	428190·68	81·1	
	Cleisham	34	39	30·130	5·42795235	267887·44	50·7	
		180	0	26·529				
6	North Rona	11	16	30·669	4·95784865	90750·42	17·2	”
	Cnoc-Ghiubhais	135	55	26·332	5·50903727	322877·12	61·2	
	Ben Hutig	32	48	6·726	5·40045073	251449·48	47·6	
		180	0	3·728				
7	Ben Hutig	4	41	31·903	4·25345221	17924·71	33·9	−10·32
	Cnoc-Ghiubhais	19	46	26·264	4·87000040	74131·09	14·0	
	Fashven	155	32	1·962	4·95784865	90750·42	17·2	
		180	0	0·129				
8	Cnoc-Ghiubhais	14	23	33·840	5·23842310	173152·39	32·8	Supplementary.
	Monach	143	2	55·783	5·62197495	418769·41	79·3	
	Cleisham	22	33	36·910	5·42692701	267255·72	50·6	
		180	0	6·533				
9	Cleisham	46	4	58·742	5·49487002	312514·39	59·2	”
	Cnoc-Ghiubhais	28	46	7·680	5·31971505	208792·58	39·5	
	Ru Rea	105	9	8·370	5·62197495	418769·41	79·3	
		180	0	14·792				

FIGURE 8—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
10	Cleisham	68 38 35·653	5·33716823	217354·30	41·2	—0·64
	Monach	63 27 46·990	5·31971505	208792·58	39·5	
	Ru Rea	47 53 45·265	5·23842310	173150·24	32·8	
		180 0 7·908				
11	Cleisham	40 49 18·700	5·16171442	145115·71	27·5	—0·02
	Ru Rea	69 2 9·412	5·31658955	207295·35	39·3	
	Storr	70 8 38·533	5·31971505	208792·58	39·5	
		180 0 6·645				
12	Cleisham	57 2 19·071	5·37556491	237446·03	45·0	—4·21
	Storr	75 52 4·812	5·43844005	274435·35	52·0	
	Ben More, S. Uist	47 5 47·328	5·31658955	207295·35	39·3	
		180 0 11·211				
13	Cleisham	78 46 54·338	5·64708656	443697·07	84·0	—0·11
	Ben More, S. Uist	63 52 19·044	5·60864501	406111·24	76·9	
	Scournalapich	37 21 12·296	5·43844005	274435·35	52·0	
		180 0 25·678				
14	Cleisham	74 5 36·480	5·63385620	430384·08	81·5	+0·97
	Ben More, S. Uist	68 5 16·559	5·61824448	415187·71	78·6	
	Mamsuil	37 49 32·699	5·43844005	274435·35	52·0	
		180 0 25·738				
15	Mamsuil	22 10 29·604	5·37556491	237446·03	45·0	+0·97
	Ben More, S. Uist	20 59 29·231	5·35288219	225362·78	42·7	
	Storr	136 50 9·764	5·63385620	430384·08	81·5	
		180 0 8·599				
16	Cnoc-Ghiubhais	26 55 35·561	5·31658955	207295·35	39·3	Supplementary.
	Cleisham	86 54 17·442	5·66003063	457120·43	86·6	
	Storr	66 10 27·356	5·62197495	418769·41	79·3	
		180 0 20·359				
17	Ben More, S. Uist	25 29 6·924	5·32107834	209449·02	39·7	+2·22
	Scournalapich	88 48 51·349	5·68726718	486706·53	92·2	
	Ben Nevis	65 42 23·553	5·64708656	443697·07	84·0	
		180 0 21·826				
18	Ben More, S. Uist	30 2 52·102	5·38877379	244778·79	46·4	—3·71
	Ben Nevis	54 32 55·283	5·60014207	398237·43	75·4	
	Ben More, Mull	95 24 35·413	5·68726718	486706·53	92·2	
		180 0 22·798				
19	Storr	46 39 45·341	5·48115202	302797·31	57·4	+0·01
	Ben More, S. Uist	98 33 60·793	5·61456360	411683·63	78·0	
	Ben Heynish	34 49 30·568	5·37556491	237446·03	45·0	
		180 0 16·702				
20	Storr	26 16 53·837	5·26569757	184373·10	34·9	—3·04
	Ben Heynish	72 22 12·519	5·59863433	396857·26	75·2	
	Ben More, Mull	81 21 10·636	5·61456360	411683·63	78·0	
		180 0 16·992				

FIGURE 8—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
	Storr	26 16 23·684	5·25055482	178055·26	33·7	
21	Mamsuil	119 39 19·684	5·54353474	349570·47	66·2	+1·18
	Ben Nevis	34 4 24·824	5·35288219	225362·78	42·7	
		180 0 8·192				
	Ben Nevis	24 25 45·473	5·28072430	190864·12	36·2	
22	Ben More, Mull	123 32 29·349	5·58509190	384673·19	72·9	+0·94
	Jura	32 1 54·327	5·38877379	244778·79	46·4	
		180 0 9·149				
	Ben More, Mull	19 46 27·730	5·04594334	111158·67	21·1	
23	Jura	124 42 46·633	5·43150942	270090·57	51·1	+3·54
	Ben Tartavil	35 30 49·735	5·28072430	190864·12	36·2	
		180 0 4·098				
	Ben More, Mull	74 40 28·113	5·45321443	283932·06	53·8	
24	Ben Tartavil	38 46 33·324	5·26569758	184373·10	34·9	−0·73
	Ben Heynish	66 33 9·847	5·43150942	270090·57	51·1	
		180 0 11·284				
	Ben Nevis	78 58 40·755	5·74794014	559680·39	106·0	
25	Ben More, S. Uist	42 25 43·455	5·58509190	384673·19	72·9	−0·42
	Jura	58 36 18·964	5·68726718	486706·53	92·2	
		180 0 43·174				
	Mamsuil	59 43 41·956	5·60014207	398237·43	75·4	
26	Ben More, S. Uist	51 19 1·511	5·55624002	359948·21	68·2	Supplementary.
	Ben More, Mull	68 57 47·962	5·63385620	430384·08	81·5	
		180 0 31·429				
	Ben Clibrig	44 35 30·134	5·49310155	311244·40	59·0	
27	Monach	79 17 65·465	5·63913345	435645·72	82·5	−2·78
	Storr	56 6 50·837	5·56589633	368041·11	69·7	
		180 0 26·436				
	Jura	73 53 29·532	5·60880215	406258·21	76·9	
28	Ben Nevis	40 38 64·847	5·44006641	275464·99	52·2	−0·62
	Ben Heynish	65 27 49·539	5·58509190	384673·19	72·9	
		180 0 23·918				

FIGURE 9.

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
	Ben Lawers	25 59 43·963	5·28072428	190864·11	36·1	
1	Jura	56 58 8·260	5·56240606	365095·15	69·1	−2·22
	Ben More, Mull	97 2 24·026	5·63568972	432204·94	81·9	
		180 0 16·249				
	Ben Lawers	36 47 39·289	5·38877377	244778·78	46·4	
2	Ben More, Mull	26 30 5·323	5·26093366	182361·71	34·5	+3·94
	Ben Nevis	116 42 24·756	5·56240606	365095·15	69·1	
		180 0 9·368				

FIGURE 9—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
	Ben Lawers	89 4 21·329	5·45432436	284658·63	53·9	
3	Ben Nevis	51 5 47·053	5·34546930	221548·75	42·0	+0·68
	Ben Macdui	39 50 1·108	5·26093366	182361·71	34·5	
		180 0 9·490				
	Ben Lawers	24 56 44·210	4·97220047	93799·49	17·8	
4	Ben Macdui	70 5 2·833	5·32036048	209103·11	39·6	−1·63
	Glashmeal	84 58 17·547	5·34546930	221548·75	42·0	
		180 0 4·590				
	Ben Nevis	76 23 34·549	5·47441148	298133·98	56·5	
5	Ben Macdui	35 29 3·975	5·25055480	178055·26	33·7	−0·38
	Mamsuil..................	68 7 33·049	5·45432436	284658·63	53·9	
		180 0 11·573				
	Ben Macdui	42 11 9·501	5·32107832	209449·01	39·7	
6	Ben Nevis	71 56 29·354	5·47207618	296535·15	56·2	+1·36
	Scournalapich	65 52 34·460	5·45432436	284658·63	53·9	
		180 0 13·315				
	Ben Macdui	110 41 34·864	5·56598393	368115·35	69·7	
7	Scournalapich	20 24 24·217	5·13735653	137200·77	26·0	+0·49
	Corryhabbie	48 54 9·858	5·47207618	296535·15	56·2	
		180 0 8·939				
	Mamsuil..................	33 27 19·346	5·32036048	209103·11	39·6	
8	Ben Lawers	88 5 58·987	5·57876005	379105·47	71·8	Supplementary.
	Glashmeal	58 26 57·535	5·50952261	323238·15	61·2	
		180 0 15·868				
	Corryhabbie	71 29 11·826	5·45614422	285853·96	54·1	
9	Ben Macdui	81 26 31·016	5·47436003	298098·66	56·5	−0·16
	Ben Wyvis	27 4 26·268	5·13735653	137200·77	26·0	
		180 0 9·110				
	Ben Wyvis..................	81 11 27·627	5·61442657	411553·75	77·9	
10	Corryhabbie	53 6 22·859	5·52252507	333061·99	63·1	−0·70
	Ben Nevis	45 42 32·560	5·47436003	298098·66	56·5	
		180 0 23·046				
	Corryhabbie	43 28 54·526	5·10806024	128250·85	24·3	
11	Glashmeal	66 3 47·392	5·23133844	170348·55	32·3	+1·04
	Mount Battock	70 27 22·918	5·24462501	175640·64	33·3	
		180 0 4·836				
	Corryhabbie	80 40 19·424	5·44233125	276905·29	52·4	
12	Mount Battock..................	61 57 11·494	5·39385588	247660·01	46·9	−0·08
	Mormonth	37 22 38·861	5·23133844	170348·55	32·3	
		180 0 9·779				
	Mount Battock..................	31 46 41·142	5·16394039	145861·40	27·6	
13	Mormonth	56 55 22·781	5·36565127	232087·24	44·0	−1·86
	Knock	91 18 4·027	5·44233125	276905·29	52·4	
		180 0 7·950				

FIGURE 9—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
14	Corryhabbie	108 28 52·985	5·55047146	355198·78	67·3	Supplementary.
	Ben Wyvis	18 46 24·802	5·08107412	120524·16	22·8	
	Knock	52 44 50·217	5·47436003	298098·66	56·5	
		180 0 8·004				
15	Mormonth	48 1 30·905	5·46788099	293684·48	55·6	"
	Knock	110 18 29·680	5·56877197	370486·14	70·2	
	Ben Cheilt	21 40 8·851	5·16394039	145861·40	27·6	
		180 0 9·436				
16	Ben Cheilt	7 24 1·748	4·62977101	42635·465	8·1	"
	Knock	55 7 28·735	5·43387808	271567·680	51·4	
	Cowhythe	117 28 31·930	5·46788099	293684·476	55·6	
		180 0 2·413				
17	Ben Cheilt	25 24 28·264	5·21084747	162497·79	30·8	"
	Cowhythe	108 47 5·048	5·55457586	358571·58	67·9	
	Corryhabbie	45 48 36·501	5·43387808	271567·68	51·4	
		180 0 9·813				
18	Corryhabbie	56 38 6·498	5·44947621	281498·58	53·3	"
	Knock	102 24 52·034	5·51742292	329172·02	62·3	
	Scarabin	20 57 9·251	5·08107412	120524·16	22·8	
		180 0 7·783				
19	Cowhythe	11 41 10·258	4·74046625	55013·12	10·4	"
	Ben Cheilt	79 18 25·241	5·42633525	266891·81	50·5	
	Scarabin	89 0 27·949	5·43387808	271567·68	51·4	
		180 0 3·448				
20	Ben Macdui	40 35 34·981	5·44033099	275632·86	52·2	+1·47
	Ben Wyvis	96 58 21·713	5·62375481	420489·17	79·6	
	Scarabin	42 26 21·675	5·45614422	285853·96	54·1	
		180 0 18·369				
21	Ben Macdui	20 0 34·795	5·31359330	205870·11	39·0	+0·91
	Ben Wyvis	131 37 35·276	5·65297290	449751·79	85·2	
	Ben Clibrig	28 22 0·261	5·45614422	285853·96	54·1	
		180 0 10·332				
22	Ben Macdui	25 29 59·948	5·30352254	201151·16	38·1	Supplementary.
	Ben Clibrig	80 14 35·046	5·66323901	460509·95	87·2	
	Ben Cheilt	74 15 45·947	5·65297290	449751·79	85·2	
		180 0 20·941				
23	Corryhabbie	22 10 41·437	5·19895844	158109·67	29·9	−1·21
	Scarabin	106 1 23·948	5·60486868	402595·28	76·2	
	Ben Clibrig	51 48 6·365	5·51742292	329172·02	62·3	
		180 0 11·750				
24	Ben Clibrig	12 52 56·459	5·16853260	147411·92	27·9	+1·40
	Ben Wyvis	148 58 38·609	5·53246544	340773·21	64·5	
	Scournalapich	18 8 28·606	5·31359330	205870·11	39·0	
		180 0 3·674				

FIGURE 9—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
25	Ben Macdui	6 42 5·526	4·54149240	34793·042	6·6	+4·28
	Mamsuil	84 1 12·924	5·47207618	296535·147	56·2	
	Scournalapich	89 16 43·973	5·47441148	298133·981	56·5	
		180 0 2·423				
26	Ben Macdui	35 57 9·374	5·25673050	180605·30	34·2	+0·45
	Mamsuil	68 18 61·754	5·45614422	285853·96	54·1	
	Ben Wyvis	75 44 0·624	5·47441148	298133·98	56·5	
		180 0 11·752				
27	Corryhabbie	75 28 33·206	5·27887092	190051·34	36·0	−0·55
	Mount Battock	44 20 4·759	5·13735653	137200·77	26·0	
	Ben Macdui	60 11 27·350	5·23133844	170348·55	32·3	
		180 0 5·315				
28	Mamsuil	26 42 40·808	5·24462501	175640·64	33·3	Supplementary.
	Glashmeal	77 19 32·638	5·58120488	381245·64	72·2	
	Corryhabbie	75 58 1·816	5·57876005	379105·47	71·8	
		180 0 15·262				
29	Corryhabbie	80 31 9·058	5·57740724	377926·41	71·6	+1·73
	Scarabin	40 16 9·671	5·39385588	247660·01	46·9	
	Mormonth	59 13 0·157	5·51742292	329172·02	62·3	
		180 0 18·886				
30	Ben Lawers	62 42 45·841	5·55624000	359948·20	68·2	Supplementary.
	Ben More, Mull	52 56 52·773	5·50952261	323238·15	61·2	
	Mamsuil	64 20 46·024	5·56240606	365095·15	69·1	
		180 0 24·638				
31	Scarabin	80 11 18·935	5·50373052	318955·81	60·4	+0·54
	Ben Wyvis	41 26 7·708	5·33083135	214205·86	40·6	
	Ben Hutig	58 22 47·020	5·44033099	275632·86	52·2	
		180 0 13·663				

FIGURE 10.

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Feet.	Miles.	Error of Δ.
1	Mount Battock	114 20 18·822	5·54283857	349010·56	66·1	Supplementary.
	Ben Macdui	35 55 2·869	5·35160461	224700·80	42·6	
	Dudwick	29 44 47·449	5·27887092	190051·34	36·0	
		180 0 9·140				
2	Mount Battock	70 0 14·062	5·36348448	230932·19	43·7	−2·35
	Corryhabbie	66 6 53·981	5·35160461	224700·80	42·6	
	Dudwick	43 53 0·407	5·23133844	170348·55	32·3	
		180 0 8·450				
3	Mount Battock	39 49 43·710	5·19240428	155741·47	29·5	−1·60
	Dudwick	72 38 32·810	5·36565127	232087·24	44·0	
	Knock	67 31 51·328	5·35160461	224700·80	42·6	
		180 0 7·848				

FIGURE 10—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
4	Knock	23 46 12·698	4·79839123	62862·44	11·9	−5·32
	Dudwick	69 16 4·677	5·16394039	145861·40	27·6	
	Mormonth	86 57 44·774	5·19240428	155741·47	29·5	
		180 0 2·149				
5	Mount Battock	36 9 60·760	5·27081059	186556·59	35·3	−2·57
	Mormonth	24 59 22·104	5·12563052	133545·89	25·3	
	Blue Hill	118 50 42·264	5·44233125	276905·29	52·4	
		180 0 5·128				
6	Mount Battock	28 6 58·191	5·09363745	124061·62	23·5	+4·15
	Dudwick	30 28 59·885	5·12563052	133545·89	25·3	
	Blue Hill	121 24 5·246	5·35160461	224700·80	42·6	
		180 0 3·322				
7	Blue Hill	19 17 3·424	4·83326885	68119·09	12·9	+2·92
	Mormonth	45 27 58·502	5·16741210	147032·08	27·8	
	Little Stirling	115 15 0·202	5·27081059	186556·59	35·3	
		180 0 2·128				
8	Mormonth	40 24 58·613	4·65813756	45513·220	8·6	−0·23
	Dudwick	76 0 51·599	4·83326885	68119·092	12·9	
	Little Stirling	63 34 10·440	4·79839123	62862·440	11·9	
		180 0 0·652				
9	Mormonth	73 6 46·589	5·26493660	184050·33	34·9	Supplementary.
	Knock	57 34 14·210	5·21044796	162348·38	30·7	
	Brimmond	49 19 4·525	5·16394039	145861·40	27·6	
		180 0 5·324				
10	Mormonth	7 55 7·998	4·60126469	39926·82	7·6	”
	Brimmond	26 9 1·455	5·10628071	127726·41	24·2	
	Over Hill	145 55 51·218	5·21044796	162348·38	30·7	
		180 0 0·671				
11	Knock	33 48 1·511	5·01040525	102424·83	19·4	”
	Brimmond	57 45 57·637	5·19240428	155741·47	29·5	
	Dudwick	88 26 4·599	5·26493660	184050·33	34·9	
		180 0 3·747				
12	Brimmond	17 42 8·342	4·81639473	65523·14	12·4	−1·59
	Dudwick	10 40 40·433	4·60126469	39926·82	7·6	
	Over Hill	151 37 11·517	5·01040525	102424·83	19·4	
		180 0 0·292				
13	Brimmond	43 44 57·080	4·90423596	80211·38	15·2	−0·49
	Dudwick	18 15 28·227	4·56039291	36340·67	6·9	
	Tarbathy	117 59 35·298	5·01040525	102424·83	19·4	
		180 0 0·605				

FIGURE 10—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
14	Brimmond	26 2 48·738	4·24398945	17538·38	3·3	+0·81
	Over Hill	65 29 7·932	4·56039291	36340·67	6·9	
	Tarbathy	88 28 3·480	4·60126469	39926·82	7·6	
		180 0 0·150				
15	Brimmond	23 51 19·571	4·73965280	54910·17	10·4	+1·94
	Dudwick	25 6 60·021	4·76064885	57630·03	10·9	
	Layton	131 1 40·969	5·01040525	102424·83	19·4	
		180 0 0·561				
16	Over Hill	94 56 21·553	4·42353693	26517·766	5·0	−0·41
	Tarbathy	43 50 33·601	4·26568559	18436·802	3·5	
	Layton	41 13 4·922	4·24398945	17538·379	3·3	
		180 0 0·076				
17	Tarbathy	24 39 15·408	4·65813756	45513·220	8·6	Supplementary.
	Dudwick	108 1 30·898	5·01599919	103752·647	19·6	
	Little Stirling	47 19 14·510	4·90423596	80211·375	15·2	
		180 0 0·816				
18	Over Hill	25 43 25·097	4·65813756	45513·220	8·6	„
	Dudwick	115 36 18·693	4·97572518	94563·858	17·9	
	Little Stirling	38 40 16·842	4·81639473	65523·145	12·4	
		180 0 0·632				
19	Brimmond	119 11 44·540	5·09363745	124061·62	23·5	„
	Dudwick	14 41 28·096	4·55680497	36041·68	6·8	
	Blue Hill	46 6 48·121	5·01040525	102424·83	19·4	
		180 0 0·757				
20	Mount Battock	9 58 32·569	4·81639473	65523·14	12·4	„
	Dudwick	26 28 12·222	5·22684577	168595·42	31·9	
	Over Hill	143 33 16·751	5·35160461	224700·80	42·6	
		180 0 1·542				
21	Over Hill	44 55 32·186	5·12563052	133545·89	25·3	„
	Mount Battock	18 8 25·623	4·76995304	58878·00	11·2	
	Blue Hill	116 56 3·838	5·22684577	168595·42	31·9	
		180 0 1·647				
22	Tarbathy	51 58 14·958	4·55680497	36041·68	6·8	„
	Brimmond	75 26 47·460	4·64628237	44287·62	8·4	
	Blue Hill	52 34 57·880	4·56039291	36340·67	6·9	
		180 0 0·298				
23	Layton	30 28 26·413	4·55680497	36041·68	6·8	„
	Brimmond	95 20 24·969	4·84978226	70759·09	13·4	
	Blue Hill	54 11 9·104	4·76064885	57630·03	10·9	
		180 0 0·486				
24	Over Hill	8 3 54·766	5·11121217	129185·02	24·5	„
	Brimmond	169 26 58·520	5·22684579	168595·43	31·9	
	Mount Battock	2 29 6·936	4·60126469	39926·82	7·6	
		180 0 0·222				

FIGURE 10—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
25	Tarbathy	39 51 29·829	5·12563053	133545·89	25·3	Supplementary.
	Blue Hill	127 52 15·004	5·21614322	164491·41	31·2	
	Mount Battock	12 16 16·264	4·64628237	44287·62	8·4	
		180 0 1·097				
26	Mormonth	15 15 30·512	4·29618278	19778·019	3·7	−12·74
	Little Stirling	49 45 23·245	4·75863866	57363·898	10·9	
	Peterhead Windmill	114 59 6·485	4·83326885	68119·092	12·9	
		180 0 0·242				
27	Mormonth	9 5 1·330	4·09114661	12335·212	2·3	Supplementary.
	Little Stirling	51 35 25·719	4·78691679	61223·307	11·6	
	Reform Monument	119 19 33·106	4·83326885	68119·092	12·9	
		180 0 0·155				
28	Mormonth	6 10 29·182	3·87271433	7459·579	1·4	"
	Peterhead Windmill	118 1 7·620	4·78691679	61223·307	11·6	
	Reform Monument	55 48 23·287	4·75863866	57363·898	10·9	
		180 0 0·089				
29	Little Stirling	58 30 53·009	4·24967090	17769·324	3·4	−1·65
	Peterhead Windmill	13 8 22·283	3·67548028	4736·748	0·9	
	Sector Station, Great Stirling	108 20 44·727	4·29618278	19778·019	3·7	
		180 0 0·019				
30	Little Stirling	56 40 50·535	4·02149213	10507·324	2·0	Supplementary.
	Reform Monument	22 7 46·426	3·67548028	4736·748	0·9	
	Sector Station	101 11 23·050	4·09114661	12335·212	2·3	
		180 0 0·011				

FIGURE 11.

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
1	Slieve Snaght	57 24 11·175	5·61166782	408947·75	77·5	Supplementary.
	Divis	76 29 53·716	5·67394491	472003·16	89·4	
	Goat Fell	46 6 27·798	5·54382111	349801·06	66·2	
		180 0 32·689				
2	Slieve Snaght	23 22 34·154	5·29102774	195446·43	37·0	"
	Goat Fell	49 59 18·204	5·57669069	377303·37	71·5	
	Jura	106 38 24·245	5·67394491	472003·16	89·4	
		180 0 16·603				
3	Slieve Donard	24 46 67·631	5·36704874	232835·25	44·1	"
	Merrick	98 9 58·144	5·74021994	549819·25	104·1	
	Goat Fell	57 3 19·474	5·66849586	466117·98	88·3	
		180 0 25·249				

FIGURE 11—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
4	South Berule	48 17 43·623	5·61366598	410833·62	77·8	Supplementary.
	Criffel	90 20 3·669	5·74060123	550302·15	104·2	
	Goat Fell	41 22 47·834	5·56081440	363759·55	68·9	
		180 0 35·126				
5	Knocklayd	38 32 59·263	5·36704874	232835·25	44·1	+12·47
	Merrick	48 1 57·200	5·44372440	277794·98	52·6	
	Goat Fell	93 25 18·709	5·57166410	372961·58	70·6	
		180 0 15·172				
6	Trostan	40 44 44·948	5·36704874	232835·25	44·1	−4·74
	Merrick	54 52 35·803	5·46504540	291773·20	55·3	
	Goat Fell	84 22 55·137	5·55025292	355020·08	67·2	
		180 0 15·888				
7	Slieve Snaght	49 23 29·150	5·32166699	209733·11	39·7	−1·22
	Knocklayd	75 26 43·511	5·42716461	267401·98	50·6	
	Ben Tartavil	55 9 58·157	5·35557056	226762·15	42·9	
		180 0 10·818				
8	Slieve Snaght	58 45 31·786	5·40744775	255533·44	48·4	−2·72
	Trostan	63 28 12·053	5·42716461	267401·98	50·6	
	Ben Tartavil	57 46 29·745	5·40283536	252833·93	47·9	
		180 0 13·584				
9	Trostan	78 7 24·923	5·62640027	423058·35	80·1	+1·15
	Merrick	46 40 32·215	5·49760615	314489·50	59·6	
	Jura	55 12 28·538	5·55025292	355020·08	67·2	
		180 0 25·676				
10	Knocklayd	79 56 32·189	5·62640027	423058·35	80·1	+15·39
	Merrick	39 49 53·612	5·43965052	275201·32	52·1	
	Jura	60 13 57·948	5·57166410	372961·58	70·6	
		180 0 23·749				
11	Slieve Snaght	47 6 22·163	5·51315835	325955·52	61·7	−1·23
	Jura	10 53 29·866	4·92461702	84065·35	15·9	
	Mount Sandy	122 0 13·432	5·57669069	377303·37	71·5	
		180 0 5·461				
12	Merrick	53 42 37·218	5·63568972	432204·94	81·9	−1·48
	Jura	74 12 39·198	5·71264570	515995·24	97·7	
	Ben Lawers	52 5 24·929	5·62640027	423058·35	80·1	
		180 0 41·345				
13	Merrick	39 2 17·670	5·56240606	365095·15	69·1	Supplementary.
	Ben Lawers	78 5 8·891	5·75374975	567217·67	107·4	
	Benmore, Mull	62 53 16·752	5·71264570	515995·24	97·7	
		180 0 43·313				
14	Jura	51 55 23·849	5·37281476	235947·16	44·7	+3·57
	Goat Fell	87 22 55·160	5·47628959	299426·06	56·7	
	Ben Lomond	40 41 51·816	5·29102774	195446·43	37·0	
		180 0 10·825				

FIGURE 11—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
·15	Ben More, Mull	67 21 19·808	5·59101976	389959·73	73·9	+0·08
	Ben Lawers	52 52 30·395	5·52748700	336889·13	63·8	
	Goat Fell	59 46 36·468	5·56240606	365095·15	69·1	
		180 0 26·671				
16	Merrick	43 9 54·062	5·47628959	299426·06	56·7	Supplementary.
	Jura	61 42 26·032	5·58592621	385412·87	73·0	
	Ben Lomond..................	75 7 66·118	5·62640027	423058·35	80·1	
		180 0 26·212				
17	Ben More, Mull	50 33 54·408	5·36908218	233927·98	44·3	+2·40
	Ben Lomond..................	53 54 63·358	5·38877377	244778·78	46·4	
	Ben Nevis......................	75 31 15·257	5·46725744	293263·11	55·5	
		180 0 13·023				
18	Ben Lawers	53 19 7·829	5·57919191	379482·64	71·9	−2·26
	Goat Fell	71 11 30·558	5·65120997	447929·82	84·8	
	Hart Fell	55 29 54·528	5·59101976	389959·73	73·9	
		180 0 32·915				
19	Ben Lawers	29 22 59·461	5·34303505	220310·43	41·7	+1·14
	Hart Fell	64 39 42·750	5·60835552	405840·63	76·9	
	Sayrs Law.....................	85 57 38·746	5·65120997	447929·82	84·8	
		180 0 20·957				
20	Ben Lawers	98 33 21·697	5·69059228	490447·23	92·9	Supplementary.
	Sayrs Law.....................	26 31 63·399	5·34546930	221548·75	42·0	
	Ben Macdui	54 54 55·793	5·60835552	405840·63	76·9	
		180 0 20·889				
21	Ben Macdui	52 10 51·384	5·60522155	402922·52	76·3	,,
	Sayrs Law.....................	21 52 39·050	5·27887091	190051·33	36·0	
	Mount Battock	105 56 46·865	5·69059228	490447·23	92·9	
		180 0 17·299				
22	Ben Lawers	73 36 37·488	5·60278467	400668·01	75·9	,,
	Sayrs Law.....................	30 2 47·162	5·32036047	209103·10	39·6	
	Glashmeal	76 20 54·478	5·60835552	405840·63	76·9	
		180 0 19·128				
23	Glashmeal	81 48 47·957	5·60522155	402922·52	76·3	,,
	Sayrs Law.....................	18 21 55·287	5·10806024	128250·85	24·3	
	Mount Battock	79 49 28·706	5·60278467	400668·01	75·9	
		180 0 11·950				
24	Mount Battock	39 7 13·900	5·40924840	256595·12	48·6	+0·73
	Sayrs Law.....................	58 41 35·718	5·54091993	347472·10	65·8	
	Ben Cleugh	82 11 31·136	5·60522155	402922·52	76·3	
		180 0 20·754				
25	Mount Battock	14 2 6·896	4·99046774	97829·03	18·5	+0·72
	Sayrs Law.....................	78 47 11·851	5·59737566	395708·76	74·9	
	Lumsden	87 10 50·337	5·60522155	402922·52	76·3	
		180 0 9·084				

FIGURE 11—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
	Goat Fell	24 13 61·169	5·20427464	160056·99	30·3	
26	Ben Lawers	66 4 31·836	5·55200635	356456·34	67·5	−3·68
	Ben Cleugh	89 41 40·399	5·59101976	389959·73	73·9	
		180 0 13·404				
	Sayrs Law	75 40 45·477	5·46828172	293955·59	55·7	
27	Ben Cleugh	46 34 4·615	5·34303505	220310·43	41·7	+2·17
	Hart Fell	57 45 22·779	5·40924840	256595·12	48·6	
		180 0 12·871				
	Goat Fell	72 30 29·854	5·58195056	381900·79	72·3	
28	Hart Fell	36 6 20·206	5·37281476	235947·16	44·7	−6·12
	Ben Lomond	71 23 30·007	5·57919191	379482·64	71·9	
		180 0 20·067				
	Ben Lomond	11 18 23·364	4·87436661	74880·13	14·2	
29	Hart Fell	77 46 20·931	5·57204007	373285·60	70·7	+2·60
	Dunrich	90 55 22·272	5·58195056	381900·79	72·3	
		180 0 6·567				
	Ben Lomond	36 25 36·299	5·40510357	254157·88	48·1	
30	Dunrich	24 16 40·849	5·24547749	175985·75	33·3	+1·16
	Ben Cleugh	119 17 52·018	5·57204007	373284·60.	70·7	
		180 0 9·166				
	Ben Lomond	20 5 20·058	5·16467961	146109·89	27·7	
31	Dunrich	98 34 26·964	5·62392543	420654·40	79·7	Supplementary.
	Sayrs Law	61 20 25·651	5·57204007	373284·60	70·7	
		180 0 12·673				
	Ben Cleugh	32 39 1·971	5·27887091	190051·33	36·0	
32	Mount Battock	66 49 32·965	5·51034299	323849·32	61·3	−1·09
	Ben Macdui	80 31 39·326	5·54091993	347472·10	65·8	
		180 0 14·262				
	Ben Cleugh	51 10 50·184	5·45432437	284658·64	53·9	
33	Ben Macdui	66 24 8·958	5·52479754	334809·32	63·4	+1·33
	Ben Nevis	62 25 20·704	5·51034299	323849·32	61·3	
		180 0 19·846				
	Glashmeal	39 0 10·737	5·40924840	256595·12	48·6	
34	Sayrs Law	40 19 40·432	5·42136168	263852·78	50·0	Supplementary.
	Ben Cleugh	100 40 24·463	5·60278467	400668·01	75·9	
		180 0 15·632				
	Glashmeal	13 13 54·686	4·99046774	97829·03	18·5	
35	Sayrs Law	97 9 7·138	5·62747262	424104·25	80·3	„
	Lumsden	69 37 7·314	5·60278467	400668·01	75·9	
		180 0 9·138				
	Knocklayd	49 33 70·707	5·58592621	385412·87	73·0	
36	Merrick	82 59 47·674	5·70119509	502568·30	95·2	„
	Ben Lomond	47 26 35·146	5·57166410	372961·58	70·6	
		180 0 33·527				

FIGURE 11—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
37	Ben More, Mull	43 17 30·723	5·37281476	235947·16	44·7	+4·32
	Goat Fell	58 27 37·172	5·46725744	293263·11	55·5	
	Ben Lomond	78 15 8·023	5·52748700	336889·13	63·8	
		180 0 15·918				
38	Ben Lomond	40 59 3·006	5·38897096	244889·95	46·4	−2·19
	Dunrich	50 28 14·673	5·45939307	288000·38	54·5	
	East Lomond	88 32 58·889	5·57204007	373284·60	70·7	
		180 0 16·568				
39	Ben Lawers	63 33 4·963	5·36616147	232360·05	44·0	−1·99
	Glashmeal	62 46 16·644	5·36316942	230764·72	43·7	
	East Lomond	53 40 48·543	5·32036047	209103·10	39·6	
		180 0 10·150				
40	Mount Battock	36 49 57·711	5·38152139	240725·11	45·6	−5·23
	Lumsden	43 22 9·407	5·44051503	275749·69	52·2	
	East Lomond	99 48 8·250	5·59737566	395708·76	74·9	
		180 0 15·368				
41	Mount Battock	22 47 50·815	5·26272065	183113·62	34·7	−8·67
	Sayrs Law	35 41 46·010	5·44051503	275749·69	52·2	
	East Lomond	121 30 33·289	5·60522155	402922·52	76·3	
		180 0 10·114				
42	Dunrich	26 11 33·824	5·05475167	113436·21	21·5	−5·08
	Ben Cleugh	72 20 25·063	5·38897096	244889·95	46·4	
	East Lomond	81 28 7·569	5·40510357	254157·88	48·1	
		180 0 6·456				
43	Mount Battock	17 52 19·024	5·23295834	170985·12	32·4	−1·80
	Lumsden	27 22 52·299	5·40865187	256242·91	48·5	
	Kellie Law	134 44 55·988	5·59737566	395708·76	74·9	
		180 0 7·311				
44	Mount Battock	18 57 38·687	4·95283913	89709·64	17·0	−2·01
	Kellie Law	92 54 18·103	5·44051503	275749·69	52·2	
	East Lomond	68 8 8·603	5·40865187	256242·91	48·5	
		180 0 5·393				
45	Mount Battock	75 59 16·578	5·41040858	257281·51	48·7	+4·11
	Kellie Law	28 55 29·234	5·10806021	128250·83	24·3	
	Glashmeal	75 5 21·679	5·40865187	256242·91	48·5	
		180 0 7·491				

FIGURE 11—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
	Kellie Law	34 46 24·426	4·99046774	97829·03	18·5	
46	Lumsden	59 47 58·037	5·17099072	148248·64	28·1	+1·08
	Sayrs Law	85 25 40·934	5·23295834	170985·12	32·4	
		180 0 3·397				
	Ben Lomond	21 1 52·482	5·14556310	139818·00	26·5	
47	Dunrich	52 19 24·052	5·48906584	308365·54	58·4	+4·91
	Calton Hill	106 38 53·173	5·57204007	373284·60	70·7	
		180 0 9·707				
	Ben Lomond	72 56 21·402	5·47942685	301596·88	57·1	
48	Calton Hill	29 15 17·971	5·18800034	154170·14	29·2	Supplementary.
	Ben Lawers	77 48 31·306	5·48906584	308365·54	58·4	
		180 0 10·679				
	Ben Cleugh	45 38 57·376	5·02218970	105242·15	19·9	
49	East Lomond	83 55 49·252	5·16539890	146352·09	27·7	−1·73
	Calton Hill	50 25 16·161	5·05475167	113436·21	21·5	
		180 0 2·789				
	Calton Hill	41 33 20·721	4·95283913	89709·64	17·0	
50	East Lomond	87 20 55·303	5·13063390	135093·33	25·6	+4·10
	Kellie Law	51 5 46·192	5·02218970	105242·15	19·9	
		180 0 2·216				

FIGURE 12.

	Precelly	54 9 6·902	5·51774790	329418·44	62·4	
1	Cradle	59 22 27·467	5·54371596	349716·37	66·2	−2·40
	Paracombe	66 28 50·476	5·57129508	372644·82	70·6	
		180 0 24·845				
	Precelly	32 7 28·868	5·27016409	186279·09	35·3	
2	Paracombe	54 31 33·026	5·45527895	285285·01	54·0	+1·79
	Lundy Island	93 21 10·584	5·54371596	349716·37	66·2	
		180 0 12·478				
	Lundy Island	86 50 43·672	5·43731551	273725·66	51·8	
3	Paracombe	50 21 5·897	5·32444565	211079·30	40·0	−1·71
	Brown Willy	42 48 19·667	5·27016409	186279·09	35·3	
		180 0 9·236				
	Lundy Island	50 4 1·739	5·26086364	182332·31	34·5	
4	Paracombe	78 21 45·188	5·36716600	232898·13	44·1	−2·91
	High Willhays	51 34 20·898	5·27016409	186279·09	35·3	
		180 0 7·825				
	Lundy Island	24 47 6·411	5·00501493	101161·42	19·2	
5	Brown Willy	94 12 20·998	5·38141360	240665·37	45·6	+7·83
	Trevose Head	61 0 37·601	5·32444565	211079·30	40·0	
		180 0 5·010				

FIGURE 12—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
6	Brown Willy	47 38 34·867	4·89271977	78112·36	14·8	+2·76
	Trevose Head	59 13 7·590	4·95815606	90814·68	17·2	
	Hensbarrow	73 8 19·140	5·00501493	101161·42	19·2	
		180 0 1·597				
7	Lundy Island	48 0 38·530	5·33808015	217811·17	41·3	+0·79
	High Wilhays	79 21 38·839	5·45940875	288010·79	54·5	
	Hensbarrow	52 37 54·359	5·36716600	232898·13	44·1	
		180 0 11·728				
8	Precelly	47 7 23·673	5·48689345	306826·91	58·1	+0·01
	Cradle	70 0 37·032	5·59492094	393478·43	74·5	
	Dunkery	62 52 24·568	5·57129508	372644·82	70·6	
		180 0 25·273				
9	Precelly	22 46 33·559	5·30083762	199911·43	37·9	−0·92
	Dunkery	107 35 36·450	5·69221059	492278·19	93·2	
	High Wilhays	49 38 7·628	5·59492094	393478·43	74·5	
		180 0 17·637				
10	Paracombe	124 50 22·808	5·49704166	314081·00	59·5	Supplementary.
	Brown Willy	9 29 35·276	4·80011112	63111·88	12·0	
	Dunkery	45 40 5·251	5·43731551	273725·66	51·8	
		180 0 3·335				
11	Cradle	46 41 4·300	5·37995750	239859·82	45·4	+0·12
	Dunkery	64 46 25·679	5·47454951	298228·75	56·5	
	Mendip	68 32 45·680	5·48689345	306826·91	58·1	
		180 0 15·659				
12	Dunkery	41 48 11·675	5·21569899	164323·24	31·1	−0·25
	Mendip	61 32 57·755	5·33595500	216747·95	41·0	
	Pillesdon	76 38 58·722	5·37995750	239859·82	45·4	
		180 0 8·152				
13	Mendip	36 38 56·475	5·44130642	276252·63	52·3	−1·61
	Pillesdon	122 33 21·276	5·59116789	390092·76	73·9	
	High Wilhays	20 47 51·250	5·21569899	164323·24	31·1	
		180 0 9·001				
14	High Wilhays	76 36 42·163	5·42949205	268838·87	50·9	Supplementary.
	Pillesdon	14 54 25·788	4·85180850	71090·00	13·5	
	Ryder's Hill	88 28 56·544	5·44130642	276252·63	52·3	
		180 0 4·495				
15	Pillesdon	22 28 57·538	5·04667169	111345·25	21·1	”
	Ryder's Hill	90 6 33·109	5·46416083	291179·51	55·1	
	Barrow Hill	67 24 36·395	5·42949205	268838·87	50·9	
		180 0 7·042				
16	Ryder's Hill	56 9 1·046	5·15100518	141581·07	26·8	+3·70
	High Wilhays	99 12 15·891	5·22603780	168282·05	31·9	
	Brown Willy	24 38 45·400	4·85180850	71090·00	13·5	
		180 0 2·337				

FIGURE 12—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
17	Ryder's Hill	103 5 38·407	5·42951546	268853·36	50·9	Supplementary.
	Barrow Hill	53 7 2·275	5·34396994	220785·19	41·8	
	Hensbarrow	23 47 24·951	5·04667169	111345·25	21·1	
		180 0 5·633				
18	High Willhays	70 49 11·940	5·37384198	236505·90	44·8	+1·09
	Ryder's Hill	92 41 15·789	5·39816723	250130·84	47·4	
	Deadman	16 29 36·222	4·85180850	71090·00	13·5	
		180 0 3·951				
19	Brown Willy...................	14 42 17·739	4·77301845	59295·05	11·2	−0·12
	Hensbarrow	142 25 1·063	5·15372540	142470·65	27·0	
	Deadman	22 52 41·971	4·95815606	90814·68	17·2	
		180 0 0·773				
20	Hensbarrow	6 53 5·554	5·08868212	122654·11	23·2	−4·48
	Barrow Hill	8 20 56·618	5·17193237	148570·43	28·1	
	Maker Church Tower	164 45 58·955	5·42951546	268853·36	50·9	
		180 0 1·127				
21	High Willhays	29 28 41·782	5·17973892	151265·16	28·6	Supplementary.
	Deadman	24 58 57·820	5·11335770	129824·81	24·6	
	Maker Church Tower	125 32 24·158	5·39816723	250130·84	47·4	
		180 0 3·760				

FIGURE 13.

1	Trevose Head	65 43 55·940	4·98724202	97105·09	18·4	+3·22
	Hensbarrow	67 6 7·276	4·99177485	98123·91	18·6	
	St. Agnes' Beacon............	47 9 58·428	4·89271977	78112·36	14·8	
		180 0 1·644				
2	Hensbarrow	77 20 32·521	5·00897351	102087·72	19·3	+2·13
	St. Agnes' Beacon............	34 31 16·068	4·77301844	59295·05	11·2	
	Deadman	68 8 12·733	4·98724202	97105·09	18·4	
		180 0 1·322				
3	Deadman	25 51 23·420	4·65772987	45470·515	8·6	−7·74
	St. Agnes' Beacon............	75 51 56·605	5·00477550	101105·667	19·1	
	Karnbonellis	78 16 41·034	5·00897351	102087·722	19·3	
		180 0 1·059				
4	Deadman	27 45 9·281	4·70742110	50982·50	9·7	Supplementary.
	Karnbonellis	84 48 39·232	5·03757542	109037·38	20·6	
	Goonhilly	67 26 12·695	5·00477550	101105·67	19·1	
		180 0 1·208				
5	Ryder's Hill	17 18 27·430	5·17326057	149025·49	28·2	,, 1
	Hensbarrow	136 32 31·074	5·53726361	344559·02	65·3	
	Goonhilly	26 9 6·820	5·34396994	220785·19	41·8	
		180 0 5·324				

FIGURE 13—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ:
		° ′ ″		Feet.	Miles.	″
6	St. Agnes' Beacon	129 34 23·620	5·31945268	208666·478	39·5	
	Karnbonellis	40 45 26·498	5·24732085	176734·300	33·5	Supplementary.
	Brown Willy	9 40 11·339	4·65772987	45470·515	8·6	
		180 0 1·457				
7	Trevose Head	82 2 58·162	5·25712417	180769·10	34·2	
	Hensbarrow	72 36 47·128	5·24100730	174183·61	33·0	−3·70
	Karnminnis	25 20 17·880	4·89271977	78112·36	14·8	
		180 0 3·170				
8	Brown Willy	32 57 4·744	5·25712417	180769·10	34·2	
	Hensbarrow	145 45 6·268	5·41646086	260892·06	49·4	+2·93
	Karnminnis	11 17 51·162	4·95815606	90814·68	17·2	
		180 0 2·174				
9	Karnminnis	26 10 22·133	5·08177803	120719·69	22·9	
	Hensbarrow	15 9 50·960	4·85487255	71593·33	13·6	+3·00
	Karnbonellis	138 39 48·250	5·25712417	180769·10	34·2	
		180 0 1·343				
10	St. Agnes' Beacon	57 46 31·855	4·85487255	71593·328	13·6	
	Karnbonellis	89 43 30·760	4·92751530	84628·238	16·0	+6·90
	Karnminnis	32 29 58·151	4·65772987	45470·515	8·6	
		180 0 0·766				
11	Trevose Head	83 59 49·405	5·30555858	202096·41	38·3	
	Hensbarrow	73 23 51·645	5·28945257	194738·84	36·9	Supplementary.
	Karn Galver	22 36 22·509	4·89271977	78112·36	14·8	
		180 0 3·559				
12	Karnbonellis	7 33 46·264	4·33218200	21487·308	4·1	
	Karnminnis	146 25 39·443	4·95559943	90281·638	17·1	„
	Karn Galver	26 0 34·473	4·85487255	71593·328	13·6	
		180 0 0·200				
13	Karn Galver	26 57 51·028	4·70742110	50982·50	9·7	
	Karnbonellis	99 37 22·711	5·04475498	110854·92	21·0	„
	Goonhilly	53 24 47·329	4·95559943	90281·64	17·1	
		180 0 1·068				
14	Karn Galver	86 39 16·380	5·04860063	111840·893	21·2	
	Goonhilly	11 39 28·018	4·35483203	22637·686	4·3	−3·04
	Pertinny	81 41 16·191	5·04475498	110854·921	21·0	
		180 0 0·589				
15	Karnbonellis	19 21 18·144	4·61717510	41416·662	7·8	
	Karnminnis	125 41 33·387	5·00643760	101493·354	19·2	Supplementary.
	Pertinny	34 57 9·036	4·85487255	71593·328	13·6	
		180 0 0·567				
16	Pertinny	112 6 10·069	5·18440256	152898·27	29·0	
	Goonhilly	25 13 55·101	4·84724846	70347·47	13·3	„
	Wolf Rock	42 39 56·545	5·04860063	111840·89	21·2	
		180 0 1·715				

FIGURE 13—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.			Log. Distances.	Distances in		Error of Δ.
						Feet.	Miles.	
		°	′	″				
17	Pertinny	37	40	18·852	5·03304592	107906·08	20·4	″
	Wolf Rock	118	50	56·206	5·18935992	154653·56	29·3	Supplementary.
	St. Martin's Head	23	28	46 506	4·84724846	70347·47	13·3	
		180	0	1·564				
18	Pertinny	128	32	14·888	5·22964053	169683·857	32·1	
	St. Martin's Head	5	59	24·084	4·35483203	22637·686	4·3	+4·75
	Karn Galver	45	28	21·672	5·18935992	154653·561	29·3	
		180	0	0·644				
19	Pertinny	148	10	27·004	5·28125072	191095·614	36·2	
	St. Martin's Head	6	33	46·270	4·61717509	41416·662	7·8	+5·54
	Karnminnis	25	15	47·520	5·18935992	154653·561	29·3	
		180	0	0·794				
20	Karnminnis	26	52	57·443	5·23559230	172025·291	32·6	
	Pertinny	146	52	4·106	5·31794619	207943·901	39·4	Supplementary.
	Beacon Hill, Trescow	6	14	59·367	4·61717510	41416·662	7·8	
		180	0	0·916				
21	Pertinny	4	44	4·669	4·16567463	14644·503	2·8	
	Beacon Hill, Trescow	71	5	45·919	5·22493035	167853·479	31·8	+4·80
	Telegraph Tower	104	10	9·973	5·23559230	172025·291	32·6	
		180	0	0·561				
22	Pertinny	127	13	51·989	5·27089849	186594·353	35·3	
	Beacon Hill	5	32	35·612	4·35483201	22637·686	4·3	+3·94
	Karn Galver	47	13	33·128	5·23559230	172025·291	32·6	
		180	0	0·729				
23	Karn Galver	6	27	46·300	4·39677832	24933·217	4·7	
	Beacon Hill	116	8	51·807	5·29851617	198845·687	37·7	Supplementary.
	St. Agnes' Lighthouse	57	23	22·876	5·27089849	186594·353	35·3	
		180	0	0·983				
24	Karn Galver	6	25	51·838	4·32285900	21030·956	4·0	
	Beacon Hill	89	58	22·267	5·27361666	187765·871	35·6	−1·13
	Peninnis Windmill	83	35	46·819	5·27089849	186594·353	35·3	
		180	0	0·924				
25	Pertinny	2	37	14·664	4·21784361	16513·671	3·1	
	Telegraph Tower	149	41	3·963	5·26078597	182299·710	34·5	Supplementary.
	St. Agnes' Lighthouse	27	41	41·702	5·22493035	167853·479	31·8	
		180	0	0·329				
26	Pertinny	31	57	29·449	5·07052199	117631·05	22·3	
	Wolf Rock	129	35	18·132	5·23367556	171267·74	32·4	”
	Peninnis Windmill	18	27	13·920	4·84724846	70347·47	13·3	
		180	0	1·501				
27	Beacon Hill	72	58	56·535	4·36588102	23221·005	4·4	
	Peninnis Windmill	47	1	1·242	4·24957321	17765·327	3·4	+7·07
	St. Martin's Head	60	0	2·307	4·32285900	21030·956	4·0	
		180	0	0·084				

FIGURE 13—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
28	Beacon Hill	59 38 55·798	4·21337511	16344·631	3·1	+9·48
	St. Martin's Head	50 38 26·210	4·16567463	14644·503	2·8	
	Telegraph Tower	69 42 38·045	4·24957321	17765·327	3·4	
		180 0 0·053				
29	Wolf Rock	3 40 25·074	3·87942187	7575·684	1·4	Supplementary.
	Peninnis Windmill	80 31 45·491	5·06678853	116624·160	22·1	
	Telegraph Tower	95 47 49·642	5·07052199	117631·054	22·3	
		180 0 0·207				
30	St. Agnes' Lighthouse	56 51 3·217	4·32285900	21030·956	4·0	„
	Beacon Hill	26 10 29·540	4·04455278	11080·332	2·1	
	Peninnis Windmill	96 58 27·297	4·39677832	24933·217	4·7	
		180 0 0·054				
31	Hensbarrow	28 13 4·604	5·04860064	111840·90	21·2	+0·86
	Goonhilly	112 43 50·924	5·33879091	218167·93	41·3	
	Pertinny	39 3 8·088	5·17326057	149025·49	28·2	
		180 0 3·616				

FIGURE 14.

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
1	Dean Hill	45 40 5·512	5·00309963	100716·27	19·1	−2·64
	Wingreen	85 26 16·998	5·14723190	140356·30	26·6	
	Westbury Down	48 53 39·996	5·02569162	106094·19	20·1	
		180 0 2·506				
2	Dean Hill	73 13 17·848	5·20089906	158817·76	30·1	−2·35
	Westbury Down	48 59 8·276	5·09747629	125163·10	23·7	
	Inkpen	57 47 37·832	5·14723190	140356·30	26·6	
		180 0 3·956				
3	Westbury Down	97 52 48·272	5·29968295	199380·62	37·8	−0·07
	Inkpen	30 1 30·841	5·00309963	100716·27	19·1	
	Wingreen	52 5 44·614	5·20089906	158817·76	30·1	
		180 0 3·727				
4	Dean Hill	66 47 53·737	5·14575579	139880·05	26·5	Supplementary.
	Inkpen	57 52 28·841	5·11020695	128886·36	24·4	
	Stoke Hill	55 19 40·910	5·09747629	125163·10	23·7	
		180 0 3·488				
5	Dean Hill	6 25 24·111	4·27735406	18938·870	3·6	+1·79
	Westbury Down	49 34 57·651	5·11020695	128886·356	24·4	
	Stoke Hill	123 59 38·714	5·14723190	140356·296	26·6	
		180 0 0·476				
6	Dean Hill	82 56 18·488	5·05888866	114521·93	21·7	−4·95
	Wingreen	30 13 33·204	4·76411607	58091·96	11·0	
	Beacon Hill	66 50 9·747	5·02569162	106094·19	20·1	
		180 0 1·439				

FIGURE 14—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
7	Beacon Hill	55 24 4·558	5·00309963	100716·27	19·1	+2·98
	Wingreen	55 12 43·793	5·00210736	100486·42	19·0	
	Westbury Down	69 23 13·877	5·05888866	114521·93	21·7	
		180 0 2·228				
8	Beacon Hill	51 1 1·577	4·89866534	79189·09	15·0	−2·10
	Westbury Down	48 26 48·320	4·88215646	76235·36	14·4	
	Milk Hill	80 32 11·504	5·00210736	100486·42	19·0	
		180 0 1·401				
9	Beacon Hill	66 16 38·563	4·94745472	88604·28	16·8	Supplementary.
	Milk Hill	61 45 6·072	4·93072338	85255·69	16·1	
	Inkpen	51 58 16·764	4·88215646	76235·36	14·4	
		180 0 1·399				
10	Beacon Hill	61 39 41·501	5·02197212	105189·43	19·9	"
	Wingreen	44 57 15·389	4·92654856	84440·06	16·0	
	Stoke Hill	73 23 5·112	5·05888866	114521·93	21·7	
		180 0 2·002				
11	Beacon Hill	44 45 24·634	4·78987136	61641·24	11·7	−3·67
	Milk Hill	74 41 29·836	4·92654856	84440·06	16·0	
	Stoke Hill	60 33 6·596	4·88215646	76235·36	14·4	
		180 0 1·066				
12	Dean Hill	39 46 48·868	4·94745472	88604·28	16·8	Supplementary.
	Inkpen	75 33 7·341	5·12742522	134098·90	25·4	
	Milk Hill	64 40 6·317	5·09747629	125163·10	23·7	
		180 0 2·526				
13	Dean Hill	79 6 34·493	5·18885754	154474·76	29·3	"
	Wingreen	58 28 50·333	5·12742522	134098·90	25·4	
	Milk Hill	42 24 38·461	5·02569162	106094·19	20·1	
		180 0 3·287				
14	Inkpen	24 56 42·025	4·58880754	38797·840	7·3	"
	Milk Hill	80 39 19·172	4·95795350	90772·335	17·2	
	Upcot Down	74 23 59·601	4·94745472	88604·284	16·8	
		180 0 0·798				
15	Inkpen	42 42 11·534	5·04449801	110789·35	21·0	+0·78
	Westbury Down	33 45 23·954	4·95795350	90772·33	17·2	
	Upcot Down	103 32 26·812	5·20089905	158817·76	30·1	
		180 0 2·300				
16	Westbury Down	33 9 34·579	4·97999505	95498·169	18·1	Supplementary.
	Upcot Down	6 13 38·522	4·27735406	18938·870	3·6	
	Stoke Hill	140 36 47·169	5·04449801	110789·349	21·0	
		180 0 0·270				

FIGURE 14—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
	Dean Hill	40 54 51·415	4·58096062	38103·127	7·2	
17	Beacon Hill	45 57 2·321	4·62133882	41815·647	7·9	−1·18
	Old Sarum Castle	93 8 6·638	4·76411607	58091·965	11·0	
		180 0 0·374				
	Dean Hill	58 40 46·570	4·59356492	39225·178	7·4	
18	Old Sarum Castle	55 43 10·163	4·57910026	37940·256	7·2	−0·92
	Old Lodge	65 36 3·586	4·62133882	41815·647	7·9	
		180 0 0·319				
	Dean Hill	39 29 1·595	4·58906622	38820·956	7·4	
19	Beacon Hill	68 25 59·202	4·75418305	56778·382	10·8	−0·62
	Four Mile Stone	72 4 59·696	4·76411607	58091·965	11·0	
		180 0 0·493				
	Beacon Hill	96 13 41·648	4·68390792	48295·639	9·1	
20	Four Mile Stone	30 43 47·919	4·39489332	24825·232	4·7	−2·25
	Old Lodge	53 2 30·658	4·58906622	38820·956	7·4	
		180 0 0·225				
	Beacon Hill	72 14 1·174	4·57307331	37417·375	7·1	
21	Old Lodge	68 34 52·961	4·56321587	36577·656	6·9	−4·50
	Old Sarum Gun	39 11 6·068	4·39489332	24825·232	4·7	
		180 0 0·203				
	Beacon Hill	23 59 40·474	4·19936067	15825·618	3·0	
22	Four Mile Stone	70 1 57·751	4·56321587	36577·656	6·9	−4·57
	Old Sarum Gun	85 58 21·911	4·58906622	38820·956	7·4	
		180 0 0·136				
	Four Mile Stone	6 2 16·653	3·25910770	1815·966	0·3	
23	Old Sarum Gun	60 24 25·413	4·17644177	15012·111	2·8	−2·10
	Old Sarum Castle	113 33 17·940	4·19936067	15825·618	3·0	
		180 0 0·006				
	Milk Hill	56 37 33·310	4·95376577	89901·26	17·0	
24	Inkpen	67 58 60·072	4·99914412	99803·12	18·9	Supplementary.
	Old Lodge	55 23 28·355	4·94745472	88604·28	16·8	
		180 0 1·737				
	Milk Hill	22 5 41·071	4·59356492	39225·178	7·4	
25	Old Lodge	84 45 44·697	5·01640072	103848·617	19·7	−3·05
	Old Sarum Castle	73 8 35·149	4·99914412	99803·122	18·9	
		180 0 0·917				
	Milk Hill	57 43 21·527	4·94457441	88018·59	16·7	
26	Old Sarum Castle	36 18 23·012	4·78987136	61641·24	11·7	+0·17
	Stoke Hill	85 58 16·734	5·01640072	103848·62	19·7	
		180 0 1·273				
	Old Lodge	34 18 36·863	4·78987136	61641·24	11·7	
27	Milk Hill	79 49 2·597	5·03195001	107634·13	20·4	−1·43
	Stoke Hill	65 52 21·964	4·99914412	99803·12	18·9	
		180 0 1·424				

FIGURE 14—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
28	Old Lodge	40 5 14·298	4·89866534	79189·09	15·0	+0·43
	Milk Hill	85 39 44·266	5·08856607	122621·34	23·2	
	Westbury Down	54 15 3·290	4·99914412	99803·12	18·9	
		180 0 1·854				
29	Old Lodge	49 12 60·754	5·00309963	100716·27	19·1	Supplementary.
	Westbury Down	63 34 58·906	5·07600127	119124·55	22·6	
	Wingreen	67 12 2·942	5·08856607	122621·34	23·2	
		180 0 2·602				
30	Westbury Down	47 23 31·861	4·87923232	75723·78	14·4	,,
	Wingreen	54 23 36·523	4·92246089	83649·03	15·8	
	Four Mile Stone	78 12 53·075	5·00309963	100716·27	19·1	
		180 0 1·459				
31	Westbury Down	70 26 30·336	4·97305452	93984·13	17·8	−1·44
	Four Mile Stone	52 33 27·296	4·89866534	79189·09	15·0	
	Milk Hill	57 0 3·836	4·92246089	83649·03	15·8	
		180 0 1·468				
32	Four Mile Stone	40 57 7·077	4·78987136	61641·24	11·7	+0·05
	Milk Hill	51 9 22·168	4·86480633	73249·78	13·9	
	Stoke Hill	87 53 31·817	4·97305452	93984·13	17·8	
		180 0 1·062				

FIGURE 15.

1	Pillesdon	44 42 44·304	5·11926316	131602·20	24·9	−1·63
	Mendip	73 49 49·971	5·25444357	179656·77	34·0	
	Wingreen	61 27 30·611	5·21569899	164323·24	31·1	
		180 0 4·886				
2	Pillesdon	44 25 16·492	5·14145421	138501·42	26·2	+3·32
	Wingreen	70 21 45·971	5·27038005	186371·73	35·3	
	Swyre Barrow	65 13 3·050	5·25444357	179656·77	34·0	
		180 0 5·513				
3	Wingreen	56 32 62·233	5·31820151	208066·19	39·4	−0·84
	Swyre Barrow	89 42 43·249	5·39683883	249366·92	47·2	
	Dunnose	33 44 21·297	5·14145421	138501·42	26·2	
		180 0 6·779				
4	Wingreen	31 58 21·912	5·14802983	140614·41	26·6	−0·44
	Dunnose	78 8 22·690	5·41478671	259888·29	49·2	
	Butser Hill	69 53 23·471	5·39683884	249366·92	47·2	
		180 0 8·073				
5	Wingreen	42 39 15·982	5·24621308	176284·08	33·4	−1·32
	Butser Hill	50 1 37·335	5·29968295	199380·62	37·8	
	Inkpen	87 19 14·942	5·41478671	259888·29	49·2	
		180 0 8·259				

FIGURE 15—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
6	Mendip	51 46 9·889	5·29968295	199380·62	37·8	+1·58
	Wingreen	97 0 3·291	5·40127580	251927·63	47·7	
	Inkpen	31 13 52·946	5·11926316	131602·20	24·9	
		180 0 6·126				
7	Wingreen	71 51 45·342	5·22760874	168891·87	32·0	Supplementary.
	Mendip	60 21 53·891	5·18885754	154474·76	29·3	
	Milk Hill	47 46 25·311	5·11926316	131602·20	24·9	
		180 0 4·544				
8	Wingreen	45 51 52·656	5·26402736	183665·41	34·8	+3·56
	Dunnose	31 8 61·672	5·12181566	132377·95	25·1	
	Coringdon	102 59 11·245	5·39683884	249366·92	47·2	
		180 0 5·573				
9	Wingreen	41 17 5·510	5·26367907	183518·17	34·8	−0·45
	Dunnose	22 25 23·334	5·02569162	106094·20	20·1	
	Dean Hill	116 17 35·262	5·39683884	249366·92	47·2	
		180 0 4·106				
10	Coringdon	74 11 33·776	5·37864326	239135·07	45·3	+4·16
	Dunnose	58 9 47·605	5·32457447	211141·92	40·0	
	Beacon Hill	47 38 47·396	5·26402736	183665·41	34·8	
		180 0 8·777				
11	Coringdon	44 13 4·158	5·27048029	186414·76	35·3	−2·89
	Beacon Hill	83 36 33·925	5·42430467	265646·85	50·3	
	Butser Hill	52 10 31·118	5·32457447	211141·92	40·0	
		180 0 9·201				
12	Swyre Barrow	66 23 6·841	5·37864326	239135·07	45·3	−2·32
	Dunnose	60 45 7·230	5·35739586	227717·22	43·1	
	Beacon Hill	52 51 56·142	5·31820151	208066·19	39·4	
		180 0 10·213				
13	Swyre Barrow	55 13 5·825	5·26367907	183518·17	34·8	+0·70
	Dunnose	56 9 44·631	5·26856279	185593·51	35·1	
	Dean Hill	68 37 17·005	5·31820151	208066·19	39·4	
		180 0 7·461				
14	Coringdon	63 10 6·859	5·26367907	183518·17	34·8	+0·87
	Dunnose	53 34 25·007	5·21873952	165477·72	31·3	
	Dean Hill	63 15 34·514	5·26402736	183365·41	34·8	
		180 0 6·380				
15	Dunnose	51 7 36·757	5·27048029	186414·76	35·3	−3·26
	Butser Hill	92 54 42·872	5·37864326	239135·07	45·3	
	Beacon Hill	35 57 46·529	5·14802983	140614·41	26·6	
		180 0 6·158				
16	Butser Hill	27 0 17·933	4·93072338	85255·69	16·1	+2·39
	Beacon Hill	69 51 47·048	5·24621308	176284·08	33·4	
	Inkpen	83 7 58·529	5·27048029	186414·76	35·3	
		180 0 3·510				

FIGURE 15—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
17	Wingreen	74 37 37·894	5·43918596	274907·11	52·1	−1·23
	Dunnose	44 22 22·618	5·29968295	199380·62	37·8	
	Inkpen	61 0 10·765	5·39683883	249366·92	47·2	
		180 0 11·277				
18	Dunnose	55 42 59·356	5·19351096	156138·85	29·6	−1·35
	Butser Hill	76 12 8·199	5·26367907	183518·17	34·8	
	Dean Hill	48 4 57·461	5·14802983	140614·41	26·6	
		180 0 5·016				
19	Butser Hill	43 42 52·607	5·09747629	125163·10	23·7	+4·95
	Dean Hill	76 44 3·916	5·24621308	176284·08	33·4	
	Inkpen	59 33 7·952	5·19351096	156138·85	29·6	
		180 0 4·475				
20	Wingreen	44 54 18·677	4·96943936	93205·03	17·7	−1·18
	Mendip	49 42 50·116	5·00309963	100716·27	19·1	
	Westbury Down	85 22 53·408	5·11926316	131602·20	24·9	
		180 0 2·201				
21	Mendip	36 36 57·863	5·05888866	114521·93	21·7	+4·61
	Wingreen	100 7 2·471	5·27651198	189021·84	35·8	
	Beacon Hill	43 16 3·156	5·11926316	131602·20	24·9	
		180 0 3·490				
22	Mendip	9 30 40·403	4·58880754	38797·840	7·4	Supplementary.
	Milk Hill	124 29 30·739	5·28673225	193522·852	36·6	
	Upcot Down	45 59 50·129	5·22760874	168891·866	32·0	
		180 0 1·271				
23	Mendip	20 9 44·178	5·04449801	110789·35	21·0	”
	Westbury Down	142 58 54·366	5·28673225	193522·85	36·6	
	Upcot Down	16 51 22·918	4·96943937	93205·03	17·7	
		180 0 1·462				
24	Pillesdon	39 25 29·705	5·12181566	132377·95	25·1	+0·32
	Wingreen	81 2 55·549	5·31367885	205910·67	39·0	
	Coringdon	59 31 40·273	5·25444357	179656·77	34·0	
		180 0 5·527				
25	Dean Hill	83 44 1·816	5·29403115	196802·75	37·3	Supplementary.
	Dunnose	28 18 22·599	4·97257616	93880·66	17·8	
	Horton's Gazebo	67 57 39·614	5·26367907	183518·17	34·8	
		180 0 4·029				
26	Horton's Gazebo	129 2 9·920	5·49740321	314342·58	59·5	”
	Dunnose	21 51 60·012	5·17817534	150721·55	28·5	
	Blackdown	29 5 55·488	5·29403115	196802·75	37·3	
		180 0 5·420				
27	Horton's Gazebo	46 33 45·948	5·03974802	109584·22	20·8	”
	Blackdown	40 31 32·179	4·99150601	98063·19	18·6	
	Swyre Barrow	92 54 44·398	5·17817534	150721·55	28·5	
		180 0 2·525				

FIGURE 15—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
28	Horton's Gazebo	60 25 37·086	5·11876983	131452·80	24·9	Supplementary.
	Blackdown......................	33 51 10·006	4·92528768	84195·27	15·9	
	Coringdon	85 43 15·504	5·17817534	150721·55	28·5	
		180 0 2·596				
29	Blackdown......................	6 40 22·173	4·41413568	25949·900	4·9	+2·84
	Coringdon	29 23 11·592	5·03974801	109584·217	20·8	
	Swyre Barrow	143 56 26·629	5·11876983	131452·796	24·9	
		180 0 0·394				
30	Horton's Gazebo	85 31 58·766	5·02569162	106094·20	20·1	Supplementary.
	Dean Hill	32 33 33·446	4·75793270	57270·73	10·8	
	Wingreen	61 54 29·049	4·97257616	93880·66	17·8	
		180 0 1·261				
31	Blackdown......................	20 13 39·547	5·04667169	111345·25	21·1	+0·94
	Barrow Hill	81 42 55·049	5·50336687	318688·85	60·4	
	Ryder's Hill	78 3 33·572	5·49842221	315081·00	59·7	
		180 0 8·168				
32	Blackdown......................	11 53 59·297	4·85180849	71089·99	13·5	+1·74
	Ryder's Hill	100 31 56·080	5·53015693	338966·62	64·2	
	High Wilhays	67 34 9·863	5·50336687	318688·85	60·4	
		180 0 5·240				
33	Pillesdon	148 18 37·929	5·64906452	445722·46	84·4	−2·16
	High Wilhays	12 41 21·869	5·27038005	186371·73	35·3	
	Swyre Barrow	19 0 6·565	5·44130642	276252·63	52·3	
		180 0 6·363				
34	Pillesdon	133 24 12·141	5·62258805	419361·01	79·4	Supplementary.
	Ryder's Hill	18 50 18·766	5·27038005	186371·73	35·3	
	Swyre Barrow	27 45 37·657	5·42949205	268838·86	50·9	
		180 0 8·564				
35	Barrow Hill	14 18 18·654	4·89829760	79122·06	15·0	+4·18
	Pillesdon	100 17 22·960	5·49842221	315081·00	59·7	
	Blackdown......................	65 24 23·719	5·46416083	291179·51	55·1	
		180 0 5·333				
36	Dean Hill	72 9 11·471	5·23658800	172420·14	32·7	+2·67
	Butser Hill	48 18 23·694	5·13115887	135256·73	25·6	
	Nodes Beacon	59 32 29·564	5·19351096	156138·85	29·6	
		180 0 4·729				
37	Nodes Beacon	30 25 29·049	4·85519032	71645·73	13·6	+5·67
	Dean Hill	42 31 9·300	4·98053460	95616·89	18·1	
	Southampton	107 3 23·192	5·13115887	135256·73	25·6	
		180 0 1·541				

FIGURE 15—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
38	Nodes Beacon	82 55 10·913	5·07098936	117757·71	22·3	−4·54
	Southampton	43 23 39·282	4·911127855	81522·70	15·4	
	Dunnose	53 41 11·625	4·98053460	95616·89	18·1	
		180 0 1·820				
39	Dunnose	98 18 7·766	5·23658800	172420·14	32·7	−3·52
	Butser Hill	27 53 44·505	4·911127855	81522·70	15·4	
	Nodes Beacon	53 48 10·398	5·14802983	140614·41	26·6	
		180 0 2·669				
40	Wingreen	83 51 41·022	5·39949127	250894·58	47·5	−1·26
	Inkpen	43 56 35·809	5·24330893	175109·19	33·2	
	Nodes Beacon	52 11 51·335	5·29968295	199380·62	37·8	
		180 0 8·166				
41	Nodes Beacon	51 25 21·411	5·14145421	138501·42	26·2	+2·49
	Wingreen	47 18 59·105	5·11472837	130235·20	24·7	
	Swyre Barrow	81 15 43·678	5·24330893	175109·19	33·2	
		180 0 4·194				
42	Nodes Beacon	11 48 0·911	5·03974801	109584·22	20·8	+4·29
	Swyre Barrow	154 8 3·322	5·36881020	233781·53	44·3	
	Blackdown	14 3 57·232	5·11472837	130235·20	24·7	
		180 0 1·465				
43	Nodes Beacon	48 54 39·360	5·12181566	132377·95	25·1	+3·50
	Wingreen	36 37 49·527	5·02034295	104795·58	19·8	
	Coringdon	94 27 34·367	5·24330893	175109·19	33·2	
		180 0 3·254				
44	Nodes Beacon	2 30 42·052	4·41413568	25949·90	4·9	−0·77
	Swyre Barrow	10 11 36·693	5·02034295	104795·58	19·8	
	Coringdon	167 17 41·396	5·11472837	130235·20	24·7	
		180 0 0·141				
45	Wingreen	10 41 9·578	4·41413568	25949·90	4·9	−0·25
	Swyre Barrow	71 4 6·984	5·12181566	132377·95	25·1	
	Coringdon	98 14 44·238	5·14145421	138501·42	26·2	
		180 0 0·800				

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Feet.	Miles.	Error of Δ.
46	Dunnose	53 49 45·700	4·87737757	75401·080	14·3	+1·11
	Nodes Beacon	6 57 23·768	4·05356865	11312·762	2·1	
	Week Down	119 12 50·707	4·911127855	81522·698	15·4	
		180 0 0·175				
47	Dunnose	45 10 21·329	3·92413940	8397·295	1·6	−0·32
	Week Down	27 39 54·765	3·74015289	5497·344	1·0	
	Boniface Down	107 9 43·917	4·05356865	11312·762	2·1	
		180 0 0·011				

FIGURE 15—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.			Log. Distances.	Distances in		Error of Δ.
		°	′	″		Feet.	Miles.	″
48	Dunnose	41	21	5·634	3·99439474	9871·763	1·9	−0·32
	Week Down	7	51	33·107	3·31029796	2043·139	0·4	
	Shanklin Down	130	47	21·262	4·05356865	11312·762	2·1	
		180	0	0·003				
49	Dunnose	99	0	7·030	4·91677805	82561·591	15·6	Supplementary.
	Nodes Beacon	3	46	14·761	3·74015287	5497·344	1·0	
	Boniface Down	77	13	38·313	4·91127855	81522·698	15·4	
		180	0	0·104				
50	Boniface Down	29	56	5·604	4·87737757	75401·080	14·3	”
	Nodes Beacon	3	11	9·006	3·92413942	8397·295	1·6	
	Week Down	146	52	45·471	4·91677805	82561·591	15·6	
		180	0	0·081				
51	Dunnose	16	51	32·927	3·27675294	1891·267	0·4	−2·34
	Boniface Down	105	40	57·155	3·79784959	6278·409	1·2	
	Wroxall Down	57	27	29·920	3·74015289	5497·344	1·0	
		180	0	0·002				
52	Dunnose	69	39	54·037	3·77002323	5888·751	1·1	Supplementary.
	Wroxall Down	18	59	8·675	3·31029796	2043·139	0·4	
	Shanklin Down	91	20	57·291	3·79784959	6278·409	1·2	
		180	0	0·003				
53	Dunnose	86	31	26·963	3·75947588	5747·459	1·1	+6·20
	Boniface Down	20	46	59·275	3·31029796	2043·139	0·4	
	Shanklin Down	72	41	33·765	3·74015289	5497·344	1·0	
		180	0	0·003				
54	Boniface Down	50	36	38·900	3·88078522	7599·504	1·4	−4·51
	Week Down	8	2	13·882	3·13824451	1374·816	0·3	
	Littletown Down	121	21	7·220	3·92413940	8397·295	1·6	
		180	0	0·002				
55	Boniface Down	113	14	32·375	3·33272617	2151·425	0·4	−24·65
	Wroxall Down	12	52	57·034	2·71769678	522·031	0·1	
	Boniface Down, South East	53	52	30·591	3·27675294	1891·267	0·4	
		180	0	0·000				
56	Boniface Down	52	5	25·662	3·17819150	1507·271	0·3	Supplementary.
	Wroxall Down	46	1	31·499	3·13824456	1374·816	0·3	
	Littletown Down	81	53	2·840	3·27675294	1891·267	0·4	
		180	0	0·001				
57	Boniface Down	61	9	6·714	3·08366940	1212·465	0·2	−42·98
	Littletown Down	22	9	20·059	2·71769678	522·031	0·1	
	Boniface Down, South East	96	41	33·227	3·13824456	1374·816	0·3	
		180	0	0·000				
58	Boniface Down, South East	78	25	67·729	3·42373085	2652·961	0·5	−35·62
	Littletown Down	74	57	55·347	3·41751129	2615·238	0·5	
	Highport Cliff	26	35	56·925	3·08366940	1212·465	0·2	
		180	0	0·001				

FIGURE 15—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
59	Nodes Beacon	2 57 44·641	3·93472638	8604·515	1·6	Supplementary.
	Week Down	150 6 33·708	4·91893096	82971·886	15·7	
	Boniface Down, South East	26 55 41·727	4·87737757	75401·080	14·3	
		180 0 0·076				

FIGURE 16.

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
1	Mendip	85 15 49·530	5·57766991	378155·06	71·6	−9·07
	Cradle	42 55 52·585	5·41236684	258444·23	48·9	
	Whitehorse Hill	51 48 35·953	5·47454950	298228·75	56·5	
		180 0 18·068				
2	Mendip	34 3 20·832	5·26738020	185088·83	35·1	−0·96
	Cradle	81 29 11·358	5·51439222	326882·91	61·9	
	Malvern	64 27 40·649	5·47454950	298228·75	56·5	
		180 0 12·839				
3	Mendip	51 12 28·698	5·41557849	260362·54	49·3	−1·20
	Malvern	50 41 8·739	5·41236684	258444·23	48·9	
	Whitehorse Hill	78 6 38·051	5·51439222	326882·91	61·9	
		180 0 15·488				
4	Whitehorse Hill	60 22 39·897	5·40978719	256913·66	48·7	Supplementary.
	Malvern	57 51 57·924	5·39840001	250264·94	47·4	
	Arbury Hill	61 45 35·501	5·41557849	260362·54	49·3	
		180 0 13·322				
5	Whitehorse Hill	46 43 12·389	5·30314247	200975·20	38·1	„
	Arbury Hill	68 14 58·794	5·40893294	256408·81	48·6	
	Dunstable	65 1 59·804	5·39840001	250264·94	47·4	
		180 0 10·987				
6	Whitehorse Hill	47 49 63·475	5·46425192	291240·60	55·2	+0·64
	Cradle	26 24 12·675	5·24236128	174727·51	33·1	
	Broadway Tower	105 45 55·369	5·57766991	378155·06	71·6	
		180 0 11·519				
7	Whitehorse Hill	52 16 40·849	5·40465519	253895·63	48·1	−1·65
	Dunstable	74 42 31·340	5·49083617	309625·10	58·6	
	Leith Tower	53 0 62·581	5·40893294	256408·81	48·6	
		180 0 14·770				
8	Whitehorse Hill	46 23 13·764	5·41262566	258598·30	49·0	−2·50
	Leith Tower	13 42 40·183	4·92766546	84657·50	16·0	
	Inkpen	119 54 10·517	5·49083617	309625·10	58·6	
		180 0 4·464				
9	Whitehorse Hill	76 7 35·049	5·40127580	251927·63	47·7	+3·06
	Mendip	19 2 27·175	4·92766546	84657·50	16·0	
	Inkpen	84 50 2·773	5·41236684	258444·23	48·9	
		180 0 4·997				

FIGURE 16—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
10	Leith Tower .... Dunstable .... Epping Poorhouse	38 34 14·749 60 20 46·311 81 5 7·256	5·20474992 5·34896601 5·40465519	160232·25 223339·74 253895·61	30·3 42·3 48·1	+0·80
		180 0 8·316				
11	Leith Tower .... Epping Poorhouse .... Wrotham	40 47 2·631 45 10 45·199 94 2 17·619	5·16509373 5·20088075 5·34896601	146249·28 158811·06 223339·74	27·7 30·1 42·3	−3·79
		180 0 5·449				
12	Leith Tower .... Dunstable .... Wrotham	79 21 17·381 34 48 7·146 65 50 44·794	5·43690983 5·20088075 5·40465519	273470·09 158811·06 253895·61	51·8 30·1 48·1	+3·66
		180 0 9·321				
13	Leith Tower .... Wrotham .... Berkhampstead	56 10 5·787 78 34 13·718 45 15 47·295	5·26884369 5·34071515 5·20088075	185713·59 219136·72 158811·06	35·2 41·5 30·1	+2·86
		180 0 6·800				
14	Leith Tower .... Dunstable .... Berkhampstead	23 11 11·594 58 41 61·0 2 98 6 52·496	5·00421072 5·34071515 5·40465519	100974·27 219136·72 253895·61	19·1 41·5 48·1	+0·26
		180 0 5·152				
15	Leith Tower .... Wrotham .... Chingford	44 21 39·706 79 27 5·428 56 11 19·774	5·12593061 5·27394406 5·20088075	133638·20 187907·48 158811·06	25·3 35·6 30·1	Supplementary.
		180 0 4·908				
16	Leith Tower .... Wrotham .... St. Paul's	44 8 46·695 58 9 4·273 77 42 12·624	5·05387480 5·14009359 5·20088075	113207·39 138068·18 158811·06	21·4 26·1 30·1	„
		180 0 3·592				
17	Wrotham .... Epping Poorhouse .... St. Paul's	35 53 13·346 50 35 19·753 93 31 29·184	4·93395150 5·05387480 5·16509373	85891·76 113207·39 146249·28	16·3 21·4 27·7	„
		180 0 2·283				
18	Epping Poorhouse .... St. Paul's .... Berkhampstead	72 52 32·749 39 40 18·882 67 27 9·515	4·94879239 4·77357083 4·93395150	88877·61 59370·52 85891·76	16·8 11·2 16·3	„
		180 0 1·146				
19	St. Paul's .... Berkhampstead .... Chingford	30 5 26·393 28 38 36·544 121 15 57·586	4·71710193 4·69760379 4·94879239	52131·705 49842·956 88877·613	9·9 9·4 16·8	„
		180 0 0·523				
20	Leith Tower .... Wrotham .... Severndroog	28 7 34·245 65 28 20·590 86 24 7·714	4·87513762 5·16066432 5·20088075	75013·19 144765·25 158811·06	14·2 27·4 30·1	−0·08
		180 0 2·549				

FIGURE 16—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.			Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		°	′	″				″
21	Leith Tower	28	2	31·541	5·05662801	113927·35	21·6	+2·77
	Berkhampstead	36	40	54·522	5·16066432	144765·25	27·4	
	Severndroog	115	16	37·445	5·34071515	219136·72	41·5	
		180	0	3·508				
22	Severndroog	30	56	17·063	4·77357083	59370·52	11·2	Supplementary.
	Berkhampstead	49	38	58·073	4·94452493	88008·56	16·7	
	Epping Poorhouse	99	24	46·076	5·05662801	113927·35	21·6	
		180	0	1·212				
23	St. Paul's	89	16	4·203	4·80268862	63487·557	12·0	„
	Severndroog	51	43	20·649	4·69760379	49842·956	9·4	
	Chingford	39	0	35·617	4·60168807	39965·759	7·6	
		180	0	0·469				
24	Leith Tower	11	48	26·081	4·71710193	52131·70	9·9	„
	Berkhampstead	47	31	19·625	5·27394406	187907·48	35·6	
	Chingford	120	40	16·276	5·34071515	219136·72	41·5	
		180	0	1·982				
25	Leith Tower	77	48	33·652	5·24632215	176328·35	33·4	−1·11
	Wrotham	40	30	26·979	5·06883549	117175·14	22·2	
	Ditchling	61	41	3·648	5·20088075	158811·06	30·1	
		180	0	4·279				
26	Leith Tower	94	9	57·502	5·30782811	203155·28	38·5	+3·09
	Ditchling	50	43	3·021	5·19773428	157664·63	29·9	
	Butser	35	7	3·811	5·06883549	117175·14	22·2	
		180	0	4·334				
27	Leith Tower	41	56	28·702	5·24621308	176284·08	33·4	+0·49
	Inkpen	36	42	38·821	5·19773428	157664·63	29·9	
	Butser	101	20	58·887	5·41262566	258598·30	49·0	
		180	0	6·410				
28	Ditchling	62	8	32·926	5·24999568	177826·17	33·7	−1·11
	Wrotham	56	36	59·427	5·22517749	167949·03	31·8	
	Fairlight	61	14	33·806	5·24632215	176328·35	33·4	
		180	0	6·159				
29	Ditchling	32	25	23·862	4·99300490	98402·22	18·6	−0·86
	Fairlight	33	47	53·853	5·00898878	102091·31	19·3	
	Beachy Head	113	46	44·447	5·22517749	167949·03	31·8	
		180	0	2·162				
30	Ditchling	30	2	52·835	5·14802983	140614·41	26·6	−2·49
	Butser	103	36	56·497	5·43605920	272934·98	51·7	
	Dunnose	46	20	17·199	5·30782811	203155·28	38·5	
		180	0	6·531				
31	Ditchling	122	59	3 708	5·53082975	339492·16	64·3	+2·87
	Dunnose	14	36	40·177	5·00898878	102091·31	19·3	
	Beachy Head	42	24	21·613	5·43605920	272934·98	51·7	
		180	0	5·498				

FIGURE 16—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
32	Leith Tower	38 57 30·167	4·99991518	99980·47	18·9	—0·70
	Wrotham	53 56 18·495	5·10905273	128544·27	24·3	
	Crowborough	87 6 14·357	5·20088075	158811·06	30·1	
		180 0 3·019				
33	Leith Tower	38 51 3·484	4·91604035	82421·47	15·6	Supplementary.
	Ditchling	78 2 57·965	5·10905273	128544·27	24·3	
	Crowborough	63 6 0·773	5·06883549	117175·14	22·2	
		180 0 2·222				
34	Ditchling	45 46 38·608	5·09782678	125264·14	23·7	”
	Fairlight	28 8 2·183	4·91604035	82421·47	15·6	
	Crowborough	106 5 21·542	5·22517749	167949·03	31·8	
		180 0 2·333				
35	Severndroog	118 12 39·105	5·10105835	126199·71	23·9	—2·27
	Chingford	35 28 25·627	4·91965088	83109·54	15·7	
	Banstead	26 18 56·362	4·80268862	63487·56	12·0	
		180 0 1·094				
36	Severndroog	66 29 18·456	4·88373766	76513·428	14·5	Supplementary.
	St. Paul's	84 53 36·254	4·91965088	83109·539	15·7	
	Banstead	28 37 6·006	4·60168807	39965·759	7·6	
		180 0 0·716				
37	Severndroog	140 15 54·211	5·20666918	160941·92	30·5	”
	Epping Poorhouse	19 16 29·633	4·91965088	83109·54	15·7	
	Banstead	20 27 37·256	4·94452493	88008·56	16·7	
		180 0 1·100				
38	Severndroog	55 15 15·610	4·89028797	77676·20	14·7	—1·54
	Banstead	63 12 14·903	4·92624594	84381·25	16·0	
	Hanger Hill Tower	61 32 30·842	4·91965088	83109·54	15·7	
		180 0 1·355				
39	Severndroog	61 12 15·907	5·10621418	127706·84	24·2	—2·07
	Leith Tower	35 22 58·725	4·92624594	84381·25	16·0	
	Hanger Hill Tower	83 24 47·886	5·16066432	144765·25	27·4	
		180 0 2·518				
40	St. Paul's	74 5 41·232	4·89028797	77676·200	14·7	Supplementary.
	Banstead	34 35 8·897	4·66131314	45847·234	8·7	
	Hanger Hill Tower	71 19 10·664	4·88373766	76513·428	14·5	
		180 0 0·793				
41	Leith Tower	63 30 32·971	5·18494765	153090·29	29·0	—0·71
	Wrotham	48 17 51·099	5·10621418	127706·84	24·2	
	Hanger Hill Tower	68 11 40·200	5·20088075	158811·06	30·1	
		180 0 4·270				

FIGURE 16—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
42	Leith Tower	27 29 6·142	4·89028797	77676·20	14·7	−3·88
	Hanger Hill Tower	21 52 17·043	4·79725490	62698·17	11·9	
	Banstead	130 38 37·684	5·10621418	127706·84	24·2	
		180 0 0·869				
43	Banstead	77 37 58·941	4·94475557	88055·31	16·7	+4·32
	Leith Tower	58 17 57·632	4·88478149	76697·55	14·5	
	St. Ann's	44 4 4·532	4·79725490	62698·17	11·9	
		180 0 1·105				
44	Severndroog	36 24 45·324	4·94475557	88055·31	16·7	Supplementary.
	Leith Tower	66 11 50·215	5·13265942	135724·87	25·7	
	St. Ann's	77 23 27·204	5·16066432	144765·25	27·4	
		180 0 2·743				
45	Severndroog	24 47 30·583	4·83821699	68899·65	13·0	”
	Hanger Hill Tower	124 18 30·117	5·13265942	135724·87	25·7	
	St. Ann's	30 54 0·429	4·92624594	84381·25	16·0	
		180 0 1·129				
46	Hanger Hill Tower	25 3 54·009	4·57699033	37756·378	7·2	+2·91
	St. Ann's	25 34 1·760	4·58503696	38462·452	7·3	
	Hampton Poorhouse	129 22 4·495	4·83821699	68899·646	13·0	
		180 0 0·264				
47	Hanger Hill Tower	37 42 5·265	4·72243576	52775·914	10·0	Supplementary.
	Banstead	26 28 2·629	4·58503696	38462·452	7·3	
	Hampton Poorhouse	115 49 52·536	4·89028797	77676·200	14·7	
		180 0 0·430				
48	Hanger Hill Tower	79 44 45·018	4·90386836	80143·509	15·2	”
	Banstead	27 45 4·321	4·57890399	37923·113	7·2	
	King's Arbour	72 30 11·343	4·89028797	77676·200	14·7	
		180 0 0·682				
49	Hanger Hill Tower	16 58 45·743	4·53728700	34457·757	6·5	”
	St. Ann's	18 44 50·548	4·57890399	37923·113	7·2	
	King's Arbour	144 16 23·888	4·83821699	68899·646	13·0	
		180 0 0·179				
50	Hanger Hill Tower	42 2 39·753	4·43785141	27406·363	5·2	”
	Hampton Poorhouse	67 55 31·052	4·57890399	37923·113	7·2	
	King's Arbour	70 1 49·425	4·58503696	38462·452	7·3	
		180 0 0·230				
51	Severndroog	83 6 26·482	4·94184086	87466·320	16·6	”
	Epping Poorhouse............	9 32 48·384	4·16471235	14612·091	2·8	
	Greenwich Transit	87 20 45·434	4·94452493	88008·564	16·7	
		180 0 0·300				
52	Severndroog	61 3 11·377	4·76227507	57846·232	11·0	”
	Chingford	12 46 13·449	4·16471235	14612·091	2·8	
	Greenwich Transit	106 10 35·365	4·80268862	63487·557	12·0	
		180 0 0·191				

FIGURE 17.

No. of Δ.	Names of Stations.	Corrected Angles.			Log. Distances.	Distances in		Error of Δ.
		°	′	″		Feet.	Miles.	″
1	Kippure	36	23	10·741	5·54986094	354699·80	67·2	−1·02
	Knockanaffrin	9	9	25·023	4·97839889	95147·83	18·0	
	Ballycreen	134	27	29·901	5·63020198	426777·96	80·8	
		180	0	5·665				
2	Knockanaffrin	37	58	23·372	5·35055683	224159·34	42·5	+1·56
	Ballycreen	38	49	52·261	5·35876256	228434·96	43·3	
	Forth	103	11	56·090	5·54986094	354699·80	67·2	
		180	0	11·723				
3	Forth	17	31	18·091	4·93750691	86597·81	16·4	−8·16
	Ballycreen	33	40	32·610	5·20274214	159493·19	30·2	
	Tara	128	48	11·831	5·35055683	224159·34	42·5	
		180	0	2·532				
4	Kippure	78	1	59·597	5·69180902	491823·21	93·1	+10·15
	Snowdon	20	35	4·862	5·24735469	176748·07	33·5	
	Tara	81	23	15·751	5·69642725	497081·10	94·1	
		180	0	20·210				
5	Kippure	48	2	54·992	5·64174764	438275·95	83·0	−0·31
	Snowdon	74	27	19·862	5·75418108	567781·29	107·5	
	Precelly	57	30	34·502	5·69642725	497081·10	94·1	
		180	0	49·356				
6	Snowdon	63	38	60·816	5·70085354	502173·21	95·1	Supplementary.
	Precelly	64	54	35·070	5·70545341	507520·29	96·1	
	Ballycreen	51	27	10·984	5·64174764	438275·95	83·0	
		180	0	46·870				
7	Forth	79	14	23·858	5·62735616	423990·54	80·3	+0·20
	Tara	79	4	28·934	5·62711621	423756·34	80·3	
	Precelly	21	41	22·825	5·20274214	159493·19	30·2	
		180	0	15·617				
8	Keeper	53	20	43·282	5·45880531	287610·88	54·5	Supplementary.
	Croghan	67	54	36·021	5·52139060	332193·09	62·9	
	Mount Leinster	58	44	59·903	5·48641948	306492·24	58·0	
		180	0	19·206				
9	Knockanaffrin	31	19	43·782	5·08098507	120499·45	22·8	„
	Forth	68	23	1·926	5·33335974	215456·57	40·8	
	Mount Leinster	80	17	20·310	5·35876256	228434·95	43·3	
		180	0	6·018				
10	Precelly	11	32	34·529	5·10837107	128342·67	24·3	„
	Tara	127	4	58·019	5·70903518	511723·29	96·9	
	Mount Leinster	41	22	37·664	5·62735616	423990·53	80·3	
		180	0	10·212				
11	Keeper	16	52	3·242	5·15497676	142881·75	27·1	„
	Mount Leinster	120	42	47·111	5·62672955	423379·23	80·2	
	Ballycreen	42	25	19·241	5·52139060	332193·09	62·9	
		180	0	9·594				

FIGURE 17—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
12	Snowdon	48 58 18·059	5·57129508	372644·81	70·6	Supplementary.
	Precelly	68 30 39·049	5·66242412	459646·67	87·1	
	Cradle	62 31 38·628	5·64174764	438275·95	83·0	
		180 0 35·736				
13	Snowdon	27 5 35·019	5·35027579	224014·32	42·4	”
	Cradle	42 2 55·895	5·51777889	329441·94	62·4	
	Longmount	110 51 45·303	5·66242412	459646·67	87·1	
		180 0 16·217				
14	Precelly	14 17 35·850	5·14507372	139660·54	26·5	”
	Snowdon	36 29 17·341	5·52685968	336402·86	63·7	
	Cader Idris	129 13 15·368	5·64174764	438275·95	83·0	
		180 0 8·559				
15	Cradle	47 23 2·012	5·37835517	238976·49	45·3	”
	Longmount	89 0 9·496	5·51147454	324694·21	61·5	
	Cader Idris	43 37 1·079	5·35027579	224014·32	42·4	
		180 0 12·587				
16	Precelly	28 33 59·798	5·35986096	229013·43	43·4	”
	Snowdon	37 39 11·932	5·46623233	292571·71	55·4	
	Plynlimmon	113 47 2·688	5·64174764	438275·95	83·0	
		180 0 14·418				
17	Cradle	52 52 21·599	5·31542799	206741·66	39·2	”
	Longmount	67 22 19·829	5·37902352	239344·54	45·3	
	Plynlimmon	59 45 28·625	5·35027579	224014·32	42·4	
		180 0 10·053				
18	Snowdon	32 11 18·332	5·28530600	192888·35	36·5	+2·14
	Longmount	33 17 31·998	5·29831997	198755·87	37·6	
	Cyrn-y-Brain	114 31 17·872	5·51777889	329441·94	62·4	
		180 0 8·202				
19	Longmount	55 8 5·923	5·32875363	213183·52	40·4	+0·17
	Cyrn-y-Brain	76 55 55·640	5·40328240	253094·32	47·9	
	Mowcopt	47 56 7·855	5·28530600	192888·35	36·5	
		180 0 9·418				
20	Longmount	59 11 13·019	5·44438327	278216·75	52·7	+6·67
	Cyrn-y-Brain	84 16 19·264	5·50829985	322329·35	61·0	
	Axedge	36 32 40·272	5·28530600	192888·35	36·5	
		180 0 12·555				
21	Whittle	102 1 58·344	5·52679449	336352·37	63·7	Supplementary.
	Holme Moss	61 52 7·841	5·48184452	303280·52	57·4	
	Cyrn-y-Brain	16 6 0·466	4·97940219	95367·89	18·1	
		180 0 6·651				
22	Cradle	57 14 32·787	5·29868196	198921·61	37·7	−2·57
	Longmount	51 29 20·618	5·26738020	185088·83	35·1	
	Malvern	71 16 14·795	5·35027579	224014·32	42·4	
		180 0 8·200				

FIGURE 17—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
23	Longmount	64 28 51·771	5·50264454	318159·24	60·3	+3·31
	Malvern........................	81 10 23·507	5·54205398	348380·62	66·0	
	Bardon Hill	34 20 59·428	5·29868196	198921·61	37·7	
		180 0 14·706				
24	Axedge	89 59 46·047	5·38966423	245281·18	46·5	−4·13
	Mowcopt	72 54 18·808	5·37003940	234444·15	44·4	
	Bardon Hill	17 5 59·120	4·85805603	72120·05	13·7	
		180 0 3·975				
25	Cradle	12 9 6·098	5·06810241	116977·52	22·2	+0·75
	Malvern........................	148 23 39·068	5·46425192	291240·60	55·2	
	Broadway Tower	19 27 17·503	5·26738020	185088·83	35·1	
		180 0 2·669				
26	Malvern........................	24 37 8·244	5·19936712	158258·53	30·0	+0·38
	Broadway Tower	137 26 52·673	5·40978719	256913·66	48·7	
	Arbury Hill	17 56 2·027	5·06810241	116977·52	22·2	
		180 0 2·944				
27	Broadway Tower	37 51 52·743	5·25641148	180472·68	34·2	+2·11
	Arbury Hill	109 34 21·528	5·44254429	277041·16	52·5	
	Bardon Hill	32 33 52·058	5·19936712	158258·53	30·0	
		180 0 6·329				

28	Whittle Hill	90 21 44·159	5·32208677	209935·93	39·8	Supplementary.
	Holme Moss	62 37 15·688	5·27049915	186422·85	35·3	
	Delamere, New..............	27 1 4·332	4·97940219	95367·89	18·1	
		180 0 4·179				
29	Whittle Hill	32 51 50·459	5·04460228	110815·95	21·0	+6·85
	Mowcopt	65 54 19·862	5·27049915	186422·85	35·3	
	Delamere, New..............	81 13 54·479	5·30498411	201829·25	38·2	
		180 0 4·800				
30	Holme Moss	31 46 25·880	5·04460228	110815·95	21·0	Supplementary.
	Mowcopt	94 0 48·410	5·32208677	209935·93	39·8	
	Delamere, New..............	54 12 50·147	5·23228182	170718·99	32·3	
		180 0 4·437				
31	Mowcopt	27 57 28·657	5·10196883	126464·56	23·9	−0·07
	Cyrn-y-Brain................	24 15 21·157	5·04460228	110815·95	21·0	
	Delamere, New..............	127 47 12·790	5·32875362	213183·51	40·4	
		180 0 2·604				

FIGURE 17—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
32	Delamere	43 7 32·762	4·98889949	97476·40	18·5	Supplementary.
	Cyrn-y-Brain	74 23 22·274	5·13777510	137333·06	26·0	
	Garreg	62 29 7·755	5·10196883	126464·56	24·0	
		180 0 2·791				
33	Cyrn-y-Brain	69 54 3·057	5·27627147	188917·19	35·8	„
	Snowdon	28 58 60·169	4·98889949	97476·40	18·5	
	Garreg	81 6 61·052	5·29831997	198755·87	37·6	
		180 0 4·278				
34	Cyrn-y-Brain	92 33 29·577	5·22743148	168822·95	32·0	„
	Garreg	52 12 56·343	5·12566706	133557·12	25·3	
	Arrenig	35 13 37·137	4·98889949	97476·40	18·5	
		180 0 3·057				
35	Cyrn-y-Brain	52 21 12·727	5·09247234	123729·24	23·4	„
	Arrenig	68 55 20·490	5·16378617	145809·62	27·6	
	Llanelian	58 43 30·408	5·12566706	133557·12	25·3	
		180 0 3·625				
36	Cyrn-y-Brain	40 12 16·850	4·97836909	95141·30	18·0	„
	Garreg	98 23 31·164	5·16378617	145809·62	27·6	
	Llanelian	41 24 14·144	4·98889949	97476·40	18·5	
		180 0 2·158				
37	Cyrn-y-Brain	7 56 63·398	4·54114933	34765·568	6·6	„
	Llanelian	27 30 35·233	5·06479815	116090·893	22·0	
	Moelfre Issa	144 32 21·920	5·16378617	145809·617	27·6	
		180 0 0·551				
38	Garreg	34 18 40·628	4·88724329	77133·545	14·6	„
	Llanelian	9 44 20·244	4·36449953	23147·257	4·4	
	Gwaunysgaer	135 56 59·420	4·97836909	95141·303	18·0	
		180 0 0·292				
39	Garreg	15 12 25·532	4·58979975	38886·581	7·4	„
	Llanelian	124 52 8·534	5·08504729	121631·844	23·0	
	Great Ormes Head	39 55 26·648	4·97836909	95141·303	18·0	
		180 0 0·714				
40	Llanelian	115 7 48·290	5·00020260	100046·661	18·9	„
	Gwaunysgaer	20 36 11·762	4·58979975	38886·581	7·4	
	Great Ormes Head	44 16 0·586	4·88724329	77133·545	14·6	
		180 0 0·638				
41	Great Ormes Head	24 51 15·776	4·67559345	47379·825	9·0	„
	Gwaunysgaer	37 42 35·400	4·83853260	68949·735	13·1	
	Moelfre Issa	117 26 9·506	5·00020260	100046·661	18·9	
		180 0 0·682				
42	Moelfre Issa	55 25 50·834	4·78330866	60716·771	11·5	„
	Great Ormes Head	13 48 46·884	4·24562642	17604·610	3·3	
	West End of Base	110 45 22·516	4·83853260	68949·735	13·1	
		180 0 0·234				

FIGURE 17—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
43	Moelfre Issa	62 0 18·672	4·62419845	42091·892	8·0	+6·60
	Gwaunysgaer..................	21 40 21·724	4·24562642	17604·610	3·3	
	West End of Base...........	96 19 19·777	4·67559345	47379·825	9·0	
		180 0 0·173				
44	Moelfre Issa	119 28 3·545	4·93004003	85121·649	16·1	Supplementary.
	Great Ormes Head	15 41 4·695	4·42211757	26431·242	5·0	
	East End of Base	44 50 52·133	4·83853260	68949·735	13·1	
		180 0 0·373				
45	Gwaunysgaer..................	24 13 54·614	4·38947788	24517·596	4·6	+0·02
	West End of Base...........	20 34 3·366	4·32193231	20986·128	4·0	
	East End of Base	135 12 2·105	4·62419845	42091·892	8·0	
		180 0 0·085				

FIGURE 18.

1	Fairlight	47 53 2·952	5·12162477	132319·78	25·1	−0·94
	Wrotham	46 37 57·496	5·11285752	129675·38	24·6	
	Frittenfield	85 29 3·576	5·24999568	177826·17	33·7	
		180 0 4·024				
2	Fairlight	31 34 5·384	4·88827416	77316·85	14·6	−2·64
	Frittenfield	87 1 37·311	5·16876477	147490·74	27·9	
	Paddlesworth.................	61 24 19·660	5·11285752	129675·38	24·6	
		180 0 2·355				
3	Beachy Head.................	79 54 17·969	5·44218593	276812·65	52·4	−2·91
	Leith Hill	50 20 48·030	5·33540257	216472·42	41·0	
	Frittenfield	49 45 4·759	5·33162235	214596·37	40·6	
		180 0 10·758				
4	Wrotham	38 43 56·576	5·01039376	102422·12	19·4	+0·70
	Epping Cupola	77 57 47·111	5·20438629	160098·14	30·3	
	Danbury Spire	63 18 19·759	5·16509372	146249·28	27·7	
		180 0 3·446				
5	Wrotham	83 28 21·903	5·29173176	195763·52	37·1	+1·97
	Danbury Spire	42 11 9·834	5·12162477	132319·78	25·1	
	Frittenfield	54 20 33·214	5·20438629	160098·14	30·3	
		180 0 4·951				
6	Danbury Spire	38 50 55·993	5·16371982	145787·34	27·6	+5·53
	Frittenfield	83 46 12·110	5·36369922	231046·41	43·8	
	St. Peter's Tower...........	57 22 58·570	5·29173176	195763·52	37·1	
		180 0 6·673				
7	Danbury Spire	52 7 22·869	5·26620760	184589·76	35·0	−8·58
	St. Peter's Tower...........	46 46 19·605	5·23145795	170395·43	32·3	
	Walton Tower	81 6 24·835	5·36369922	231046·41	43·8	
		180 0 7·309				

FIGURE 18—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
	Danbury Spire	90 58 18·862	5·41780956	261703·52	49·6	
8	Frittenfield	40 37 5·998	5·23145795	170395·43	32·3	Supplementary.
	Walton Tower	48 24 42·985	5·29173176	195763·52	37·1	
		180 0 7·845				
	Frittenfield	92 31 39·901	5·44112305	276136·01	52·3	
9	Walton Tower	16 14 38·849	4·88827416	77316·85	14·6	„
	Paddlesworth	71 13 46·004	5·41780957	261703·52	49·6	
		180 0 4·754				
	Walton Tower	39 29 47·507	5·09745610	125157·28	23·7	
10	Danbury Spire	80 30 60·516	5·28800512	194090·88	36·8	„
	Norwood	59 59 16·925	5·23145795	170395·43	32·3	
		180 0 4·948				
	Norwood	65 25 3·611	5·26620760	184589·76	35·0	
11	Walton Tower	41 36 37·329	5·12967533	134795·48	25·5	„
	St. Peter's Church	72 58 24·656	5·28800512	194090·88	36·8	
		180 0 5·596				
	Danbury Spire	52 38 28·180	5·11494225	130299·35	24·7	
12	Wrotham	49 46 20·159	5·09745610	125157·28	23·7	−2·55
	Norwood	77 35 15·407	5·20438629	160098·14	30·3	
		180 0 3·746				
	Epping Cupola	56 16 59·534	5·05351464	113113·55	21·4	
13	Danbury Spire	74 51 5·980	5·11814186	131262·86	24·9	−2·06
	Gads Hill	48 51 57·116	5·01039376	102422·12	19·4	
		180 0 2·630				
	Danbury Spire	41 5 41·960	4·92626915	84385·76	16·0	
14	Gads Hill	77 7 58·594	5·09745610	125157·28	23·7	+1·05
	Norwood	61 46 21·635	5·05351464	113113·55	21·4	
		180 0 2·189				
	Danbury Spire	39 21 54·991	4·97342235	94063·76	17·8	
15	Gads Hill	90 55 53·697	5·17109837	148285·39	28·1	Supplementary.
	Severndroog	49 42 13·815	5·05351464	113113·55	21·4	
		180 0 2·503				
	Danbury Spire	35 29 10·989	4·94452493	88008·56	16·7	
16	Severndroog	42 29 57·547	5·01039376	102422·12	19·4	„
	Epping Cupola	102 0 53·538	5·17109837	148285·39	28·1	
		180 0 2·074				
	Severndroog	35 10 46·416	4·73420839	54226·10	10·3	
17	Gads Hill	52 50 39·875	4·87513762	75013·19	14·2	„
	Wrotham	91 58 34·665	4·97342235	94063·76	17·8	
		180 0 0·956				
	Epping Cupola	72 16 7·329	5·12447598	133191·34	25·2	
18	Gads Hill	37 53 48·908	4·93395150	85891·76	16·3	„
	St. Paul's	69 50 6·289	5·11814186	131262·86	24·9	
		180 0 2·526				

FIGURE 18—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
19	Fairlight	80 59 34·574	5·21903782	165591·42	31·4	Supplementary.
	Frittenfield	48 20 35·851	5·09782678	125264·15	23·7	
	Crowborough	50 39 53·349	5·11285752	129675·38	24·6	
		180 0 3·774				
20	Fairlight	112 33 39·958	5·35641398	227202·96	43·0	”
	Paddlesworth	30 36 24·955	5·09782678,	125264·15	23·7	
	Crowborough	36 49 59·100	5·16876477	147490·74	27·9	
		180 0 4·013				
21	Frittenfield	37 8 27·725	4·99991518	99980·472	18·9	”
	Wrotham	89 49 5·408	5·21903782	165591·416	31·4	
	Crowborough	53 2 29·979	5·12162477	132319·780	25·1	
		180 0 3·112				
22	Frittenfield	49 22 33·790	5·04938793	112043·83	21·2	+0·30
	St. Peter's Church	31 35 4·223	4·88827416	77316·85	14·6	
	Paddlesworth	99 2 23·999	5·16371982	145787·34	27·6	
		180 0 2·012				
23	Frittenfield	66 25 3·352	5·12967533	134795·48	25·5	+4·12
	St. Peter's Church	31 10 53·520	4·88166881	76149·81	14·4	
	Norwood	82 24 5·521	5·16371982	145787·34	27·6	
		180 0 2·393				

FIGURE 19.

1	Dunstable	47 34 45·525	5·07797421	119666·95	22·7	+3·81
	Epping Cupola	51 8 17·359	5·10114199	126224·01	23·9	
	Tharfield	81 16 60·628	5·20474992	160232·25	30·3	
		180 0 3·512				
2	Epping Cupola	45 41 15·573	4·94429699	87962·38	16·7	−0·66
	Tharfield	57 32 47·274	5·01591555	103732·67	19·6	
	Thaxted Spire	76 45 59·242	5·07797421	119666·95	22·7	
		180 0 2·089				
3	Dunstable	49 13 30·774	4·98841660	97368·08	18·4	Supplementary.
	Tharfield	51 45 4·204	5·00421072	100974·27	19·1	
	Berkhampstead	79 1 27·292	5·10114199	126224·01	23·9	
		180 0 2·270				
4	Thaxted Spire	49 31 4·667	4·98841660	97368·08	18·4	”
	Tharfield	87 4 43·698	5·10669127	127847·21	24·2	
	Berkhampstead	43 24 13·647	4·94429698	87962·38	16·7	
		180 0 2·012				
5	Epping Cupola	58 56 47·503	5·00620001	101437·84	19·2	−0·66
	Thaxted Spire	59 52 57·261	5·01039376	102422·12	19·4	
	Danbury Spire	61 10 17·377	5·01591555	103732·67	19·6	
		180 0 2·141				

FIGURE 19—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
6	Danbury Spire	66 42 47·863	5·09517157	124500·64	23·6	Supplementary.
	Thaxted Spire	64 50 11·635	5·08877019	122678·99	23·2	
	Stoke Tower	48 27 3·190	5·00620001	101437·84	19·2	
		180 0 2·688				
7	Danbury Spire	35 39 6·305	5·00243148	100561·44	19·0	+3·02
	Stoke Tower	99 1 43·139	5·23145795	170395·43	32·3	
	Walton Tower	45 19 13·422	5·08877019	122678·99	23·2	
		180 0 2·866				
8	Walton Tower	54 28 28·535	4·97370309	94124·59	17·8	+2·02
	Stoke Tower	65 7 25·350	5·02086658	104922·00	19·9	
	Otley Tower	60 24 8·135	5·00243148	100561·44	19·0	
		180 0 2·020				
9	Walton Tower	32 41 53·678	4·79746879	62729·06	11·9	−1·21
	Otley Tower	82 40 13·844	5·06134083	115170·39	21·8	
	Naughton Tower	64 37 54·013	5·02086658	104922·01	19·9	
		180 0 1·535				
10	Thaxted Spire	34 29 3·462	4·85277347	71248·13	13·5	Supplementary.
	Stoke Tower	63 53 4·716	5·05305260	112993·28	21·4	
	Lawshall Tower..............	81 37 53·695	5·09517157	124500·64	23·6	
		180 0 1·873				
11	Stoke Tower	50 7 45·635	4·73829143	54738·316	10·4	−4·35
	Lawshall Tower..............	37 16 54·184	4·63549885	43201·502	8·2	
	Naughton Tower	92 35 20·737	4·85277347	71248·130	13·5	
		180 0 0·556				
12	Thaxted Spire	53 59 37·427	4·96099533	91410·34	17·3	+0·31
	Lawshall Tower..............	35 27 7·452	4·81651580	65541·41	12·4	
	Balsham Tower..............	90 33 16·530	5·05305260	112993·28	21·4	
		180 0 1·409				
13	Tharfield	43 12 24·909	4·81651580	65541·41	12·4	+0·65
	Thaxted Spire	70 2 10·972	4·95414333	89979·45	17·0	
	Balsham Tower..............	66 45 25·393	4·94429698	87962·38	16·7	
		180 0 1·274				
14	Tharfield	36 21 30·859	4·99007870	97741·43	18·5	−4·51
	Balsham Tower..............	110 33 55·795	5·18854762	154364·57	29·2	
	Ely Minster	33 4 35·283	4·95414333	89979·45	17·0	
		180 0 1·937				
15	Danbury Spire	40 11 35·458	5·06134083	115170·39	21·8	−1·48
	Walton Tower	67 5 48·280	5·21587310	164389·13	31·1	
	Naughton Tower	72 42 40·514	5·23145795	170395·43	32·3	
		180 0 4·252				
16	Danbury Spire	62 10 18·710	5·16867489	147460·22	27·9	Supplementary.
	Naughton Tower	37 28 12·538	5·00620001	101437·84	19·2	
	Thaxted Spire	80 21 32·220	5·21587310	164389·13	31·1	
		180 0 3·468				

FIGURE 19—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
17	Danbury Spire	19 12 41·754	4·73829143	54738·31	10·4	Supplementary.
	Naughton Tower	79 35 43·054	5·21382252	163614·78	31·0	
	Lawshall Tower	81 11 37·273	5·21587310	164389·13	31·1	
		180 0 2·081				
18	Balsham Tower	61 9 10·504	5·25565212	180157·40	34·1	„
	Lawshall Tower...............	92 27 39·775	5·31279324	205491·21	38·9	
	Swaffham Spire...............	26 23 13·590	4·96099533	91410·34	17·3	
		180 0 3·869				
19	Lawshall Tower...............	37 4 43·021	5·05944028	114667·48	21·7	−4·64
	Swaffham Spire...............	34 13 40·684	5·02929982	106979·32	20·3	
	South Lopham Tower	108 41 39·028	5·25565212	180157·40	34·1	
		180 0 2·733				
20	Lawshall Tower...............	48 45 58·349	4·92390084	83926·83	15·9	+0·35
	South Lopham Tower	57 46 52·628	4·97504827	94416·58	17·9	
	Brandon........................	73 27 10·810	5·02929982	106979·32	20·3	
		180 0 1·787				
21	Brandon........................	82 33 23·173	5·05944028	114667·48	21·7	+0·18
	South Lopham Tower	50 54 46·400	4·95308184	89759·79	17·0	
	Swaffham Spire...............	46 31 52·184	4·92390084	83926·83	15·9	
		180 0 1·757				
22	Brandon........................	101 33 15·645	5·11948288	131668·80	24·9	−2·81
	Swaffham Spire...............	36 32 28·279	4·90318158	80016·87	15·2	
	Ely Minster	41 54 17·731	4·95308184	89759·79	17·0	
		180 0 1·655				
23	Tharfield	20 17 0·972	4·90318158	80016·87	15·2	−4·51
	Ely Minster	117 44 42·017	5·31023174	204282·77	38·7	
	Brandon........................	41 58 19·582	5·18854762	154364·57	29·2	
		180 0 2·571				
24	Lawshall Tower...............	42 2 30·025	4·86177302	72739·95	13·8	−4·05
	South Lopham Tower	57 55 54·994	4·96400979	92047·03	17·4	
	Mickfield Tower	80 1 36·532	5·02929982	106979·32	20·3	
		180 0 1·551				
25	Lawshall Tower...............	34 3 11·847	4·74707502	55856·67	10·6	+0·04
	Mickfield Tower	33 16 53·244	4·73829143	54738·31	10·4	
	Naughton Tower	112 39 55·573	4·96400979	92047·03	17·4	
		180 0 0·664				
26	Lawshall Tower...............	76 5 41·873	5·03272163	107825·54	20·4	Supplementary.
	South Lopham Tower	29 31 24·778	4·73829143	54738·31	10·4	
	Naughton Tower	74 22 54·686	5·02929982	106979·32	20·3	
		180 0 1·337				
27	Naughton Tower	30 23 46·845	4·50225966	31787·741	6·0	+6·16
	Mickfield Tower	86 50 39·297	4·79746879	62729·062	11·9	
	Otley Tower	62 45 34·275	4·74707502	55856·667	10·6	
		180 0 0·417				

FIGURE 19—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
28	Ely Minster	36 31 35·533	4·76489460	58196·19	11·0	Supplementary.
	Balsham Tower...............	51 55 48·690	4·88635394	76975·75	14·6	
	Cambridge Obs^{y} Dome......	91 32 36·830	4·99007870	97741·43	18·5	
		180 0 1·053				
29	Ely Minster	121 11 42·267	5·13601911	136778·90	25·9	”
	Brandon........................	28 46 37·951	4·88635394	76975·75	14·6	
	Cambridge Obs^{y} Dome......	30 1 41·021	4·90318158	80016·87	15·2	
		180 0 1·239				
	FIGURE 20.					
1	Walton Tower	41 24 14·662	4·86111526	72629·87	13·8	+2·62
	Otley Tower	65 46 12·132	5·00062260	100143·46	19·0	
	Orford Castle.................	72 49 34·840	5·02086658	104922·01	19·9	
		180 0 1·634				
2	Orford Castle.................	48 4 45·204	4·81191126	64850·19	12·3	−1·60
	Otley Tower	75 28 43·276	4·92619812	84371·96	16·0	
	Laxfield Tower	56 26 32·592	4·86111526	72629·87	13·8	
		180 0 1·072				
3	Orford Castle.................	47 0 50·713	4·84828369	70515·35	13·4	+3·49
	Laxfield Tower	71 54 34·442	4·96204025	91630·54	17·4	
	Southwold Tower	61 4 36·175	4·92619812	84371·95	16·0	
		180 0 1·330				
4	Southwold Tower	77 19 31·894	4·93547002	86192·61	16·3	−4·60
	Laxfield Tower	49 43 5·590	4·82863613	67396·31	12·8	
	Tofts Tower	52 57 23·607	4·84828369	70515·35	13·4	
		180 0 1·091				
5	Laxfield Tower..............	69 40 51·417	4·99554995	98980·57	18·7	−0·08
	Tofts Tower	55 34 19·005	4·93981963	87060·19	16·5	
	Bunwell Tower	54 44 51·233	4·93547002	86192·61	16·3	
		180 0 1·655				
6	Laxfield Tower	39 10 37·852	4·88674290	77044·72	14·6	+0·87
	Tofts Tower	95 51 14·336	5·08394854	121324·51	23·0	
	Norwich Spire	44 58 9·366	4·93547002	86192·61	16·3	
		180 0 1·554				
7	Laxfield Tower	30 30 13·565	4·80627646	64014·22	12·1	−1·32
	Norwich Spire	43 39 23·907	4·93981963	87060·19	16·5	
	Bunwell Tower..............	105 50 23·789	5·08394854	121324·51	23·0	
		180 0 1·261				
8	Laxfield Tower	30 1 59·473	4·66092175	45805·934	8·7	+0·18
	Bunwell Tower..............	77 55 40·856	4·95180537	89496·359	16·9	
	South Lopham Tower	72 2 20·588	4·93981963	87060·194	16·5	
		180 0 0·917				

FIGURE 20—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
9	South Lopham Tower	42 59 51·047	4·66154706	45871·935	8·7	+1·16
	Bunwell Tower	94 4 54·950	4·82668126	67093·625	12·7	
	Hingham Tower	42 55 14·496	4·66092175	45805·935	8·7	
		180 0 0·493				
10	South Lopham Tower	44 36 57·653	4·80279878	63503·66	12·0	+6·85
	Mickfield Tower	81 49 18·416	4·95180537	89496·36	16·9	
	Laxfield Tower	53 33 45·006	4·86177302	72739·95	13·8	
		180 0 1·075				
11	Norwich Spire	38 11 55·911	4·66154706	45871·935	8·7	+0·26
	Bunwell Tower	82 9 0·406	4·86619434	73484·263	13·9	
	Hingham Tower	59 39 4·367	4·80627646	64014·220	12·1	
		180 0 0·684				
12	Norwich Spire	86 33 59·266	5·07592535	119103·73	22·6	+3·84
	Hingham Tower	55 25 7·429	4·99227458	98236·88	18·6	
	Baconsthorpe Tower	38 0 55·000	4·86619434	73484·26	13·9	
		180 0 1·695				
13	Baconsthorpe Tower	30 48 34·891	4·84296670	69657·31	13·2	+3·48
	Hingham Tower	88 3 19·557	5·13328924	135921·84	25·7	
	Swaffham Spire	61 8 7·502	5·07592535	119103·73	22·6	
		180 0 1·950				
14	Baconsthorpe Tower	56 9 36·622	4·93947357	86990·85	16·5	+3·95
	Norwich Spire	54 7 33·750	4·92873302	84865·86	16·1	
	Happisburgh Tower	69 42 51·257	4·99227458	98236·88	18·6	
		180 0 1·629				
15	Happisburgh Tower	61 27 4·117	4·98234618	96016·57	18·2	−8·34
	Norwich Spire	65 48 53·410	4·99875162	99712·96	18·9	
	Gorleston Tower	52 44 4·265	4·93947357	86990·85	16·5	
		180 0 1·792				
16	Happisburgh Tower	40 26 9·157	4·88674290	77044·71	14·6	−7·38
	Norwich Spire	92 28 57·800	5·07436170	118675·67	22·5	
	Tofts Tower	47 4 54·618	4·93947357	86990·85	16·5	
		180 0 1·575				
17	Southwold Tower	26 43 35·598	4·64320396	43974·809	8·3	−1·58
	Tofts Tower	109 42 5·896	4·96405257	92056·100	17·4	
	Gorleston Tower	43 34 19·162	4·82863613	67396·312	12·8	
		180 0 0·656				
18	Swaffham Spire	66 33 10·847	5·21825628	165293·69	31·3	Supplementary.
	Lawshall Tower	22 44 40·073	4·84296670	69657·31	13·2	
	Hingham Tower	90 42 11·788	5·25565212	180157·40	34·1	
		180 0 2·708				
19	Swaffham Spire	78 51 22·347	5·01043112	102430·93	19·4	−0·49
	Brandon	41 51 10·627	4·84296670	69657·31	13·2	
	Hingham Tower	59 17 28·468	4·95308184	89759·79	17·0	
		180 0 1·442				

FIGURE 20—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
20	Brandon	64 35 37·694	5·05417261	113285·05	21·5	Supplementary.
	Swaffham Spire	69 42 19·639	5·07051280	117628·57	22·3	
	Bunwell Tower	45 42 4·910	4·95308184	89759·79	17·0	
		180 0 2·243				
21	South Lopham Tower	76 43 7·737	5·05417261	113285·053	21·5	”
	Swaffham Spire	23 10 27·455	4·66092175	45805·934	8·7	
	Bunwell Tower	80 6 26·010	5·05944028	114667·483	21·7	
		180 0 1·202				
22	Swaffham Spire	70 17 10·210	5·16028047	144637·36	27·4	”
	Bunwell Tower	62 12 36·729	5·13328924	135921·84	25·7	
	Baconsthorpe Tower	47 30 16·470	5·05417261	113285·05	21·5	
		180 0 3·409				
23	Bunwell Tower	38 16 34·027	4·80279878	63503·66	12·0	+1·16
	Laxfield Tower	83 35 44·479	5·00807356	101876·39	19·3	
	Mickfield Tower	58 7 42·786	4·93981963	87060·19	16·5	
		180 0 1·292				
	FIGURE 21.					
1	Great Whernside	44 55 11·738	5·21002890	162191·80	30·7	−2·73
	Botton Head	65 58 43·267	5·32181280	209803·53	39·7	
	York Minster	69 6 12·468	5·33160691	214588·73	40·6	
		180 0 7·473				
2	Botton Head	26 11 14·503	4·85478165	71578·34	13·6	+5·99
	York Minster	63 20 41·386	5·16124532	144959·04	27·5	
	Acklam Wold	90 28 6·550	5·21002890	162191·80	30·7	
		180 0 2·439				
3	Great Whernside	33 21 18·282	5·16124532	144959·04	27·5	Supplementary.
	Botton Head	92 9 57·770	5·42071759	263461·76	49·9	
	Acklam Wold	54 28 51·256	5·33160691	214588·73	40·6	
		180 0 7·308				
4	Great Whernside	36 35 59·612	5·27097898	186628·93	35·3	”
	York Minster	101 18 56·589	5·48705434	306940·61	58·1	
	Clifton Beacon	42 5 12·826	5·32181280	209803·53	39·7	
		180 0 9·027				
5	Great Whernside	48 9 53·068	5·37310914	236107·15	44·7	”
	Clifton Beacon	56 14 27·573	5·42071759	263461·76	49·9	
	Acklam Wold	75 35 53·523	5·48705434	306940·61	58·1	
		180 0 14·164				
6	Great Whernside	37 5 48·307	5·20353963	159786·33	30·3	”
	Acklam Wold	46 53 37·832	5·28648277	193411·71	36·6	
	Garforth Cliff	96 0 41·087	5·42071759	263461·76	49·9	
		180 0 7·226				

FIGURE 21—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
	Great Whernside	27 15 26·268	5·16845706	147386·28	27·9	
7	Clifton Beacon	45 15 10·054	5·35900153	228560·69	43·3	Supplementary.
	Holme Moss	107 29 31·232	5·48705434	306940·61	58·1	
		180 0 7·554				
	Great Whernside	38 19 31·029	5·15361486	142434·39	27·0	
8	Holme Moss	57 21 30·977	5·28648277	193411·71	36·6	„
	Garforth Cliff	84 18 64·439	5·35900153	228560·69	43·3	
		180 0 6·445				
	Great Whernside	25 31 54·851	4·95674278	90519·63	17·1	
9	York Minster	67 3 44·191	5·28648277	193411·71	36·6	„
	Garforth Cliff	87 24 25·070	5·32181280	209803·53	39·7	
		180 0 4·112				
	Clifton Beacon	33 26 44·606	5·20676564	160977·67	30·5	
10	Acklam Wold	20 29 28·839	5·00964557	102245·82	19·4	−4·95
	Crowle	126 3 49·683	5·37310914	236107·15	44·7	
		180 0 3·128				
	Acklam Wold	49 11 44·530	5·12554721	133520·27	25·3	
11	Crowle	64 56 16·611	5·20353963	159786·33	30·3	−4·54
	Garforth Cliff	65 52 3·436	5·20676564	160977·67	30·5	
		180 0 4·577				
	Holme Moss	48 19 5·713	4·93230992	85567·71	16·2	
12	Axedge	31 22 19·947	4·77557631	59645·31	11·3	Supplementary.
	Back Tor	100 18 35·521	5·05200814	112721·86	21·3	
		180 0 1·181				
	Bardon Hill	17 52 13·991	4·93230993	85567·71	16·2	
13	Axedge	104 54 36·554	5·43049548	269460·73	51·0	„
	Back Tor	57 13 14·013	5·37003940	234444·15	44·4	
		180 0 4·558				
	Holme Moss	37 41 1·543	5·02780527	106611·80	20·2	
14	Clifton Beacon	19 59 56·027	4·77557631	59645·31	11·3	„
	Back Tor	122 18 63·694	5·16845706	147386·28	27·9	
		180 0 1·264				
	Bardon Hill	77 58 23·981	5·49065668	309497·17	58·6	
15	Axedge	54 13 25·153	5·40947466	256728·84	48·6	+5·48
	Lincoln Minster	47 48 24·707	5·37003940	234444·15	44·4	
		180 0 13·841				
	Bardon Hill	60 6 9·990	5·42115715	263728·55	49·9	
16	Back Tor	57 33 20·301	5·40947466	256728·84	48·6	Supplementary.
	Lincoln Minster	62 20 43·810	5·43049548	269460·73	51·0	
		180 0 14·101				
	Crowle	94 44 50·967	5·42115715	263728·55	49·9	
17	Back Tor	34 51 39·557	5·17972460	151260·18	28·6	„
	Lincoln Minster	50 23 36·702	5·30938462	203884·69	38·6	
		180 0 7·226				

FIGURE 21—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in Feet.	Distances in Miles.	Error of Δ.
		° ′ ″				″
18	Clifton Beacon	61 34 32·116	5·17972459	151260·18	28·6	+0·28
	Crowle	81 56 61·227	5·23121621	170300·61	32·2	
	Lincoln Minster	36 28 30·257	5·00964557	102245·82	19·4	
		180 0 3·600				
19	Bardon Hill	24 47 23·580	4·90392124	80153·27	15·2	−7·17
	Arbury Hill	45 57 24·374	5·13802435	137411·90	26·0	
	Naseby Tower	109 15 14·491	5·25641148	180472·68	34·2	
		180 0 2·445				
20	Arbury Hill	92 23 41·831	5·34135081	219457·69	41·6	Supplementary.
	Dunstable	21 24 9·599	4·90392124	80153·27	15·2	
	Naseby Tower	66 12 12·355	5·30314247	200975·20	38·1	
		180 0 3·785				
21	Arbury Hill	76 57 40·328	5·04062719	109806·28	20·8	,,
	Naseby Tower	57 42 43·557	4·97901999	95284·00	18·0	
	Hanslope Spire	45 19 37·865	4·90392124	80153·27	15·2	
		180 0 1·750				
22	Arbury Hill	15 26 1·503	5·04935185	112034·51	21·2	,,
	Dunstable	13 4 53·127	4·97901999	95284·00	18·0	
	Hanslope Spire	151 29 6·568	5·30314247	200975·20	38·1	
		180 0 1·198				
23	Dunstable	99 15 3·892	5·25946006	181743·98	34·4	,,
	Tharfield	37 28 33·397	5·04935185	112034·51	21·2	
	Hanslope Spire	43 16 25·994	5·10114199	126224·02	23·9	
		180 0 3·283				
24	Dunstable	53 45 36·563	5·08849418	122601·04	23·2	+2·27
	Tharfield	70 6 9·565	5·15513234	142932·94	27·1	
	Keysoe Spire	56 8 17·295	5·10114199	126224·02	23·9	
		180 0 3·423				
25	Naseby Tower	39 17 9·268	5·15513234	142932·94	27·1	Supplementary.
	Dunstable	37 10 10·858	5·13476222	136383·62	25·8	
	Keysoe Spire	103 32 44·331	5·34135081	219457·69	41·6	
		180 0 4·457				
26	Dunstable	45 29 27·330	5·01121272	102615·43	19·4	,,
	Keysoe Spire	51 7 56·573	5·04935185	112034·51	21·2	
	Hanslope Spire	83 22 38·783	5·15513234	142932·94	27·1	
		180 0 2·686				
27	Tharfield	32 37 36·168	5·01121272	102615·43	19·4	,,
	Hanslope Spire	40 6 12·789	5·08849418	122601·04	23·2	
	Keysoe Spire	107 16 13·868	5·25946006	181743·98	34·4	
		180 0 2·825				
28	Tharfield	71 30 6·766	5·21447396	163860·38	31·0	+5·73
	Ely Minster	45 11 54·178	5·08849418	122601·04	23·2	
	Keysoe Spire	63 18 3·277	5·18854762	154364·57	29·2	
		180 0 4·221				

FIGURE 21—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
29	Balsham Tower..............	68 7 2·809	5·21447396	163860·38	31·0	Supplementary.
	Ely Minster	78 16 29·461	5·23779199	172898·81	32·7	
	Keysoe Spire..................	33 36 31·418	4·99007870	97741·43	18·5	
		180 0 3·688				
30	Ely Minster	45 39 33·226	5·14656688	140141·53	26·5	−11·14
	Keysoe Spire..................	77 35 49·577	5·28188890	191376·63	36·2	
	Easton Tower	56 44 42·471	5·21447396	163860·38	31·0	
		180 0 5·274				
31	Naseby Tower	61 39 20·814	5·14656688	140141·53	26·5	−4·23
	Keysoe Spire..................	59 25 5·519	5·13698375	137083·04	26·0	
	Easton Tower	58 55 37·536	5·13476222	136383·62	25·8	
		180 0 3·869				
32	Easton Tower	101 14 18·513	5·24153707	174396·22	33·0	+3·65
	Keysoe Spire..................	26 44 48·893	4·90320291	80020·80	15·2	
	Tilton..........................	52 0 55·181	5·14656688	140141·53	26·5	
		180 0 2·587				
33	Naseby Tower	96 18 59·468	5·24153707	174396·22	33·0	−4·95
	Keysoe Spire..................	32 40 16·627	4·97642582	94716·54	17·9	
	Tilton..........................	51 0 46·925	5·13476222	136383·62	25·8	
		180 0 3·020				
34	Bardon Hill	43 31 25·417	4·97642582	94716·54	17·9	+2·19
	Naseby Tower	48 56 24·418	5·01580934	103707·30	19·6	
	Tilton	87 32 12·473	5·13802435	137411·90	26·0	
		180 0 2·308				
35	Bardon Hill	26 36 13·864	4·82755280	67228·40	12·7	Supplementary.
	Tilton..........................	109 42 8·523	5·15025296	141336·05	26·8	
	Buckminster Spire	43 41 39·156	5·01580934	103707·30	19·6	
		180 0 1·543				
36	Bardon Hill	35 15 36·233	5·21267626	163183·51	30·9	„
	Lincoln Minster	30 0 0·220	5·15025297	141336·05	26·8	
	Buckminster Spire	114 44 28·472	5·40947466	256728·84	48·6	
		180 0 4·925				
37	Lincoln Minster	63 3 55·211	5·21044044	162345·56	30·7	„
	Buckminster Spire	53 16 57·852	5·16426204	145969·47	27·6	
	Boston Tower	63 39 11·930	5·21267626	163183·51	30·9	
		180 0 4·993				
38	Buckminster Spire	79 32 41·836	5·21955800	165789·87	31·4	„
	Boston Tower	26 5 50·139	4·87017548	74160·98	14·0	
	Easton Tower	74 21 30·809	5·21044044	62345·56	30·7	
		180 0 2·784				
39	Boston Tower	71 22 34·564	5·22003868	165973·47	31·4	+3·01
	Easton Tower	37 26 8·598	5·02720579	106464·74	20·2	
	Walpole, St. Peter's.........	71 11 20·772	5·21955800	165789·87	31·4	
		180 0 3·934				

FIGURE 21—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.			Log. Distances.	Distances in		Error of Δ.
		°	′	″		Feet.	Miles.	″
40	Easton Tower	38	41	28·098	5·08198970	120778·52	22·9	+2·96
	Ely Minster	59	12	30·837	5·22003868	165973·47	31·4	
	Walpole, St. Peter's.........	82	6	5·735	5·28188890	191376·63	36·2	
		180	0	4·670				
41	Ely Minster	50	17	2·012	5·03224369	107706·94	20·4	−0·50
	Swaffham Spire...............	59	36	24·346	5·08198970	120778·52	22·9	
	Walpole, St. Peter's.........	70	6	36·519	5·11948288	131668·80	24·9	
		180	0	2·877				
42	Boston Tower	21	49	58·533	5·03224369	107706·94	20·4	+1·47
	Walpole, St. Peter's.........	136	35	56·974	5·29883936	198993·71	37·7	
	Swaffham Spire...............	21	34	6·345	5·02720579	106464·74	20·2	
		180	0	1·852				
43	Ely Minster	17	24	13·150	4·60131725	39931·649	7·6	−2·69
	Walpole, St. Peter's.........	97	49	5·140	5·12144542	132265·147	25·1	
	Lynn Tower	64	46	42·834	5·08198970	120778·519	22·9	
		180	0	1·124				
44	Boston Tower	17	33	34·183	4·60131725	39931·649	7·6	+2·12
	Walpole, St. Peter's.........	108	53	28·353	5·09770320	125228·506	23·7	
	Lynn Tower	53	32	58·410	5·02720579	106464·738	20·2	
		180	0	0·946				
45	Walpole, St. Peter's.........	27	42	28·621	4·87331191	74698·505	14·1	−2·23
	Swaffham Spire...............	14	23	31·722	4·60131725	39931·649	7·6	
	Lynn Tower	137	53	60·127	5·03224369	107706·941	20·4	
		180	0	0·470				
46	Boston Tower	47	57	6·336	5·03597279	108635·75	20·6	−5·85
	Walpole, St. Peter's.........	85	21	5·834	5·16379957	145814·11	27·6	
	Docking Tower...............	46	41	50·541	5·02720579	106464·74	20·2	
		180	0	2·711				
47	Walpole, St. Peter's.........	51	14	51·141	4·97110385	93562·94	17·7	Supplementary.
	Swaffham Spire...............	64	53	21·776	5·03597279	108635·75	20·6	
	Docking Tower...............	63	51	49·229	5·03224369	107706·94	20·4	
		180	0	2·146				
48	Swaffham Spire...............	58	58	15·751	5·07488551	118818·89	22·5	„
	Baconsthorpe Tower.........	42	26	9·389	4·97110385	93562·94	17·7	
	Docking Tower...............	78	35	37·423	5·13328924	135921·84	25·7	
		180	0	2·563				
49	Lynn Tower	78	7	27·454	4·97110385	93562·94	17·7	„
	Swaffham Spire...............	50	29	50·055	4·86788852	73771·48	14·0	
	Docking Tower...............	51	22	43·759	4·87331191	74698·50	14·1	
		180	0	1·268				
50	Boston Tower	30	23	32·154	4·86788851	73771·48	14·0	−1·47
	Lynn Tower	90	25	34·008	5·16379957	145814·11	27·6	
	Docking Tower...............	59	10	56·010	5·09770320	125228·50	23·7	
		180	0	2·172				

FIGURE 21—*continued.*

No. of Δ.	Names of Stations.	Corrected Angles.	Log. Distances.	Distances in		Error of Δ.
				Feet.	Miles.	
		° ′ ″				″
51	Crowle Beacon	34 51 17·498	4·99778388	99491·02	18·8	Supplementary.
	Lincoln Minster	25 28 23·073	4·87432350	74872·70	14·2	
	Gringley Beacon	119 40 20·951	5·17972459	151260·17	28·6	
		180 0 1·522				
52	Crowle Beacon	47 5 43·729	4·87551675	75078·70	14·2	−2·55
	Clifton Beacon	46 55 35·760	4·87432350	74872·70	14·2	
	Gringley Beacon	85 58 41·829	5·00964557	102245·82	19·4	
		180 0 1·318				
53	Clifton Beacon	34 44 43·693	4·64684011	44344·535	8·4	+9·25
	Gringley Beacon	70 28 19·615	4·86528943	73331·308	13·9	
	North End of Base	74 46 57·430	4·87551675	75078·701	14·2	
		180 0 0·738				
54	Clifton Beacon	20 47 13·410	4·42067955	26343·869	5·0	+5·49
	North End of Base	60 17 18·138	4·80936456	64471·023	12·2	
	South End.....................	98 55 28·847	4·86528943	73331·308	13·9	
		180 0 0·395				
55	Gringley Beacon	19 17 21·488	4·42067955	26343·869	5·0	+0·90
	North End of Base	14 29 39·293	4·30015176	19959·597	3·8	
	South End.....................	146 12 59·288	4·64684011	44344·535	8·4	
		180 0 0·069				

The triangles for the connection of special points with the general triangulation will be found in the divisions or Figures into which they naturally fall. Thus, the triangles for the connection of the extremities of the Misterton Carr Base are given at the end of Figure 21.

DETERMINATION OF THE MERIDIONAL DISTANCES

OF

HENSBARROW AND BEN HUTIG,

DUNNOSE AND SAXAVORD.

The first of the following tables contains the calculation of the successive distances Hensbarrow — Precelly, Precelly — South Berule, South Berule — Ben Lomond, Ben Lomond—Ben Wyvis.

The second contains the calculation of the successive distances Dunnose—Arbury, Arbury—Clifton, and Clifton—Burleigh Moor.

HENSBARROW TO BEN WYVIS.

Names of Stations.	Spherical Angles.	Logarithmic Sines of Reduced Angles.	Log. Distances.	Distances in Feet.	References.
Hensbarrow to Precelly.					
	° ′ ″				
Precelly	5 44 35·498	9·00027660	5·45940875	288010·79	△ 7, page 461
Lundy Island	168 34 9·147	9·29708219	5·75621434	570445·74	
Hensbarrow.........	5 41 19·185	8·99614680	5·45527895	285285·01	△ 2 „ 460
	180 0 3·830				
Precelly	22 7 14·034	9·57578754	5·33808015	217811·17	△ 7 „ 461
High Willhays	99 33 57·294	9·99392173	5·75621434	570445·74	
Hensbarrow.........	58 19 13·545	9·92991798	5·69221059	492278·19	△ 9 „ 461
	180 0 24·873				
Precelly to South Berule.					
South Berule	44 27 23·532	9·84527835	5·75418108	567781·29	△ 5, page 479
Kippure	96 57 34·964	9·99679384	5·90569657	804815·95	
Precelly	38 36 8·536	9·79506436	5·70396709	505786·34	△ 1 „ 433
	180 0 67·032				
South Berule	20 0 1·984	9·53401134	5·64174764	438275·95	△ 5 „ 479
Snowdon	141 5 58·925	9·79796027	5·90569657	804815·95	
Precelly	18 54 25·964	9·51053887	5·61827517	415217·05	△ 11 „ 434
	180 0 26·873				

HENSBARROW TO BEN WYVIS—*continued.*

Names of Stations.	Spherical Angles.	Logarithmic Sines of Reduced Angles.	Log. Distances.	Distances in Feet.	References.
SOUTH BERULE TO BEN LOMOND.					
	° ′ ″				
Merrick	117 1 14·715	9·94980619	5·70027911	501509·43	
Ben Lomond	19 46 33·247	9·52932612	5·27979904	190457·92	△ 33, page 440
Criffel	43 12 27·406	9·83545330	5·58592621	385412·87	△ 16 „ 457
	180 0 15·368				
Criffel	118 6 10·698	9·94553320	5·87241318	745440·84	
Ben Lomond	25 30 1·296	9·63393441	5·56081440	363759·55	△ 16 „ 435
South Berule	36 24 25·830	9·77339912	5·70027911	501509·43	
	180 0 37·824				
Ben Lomond	5 43 28·049	8·99884183	5·56108339	363984·92	△ 10 „ 434
South Berule	6 3 45·541	9·02368465	5·58592621	385412·87	△ 16 „ 457
Merrick	168 12 53·144	9·31017162	5·87241318	745440·84	
	180 0 6·734				
Merrick	85 13 5·470	9·99848388	5·69804595	498937·26	
Knocklayd	46 38 11·740	9·86152132	5·56108339	363984·92	△ 10 „ 434
South Berule	48 9 14·586	9·87210203	5·57166410	372961·58	△ 10 „ 456
	180 0 31·796				
Knocklayd	96 12 22·447	9·99745167	5·87241318	745440·84	
Ben Lomond	41 43 7·098	9·82308444	5·69804595	498937·26	
South Berule	42 5 29·044	9·82623358	5·70119509	502568·30	△ 36 „ 458
	180 0 58·589				
BEN LOMOND TO BEN WYVIS.					
Ben Nevis	92 16 56·552	9·99965577	5·57469157	375570·59	
Ben Macdui.........	38 29 26·235	9·79404638	5·36908218	233927·98	△ 17, page 457
Ben Lomond	49 13 52·846	9·87928856	5·45432436	284658·63	△ 3 „ 450
	180 0 15·633				
Ben Lomond	29 36 23·309	9·69373293	5·45614422	285853·96	△ 9 „ 450
Ben Macdui.........	109 55 39·585	9·97319102	5·73560231	544004·27	
Ben Wyvis	40 28 20·817	9·81228028	5·57469157	375570·59	
	180 0 23·711				
Ben Nevis	146 43 59·963	9·73921637	5·73560231	544004·27	
Ben Wyvis	13 38 40·542	9·37269624	5·36908218	233927·98	△ 17 „ 457
Ben Lomond	19 37 29·536	9·52613913	5·52252507	333061·99	△ 10 „ 450
	180 0 10·041				

DUNNOSE TO BURLEIGH MOOR.

Names of Stations.	Spherical Angles.	Logarithmic Sines of Reduced Angles.	Log. Distances.	Distances in Feet.	References.
DUNNOSE TO GREENWICH.					
	° ′ ″				
Leith Hill	10 50 46·254	9·27455308	4·76227507	57846·23	△ 52, page 478
Chingford............	26 50 3·477	9·65457130	5·14229329	138769·27	
Greenwich Transit	142 19 11·423	9·78622207	5·27394406	187907·48	△ 15 „ 475
	180 0 1·154				
Leith Hill	19 24 2·634	9·52135770	5·14802983	140614·41	△ 15 „ 469
Butser	138 44 0·308	9·81925928	5·44593140	279210·28	
Dunnose	21 52 0·498	9·57106216	5·19773428	157664·63	△ 26 „ 476
	180 0 3·440				
Dunnose	2 1 15·449	8·54732536	5·14229329	138769·27	
Leith Hill	173 54 38·028	9·02564294	5·62061087	417456·16	
Greenwich	4 4 7·490	8·85096347	5·44593140	279210·28	
	180 0 0·967				
Leith Hill	100 16 40·258	9·99297571	5·44021939	275562·04	
Beachy Head	29 42 13·726	9·69504961	5·14229329	138769·27	
Greenwich	50 1 12·909	9·88437867	5·33162235	214596·36	△ 3 „ 483
	180 0 6·893				
Dunnose	41 6 12·327	9·81782547	5·44021939	275562·04	
Beachy Head	84 48 49·191	9·99821695	5·62061087	417456·16	
Greenwich	54 5 20·399	9·90843583	5·53082975	339492·16	△ 31 „ 476
	180 0 21·917				
DUNNOSE TO ARBURY.					
Leith Hill	35 19 44·621	9·76211397	5·39840001	250264·94	△ 5, page 474
Whitehorse Hill ...	98 59 53·238	9·99462418	5·63091023	427474·52	
Arbury	45 40 40·142	9·85455012	5·49083617	309625·10	△ 7 „ 474
	180 0 18·001				
Arbury	18 45 59·399	9·50744165	5·14229329	138769·27	
Leith Hill	63 31 41·888	9·95189371	5·58674535	386140·50	
Greenwich Transit	97 42 31·203	9·99605859	5·63091023	427474·52	
	180 0 12·490				
Arbury	26 30 47·694	9·64969176	5·44593140	279210·28	
Leith Hill	110 22 56·139	9·97192712	5·76816675	586363·25	
Dunnose	43 6 42·485	9·83467059	5·63091023	427474·52	
	180 0 26·318				
Dunnose	41 5 27·036	9·81770332	5·58674535	386140·50	
Greenwich	93 38 23·712	9·99912472	5·76816675	586363·25	
Arbury	45 16 47·093	9·85156883	5·62061087	417456·16	
	180 0 37·841				

DUNNOSE TO BURLEIGH MOOR—*continued.*

Names of Stations.	Spherical Angles.	Logarithmic Sines of Reduced Angles.	Log. Distances.	Distances in Feet.	References.
ARBURY TO CLIFTON.					
	° ′ ″				
Bardon...............	130 10 39·096	9·88312623	5·59968937	397822·53	
Arbury...............	29 32 36·796	9·69291151	5·40947466	256728·84	Δ 15, page 491
Lincoln Minster ...	20 16 52·433	9·53984834	5·25641148	180472·68	Δ 19 „ 492
	180 0 8·325				
Lincoln Minster ...	76 15 50·254	9·98739636	5·43504841	272300·48	
Bardon...............	37 24 42·065	9·78356417	5·23121621	170300·61	Δ 18 „ 492
Clifton	66 19 37·667	9·96182261	5·40947466	256728·84	Δ 16 „ 491
	180 0 9·986				
Lincoln Minster ...	96 32 42·687	9·99716139	5·65343405	450229·61	
Arbury...............	22 4 28·287	9·57494355	5·23121621	170300·61	Δ 18 „ 492
Clifton	61 22 64·852	9·94341671	5·59968937	397822·53	
	180 0 15·826				
Arbury...............	7 28 8·509	9·11389756	5·43504841	272300·48	
Clifton	4 56 32·815	8·93526062	5·25641148	180472·68	Δ 19 „ 492
Bardon...............	167 35 21·160	9·33228320	5·65343405	450229·61	
	180 0 2·484				
CLIFTON TO BURLEIGH MOOR AND EASINGTON.					
Easington............	70 41 35·082	9·97485951	5·44598710	279246·09	
Wordeslow	76 17 2·411	9·98743342	5·45856102	287449·15	
Great Whernside...	33 1 32·790	9·73639836	5·20752595	161259·74	Δ 43, page 441
	180 0 10·283				
Great Whernside...	87 48 64·102	9·99968438	5·61537072	412449·44	
Clifton	44 8 34·230	9·84287468	5·45856102	287449·15	
Easington............	48 2 42·394	9·87136801	5·48705434	306940·61	Δ 4 „ 490
	180 0 20·726				
Easington............	81 36 50·869	9·99533125	5·61111831	408430·64	
Clifton	5 52 41·438	9·01033028	4·62611735	42278·28	Δ 40 „ 440
Burleigh Moor......	92 30 31·748	9·99958365	5·61537072	412449·44	
	180 0 4·055				

The distance of Easington and Saxavord, which are very nearly on the same meridian, is obtained in the following manner:—

Through Easington draw a meridian line extending to Saxavord. The direction of this line is determined by the azimuthal observations at Easington, from which we have the angle between Cheviot and the north meridian line, 38° 48′ 58″·68 approximately (this was the angle actually assumed in the calculations). On this meridian line take a point, A (*see* Figure 1, plate xviii.), whose distance from Easington is equal to the side Easington — Cheviot. Join the point A with Cheviot and Mount Battock, and it is evident that we can determine the distance of A from Mount Battock, and also the angle at A between Mount Battock and the north meridian. Next take the point, D, in the meridian line whose distance from A is equal to the side A — Mount Battock. Join D with Mount Battock and Scarabin, then we can determine the side D—Scarabin, and its inclination to the north meridian at D. Next take a point, G, in the meridian line whose distance from D is equal to the side D—Scarabin, and join G with Scarabin, Fitty, and Foula. Next take a point, H, in the meridian line whose distance from G is equal to the side G—Foula, and join H with Foula, Yell, and Saxavord. From Saxavord drop a perpendicular upon the meridian line, meeting it in S, then we can obtain HS, or the distance from Easington to S, and the perpendicular just mentioned, which is only about two hundred feet.

The following table contains the calculation :—

Names of Stations.	Spherical Angles.	Logarithmic Sines of Reduced Angles.	Log. Distances.	Distances in Feet.	References.
Hart Fell…………	98° 36′ 51″·175	9·99507489	5·62539455	422079·79	
Cheviot …………	43 31 13·533	9·83796207	5·46828172	293955·59	△ 27, page 458
Ben Cleugh………	37 52 13·190	9·78806486	5·41838451	262050·21	△ 4 „ 437
	180 0 17·898				
Dunrich …………	126 30 44·697	9·90511451	5·62539455	422079·79	
Cheviot …………	28 56 46·871	9·68482354	5·40510357	254157·88	△ 30 „ 458
Ben Cleugh………	24 32 38·903	9·61844427	5·33872431	218134·48	△ 27 „ 439
	180 0 10·471				
Sayrs Law ………	142 58 33·534	9·77971369	5·74102635	550841·12	
Cheviot …………	26 8 3·492	9·64390889	5·60522155	402922·52	△ 21 „ 457
Mount Battock …	10 53 32·828	9·27634792	5·23766058	172846·50	△ 24 „ 439
	180 0 9·854				
Ben Cleugh………	90 53 22·560	9·99994802	5·74102635	550841·12	
Cheviot …………	39 6 25·168	9·79984162	5·54091993	347472·10	△ 32 „ 458
Mount Battock …	50 0 46·728	9·88431622	5·62539455	422079·79	
	180 0 34·456				
Ben Macdui………	101 2 23·384	9·99189031	5·69327789	493489·46	
Scarabin …………	22 12 39·972	9·57748334	5·27887091	190051·33	△ 1 „ 452
Mount Battock …	56 45 15·069	9·92236723	5·62375481	420489·17	△ 20 „ 451
	180 0 18·425				
Mount Battock …	49 31 61·184	9·88124869	5·57740724	3 77926·41	△ 29 „ 452
Scarabin …………	33 52 44·216	9·74617270	5·44233125	276905·29	△ 5 „ 453
Mormonth ………	96 35 39·019	9·99711934	5·69327789	493489·46	
	180 0 24·419				
Ben Hutig ………	77 51 53·492	9·99018249	5·60784005	405359·21	
Fitty Hill ………	31 6 28·041	9·71317378	5·33083134	214205·85	△ 3 „ 444
Scarabin …………	71 1 57·750	9·97575070	5·59340827	392110·33	△ 19 „ 446
	180 0 19·283				
Ben Clibrig………	57 22 6·128	9·92538537	5·60784005	405359·21	
Fitty Hill ………	19 10 39·581	9·51650375	5·19895844	158109·67	△ 9 „ 445
Scarabin …………	103 27 28·929	9·98791022	5·67036490	468128·30	△ 19 „ 446
	180 0 14·638				

Names of Stations.	Spherical Angles.	Logarithmic Sines of Reduced Angles.	Log. Distances.	Distances in Feet.	References.
Easington…………	38° 48′ 58″·680	9·79712273	5·45746797	286726·59	
Cheviot …………	70 35 44·371	9·97459586	5·63494110	431460·56	
A …………………	70 35 44·371	9·97459586	5·63494110	431460·56	△ 8, page 437
	180 0 27·422				
Cheviot …………	82 0 12·512	9·99575285	5·76680739	584530·79	
Mount Battock……	29 3 54·253	9·68641343	5·45746797	286726·59	
A …………………	68 56 29·986	9·96997181	5·74102635	550841·12	
	180 0 36·751				
A …………………	40 27 45·643	9·81217011	5·60662719	404228·74	
Mount Battock …	69 46 33·223	9·97235031	5·76680739	584530·79	
D …………………	69 46 33·223	9·97235031	5·76680739	584530·79	
	180 0 52·089				
Mount Battock……	87 33 57·721	9·99960662	5·79550966	624467·25	
Scarabin …………	40 17 59·898	9·81072415	5·60662719	404228·74	
D …………………	52 8 49·197	9·89737484	5·69327789	493489·46	
	180 0 46·816				
Scarabin …………	60 58 20·096	9·94167231	5·79550966	624467·25	
D …………………	58 4 37·540	9·92875110	5·78258844	606161·63	
G …………………	60 58 20·095	9·94167231	5·79550966	624467·25	
	180 0 77·731				
G …………………	41 44 38·159	9·82331466	5·60784005	405359·21	
Scarabin …………	42 51 21·675	9·83258037	5·61710576	414100·51	
Fitty Hill ………	95 24 39·410	9·99806305	5·78258844	606161·63	
	180 0 39·244				
Fitty Hill ………	73 1 48·585	9·98065909	5·66241275	459634·64	
G …………………	47 28 7·794	9·86739308	5·54914673	354116·96	△ 26 „ 446
Foula ……………	59 30 36·553	9·93535210	5·61710576	414100·51	
	180 0 32·932				
G …………………	29 48 53·953	9·69650172	5·37378112	236472·76	
Foula ……………	75 5 45·354	9·98513336	5·66241275	459634·64	
H …………………	75 5 45·354	9·98513336	5·66241275	459634·64	
	180 0 24·661				
Foula ……………	23 35 5·470	9·60216749	4·98412328	96410·26	
H …………………	77 29 53·162	9·98957751	5·37153330	235251·99	△ 5 „ 443
Yell ………………	78 55 6·593	9·99182532	5·37378112	236472·76	
	180 0 5·225				
Yell ………………	129 7 46·789	9·88970599	5·27492916	188334·19	
Saxavord …………	23 23 49·939	9·59890012	4·98412328	96410·26	
H …………………	27 28 25·238	9·66401945	5·04924262	112006·34	△ 10 „ 443
	180 0 1·966				
H …………………	0 4 3·755	7·0725221–	2·34745125	222·56	
Saxavord …………	89 55 56·255	9·99999970	5·27492886	188334·05	
S …………………	89 59 60·000		5·27492916	188334·19	
	180 0 0·010				

In order that no doubt might rest upon this determination, another calculation was made, using a different series of points (*see* Figure 2, plate xviii.) The distance Easington—Hartfell was first obtained, and the angle Hartfell—Easington—Cheviot. Take a point, K, in the meridian line (the same as previously used) passing through Easington, so that the distance Easington to K may be equal to the distance Easington—Hartfell, and join K with Hartfell and Ben Macdui. Take L in the same meridian line at a distance from K equal to the distance K—Ben Macdui, and join L with Ben Macdui, Ben Clibrig, and Fitty Hill. Next take a point, M, in the meridian line, so that LM shall be equal to the line L—Fitty Hill, and join M with Fitty Hill, Ronas, and Saxavord. The right-angled triangle Saxavord—M—S will give MS, or with it the distance Easington to Saxavord, and the perpendicular from Saxavord upon the assumed meridional line through Easington.

The calculation is shown in the following table:—

Names of Stations.	Spherical Angles.	Logarithmic Sines of Reduced Angles.	Log. Distances.	Distances in Feet.	References.
	° ′ ″				
Cheviot	124 46 24·416	9·91457252	5·79204717	619508·36	
Easington	20 20 3·494	9·54090987	5·41838451	262050·21	△ 2, page 437
Hart Fell	34 53 53·918	9·75746645	5·63494110	431460·56	△ 8 „ 437
	180 0 21·828				
Cheviot	54 12 16·974	9·90906864	5·54678995	352200·49	
Easington	42 16 33·386	9·82780409	5·46552539	292095·85	△ 5 „ 437
Cross Fell	83 31 33·665	9·99721979	5·63494110	431460·56	△ 8 „ 437
	180 0 24·025				
Cross Fell	133 51 44·455	9·85795218	5·79204717	619508·36	
Easington	21 56 29·893	9·57244548	5·50654047	321026·19	△ 3 „ 437
Hart Fell............	24 12 4·815	9·61269496	5·54678995	352200·49	
	180 0 19·163				
Ben Lawers.........	127 56 21·158	9·89690181	5·78512472	609711·97	
Hart Fell............	16 39 18·139	9·45724638	5·34546930	221548·75	△ 3 „ 450
Ben Macdui.........	35 24 39·093	9·76298706	5·65120997	447929·82	△ 19 „ 457
	180 0 18·390				
Sayrs Law	112 29 42·145	9·96563773	5·78512472	609711·97	
Ben Macdui.........	19 30 16·701	9·52354806	5·34303505	220310·43	△ 24 „ 439
Hart Fell............	48 0 24·611	9·87110529	5·69059228	490447·23	△ 21 „ 457
	180 0 23·457				
Foula	169 34 57·385	9·25725165	5·73084564	538078·50	
Fitty Hill............	3 35 0·334	8·79586133	5·26945531	185975·32	△ 5 „ 443
Ronas	6 50 5·077	9·07555274	5·54914673	354116·96	△ 26 „ 446
	180 0 2·796				
Fair Isle	115 7 20·954	9·95684847	5·73084564	538078·50	
Fitty Hill............	37 36 54·821	9·78556389	5·55956106	362711·28	△ 3 „ 443
Ronas	27 16 5·218	9·66098383	5·43498100	272258·22	△ 26 „ 446
	180 0 20·993				

Names of Stations.	Spherical Angles.			Logarithmic Sines of Reduced Angles.	Log. Distances.	Distances in Feet.	References.
	°	′	″				
Easington…………	59	9	2·174	9·93371701	5·78637541	611470·37	
Hart Fell…………	60	26	7·627	9·93938877	5·79204717	619508·36	
K …………………	60	26	7·627	9·93938877	5·79204717	619508·36	
	180	0	77·428				
Hart Fell…………	63	19	39·298	9·95110953	5·80684175	640975·98	
Ben Macdui………	58	28	51·122	9·93064319	5·78637541	611470·37	
K …………………	58	12	47·855	9·92939250	5·78512472	609711·97	
	180	0	78·275				
Ben Macdui………	59	20	10·088	9·93455111	5·80684175	640975·98	
K …………………	61	21	4·519	9·94325196	5·81554260	653947·08	
L …………………	59	20	10·088	9·93455111	5·80684175	640975·98	
	180	0	84·695				
Ben Macdui………	75	29	30·444	9·98591334	5·84174859	694622·09	
L …………………	38	49	17·535	9·79713766	5·65297290	449751·79	Δ 21, page 451
Ben Clibrig………	65	42	18·901	9·95970736	5·81554260	653947·08	
	180	0	66·880				
Ben Clibrig………	60	52	53·226	9·94129387	5·79279421	620574·90	
L …………………	41	13	38·707	9·81886456	5·67036490	468128·30	Δ 19 „ 446
Fitty Hill ………	77	54	34·779	9·99024825	5·84174859	694622·09	
	180	0	66·712				
L …………………	40	36	53·670	9·81351397	5·63417002	430695·19	
Fitty Hill…………	69	42	2·597	9·97213815	5·79279421	620574·90	
M …………………	69	42	2·597	9·97213815	5·79279421	620574·90	
	180	0	58·864				
Fitty Hill…………	36	25	29·867	9·77358717	5·50444385	319480·13	
Ronas ……………	53	10	20·676	9·90331334	5·63417002	430695·19	
M …………………	90	24	41·763	9·99998896	5·73084564	538078·50	
	180	0	32·306				
Ronas ……………	114	35	40·908	9·95869842	5·60998713	407368·21	
M …………………	19	55	8·025	9·53233876	5·18362746	152625·63	Δ 8 „ 443
Saxavord …………	45	29	21·476	9·85315515	5·50444385	319480·13	
	180	0	10·409				
M …………………	0	1	52·385	6·7362569–	2·34624405	221·94	
Saxavord…………	89	58	7·636	9·99999994	5·60998706	407368·14	
S …………………	90	0	0·000		5·60998713	407368·21	
	180	0	0·021				

If s be the distance of two points, A B, whose reciprocal azimuths are $\alpha\ \alpha'$ measured from the north, and if the line be very nearly in the direction of the meridian, the distance of the parallels of the two points will be

$$S = s\,\frac{\sin\frac{1}{2}(\alpha' - \alpha)}{\sin\frac{1}{2}(\alpha' + \alpha)}$$

When the azimuth is given at only one extremity, A,*

$$S = s - 2\,v\,\frac{\cos\lambda\,\sin\theta}{\cos(\lambda+\theta)}\,\sin^2\frac{\alpha}{2}$$

where v is the normal at A, whose latitude is λ, and θ results from the equation $s = v\,\theta$. The reciprocal azimuth α' is given by the equation

$$\frac{\sin\alpha'}{\sin\alpha} = \frac{\cos\lambda}{\cos(\lambda+\theta)}$$

with sufficient exactness for the purposes required at present.

Distance of Parallels of Hensbarrow and Ben Hutig.

	°	′	″
At Precelly, Lundy reads	355	17	53·498
„ Snowdon reads	200	30	38·680
∴ Angle Snowdon—Precelly—Lundy	154	47	14·818
„ Snowdon—Precelly—South Berule . .	18	54	25·964
„ Lundy—Precelly—Hensbarrow . . .	5	44	35·498
∴ Angle Hensbarrow—Precelly—South Berule .	179	26	16·280

	°	′	″
At South Berule, Snowdon reads	341	41	50·434
„ Merrick „	186	38	19·850
∴ Angle Merrick—South Berule—Snowdon . .	155	3	30·584
„ Snowdon—South Berule—Precelly . .	20	0	1·984
„ Merrick—South Berule—Ben Lomond .	6	3	45·541
∴ Angle Precelly—South Berule—Ben Lomond .	181	7	18·109

	°	′	″
At Ben Lomond, Ben Nevis reads	161	28	57·615
„ Knocklayd „	42	19	7·078
∴ Angle Ben Nevis—Ben Lomond—Knocklayd .	119	9	50·537
„ Knocklayd—Ben Lomond—Merrick . .	47	26	35·146
„ Ben Nevis—Ben Lomond—Merrick . .	193	23	34·317
„ Merrick—Ben Lomond—South Berule .	5	43	28·049
„ Ben Nevis—Ben Lomond—Ben Wyvis .	19	37	29·537
∴ Angle Ben Wyvis—Ben Lomond—South Berule	179	29	32·829

---

* If $90° - \lambda - \theta + x$ be the third side of a spherical triangle of which the other sides, $90° - \lambda$, and θ, include the small angle α, then $\sin(\lambda + \theta - x) = \cos\theta \sin\lambda + \cos\lambda \sin\theta\,(1 - 2\sin^2\tfrac{1}{2}\alpha)$, therefore, neglecting the square of x,

$$x = 2\,\frac{\cos\lambda\,\sin\theta}{\cos(\lambda+\theta)}\,\sin^2\tfrac{1}{2}\alpha$$

	°	′	″
At Ben Wyvis, Ben Hutig reads . . .	182	16	13·455
„ Ben Nevis „ . . .	14	47	44·235
Angle Ben Hutig—Ben Wyvis—Ben Nevis . .	192	31	30·780
„ Ben Nevis —Ben Wyvis—Ben Lomond . .	13	38	40·542
∴ Angle Ben Hutig—Ben Wyvis—Ben Lomond .	178	52	50·238

If to the reading of Lundy at Precelly we add the angle Lundy—Precelly—Hens barrow, we get 1° 2′ 28″·996 for the reading of Hensbarrow at Precelly. Now the true azimuth of the referring-object at Precelly was 211° 58′ 46″·38, whereas in the list of bearings (page 144) it is given as 211° 58′ 30″·09; therefore all readings at Precelly have to be corrected by the quantity + 16″·29, so that the true azimuth, measured from the north, of Hensbarrow at Precelly is 178° 57′ 14″·714. The angle Lundy—Hensbarrow—Precelly, 5° 41′ 19″·185, when applied to the reading of Lundy at Hensbarrow gives 181° 0′ 43″·591 for the reading of Precelly at Hensbarrow. The correction to reduce the readings at Hensbarrow to the true meridian is − 5″·19 (*see* pages 118, 197). Therefore the azimuth, measured from the north, of Precelly at Hensbarrow is 1° 0′ 38″·761.

The calculation* of the successive distances of parallels from Hensbarrow to Ben Hutig is as follows:—

HENSBARROW TO PRECELLY.

$\alpha = 1^\circ\ 0'\ 39''$	$\alpha' = 178^\circ\ 57'\ 15''$	
$\alpha' - \alpha = 177^\circ\ 56'\ 36''$	Log sin (88° 58′ 18″) . .	9·99993004
$\alpha' + \alpha = 179^\circ\ 57'\ 54''$	− Log sin (89° 58′ 57″) . .	0· 2
	Log Distance	5·75621434
	Distance of parallels = 570353·88	5·75614440

PRECELLY TO SOUTH BERULE.

$\alpha = 1^\circ\ 36'\ 29''$		Log Distance . . .	5·905697
		− Log v sin 1″	7·992886
Log Sin θ . . .	8·584029	Log ($\theta = 7917''$) . .	3·898583
v	7·321539		
„ Cos λ . . .	9·789864	. . . .	9·789864
„ Sec ($\lambda + \theta$) .	0·232301	. . . .	0·232301
„ Sin $^2 \frac{1}{2}\alpha$ . .	6·294268	Sin α . . .	8·448121
$\frac{1}{2}$ Correction = 166·73 . .	2·222001	Sin α' . . .	8·470286

	°	′	″
$\alpha' =$	178	18	28
	181	7	18
	359	25	46

---

* The approximate latitudes used in these calculations are—

	°	′	″		°	′	″
Precelly	51	56	46	Ben Lomond . . .	56	11	25
South Berule . . .	54	8	58	Ben Wyvis . . .	57	40	45

SOUTH BERULE TO BEN LOMOND.

$\alpha = 0^\circ\ 34'\ 14''$		Log Distance . . .	5·872413
		− Log v sin 1″ . . .	7·992833
Log Sin θ . . .	8·550706	Log (θ = 7332″) . .	3·865246
„ v	7·321592		
„ Cos λ . . .	9·767655	. . . .	9·767655
„ Sec $(\lambda + \theta)$.	0·254537	. . . .	0·254537
„ Sin $^2 \frac{1}{2} \alpha$ . .	5·394286	Sin α . . .	7·998168
$\frac{1}{2}$ Correction = 19·44 . . .	1·288776	Sin α' . . .	8·020360

$$\begin{array}{rrrr} & {}^\circ & {}' & {}'' \\ \alpha' = & 179 & 23 & 58 \\ & 179 & 29 & 33 \\ \hline & 358 & 53 & 31 \end{array}$$

BEN LOMOND TO BEN WYVIS.

$\alpha = 1^\circ\ 6'\ 29''$.		Log Distance . . .	5·735602
		− Log v sin 1″ . . .	7·992784
Log Sin θ . . .	8·413880	Log (θ = 5350″) . .	3·728386
„ v	7·321641		
„ Cos λ . . .	9·745416	. . . .	9·745416
„ Sec $(\lambda + \theta)$.	0·271889	. . . .	0·271889
„ Sin $^2 \frac{1}{2} \alpha$ . .	5·970804	Sin α . . .	8·286412
$\frac{1}{2}$ Correction = 52·92 . . .	1·723630	Sin α' . . .	8·303717

$$\begin{array}{rrrr} & {}^\circ & {}' & {}'' \\ \alpha' = & 178 & 50 & 49 \\ & 178 & 52 & 50 \\ \hline & 357 & 43 & 39 \end{array}$$

BEN WYVIS TO BEN HUTIG.

$\alpha = 2^\circ\ 16'\ 21''$		Log Distance . . .	5·503730
		− Log v sin 1″ . . .	7·992749
Log Sin θ	8·182073	Log (θ = 3137″) . .	3·496479
„ v	7·321676		
„ Cos λ . . .	9·728077	. . . .	9·728077
„ Sec $(\lambda + \theta)$.	0·282541	. . . .	0·282541
„ Sin $^2 \frac{1}{2} \alpha$ . .	6·594646	Sin α . . .	8·598267
$\frac{1}{2}$ Correction = 128·53 . .	2·109013	Sin α' . . .	8·608885

Azimuth of Ben Wyvis . . . . $\alpha' = 177^\circ\ 40'\ 16''$ . . . . at Ben Hutig.

Summation.

Hensbarrow to Precelly	570353·88	− ....
Precelly to South Berule	804815·95	− 333·46
South Berule to Ben Lomond	745440·84	− 38·88
Ben Lomond to Ben Wyvis	544004·27	− 105·84
Ben Wyvis to Ben Hutig	318955·81	− 257·06

∴ *Distance of Parallels of Hensbarrow and Ben Hutig* . . . . } = 2983570·75 − 735·24 = 2982835·51

Distance of the Parallels of Dunnose and Saxavord.

		°	′	″	
	At Dunnose, Beachy Head bears	261	55	18·383	By Azim. Obs. at Dunnose.
Angle	Beachy—Dunnose—Arbury	82	11	39·363	
∴	At Dunnose—Arbury bears	179	43	39·020	
	At Arbury, Dunstable bears	310	37	52·253	By Azim. Obs. at Arbury.
Angle	Dunstable—Arbury—Whitehorse	68	14	58·794	
„	Whitehorse—Arbury—Leith	45	40	40·142	
	Leith—Arbury—Dunnose	26	30	47·694	
∴	At Arbury, Dunnose bears	359	42	58·599	
	At Arbury, Bardon bears	172	16	46·048	By Azim. Obs. at Arbury.
Angle	Bardon—Arbury—Clifton	7	28	8·509	
∴	At Arbury, Clifton bears	179	44	54·557	
	At Clifton, Lincoln Minster bears	298	21	27·421	By Azim. Obs. at Clifton.
Angle	Lincoln—Clifton—Bardon	66	19	37·667	
„	Bardon—Clifton—Arbury	4	56	32·815	
∴	At Clifton, Arbury bears	359	44	32·273	
	At Clifton, Great Whernside bears	147	5	43·126	By Azim. Obs. at Clifton.
Angle	Great Whernside—Clifton—Easington	44	8	34·230	
∴	At Clifton, Easington bears	191	14	17·356	
	At Easington, Wordeslow bears	130	17	2·505	By Azim. Obs. at Easington.
Angle	Wordeslow—Easington—Whernside	70	41	35·082	
„	Whernside—Easington—Clifton	48	2	42·394	
∴	At Easington—Clifton bears	11	32	45·029	

DUNNOSE TO ARBURY.

	° ′ ″	° ′ ″		
$\alpha =$	0 16 21			
$\alpha' =$	179 42 59		Log Dist.	5·76816675
$\alpha' + \alpha =$	179 59 20	89 59 40	Cosec.	0·...... .....
$\alpha' - \alpha =$	179 26 38	89 43 19	Sine	9·99999489
Distance of parallels =	586356·36		Log	5·76816164

ARBURY TO CLIFTON.

	° ′ ″	° ′ ″		
$\alpha =$	0 15 5			
$\alpha' =$	179 44 32		Log Dist.	5·65343405
$\alpha' + \alpha =$	179 59 37	89 59 48·5	Cosec.	0·............
$\alpha' - \alpha =$	179 29 27	89 44 43·5	Sine	9·99999571
Distance of parallels =	450225·16		Log	5·65342976

CLIFTON TO EASINGTON.

	° ′ ″	° ′ ″		
$\alpha =$	11 14 17			
$\alpha' =$	168 27 15		Log Dist.	5·61537072
$\alpha' + \alpha =$	179 41 32	89 50 46	Cosec.	0·........157
$\alpha' - \alpha =$	157 12 58	78 36 29	Sine	9·99135854
Distance of parallels =	404325·22		Log	5·60673083

EASINGTON TO BURLEIGH MOOR.

$$\alpha = 86^\circ\ 50'\ 24''{\cdot}10$$

Log Dist.	4·6261173	$(\text{Dist.})^2$	9·25223
Cos α	8·7413405	$\text{Sin}^2\alpha$	9·99868
2330·55	3·3674578	Tan λ	0·14787
		$(2v)^{-1}$	2·37736
−59·72			1·77614

2270·83 = Distance of parallels.

Summation.

Dunnose to Arbury	586356·36
Arbury to Clifton	450225·16
Clifton to Easington	404325·22
Easington to Burleigh Moor	2270·83

Distance of parallels of *Dunnose and Burleigh Moor* = 1443177·57

Distance of parallels of *Dunnose and Easington* = 1440906·74

For the distance of the parallels of Easington and Saxavord, we have—

By first Calculation, pages 500, 501.		By second Calculation, pages 502, 503.	
Easington to A	431460·56	Easington to K	619508·36
A to D	584530·79	K to L	640975·98
D to G	624467·25	L to M	620574·90
G to H	459634·64	M to S	407368·14
H to S	188334·05		
Easington to S	2288427·29	Easington to S	2288427·38

Distance of parallels of *Easington and Saxavord* = 2288427·33

Distance of parallels of *Dunnose and Saxavord* = 3729334·07

By the first calculation of the connection of Easington and Saxavord, the bearing of the latter is 0° 0′ 20″·130 east of the assumed meridian line at Easington. By the second calculation it is 0° 0′ 20″·044, which differ by a tenth of a second only. The mean is 0° 0′ 20″·087; so that the

$$\text{Angle : Cheviot—Easington—Saxavord} = 38^\circ\ 49'\ 18''{\cdot}767$$

By the first calculation the distance of Saxavord from the assumed meridian at Easington is 222·56; by the second calculation the same distance is found to be 221·94.

SECTION IX.

OBSERVED ZENITH DISTANCES

AND

ALTITUDES.

As the position of a point in space is determined by three co-ordinates, so the position of a point on the earth's physical surface is incompletely given without its three co-ordinates of latitude, longitude, and altitude; the latter co-ordinate being the distance of the point in question from the mean surface of the earth, or rather from the mean surface of the sea.

Regarded as a part of the principal triangulation, the accurate determination of the heights of the trigonometrical stations by vertical angles has been considered a matter of secondary importance, inasmuch as it belongs more properly to subsequent operations. The co-efficient of refraction is a quantity so variable, that at great distances the average of any number of observations, however large, cannot be entirely relied upon.

When the determination of altitudes becomes a matter of primary importance, the stations must be within a short distance of each other, or no very consistent or accurate results can be obtained; for the uncertainty of refraction being as the distance, the uncertainty of difference of height is as the square of the distance. If we suppose, for instance, an object at fourteen miles' distance to be observed, and that in the calculation of the difference of height between this point and the point of observation an error of $\cdot 02$ is made in the assumption of the co-efficient of refraction that ought to be used—as is quite possible if the number of observations be small and taken under similar circumstances—then the error in the assumed zenith distance will be $\cdot 02 \sin 12'$ and the error in height $\cdot 02 \sin^2 12'$ multiplied by the radius of the earth, or $5 \cdot 1$ feet, whereas had the distance been about seventy miles, or one degree, the error in the assumption of the true zenith distance would have been $\cdot 02 \sin 1^\circ$, and the resulting error in the computed difference of height $\cdot 02 \sin^2 1^\circ$ multiplied by the radius of the earth, or $127 \cdot 4$ feet. But if the observations had been so numerous that the mean refraction, due to them all, approximated to a mean refraction derived from a combination of a great many zenith distances, then the errors would probably be much smaller than those exhibited above, which are only *possible* errors.

The precision of the results is very much increased by simultaneous and reciprocal zenith distances. This case, however, does not exist in the Ordnance Survey, for the observations, though generally reciprocal, are not simultaneous.

The vertical observations with the larger theodolites are generally numerous (at least in the more modern observations), and taken under all circumstances. In the vertical circles of the two 3-ft. theodolites one division of the micrometer is equivalent to $3''$; in the 2-ft. theodolite one division of the micrometer is equivalent to $2''$. In the 18-inch theodolite the vertical circle is read by verniers to 10″. The practice has been to record in each observation the vertical reading of the object after the horizontal reading: this applies more especially to those stations visited since 1839, the older observations not being so numerous.

In some cases the vertical observation is taken to the top of the pile, sometimes the base of the pile or surface of the ground, and sometimes to a heliostat. The observations to a heliostat are generally more regular than the observations to the ground at the same point, the former being a better object for bisection. In some cases the height of the observing telescope above the ground has been omitted to be recorded, and consequently a mean height of 5·5 feet has been assumed.

Let h h' be the heights of two points AB above the surface of a sphere whose radius is r, and let the distance of their projections on the sphere be s, this arc subtending an angle v at the centre C. Let $z + \Delta z$ be the true zenith-distance of the point B at the point A, and $z' + \Delta z'$ the true zenith-distance of A at B, then the angles of the triangle ABC are $180° - (z + \Delta z)$, $180° - (z' + \Delta z')$ and v, and the two sides including the angle v, are $r + h$ and $r + h'$; hence we obtain immediately,

$$z' + z + \Delta z' + \Delta z = 180 + v$$

$$\frac{h' - h}{2r + h' + h} = \tan \tfrac{1}{2} (z' - z + \Delta z' - \Delta z) \tan \tfrac{1}{2} v$$

$$\therefore h' - h = s \tan \tfrac{1}{2} (z' - z + \Delta z' - \Delta z) \left(1 + \frac{h' + h}{2r} + \frac{s^2}{12r^2}\right).$$

If we here take z and z' to be the observed zenith-distances, then Δz and $\Delta z'$ are the refractions at A and B respectively. The usual assumption is, that $\Delta z = \Delta z'$, which, however, is not true, except in particular cases that *may* occur. If k be the co-efficient of refraction, then $\Delta z = \Delta z' = k v$, and the above equation gives the two following:—

$$\left.\begin{aligned} h' - h &= s \cot \left(z - \frac{1 - 2k}{2} v\right) \left(1 + \frac{h' + h}{2r} + \frac{s^2}{12r^2}\right) \\ h - h' &= s \cot \left(z' - \frac{1 - 2k}{2} v\right) \left(1 + \frac{h' + h}{2r} + \frac{s^2}{12r^2}\right) \end{aligned}\right\} \cdots\cdots (1)$$

or, if the reciprocal observations be used,

$$h' - h = s \tan \tfrac{1}{2} (z' - z) \left(1 + \frac{h' + h}{2r} + \frac{s^2}{12r^2}\right) \cdots\cdots\cdots (2)$$

In general, when z' and z are given and not simultaneous, it will be more advantageous to obtain the two independent values of $h' - h$ by means of a co-efficient of mean refraction, and to combine them with reference to their weights; so that if z observed n times gives $h' - h = e$, and z' observed n' times gives $h' - h = e'$, then

$$h' - h = \frac{ne + n'e'}{n + n'} \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

The co-efficient of refraction (which is sometimes and perhaps more properly referred to the whole ray, and then expresses the ratio of $\Delta z + \Delta z'$ to v) may be obtained from the reciprocal observed zenith distances z and z' of two points; for

$$z + z' + \Delta z + \Delta z' = 180 + v$$

$$\therefore 1 - 2k = \frac{z' + z - 180^\circ}{v} \quad \dots\dots\dots\dots\dots \quad (4)$$

It may also be obtained by means of the observed mean zenith distances of two points whose heights have been previously ascertained by some more precise operation, as levelling; for if H' H be the heights of the points, Z' Z the true reciprocal zenith distances, and h the mean of H' and H,

$$\left.\begin{aligned} \tfrac{1}{2}(Z' + Z) &= 90^\circ + \tfrac{1}{2}v \\ \tfrac{1}{2}(Z' - Z) &= \tan^{-1}\left(\frac{H' - H}{s}\left\{1 - \frac{h}{r} - \frac{s^2}{12r^2}\right\}\right) \end{aligned}\right\} \quad \dots\dots \quad (5)$$

and if z' and z be the observed zenith distances, then

$$k = \frac{Z - z}{v}, \text{ and } k = \frac{Z' - z'}{v} \quad \dots\dots\dots\dots\dots\dots \quad (6)$$

The above equations have been deduced on the supposition of the earth being a sphere; but it is easily proved, that if the normal for the mean latitude of the two points be used as the radius of the sphere, the remaining errors due to the eccentricity are quite inappreciable.

The path of a ray of light joining two given points, inasmuch as it depends upon the refractive power of the atmosphere at every point through which it passes, is necessarily very variable. It is not, however, without some obvious laws; for refraction by night, for instance, is well known to be greater than by day. When observations are recorded as having been taken at some early hour in the morning, as 5 A.M., and from that time until 8 A.M., they almost invariably show a smaller zenith distance than during any other part of the day; sometimes the difference is very considerable, and shows that no vertical observations should be taken early in the morning. Towards the middle of the day the zenith distances seem to diminish, and again increase towards evening. These rules, however, are not invariable, for at the station on Hart Fell, at which the 2-foot theodolite was used, a remarkable case of refraction is shown, when from 1 P.M. till 3 P.M. the refraction was double its mean amount. This was in the month of September.*

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* A series of observations for determining the daily variations of refraction were carried out by M. Hossard, at Angoulême, during the months of May and June 1844. The observations were made half-hourly, from daybreak till twilight; and the object observed was at a distance of about 13 miles. To exhibit more clearly the laws of the variation of refraction, daily curves were constructed from the observations, having the time as abscissa, and the corresponding ordinate being the refraction. The consideration of these curves, which were very consistent with one another, lead to the following conclusions:—The refraction is greatest about daybreak; from 5 or 6 A.M. until 8 A.M. it diminishes very rapidly; from 8 A.M. until 10 A.M. the diminution is slow; from 10 A.M to 4 P.M. the curve remains nearly parallel to the axis of abscissas, or the refraction is nearly constant; from 4 P.M. the refraction commences to increase. The comparison of the curves with one another shows that the maximum refraction about daybreak is very variable, both in magnitude and as to position or time: during the daytime, however, from 10 A.M. until 4 P.M., the variations from day to day are very small, notwithstanding any changes in the thermometer and barometer, upon which the variations do not depend so much as on the rapidity of the diminution of temperature.

The co-efficient of refraction has been observed of all magnitudes from ·50 to ·00, and even negative refraction has been observed in a few instances. The smaller the altitude the larger, generally, will be the mean co-efficient of refraction. The large co-efficients exhibited in the operations at the Lough Foyle base are due to the smallness of the altitude of some of the stations. Delambre determined the mean co-efficient of refraction at ·0786, which is probably larger than it ought to be, as his instrument gave zenith distances too small. The determination of Bessel, from his operations in Prussia, is ·0685. Gauss, from his observations, obtained ·0653; Coraboeuf, ·0642; Struve, ·0618; and the mean co-efficient derived from the operations in France is ·0665.

M. Struve has given the following formula for the co-efficient of "normal refraction," that which exists previous to and just after mid-day,

$$k = \left(\cdot 07238 + \frac{\cdot 1298}{A}\right) \frac{B}{736\cdot 586} \, 1\cdot 011838^{(20 - T)}$$

in which A is the mean height of the ray above the ground, expressed in metres; B the height of the barometer in millimetres, and reduced to the freezing point; and T the temperature of the air in centesimal degrees.

When a number of points, connected in a network of triangulation, afford by the vertical observations the means of ascertaining their relative altitudes, these observations should be so combined as to furnish the best and most probable results.

The first operation is the ascertaining the value of the co-efficient of refraction: this, as before stated, may be determined either by reciprocal zenith distances, or by the comparison of observed zenith distances with the corresponding true zenith distances, calculated from known heights of mutually visible points.

Each pair of reciprocal mean zenith distances will afford a value of k, the co-efficient of refraction, and all values of k thus obtained must be combined with regard to their weights in obtaining the mean value of k. Now the value of k in each case will depend on the number of observations at each end: if one zenith distance be observed a times, and the other b times, then, so far as the number of observations goes, the weight is the product divided by the sum of those quantities. But each value of k is affected both by variability in k itself, and by errors of observation. If we suppose, first, k constant, and ascribe the different values of k to errors of observation, and, secondly, suppose k to have slightly different values, or to be variable within small limits, and suppose no errors of observation, then the weight in these separate cases would be

$$\frac{abs}{a+b} \quad \text{and} \quad \frac{ab}{a+b}$$

Now, neither of these suppositions is true, but the truth probably lies between; we shall therefore assume for the weight of each value of k, as M. Bessel has done,

$$\omega = \frac{ab\sqrt{s}}{a+b}$$

the mean value will then be

$$k = \frac{k_1 \omega + k_2 \omega_2 + k_3 \omega_3 + \cdots\cdots}{\omega_1 + \omega_2 + \omega_3 + \cdots\cdot}$$

When a value of k is derived by means of a zenith distance taken a-times, and compared with its true value calculated from the known heights of the two points, then its weight is $a\sqrt{s}$; but such values should be combined with similar, and kept separate from the determinations arising from reciprocal azimuths.

The mean coefficient of refraction being determined, each zenith distance will give a difference of height. If $x_1\ x_2\ x_3 \ldots\ldots x_n$ be the heights of the various points, the observed zenith distances will give, each, an equation of the form

$$x_i - x_n + q = 0$$

Should the points $x_i\ x_n$ mutually observe one another, there will be two equations in x_i and x_n,

$$\begin{aligned} x_i - x_n + q &= 0 \\ - x_i + x_n + q' &= 0 \end{aligned}$$

All equations being formed, they must be solved by the method of least squares, regard being had to the weight of each, which will be as the number of observations and inversely as the distance, or, in other words, the quantity $\Sigma\ (\omega\ (x - x' + r)^2)$ must be made a minimum.

If, however, the points are very numerous, the solution of the above equations will become unmanageable, from the number of eliminations required, and in this case a small number of points must be taken at a time and determined as above. These results must then be used for the determination of others by the same method.

The following pages contain the Mean Observed Zenith Distances, showing in each case the number of observations on which the mean depends. When the fourth column is blank, the object observed is the surface of the ground; though there are, indeed, a few cases in which there is no record by the observer as to whether the observation was to the surface of the ground or the top of the pile. Those observations in which the object was a heliostat, are distinguished by the letter H.

OBSERVED ZENITH DISTANCES.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
SAXAVORD. Telescope above surface 5·5 feet.				
Brassa . . .	90° 20′ 48″·5	2	...	− 4″·4
Yell	90 16 3·5	2	...	−10·1
Ronas . . .	89 58 45·5	2	...	− 7·4
Balta . . .	91 34 59·5	2	...	−38·4
Fetlar . . .	90 24 14·5	2	...	−14·9
With a 7-in. Theodolite. Telescope 5·0 feet.				
Gerth of Scaw .	93 48 6·3	10	...	−78·7
Nive Hill . .	91 29 15·8	23	...	−63·3
GERTH OF SCAW. 7-in. Theodolite. Telescope above surface 5·0 feet.				
Balta . . .	89° 51′ 21″·7	12	...	− 43″·0
Nive Hill . .	87 10 53·8	10	...	−114·3
Saxavord . .	86 14 10·0	10	...	− 78·7
BALTA. Telescope above surface 5·5 feet.				
Fetlar . . .	89° 37′ 34″·0	1	...	−22″·7
Yell . . .	89 46 48·0	2	...	−12·3
Saxavord . .	88 30 6·0	1	...	−38·4
With a 7-in. Theodolite. Telescope 5·0 feet.				
Gerth of Scaw .	90 12 28·6	15	...	−43·0
Nive Hill . .	88 35 7·0	16	...	−68·2
FETLAR. Telescope above surface 5·5 feet.				
Brassa . . .	90° 8′ 20″·5	2	...	− 6″·2
Yell	89 52 53·3	2	...	−23·4
Ronas . . .	89 37 29·5	2	...	−10·4
Saxavord . .	89 46 28·0	1	...	−14·9
Balta . . .	90 29 44·0	1	...	−22·7
YELL. Telescope above surface 5·5 feet.				
Foula . . .	90° 5′ 19″·7	3	...	− 4″·8
Ronas . . .	89 21 17·7	3	...	−17·6
Saxavord . .	89 59 14·0	4	...	−10·1
Balta . . .	90 25 45·7	3	...	−12·3
Fetlar . . .	90 13 48·3	3	...	−23·4
Brassa . . .	90 8 25·0	4	...	− 7·5
RONAS. Telescope above surface 5·5 feet.				
Saxavord . .	90° 22′ 55″·5	2	...	− 7″·4
Fetlar . . .	90 37 45·0	2	...	−10·4
Yell	90 47 28·7	3	...	−17·6
Brassa . . .	90 27 0·0	3	...	− 7·1
Foula . . .	90 14 53·0	1	...	− 6·1
BRASSA. Telescope above surface 5·5 feet.				
Fair Isle . .	90° 16′ 37″·3	7	...	− 4″·8
Foula . . .	90 0 36·8	5	...	− 6·2
Ronas . . .	89 55 25·3	6	...	− 7·1
Yell	90 11 51·7	6	...	− 7·5
Saxavord . .	90 15 21·0	2	...	− 4·4
Fetlar . . .	90 16 45·8	6	...	− 6·2
FOULA. Telescope above surface 5·5 feet.				
Ronas . . .	90° 10′ 39″·0	2	...	− 6″·1
Yell	90 26 11·0	1	...	− 4·8
Brassa . . .	90 24 6·3	3	...	− 6·2
Fair Isle . .	90 25 28·5	2	...	− 4·9
Fitty Hill . .	90 32 11·0	1	...	− 3·2
FAIR ISLE. Telescope above surface 5·5 feet.				
Foula . . .	90° 6′ 10″·5	2	...	− 4″·9
Brassa . . .	90 15 38·0	2	...	− 4·8
Stronsay . .	90 24 16·0	3	...	− 4·8
Fitty Hill . .	90 20 31·5	2	...	− 4·2
Ronas . . .	90 17 34·0	2	...	− 3·1
FITTY HILL. Telescope above surface 5·5 feet.				
Wart Hill Hoy .	89° 48′ 12″·0	4	...	− 7″·3
Foula . . .	90 16 25·5	2	...	− 3·2
Fair Isle . .	90 16 37·0	3	...	− 4·2
Stronsay . .	90 20 5·8	5	...	−10·2
Deerness . .	90 16 8·3	3	...	− 8·6
NorthRonaldshay Lighthouse .	90 23 31·0	1	...	− 9·2
STRONSAY. Telescope above surface 5·5 feet.				
Deerness . .	89° 57′ 44″·0	6	...	−17″·5
Wart Hill Hoy .	89 41 33·0	2	...	− 6·8
Fitty Hill . .	89 55 21·5	6	...	−10·2
Fair Isle . .	90 8 30·5	2	...	− 4·8

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
DEERNESS. Telescope above surface 5·5 feet.				
SouthRonaldshay	90° 0′ 9″·0	1	...	−15″·8
Wart Hill Hoy .	89 28 36·0	2	..	−10·0
Fitty Hill . .	90 2 6·0	2	...	− 8·6
Stronsay . .	90 11 37·0	3	...	−17·5
WART HILL HOY. Telescope above surface 5·5 feet.				
Ben Clibrig .	90° 3′ 33″·3	3	...	− 3″·6
Fitty Hill . .	90 32 36·5	2	...	− 7·3
Stronsay . .	90 40 34·0	2	...	− 6·8
Deerness . .	90 46 39·7	3	...	−10·0
SouthRonaldshay	90 52 32·3	3	...	−13·1
Ben Cheilt . .	90 24 33·5	2	...	− 5·4
SOUTH RONALDSHAY. Telescope above surface 5·5 feet.				
Ben Cheilt . .	90° 2′ 36″·0	2	...	− 6″·0
Dunnet Head .	90 5 1·0	1	...	−12·4
Ben Hutig . .	90 10 48·0	1	...	− 3·7
Wart Hill Hoy .	89 19 3·3	6	...	−13·1
Deerness . .	90 9 47·3	3	...	−15·8
DUNNET HEAD. Telescope above surface 5·5 feet.				
Scarabin . .	89° 38′ 42″·0	1	...	− 6″·7
Ben Clibrig .	89 40 42·5	2	...	− 4·5
Ben Hutig . .	90 1 10·0	2	...	− 5·1
Fashven . .	90 7 52·0	1	...	− 3·9
Wart Hill Hoy .	89 19 23·0	4	...	−13·4
SouthRonaldshay	90 7 50·3	6	...	−12·4
Ben Cheilt . .	89 54 52·0	2	...	− 8·9
Brock Tide Station:—				
Upper Cross	92 3 12·3	8	11·8	+145·2
Lower Cross	92 5 9·1	8	6·8	+ 30·6
Base . .	92 7 48·5	8	...	−126·1
CNOCGHUBHAIS. Telescope above surface 5·5 feet.				
N. Rona Island	90° 25′ 46″·0	2	...	− 4″·5
Ben Hutig . .	89 52 46·9	5	...	−12·5
Fashven . .	88 22 35·7	6	...	−63·3
Monach . . .	90 20 20·8	4	...	− 4·2
Clash Carnagh Tide Station:				
Upper Cross	93 41 38·2	9	7·2	+29·7
Lower Cross	93 43 6·8	12	2·2	−59·3

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
FASHVEN. Telescope above surface 5·5 feet.				
Monach . . .	90° 27′ 57″·0	2	...	− 4″·1
Cnocghuibhais .	91 42 10·0	5	...	−63·3
Ben Hutig . .	90 12 34·5	5	...	−15·3
Ben Clibrig .	89 33 6·7	7	...	− 7·5
Clash Carnagh Tide Station:				
Upper Cross	92 56 50·9	6	7·2	+13·9
Lower Cross	92 57 37·8	6	2·2	−27·7
Rispond Tide Station:—				
Upper Cross	91 46 37·6	3	20·0	+63·8
Base . .	91 48 4·7	6	...	−24·2
Lower Cross	91 47 13·8	3	15·0	+41·8
BEN HUTIG. Telescope above surface 5·5 feet.				
Fashven . .	89° 58′ 8″·8	5	...	−15″·3
Cnocghuibhais .	90 20 17·2	5	...	−12·5
Wart Hill Hoy .	90 14 32·5	3	...	− 4·4
SouthRonaldshay	90 31 51·5	1	...	− 3·7
Dunnet Head .	90 29 35·5	2	...	− 5·1
Ben Cheilt . .	90 22 23·4	4	...	− 4·9
Scarabin . .	90 3 35·5	1	...	− 5·3
Ben Clibrig .	89 15 6·3	8	...	− 9·7
Roan Island Tide Station:—				
Upper Cross	91 56 0·6	10	18·0	+78·9
Lower Cross	91 56 39·7	14	13·0	+47·3
Base . .	91 58 13·4	4	...	−34·7
Rispond Tide Station:—				
Upper Cross	92 39 51·9	9	20·0	+109·5
Lower Cross	92 40 36·9	9	15·0	+ 71·8
Base . .	92 42 22·3	5	...	− 41·5
BEN CHEILT. Telescope above surface 5·5 feet.				
Scarabin . .	88° 54′ 18″·0	1	...	−20″·6
Ben Clibrig .	89 36 11·0	3	...	− 5·6
Ben Hutig . .	90 10 9·5	2	...	− 4·9
Wart Hill Hoy	90 4 23·0	3	...	− 5·4
SouthRonaldshay	90 23 38·3	3	...	− 6·0
Knock . . .	90 14 39·7	3	...	− 3·9
Corryhabbie .	90 9 5·0	2	...	− 3·2
MONACH. Telescope above surface 5·5 feet.				
Cleisham . .	89° 36″ 20′·1	19	...	− 6″·6
N. Rona Island	90 25 43·3	7	...	− 3·9
Cnocghuibhais .	90 16 5·3	1	...	− 4·2
Fashven . .	90 10 57·6	11	...	− 4·1
Ben Clibrig .	90 3 27·5	8	...	− 3·1
Scournalapich .	90 6 22·7	2	...	− 2·6
Ru Rea . . .	90 12 44·5	8	...	− 5·2
Storr . . .	90 4 35·2	9	...	− 3·6

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
WISP. Telescope above surface 5·5 feet.				
Hart Fell . .	89° 42′ 43″·7	3	...	−11″·2
Criffel . . .	90 14 25·7	3	...	− 6·1
Dunrich . .	89 53 43·0	3	...	− 9·9
Cheviot . . .	89 59 45·3	3	...	− 6·1
Cross Fell . .	90 2 31·0	1	...	− 4·8
CALTON HILL. Telescope above surface 5·5 feet.				
Ben Lomond .	89° 49′ 16″·7	3	...	− 3″·7
Ben Cleugh .	89 22 26·5	2	...	− 7·8
Ben Lawers .	89 38 12·0	1	...	− 3·8
East Lomond .	89 30 26·5	2	...	−10·8
Kellie Law . .	90 2 40·0	2	...	− 8·4
BEN CLIBRIG. Telescope above surface 5·5 feet.				
Ben Wyvis . .	90° 9′ 54″·4	20	...	− 5″·5
Scournalapich .	90 17 36·4	11	...	− 3·3
Storr . . .	90 36 37·2	9	...	− 2·6
Monach . . .	90 47 29·8	10	...	− 3·1
Fashven . .	90 48 14·2	17	...	− 7·5
Ben Hutig . .	91 1 40·7	33	...	− 9·7
Fitty Hill . .	90 51 35·6	3	...	− 2·4
Wart Hill Hoy .	90 39 19·0	4	...	− 3·6
Dunnet Head .	90 54 54·3	4	...	− 4·5
SouthRonaldshay	90 51 44·1	2	...	− 3·3
Ben Cheilt . .	90 52 7·0	11	...	− 5·6
Scarabin . .	90 35 10·7	32	...	− 7·2
Corryhabbie .	90 33 7·4	6	...	− 2·8
Ben Macdui .	90 22 48·6	4	...	− 2·5
Roan Island Tide Station . .	91 32 57·1	7	...	− 9·6
SCARABIN. Telescope above surface 6·0 feet.				
Ben Macdui .	90° 11′ 9″·3	1	...	− 2″·9
Ben Wyvis . .	90 2 17·7	4	...	− 4·5
Ben Clibrig . .	89 47 28·2	12	...	− 7·8
Ben Hutig . .	90 26 54·5	10	...	− 5·8
Berridale Tower Point (4)	94 40 54·8	8	0·0	−53·8
Berridale Tower Point (3)	94 39 47·3	8	8·0	+17·9
Berridale Tower Point (2)	94 38 43·8	8	16·0	+89·6
Berridale Tower Point (1)	94 38 23·9	8	18·0	+107·5
Wart Hill Hoy .	90 24 28·5	4	...	− 4·9
Dunnet Head .	90 45 13·8	13	...	− 7·3
SouthRonaldshay	90 40 45·2	6	...	− 5·2
Ben Cheilt . .	91 13 44·7	13	...	−22·5
Mormonth . *H	90 37 38·1	3	2·0	− 2·2
Cowhythe . .	90 41 23·9	4	...	− 4·6
Knock . . .	90 27 32·4	4	...	− 4·4
Corryhabbie .	90 17 44·0	4	...	− 3·8

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
CLEISHAM. Telescope above surface 5·5 feet.				
Ben More, S. Uist	90° 26′ 31″·0	4	...	− 4″·1
NorthRonaIsland	90 48 59·5	3	...	− 2·5
Monach . . .	90 48 8·7	18	...	− 6·6
Cnocghiubhais .	90 42 20·7	3	...	− 2·7
Fashven . . .	90 38 36·3	4	...	− 2·7
Ru Rea . . .	90 41 49·5	3	...	− 5·4
Scournalapich .	90 18 10·5	3	...	− 2·8
Mamsuil . .	90 18 44·5	1	...	− 2·7
Storr . . .	90 18 46·1	11	...	− 5·5
S.W. corner Sir Fred. Johnstone's cottage, near Tide Pole Loch Seaforth, Is^d of Lewis : Ground	97 22 15·4	8	0·0	−56·3
Eaves of roof .	97 20 21·8	8	11·2	+58·9
Chimney band .	97 18 53·0	8	19·4	+142·5
RU REA. Telescope above surface 8·1 feet.				
Storr . . .	89° 37′ 23″·9	29	...	−11″·5
Cleisham . .	89 47 7·2	21	...	− 8·0
Monach . . .	90 17 15·7	11	...	− 7·7
Cnocghiubhais .	90 21 36·3	8	...	− 5·3
COWHYTHE. Telescope above surface 8·8 feet.				
Knock . . .	88° 30′ 4″·0	41	12·5	+17″·7
Corryhabbie .	89 22 5·9	22	13·0	+14·0
Scarabin . .	89 54 43·5	24	11·5	+ 2·1
Ben Cheilt . *H	90 9 52·0	19	1·0	− 6·0
BEN WYVIS. Telescope above surface 5·5 feet.				
Ben Nevis . .	90° 12′ 19″·0	2	...	− 3″·4
Mamsuil . .	90 4 4·0	1	...	− 6·3
Scournalapich .	90 1 43·0	2	...	− 7·7
Ben Hutig . .	90 44 29·5	2	...	− 3·6
Ben Clibrig . .	90 18 31·7	3	...	− 5·5
Scarabin . .	90 35 51·0	2	...	− 4·1
Corryhabbie .	90 30 10·0	1	...	− 3·8
Ben Macdui .	90 8 26·0	3	...	− 4·0
MORMONTH. Telescope above surface 6·2 feet.				
Dudwick . .	90° 14′ 13″·0	21	10·0	+12″·6
Blue Hill . .	90 18 11·0	10	9·0	+ 3·1
Brimmond . .	90 8 47·3	15	8·0	+ 2·3
Mount Battock H	89 56 49·7	21	1·2	− 3·7
Corryhabbie H	89 52 9·2	25	2·3	− 3·3
Knock . . .	89 54 29·5	13	12·5	+ 9·1

* H signifies that the object observed was a heliostat.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
MORMONTH—*continued.*				
Scarabin . H	90° 14′ 2″·6	19	3·3	− 1″·6
Ben Cheilt . H	90 23 58·0	19	0·2	− 3·3
Peterhead Old Windmill . .	90 42 22·0	4	27·0	+74·9
Reform Monument	90 32 26·0	10	Top.	−20·8
Little Stirling .	90 26 4·5	21	10·0	+11·6
KNOCK. Telescope above surface 5·5 feet.				
Ben Cheilt . .	90° 26′ 26″·0	2	...	− 3″·9
Cowhythe . .	91 36 42·0	3	...	−26·6
Mormonth . .	90 26 15·0	2	...	− 7·8
Dudwick . .	90 29 47·0	2	...	− 7·3
Mount Battock .	90 0 0·0	1	...	− 4·9
LITTLE STIRLING. Telescope above surface 5·5 feet.				
Blue Hill . .	90° 6′ 22″·0	2	...	− 7″·7
Dudwick . .	89 44 29·7	3	...	−24·9
Mormonth . .	89 42 52·6	5	...	−16·7
With a 7-inch Theodolite. Telescope above surface 5 feet.				
Mormonth . .	89 42 0·0	6	...	−15·1
Peterhead Old Windmill . .	90 37 23·3	6	27·0	+229·4
Reform Monument	90 21 33·1	8	...	−83·6
Sector Station .	90 37 58·8	4	...	−217·7
PETERHEAD OLD WINDMILL. 7-inch Theodolite. Telescope above surface 32·0 feet.				
Little Stirling .	89° 24′ 47″·5	8	...	−333″·7
Mormonth . .	89 25 27·5	8	...	−115·1
Sector Station .	89 30 5·0	8	...	−371·5
Reform Monument	88 59 40·0	8	...	−884·8
DUDWICK. Telescope above surface 5·5 feet.				
Tarbathy . .	90° 22′ 35″·8	4	...	−14″·1
Blue Hill . .	90 11 24·0	4	...	− 9·1
Over Hill . .	90 10 26·7	3	...	−17·3
Brimmond . .	89 56 56·5	2	...	−11·1
Mount Battock .	89 45 6·3	3	...	− 5·1
Corryhabbie .	89 45 50·5	2	...	− 4·9
Knock . . .	89 52 34·7	6	...	− 7·3
Mormonth . .	89 55 7·0	2	...	−18·1
Little Stirling .	90 22 7·5	2	...	−24·9
Layton . . .	90 31 47·5	4	...	−20·7

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
LAYTON. Telescope above surface 5·5 feet.				
Blue Hill . .	89° 48′ 9″·8	5	...	−16″·0
Tarbathy . .	89 55 16·12	8	...	−42·8
Brimmond . .	89 19 24·4	5	...	−19·7
Over Hill . .	88 59 35·2	5	...	−61·5
Dudwick . .	89 35 49·7	6	...	−20·7
GREAT STIRLING SECTOR STATION. Telescope above surface 5·0 feet.				
Little Stirling .	89° 24′ 51″·0	11	...	−217″·7
Reform Monument	90 9 6·0	11	...	− 98·2
Peterhead Old Windmill . .	90 32 8·5	11	27·0	+255·4
STORR. Telescope above surface 6·1 feet.				
Ben Heynish H	90° 41′ 17″·0	4	5·8	− 0″·2
Ben More, S.U. .	90 20 32·5	9	...	− 5·3
Ben More, S.U. H	90 20 38·3	13	5·0	− 0·9
Cleisham . .	90 9 47·3	16	...	− 6·1
Cleisham . H	90 9 53·0	17	2·3	− 3·7
Monach . . H	90 38 11·1	23	1·3	− 3·2
Cnocghiubhais. H	90 41 46·5	12	2·0	− 1·8
Ru Rea . . .	90 42 45·6	15	...	− 8·7
Ru Rea . . H	90 42 47·4	4	1·5	− 6·5
Ben Clibrig . H	90 23 27·3	17	2·0	− 1·9
Mamsuil . . .	89 52 19·5	8	...	− 5·6
Mamsuil . . H	89 52 22·5	14	0·7	− 5·0
Ben Nevis . .	90 3 34·9	13	...	− 3·6
Ben Nevis . H	90 3 45·7	21	1·5	− 2·7
Ben More, Mull .	90 19 42·1	5	...	− 3·2
Ben More, Mull. H	90 19 46·3	10	0·6	− 2·9
SCOURNALAPICH. Telescope above surface 7·5 feet.				
Mamsuil . . .	89° 52′ 8″·7	11	26·8	+114″·1
Ben More, S.U. H	90 44 18·0	22	5·0	− 1·2
Cleisham . H	90 37 55·1	15	...	− 3·8
Monach . . H	90 53 51·2	11	2·3	− 2·5
Ben Clibrig . .	90 29 59·5	9	12·0	+ 2·7
Ben Wyvis . .	90 18 22·2	22	12·0	+ 6·3
Corryhabbie . H	90 37 23·0	26	0·9	− 3·7
Corryhabbie .	90 37 0·8	3	16·8	+ 5·2
Ben Nevis . .	90 4 6·2	26	8·0	+ 0·5
Ben Nevis . H	90 4 24·0	7	1·5	− 5·9
Ben Macdui . H	90 15 3·0	17	1·0	− 4·5
Ben Macdui . .	90 14 26·4	12	20·0	+ 8·7

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
CORRYHABBIE. Telescope above surface 7·2 feet.				
Glashmeal	89° 53′ 59″·9	16	...	− 8″·5
Ben Macdui	89 26 16·8	24	...	−10·8
Ben Nevis	90 12 15·3	19	...	− 3·6
Mamsuil	90 13 58·2	20	...	− 3·9
Scournalapich	90 13 32·4	23	...	− 4·0
Ben Wyvis	90 10 15·7	27	...	− 5·0
Ben Clibrig H	90 22 41·6	31	...	− 3·7
Scarabin H	90 28 10·5	35	1·0	− 3·9
Ben Cheilt H	90 40 8·2	28	1·5	− 3·3
Ben Cheilt	90 40 21·3	4	...	− 4·2
Cowhythe H	91 0 23·1	32	1·0	− 7·9
Knock	90 41 30·1	21	...	−12·3
Mormonth H	90 42 34·5	44	1·7	− 4·6
Dudwick	90 45 48·3	24	...	− 6·4
Mount Battock	90 12 18·9	49	...	− 8·7
MAMSUIL. Telescope above surface 14·0 feet.				
Ben More, Mull	90° 32′ 10″·0	9	...	− 8″·0
Ben More, S.U. H	90 44 34·8	15	1·5	− 6·0
Ben More, S.U.	90 44 25·5	4	...	− 6·7
Storr	90 38 54·4	32	...	−12·8
Cleisham	90 39 19·9	13	...	− 7·0
Scournalapich	90 12 40·5	43	...	−83·0
Ben Wyvis	90 21 5·4	34	...	−16·0
Corryhabbie H	90 38 36·5	27	0·9	− 7·1
Ben Macdui	90 16 5·2	12	...	− 9·7
Glashmeal	90 30 2·0	3	...	− 7·6
Ben Lawers H	90 21 38·6	21	1·5	− 8·0
Ben Nevis	90 2 14·2	23	...	−16·2
OVER HILL. Telescope above surface 5·5 feet.				
Brimmond	89° 27′ 25″·5	2	...	−28″·4
Mount Battock	89 28 50·0	3	...	− 6·7
Mormonth	90 1 0·7	3	...	− 8·9
Dudwick	89 58 46·0	4	...	−17·3
Little Stirling	90 11 18·5	2	...	−12·0
Layton	91 4 7·7	3	...	−61·5
Tarbathy	90 57 14·3	3	...	−64·7
Blue Hill	90 3 35·5	2	...	−19·3
TARBATHY. Telescope above surface 5·5 feet.				
Blue Hill	89° 40′ 35″·8	4	...	−25″·6
Mount Battock	89 21 43·0	1	...	− 6·9
Brimmond	88 57 17·0	3	...	−31·2
Over Hill	89 6 23·0	3	...	−64·7
Dudwick	89 49 0·0	2	...	−14·1
Layton	90 9 27·7	3	...	−42·8
Little Stirling	90 2 0·0	2	...	−10·9

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
BRIMMOND. Telescope above surface 5·5 feet.				
Mount Battock	89° 23′ 35″·5	2	...	− 8″·8
Knock	90 1 15·5	2	...	− 6·2
Mormonth	90 11 32·0	2	...	− 7·0
Dudwick	90 15 36·0	2	...	−11·1
Over Hill	90 38 35·0	2	...	−28·4
Layton	90 48 41·5	2	...	−19·7
Tarbathy	91 8 41·3	3	...	−31·2
Blue Hill	90 41 2·5	2	...	−31·5
BLUE HILL. Telescope above surface 5·5 feet.				
Mount Battock	89° 15′ 48″·5	2	...	− 8″·5
Dudwick	90 6 23·3	4	...	− 9·1
Little Stirling	90 13 39·0	1	...	− 7·7
BEN MACDUI. Telescope above surface 8·2 feet.				
Ben Cleugh H	90° 43′ 25″·3	19	1·3	− 4″·4
Ben Lawers	90 20 30·4	8	...	− 7·6
Ben Lawers H	90 20 39·2	34	2·5	− 5·3
Ben Nevis	90 18 16·6	8	...	− 6·0
Ben Nevis H	90 18 37·4	31	1·5	− 4·9
Mamsuil H	90 25 54·5	7	4·0	− 2·9
Scournalapich H	90 26 59·5	28	3·0	− 3·6
Ben Wyvis H	90 30 26·9	10	1·0	− 5·2
Ben Clibrig H	90 40 7·2	10	2·8	− 2·5
Scarabin H	90 48 1·6	17	3·0	− 2·6
Ben Cheilt H	90 57 24·0	18	2·0	− 2·8
Corryhabbie	90 53 22·9	21	...	−12·3
Dudwick H	91 1 32·0	13	1·7	− 3·9
MountBattock H	90 45 9·3	39	1·0	− 7·8
Glashmeal	90 35 58·6	22	...	−18·1
Sayrs Law H	90 52 1·2	8	2·5	− 2·4
BEN MORE, SOUTH UIST. Telescope above surface 5·4 feet.				
Cleisham	90° 11′ 40″·4	8	...	− 4″·1
Cleisham H	90 11 14·5	34	3·0	− 1·8
Storr	90 11 44·5	9	...	− 4·7
Storr H	90 11 32·7	37	6·8	+ 1·2
Scournalapich H	90 17 4·3	25	0·8	− 2·2
Mamsuil H	90 14 31·7	19	14·0	+ 4·1
Ben Nevis H	90 16 36·8	15	1·5	− 1·7
BenMore, Mull H	90 17 29·0	27	0·0	− 2·8
Jura H	90 34 1·7	13	2·1	− 1·2
Ben Heynish H	90 38 42·5	39	...	− 3·7
Callernish Tide Station	98 23 31·3	14	...	−82·5

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
MOUNT BATTOCK. Telescope above surface 5·8 feet.				
	° ′ ″			″
Glashmeal . .	89 43 25·5	28	13·0	1 1·7
Kellie Law . H	90 44 18·0	8	1·3	− 3·6
East Lomond .	90 32 34·5	13	11·0	+ 3·9
Ben Cleugh . H	90 26 7·6	15	1·0	− 2·8
Ben Macdui . .	89 41 49·5	11	8·0	+ 2·4
Ben Macdui . H	89 41 56·9	15	1·8	− 4·3
Ben Macdui . .	89 42 2·0	6	...	− 6·2
Corryhabbie .	90 11 38·1	24	16·8	+13·4
Knock . . .	90 33 8·6	13	12·5	+ 6·0
Mormonth . .	90 41 48·1	15	...	− 4·3
Dudwick . .	90 45 56·5	7	10·0	+ 3·9
Over Hill . .	90 54 33·0	7	8·0	+ 2·8
Brimmond . .	90 53 53·5	16	8·0	+ 3·6
Blue Hill . .	91 3 2·8	16	9·0	+ 5·0
Lumsden . H	90 43 23·9	17	1·7	− 2·1
Sayrs Law . H	90 34 52·2	14	2·5	− 1·7
GLASHMEAL. Telescope above surface 5·5 feet.				
	° ′ ″			″
Ben Cleugh . .	90 34 3·0	2	...	− 4·3
Ben Lawers . .	90 7 27·0	3	...	− 5·4
Ben Macdui .	89 38 30·3	4	...	−12·1
Corryhabbie .	90 31 46·3	3	...	− 6·5
Mount Battock .	90 35 20·5	4	...	− 8·8
East Lomond .	90 47 7·5	2	...	− 4·9
EAST LOMOND. Telescope above surface 5·5 feet.				
	° ′ ″			″
Dunrich . .	90 3 37·0	2	...	− 4·6
Calton Hill . .	90 44 46·0	1	...	−10·8
Ben Cleugh . .	89 41 49·5	2	...	−10·0
Ben Lomond .	89 59 51·5	2	...	− 3·9
Ben Lawers . .	89 39 16·0	3	....	− 4·9
Glashmeal . .	89 46 23·0	3	...	− 4·9
Mount Battock .	90 5 48·7	3	...	− 4·1
Kellie Law . .	90 40 19·0	2	...	−12·7
Lumsden . .	90 26 44·0	1	...	− 4·7
Sayrs Law . .	90 7 28·0	2	...	− 6·2
BEN CLEUGH. Telescope above surface 6·0 feet.				
	° ′ ″			″
Merrick . . .	90 24 31·7	13	...	− 3·0
Goat Fell . .	90 19 37·3	8	...	− 3·5
Ben Lomond .	89 55 55·0	45	...	− 7·0
Ben Nevis . H	90 2 13·9	27	1·5	− 2·8
Ben Lawers . .	89 36 17·4	37	...	− 7·7
Ben Macdui .	90 1 46·6	21	...	− 3·8
Glashmeal . .	90 3 22·0	20	...	− 4·7
Mount Battock .	90 22 0·5	10	...	− 3·6
East Lomond .	90 35 10·5	22	...	−10·9
Sayrs Law . .	90 25 46·9	15	...	− 4·8
Calton Hill Obsʸ	90 56 59·5	7	Top.	− 8·5
Dunrich . . .	90 16 25·0	9	...	− 4·9
Hart Fell . .	90 16 22·8	8	...	− 4·2
BEN NEVIS. Telescope above surface 5·5 feet.				
	° ′ ″			″
Jura . . . H	90 43 0·2	9	1·5	− 2·1
Ben More, Mull .	90 34 15·7	14	...	− 4·6
BenMore,Mull. H	90 34 20·3	14	1·8	− 3·2
Ben Heynish H	91 1 17·7	23	1·3	− 2·2
Ben More, S.U. H	90 49 57·8	13	5·3	− 0·1
Storr . . .	90 44 13·6	7	...	− 3·3
Storr . . H	90 44 18·0	7	1·3	− 2·5
Mamsuil . . .	90 23 13·7	8	...	− 6·4
Scournalapich. H	90 25 9·4	24	1·0	− 4·4
Scournalapich .	90 24 50·1	23	...	− 5·4
Ben Wyvis . H	90 32 7·8	15	1·5	− 2·5
Ben Wyvis . .	90 32 41·8	27	...	− 3·4
Corryhabbie H	90 44 51·0	14	2·3	− 1·6
Ben Macdui . H	90 21 52·8	13	2·5	− 2·2
Ben Macdui . .	90 21 6·1	21	...	− 4·0
Ben Lawers . .	90 20 52·5	26	...	− 6·2
Ben Cleugh . H	90 44 37·2	14	1·5	− 2·4
Ben Lomond .	90 34 10·3	9	...	− 4·9
Ben Lomond H	90 32 50·0	10	4·6	− 0·8
BEN LAWERS. Telescope above surface 6·0 feet.				
	° ′ ″			″
Merrick . . . H	90 44 29·8	16	...	− 2·4
Goat Fell . . H	90 37 29·8	5	1·0	− 2·6
Goat Fell . . .	90 37 29·3	5	...	− 3·2
Ben Lomond .	90 28 40·9	15	...	− 8·0
Ben Lomond H	90 28 40·2	7	1·2	− 6·4
Jura . . . H	90 41 27·8	19	1·6	− 2·1
Ben More,Mull. H	90 33 9·4	14	8·5	+ 1·4
Ben More, Mull .	90 33 17·8	5	...	− 3·4
Ben Nevis . H	90 5 1·5	20	1·2	− 5·4
Ben Nevis . .	90 4 59·3	7	...	− 6·7
Mamsuil . . H	90 24 11·3	20	1·5	− 2·8
Ben Macdui . H	90 10 57·4	33	1·2	− 4·5
Glashmeal . H	90 22 50·2	35	1·2	− 4·7
East Lomond .	90 53 49·6	11	...	− 5·3
Sayrs Law . . H	90 47 27·5	14	1·3	− 2·4
Ben Cleugh . H	90 46 21·3	27	1·2	− 6·2
Hart Fell . H	90 41 44·6	12	0·7	− 2·4
BEN MORE, MULL. Telescope above surface 5·5 feet.				
	° ′ ″			″
Jura	90 24 14·3	7	...	− 5·9
Ben Tartevil .	90 49 40·3	4	...	− 4·2
Ben Heynish .	91 3 39·0	2	...	− 6·2
Ben More, S. Uist	90 37 35·0	2	...	− 2·9
Storr . . .	90 34 58·5	2	...	− 2·9
Ben Nevis . .	89 59 58·0	4	...	− 4·6
Ben Lawers . .	90 18 21·0	2	...	− 3·1
Ben Lomond .	90 20 28·3	3	...	− 3·9
Goat Fell . .	90 25 39·0	1	...	− 3·4

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
BEN HEYNISH. Telescope above surface 5·5 feet.				
Ben More, S. Uist	90° 3′ 9″·0	1	...	− 3″·8
Ben Nevis	89 55 39·0	1	...	− 2·8
Ben More, Mull	89 22 21·3	4	...	− 6·2
Jura	89 52 45·5	2	...	− 4·1
BURNSWARK. Telescope above surface 5·6 feet.				
Criffel	89° 31′ 55″·0	12	...	−12″·4
Merrick H	89 52 0·5	22	1·6	− 3·3
Hart Fell	89 17 44·8	15	...	− 9·8
Wisp	89 29 59·5	10	...	−12·1
Cross Fell H	89 44 6·7	16	1·0	− 4·3
Sea Fell H	89 42 42·1	11	−3.4	7·9
BEN LOMOND. Telescope above surface 5·5 feet.				
Goat Fell	90° 21′ 24″·0	1	...	− 4″·8
Knocklayd	90 45 20·0	1	...	− 2·3
Jura	90 28 26·0	1	...	− 3·8
Ben More, Mull	90 21 11·5	2	...	− 3·9
Ben Nevis	89 59 5·7	3	...	− 4·9
Ben Lawers	89 53 53·3	3	...	− 7·4
East Lomond	90 41 1·0	3	...	− 3·9
Ben Cleugh	90 29 6·5	4	...	− 6·5
Dunrich	90 33 19·0	2	...	− 3·0
Hart Fell	90 31 39·0	1	...	− 3·0
BEN LOMOND. Telescope above surface 6·2 feet. (1855.)				
Ben Cleugh	90° 28′ 54″·5	1	...	− 7″·3
Jura	90 27 21·0	1	...	− 4·3
Ben Nevis	89 58 3·8	2	...	− 5·5
Ben Lawers	89 53 10·9	4	...	− 8·3
Goat Fell	90 20 39·5	4	...	− 5·4
KELLIE LAW. Telescope above surface 5·8 feet.				
Sayrs Law	89° 43′ 33″·5	15	...	− 8″·1
Sayrs Law H	89 43 26·9	14	2·6	− 4·5
Calton Obsy	90 15 6·8	6	Top.	− 8·9
East Lomond	89 32 51·0	11	...	−13·4
East Lomond H	89 33 2·0	14	1·2	−10·7
Glashmeal	89 38 29·2	4	...	− 4·7
Glashmeal H	89 38 50·5	13	1·3	− 3·6
Mount Battock H	89 51 23·3	20	1·2	− 3·8
Lumsden	90 8 54·2	12	...	− 7·0
Lumsden H	90 8 55·3	18	1·1	− 5·7

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
MORDINGTON. Telescope above surface 6·1 feet.				
Cheviot	89° 11′ 11″·9	11	...	−10″·3
Cheviot H	89 11 11·0	30	2·5	− 6·1
Wisp H	90 1 54·7	12	1·5	− 3·6
Dunrich H	89 52 14·6	21	2·3	− 3·2
Sayrs Law	89 38 16·1	15	...	−10·1
Blackheddon	90 4 36·1	11	...	−18·3
MERRICK. Telescope above surface 6·3 feet.				
Burnswark	90° 42′ 6″·6	2	...	− 5″·2
Jura H	90 30 31·7	17	1·5	− 2·3
S. Donard H	90 31 39·7	23	1·6	− 2·1
Ben More, Mull H	90 36 26·3	12	1·7	− 1·7
Goat Fell H	90 14 53·9	13	1·0	− 4·7
Wisp H	90 31 9·9	15	1·3	− 3·3
Hart Fell H	90 18 50·6	28	1·4	− 4·1
Sea Fell H	90 21 17·7	32	...	− 3·6
Criffel	90 29 41·6	20	...	− 6·8
Knocklayd H	90 35 20·0	26	10·5	+ 2·3
Blackcomb H	90 34 46·4	35	...	− 3·2
Divis	90 36 51·6	6	...	− 3·4
Divis H	90 37 26·3	5	1·5	− 2·6
Ben Lomond H	90 22 51·5	29	...	− 3·4
Ben Lawers H	90 27 28·5	16	1·0	− 2·1
South Berule H	90 36 19·5	35	...	− 3·6
Trostan H	90 33 26·6	31	1·5	− 2·8
Ben Cleugh H	90 31 51·1	10	1·2	− 2·6
Cross Fell H	90 29 52·8	10	1·6	− 2·2
HART FELL. Telescope above surface 5·6 feet.				
Dunrich H	90° 14′ 56″·0	23	3·5	− 5″·7
Ben Cleugh H	90 23 13·8	13	1·6	− 2·8
Merrick H	90 14 41·8	17	0·3	− 4·5
Cross Fell H	90 18 40·9	8	0·8	− 3·1
Criffel	90 27 12·5	16	...	− 6·5
Sea Fell H	90 17 29·6	19	5·0	− 0·3
Wisp	90 30 36·3	20	...	−11·4
Cheviot H	90 16 57·3	12	2·5	− 2·4
Sayrs Law H	90 29 6·6	10	4·0	− 1·5
Goat Fell	90 23 52·0	2	...	− 3·0
Ben Lomond H	90 20 41·4	13	1·3	− 2·3
Ben Lawers H	90 20 26·6	15	0·5	− 2·3
Burnswark	90 58 7·9	11	...	− 9·8
JURA. Telescope above surface 6·0 feet.				
Ben Tartevil	91° 3′ 49″·7	12	15·0	+16″·7
Ben More, Mull	90 2 3·1	16	20·0	+15·1
Ben More, Mull H	90 1 59·3	4	...	− 6·5
Knocklayd	90 30 21·3	9	10·0	+ 3·0
Goat Fell	90 8 10·0	13	16·0	+10·6

JURA—*continued.*

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Ben Lomond	90° 13′ 22″·9	6	19·0	+ 9″·0
Ben Lomond H	90 13 40·9	5	0·8	− 3·6
Ben Heynish H	90 45 21·5	26	1·0	− 3·7
Ben More, S.U. H	90 42 13·1	8	5·0	− 0·4
Merrick H	90 28 1·6	14	1·9	− 2·0
S. Snaght H	90 31 29·9	15	3·8	− 1·2
Ben Nevis H	90 10 4·6	16	0·8	− 2·8
Ben Nevis	90 9 25·0	12	21·5	+ 8·3
Ben Lawers H	90 18 2·2	21	3·0	− 1·4
Trostan H	90 30 21·3	12	1·5	− 3·0
Mount Sandy H	90 49 20·6	10	1·3	− 3·0

GOATFELL. Telescope above surface 7 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Jura	90° 19′ 4″·4	5	...	− 7″·4
Merrick	90 17 49·3	6	...	− 6·2
Ben Lomond	90 11 55·1	8	...	− 6·1
Knocklayd	90 33 48·4	6	...	− 5·2
Ben Cleugh H	90 29 50·3	6	1·2	− 3·4
Slieve Donard	90 38 24·8	4	...	− 2·6
Hart Fell H	90 28 18·7	6	1.4	− 3·0
Trostan H	90 33 10·0	4	1·5	− 3·9
Ben More, Mull H	90 20 39·3	4	1·7	− 3·3
Divis H	90 39 23·0	4	1·5	− 2·8
South Berule H	90 46 11·3	6	...	− 2·6
Criffel H	90 37 9·0	6	1·5	− 2·8
Slieve Snaght	90 38 45·0	5	...	− 3·1
Ben Lawers H	90 17 27·5	8	1·0	− 3·2

LUMSDEN. Telescope above surface 7 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Sayrs Law	89° 31′ 23″·4	12	...	−14″·8
East Lomond	90 6 10·0	4	...	− 6·0
Kellie Law H	90 14 53·6	14	1·0	− 7·2
Glashmeal H	90 6 8·0	6	1·5	− 2·7
Mount Battock H	90 10 46·5	6	1·7	− 2·8
Blackheddon	90 10 43·4	8	...	−12·7
Cheviot H	89 28 39·0	25	2·0	− 6·6

BLACKHEDDON. Telescope above surface 5·8 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Cheviot	88° 31′ 4″·6	18	...	−16″·3
Dunrich	89 54 52·1	5	...	− 4·6
Sayrs Law	89 49 58·1	22	...	− 7·1
Lumsden	90 5 39·8	18	...	−10·6
Mordington	90 5 54·6	6	...	−17·6
Mordington H	90 5 39·7	7	1·2	−13·9

SAYRS LAW. Telescope above surface 5·5 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Wisp	90° 11′ 57″·1	6	...	− 5″·3
Hart Fell	90 0 23·5	3	...	− 5·2
Dunrich	89 54 21·1	6	...	− 7·8
Lumsden	90 42 41·6	13	...	−11·6
Ben Lomond	90 15 52·0	1	...	− 2·7
Ben Cleugh	90 9 41·5	6	...	− 4·4
Ben Lawers H	90 5 55·7	8	3·5	− 1·0
East Lomond	90 17 55·0	8	...	− 6·2
Ben Macdui	90 15 41·0	2	...	− 2·3
Glashmeal H	90 13 16·0	9	1·5	− 2·1
Mount Battock H	90 21 3·0	3	1·7	− 2·0
Kellie Law H	90 36 49·7	7	1·0	− 6·3
Blackheddon	90 34 14·2	8	...	− 6·7
Cheviot	89 53 47·2	8	...	− 6·6
Cheviot H	89 53 50·9	3	2·0	− 4·2
Mordington	90 39 27·6	10	...	− 9·2

DUNRICH. Telescope above surface 7·7 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Mordington H	90° 42′ 12″·5	1	1·3	− 5″·4
Ben Lomond	90 18 46·8	5	...	− 4·3
Hart Fell	89 55 39·0	3	...	−21·2
Sayrs Law	90 26 28·0	2	...	−10·9
Ben Cleugh	90 18 45·8	4	...	− 6·3
East Lomond H	90 30 44·5	4	...	− 6·5
Blackheddon	90 41 38·5	2	...	− 6·1
Wisp	90 22 44·0	3	...	−13·9
Cheviot	90 11 4·0	4	...	− 7·3
Criffel H	90 25 29·8	8	1·3	− 5·3

CHEVIOT. Telescope above surface 8·9 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Lumsden	90° 53′ 16″·2	11	...	−11″·7
Blackheddon	91 39 27·6	21	...	−25·0
Sayrs Law	90 30 23·2	12	...	−10·6
Sayrs Law H	90 30 25·5	2	4·0	− 5·9
Wisp	90 26 15·0	13	...	− 9·9
Collier Law H	90 31 22·1	17	1·5	− 5·9
Sca Fell H	90 25 49·8	11	0·5	− 4·0
Cross Fell H	90 17 39·9	12	1·5	− 5·2
Cross Fell	90 17 15·7	5	...	− 6·3
Easington H	90 46 7·2	12	2·8	− 3·0
Botton Head H	90 40 41·0	15	...	− 4·1
Dunrich H	90 18 54·8	10	0·8	− 7·6
Dunrich	90 19 2·0	4	...	− 8·4
Wordeslow H	90 44 46·7	20	1·3	− 5·8
Hart Fell H	90 18 36·8	13	2·5	− 5·1
Hart Fell	90 18 50·0	4	...	− 7·0
Mordington H	91 5 52·2	18	3·7	− 8·9

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
BEN TARTEVIL. Telescope above surface 5·5 feet.				
Slieve Snaght .	90° 2′ 33″·0	4	...	− 4″·2
Jura	89 11 48·5	12	...	−10·2
Knocklayd . .	89 59 24·9	9	...	− 5·4
Ben More, Mull .	89 48 8·0	4	...	− 4·2
Trostan . . .	90 3 26·0	2	...	− 4·4
Ben Heynish .	90 23 26·0	1	...	− 4·0
TROSTAN. Telescope above surface 5·0 feet.				
Divis . . .	90° 16′ 42″·5	8	...	− 6″·4
Knocklayd . .	90 12 53·5	7	...	−21·9
Ben Tartevil .	90 31 40·0	2	...	− 4·0
Slieve Donard .	90 11 35·0	3	...	− 3·2
Jura	90 13 35·4	1	...	− 3·3
Goat Fell . .	90 7 49·4	1	...	− 3·5
Sawel . . .	90 7 3·5	2	...	− 5·1
KNOCKLAYD. Telescope above surface 5·0 feet.				
Slieve Snaght .	90° 10′ 36″·8	8	...	− 4″·6
Divis . . .	90 16 37·5	8	...	− 5·0
Sawel . . .	90 5 24·6	8	...	− 5·0
Trostan . . .	89 54 24·8	13	...	−21·9
SLIEVE SNAGHT. Telescope above surface 5·0 feet.				
Sawel . . .	90° 5′ 30″·1	5	...	− 6″·8
Slieve League .	90 24 45·0	1	...	− 2·9
Cuilcagh . . .	90 24 39·0	1	...	− 2·7
MOUNT SANDY. Telescope above surface 5·0 feet.				
Slieve Snaght .	88° 46′ 12″·0	4	...	−12″·3
Cundtham . .	89 2 0·7	6	16·7	+59·6
S. End of Base .	90 1 22·2	5	14·5	+36·8
Drung Point .	90 3 32·5	3	...	−22·1
N. End of Base .	90 13 24·0	4	...	−89·2
Do. Top of Staff	90 12 15·3	6	5·3	+ 5·2
CUNDTHAM. Telescope above surface 5·0 feet.				
Mount Sandy .	91° 1′ 30″·0	4	14·5	+48″·5
N. End of Base .	91 7 10·0	4	14·0	+47·8
S. End of Base .	91 43 47·0	3	14·5	+34·2

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
SOUTH END OF BASE. Telescope above surface 5·0 feet.				
N. End of Base .	90° 8′ 22″·5	8	...	−24″·8
Slieve Snaght .	88 51 9·5	8	...	−11·7
Mount Sandy .	90 5 26·8	8	...	−19·4
Cundtham . .	89 23 29·0	10	...	−18·0
Do. Top of Staff	89 22 34·0	2	16·7	+42·1
DRUNG POINT. Telescope above surface 5·0 feet.				
Mount Sandy, Top of Staff . .	90° 2′ 28″·3	6	14·5	+42″·1
N. End of Base, Top of Staff .	90 3 59·7	6	14·0	+44·2
S. End of Base, Top of Staff .	90 1 0·0	6	14·5	+39·1
NORTH END OF BASE. Telescope above surface 4·5 feet.				
Mount Sandy .	89° 50′ 16″·8	8	...	−80″·3
S. End of Base, Top of Pole .	89 56 21·0	4	14·5	+49·5
Cundtham . .	88 57 13·7	6	...	−23·9
STOKE HILL. Telescope above surface 11·5 feet.				
Wingreen . .	90° 1′ 34″·4	24	...	− 22″·6
Westbury Down	89 58 47·7	30	...	−125·3
Westbury Down	89 57 53·2	13	14·0	+ 27·2
Milk Hill . H	89 51 49·2	23	1·5	− 33·5
Inkpen . . . H	90 3 48·5	25	1·5	− 14·7
Beacon Hill . H	90 8 45·3	25	1·5	− 24·4
Dean Hill . H	90 15 0·4	21	1·0	− 16·8
Old Lodge . H	90 13 15·1	4	1·5	− 19·2
Old Lodge . .	90 12 15·3	6	27·3	+ 30·2
Old Sarum Cas. H	90 19 7·5	11	2·8	− 20·3
Four Mile Stone	90 15 49·0	26	...	− 32·4
UPCOT DOWN. Telescope above surface 10·9 feet.				
Milk Hill . .	89° 54′ 28″·3	21	25·4	+77″·1
Stoke Hill . .	90 12 46·7	14	...	−23·6
Westbury Down	90 12 11·9	14	...	−20·3
Mendip . . . H	90 12 20·5	17	3·0	− 8·4
White Horse Hill	90 6 34·3	29	...	−34·6
Inkpen . . .	90 3 34·2	12	...	−24·8

MILK HILL. Telescope above surface 21·7 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
	° ′ ″			″
Wingreen . . H	90 12 11·3	3	10·8	−14·5
Stoke Hill . .	90 18 17·3	4	...	−72·5
Westbury Down .	90 15 13·5	4	14·0	−20·0
Mendip . . H	90 11 59·0	9	3·0	−22·8
Upcot Down . .	90 10 59·6	7	5·4	−86·4
Inkpen . . .	90 6 38·1	4	...	−50·4
Beacon Hill . H	90 19 37·2	7	2·0	−53·2
Dean Hill . H	90 21 27·6	6	1·5	−31·0
Four Mile Stone. H	90 23 54·7	9	1·5	−44·3
Old Lodge . H	90 21 30·6	4	4·7	−35·1
Old Sarum Castle. H	90 26 36·0	5	2·5	−38·1
White Horse Hill	90 11 26·5	2	...	−45·9

FOUR MILE STONE. Telescope above surface 6·4 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
	° ′ ″			″
Wingreen . H	89 47 12·3	2	1·3	−14·0
Westbury Down. H	89 55 44·5	2	2·5	− 9·6
Westbury Down .	89 55 29·5	1	14·0	+18·8
Stoke Hill . H	89 54 25·3	3	6·0	− 1·1
Dean Hill . H	90 4 29·6	4	1·5	−17·8

OLD SARUM GUN. Telescope above surface 37·5 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
	° ′ ″			″
Beacon Hill . H	89 25 53·7	6	1·0	...
Old Lodge . .	89 33 51·8	2	27·3	...

OLD SARUM CASTLE. Telescope above surface 37·8 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
	° ′ ″			″
Stoke Hill . .	89 54 1·1	4	25·1	− 29·9
Milk Hill . .	89 49 45·9	3	...	− 75·1
Beacon Hill . .	89 41 56·0	2	6·3	−171·0
Old Lodge . .	89 49 39·3	2	27·3	− 55·7
Dean Hill . .	89 56 29·3	2	...	−186·6

HAPPISBURGH TOWER.

Telescope above surface . 116·6 feet.
„ parapet . 5·3 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
	° ′ ″			″
Norwich Sp., Top	90 0 12·3	7	307·0	...
Baconsthorpe Tower . „	89 59 58·5	4	59·5	...
Gorleston Tr. „	90 8 47·3	4	87·7	...
Tofts Tower „	90 9 3·5	3	57·0	...

GORLESTON TOWER.

Telescope above surface . 91·3 feet.
„ battlement . 3·6 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
	° ′ ″			″
Norwich Sp. Top	89 59 30·9	8	307·0	...
Happisburgh Tower . „*	90 5 19·0	13	111·3	+ 41·4
Southwold Tower Pole . . Top	90 5 31·0	1	118·0	+ 59·9
Toft's Tower „	90 0 35·0	6	57·0	−160·7

TOFT'S TOWER.

Telescope above surface . 72·0 feet.
„ tower . 15·0 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
	° ′ ″			″
Happisburgh Tower . Top	90 8 15·5	9	111·6	+68·2
Norwich Sp. „	89 58 17·4	12	307·0	...
Gorleston Tr. „	90 6 39·0	10	87·7	+73·5
Southwold Tr. „	90 6 41·9	15	95·0	+70·4
Laxfield Ch. Tower . „	90 2 56·4	11	93·8	+52·3
Bunwell Tr. „	90 3 18·8	10	75·0	+ 6·3
Do. . H	90 3 42·9	8	80·0	+16·7

NORWICH SPIRE.

Telescope above surface . 305·0 feet.
Vane above telescope . 2·0 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
	° ′ ″			″
Hingham Tr., Top	90 6 1·5	6	...	...
Gorleston Tr. „	90 13 59·0	4	87·7	...
Baconsthorpe Tower . „	90 6 45·0	4	59·5	...
Happisburgh Tower . „	90 12 0·4	2	111·3	...
Laxfield Ch. Tower . „	90 10 15·4	1	93·8	...
Toft's Tower „	90 12 45·4	2	57·0	...
Bunwell Tr. „	90 7 49·4	7	75·0	...

BACONSTHORPE TOWER.

Telescope above surface . 65·0 feet.
„ battlement 5·5 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
	° ′ ″			″
Happisburgh Tower . Top	90 12 10·5	8	111·3	+112·4
Norwich Sp. „	90 6 40·0	5	307·0	...
Hingham Tr. „	90 8 45·5	5	...	−112·6
Swaffham Sp. „	90 7 52·8	9	154·8	+136·2
Docking Tr. „	90 7 25·8	8	...	−112·8

* Heliostat Top.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
DOCKING TOWER.				
	° ′ ″			″
Walpole, St. Peter's Tower . Top	90 15 53·2	5	81·0	...
Boston Tower ,,	90 11 59·5	6	...	...
Baconsthorpe Tower . . ,,	90 9 5·3	6	59·5	...
Lynn Tower . ,,	90 16 56·9	7	88·2	...
SOUTHWOLD TOWER. { Telescope above surface 123·3 feet. ,, tower 28·3 feet.				
	° ′ ″			″
Orford Cas. (¹) Top	90 7 58·8	9	84·7	− 86·9
Laxfield Tr. ,,	90 0 49·4	10	93·8	− 86·3
Toft's Tower ,,	90 4 58·9	10	57·0	−203·0
Gorleston Tr. ,,	90 8 4·5	10	87·7	− 79·9
ORFORD CASTLE. { Telescope above surface . 94·5 feet. ,, coping of tr. 9·8 feet.				
	° ′ ″			″
Walton Tr. Top	90 6 2·5	3	91·5	− 6·2
Otley Tower ,,	89 59 10·0	8	72·8	−61·9
Laxfield Tr. ,,	90 1 10·2	12	93·8	− 1·7
Southwold Tr. ,,	90 6 23·0	6	95·0	+ 1·0
LAXFIELD TOWER. { Telescope above surface . 107·1 feet. ,, battlements 13·3 feet.				
	° ′ ″			″
Otley Tower* .	90 4 36·5	9	72·8	−109·2
Mickfield Ch. Tower* . .	90 4 27·2	12	58·7	−157·3
South Lopham Tower . Top	90 7 13·4	9	69·8	− 86·0
Bunwell Tower ,,	90 5 38·3	19	75·0	− 76·0
Toft's Tower* .	90 10 10·5	16	62·5	−106·7
Norwich Sp. Top	90 6 18·6	12	307·0	...
Southwold Tr. ,,	90 11 1·1	19	95·0	− 35·3
Orford Castle ,,	90 11 13·2	15	84·7	− 54·7
BUNWELL TOWER. { Telescope above surface . 94·7 feet. ,, battlements 19·7 feet.				
	° ′ ″			″
Norwich Sp. Top	90 2 4·7	13	307·0	...
Hingham Tr. ,,	90 1 29·8	10	...	...
Laxfield Tr.* .	90 7 28·6	12	94·3	− 1·0
Toft's Tower* .	90 10 26·5	7	85·0	− 20·1
Baconsthorpe* .	90 8 37·3	9	60·5	− 48·7
Swaffham Sp. Top	90 4 29·3	10	154·8	+109·4
South Lopham Tower . . ,,	90 6 58·7	12	69·8	−112·2
Brandon . . H	90 10 38·5	10	36·0	−102·9
Mickfield Tr.* .	90 7 52·4	6	57·4	− 75·5
Do. . Top	90 7 50·4	5	57·0	− 76·3

(¹) Flag. * Heliostat Top.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
HINGHAM TOWER.				
	° ′ ″			″
Bunwell Tr. Top	90 7 2·0	3	75·0	...
Norwich Sp. ,,	90 4 34·5	5	307·0	...
Baconsthorpe Tower . . ,,	90 7 54·1	7	59·5	...
Swaffham Sp. ,,	90 1 30·7	6	154·8	...
South Lopham Tower . . ,,	90 8 59·6	5	69·8	...
Brandon . . .	90 11 26·6	4	36·0	...
BRANDON. Telescope above surface 39·2 feet.				
	° ′ ″			″
Ely Minster Top	90 2 56·0	16	204·6	...
Hingham Tr. ,,	90 3 24·1	9	...	− 78·9
Swaffham Spire (¹)	90 0 54·5	9	104·0	+149·0
Tharfield . . H	90 8 29·8	11	10·0	− 29·5
Bunwell Tower* .	90 6 8·0	8	78·8	+ 69·6
S. Lopham Tr. Top	90 4 17·2	8	69·8	+ 75·2
Lawshall Tr. ,,	89 58 48·7	11	75·2	+ 78 7
Lawshall Tr.* .	89 58 59·5	5	75·7	+ 79·7
STOKE TOWER { Telescope above surface 125·3 feet. ,, roof . 19·3 feet.				
	° ′ ″			″
Walton Tr. Top	90 11 40·5	13	91·5	− 69·2
Lawshall Tr. ,,	89 59 51·6	13	75·2	−145·0
Otley Tower* .	90 8 20·3	10	72·0	−116·7
Danbury Sp. Top	90 3 29·9	9	119·9	− 9·0
Naughton Tr. ,,	90 1 14·8	9	45·0	...
WALTON TOWER { Telescope above surface . 97·3 feet. ,, battlement 5·8 feet.				
	° ′ ″			″
Danbury Sp. Top	90 5 16·3	7	119·9	+27·4
Stoke Tower ,,	90 2 16·6	8	111·3	+28·7
Naughton Tr.* .	90 3 18·3	7	45·0	−93·6
Otley Tower Top	90 4 3·6	6	72·8	−48·2
St. Peter's Tr. ,,	90 12 2·3	3	77·3	−22·3
Orford Cas. Tr. ,,	90 8 23·2	9	84·7	−25·8
Orford Obsy. ,,	90 7 51·8	4	97·5	+ 0·6
ST. PETER'S TOWER. { Telescope above surface . 86·6 feet. ,, battlement 9·3 feet.				
	° ′ ″			″
Paddlesworth . .	89 56 23·0	10	...	−159·4
Frittenfield . H	90 0 28·3	5	...	−122·5
Walton Tr. Top	90 14 21·9	9	91·5	+ 5·5
Danbury Sp.* .	90 14 13·3	5	...	− 77·3
Norwood . . H	90 8 56·5	2	...	−132·5

(¹) Top of Tower. * Heliostat Top.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
FRITTENFIELD. Telescope above surface 37·5 feet.				
Fairlight Down. H	90° 11′ 32″·2	11	2·4	−55″·8
Beachy Head . H	90 17 15·4	12	0·5	−35·3
Crowborough . H	90 8 55·0	11	2·3	−43·8
Leith Hill Tower	90 15 2·5	9	45·8	+ 6·2
Leith Hill Tr. H	90 15 50·4	9	0·9	−27·3
Wrotham . . H	90 5 45·7	15	23·5	−21·8
Danbury Spire .	90 16 35·5	7	119·9	+86·8
Norwood . . .	90 21 40·0	6	29·5	−21·7
Walton Tower H	90 24 59·6	10	91·5	+42·6
St. Peter's Tr. Top	90 20 31·0	11	77·3	+56·4
Paddlesworth „	90 6 7·5	15	32·3	−14·0
PADDLESWORTH. Telescope above surface 37·5 feet.				
Fairlight Down. H	90° 12′ 19″·2	13	2·4	−49″·1
Crowborough H	90 13 40·4	12	2·3	−31·9
Frittenfield. Top of Stage . .	90 5 13·5	9	39·8	+ 6·2
Walton Tower* .	90 23 43·5	10	91·5	+42·3
St. Peter's Tr. Top	90 20 45·2	12	77·3	+73·3
NORWOOD. Telescope above surface 31 feet.				
Wrotham . . H	89° 56′ 13″·3	4	15·4	− 24″·7
Gad's Hill (1) . .	90 4 6·9	1	...	− 75·8
Danbury Sp. Top	90 3 46·4	11	119·9	+146·5
Walton Tower* .	90 16 13·8	14	92·0	+ 64·8
St. Peter's Tr. Top	90 11 13·3	9	77·3	+ 70·9
Frittenfield . H	89 49 16·0	10	4·5	− 71·8
GAD'S HILL. Telescope above surface 5·8 feet.				
Severndroog (2) .	90° 0′ 22″·4	18	62·9	+125″·2
Epping Cupola. H	90 5 47·9	12	60·3	+ 85·6
Danbury Sp. Top	90 2 2·8	14	119·9	+208·0
Norwood . . H	90 7 39·8	19	4·8	− 2·7
DANBURY SPIRE { Telescope above surface . 124·8 feet. Parapet „ . 57·0 feet.				
Gad's Hill (1) . .	90° 12′ 5″·0	2	...	−227″·5
Wrotham . . .	90 3 55·5	9	42·0	−106·6
Severndroog (2) .	90 10 9·8	10	67·9	− 79·1
Epping Cupola .	90 8 47·9	14	66·5	−117·3
Thaxted Sp. Top	90 7 24·2	15	174·0	+100·2

* Heliostat Top. (1) Top of Obelisk.
(2) Top of Tower.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
DANBURY SPIRE—*continued.*				
Lawshall Tower* .	90° 12′ 29″·4	4	101·4	− 29″·4
Naughton Tr.* .	90 14 50·0	4	45·0	−100·1
Stoke Tower Top	90 13 52·6	17	111·3	− 22·7
Walton Tower* .	90 18 45·5	12	92·5	− 39·0
St. Peter's Tr. Top	90 20 1·6	11	77·3	− 42·3
Norwood . . H	90 14 43·0	17	28·5	−158·6
Frittenfield . H	90 10 27·0	11	32·0	− 97·7
SEVERNDROOG TOWER . { Telescope above surface . 68·9 feet. „ top of battlement 6·0 feet.				
Leith Hill . H	89° 58′ 37″·4	11	4·0	− 92″·5
Leith Hill Tr. Top	89 57 28·4	11	45·8	− 33·0
Banstead . . H	90 0 59·3	4	24·3	−110·7
St. Ann's . . H	90 16 26·7	7	1·4	−102·6
Hanger Hill Tr. H	90 15 33·5	15	38·8	− 73·5
St. Paul's Cath. (1)	90 7 23·4	6	352·8	...
Berkhampstead. H	90 8 26·5	11	104·0	+ 63·5
Chingford . H	90 13 56·5	11	1·5	−219·0
EppingCupola. Top	90 7 48·8	10	59·3	− 22·5
Gad's Hill . H	90 12 56·5	11	1·3	−148·4
Wrotham . . .	89 50 6·0	19	33·6	− 97·2
THAXTED SPIRE { Telescope above surface 178·3 feet. „ spire . 4·3 feet.				
Epping Cupola H	90° 10′ 37″·2	11	60·7	...″
Berkhampstead. H	90 10 46·0	21	96·9	...
Tharfield. Top of Pole . . .	90 3 39·0	11	30·0	...
Balsham. Top of Observatory .	90 7 0·2	17	77·0	...
Stoke Tower . H	90 12 53·0	8	142·8	...
Danbury Sp. Top	90 8 29·2	16	119·9	...
Lawshall Tr. „	90 11 9·5	18	75·2	...
BALSHAM TOWER { Telescope above surface 73·5 feet. „ tower 5·5 feet.				
Tharfield . . .	90° 1′ 34″·0	14	30·0	...″
Cambridge Obs. (2)	90 24 22·6	11	...	...
Ely Minster Top	90 13 42·0	14	204·6	...
Swaffham Tr. „	90 16 22·2	8	104·0	...
Swaffham Sp. „	90 15 12·5	4	154·8	...
Lawshall Tr. H	90 7 29·5	10	103·9	...
Thaxted Sp. Top	90 3 8·5	9	174·0	...
Keysoe Sp. . „	90 13 46·5	5	...	...

* Heliostat Top. (1) Top of Cross.
(2) Top of Dome.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
ELY MINSTER { Telescope above surface . 219·3 feet. " battlement 14·7 feet.				
	° ′ ″			″
Keysoe Spire Top	90 9 5·2	15	...	...
Tharfield . . H	90 4 30·8	15	2·5	...
Lynn Old Tr. Top	90 13 18·9	11	88·2	...
Swaffham Spire .	90 7 19·9	12	104·0	...
Brandon . . .	90 9 44·0	21	...	...
Cambridge Obs. (1)	90 12 10·7	7	...	...
BalshamCh.Tr.Top	90 1 5·1	16	68·0	...
Easton Tower H	90 11 42·3	23	86·8	...
Walpole, Saint Peter's . Top	90 13 24·8	11	81·0	...
EPPING CUPOLA. Telescope above surface 63·0 feet.				
	° ′ ″			″
Severndroog . H	90 4 59·2	8	62·9	− 0·2
LeithHillTower.H	90 7 22·1	13	...	− 58·2
St. Paul's . . .	90 6 42·3	7	352·8	...
Berkhampstead .	90 2 58·7	8	106·5	+151·1
Dunstable. . .	90 2 44·1	6	10·0	− 68·2
Dunstable . . H	90 3 32·3	5	...	...
Danbury Spire .	90 4 54·2	13	128·5	+131·9
Gad's Hill (2) . .	90 10 45·0	2	...	− 99·0
Wrotham . . .	90 1 0·7	11	42·0	− 29·6
Wrotham . . H	90 1 22·0	4	24·9	− 53·7
Thaxted Spire .	90 5 15·4	17	174·0	+220·7
Tharfield . . H	90 5 14·3	5	...	−108·6
Tharfield . . .	90 3 51·6	9	30·0	− 56·9
THARFIELD. Telescope above surface 37·5 feet.				
	° ′ ″			″
Keysoe Spire . .	90 14 43·8	11	...	− 63·1
Berkhampstead. H	90 11 25·6	10	101.3	+135·2
Dunstable . . H	90 3 7·3	15	...	− 61·3
Ely Minster . .	90 17 36·2	13	204·6	+223·3
Brandon . . H	90 20 59·1	6	36·0	− 1·5
Balsham Tower .	90 11 50·3	8	68·0	+ 69·9
Thaxted Spire .	90 10 6·3	5	174·0	+320·1
Epping Cupola .	90 12 48·8	5	59·3	+ 37·6
Hanslope Spire .	90 13 19·8	4	...	− 42·6
LEITH HILL TOWER . { Telescope above surface . 49·5 feet. " battlement 3·8 feet.				
	° ′ ″			″
Butser. : . H	90 13 41·4	10	2·9	−61·0
Inkpen . . H	90 18 30·5	13	4·3	−36·1
WhiteHorseHill.H	90 23 1·5	11	24·6	−16·6
Dunstable . . H	90 20 41·3	9	...	−40·2
Berkhampstead H	90 24 6·0	11	101·5	+48·9
Severndroog . H	90 23 12·6	13	62·9	+19·1
Frittenfield . H	90 23 38·3	8	41 7	− 5·8

(1) Top of Dome. (2) Top of Obelisk.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
LEITH HILL TOWER—*continued.*				
	° ′ ″			″
Epping Poor Ho. H	90 24 45·5	9	60·7	+10·3
Wrotham . . H	90 16 4·8	10	15·4	−44·3
Crowborough . .	90 14 45·5	9	...	−79·4
Beachy Head . .	90 22 26·7	7	...	−47·6
Ditchling . . .	90 13 44·9	9	10·0	−69·5
St. Paul's (1) . .	90 24 35·5	8	352·8	...
Hanger Hill Tr. H	90 29 58 4	6	40·5	−14·5
CHINGFORD. Telescope above surface 6·3 feet.				
	° ′ ″			″
St. Paul's (1) . .	89 55 26·8	6	352·8	...
Leith Hill. . H	90 1 20·8	3	4·1	− 2·4
Berkhampstead .	89 53 53·5	4	97·0	+359·1
Wrotham . . H	89 56 36·4	7	34·4	+ 43·5
Severndroog . .	89 55 42·0	6	63·1	+184·6
Banstead . . H	90 2 2·2	2	1·9	− 7·1
BROADWAY TOWER. Telescope above surface 56·1 feet.				
	° ′ ″			″
Cradle. . . H	90 2 11·8	27	0·3	−39·6
Malvern . . .	89 58 56·6	15	5·8	−88·7
Malvern . . H	89 59 4·3	33	1·5	−96·3
Arbury Hill . H	90 18 42·3	20	4·0	−67·9
WhiteHorseHill.H	90 16 52·5	24	1·7	−64·3
Bardon Hill . H	90 21 46·6	23	0·0	−41·8
MALVERN. Telescope above surface 5·5 feet.				
	° ′ ″			″
Cradle. . . .	89 49 47·9	8	...	− 6·1
Arbury . . H	90 26 8·0	8	5·0	− 0·4
Broadway Tr. Top	90 17 6·3	4	44·9	+69·5
Broadway Tower.	90 18 59·0	7	...	− 9·7
WhiteHorseHill.H	90 25 14·1	7	...	− 4·3
Long Mount Pole.	90 8 32·7	8	...	− 5·7
CRADLE. Telescope above surface 5·5 feet.				
	° ′ ″			″
DunkeryBeacon.H	90 31 58·4	6	...	− 3·7
Paracombe . H	90 33 59·2	6	...	− 3·5
Precelly . . H	90 34 20·5	6	...	− 3·0
Mendip . . H	90 40 9·8	2	...	− 3·8
Plynlimmon . .	90 19 10·3	9	...	− 4·7
Cader Idris . .	90 19 23·4	6	...	− 3·5
Snowdon . . .	90 24 19·3	4	...	− 2·5
Long Mount Pole.	90 30 9·8	6	...	− 5·1
Malvern . . .	90 36 13·0	5	...	− 6·1
Broadway Tower.	90 39 35·4	4	...	− 3·9
WhiteHorseHill.H	90 42 42·9	8	...	− 3·0
Arrenig . . .	90 24 44·4	2	...	− 3·0

(1) Top of Cross.

PRECELLY. Telescope above surface 5·5 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Snowdon . . .	90° 15′ 51″·8	7	...	− 2″·6
Cader Idris . .	90 11 11·3	8	...	− 3·4
Plynlimmon . .	90 11 55·4	7	...	− 3·9
Cradle. . . H	90 18 10·4	5	...	− 3·0
DunkeryBeacon.H	90 27 47·3	5	...	− 2·9
Paracombe . H	90 26 14·5	2	...	− 3·2
Paracombe . .	90 26 22·0	2	...	− 3·2
High Wilhays H	90 31 38·8	6	...	− 2·3
Lundy Island . H	90 35 9·8	5	...	− 4·0
Tara . . . H	90 36 53·0	4	...	− 2·7
Kippure . . H	90 35 12·3	2	...	− 2·0
Forth	90 37 37·1	3	...	− 2·7
Mount Leinster .	90 29 30·9	1	...	− 2·2

LUNDY ISLAND. Telescope above surface 5·5 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Hensbarrow . H	90° 13′ 31″·7	11	3·8	− 1″·3
Trevose Head H	90 20 21·4	11	2·0	− 3·0
Precelly . . H	90 4 15·0	21	1·3	− 3·1
Paracombe . H	89 52 44·3	8	1·0	− 5·0
Paracombe . .	89 52 42·3	3	...	− 6·1
High Wilhays H	89 53 20·6	19	−7·0	−11·1
Brown Willy . H	90 0 3·3	8	12·0	+ 6·4
Brown Willy . .	90 0 14·9	3	...	− 5·4

PARACOMBE. Telescope above surface 5·5 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
High Wilhays .	90° 3′ 51″·1	11	...	− 6″·2
Brown Willy . H	90 21 8·5	7	13·0	+ 5·7
Brown Willy . .	90 20 56·9	4	...	− 4·1
Lundy Island . .	90 33 5·6	7	...	− 6·1
Precelly . . H	90 21 56·4	10	1·0	− 2·7
Cradle. . . H	90 11 32·6	12	1·3	− 2·7
Dunkery Beacon .	89 57 32·2	16	...	−18·0

WROTHAM. Telescope above surface 37·5 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Crowborough . H	90° 7′ 19″·1	11	2·3	−72″·6
Ditchling . . H	90 11 2·9	10	41·4	+ 4·6
Leith Hill Tr. H	90 6 48·1	20	45·8	+10·7
Epping Cupola .	90 18 41·1	9	66·5	+40·9
Hanger Hill Tr. H	90 23 30·0	10	40·5	+ 4·0
Fairlight . . H	90 17 0·5	14	2·4	−40·7
Berkhampstead H	90 19 40·5	11	101·5	+71·1
Danbury Spire Top	90 18 5·3	4	119·9	+106·2
Frittenfield . H	90 12 52·6	11	41·7	+ 6·5
Dunstable . . H	90 18 40·0	2	29·8	− 5·9
Severndroog . H	90 21 6·2	17	62·9	+69·9
St. Paul's (1) . .	90 19 41·7	10	352·8	...
Norwood . . H	90 23 15·8	9	29·5	−12·7
Gad's Hill (2) . .	90 32 43·9	6	...	−142·6

(1) Top of Cross. (2) Top of Obelisk.

BANSTEAD. Telescope above surface 37·8 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Leith Hill. . H	89° 44′ 42″·6	10	4·2	−110″·5
St. Ann's . . H	90 23 14·8	6	1·4	−97·7
King's Arbour H	90 27 1·5	7	23·4	−36·9
Hampton Poor-house . . H	90 37 35·5	10	30·0	−30·3
HangerHillTr.Top	90 22 0·0	7	31·8	−15·7
St. Paul's (1) . .	90 13 27·0	1	362·8	...
Chingford . . H	90 17 23·5	10	1·2	−59·8
Epping Cupola H	90 15 5·5	8	60·3	+28·9
SeverndroogCastle	90 11 36·0	7	62·9	+62·5
Berkhampstead H	90 11 39·0	2	110·2	...

HANGER HILL TOWER. Telescope above surface 41·8 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Leith Hill Tower.	89° 48′ 19″·7	12	45·8	+ 6″·5
Hampton Poor-house . . H	90 16 23·0	3	35·0	− 36·2
St. Ann's . . H	90 6 31·0	11	1·2	−121·5
King's Arbour H	90 13 54·8	2	35·5	− 34·0
St. Paul's (2) . .	89 50 3·5	5	352·8	...
Severndroog Tr. H	89 56 39·5	12	69·7	+ 68·2
Wrotham . . H	89 58 3·4	7	34·4	− 9·9
Banstead . . .	89 50 52·0	8	...	−110·9

WHITE HORSE HILL. Telescope above surface 37·5 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Mendip . . H	90° 16′ 20″·0	3	43·7	+ 4″·9
Cradle . . . H	90 10 28·1	10	1·0	−19·9
Malvern . . .	90 11 26·9	7	...	−29·7
Broadway Tower .	90 8 18·0	11	44·9	+ 8·8
Arbury Hill . H	90 18 46·0	9	80·8	+35·7
Dunstable . . H	90 19 6·8	11	...	−30·2
Leith Hill . . H	90 20 46·9	14	1·6	−23·9
Inkpen . . .	90 2 39·5	15	...	−91·4

DUNSTABLE. Telescope above surface 37·5 feet.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
Hanslope Sp. Top	90° 15′ 55″·7	5	...	−69″·0
Leith Hill Tr. „	90 15 23·9	6	45·8	+ 6·7
Leith Hill. Surf.	90 15 57·6	8	...	−30·5
Arbury Hill . H	90 15 38·4	8	...	−38·5
Tharfield . . .	90 14 59·1	7	30·0	−12·3
Tharfield . . .	90 15 42·9	3	...	−61·3
WhiteHorseHill.H	90 17 46·1	9	...	−30·2
NasebyCh.Tr.Top	90 16 47·5	7	...	−35·3
Epping Cupola .	90 19 46·1	3	59·3	+28·1
Berkhampstead H	90 19 40·6	5	101·5	+130·7
Wrotham . . H	90 19 53·5	6	...	−28·3
Keysoe Spire. Top	90 21 20·2	9	...	−54·1

(1) Top of Observatory. (2) Top of Cross.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
INKPEN. Telescope above surface 20·4 feet.				
	° ′ ″			″
Dean Hill . .	90 20 43·4	4	38·0	+29·0
Beacon Hill . .	90 18 37·0	8	...	−49·4
Wingreen . H	90 15 40·6	19	1·5	−19·6
Westbury Down .	90 16 3·5	8	...	−26·5
Upcot Down . .	90 9 47·6	1	...	−46·4
Butser . . H	90 14 17·5	13	1·5	−22·1
Dunnose . . H	90 22 1·9	15	1·5	−14·2
White Horse Hill	90 11 15·6	15	...	−49·7
Leith Hill . H	90 18 20·0	14	5·4	−12·0
Nodes Beacon .	90 24 43·8	2	...	−16·8
SOUTH LOPHAM TOWER. Telescope above surface 89·3 feet.				
	° ′ ″			″
Lawshall Tr. Top	90 2 2·0	3	75·2	−27·2
Lawshall Tr.* .	90 2 6·0	8	75·7	−26·2
Brandon . . .	90 7 18·2	2	36·0	...
Brandon . . .	90 7 21·8	3	33·7	...
Brandon . . H	90 7 28·0	4	36·0	...
Brandon . . .	90 8 46·2	1	...	...
Swaffham Sp. Top	90 3 27·7	4	154·8	...
Hingham Ch. Tr. „	90 1 41·3	6	...	...
Bunwell Tower „	90 1 47·0	13	75·2	−63·4
Laxfield Tower „	90 6 6·9	3	93·8	+10·6
Laxfield Tower* .	90 6 9·3	6	94·3	+11·5
Mickfield Tower *	90 4 23·4	10	60·0	−82·9
MICKFIELD TOWER. Telescope above surface 84·7 feet.				
	° ′ ″			″
Naughton Tr. Top	90 1 7·1	21	45·0	−146·5
Lawshall Tr.* .	90 1 40·5	9	75·7	−20·2
Lawshall Tr. Top	90 1 30·5	12	75·2	−21·3
S. Lopham Tr.* .	90 7 22·2	12	73·3	−32·4
Bunwell Tr. Top	90 7 15·3	10	75·0	−19·6
Bunwell Tower* .	90 7 32·9	18	76·2	−17·2
Laxfield Tower (1)	90 5 40·6	20	93·8	+29·8
Otley Tower. Top	90 5 12·3	27	72·8	−77·3
LONG MOUNT. Telescope above surface 5·5 feet.				
	° ′ ″			″
Cradle . . .	90 1 23·5	15	...	− 5·1
Plynlimmon . .	90 1 25·4	11	...	− 5·5
Cader Idris . .	89 58 44·8	8	...	− 4·8
Snowdon . . .	90 3 36·1	4	...	− 3·4
Cyrn-y-Brain .	90 10 53·2	7	...	− 5·9
Axedge . . H	90 21 5·4	7	...	− 3·5
Malvern . . .	90 19 8·6	10	...	− 5·7
Mowcopt . . H	90 25 59·8	1	1·5	− 3·3
Bardon Hill . H	90 31 32·6	4	...	− 3·3
Arrenig . . .	90 0 29·8	8	...	− 4·8

* Heliostat Top. (1) Top of Pinnacles.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
BARDON HILL. Telescope above surface 5·5 feet.				
	° ′ ″			″
Tilton	90 12 37·2	3	...	−10·9
Broadway Tower	90 18 1·0	3	...	− 4·1
Malvern . . .	90 16 57·8	3	...	− 3·6
Mowcopt . . H	90 14 11·9	3	...	− 4·6
Axedge . . .	90 3 25·3	3	...	− 4·8
Back Tor . . .	90 6 32·5	1	...	− 4·2
Lincoln Minst. Top	90 24 10·0	6	...	...
Arbury Hill . H	90 16 35·5	7	...	− 6·3
Long Mount . H	90 17 9·5	1	...	− 3·3
Buckminster Spire, Top	90 18 1·3	3	...	− 8·0
Naseby Tower „	90 14 19·9	3	...	− 8·3
ST. MARTIN'S HEAD. Telescope above surface 5·1 feet.				
	° ′ ″			″
Peninnis Windmill	90 4 28·2	18	25·8	...
Telegraph Tower	89 51 13·2	17	41·5	...
Beacon Hill, Trescow	90 3 2·0	7	...	−59·0
Karnminnis . H	90 1 2·7	12	...	− 5·5
Karn Galver . H	89 58 19·1	22	2·5	− 3·1
Pertinney . H	89 57 58·8	17	2·5	− 3·5
Wolf Rock . .	90 11 48·3	7	...	− 9·7
PENINNIS WINDMILL. Telescope above surface 31·8 feet.				
	° ′ ″			″
St. Agnes' Lighthouse. . Top	89 54 36·5	2	...	...
Beacon Hill, Trescow	90 0 44·1	12	...	...
Telegraph Tower	89 31 20·3	11	41·5	...
Karn Galver . H	90 0 29·3	15	2·5	...
St. Martin's Head Day-mark . .	89 53 32·1	12	40·0	...
Pertinny . . H	89 59 56·2	15	...	...
Wolf Rock . .	90 11 15·7	5	...	...
OTLEY TOWER. Telescope above surface 88·6 feet.				
	° ′ ″			″
Stoke Tower. Top	90 4 46·4	13	111·3	+49·7
Naughton Tr. „	90 1 6·9	13	45·0	...
Mickfield Tr. „	90 4 0·5	13	57·0	...
Laxfield Tr. „	90 5 35·0	3	93·8	+16·7
Laxfield Tr.* .	90 6 0·4	13	94·3	+18·0
Orford Castle. Top	90 11 40·5	23	84·7	−11·0
Walton Tower „	90 10 46·2	22	91·5	+ 5·7

* Heliostat Top.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
TILTON. Telescope above surface 5·5 feet.				
Naseby Tr. Top	90° 8′ 13″·7	10	...	−12″·0
Bardon . . .	90 1 57·1	4	...	−10·9
Buckminster Spire	90 17 44·0	2	...	−16·9
Easton Ch. Tr. Top	90 23 35·2	1	72·8	...
Keysoe Spire „	90 20 28·3	8	...	...
Arbury Hill (1)	90 12 6·3	2	27·3	...
WALPOLE, ST. PETER'S TOWER. Telescope above surface 91·5 feet.				
Easton Tower. Top	90° 5′ 35″·8	7	72·8	...″
Boston Tower „	90 1 44·3	9	...	...
Docking Tr. „	89 59 18·7	12	...	...
Lynn Old Tr. „	90 2 22·3	15	88·2	...
Swaffham Spire (1)	89 58 34·7	9	141·8	...
Ely Minster. Top	90 3 36·0	9	204·6	...
SWAFFHAM SPIRE. Telescope above surface 135·8 feet.				
Lynn Old Tr. Top	90° 17′ 25″·8	15	88·2	...″
Hingham Tr. „	90 7 52·9	21	...	...
Walpole St. Peter's Tower. Top	90 16 33·9	10	81·0	...
Boston Tower „	90 15 43·9	5	...	...
BaconsthorpeTr. „	90 11 11·8	3	59·5	...
Ely Minster. „	90 12 15·8	13	204·6	...
Brandon . . H	90 13 42·1	8	36·0	...
S. Lopham Tr. Top	90 12 12·5	5	69·8	...
Bunwell Tr. „	90 11 30·2	1	75·0	...
Lawshall Tr. „	90 11 38·5	3	75·2	...
Lawshall Tr. *	90 12 4·5	5	75·7	...
DEAN HILL. Telescope above surface 38·3 feet.				
Southampton . H	90° 25′ 10″·7	7	52·1	+ 39″·7
Wingreen . . .	89 55 44·5	5	...	− 74·5
WestburyDown H	90 4 57·3	8	2·5	− 52·7
Stoke Hill. . H	90 4 8·3	2	2·5	− 57·4
Milk Hill . . H	89 58 30·2	6	3·3	− 54·0
Beacon Hill . H	89 57 1·7	6	1·5	−130·8
Inkpen . . H	89 56 47·2	5	2·0	− 59·9
Old Sarum Cas. H	90 14 28·4	3	1·8	−180·5
Four Mile Stone .	90 6 19·6	4	...	−139·3
Old Lodge . H	89 59 10·8	5	27·3	− 60·3
Coringdon . H	90 9 23·4	3	4·5	− 42·2
Swyre Barrow H	90 10 33·1	4	1·5	− 40·9
Horton's Gazebo. Top	90 13 36·8	4	...	− 84·2
Butser Hill . H	90 3 20·4	8	3·0	− 46·7
Dunnose . . H	90 8 36·3	9	2·3	− 40·5
Nodes Beacon .	90 10 55·8	2	...	− 58·5

(1) Top of Observatory. * Heliostat Top.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
WINGREEN. Telescope above surface 5·5 feet.				
Westbury Down .	90° 12′ 36″·7	8	...	− 11″·3
Inkpen . . H	90 13 4·4	16	3·3	− 2·2
Inkpen . . .	90 12 56·6	4	0·0	− 5·7
Beacon Hill . H	90 15 42·0	5	1·0	− 8·1
Dean Hill . H	90 20 33·8	7	2·5	− 5·8
Pillesdon . . H	90 12 19·7	8	31·0	+ 29·3
Mendip. Top of Stage . . .	90 5 11·7	11	97·5	+144·2
Mendip . . .	90 6 50·0	4	0·0	− 8·6
Butser . . H	90 18 45·6	15	...	− 4·4
Dunnose . . H	90 19 33·7	12	1·3	− 3·5
Horton's Gazebo, Top	90 37 58·0	10	...	− 19·8
Coringdon . .	90 16 12·3	18	...	− 8·6
Swyre Barrow .	90 15 43·1	16	...	− 8·2
Nodes Beacon .	90 20 28·4	6	...	− 6·5
WESTBURY DOWN. Telescope above surface 10·3 feet.				
Upcot Down . H	90° 3′ 44″·7	27	0·5	− 18″·3
Upcot Down . .	90 3 49·3	4	...	− 19·2
Milk Hill . . .	89 56 41·2	21	...	− 26·9
Stoke Hill . .	90 6 58·8	32	...	−112·5
Inkpen . . H	90 6 33·6	22	1·5	− 11·5
Beacon Hill . H	90 10 8·1	10	2·3	− 16·5
Beacon Hill . .	90 10 14·3	4	...	− 21·2
Dean Hill . . H	90 15 60·6	13	1·5	− 13·0
Dean Hill . . .	90 15 53·1	6	...	− 15·2
Wingreen . . .	90 1 57·2	16	...	− 21·2
Old Lodge . H	90 14 4·6	12	1·5	− 14·9
Four Mile Stone .	90 16 12·0	7	...	− 25·5
Mendip . . H	89 58 40·1	17	3·3	− 15·7
Mendip . . .	89 58 23·8	18	...	− 22·9
OLD LODGE. Telescope above surface 22·6 feet.				
Dean Hill . . H	90° 6′ 13″·8	17	33·5	+ 59″·3
Wingreen . . .	89 58 50·1	8	...	− 39·1
Old Sarum Castle .	90 18 38·7	19	...	−118·7
Old Sarum Gun .	90 33 40·3	21	6·5	− 88·9
Four Mile Stone .	90 8 27·2	5	...	− 96·4
Westbury Down .	90 3 28·3	4	14·0	− 14·4
Westbury Down .	90 3 31·9	5	...	− 38·0
Stoke Hill. . H	90 2 49·2	24	3·0	− 37·5
Stoke Hill. . .	90 2 46·0	3	...	− 43·3
Beacon Hill . .	89 50 5·5	30	...	−187·6
Milk Hill . . H	89 53 46·9	15	3·8	− 38·9
Milk Hill . . .	89 53 48·2	11	...	− 46·7
Inkpen . . .	89 51 15·8	15	...	− 51·8

Stations observed.		Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
DUNKERY BEACON. Telescope above surface 5·3 feet.					
High Willhays .		90° 8′ 21″·7	9	...	− 5″·5
Brown Willy . .		90 25 32·8	5	...	− 3·5
Paracombe . .		90 11 59·3	11	...	−17·5
Precelly . .	H	90 26 58·5	4	0·8	− 2·4
Cradle . . .	H	90 9 31·6	13	1·2	− 2·8
Mendip . .	H	90 26 53·6	12	21·3	+13·8
Pillesdon . .	H	90 27 48·3	18	1·9	− 3·3
MENDIP. Telescope above surface 37·5 feet.					
Pillesdon . .	H	90° 13′ 41″·8	10	15	−45″·4
Pillesdon . . .		90 13 51·8	2	...	−47·3
High Willhays	H	90 18 32·1	6	...	−19·8
Dunkery . .	H	90 6 9·3	3	1·3	−31·2
Dunkery . . .		90 6 26·1	9	...	−32·3
Cradle . . .	H	90 2 15·8	11	1·0	−25·3
Malvern . .	H	90 18 53·9	14	1·5	−22·7
WhiteHorseHill.	H	90 19 36·3	7	34·1	− 2·7
Inkpen . .	H	90 17 18·9	8	45·5	+ 6·6
Westbury Down .		90 16 8·0	11	...	−83·0
Beacon Hill .	H	90 19 31·5	12	2·0	−38·7
Wingreen . . .		90 11 56·8	15	...	−58·9
RYDER'S HILL. Telescope above surface 5·8 feet.					
Deadman . .	H	90° 35′ 55″·0	6	5·0	− 0″·7
Hensbarrow .	H	90 26 13·2	14	1·2	− 4·4
Brown Willy .	H	90 18 30·6	6	1·3	− 5·6
Brown Willy . .		90 18 13·9	6	13·5	+ 9·4
High Willhays .		89 47 51·7	11	19·0	+38·2
Pillesdon . .	H	90 29 4·6	15	1·3	− 3·5
Barrow Hill . .		90 46 13·6	10	10·1	+ 7·9
Barrow Hill . .		90 46 32·3	3	...	−10·8
Blackdown .	H	90 32 11·8	12	1·4	− 2·9
Swyre Barrow	H	90 37 34·4	9	3·5	− 1·2
Goonhilly . .	H	90 37 36·2	20	—	—
HIGH WILHAYS. Telescope above surface 7·0 feet.					
Paracombe . .		90° 21′ 38″·6	7	10·5	+ 4″·0
Paracombe .	H	90 21 51·7	5	2·3	− 5·3
Lundy Island .	H	90 39 46·4	15	0·8	− 5·4
Deadman . .	H	90 40 50·8	4	5·5	− 1·2
Hensbarrow .	H	90 31 38·3	12	1·2	− 5·5
Brown Willy . .		90 26 7·0	11	13·5	+ 9·5
Precelly . .	H	90 36 16·6	14	1·5	− 2·3
Dunkery . . .		90 19 32·3	10	20·0	+13·5
Mendip . .	H	90 36 38·0	12	21·2	+ 7·5
Pillesdon . .	H	90 33 35·6	17	1·4	− 4·1
Ryders Hill . .		90 21 55·7	14	11·5	+13·2
Blackdown .	H	90 36 13·0	6	5·6	− 0·8
Swyre Barrow	H	90 41 41·9	8	2·5	− 2·1

Stations observed.		Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
HENSBARROW. Telescope above surface 5·5 feet.					
Trevose Head .		90° 39′ 57″·9	8	...	−14″·5
Lundy Island	H	90 27 1·1	14	...	− 3·9
Brown Willy . .		89 53 12·3	9	...	−12·5
High Willhays	H	89 59 28·3	9	1·5	− 3·8
High Willhays .		89 59 20·4	8	...	− 5·2
Maker Tower. Top		90 24 36·8	3	69·4	+88·7
Barrow Hill . .		90 25 9·0	10	19·5	+10·7
Deadman . . .		90 42 11·0	6	...	−19·1
Goonhilly . .	H	90 25 56·3	11	2·3	− 4·5
Karnbonellis . .		90 14 28·6	10	...	− 9·4
Pertinny . .	H	90 19 55·8	6	2·0	− 3·3
Pertinny . . .		90 19 52·5	3	...	− 5·2
Karnminnis .	H	90 17 1·9	15	2·0	− 4·0
St. Agnes' Beacon	H	90 22 7·7	7	11·0	+11·7
TREVOSE HEAD. Telescope above surface 6·8 feet.					
Lundy Island	H	90° 13′ 49″·5	33	2·6	− 3″·6
Brown Willy, Top of Pile . . .		89 28 34·5	20	13·0	+12·6
Hensbarrow, Top of Pile . . .		89 30 48·1	13	11·7	+12·8
St. Agnes' Beacon, Top of Pile .		89 54 14·3	12	11·0	+ 8·8
Karnminnis .	H	90 1 12·3	20	1·3	− 6·6
Karn Galver, Top of Pile . . .		90 3 12·7	7	...	− 7·2
DEADMAN. Telescope above surface 9·0 feet.					
Hensbarrow, Top of Pile . . .		89° 26′ 14″·8	18	11·7	+ 9″·4
Brown Willy, Top of Pile . . .		89 45 38·6	14	13·0	+ 5·8
High Willhays	H	89 54 34·7	24	1·5	− 6·2
Ryder's Hill .	H	89 57 20·6	22	1·3	− 6·8
Goonhilly . . .		90 7 57·7	18	...	−17·0
Karnbonellis, Top of Pile . . .		89 51 46·0	14	11·0	+ 4·1
St. Agnes, Top of Pile . . .		89 59 26·9	14	11·0	+ 4·0
KARN GALVER. Telescope above surface 5·1 feet.					
Pertinny . . .		90° 15′ 29″·2	22	9·8	+43″·3
St. Agnes Lighthouse . Top		90 25 34·0	11	...	− 5·3
Peninnis Windmill . . Top		90 25 42·6	12	25·8	+22·7
Telegraph Tr. „		90 24 27·0	12	41·5	+40·9

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
KARN GALVER—*continued.*				
	° ′ ″			″
St. Martin's Head Daymark Top	90 24 42·9	12	40·0	+42·5
Beacon Hill, Trescow . . Top	90 25 48·9	10	...	− 5·6
Trevose Head H	90 24 1·7	14	1·5	− 3·8
Karnminnis . .	90 4 43·6	25	11·0	+56·8
Hensbarrow . H	90 11 26·9	14	...	− 5·2
Karnbonellis . .	90 6 50·8	21	11·0	+13·5
Goonhilly . . H	90 21 57·5	15	1·5	− 6·7
TELEGRAPH TOWER. Telescope above surface 43·6 feet.				
	° ′ ″			″
Peninnis Windmill . . Top	90 31 40·3	15	25·8	...
St. Agnes Lighthouse . Top	90 10 1·9	12	...	...
Beacon Hill, Trescow . . Top	90 14 22·5	12	...	...
St. Martin's Head Daymark Top	90 3 41·5	19	40·0	...
St. Martin's Head Daymark . .	90 12 36·5	3	...	...
Karn Galver . H	90 1 3·9	22	1·5	...
Pertinny . . H	89 59 53·7	14	1·5	...
Wolf Rock Top	90 13 34·4	15	...	...
BEACON HILL. Telescope above surface 6·3 feet.				
	° ′ ″			″
Coringdon. . H	90 15 5·2	15	4·2	− 2·1
Swyre Barrow H	90 15 42·7	9	23·5	+15·6
Wingreen . . .	90 0 52·2	15	...	−11·3
Mendip . . H	90 7 29·3	15	17·5	+12·3
Westbury . . H	90 4 15·5	14	2·5	− 7·7
Milk Hill, Top of Beacon . . .	89 51 31·8	6	...	−17·0
Milk Hill . . H	89 52 8·5	6	4·5	− 4·8
Inkpen . . H	89 53 55·1	18	1·5	−11·5
Butser Hill . . H	90 9 3·0	19	1·5	− 5·3
Dunnose . . H	90 15 16·3	15	2·7	− 3·1
Dean Hill . . .	90 13 22·5	11	...	−22·3
Stoke Hill . H	90 3 36·3	6	2·0	−10·4
Stoke Hill, Top of Beacon . . .	90 2 35·0	2	25·1	+46·0
Old Sarum Gun H	90 43 2·3	5	6·5	− 1·3
Old Sarum Gun, Top of Beacon .	90 40 54·1	13	24·2	+101·2
Sarum Old Castle, Axis of Inst .	90 23 47·6	4	37·8	...
Old Lodge, Top of Beacon . . .	90 13 23·1	6	27·3	...
Four-mile Stone H	90 16 44·6	6	2·0	−22·7
Four-mile Stone .	90 16 55·5	3	...	−33·3
NASEBY TOWER.				
	° ′ ″			″
Arbury Hill . .	90 4 45·5	6	...	...
Arbury Hill . .	90 3 55·2	4	25·1	...
Bardon Hill Obsy.	90 3 55·5	5	8·5	...
Tilton	90 5 18·2	10	...	...
Easton Tower Top	90 18 15·1	6	72·8	...
Keysoe Spire . „	90 18 14·8	4	...	...
Hanslope Spire „	90 12 39·3	5	...	...
BOSTON TOWER.				
	° ′ ″			″
Easton Tower Top	90 10 0·1	10	72·8	...
Buckminster . „	90 4 57·3	12	...	...
Lincoln Minster „	90 6 9·8	21	...	...
Docking Tower „	90 8 27·2	6	...	...
Swaffham Spire „	90 12 8·0	3	154·8	...
Lynn Old Tower „	90 13 11·8	3	88·2	...
Walpole, Saint Peter's . Top	90 13 54·8	6	81·0	...
EASTON TOWER. Telescope above surface 79·0 feet.				
	° ′ ″			″
Naseby Tower Top	90 1 14·9	8	...	...
Tilton	89 49 3·7	7	...	...
Buckminster Top	89 54 53·8	9	...	...
Boston Tower „	90 13 14·1	9	...	...
Walpole, Saint Peter's . Top	90 17 18·0	6	81·0	...
Ely Minster . „	90 15 51·8	5	204·6	...
Keysoe Spire . „	90 10 4·4	8	...	...
CROWBOROUGH. Telescope above surface 5·0 feet.				
	° ′ ″			″
Leith Hill . . .	90 4 35·5	2	...	− 8·0
Wrotham . . .	90 8 0·5	2	...	−10·3
Fairlight . . .	90 15 29·0	1	...	− 8·2
BERKHAMPSTEAD GAZEBO. Telescope above surface 98·8 feet.				
	° ′ ″			″
Leith Hill Tower.	90 7 26·0	6	...	...
Dunstable . . .	89 53 21·0	1	44·0	...
Chingford . . .	90 13 47·5	2	...	...
Wrotham . . .	90 6 49·5	4	...	...
St. Paul's (1) . .	90 7 35·3	3	352·8	...
Tharfield . . .	90 1 23·3	2	50·5	...

(1) Top of Cross.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
NAUGHTON TOWER. Telescope above surface 64·3 feet.				
	° ′ ″			″
Stoke Tower Top	90 5 41·4	15	111·3	...
Otley Tower . „	90 8 41·9	7	72·8	...
Walton Tower „	90 12 57·8	8	91·5	...
Walton Tower H	90 12 39·1	10	92·3	...
LawshallTowerTop	89 59 33·9	19	75·2	...
MickfieldTower „	90 8 43·3	17	57·0	...
Thaxted Spire „	90 6 58·8	11	174·0	...
Danbury Spire „	90 8 11·5	3	119·9	...
MAKER TOWER { Telescope above surface . 86·2 feet. „ battlements 16·8 feet. „ pinnacles 0·8 feet.				
	° ′ ″			″
Deadman . . H	90 12 15·5	20	5·7	...
Hensbarrow . H	89 57 0·1	28	1·3	...
High Willhays .	89 26 21·3	11	16·0	...
Barrow Hill . H	90 9 4·0	24	3·0	...
ARBURY HILL. Telescope above surface 23·8 feet.				
	° ′ ″			″
Broadway Tower	90 3 47·4	2	44·9	+27·5
Broadway Tower	90 4 57·1	4	...	−31·1
Malvern . . .	90 9 28·5	6	...	−19·1
Bardon Hill . H	90 9 35·9	7	...	...
Naseby Tr. Top	90 6 45·0	1	...	...
Hanslope Sp. „	90 13 48·0	1	...	−51·6
Dunstable . H	90 13 8·7	7	...	...
Dunstable . .	90 13 16·1	3	...	−24·5
CLIFTON BEACON. Telescope above surface 5·5 feet.				
	° ′ ″			″
Back Tor . . .	89 25 33·7	5	...	−10·6
Holme Moss . .	89 36 37·2	5	...	− 7·7
Gt. Whernside. H	90 1 7·0	1	...	− 3·7
York Minster. Top	90 16 58·8	3	199·3	...
Acklam Wold .	90 11 55·0	1	...	− 4·8
Crowle . . H	90 20 43·0	5	...	−11·1
Gringley Beacon .	90 14 0·7	7	...	−15·1
Lincoln Minst. Top	90 12 32·8	5	...	− 6·7
With the 3-ft. Theodolite. Telescope above surface 5·5 ft.				
	° ′ ″			″
S. End of Base .	90 28 51·0	1	...	−17·6
Gringley Beacon .	90 13 40·0	1	...	−15·1
N. End of Base .	90 32 57·0	1	...	−15·5

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
BARROW HILL { Telescope above centre stone 23·6 feet. „ surface . 21·5 feet.				
	° ′ ″			″
Hensbarrow . H	90 11 42·7	18	1·5	− 15·3
Maker Ch. Tr. (1)	90 9 10·5	8	85·4	+107·5
Ryder's Hill . .	89 29 32·9	20	11·5	− 18·5
Pillesdon . . H	90 15 3·3	12	1·5	− 14·2
Blackdown . H	90 18 22·3	12	1·5	− 13·1
BLACKDOWN { Telescope above surface . 73·3 feet. „ top of monument 2·0 feet.				
	° ′ ″			″
Barrow Hill . H	90 26 24·1	21	2·7	− 46·2
Ryder's Hill . H	90 12 59·6	6	1·5	− 46·4
High Willhays H	90 11 30·5	16	2·0	− 43·4
Pillesdon . . .	90 3 17·2	9	...	−191·0
Horton's Gazebo .	90 21 57·0	1	...	−100·2
Nodes Beacon H	90 21 35·8	13	1·3	− 63·5
Dunnose . . H	90 22 29·2	17	4·5	− 45·1
Coringdon . .	90 14 13·0	10	...	−114·9
Swyre Barrow .	90 13 1·2	13	...	−137·9
PILLESDON. Telescope above surface 38·4 feet.				
	° ′ ″			″
Barrow Hill . H	90 23 48·2	6	2·3	−25·6
High Willhays H	90 5 51·3	13	−6·0	−33·1
Dunkery . . H	90 3 18·3	17	1·3	−35·3
Mendip . . H	90 10 30·3	15	21·6	−21·0
Coringdon . H	90 19 10·2	18	3·5	−34·9
Wingreen . . H	90 13 11·3	16	1·1	−42·8
Swyre Barrow .	90 17 17·6	13	18·3	−22·2
Blackdown . H	90 12 7·6	20	1·5	−96·0
HOLME MOSS. Telescope above surface 5·5 feet.				
	° ′ ″			″
Axedge . . .	90 12 30·0	4	...	−10·1
Mowcopt . . .	90 29 15·0	3	...	− 6·6
Cyrn-y-Brain .	90 25 39·0	3	...	− 3·4
Whittle Hill . .	90 21 24·8	7	...	−11·9
Pendle Hill . .	90 13 2·7	4	...	− 7·6
Gt. Whernside H	90 11 14·8	8	...	− 5·0
Rumbles Moor .	90 25 55·0	2	...	− 8·5
Garforth Cliff H	90 49 15·5	2	...	− 8·0
Clifton Beacon H	90 47 39·8	3	...	− 7·7
Back Tor . . .	90 14 2·1	5	...	−19·0

(1) Top of Pinnacles.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
LINCOLN MINSTER. Telescope above surface 238·5 feet.				
	° ′ ″			″
Buckminster Spire, Top	90 6 57·5	4	...	...
Bardon Hill . .	90 11 9·3	8	...	...
Axedge . . H	90 6 0·3	4	...	...
Back Tor . . .	90 0 33·0	1	...	...
Clifton Beacon .	90 11 11·0	6	...	...
Crowle . . H	90 19 10·2	6	...	...
Boston Tr. Top	90 14 12·7	6	...	...
MOWCOPT. Telescope above surface 7·4 feet.				
	° ′ ″			″
Long Mount Pole. H	90 9 42·4	15	1·5	− 4·8
Cyrn-y-Brain .	90 2 43·5	8	...	− 7·2
Snowdon . . .	90 7 31·0	3	15·5	+ 4·1
Delamere . . .	90 24 1·8	9	...	−13·8
Whittle Hill . H	90 6 48·0	19	1·8	− 5·8
Holme Moss . .	89 55 47·5	3	...	− 9·0
Holme Moss . H	89 55 42·6	14	5·0	− 2·9
Bardon Hill . H	90 20 6·6	14	2·5	− 4·1
Axedge . . .	89 31 38·8	31	...	−21·2
PENDLE HILL. Telescope above surface 5·5 feet.				
	° ′ ″			″
Snowdon . . .	90 20 29·5	4	...	− 2·3
Black Comb . .	90 15 29·5	9	...	− 4·3
Sca Fell . . .	90 2 56·8	8	...	− 3·9
Ingleborough . .	89 50 25·7	9	...	−10·2
Gt. Whernside .	89 54 54·4	8	...	− 9·1
Rumbles Moor .	90 23 46·9	7	...	−11·1
Holme Moss . .	90 7 36·7	7	...	− 7·6
Axedge . . .	90 15 59·7	4	...	− 4·6
Whittle Hill . .	90 18 30·2	8	...	−15·3
SNOWDON. Telescope above surface 5·5 feet.				
	° ′ ″			″
Precelly . . .	90 44 40·4	5	...	− 2·6
Tara	90 53 28·0	3	...	− 2·3
Kippure . . .	90 41 40·0	1	...	− 2·3
Howth . . .	90 53 56·0	1	...	− 2·5
Slieve Donard .	90 43 50·0	3	...	− 2·0
South Berule . .	90 45 5·8	3	...	− 2·7
Sea Fell . . .	90 38 27·5	2	...	− 2·1
Black Comb . .	90 44 16·5	2	...	− 2·5
Ingleborough . .	90 45 23·5	4	...	− 2·1
Whittle Hill . .	90 46 58·5	4	...	− 2·5
Axedge . . .	90 45 41·5	2	...	− 2·4
Cyrn-y-Brain .	90 43 32·4	8	...	− 5·7
Long Mount Pole	90 42 25·5	4	...	− 3·4
Plynlimmon . .	90 29 50·5	4	...	− 5·0
Cader Idris . .	90 25 11·3	15	...	− 8·1
Arrenig . . .	90 35 12·0	7	...	−12·4
WHITTLE HILL. Telescope above surface 5·5 feet.				
	° ′ ″			″
Axedge . . .	90 6 10·0	2	...	− 6·6
Delamere . . .	90 30 23·0	1	...	− 6·1
Black Comb . .	90 17 2·5	2	..	− 3·6
Scafell . . .	90 7 47·0	1	...	− 3·2
Pendle Hill . .	89 51 38·0	11	...	−15·3
Ingleborough . .	89 56 57·5	2	...	− 6·1
Mowcopt . . .	90 21 21·0	1	...	− 5·6
Gt. Whernside .	89 58 57·2	6	...	− 6·0
Holme Moss . .	89 53 10·6	7	...	−11·9
YORK MINSTER { Telescope above outside base of building . 203·8 ft. „ battlements . 4·5 ft.				
	° ′ ″			″
Clifton . . H	90 8 34·2	14	9·0	...
Garforth Cliff H	90 2 22·8	20	7·0	...
Gt. Whernside H	89 40 8·8	12	4·7	...
Botton Head . .	89 44 31·7	13	...	...
Acklam Wold .	89 40 4·5	12	...	...
LLANELIAN. Telescope above surface 5·5 feet.				
	° ′ ″			″
Arrenig . . .	89 21 48·0	1	...	− 9·2
Garreg . . .	90 17 35·0	1	...	−11·9
Gt. Orme's Head .	90 40 57·0	1	...	−29·2
Cyrn-y-Brain .	89 52 57·0	1	...	− 7·8
GARREG. Telescope above surface 5·5 feet.				
	° ′ ″			″
Snowdon . . .	89 23 2·0	1	...	− 6·0
Llanelian . . .	89 56 39·0	1	...	−11·9
Great Orme's Head	90 12 41·0	1	...	− 9·3
Gwaunysgaer . .	90 19 36·0	1	...	−49·0
Arrenig . . .	89 31 6·0	1	...	− 6·7
Cyrn-y-Brain .	89 30 14·0	1	...	−11·6
Delamere . . .	90 14 57·0	1	...	− 8·3
MOELFRE ISSA. Telescope above surface 5·6 feet.				
	° ′ ″			″
Gwaunysgaer . .	90 28 36·0	1	...	−24·3
Great Orme's Head	90 24 6·0	1	...	−16·7
Llanelian . . .	89 58 0·0	1	...	−33·1
Cyrn-y-Brain .	89 43 39·0	1	...	− 9·9
EAST END OF BASE. Telescope above surface 5·5 feet.				
	° ′ ″			″
West End of Base	89 59 51·0	1	22·4	...
Moelfre Issa . .	87 50 34·0	1	...	−42·9
Gwaunysgaer . .	88 13 15·0	1	...	−54·1

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
WEST END OF BASE. Telescope above surface 5.5 feet.				
Moelfre Issa . .	86° 43′ 3″·0	1	...	−64″·4
Orme's Head . .	89 27 37·0	1	...	−18·7
East End of Base	90 0 6·0	1	21·5	...
Gwaunysgaer . .	89 8 7·0	1	...	−27·0
GWAUNYSGAER. Telescope above surface 5·5 feet.				
Garreg . . .	89° 45′ 47″·0	1	...	−49″·0
Moelfre Issa . .	89 39 56·0	1	...	−23·9
West End of Base	90 59 12·0	1	...	−27·0
DUBLIN OBSERVATORY.				
Howth . . .	89° 49′ 58″·0	1	...	−19″·2
Kippure . . .	88 28 18·0	1	...	−14·9
Lyons Hill . .	89 43 21·0	1	...	−20·4
HOWTH.				
Lyons Hill . .	90° 4′ 28″·4	8	...	−10″·7
Kippure . . .	88 54 42·4	7	...	−12·4
Slieve Donard .	89 54 41·0	1	...	− 3·8
Dublin Obs^{y} (1) .	90 17 58·3	3	...	−19·2
Dublin Obs^{y} (2) .	90 18 28·0	1	...	...
KIPPURE.				
Howth . . .	91° 18′ 11″·0	11	...	−12″·4
Lyons Hill . .	91 47 46·7	7	...	−18·7
South Berule . .	90 40 45·5	2	...	− 2·2
Slieve Donard .	90 23 33·6	5	...	− 3·0
Croghan . . .	90 42 20·7	3	...	− 5·3
Dublin Obs^{y} (1) .	91 42 24·0	1	...	−14·9
Snowdon . . .	90 27 22·0	1	...	− 2·3
Ballycreen . .	90 17 30·7	3	...	...
Keeper . . .	90 33 11·0	1	...	...
Tara	90 44 33·3	3	...	...
LYONS HILL.				
Kippure . . .	88° 20′ 51″·0	2	...	...″
Dublin Obs^{y} (1) .	90 23 48·0	2	...	...
Howth . . .	90 10 16·5	2	...	...

(1) Top of Dome. (2) Base of Dome.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
CROGHAN.				
Kippure . . .	89° 47′ 23″·5	2	...	...″
Lyons Hill . .	90 13 49·2	5	...	...
Mount Leinster .	89 57 43·0	1	...	...
BENCORR.				
Nephin . . .	90° 9′ 27″·4	5	...	...″
Slieve More, Achill	90 16 0·6	5	...	...
Keeper . . .	90 30 24·0	2	...	..
Baurtregaum . .	90 29 43·5	2	...	...
WORDESLOW. Telescope above surface 5·8 feet.				
Gt. Whernside H	89° 58′ 52″·1	15	4·2	− 1″·2
Merrington Tower, Top	90 1 53·3	4	...	−16·4
Water Crag . H	89 49 14·5	4	1·1	− 4·6
Brandon . . H	89 51 50·3	6	7·0	+ 4·1
Collier Law . .	89 38 5·5	10	...	− 9·9
Cheviot . . H	89 53 27·8	22	2·5	− 2·4
Easington . .	90 9 49·8	3	...	− 7·4
Easington . H	90 9 37·1	10	3·0	− 3·5
Burleigh Moor H	90 10 38·3	8	1·3	− 7·1
Botton Head . .	89 55 28·8	7	...	− 6·7
Botton Head . H	89 55 47·1	6	1·0	− 5·5
EASINGTON. Telescope above surface 5·7 feet.				
Botton Head . .	89° 30′ 17″·0	9	...	−15″·0
Water Crag . .	90 0 0·5	2	...	− 4·3
Collier Law . H	90 3 56·0	6	1·8	− 3·2
Wordeslow .. H	90 12 15·0	17	2·5	− 4·1
Cheviot . . H	90 13 40·0	10	3·0	− 1·3
GARFORTH CLIFF. Telescope above surface 5·5 feet.				
Holme Moss . .	89° 32′ 0″·0	2	15·0	+13″·8
Great Whernside	89 38 54·0	3	...	− 5·9
York Minster .	90 10 9·0	16	199·3	...
Acklam Wold H	90 2 42·0	3	...	− 7·1
Crowle . . H	90 16 8·0	2	...	− 8·5
GRINGLEY BEACON. Telescope above surface 5·4 feet.				
South End of Base	90° 45′ 42″·5	4	17·3	...″
South End of Base	90 46 27·2	4	13·1	...
North End of Base	90 21 47·5	2	33·5	...
North End of Base	90 22 59·4	2	20·0	...

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
MOELFRE ISSA. Telescope above surface 5·6 feet.				
Gwaunysgaer .	90° 27′ 47″·3	1	...	−24″·3
Llanelian . . .	89 57 35·6	6	...	−33·1
Llanelian . . .	89 56 1·5	4	14·0	+49·9
Great Orme's Head	90 22 55·7	9	...	−16·7
NORTH END OF BASE. Telescope above surface 5·5 feet.				
Gringley Beacon .	89° 43′ 13″·0	1	...	−25″·6
South End of Base	90 1 3·0	3	...	−43·1
Clifton Beacon .	89 44 12·5	2	...	−15·5
SOUTH END OF BASE. Telescope above surface 5·5 feet.				
Clifton Beacon .	89° 40′ 58″·5	1	...	−17″·6
Gringley Beacon .	89 16 27·0	1	...	−56·8
North End of Base	90 2 10·7	3	14·0	+66·6
Telescope above surface 16 feet.				
Gringley Beacon .	89° 16′ 0″·9	4	11·3	− 48″·7
Gringley Beacon .	89 16 55·0	4	5·3	−110·7
GREAT WHERNSIDE. Telescope above surface 5·5 feet.				
Pendle Hill . .	90° 21′ 54″·0	2	...	− 9″·1
Ingleborough . .	90 3 36·7	6	...	−13·3
Sca Fell . . .	90 8 55·0	2	...	− 4·1
Cross Fell . .	90 5 53·0	1	...	− 5·1
Water Crag . .	90 11 21·0	2	...	−11·0
Botton Head . .	90 27 21·0	1	...	− 5·3
INGLEBOROUGH. Telescope above surface 5·5 feet.				
Pendle Hill . .	90° 25′ 19″·0	2	...	−10″·2
Great Whernside .	90 9 17·5	2	...	−13·3
Black Comb . .	90 22 24·0	1	...	− 5·6
Sca Fell . . .	90 1 33·0	1	...	− 5·6
SCA FELL. Telescope above surface 5·5 feet.				
Black Comb . .	91° 1′ 33″·5	2	...	−14″·9
Ditto . . .	91 1 32·8	3	4·0	− 4·1
South Berule . .	90 39 50·8	3	...	− 3·4
Merrick . . .	90 29 2·0	1	...	− 3·1
Criffel . . .	90 36 34·0	3	...	− 5·7
Hart Fell . . .	90 30 1·3	3	...	− 3·2
Wisp	90 35 43·8	3	...	− 3·7
Cheviot . . .	90 34 21·0	1	...	− 2·6
Cross Fell . .	90 17 56·0	5	...	− 6·4
Water Crag . .	90 31 35·0	2	...	− 4·8
Great Whernside .	90 30 48·0	4	...	− 4·1
Ingleborough . .	90 28 41·0	1	...	− 5·6
Whittle Hill . H	90 40 47·5	2	...	− 3·2

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
SOUTH BERULE. Telescope above surface 5·5 feet.				
Kippure . . H	90° 28′ 47″·1	16	2·5	− 1″·2
Howth . . H	90 36 43·6	7	1·0	− 2·2
Slieve Donard H	90 3 13·4	21	0·3	− 4·0
Ditto . . .	90 3 10·7	3	...	− 4·2
Divis . . . H	90 23 6·0	9	14·4	+ 5·5
Trostan . . .	90 29 5·5	4	...	− 2·5
Trostan . . H	90 29 25·0	12	11·0	+ 2·5
Merrick . . H	90 14 18·4	16	2·0	− 2·0
Ditto . . .	90 14 9·5	3	...	− 3·1
Criffel . . . H	90 22 43·9	18	...	− 3·1
Sca Fell . . H	90 6 4·3	17	0·8	− 3·0
Ingleborough . H	90 27 36·9	10	3·2	− 1·0
Black Comb . H	90 15 31·4	24	...	− 3·9
Snowdon . . H	90 12 24·7	20	1·5	− 2·0
WATERCRAG. Telescope above surface 6·8 feet.				
Cross Fell . .	89° 49′ 10″·0	1	...	−11″·1
Collier Law . .	90 22 12·0	2	...	−11·1
Brandon Down H	90 40 21·5	3	2·0	− 6·7
Wordeslow . H	90 39 54·0	1	2·0	− 4·7
Merrington Tower, Top	90 46 1·0	1	...	− 9·9
Easington . . H	90 37 56·3	3	2·0	− 3·6
Botton Head . .	90 26 8·0	1	...	− 6·4
Great Whernside .	90 3 21·0	2	...	−13·5
SLIEVE DONARD. Telescope above surface 5·5 feet.				
Sawel	90° 30′ 0″·0	2	...	″ ...
Divis	90 38 12·0	2	...	...
Vicar's Carn . .	90 56 18·7	3	...	...
Cuilcagh . . .	90 34 2·0	2	...	...
Trostan . . .	90 32 35·0	1	...	...
Howth . . .	90 46 21·0	1	...	...
South Berule . H	90 34 4·4	13	1·5	− 5·4
South Berule . .	90 34 4·8	5	...	− 4·2
Kippure . . .	90 29 15·0	1	...	...
DIVIS.				
Slieve Donard .	89° 44′ 12″·5	4	...	″ ...
Sawel	90 5 1·0	4	...	...
Knocklayd . .	90 12 13·0	3	...	...
VICAR'S CARN.				
Slieve Donard .	89° 25′ 27″·3	4	...	″ ...
Divis	89 56 21·0	4	...	...
Sawel	89 51 46·0	2	...	...

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
SAWEL.				
	° ′ ″			″
N. End of Lough Foyle Base . .	91 10 29·1	7	...	...
Slieve Snaght .	90 15 33·0	2	...	...
Knocklayd . .	90 23 42·5	2	...	...
Trostan . . .	90 21 23·5	2	...	...
CUILCAGH.				
	° ′ ″			″
Slieve League .	90 20 55·3	6	...	...
Sawel	90 19 3·7	6	...	...
Slieve Donard .	90 23 6·5	2	...	...
Slieve Snaght .	90 27 58·7	3	...	...
Nephin . . .	90 19 22·7	6	...	...
Vicar's Carn . .	90 27 54·0	1	...	...
Croghan . . .	90 38 16·0	1	...	...
Kippure . . .	90 32 36·0	1	...	...
SLIEVE LEAGUE.				
	° ′ ″			″
Nephin . . .	90 8 34·3	3	...	...
Slieve Snaght .	90 21 31·3	3	...	...
Croghan . . .	90 2 9·5	2	...	...
Slieve More (in Achill) . .	90 22 5·5	2	...	...
Cuilcagh . . .	90 11 23·0	1	...	...
Knockalongy . .	90 15 35·0	1	...	...
NEPHIN.				
	° ′ ″			″
Croghan . . .	91 28 16·5	4	...	...
Knockalongy . .	90 30 17·0	3	...	...
Cuilcagh . . H	90 27 42·0	2	...	...
Slieve League .	90 27 11·5	2	...	...
Slieve More (in Achill) . . .	90 20 32·5	2	...	...
Tawnaghmore .	90 53 33·0	1	...	...
ST. AGNES BEACON. Telescope above surface 5·5 feet.				
	° ′ ″			″
Karnbonellis . .	89 47 15·0	1	...	−25·0
Deadman . . .	90 14 15·0	1	...	−11·1
Karnminnis . .	89 57 45·0	1	...	−13·4
Trevose Head. .	90 19 30·0	1	...	−11·6
Hensbarrow . .	89 51 52·0	1	...	−11·7

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
PERTINNY. Telescope above surface 6·1 feet.				
	° ′ ″			″
Wolf Rock . .	90 40 7·1	8	...	−17·8
Telegraph Tower, Top	90 22 41·4	8	41·5	+43·5
St. Martin's Day Mark . Top	90 22 52·5	10	40·0	+45·2
Beacon Hill, Trescow	90 23 49·3	3	...	− 7·3
Karnbonellis . .	90 4 3·3	10	12·2	+12·4
Karnminnis . .	89 57 3·9	18	11·0	+24·5
Goonhilly . . .	90 19 8·5	6	...	−11·2
Goonhilly . . H	90 19 15·6	24	1·5	− 8·5
Hensbarrow . H	80 10 43·4	20	1·5	− 4·3
Karn Galver . .	89 47 34·9	5	...	−55·4
GOONHILLY. Telescope above surface 5·8 feet.				
	° ′ ″			″
Wolf Rock . .	90 18 33·2	6	...	...
Ditto Surface of Water. .	90 18 53·5	3	...	− 7·8
Pertinny . . H	89 56 44·9	14	1·4	− 8·0
Pertinny . . .	89 56 39·0	5	...	−10·6
Karnbonellis . .	89 32 49·9	14	11·5	+23·2
Hensbarrow . H	89 55 23·0	7	1·2	− 6·3
Hensbarrow . .	89 55 8·4	7	11·7	+ 8·2
Deadman . . .	90 7 35·9	5	...	−10·9
Karn Galver . .	89 53 24·3	5	...	−10·7
BROWN WILLY. Telescope above surface 7 feet.				
	° ′ ″			″
Deadman . . H	90 33 58·6	12	1·5	− 8·0
Hensbarrow . .	90 19 9·3	21	...	−15·9
Karnbonellis . H	90 23 51·8	9	2·0	− 4·9
St. Agnes Beacon . . . H	90 26 55·8	11	...	− 8·2
Karnminnis . H	90 25 32·0	15	1·5	− 4·3
Trevose Head H	90 45 19·0	14	1·5	−11·2
Ryder's Hill . .	90 4 31·5	10	11·5	+ 5·5
High Willhays .	89 52 58·8	17	19·0	+17·5
Lundy Island Light House . Top	90 27 34·2	13	...	− 6·8
Paracombe . H	90 15 45·0	2	1·5	− 4·1
BEACHY HEAD. Telescope above surface 5·5 feet.				
	° ′ ″			″
Dunnose . . H	90 21 25·0	10	1·0	− 2·7
Ditchling . . .	89 57 31·3	28	...	−11·1
Frittenfield . H	90 13 0·0	15	36·2	+29·2
Leith Hill Tower, Top	90 7 7·5	6	45·8	+38·7
Leith Hill. . .	90 8 26·0	12	...	− 5·3
Fairlight . . .	90 5 8·6	27	...	−11·5

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
BONIFACE DOWN. Telescope above surface 5·5 feet.				
Littletown Down .	95° 25′ 15″·0	4	...	″ ...
Week Down . .	90 40 38·3	6	...	...
Wroxall Down .	91 58 5·0	2	...	...
Nodes Beacon .	90 17 54·2	5	...	...
Shanklin Down .	90 42 16·0	3	...	...
Dunnose . . .	90 9 38·3	6	...	...
BonifaceDown,S.E.	92 55 31·5	4	...	...
WEEK DOWN. Telescope above surface 5·5 feet.				
Nodes Beacon .	90° 14′ 43″·0	3	...	− 15″·1
Shanklin Down .	89 52 10·7	6	...	−114·9
Boniface Down .	89 24 1·8	4	...	−135·1
Littletown Down	90 16 55·5	6	...	−149·3
BonifaceDown,S.E.	89 34 33·0	5	...	−131·8
Dunnose . . .	89 36 38·3	4	...	−100·3
HIGH PORT CLIFF. Telescope above surface 5·5 feet.				
BonifaceDown,S.E.	76° 39′ 46″·0	10	...	−433″·8
Littletown Down .	78 54 27·0	11	...	−427·6
LITTLETOWN DOWN. Telescope above surface 5·5 feet.				
Week Down . .	89° 47′ 25″·5	6	...	−149″·3
Boniface Down .	84 47 17·9	7	3·8	−262·6
BonifaceDown,S.E.	85 23 34·0	7	...	−935·7
High Port Cliff .	101 9 0·7	7	3·8	−136·1
BEACON HILL, TRESCOW. Telescope above surface 5·1 feet.				
St. Agnes Lighthouse . Top	89° 57′ 34″·2	14	...	− 42″·1
Karnminnis . .	90 2 38·3	4	11·0	+ 5·9
Karnminnis . H	90 2 53·2	25	2·5	− 2·6
Pertinny . . .	89 59 15·0	2	9·8	+ 5·7
Pertinny . . H	89 59 35·1	18	2·5	− 3·1
Telegraph Tr. Top	89 44 34·7	17	41·5	+512·9
BUTSER HILL. Telescope above surface 12·8 feet.				
Dunnose . . H	90° 12′ 45″·4	17	1·0	−17″·2
Dunnose . . .	90 12 25·0	5	...	...
Nodes Beacon .	90 19 18·8	3	...	−15·3
Coringdon . H	90 21 28·7	12	3·8	− 7·0
Ditchling . . H	90 15 25·8	24	3·3	− 9·6
Ditchling . . .	90 15 24·0	1	...	−12·9
Wingreen . . H	90 17 46·8	18	4·3	− 6·7
Inkpen . . H	90 10 43·4	13	4·3	−10·0
Dean Hill . . H	90 19 19·8	5	2·5	−13·5
BUTSER HILL—*continued.*				
Beacon Hill . H	90° 16′ 59″·8	10	2·0	−11″·9
Leith Hill Tr. Top	90 8 18·6	12	45·8	+43·2
Crowborough . H	90 19 10·7	10	3·5	...
CORINGDON. Telescope above surface 6·2 feet.				
Swyre Barrow (1)	89° 58′ 46″·8	14	...	−49″·0
Black Down . H	90 5 40·6	7	3·3	− 4·5
Pillesdon . . H	90 10 11·2	15	3·0	− 3·2
Beacon Hill . H	90 14 23·2	9	...	− 6·0
Wingreen . . H	90 2 29·5	13	1·7	− 7·0
Horton's Gazebo, Top	90 18 13·0	4	...	−15·1
Dean Hill . . H	90 14 38·0	8	3·3	− 3·6
Butser . . H	90 15 19·9	15	3·5	− 2·1
Nodes Beacon, Top	90 12 4·7	4	...	−12·1
Dunnose . . H	90 10 34·5	16	2·5	− 4·1
DITCHLING. Telescope above surface 20·7 feet.				
Dunnose . . H	90° 19′ 18″·6	17	2·5	−14″·5
Butser . . H	90 13 15·4	11	4·5	−16·4
Leith Hill Tower	90 2 47·3	17	45·8	+44·2
Wrotham . . H	90 12 21·1	6	24·0	+ 3·9
Crowborough (1) .	90 5 36·7	9	18·0	− 6·7
Fairlight . . H	90 16 45·3	13	2·4	−22·4
Fairlight . . .	90 16 49·3	4	...	−25·4
Beachy Head, Top of Pile . . .	90 16 59·0	10	10·0	−21·6
DUNNOSE. Telescope above surface 6·6 feet.				
Week Down. . .	90° 27′ 11″·5	2	...	−120″·0
Swyre Barrow H	90 16 8·7	8	2·8	− 3·8
Coringdon . H	90 15 1·7	12	2·5	− 4·6
Black Down . H	90 21 36·6	11	1·0	− 3·7
Nodes Beacon .	90 17 56·0	2	...	− 16·7
Horton's Gazebo, Top	90 21 6·0	2	...	− 6·9
Wingreen . H	90 15 54·8	13	1·1	− 4·6
Dean Hill . H	90 18 0·9	15	2·8	− 4·2
Beacon Hill . H	90 18 39·3	5	1·5	− 4·4
Ditchling . . H	90 17 52·9	10	...	− 5·0
Southampton . H	90 27 8·4	17	...	−11·5
Inkpen . . H	90 17 2·4	18	4·3	− 1·8
Butser. . . H	90 7 12·1	19	2·9	− 5·4
Beachy Head H	90 26 3·4	13	0·5	− 3·7
DUNNOSE. 18-in. Theodolite.				
Boniface Down .	89° 52′ 29″·5	3	...	−247″·0
Week Down . .	90 26 38·3	4	...	−120·0
Shanklin Down .	91 37 40·0	2	...	...
Wroxall Down .	90 28 8·0	2	...	−216·3

(1) Top of Cross.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
FAIRLIGHT. Telescope above surface 7·1 feet.				
Beachy Head . .	90° 8′ 15″·3	8	10·0	+ 6″·1
Beachy Head . .	90 8 30·3	5	...	−14·9
Ditchling . . H	90 7 3·1	12	0·5	− 8·1
Paddlesworth . H	90 9 31·1	13	1·3	− 8·2
Crowborough . H	90 2 32·6	7	2·3	− 7·8
Crowborough . .	90 2 46·5	3	...	−11·7
Frittenfield . H	90 7 54·7	17	4·5	− 4·1
Frittenfield . .	90 7 2·8	3	39·8	+52·1
Wrotham . . H	90 8 58·7	14	2·5	− 5·3
KARNMINNIS. Telescope above surface 6·9 feet.				
Pertinny . . .	90° 8′ 38″·6	13	9·8	+14″·5
St. Martin's Day Mark . Top	90 24 12·8	6	40·0	+35·7
Trevose Head .	90 23 4·6	10	11·0	+ 4·8
Brown Willy . .	90 10 21·0	2	13·5	+ 5·2
St. Agnes Beacon	90 14 3·4	10	11·0	+10·0
Hensbarrow . H	90 8 28·9	10	1·5	− 6·2
Hensbarrow . .	90 8 1·9	5	11·7	+ 5·5
Karnbonellis . .	90 4 2·3	13	11·0	+11·8
Karn Galver . .	89 57 56·5	6	...	−66·4
NODES BEACON. Telescope above surface 5·6 feet.				
Swyre Barrow H	90° 3′ 56″·5	10	1·3	− 6″·9
Swyre Barrow .	90 3 57·5	2	...	− 8·8
Coringdon . H	90 1 51·9	14	3·8	− 3·4
Black Down . H	90 10 49·3	6	1·2	− 3·9
Dean Hill . H	90 8 37·1	8	...	− 8·5
Wingreen . . H	90 3 53·6	13	1·0	− 5·4
Inkpen . . H	90 10 29·3	8	4·3	− 1·1
Southampton . H	90 19 39·3	5	52·1	+100·3
Week Down . .	89 55 38·0	2	...	−15·3
Butser . . H	90 4 1·3	21	3·4	− 2·6
Dunnose . . .	89 53 31·8	17	...	−14·1
SHANKLIN DOWN. Telescope above surface 5·5 feet.				
Week Down . .	90° 12′ 13″·0	2	...	−114″·9
Dunnose . . .	88 34 46·5	2	...	−555·3
Boniface Down .	89 21 14·8	4	...	−197·4
BONIFACE DOWN, S.E. Telescope above surface 5·5 ft.				
Littletown Down .	94° 57′ 1″·2	6	...	...″
Week Down . .	90 29 35·7	6	...	...
Wroxall Down .	91 5 20·0	1	...	...
Nodes Beacon .	90 16 57·7	6	...	...
Boniface Down .	87 16 25·5	6	3·8	...
High Port Cliff .	103 24 10·2	6	3·8	...

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
SWYRE BARROW. Telescope above surface 6·3 feet.				
Black Down . H	90° 4′ 9″·8	13	3·7	− 4″·9
Wingreen . . H	90 4 0·5	12	1·0	− 7·8
Dunnose . . H	90 12 52·0	6	3·2	− 3·1
Dunnose . . H	90 12 10·0	9	2·1	− 4·1
High Willhays H	90 18 25·7	10	−6·0	− 5·7
Pillesdon . . H	90 7 9·1	5	35·5	+32·4
Horton's Gazebo, Top	90 18 20·5	4	...	−13·1
Beacon Hill . H	90 15 35·9	10	1·3	− 4·5
Dean Hill . H	90 16 6·8	12	3·0	− 3·6
Coringdon. . .	90 5 1·9	11	6·6	+ 2·7
KEYSOE CHURCH SPIRE.				
Dunstable . H	89° 58′ 53″·3	4	...	...″
Dunstable . .	89 58 47·0	1	...	...
Dunstable . .	89 59 20·0	1	40·5	+58·5
Hanslope Sp. Top	89 59 57·5	5	...	...
Naseby Ch. Tr. H	90 0 34·8	6	...	...
Tilton . . . H	90 4 34·0	11	...	...
Tilton	90 4 33·0	1	8·5	+10·1
Tilton	90 6 7·0	1	...	...
Easton Tower H	90 9 27·3	5	72·0	...
Ely Minster. Top	90 13 11·6	10	204·6	...
Tharfield . . .	90 2 47·1	6	...	...
LAWSHALL CH. TR. { Telescope above surface 79·1 feet. „ „ tower 3·9 feet.				
Thaxted Spire. Top	90° 5′ 24″·0	1	174·0	...″
Balsham Tr. . H	90 4 56·4	5	83·0	...
Brandon . . H	90 14 41·3	12	36·0	...
Swaffham Spire. H	90 13 12·3	3	132·3	...
Hingham Tr. . H	90 13 56·4	10	1·3	...
S. Lopham Tr. H	90 13 0·2	8	70·3	...
Mickfield Tr. Top	90 12 7·0	2	57·0	...
Naughton Tr. „	90 9 3·1	12	45·0	...
Stoke Tower (1) „ .	90 10 37·8	12	111·3	...
LYNN TOWER. { Telescope above surface 94·1 feet. „ „ tower 5·9 feet.				
Ely Minster. Top	90° 5′ 22″·5	11	204·6	...″
Swaffham Sp. „	89 52 22·5	7	154·8	...
Walpole, Saint Peter's (1) . .	90 4 13·3	5	91·5	...
Boston Tower. Top	90 4 18·5	8	...	...
Docking Ch. Tr. „	89 53 53·5	9	...	...
ACKLAM WOLD. Telescope above surface 5·5 feet.				
Crowle . . H	90° 25′ 28″·0	3	...	− 7″·1
Clifton Beacon H	90 20 33·0	2	...	− 4·8
York Minster. Top	90 29 52·0	6	199·3	+558·3
Botton Head . .	89 53 0·0	5	...	− 7·8

(1) Top of Battlement.

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
AXEDGE. Telescope above surface 5·5 feet.				
	° ′ ″			″
Mowcopt . . .	90 39 20·5	6	...	—15·7
Mowcopt. Cross	90 38 11·9	3	33·0	+78·7
Pendle Hill . .	90 16 23·0	3	...	— 4·6
Whittle Hill . .	90 17 20·5	9	...	— 6·6
Holme Moss . .	90 4 30·0	13	...	—10·1
Back Tor . . .	90 7 51·0	10	...	—13·3
LincolnMinsterTop	90 36 3·0	1	...	...
Bardon Hill . .	90 29 28·0	6	...	— 4·8
Snowdon . . .	90 19 35·0	1	21·0	+ 6·8
Cyrn-y-Brain . .	90 19 12·0	6	...	— 4·1
BLACKCOMB. Telescope above surface 5·5 feet.				
	° ′ ″			″
Snowdon . . .	90 20 12·0	3	...	— 2·5
Criffel	90 18 57·0	5	...	— 4·4
Sea Fell . . .	89 8 48·0	7	...	—14·9
Ingleborough . .	90 6 52·0	11	...	— 5·6
Water Crag . .	90 9 49·0	3	...	...
Pendle Hill . .	90 20 22·0	1	...	— 4·3
Whittle Hill . H	90 26 30·0	3	...	— 3·6
South Berule . .	90 24 32·5	10	...	— 3·9
Merrick . . .	90 20 11·0	1	...	— 2·8
BRANDON DOWN. Telescope above surface 34·3 feet.				
	° ′ ″			″
Great Whernside .	89 55 5·0	1	...	...
Water Crag . .	89 39 50·0	2	...	...
Collier Law . .	89 22 6·0	3	...	...
Wordeslow . .	90 18 57·0	2	...	...
DurhamObsy Dome	91 34 47·0	3	...	...
Easington . . .	90 17 1·0	3	...	...
Burleigh Moor. H	90 18 50·0	1	...	...
Botton Head . .	90 1 23·0	3	...	...
MerringtonChurch Tower . Top	90 23 52·0	1	...	...
COLLIER LAW. Telescope above surface 7·1 feet.				
	° ′ ″			″
Water Crag . .	89 55 14·0	4	...	—11·8
Cross Fell . .	89 29 30·0	6	...	—13·3
Cheviot . . .	90 4 38·0	3	...	— 5·7
Cheviot . . H	90 4 20·0	2	2·3	— 3·8
Wordeslow . H	90 38 37·0	3	1·4	— 9·9
DurhamObsy Dome	91 1 28·0	3	...	—17·9
Brandon Down .	90 47 23·0	3	...	—23·3
Easington . . H	90 31 20·0	5	1·3	— 4·8
MerringtonChurch Tower . Top	90 44 57·0	1	...	—16·8
Botton Head . H	90 19 19·0	4	1·5	— 5·0

Stations observed.	Mean Zenith Distance.	No. Obs.	Object observed above Surface.	Reduction.
CRIFFEL. Telescope above surface 5·5 feet.				
	° ′ ″			″
Goat Fell . . .	90 19 49·0	3	...	— 2·8
Hart Fell . . .	89 57 3·0	6	...	— 6·4
Dunrich . . .	90 9 14·0	5	...	— 4·6
Wisp	90 11 16·0	6	...	— 6·1
Cross Fell . .	90 3 10·0	7	..	— 4·4
Sea Fell . . .	89 50 18·0	9	...	— 5·7
Blackcomb . .	90 16 14·0	4	...	— 4·4
South Berule . .	90 27 58·0	1	...	— 3·1
Merrick . . .	89 56 57·0	11	...	— 6·0
Burnswark . .	90 41 4·5	2	...	—12·3
CROSS FELL. Telescope above surface 5·5 feet.				
	° ′ ″			″
Sea Fell . . .	90 7 12·0	8	...	— 6·4
Criffel	90 31 55·0	2	...	— 4·4
Merrick . . .	90 30 56·0	1	...	— 2·5
Hart Fell . . .	90 24 8·0	3	...	— 3·5
Wisp	90 30 15·0	3	...	— 4·8
Dunrich . . .	90 27 36·0	1	...	— 3·2
Cheviot . . .	90 23 8·0	4	...	— 3·9
Collier Law . .	90 46 38·0	2	...	—10·2
Botton Head . .	90 37 2·0	6	...	— 3·6
Water Crag . .	90 29 19·0	4	...	— 9·0
Great Whernside	90 25 10·0	6	...	— 5·1
Ingleborough . .	90 23 46·0	2	...	— 5·8
Burnswark . .	90 46 1·8	1	...	— 5·2
CROWLE. Telescope above surface 5·5 feet.				
	° ′ ″			″
Clifton Beacon H	89 53 53·0	6	...	—11·1
Garforth Cliff .	90 1 46·0	6	...	— 8·5
Great Whernside	89 57 16·7	1	...	— 3·5
Acklam Wold .	89 56 33·0	6	...	— 7·1
LincolnMinsterTop	90 1 28·0	4	...	— 7·5
CYRN-Y-BRAIN. Telescope above surface 9·3 feet.				
	° ′ ″			″
Snowdon . . .	89 43 4·0	16	15·5	+ 6·5
Delamere . . .	90 42 48·0	9	9·0	— 0·4
Axedge . . .	90 18 33·0	7	...	— 6·9
Mowcopt . . H	90 26 42·0	13	5·5	— 3·6
Long Mount . .	90 15 47·0	16	20·0	+11·5
Arrenig . . .	89 44 11·0	17	20·0	+16·6
Llanelian . . .	90 27 7·0	11	20·0	+15·2
Moelfre Issa . .	90 31 35·3	18	20·0	+19·1
DELAMERE. Telescope above surface 5·5 feet.				
	° ′ ″			″
Cyrn-y-Brain . .	89 34 9·0	8	...	— 9·0
Arrenig . . .	89 48 36·0	1	...	...
Whittle Hill . .	89 55 6·0	4	...	— 6·1
Holme Moss . .	89 52 29·0	3	...	— 5·4
Mowcopt . . .	89 51 6·1	6	...	—10·2

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Coefficient of Refraction.

The coefficient of refraction may be determined either by direct observation, as when the heights of the observing point and the point observed are known with exactness, or it may be deduced from pairs of reciprocal observations. In the former case, the coefficient required is simply the ratio of the excess of the observed above the true zenith distance to the angle subtended by the two stations at the centre of the earth. The following table contains the values of coefficients of refraction observed in this manner, the weights in the last column but one being proportional to the square root of the distance and to the number of observations :—

No.	Stations.	Levelled Height.	Observed Zenith Distance z.	No. Obs.	Logarithmic Distance.	v.	True Zenith Distance Z.	$Z-z$.	Coeff. of Refraction k.	w.	$k\ w$.
1	Ben Lomond .	3192·2	89° 58′ 35″·8	5	5·36908	2300″·7	90° 1′ 21″·2	165″·4	0·0719	24	1·7256
	Ben Nevis . .	4406·3	90 33 25·3	19			90 37 2·0	216·7	0·0942	92	8·6664
2	Merrick . .	2764·8	90 22 48·1	29	5·58593	3790·7	90 27 48·7	300·6	0·0793	180	14·2740
	Ben Lomond .	3192·2	...	...			...	...	...	...	...
3	Arbury Hill .	735·3	90 12 46·5	10	5·30314	1977·1	90 15 23·1	156·6	0·0792	45	3·5640
	Dunstable . .	800·2	90 14 59·9	8			90 17 36·5	156·6	0·0792	36	2·8512
4	Dunkery . .	1706·4	90 26 56·1	4	5·59492	3870·9	90 31 51·0	294·9	0·0762	25	1·9050
	Precelly . .	1757·9	90 27 44·4	5			90 32 45·0	300·6	0·0777	31	2·4087
5	High Willhays .	2039·6	90 19 45·8	10	5·30084	1966·8	90 22 8·4	142·6	0·0725	45	3·2625
	Dunkery . .	1706·4	90 8 16·2	9			90 10 41·0	144·8	0·0736	40	2·9440
6	High Willhays .	2039·6	90 36 14·3	14	5·69221	4843·0	90 42 22·7	368·4	0·0761	98	7·4578
	Precelly . .	1757·9	90 31 36·4	6			90 38 26·7	410·3	0·0847	42	3·5574
7	High Willhays .	2039·6	90 31 32·8	12	5·33808	2142·9	90 33 51·8	139·0	0·0649	56	3·6344
	Hensbarrow .	1027·0	89 59 20·1	17			90 1 54·0	153·9	0·0718	79	5·6722
8	Hensbarrow .	1027·0	90 19 50·7	9	5·33879	2146·4	90 22 30·3	159·6	0·0744	42	3·1248
	Pertinny . .	735·5	90 10 39·1	20			90 13 19·1	160·0	0·0745	93	6·9285
9	Hensbarrow .	1027·0	90 16 58·0	15	5·25712	1778·5	90 19 9·7	131·7	0·0741	64	4·7424
	Karnminnis .	799·8	90 8 17·6	15			90 10 31·3	133·7	0·0752	64	4·8128
10	Coringdon . .	655·6	90 10 30·4	16	5·26403	1806·9	90 12 54·0	143·6	0·0795	69	5·4855
	Dunnose . .	771·9	90 14 57·1	12			90 17 15·4	138·3	0·0765	51	3·9015
11	Dunnose . .	771·9	90 17 47·9	10	5·43606	2685·2	90 21 52·5	244·6	0·0911	52	4·7372
	Ditchling . .	814·2	90 19 4·1	17			90 22 56·3	232·2	0·0865	89	7·6985
12	Malvern . .	1396·2	90 26 7·6	8	5·40979	2527·4	90 29 55·9	228·3	0·0903	41	3·7023
	Arbury Hill .	735·3	90 9 9·4	6			90 12 14·7	185·3	0·0733	30	2·1990
13	Trevose Head .	242·6	90 1 5·6	20	5·24101	1713·7	90 3 18·2	132·6	0·0774	83	6·4242
	Karnminnis .	799·8	90 23 9·5	10			90 25 17·8	128·3	0·0749	42	3·1458
14	Merrick . .	2764·8	90 29 34·7	20	5·27980	1873·3	90 31 50·3	135·6	0·0724	87	6·2988
	Criffel . . .	1866·6	89 56 51·0	11			89 59 25·1	154·1	0·0823	48	3·9504
15	High Willhays .	2039·6	90 41 39·8	8	5·64907	4385·1	90 47 7·3	327·5	0·0747	53	3·9591
	Swyre Barrow .	674·4	90 18 20·0	10			90 26 3·9	463·9	0·1058	67	7·0886
16	Black Down .	790·0	90 10 47·1	16	5·53016	3334·8	90 15 9·4	262·3	0·0787	93	7·3191
	High Willhays .	2039·6	90 36 12·2	6			90 40 30·0	257·8	0·0773	35	2·7055

Mean value of $k = 0{\cdot}0792$.

The largest value of the coefficient of refraction is at Swyre Barrow, in the observed zenith distance of High Wilhays, which slightly exceeds one-tenth. The next in magnitude is at Ben Nevis. The mean reading of the barometer at this station was 25·233; and it is worthy of notice, that for about a fortnight during the stay of the observers on the hill (when the greater part of the observations were made) the state of the atmosphere was unusually calm; so much so, that a lighted candle could often be carried from the tents of the men to the observatory, whilst at the foot of the hill the weather was wild and stormy.

The coefficient resulting from the observations at Dunnose to Ditchling is considerably greater than the mean, as is also the coefficient at Malvern. The range of the whole is from $\frac{1}{9\cdot4}$ to $\frac{1}{15\cdot4}$ and the mean $\frac{1}{12\cdot6}$

When the coefficient of refraction, *k*, is derived from reciprocal observations of zenith distance, we have (see page 511)—

$$1 - 2k = \frac{z' + z - 180^\circ}{v}$$

where z and z' are the observed zenith distances, and v the angle subtended by the two stations at the centre of the earth. The weights assumed for the separate determinations are directly as the square root of the distance and inversely as the sum of the reciprocals of the numbers of observations at the two observing stations.

The following table contains the calculation of 112 values of *k*:—

No.	Stations.	Observed Zenith Distance.	No. Obs.	Log. Distance.	*v*.	$z + z' - 180^\circ$.	$1 - 2k$.	Coeff. of Refraction *k*.	*w*.	*k w*.
		° ′ ″			″	″				
1	Monach . . . . Ben Clibrig . . .	90 3 24·4 90 47 26·7	8 10	5·56590	3619·3	3051·1	0·8430	0·0785	27	2·1195
2	Corryhabbie . . . Ben Clibrig . . .	90 22 37·9 90 33 4·6	31 6	5·60487	3959·2	3342·5	0·8442	0·0779	32	2·4928
3	Corryhabbie . . . Scarabin	90 28 6·6 90 17 40·2	35 4	5·51742	3237·1	2746·8	0·8485	0·0758	21	1·5918
4	Scournalapich . . . Cleisham	90 37 51·3 90 18 7·7	15 3	5·60864	3993·8	3359·0	0·8410	0·0795	16	1·2720
5	Scournalapich . . Ben More, South Uist	90 44 16·8 90 17 2·1	22 25	5·64709	4363·6	3678·9	0·8431	0·0785	78	6·1230
6	Ben Nevis . . . Ben More, South Uist	90 49 57·7 90 16 35·1	13 15	5·68727	4786·6	3992·8	0·8342	0·0829	49	4·0621
7	Ben Nevis . . . Jura	90 42 58·1 90 9 49·6	9 28	5·58509	3783·3	3167·7	0·8373	0·0814	42	3·4188
8	Sayrs Law . . . Kellie Law . . .	90 36 43·5 89 43 23·9	7 29	5·17099	1458·1	1207·4	0·8281	0·0860	22	1·8920
9	Lumsden . . . . Kellie Law . . .	90 14 46·4 90 8 48·6	14 30	5·23296	1681·6	1415·0	0·8415	0·0793	39	3·0927
10	Lumsden . . . . Mount Battock . .	90 10 43·7 90 43 21·7	6 17	5·59737	3891·6	3245·4	0·8340	0·0830	28	2·3240
11	Ben Clibrig . . . Storr	90 36 34·6 90 23 25·4	9 17	5·63913	4284·0	3600·0	0·8403	0·0799	39	3·1161

No.	Stations.	Observed Zenith Distance.	No. Obs.	Log. Distance.	v.	z + z′ − 180°.	1 − 2 k.	Coeff. of Refraction k.	w.	k w.
		° ′ ″			″	″				
12	Ben More, South Uist Storr	90 11 35·0 90 20 33·2	46 22	5·37556	2335·1	1928·2	0·8257	0·0872	73	6·3656
13	Ben More, South Uist Jura	90 34 0·5 90 42 12·7	13 8	5·74794	5504·5	4573·2	0·8308	0·0846	37	3·1302
14	Ben More, South Uist Mamsuil	90 14 35·8 90 44 26·7	19 19	5·63386	4232·7	3542·5	0·8369	0·0816	62	5·0592
15	Ru Rea Storr	89 37 12·5 90 42 37·7	29 19	5·16171	1427·1	1190·2	0·8340	0·0830	44	3·6520
16	Ru Rea Cleisham	89 46 59·2 90 41 44·1	21 3	5·31971	2053·3	1723·3	0·8393	0·0804	12	0·9648
17	Ru Rea Monach	90 17 8·0 90 12 39·2	11 8	5·33717	2137·5	1787·2	0·8361	0·0820	22	1·8040
18	Storr Cleisham	90 9 45·4 90 18 40·6	33 11	5·31659	2038·6	1706·0	0·8368	0·0816	38	3·1008
19	Storr Monach	90 38 8·0 90 4 31·6	23 9	5·49310	3060·8	2559·6	0·8362	0·0819	36	2·9484
20	Mormonth Scarabin	90 14 1·0 90 37 35·9	19 3	5·57741	3716·6	3096·9	0·8332	0·0834	16	1·3344
21	Merrick Goat Fell	90 14 49·3 90 17 43·1	13 6	5·36705	2290·0	1952·4	0·8526	0·0737	20	1·4740
22	South Berule Slieve Donard	90 3 9·4 90 33 59·4	21 18	5·42890	2640·8	2228·8	0·8440	0·0780	50	3·9000
23	Dunnose Coringdon	90 14 57·1 90 10 30·4	12 16	5·26403	1807·0	1527·5	0·8453	0·0774	29	2·2446
24	Deadman Ryder's Hill	89 57 13·9 90 35 54·3	22 6	5·37384	2326·9	1988·2	0·8545	0·0728	23	1·6744
25	Hensbarrow Barrow Hill	90 25 19·7 90 11 27·3	10 18	5·42952	2645·2	2207·0	0·8344	0·0828	33	2·7324
26	Barrow Hill Pillesdon	90 14 49·2 90 23 22·6	12 6	5·46416	2864·7	2291·8	0·8000	0·1000	22	2·2000
27	Lundy Island Trevose Head	90 20 18·4 90 13 45·9	11 33	5·38141	2367·7	2044·3	0·8634	0·0683	40	2·7320
28	Lundy Island Paracombe	89 52 38·4 90 32 59·5	11 7	5·27016	1832·6	1537·9	0·8392	0·0804	18	1·4472
29	Lundy Island Hensbarrow	90 13 30·5 90 26 57·2	11 14	5·45941	2833·6	2427·7	0·8568	0·0716	33	2·3628
30	Lundy Island Precelly	90 4 12·0 90 35 5·8	21 5	5·45528	2806·6	2357·8	0·8401	0·0800	22	1·7600
31	Paracombe Precelly	90 21 53·7 90 26 15·0	10 4	5·54372	3440·5	2888·7	0·8396	0·0802	17	1·3634
32	Black Down Barrow Hill	90 25 38·0 90 18 9·2	21 12	5·49842	3099·9	2627·2	0·8475	0·0763	43	3·2809

No.	Stations.	Observed Zenith Distance.	No. Obs.	Log. Distance.	*v.*	*z* + *z'* − 180°.	1 − 2 *k.*	Coeff. of Refraction *k.*	*w.*	*kw.*
		° ' ''			''	''				
33	Black Down Dunnose	90 21 44·1 90 21 32·9	17 11	5·49740	3092·6	2597·0	0·8398	0·0801	37	2·9637
34	Black Down Ryder's Hill	90 12 13·2 90 32 9·0	6 12	5·50337	3135·4	2662·2	0·8491	0·0755	23	1·7365
35	Beachy Head Dunnose	90 21 22·3 90 25 59·7	10 13	5·53083	3340·0	2842·0	0·8509	0·0746	33	2·4596
36	Ben Lawers Jura	90 41 25·7 90 18 0·8	19 21	5·63569	4250·8	3566·5	0·8390	0·0805	66	5·3130
37	Sayrs Law Mount Battock	90 21 1·0 90 34 50·5	3 14	5·60522	3962·8	3351·5	0·8457	0·0772	16	1·2352
38	Kellie Law Mount Battock	89 51 19·5 90 44 14·4	20 8	5·40865	2520·1	2133·9	0·8467	0·0767	29	2·2243
39	Ben Cleugh Sayrs Law	90 25 42·0 90 9 37·1	15 6	5·40925	2523·7	2119·1	0·8397	0·0802	22	1·7644
40	Ben Lawers Goat Fell	90 37 26·7 90 17 24·3	10 8	5·59102	3835·3	3291·0	0·8581	0·0710	28	1·9880
41	Ben Lawers Sayrs Law	90 47 25·1 90 5 54·7	14 8	5·60836	3991·5	3199·8	0·8016	0·0992	32	3·1744
42	Ben Macdui Ben Clibrig	90 40 4·7 90 22 46·1	10 4	5·65297	4423·0	3770·8	0·8525	0·0738	19	1·4022
43	Mamsuil Storr	90 38 41·6 89 52 16·2	32 22	5·35288	2216·4	1857·8	0·8382	0·0809	62	5·0158
44	Wingreen Dunnose	90 19 30·2 90 15 50·3	12 13	5·39684	2453·3	2120·5	0·8644	0·0678	31	2·1048
45	Coringdon Butser Hill	90 15 17·9 90 21 21·7	15 12	5·42430	2613·4	2199·6	0·8416	0·0792	34	2·6928
46	Dunnose Inkpen	90 17 0·7 90 21 47·7	18 15	5·43919	2704·6	2328·4	0·8609	0·0696	43	2·9948
47	Dunkery Mendip	90 27 7·4 90 5 49·9	12 12	5·37996	2359·8	1977·3	0·8379	0·0811	29	2·3519
48	Lundy Island High Willhays	89 53 9·4 90 39 41·0	19 15	5·36717	2291·3	1970·4	0·8599	0·0701	40	2·8040
49	Corryhabbie Ben Macdui	89 26 5·9 90 53 10·5	24 21	5·13736	1349·3	1156·4	0·8570	0·0715	41	2·9315
50	Corryhabbie Mamsuil	90 13 54·3 90 38 29·4	20 27	5·58120	3749·3	3143·7	0·8385	0·0808	71	5·7368
51	Corryhabbie Scournalapich	90 13 28·3 90 37 17·9	23 29	5·56598	3620·2	3046·2	0·8415	0·0793	78	6·1854
52	Corryhabbie Mormonth	90 42 29·9 89 52 5·9	44 25	5·39386	2435·6	2075·8	0·8523	0·0739	79	5·8381
53	Corryhabbie Ben Nevis	90 12 11·7 90 44 49·4	19 14	5·61443	4047·5	3421·1	0·8452	0·0774	52	4·0248

No.	Stations.	Observed Zenith Distance.	No. Obs.	Log. Distance.	v.	z + z′ − 180°.	1 − 2 k.	Coeff. of Refraction k.	w.	k w.
		° ′ ″			″	″				
54	Scournalapich . .	90 4 9·3	33	5·32108	2059·9	1744·3	0·8468	0·0766	89	6·8174
	Ben Nevis . . .	90 24 55·0	47							
55	Scournalapich . .	90 14 48·8	29	5·47208	2916·3	2504·7	0·8589	0·0706	78	5·5068
	Ben Macdui . . .	90 26 55·9	28							
56	Scournalapich . .	89 54 2·8	11	4·54149	342·2	320·3	0·9361	0·0320	16	0·5120
	Mamsuil	90 11 17·5	43							
57	Ben Nevis . . .	90 20 46·3	26	5·26093	1793·5	1541·5	0·8595	0·0703	57	4·0071
	Ben Lawers . . .	90 4 55·2	27							
58	Ben Nevis . . .	90 44 34·8	14	5·52480	3292·9	2805·9	0·8521	0·0740	53	3·9220
	Ben Cleugh . . .	90 2 11·1	27							
59	Ben Nevis . . .	90 21 20·6	34	5·45432	2799·5	2388·7	0·8533	0·0734	97	7·1198
	Ben Macdui . . .	90 18 28·1	39							
60	Ben Macdui . . .	90 20 31·8	42	5·34547	2178·9	1884·7	0·8650	0·0675	87	5·8725
	Ben Lawers . . .	90 10 52·9	33							
61	Ben Lawers . . .	90 44 27·4	16	5·71265	5075·0	4313·8	0·8500	0·0750	57	4·2750
	Merrick	90 27 26·4	16							
62	Ben Lawers . . .	90 41 42·2	12	5·65121	4405·5	3726·5	0·8459	0·0771	45	3·4695
	Hart Fell. . . .	90 20 24·3	15							
63	Sayrs Law . . .	90 42 30·0	13	4·99047	962·2	818·6	0·8508	0·0746	20	1·4920
	Lumsden. . . .	89 31 8·6	12							
64	Lumsden. . . .	89 28 32·4	25	5·19654	1546·4	1296·9	0·8386	0·0807	30	2·4210
	Cheviot	90 53 4·5	11							
65	Lumsden. . . .	90 10 30·7	8	5·05542	1117·4	959·9	0·8591	0·0705	19	1·3395
	Blackheddon. . .	90 5 29·2	18							
66	Cheviot	91 39 2·6	21	4·86730	724·6	590·9	0·8155	0·0923	26	2·3998
	Blackheddon. . .	88 30 48·3	18							
67	Cheviot	90 30 13·5	14	5·23766	1700·0	1435·8	0·8446	0·0777	26	2·0202
	Sayrs Law . . .	89 53 42·3	11							
68	Cheviot	90 18 49·0	14	5·33872	2145·5	1785·7	0·8323	0·0839	15	1·2585
	Dunrich	90 10 56·7	4							
69	Ben Hutig . . .	89 14 56·6	8	5·06847	1151·3	987·6	0·8578	0·0711	22	1·5642
	Ben Clibrig . . .	91 1 31·0	33							
70	Ben Hutig . . .	89 57 53·5	5	4·87000	729·0	612·7	0·8405	0·0798	7	0·5586
	Fashven	90 12 19·2	5							
71	Ben Hutig . . .	90 20 4·7	5	4·95785	892·4	759·1	0·8506	0·0747	8	0·5976
	Cnoc-ghiubhais . .	89 52 34·4	5							
72	Ben Clibrig . . .	90 48 6·7	17	5·18161	1493·9	1265·9	0·8474	0·0763	19	1·4497
	Fashven	89 32 59·2	7							
73	Ben Clibrig . . .	90 35 3·5	32	5·19896	1554·9	1343·9	0·8643	0·0679	35	2·3765
	Scarabin	89 47 20·4	12							
74	Cnoc-ghiubhais . .	88 21 32·4	6	4·25345	176·3	159·1	0·9026	0·0487	4	0·1948
	Fashven	91 41 6·7	5							

No.	Stations.	Observed Zenith Distance.	No. Obs.	Log. Distance.	v.	z + z′ − 180°.	1 − 2k.	Coeff. of Refraction k.	w.	k w.
		° ′ ″			″	″				
75	Ben Lawers . . . Ben Cleugh . . .	90 46 15·1 89 36 9·7	27 37	5·20427	1574·2	1344·8	0·8543	0·0729	62	4·5198
76	Ben Cleugh . . . Mount Battock . .	90 21 56·9 90 26 4·7	10 15	5·54092	3417·4	2881·6	0·8432	0·0784	35	2·7440
77	Ben Cleugh . . . Dunrich	90 16 20·1 90 18 39·5	9 4	5·40510	2499·7	2099·6	0·8399	0·0801	14	1·1214
78	Ben Cleugh . . . Goat Fell . . .	90 19 33·8 90 29 46·9	8 6	5·55201	3505·7	2960·7	0·8445	0·0778	20	1·5560
79	Ben Cleugh . . . Hart Fell . . .	90 16 18·5 90 23 11·0	8 13	5·46828	2891·2	2369·5	0·8196	0·0902	27	2·4354
80	Ben Cleugh . . . Merrick	90 24 28·7 90 31 48·5	13 10	5·61129	4018·6	3377·2	0·8404	0·0798	36	2·8728
81	Ben Macdui . . . Ben Cleugh . . .	90 43 20·8 90 1 42·8	19 21	5·51034	3185·0	2703·6	0·8489	0·0756	57	4·3092
82	Ben Macdui . . . Mamsuil	90 25 51·6 90 15 55·5	7 12	5·47441	2932·0	2507·1	0·8551	0·0725	24	1·7400
83	Ben Macdui . . . Mount Battock . .	90 45 1·4 89 41 52·9	39 32	5·27887	1869·1	1614·3	0·8637	0·0682	77	5·2514
84	Mamsuil . . . . Ben Lawers . . .	90 21 30·6 90 24 8·4	21 20	5·50952	3179·0	2739·0	0·8616	0·0692	58	4·0136
85	Mormonth . . . Mount Battock . .	89 56 45·9 90 41 43·8	21 15	5·44233	2723·2	2309·7	0·8482	0·0759	46	3·4914
86	Monach . . . . Cleisham	89 36 13·6 90 48 2·2	19 18	5·23842	1702·8	1455·8	0·8549	0·0726	38	2·7588
87	Merrick . . . . Hart Fell . . .	90 18 46·5 90 14 37·2	28 17	5·38595	2392·0	2003·7	0·8377	0·0812	52	4·2224
88	Hart Fell . . . Cheviot	90 16 54·9 90 18 33·2	12 17	5·41838	2577·4	2128·1	0·8257	0·0872	36	3·1392
89	Sayrs Law . . . Blackheddon . . .	90 34 7·6 89 49 51·1	8 22	5·23217	1678·6	1438·7	0·8571	0·0715	24	1·7160
90	Corryhabbie . . . Mount Battock . .	90 12 10·1 90 11 51·5	49 24	5·23134	1675·3	1441·6	0·8605	0·0698	66	4·6068
91	Pillesdon . . . . Coringdon . . .	90 18 35·3 90 10 8·0	18 15	5·31368	2025·8	1723·3	0·8507	0·0747	37	2·7639
92	Wingreen . . . Inkpen	90 12 59·9 90 15 21·1	20 19	5·29968	1961·5	1700·0	0·8667	0·0667	44	2·9348
93	Wingreen . . . Butser Hill . . .	90 18 41·2 90 17 40·2	15 18	5·41479	2556·8	2181·4	0·8532	0·0734	42	3·0828
94	Inkpen . . . . Butser Hill . . .	90 13 55·3 90 10 33·5	13 13	5·24621	1734·3	1468·8	0·8469	0·0766	27	2·0682
95	Beachy Head . . Frittenfield . . .	90 13 29·2 90 16 40·1	15 12	5·33540	2129·7	1809·3	0·8496	0·0752	31	2·3312

No.	Stations.	Observed Zenith Distance.	No. Obs.	Log. Distance.	v.	$z + z' - 180°$.	$1 - 2k$.	Coeff. of Refraction k.	w.	kw.
		° ′ ″			″	″				
96	Leith Hill Tower . Inkpen	90 17 54·5 90 18 8·1	13 14	5·41263	2544·1	2162·6	0·8501	0·0749	34	2·5466
97	Wrotham . . . Fairlight . . .	90 16 19·8 90 8 53·4	14 14	5·25000	1749·5	1513·2	0·8649	0·0676	30	2·0380
98	Beacon Hill . . . Mendip	90 7 41·6 90 18 52·8	15 12	5·27651	1859·6	1594·4	0·8574	0·0713	29	2·0677
99	Pillesdon . . . . High Willhays . .	90 5 18·2 90 33 31·5	13 17	5·44131	2717·9	2329·7	0·8572	0·0714	39	2·7846
100	Pillesdon . . . Dunkery	90 2 43·0 90 27 45·0	17 18	5·33595	2132·4	1828·0	0·8573	0·0714	41	2·9274
101	Pillesdon . . . Mendip	90 10 9·3 90 12 57·7	15 12	5·21570	1616·6	1387·0	0·8580	0·0710	27	1·9170
102	Paracombe . . . High Willhays . .	90 3 44·9 90 21 44·2	11 12	5·26086	1793·8	1529·1	0·8525	0·0738	25	1·8450
103	Swyre Barrow . . Pillesdon . . .	90 7 41·5 90 16 55·4	5 13	5·27038	1833·6	1476·9	0·8055	0·0973	16	1·5568
104	White Horse Hill . Leith Hill Tower .	90 20 23·0 90 22 44·9	14 11	5·49084	3046·1	2587·9	0·8496	0·0752	34	2·5568
105	Butser Hill . . . Leith Hill Tower .	90 9 1·8 90 12 40·4	12 10	5·19773	1551·1	1302·2	0·8395	0·0803	22	1·7666
106	Butser Hill . . . Ditchling . . .	90 15 16·0 90 12 59·0	25 11	5·30783	1998·7	1695·0	0·8481	0·0760	34	2·5840
107	Leith Hill Tower . Wrotham . . .	90 15 20·5 90 6 58·8	10 20	5·20088	1562·4	1339·3	0·8572	0·0714	27	1·9278
108	Cheviot . . . . Wordeslow . . .	90 44 40·9 89 53 25·3	20 22	5·43903	2703·0	2286·2	0·8458	0·0771	55	4·2405
109	Fairlight . . . . Ditchling . . .	90 6 55·0 90 16 23·1	12 17	5·22518	1652·3	1398·1	0·8462	0·0769	29	2·2301
110	Wrotham . . . Ditchling . . .	90 11 7·5 90 12 25·0	10 6	5·24632	1734·7	1412·5	0·8143	0·0929	16	1·4864
111	Severndroog . . . Leith Hill Tower .	89 57 0·2 90 23 31·7	22 13	5·16066	1424·2	1231·9	0·8650	0·0675	31	2·0925
112	Leith Hill Tower . Dunstable . . .	90 20 1·1 90 15 28·6	9 14	5·40466	2497·8	2129·7	0·8526	0·0737	28	2·0636

In the first 35 of these cases it will be seen, on reference to the map, that the ray passes for a considerable portion of its length over the sea, and in the next 13 cases the ray passes also over the sea, but for a smaller proportion of its length; in the remaining cases the ray is inland. From the first 35 we have $k = 0·0801$; from the next 13 we have $k = 0·0778$; and from the remainder we have $k = 0·0744$.

In the preceding table of 31 coefficients obtained from levelled heights (page 542), if we separate those rays which cross the sea from those which do not, we get for the former

$k = \cdot 0817$, and for the latter $k = \cdot 0772$; or if we omit the case of Ben Nevis, in which the atmosphere* was apparently in an abnormal state, $k = \cdot 0756$ for rays not crossing the sea. These

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* It is much to be regretted that the observations of the barometer and thermometer on the tops of the hills in Scotland were not made with systematic regularity. The following results of the observations at some of the highest points may be interesting, but it is necessary to premise that the index error of the instrument in each case is not exactly known. It is an almost unavoidable result of travelling that small bubbles of air are introduced into the vacuum, and thus all readings will require a positive correction, which at some of the stations may amount to nearly a tenth of an inch.

Name.	Height.	Period.		Maximum Pressure.		Corres. Temp.	Minimum Pressure.		Corres. Temp.	Mean Pressure.	Mean Temp.	No. Obs.
		From	To	Date.	Amount.		Date.	Amount.				
Ben Nevis . . .	4406	Aug. 1	Nov. 14	1st to 6th August.	25·62	58°	15th & 16th October.	24·37	35°	25·233	45°	116
Ben Macdui . . .	4296	June 6	Aug. 16	June 29 to July 2	26·10	58	21st to 24th June.	25·15	38	25·713	47	71
Goat Fell . . .	2875	Aug. 13	Dec. 2	21st to 22nd August.	27·37	56	Oct. 5	26·07	37	26·754	42	100
Scournalapich . .	3773	July 15	Oct. 25	12th to 14th September.	26·63	54	Oct. 6	25·15	38	26·052	48	223
Corryhabbie . . .	2569	Sept. 30	Nov. 21	12th to 13th October.	27·43	40	Oct. 28	26·60	35	27·004	38	109
Hart Fell . . .	2638	Sept. 30	March 4	2nd to 4th March.	27·68	30	6th to 10th October.	26·54	43	27·096	29	98
Cheviot	2669	June 16	Sept. 13	11th to 13th September.	27·65	55	16th to 18th July.	26·61	47	27·211	51	100
Mount Battock . .	2555	May 10	July 3	May 31 to June 4	27·81	59	June 14	26·72	45	27·231	50	168
Jura	2566	Sept. 30	Nov. 12	1st to 2nd October.	27·53	48	23rd to 24th October.	26·67	36	27·288	41	89

N.B.—The maxima and minima here given are not single readings, but the mean of the readings during the *period* of maximum or minimum pressure.

From the usual formulæ connecting the heights of stations with the readings of the barometer, and assuming that the thermometer falls 1° for every 275 feet of altitude, we may derive this formula—

$$b' = \frac{b\left(1 + \frac{k\tau}{205}\right)}{\log^{-1} k \left(1 + \frac{k}{4\cdot 2}\right)}$$

which gives the height, b', of the barometer at the altitude, h $(= 62137\,k)$, above the level of the sea, when the height of the barometer at the level of the sea is b, and the temperature $50° + \tau$.

If we assume 29·850 as the mean height of the barometer in the latitude of Scotland at the level of the sea and temperature of 50° Fahrenheit, the corresponding height due to different altitudes will be as follows :—

Altitude in Feet.	Height of Barometer.	Altitude in Feet.	Height of Barometer.
	in.		in.
2000	$27{\cdot}702 + {\cdot}0072\,\tau$	3500	$26{\cdot}173 + {\cdot}0100\,\tau$
2500	$27{\cdot}185 + {\cdot}0082\,\tau$	4000	$25{\cdot}680 + {\cdot}0108\,\tau$
3000	$26{\cdot}675 + {\cdot}0091\,\tau$	4500	$25{\cdot}193 + {\cdot}0116\,\tau$

At Storr in the Isle of Skye (2341 feet high) the mean height of the barometer was 27·464 inches. The most severe weather ever encountered by any of the observing parties was at this station, which was only completed after the most extraordinary perseverance. Shortly after the commencement of the observing, a storm of wind carried away the wooden houses of the men, and left the observatory roofless. Three observatory roofs were subsequently demolished at different periods, and for some time the observatory was used without a roof, being filled with snow every night and emptied every morning. The instrument was protected under shelter of the cooking-house.

determinations agree satisfactorily with those from the second table. The individual determinations in the first table would seem to be entitled to the most confidence, being directly observed, but the number of determinations in the second table is considerably greater; we may make these respective advantages counterbalance, and assume the mean as true; namely,

$$\textit{Coefficient of Refraction}\begin{cases} = 0{\cdot}0809 \text{ for rays crossing the sea.} \\ = 0{\cdot}0750 \text{ for rays not crossing the sea.}\end{cases}$$

With one or two exceptions, all the coefficients in the preceding tables are derived from observed zenith distances of very distant objects; and the results are, upon the whole, tolerably consistent. The same consistency, however, is not found to exist among coefficients of refraction deduced from the observations of objects at short distances: the values are very irregular, and, indeed, cannot be represented with precision by any mean quantity. A great deal depends upon the nature of the ground over which the ray passes: as a general rule, when the distances are small the ray passes near the ground, which would tend to increase the coefficient of refraction, as is evident from Struve's formula (page 512); and this agrees with observation, for the mean value at short distances is certainly not under 0·1000, one-tenth of the arc-distance, with an uncertainty of at least half that quantity. Fortunately, however, the shorter the distance the less will the resulting altitudes be affected by error in the assumed refraction.

Altitudes above the Mean Level of the Sea.

In the summer of 1842 a system of tidal observations was instituted by the late Major-General Colby round the coast of Ireland, in order both to improve the theory of the tides and to ascertain the relative advantages of using high water, low water, or the mean of high and low water, as the plane of reference for altitudes. Observations of the tides were made with great regularity for the space of two complete lunations (during the months of June, July, and August) at 22 stations round the coast of the island, the stations being connected by lines of spirit-levelling. These observations have been fully discussed by the Astronomer Royal in a paper which will be found in the Philosophical Transactions, Part I., for 1845.

The relative heights of the surface of the sea round Ireland, which have been deduced from them by means of the connecting system of levels, are exhibited in the following table*:—

Stations.	Mean Height of Mean Water.	Spring Tides.		Neap Tides.	
		Mean Height of High Water.	Mean Height of Low Water.	Mean Height of High Water.	Mean Height of Low Water.
	Feet.	Feet.	Feet.	Feet.	Feet.
Kingstown . . .	8·395	14·353	2·553	11·213	5·363
Clogher Head . .	8·184	15·658	0·598	11·908	4·108
Ardglass . . .	8·638	16·431	1·005	12·531	4·503
Donaghadee . .	8·745	15·046	3·061	11·910	5·391
Glenarm . . .	8·709	12·681	5·336	10·681	6·700
Ballycastle . .	8·479	10·864	6·601	8·651	7·726
Portrush . . .	8·682	12·234	5·674	9·034	7·634
Carrowkeel . . .	9·029	13·447	5·187	10·107	7·532
Buncrana . . .	8·369	15·091	1·911	10·566	5·791
Mullaghmore . .	9·047	15·538	3·078	10·993	6·668
Old Head . . .	8·725	15·280	2·460	10·810	6·260
Cashla Bay . . .	8·899	15·980	1·820	11·735	5·998
Galway	8·480	15·853	1·013	11·403	5·391
Kilbaha	6·578	13·199	—0·101	9·279	3·709
Kilrush	7·358	14·412	0·079	10·423	4·016
Foynes Island . .	8·092	15·627	—0·603	11·672	3·809
Limerick . . .	8·567	17·502	—2·088	12·935	2·516
Castletownsend . .	6·660	11·954	1·984	9·551	4·014
Passage West . .	6·934	13·601	1·311	10·514	3·601
Dunmore East . .	6·753	13·099	0·699	9·879	3·629
New Ross . . .	7·667	14·822	1·256	11·300	3·737
Courtown . . .	7·598	9·849	6·139	7·732	7·407

A glance at this table shows that mean water is more nearly on a uniform level round the coast of Ireland than low water of spring tides, to which the altitudes on the Ordnance maps in that country have been referred. The mean sea level appears, indeed, to be rather higher round the northern than round the southern coast of the island, but the difference is small, and it would be difficult, considering the very small number of stations, to say how far it is real.

A very close approximation to the true height of mean water at any station may be obtained from a few observations of high and low water at any period of the lunation, and Mr. Airy has shown that the irregularities in its height from day to day, which appear to be chiefly caused by irregularities in the atmospheric pressure, are nearly the same on every part of the Irish coast at the same time.

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* See the volume entitled "Abstracts of the Principal Lines of Spirit-levelling in Ireland," published by Order of the Master General and Board of Ordnance, 1855.

In consideration of the advantage of mean water as the plane of reference for altitudes, it was adopted by Major-General Colby for the levels and altitudes of the Ordnance survey of the north of England and Scotland, which are all referred to mean water at Liverpool.

When the observations of the general triangulation were resumed in the north of Scotland, in the summer of 1838, by the late Lieutenant-Colonel Robe, R.E., several tidal stations were formed and the height of mean water ascertained for the determination of the heights of the trigonometrical stations.[*] The distances of these tidal stations with respect to the trigonometrical stations were determined by special triangulations, observed with 5 and 7-inch theodolites. It seems unnecessary to give more than the results of these operations, which are as follows:—

Names.	Height above Mean Level of the Sea.	Log. Distance.	From
Roan Island	243·88	4·5143882 5·0733199	Ben Hutig. Ben Clibrig.
Rispond	70·13	4·6709784 4·4363163	Fashven. Ben Hutig.
Clashcarnach	228·12	4·0643398 4·3947377	Cnoc-ghiubhais. Fashven.
Berriedale Tower . . .	187·76	4·3621023	Scarabin.
Brock	91·02	3·9539667	Dunnet Head.
Loch Seaforth, Cottage .	33·65	4·3041556	Cleisham.
Callernish	46·86	4·1315796	Ben More, South Uist.

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[*] The descriptions of these stations are as follows:—

Roan Island Tide Station.—The graduated tide-pole was erected in a quiet inlet on the eastern coast of Roan Island, in the Kyle of Tongue: a Bench-mark was cut on a rock, at the level of 8·250 feet above the zero of the tide-pole. From the observations of 14 high and 14 low waters, the reading of mean water on the tide-pole was found to be 6·63 feet. Thence, by levelling, the height of the Tide Station was found to be 243·88 feet above the mean level of the sea.

Rispond Tide Station.—The graduated tide-pole was erected in a quiet inlet about 100 yards west of the Tide Station, which was on the small headland forming the western promontory of the entrance to Loch Eriboll. From the observations of 34 tides, the reading of mean water on the tide-pole was found to be 7·16 feet. Thence, by levelling, the height of the Tide Station was found to be 70·13 feet above the mean level of the sea.

Clashcarnach Tide Station.—The tide-pole was erected in a small inlet to the east of Cape Wrath Lighthouse; the Tide Station being on the cliff above it. At this point only three tides were observed, and, by levelling, the height of the Tide Station was found to be 228·12 above the mean level of the sea.

Berriedale Lower Tower.—The tide-pole was erected in a small rocky inlet on the east side and immediately under the ruins of the old Castle of Berriedale, in Caithness-shire. The mean of 41 tides gave 8·911 for the reading of the pole, corresponding to mean water. A hole was jumped in the rock corresponding to 10·75 feet on the pole. The height of the bottom of the tower was found by levelling to be 187·76 feet above the mean level of the sea.

Brock Tide Station, near Dunnet Head.—Record of the tidal observations defective. The Tide Station is found by levelling to be 99·93 feet above lowest tide, or 91·02 above mean water.

Loch Seaforth Tide Station, Island of Lewis.—The graduated tide-pole was erected about a quarter of a mile south-west of Sir Frederic Johnstone's cottage, at the first rocky point projecting a little into the Loch. The mean of 51 tides

Of the stations composing the principal triangulation, the heights of the following have been obtained by spirit-levelling:—

Names.	Height above Mean Level of the Sea.	Remarks.	Names.	Height above Mean Level of the Sea.	Remarks.
	Feet.			Feet.	
Arbury Hill	735·26	Surface.	Hensbarrow	1026·99	Surface.
Beacon Hill	668·74	,,	High Willhays. . . .	2039·61	,,
Blackdown	790·00	Monument 861·38.	Ingleboro'	2373·40	,,
Ben Lomond	3192·16	Surface.	Karnminnis	799·78	,,
Ben Nevis	4406·31	,,	Lincoln Minster . . .	475·35	Pinnacles.
Botton Head	1489·00	,,	Maker Church . . .	366·35	Surface.
Burleigh Moor . . .	581·10	,,	Malvern	1396·20	,,
Calton Hill	349·60	,,	Merrick	2764·80	,,
Clifton Beacon . . .	460·90	,,	Monach	813·83	,,
Coringdon	655·55	,,	North Rona	355·20	,,
Cowhythe	272·75	Above low water.	Old Sarum Castle. . .	404·68	,,
Criffel	1866·60	Surface.	Old Sarum Gun . . .	241·31	,,
Ditchling	814·17	,,	Pertinney	735·47	,,
Dunkery	1706·44	,,	Precelly	1757·90	,,
Dunnose	771·90	,,	Sayrs Law.	1753·07	,,
Dunstable	800·25	,,	S. End Lough Foyle Base	88·00	,,
East Lomond	1471·30	,,	Southampton	78·00	,,
Garforth Cliff. . . .	342·04	,,	Saint Paul's	417·91	Top of Cross.
Gerth of Scaw . . .	70·20	,,	Swaffham	372·60	Telescope.
Great Stirling . . .	260·50	,,	Swyre	674·40	Surface.
Great Whernside . . .	2310·30	,,	Trevose	242·58	,,
Hanger Hill	201·12	,,	York Minster	53·10	,,

In order to show the degree of reliance to be placed upon the determinations of heights by means of observed zenith distances, we shall now give in detail part of the calculation by which the heights of those of the principal stations which have not been determined by levelling have been ascertained.

We shall commence with the stations on the north coast of Scotland, whose heights are determinable with sufficient accuracy by means of the tide stations described in the last page.

The heights of the tide stations at Rispond, Clashcarnach, and Roan Island are 70·1, 228·1, and 243·9 feet respectively above mean water. Assume the height of Ben Hutig $1340 + x$, of Fashven $1495 + y$, and of Cnoc-ghiubhais $976 + z$, then by means of the reciprocal observations of these stations, and their observations of the tide stations, we have eleven equations for the determination of x, y, z. These equations are thus obtained: the difference of height of Ben

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gave 8·230 feet as the reading of mean water on the tide-pole, and, by levelling, the height of the ground at south-west corner of the cottage was found to be 33·65 feet above the mean level of the sea.

Callernish Point, in South Uist.—The graduated tide-pole was erected in a sheltered situation about 200 yards north-east of the Ru Callernish Pile. The result of 28 observed tides gave 10·48 feet for the reading of mean water on the pole; and, by levelling, the centre stone in the Callernish Pile was found to be 46·86 feet above the mean level of the sea.

Hutig and Fashven is, by the observations at the former station, 156·3, and by the observations at the latter, 154·7; these quantities supply two equations, namely,

$$\left.\begin{array}{r} x - y - 155 + 156{\cdot}3 = 0 \\ x - y - 155 + 154{\cdot}7 = 0 \end{array}\right\} \text{or} \left\{\begin{array}{l} x - y + 1{\cdot}3 = 0 \\ x - y - 0{\cdot}3 = 0 \end{array}\right.$$

In general, it is convenient to assume for the weight of an equation the number of observations upon which it depends, divided by the arc $(\frac{1}{2} - k)\, v$, expressed in minutes: this quantity being proportional to the distance, and of a convenient magnitude. In thus estimating the weight, it is by no means necessary to calculate it with precision. Thus the weights of the equations just written down are each taken equal to unity. The reciprocal observations at Ben Hutig and Cnoc-ghiubhais give the equations—

$$\begin{array}{c} x - z + 1{\cdot}7 = 0 \\ x - z + 0{\cdot}1 = 0 \end{array}$$

The weight of either may be taken at ·75. From the eleven equations that may be deduced in this manner between $x\ y$ and z, we have finally, by the method of least squares, the following—

$$\begin{array}{rrrrl} 27{\cdot}5\,x & - \;\; 2{\cdot}0\,y & - \;\; 1{\cdot}5\,z & + \;\; 0{\cdot}0 & = 0 \\ - \;\; 2{\cdot}0\,x & + 22{\cdot}0\,y & - \;\; 9{\cdot}0\,z & - 18{\cdot}0 & = 0 \\ - \;\; 1{\cdot}5\,x & - \;\; 9{\cdot}0\,y & + 36{\cdot}5\,z & + 18{\cdot}8 & = 0 \end{array}$$

From these we have $x = +\,0{\cdot}03$; $y = +\,0{\cdot}68$; $z = -\,0{\cdot}35$; whence,

Height of *Ben Hutig*	1340·0	Feet above Mean Level of Sea.
„ *Fashven*	1495·7	„ „
„ *Cnoc-ghiubhais*	975·7	„ „

These values satisfy the observations with much precision, and therefore the determinations may be considered as satisfactory.

The height of the tide station Brock, near Dunnet Head, is 91·0 feet above mean water, and the height of Berriedale Tower, near Scarabin, was found by levelling to be 187·8 feet. By means of these data we may fix the heights of Dunnet Head, Ben Chielt, and Scarabin. Assuming the heights of these three stations to be $420 + x$, $940 + y$, $2054 + z$, we have six equations for the determination of x, y, z, which when combined, having respect to their weights,* form the group—

$$\begin{array}{rrrrl} 41{\cdot}25\,x & - \;\; 0{\cdot}25\,y & - \;\; 1{\cdot}00\,z & + 77{\cdot}52 & = 0 \\ - \;\; 0{\cdot}25\,x & + \;\; 4{\cdot}00\,y & - \;\; 3{\cdot}75\,z & - \;\; 2{\cdot}55 & = 0 \\ - \;\; 1{\cdot}00\,x & - \;\; 3{\cdot}75\,y & + 24{\cdot}75\,z & + \;\; 1{\cdot}03 & = 0 \end{array}$$

Whence we have $x = -\,1{\cdot}88$; $y = +\,0{\cdot}48$; $z = -\,0{\cdot}04$. Applying these corrections, there results—

Height of *Dunnet Head*	418·1	Feet above Mean Level of Sea.
„ *Ben Chielt*	940·5	„ „
„ *Scarabin*	2054·0	„ „

We may compare these results with what would have been derived from Ben Hutig. The reciprocal observations at Dunnet Head and Ben Hutig (two observations at each) give 917 feet for the difference of height: this applied to the height of Ben Hutig as just determined, namely

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* When the *weight* deduced, as explained above, does not much exceed $\frac{1}{10}$, the corresponding equation is rejected.

1340 feet, would give the height of Dunnet Head 423 feet, showing a discrepancy of 5 feet. The reciprocal observations at Ben Chielt and Ben Hutig give 415 feet as the difference of height of those stations: this quantity applied to the height of Ben Hutig gives 925 feet for the height of Ben Chielt, which is probably too small by 15 feet. And finally, the ten observations at Scarabin give 744 feet for the difference of height of that station and Ben Hutig: this applied to the height of Ben Hutig would give the height of Scarabin too great by 30 feet.

The height of Ben Clibrig may be determined very satisfactorily from Fashven, Ben Hutig, Scarabin, Dunnet Head, and Ben Chielt; and also by means of the observations at Ben Clibrig of the Roan Island Tide Station. We have thus ten determinations of the required height; of these, six are between 3151 and 3158 feet; the others are 3140·6; 3162·1; 3170·6; and 3168·0. Thus the height of Ben Clibrig is found, by giving to each determination its proper weight, to be 3157·6 feet.

From the levelled heights of Ben Nevis (4406·3 feet) and Ben Lomond (3192·2 feet) we may determine the heights of Scournalapich ($3774 + x$), Mamsuil ($3862 + y$), Ben Lawers ($3984 + z$), and Ben Macdui ($4292 + w$). This system of points supplies twenty equations for the determination of the four unknown heights. These equations resolve themselves into the following system—

$$\begin{array}{rrrrr}
30{\cdot}5x - & 23{\cdot}0\,y + & \cdots & - 2{\cdot}50w + 42{\cdot}70 = 0 \\
-23{\cdot}0x + & 28{\cdot}5\,y - & 2{\cdot}0\,z - & 0{\cdot}75w - 20{\cdot}35 = 0 \\
\cdots\cdots - & 2{\cdot}0\,y + & 13{\cdot}25z - & 5{\cdot}0\,w + 16{\cdot}40 = 0 \\
-2{\cdot}5x - & 0{\cdot}75y - & 5{\cdot}0\,z + & 11{\cdot}75w - 45{\cdot}45 = 0
\end{array}$$

Whence—

$$x = -1{\cdot}24;\; y = -0{\cdot}18;\; z = +0{\cdot}10;\; w = +3{\cdot}64$$

So that we thus have—

Height of *Scournalapich*	3772·8	Feet above Mean Level of Sea.	
" *Mamsuil*	3861·8	"	"
" *Ben Lawers*	3984·1	"	"
" *Ben Macdui*	4295·6	"	"

The greatest discrepancy here brought out is in the connection of Mamsuil and Ben Lawers. We have obtained, as most probable, their difference of height, 122·3 feet; but the observations (twenty in number) at Ben Lawers give 161·4 feet for the difference of height, and the observations (twenty-one in number) at Mamsuil give 85·4 feet for the difference of height.

If, however, we calculate the difference of height by means of the difference of the observed zenith distances, instead of using a value of the coefficient of refraction, we get 124 feet, which agrees satisfactorily with the heights we have assigned. And similarly with respect to Ben Lawers and Ben Macdui:* the differences of height calculated independently are 289·1 and 332·4 feet, but if we combine the zenith distances and thus eliminate the coefficient of refraction, the result is 311 feet, which agrees exactly with the heights assigned as most probable.

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* Ben Macdui, or, as more correctly spelt, Beinn-muic-dhuibh, has been sometimes stated as the highest mountain in Great Britain; but as the probable error of the height here given can scarcely amount to 10 feet, Ben Nevis has the advantage by at least 100 feet of height.

The difference of height of Ben Clibrig and Ben Macdui is thus 1138 feet. These points are mutually visible, and ten observed zenith distances at Ben Macdui give 1162 feet for the difference of height, while four observations at Ben Clibrig give 1103 feet; the mean of these results, having respect to their relative weights, gives 1146 feet for the difference of height. This is sufficiently satisfactory when the distance, 85 miles, is considered.

The points Ben Macdui and Scarabin are also mutually visible. By means of seventeen observations at Ben Macdui, the difference of height is found to be 2301 feet, while a single observation at Scarabin gives 2208; the former exceeds the probable truth by 60 feet, the latter is in defect by 33 feet: the distance is 80 miles. By eight observations at Sayrs Law, distant 93 miles, the height of Macdui is 4314·0 feet.

The station Knock is well observed from Cowhythe, whose levelled height is 267·7 feet, and may be safely determined from that point alone. We thus obtain 1416·9 feet for the height of Knock.

The heights of Mount Battock and Corryhabbie may be safely determined by their connection with Ben Macdui, Knock, and Cowhythe. Putting $2557 + x$ and $2555 + y$ for the required heights, we have ten equations for determining x and y, which resolve themselves into the equations—

$$\begin{aligned} 12{\cdot}25x - 6{\cdot}0y + 109{\cdot}2 &= 0 \\ -6{\cdot}0\ x + 18{\cdot}5y - 270{\cdot}1 &= 0 \end{aligned}$$

from which there results $x = -2{\cdot}10$, $y = +13{\cdot}92$. Consequently,

Height of *Mount Battock* . . . . . 2554·9 Feet above Mean Level of Sea.
„ *Corryhabbie* . . . . 2568·9 „ „

These determinations represent the observations with tolerable exactness. The heights of Corryhabbie, as deduced from Knock, Cowhythe, and Ben Macdui separately, are 2582, 2559, and 2571, the mean differing by 3 feet from the most probable height just deduced. Yet on comparing this with the height of Ben Nevis a very large discrepancy is brought out. The true difference of height is 1837·4 feet, whereas nineteen observations at Corryhabbie give 1957·4 feet for the difference of height, and fourteen observations at Ben Nevis give 1948·4, either of which, notwithstanding the great number of observations, must be more than a hundred feet greater than the truth: the distance is very nearly 80 miles. A similar effect is observed in comparing Corryhabbie with other distant stations, all tending to show an unusual amount of refraction at Corryhabbie. This extraordinary refraction may perhaps be satisfactorily explained by the circumstance that the top of the hill is an extensive flat.

The height of Ben Wyvis may be ascertained from Ben Clibrig, Scournalapich, and Mamsuil, without having recourse to more distant stations. We thus have five determinations, all lying between 3402·6 and 3426·6, which, when combined with respect to their weights, give—

Height of *Ben Wyvis* . . . . 3422·5 Feet above Mean Level of Sea.

The observations at Scarabin give 3410·4, at Ben Macdui 3419·7, while from Ben Nevis forty-two observations give 3501·2, and twenty-seven observations at Corryhabbie give 3479·4.

From Ben Lawers, Ben Lomond, Ben Macdui, Mount Battock, East Lomond, Ben Nevis, and Sayrs Law, we may obtain fourteen determinations of the height of Ben Cleugh, ranging between

2327 and 2366, from which the height required is found to be 2352·5 feet. The greatest discrepancies are shown by the distant points Ben Nevis and Ben Macdui.

Nine determinations of the height of Glashmeal may be obtained from Corryhabbie, Ben Macdui, Ben Lawers, Ben Cleugh, and Mount Battock. The values range between 3564 and 3482, the greatest value resulting in each case from the observations at Glashmeal, which are few in number, and therefore of small weight. The greatest discrepancy amounts to 62 feet, and results from the three observations of Corryhabbie at Glashmeal, a distance of little more than 33 miles. The most probable height of Glashmeal from these determinations is 3501·7 feet.

By means of the known heights of Sayrs Law, Ben Cleugh, Merrick, and Criffel, and the mutual observations of Hart Fell, Dunrich, Wisp, and Cheviot, we may determine the heights of the last four points. Putting Hart Fell $= 2640 + x$, Dunrich $= 2440 + y$, Wisp $= 1950 + z$, Cheviot $= 2670 + w$, we get twenty-eight equations for determining x, y, z, w, which resolve themselves into the following—

$$\begin{aligned} 14{\cdot}6x - 5{\cdot}0y - 3{\cdot}4z - 1{\cdot}0w - 16{\cdot}1 &= 0 \\ -5{\cdot}0x + 9{\cdot}3y - 0{\cdot}8z - 1{\cdot}2w + 63{\cdot}6 &= 0 \\ -3{\cdot}4x - 0{\cdot}8y + 6{\cdot}5z - 1{\cdot}2w - 13{\cdot}5 &= 0 \\ -1{\cdot}0x - 1{\cdot}2y - 1{\cdot}2z + 5{\cdot}4w - 4{\cdot}6 &= 0 \end{aligned}$$

From which we obtain—

$$x = -1{\cdot}6;\ y = -7{\cdot}8;\ z = +0{\cdot}1;\ w = -1{\cdot}2$$

So that we have—

Height of *Hart Fell*	. . . .	2638·4	Feet above Mean Level of Sea.	
„ *Dunrich*	. . . .	2432·2	„	„
„ *Wisp*	. . . .	1950·1	„	„
„ *Cheviot*	. . . .	2668·8	„	„

The largest discrepancies here occur in the observations of Merrick at Hart Fell; Cheviot at Dunrich; and Hart Fell at Wisp. Seventeen observations at Hart Fell give 159·2 feet for the difference of height of Merrick and Hart Fell, which probably exceeds the truth by 30 feet at least. In the other two cases the discrepancies are also about 30 feet each, but depend upon a very small number of observations.

The observations at Hart Fell present great irregularities; the range in many cases being so large as to render it difficult to say what is the most probable mean zenith distance of the object observed.

ALTITUDES

ABOVE THE MEAN LEVEL OF THE SEA

OF

THE STATIONS COMPOSING THE PRINCIPAL TRIANGULATION.

Names.	Feet.	Names.	Feet.	Names.	Feet.
Acklam Wold	765·9	Bunwell Tower Ground	189·7	Epping Poorhouse	371·2
Arbury	735·3	Burleigh Moor	581·1	Fairlight Down	583·7
Arrenig	2816·8	Burnswark	929·2	Faira	711·5
Axedge	1809·6	Butser	882·6	Fashven	1495·7
Back Tor	1764·2	Cader Idris	2959·4	Feaghmaan	880·0
Baconsthorpe Tr. Ground	263·3	Caherbarnagh	2231·0	Fetlar	521·6
Balsham Tower Ground	378·1	Calton Observatory (B.M.)	349·6	Fitty	550·0
Balta	143·5	Carn Galver	826·7	Foula	1372·6
Ballycreen	2167·2	Carrigfadda	1019·0	Four Mile Stone	513·1
Banstead	572·1	Cheviot	2668·8	Forth	768·0
Barrow Hill	447·5	Chingford	299·8	Frittenfield	632·3
Bardon	902·3	Cleisham	2622·1	Gads Hill (Base of Obelisk)	297·2
Baurtregaum	2788·0	Clifton	460·9	Galtymore	3007·0
Beacon Hill	668·7	Cnoc-ghiubhais	975·7	Garreg	808·6
Beacon Hill, Trescow	122·4	Collier Law	1685·7	Garforth Cliff	342·0
Beachy Head	532·4	Coringdon	655·6	Gerth of Scaw	70·2
Ben Cheilt	940·5	Corryhabbie	2568·9	Goonhilly	367·5
Ben Clibrig	3157·6	Cowhythe*	267·7	Gorleston Tower Ground	34·9
Ben Corr	2328·0	Cradle	2660·5	Great Ormes Head	683·1
Ben Heynish	473·6	Criffel	1866·6	Great Stirling	260·5
Ben Cleugh	2352·5	Croghan	761·0	Great Whernside	2310·3
Ben Lomond	3192·2	Cyrn-y-Brain	1843·9	Greenwich Obs^{y} (Vane)	214·0
Ben Hutig	1340·0	Cross Fell	2927·8	Gwaunysgaer	694·8
Ben Lawers	3984·1	Crowborough	803·1	Gringley	275·1
Ben More, South Uist	2034·5	Crowle	63·2	Glashmeal	3501·7
Ben More, Mull	3185·5	Cuilcagh	2180·0	Goat Fell	2874·9
Ben Macdui	4295·6	Cundtham	756·8	Hampton Poorhouse	57·6
Ben Nevis	4406·3	Danbury Spire Ground	366·7	Hanger Hill	201·1
Ben Tartevil	743·4	Dean Hill	512·0	Hanslope Spire Top	555·3
Ben Wyvis	3422·5	Deerness	282·8	Happisburgh Tr. Ground	61·2
Berkhampstead Gazebo.		Ditchling	814·2	Hensbarrow	1027·0
Ground	352·8	Divis	1559·0	Hart Fell	2638·4
Black Comb	1974·3	Docking Tower Top	357·1	High Port Cliff	134·3
Blackdown Mon^{t} Top	861·4	Doolieve	592·0	High Wilhays	2039·6
Ground	790·0	Delamere	572·0	Holme Moss	1925·8
Blackheddon	656·1	Deadman	372·7	Hingham Tower Top	312·3
Blue Hill	465·3	Drung Point	55·7	Hortons Gazebo Top	354·1
Boniface Down	483·9	Dudwick	562·2	Howth	555·0
Boniface, South East	759·2	Dunkery	1706·4	Hungry Hill	2243·0
Boston Tower Top	286·5	Dublin Observatory	272·7	Ingleborough	2373·4
Botton Head	1489·0	Dunnet Head	418·1	Inkpen	972·8
Brandon Down	866·6	Dunnose	771·9	Jura	2566·7
Brandon Hill	3119·3	Dunrich	2432·2	Karnbonellis	808·1
Brandon, Suffolk	163·9	Dunstable	800·2	Karnminnis	799·8
Brassa	737·3	Durham Observatory (Dome)	360·1	Keeper	2270·0
Brimmond	859·4	Easington	683·8	Keysoe Church Spire. Top	358·5
Broadway Tower	1045·2	East Lomond	1471·3	King's Arbour	84·8
Brown Willy	1364·1	Easton Tower Ground	287·9	Kellie Law	595·0
Buckminster Spire. Top	586·6	Ely Minster Ground	51·6	Kippure	2465·0

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* The altitude of this point was found, by levelling, on August 25, 1813, to be 272·75 feet above low water: 5 feet has been deducted to refer it to the mean level of the sea.

Names.	Feet.	Names.	Feet.	Names.	Feet.
Knock	1149·2	Norwich Spire. Ground	19.7	Slieve More in Achil	2196·4
Knockalongy	1770·0	Nodes Beacon	483·0	Slieve Snaght	2011·0
Knocklayd	1677·0	North Ronaldshay Lighthouse. Ground	25·0	Snowdon	3590·1
Knockmealdown	2601·0			South Berule	1598·3
Knocknadober	2258·2	Norwood	258·3	South Lopham Tr. Ground	160·4
Knocknagante	2178·0	Old Lodge	562·9	South Ronaldshay	383·3
Knocknaskagh	1398·0	Ordnance Map Office	78·0	Southwold Tower. Ground	38·4
Knockanaffrin	2470·0	Orford Castle. Ground	39·5	Stoke Tower. Ground	188·1
Lawshall Tower. Ground	335·5	Otley Tower. Ground	179·8	Stoke Hill	731·4
Laxfield Tower. Ground	152·0	Over Hill	449·0	Stronsay	148·4
Layton	114·2	Paddlesworth	626·3	Storr	2341·5
Leith Hill	967·0	Paracombe	1572·4	Swaffham Spire Ground	236·8
Lincoln Minster. Top	475·3	Pendle Hill	1816·4	Swyre Barrow	674·4
Littletown Down	657·6	Peterhead Old Windmill	74·7	Start Lighthouse (Base of Cupola)	108·0
Little Stirling	312·5	Peninnis Windmill	107·5		
Long Mount	1696·5	Pertinney	735·5	Taur	1321·0
Llanelian	1108 4	Pillesdon	909·5	Tarbathy	168·3
Lough Foyle Base, N. End	17·0	Precelly	1757·9	Tawnaghmore	1109·0
Lough Foyle Base, S. End	80·0	Plynlimmon	2481·3	Thaxted Spire. Ground	324·8
Lundy Island	466·0	Reform Monument. Top	243·0	Telegraph Tower. Ground	159·7
Lynn Church Tr. Ground	18·5	Rona, North	355·2	Tharfield	550·0
Lyons Hill	644·0	Ronas	1474·8	Tilton	755·0
Lumsden	737·7	Ru Rea	953·6	Tofts Tower	98·0
Maker Church Tr. Ground	366·3	Rhuddlan Base, East End	21·3	Tara	817·0
Malvern	1396·2	Rhuddlan Base, West End	20·2	Trostan	1802·0
Mamsuil	3861·8	Ryders Hill	1696·0	Trevose Head	242·6
Mendip	979·2	Old Sarum Castle	404·7	Upcot Down	890·0
Merrick	2764·8	Old Sarum Gun	241·3	Vicars Carn	806·0
Merrington Ch. Tr. Top	698·1	Sawel	2228·0	Walpole, St. Peter's Tower. Ground	8·5
Mickfield Ch. Tr. Ground	194·6	Saxavord	937·1		
Milk Hill	966·7	Sayrs Law	1753·1	Walton Tower	74·0
M. C. Base, North End	8·7	St. Agnes Beacon	597·3	Wart Hill Hoy	1559·1
M. C. Base, South End	8·6	St. Agnes Lighthouse. Top	157·6	Water Crag	2187·1
Moelfre Issa	1046·1	St. Anns	207·1	Week Down	690·5
Monach	813·8	St. Martin's Head	150·3	Westbury Down	754·0
Mordington	649·4	St. Paul's Cathedral, Top of Cross	417·9	Whitehorse Hill	859·6
Mormonth	743·8			Wingreen	913·5
Mowcopt	1101·1	St. Peter's Tower	154·6	Wittle Hill	1529·5
Mount Battock	2554·9	Sea Fell	3229·6	Wolf Rock	20·5
Mount Leinster	2602·1	Scarabin	2054·0	Wordeslow	635·0
Mount Sandy	56·0	Scournalapich	3772·8	Wrotham	774·3
Nivé Hill	524·0	Severndroog Castle. Ground	406·4	Wroxall Down	725·0
Naseby Ch. Tower. Top	714·6	Shanklin Down	717·7	Wisp	1950·1
Naughton Ch. Tr. Ground	277·1	Slieve Donard	2788·0	Yell	665·5
Nephin	2638·0	Slieve League	1956·0	York Minster. Ground	53·0

SECTION X.

CONNECTION OF GEODETICAL AND ASTRONOMICAL OBSERVATIONS.

The information we possess as to the figure of the earth has been obtained by methods singularly dissimilar, but agreeing with great precision in the resulting values of the eccentricity of the meridian. The theorem connecting the figure of the earth with the law of gravity upon its surface, enables us, after ascertaining by means of the vibration of a pendulum the actual or rather relative amounts of gravity at a great number of stations, to determine that figure or amount of eccentricity which will best represent the observed variations of gravity. But we cannot by this method, nor by the observed effects of the eccentricity of the earth in producing a disturbance both in the latitude and longitude of the moon, assign anything beyond the ratio of the axes. The actual dimensions of the earth, or the nature of the irregular surface within any given limits, can only be determined by the connection of geodetical and astronomical observations, whereby we obtain respectively the lengths and curvatures of lines traced upon the true surface.

§ I.

1. The comparison of the results of corresponding geodetical and astronomical observations has almost invariably brought to light discrepancies that from their magnitude could not, even in the earlier operations, be ascribed to the errors of observation alone; nor has the greater perfection to which instruments and modes of observing and calculating have been brought, up to the present time, removed these discrepancies, but on the contrary, they are exhibited in more unmistakeable reality. The source of these discrepancies may be a combination of errors of observation with either of the following: first, deflection of the plumb-line by local attraction, caused by inequalities of the visible surface of the earth at or near the station; second, deflection of the plumb-line by unequal distribution of density at small or very great depths below the surface; third, irregularities of the earth's figure or surface. The third of these sources differs from the other two in the quality of extent only; for the local attraction, whether originating at or below the surface, that produces a deflection of the plumb-line also produces an irregularity in the mathematical surface: there may be irregularities of different orders of magnitude.

That stations may be chosen unsuspectingly in such positions that the astronomical results shall be very materially affected by the first of the above sources of error, namely, that of the inequalities of the surface, is proved by the selection of the position of several stations even in the present triangulation; for instance, Burleigh Moor, or the south end of the Lough Foyle Base. It will be proved further on that the high ground to the south of each of these stations produces a disturbance that renders the astronomical observations quite erroneous. As another instance, we may allude to the station Takal Khera, in the vicinity of the Mahadeo Range of Mountains. The face of this range, which lies east and west, is distant about twenty miles north (very nearly) of the astronomical station, and in consequence of this great distance it was considered by Colonel

Lambton that no effect could be produced upon the plumb-line at Takal Khera. It was, however, proved by Colonel Everest, by actual calculation, that the mass was sufficient to produce a deflection of the plumb-line to an amount far exceeding the possible errors of observation.*

When we consider the heterogeneous nature of the crust of the globe, it is in no way remarkable that the surface to which geodetical operations are referred (that of the sea, produced in imagination) should present small irregularities, but, on the contrary, the *à priori* probability that the surface of the earth should be found to agree precisely with any single formula is really very small.

3. The discrepancies between geodetical and astronomical observations are brought out principally in one or other of the following ways;—either in the determination of the figure of the earth from different combinations of measures, whether of the meridian or of parallels; or in the comparison of the observed latitude, longitude, or bearing of the meridian at any point, B, by means of the observed latitude, longitude, and bearing of the meridian at another point, A, transferred geodetically on assumed elements of the earth's figure to B. Suppose A and B to be the trigonometrical stations, and suppose that at A there is a disturbing force acting which draws the vertical through an angle, δ, then it is evident that the apparent zenith of A will be really that of some other place, A′, whose distance from A is $\delta \times$ earth's radius; and similarly if there be a disturbance at B to the amount δ', the apparent zenith of B will be really that of some other place, B′, whose distance from B is $\delta' \times$ earth's radius. Hence we have the discrepancy, that while the geodetical operations deal with the points A and B, the astronomical observations belong to the points A′ and B′. Should δ, δ' be equal and parallel, the displacements AA′, BB′ will be equal and parallel, and no discrepancy will appear.

4. By the comparison of two meridian measures we may determine values of the axes of the spheroid, affected with errors of observation and the probable effects of local attraction. If $\sigma_1\ \sigma_2$ be the measured lengths of two meridian arcs, whose astronomical amplitudes are $\phi_1\ \phi_2$ and their mean latitudes $\lambda_1\ \lambda_2$, we shall have—

$$\sigma_1 = a\,(1 - n)\,\left(\phi_1 - 3\,n \sin \phi_1 \cos 2\,\lambda_1\right)$$

$$\sigma_2 = a\,(1 - n)\,\left(\phi_2 - 3\,n \sin \phi_2 \cos 2\,\lambda_2\right)$$

omitting the square of n, which is the ratio of $a - b : a + b$. If we suppose the arcs small, so that $\sin \phi$ may be replaced by ϕ, we shall have—

$$3\,n = \frac{\dfrac{\sigma_2\,\phi_1}{\sigma_1\,\phi_2} - 1}{\cos 2\,\lambda_1 - \cos 2\,\lambda_2}$$

$$a\,(1 - n) = \frac{\dfrac{\sigma_2}{\phi_2} \cos 2\,\lambda_1 - \dfrac{\sigma_1}{\phi_1} \cos 2\,\lambda_2}{\cos 2\,\lambda_1 - \cos 2\,\lambda_2}$$

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* In connection with this it is worthy of remark, that the correction to the latitude of Takal Khera, deduced by M. Bessel in his determination of the Figure of the Earth from meridian measures, is $+ 3''\cdot537$.—See "Astronomische Nachrichten," No. 438, p. 115.

Now suppose that, in consequence of local attraction, the terminal latitudes of the arc σ_1 are affected by errors x_1' and x_1, then the arc ϕ_1 is affected by the error $x_1' - x_1$; and similarly suppose ϕ_2 to be affected with the error $x_2' - x_2$, then the deduced value of $3\,n$ will be affected with the error—

$$\frac{\dfrac{\sigma_2(\phi_1 + x_1' - x_1)}{\sigma_1(\phi_2 + x_2' - x_2)} - \dfrac{\sigma_2\,\phi_1}{\sigma_1\,\phi_2}}{\cos 2\lambda_1 - \cos 2\lambda_2}$$

$$= \frac{\dfrac{x_1' - x_1}{\phi_1} - \dfrac{x_2' - x_2}{\phi_2}}{\cos 2\lambda_1 - \cos 2\lambda_2}$$

Let the arcs $\phi_1\ \phi_2$ expressed in degrees be $r_1\ r_2$, and suppose $x_1\ x_1'\ x_2\ x_2'$ expressed in seconds, then the error of the *ellipticity* will be—

$$\frac{2}{3}\cdot\frac{1}{3600}\cdot\frac{-\dfrac{x_2' - x_2}{r_2} + \dfrac{x_1' - x_1}{r_1}}{2\sin(\lambda_2 + \lambda_1)\sin(\lambda_2 - \lambda_1)}$$

$$= \frac{-\dfrac{x_2' - x_2}{r_2} + \dfrac{x_1' - x_1}{r_1}}{10800\sin(\lambda_2 + \lambda_1)\sin(\lambda_2 - \lambda_1)}$$

If we suppose each of the independent errors x to have a probable amount ϵ, then the probable error of the deduced value of the ellipticity is—

$$\pm\left(\frac{\epsilon}{10800}\right)\cdot\frac{\sqrt{\dfrac{2}{r_1^2} + \dfrac{2}{r_2^2}}}{\sin(\lambda_2 + \lambda_1)\sin(\lambda_2 - \lambda_1)}$$

Suppose, now, that we compare two arcs of a degree each, situated in mean latitudes, and close to each other, say about five degrees apart, the probable error of the resulting value of the compression will be approximately—

$$\pm\frac{\epsilon}{10800}\ \frac{\sqrt{4}}{\sin 5^\circ} = \pm\frac{\epsilon}{500}$$

so that if $\epsilon = 2''$ the probable error of the deduced value of the compression will be actually greater than the compression itself. Hence, in comparing small arcs not widely separated in latitude, it will be nothing remarkable if negative values of the ellipticity should be obtained, provided the probable disturbance due to local attraction should amount at each station to $2''$ or even a smaller quantity.

Should the two arcs be continuous, that is, suppose the northern terminus of ϕ_1 to be the southern terminus of ϕ_2, then $x_1' = x_2$, and the error of the deduced value of the ellipticity—

$$-\frac{\dfrac{x_2'}{r_2} - x_2\left(\dfrac{1}{r_2} + \dfrac{1}{r_1}\right) + \dfrac{x_1}{r_1}}{10800\sin(\lambda_2 + \lambda_1)\sin(\lambda_2 - \lambda_1)}$$

and the probable error—

$$\pm\frac{\epsilon\sqrt{2\left(\dfrac{1}{r_1^2} + \dfrac{1}{r_2^2} + \dfrac{1}{r_1 r_2}\right)}}{10800\sin(\lambda_2 + \lambda_1)\sin(\lambda_2 - \lambda_1)}$$

which is approximately equal to—

$$\frac{\pm \varepsilon \sqrt{\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_1 r_2}}}{15 \pi (r_1 + r_2) \sqrt{2}} \cdot \operatorname{cosec} (\lambda_1 + \lambda_2)$$

by putting $\lambda_2 - \lambda_1 =$ half the combined length of the two arcs.

Let us estimate from this the probable error of the ellipticity that would result from a comparison of the arcs Dunnose to Arbury and Arbury to Clifton. Their lengths expressed in degrees are $r_1 = 1 \cdot 6$ $r_2 = 1 \cdot 2$, and $\lambda_1 = 51° 25'$ $\lambda_2 = 52° 50'$; so that the probable error of the deduced value of the ellipticity is—

$$\pm \frac{\varepsilon \sqrt{1 \cdot 606}}{42 \pi \sqrt{2}} \sec (14° 15')$$
$$= \pm \frac{\varepsilon}{142 \cdot 7}$$

Therefore if we suppose the probable disturbance at each station to be even less than a second, the probable error of the resulting ellipticity will be twice as great as the ellipticity itself.

5. The anomalous results deduced from the English arc extending from Dunnose to Clifton, as given in the second volume of the "Account of the Trigonometrical Survey," have given rise to much discussion. It was found that in proceeding from the southern to the northern extremity, the length of a degree diminished, which seemed to indicate a negative value of the ellipticity. General Mudge remarks upon this discrepancy between the expected and the obtained results:—"I think it is probable that an error in the whole distance, of 197 "miles nearly, does not subsist to an amount of more than 100 feet, corresponding to one "second in the amplitude of the whole arc, and I also think it probable it cannot amount to "half that quantity. The supposition of the zenith distance of the stars being generally "erroneous at any one station cannot be admitted, unless it should be imagined that the plane "of the sector's limb was not got into that of the meridian. Such an idea, however, can "scarcely be entertained after a careful examination of the several observations, and a due "attention to the means by which the instrument was made to assume its right position. "Perhaps, also, I should not fail to observe in this place, that although the instrument was "always brought into the plane of each meridian by means of the telescope attached to the "side of the great tube and the azimuth circle, yet, having two good chronometers in my "possession, I repeatedly verified the truth of the sector's position by observing the transits "of two stars north and south of the zenith at the greatest distances my arc would admit of. "But, to return: if there be an error in the amplitude of the total arc from a deflection of "the plumb-line at either of the stations, it is not probable that any such deflection existed "at Dunnose, as the deviation of it towards the north, from a deficiency of matter towards "the channel, would tend to diminish the inequality between the lengths of the two degrees; "this will be evident on consideration. I am therefore disposed to believe that the plumb-line "was drawn towards the south from the action of matter, both at the northern extremity of "the arc and at Arbury Hill, but more particularly at the first-mentioned station. If this

" were partly the case, and both Dunnose and Arbury Hill were free from any such prevailing " cause, the total arc must be too great, if taken at 2° 50′ 23″·38, by about 8″, nearly answering " to 2″ on each degree. A deviation of 8″ from the true vertical is a large quantity, nor can " the cause of it be assigned, unless it be also supposed that the matter producing that " deflection extends in a southern direction beyond Arbury Hill. If the error, though not " probable, as above observed, be supposed to exist at Dunnose, it must amount to more than " 10″, and that, too, from the effects of attraction in a southern direction, where the deficiency " of matter would lead us to believe the reverse would happen . . . . . . In short, the " general tenor of the observations seems to prove that the plumb-line of the sector has been " drawn towards the south at all the stations, and that by attractive forces which increase as " we proceed northwards."

In the Philosophical Transactions for 1812 there is a paper entitled " Observations on the " Measurement of Three Degrees of the Meridian, conducted in England by Lieutenant-Colonel " William Mudge," by Don *Joseph Rodriguez*, in which the author endeavours to prove that the discrepancies in question arose from errors in the astronomical observations at Arbury. Assuming certain elements of the earth's figure, which are far from the true or most probable values, he finds that the arc Dunnose to Arbury should, from its geodetical magnitude, have an amplitude of 1° 36′ 23″·34, and the arc Clifton to Arbury an amplitude of 1° 13′ 58″·63, and comparing these with the observed values the resulting errors of the observed amplitudes are—on the southern portion − 3″·39; on the northern portion + 4″·77: on the total arc + 1″·38. From these quantities the inference of the author is that the observations at Arbury Hill are erroneous to the extent of 5″. But the only true inference is this: If the earth be a perfect spheroid, without appreciable irregularities, and if the elements used be the true values, then the observed amplitudes are from some cause erroneous. But even admitting the hypotheses, which are false, the conclusion that error is at Arbury Hill is not legitimate. It should also be remarked, that had the station Blenheim been allowed an influence in the deduction, it would have been necessary to ascribe to that station an error corresponding in amount to that ascribed to Arbury.

Of this paper, as it has been considered by the highest authorities to throw no light whatever on the subject, and to be deficient in demonstration,* no further notice need be taken. It would appear, however, that the discrepancy was generally attributed to a deflection at Arbury Hill† to the amount of 5″, as is evident from Captain Kater's remark at page 188 of the account of his operations to determine the difference of longitude of Greenwich and Calais in

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* DELAMBRE, Conn. des Tems, 1816, Additions: AIRY, Encyclopædia Metrop.; " Figure of the Earth," page 237.

† It is worthy of remark, that in a letter to Colonel Mudge (in 1816) the illustrious Schumacher wrote :—" You, " who know the ground, are best able to judge where local attractions are most probable; but on my part I must confess " that I should suspect them rather at Dunnose than at Arbury Hill. And further, it seems to me that in any " hypothesis it would be very difficult to establish a perfectly regular figure for the earth."—*Memoirs of Major-General Colby*, Royal Engineers, by Colonel Portlock, Royal Engineers.—*Professional Papers of the Corps of Royal Engineers*, Vol. III. New Series, page xxvii.

1821 (Philosophical Transactions, 1828); and similarly at page 236 of the treatise on the Figure of the Earth by the Astronomer Royal, in the Encyclopædia Metropolitana.

6. Let us now examine the data of this arc, in order to ascertain as far as possible what are the deflections most probably due to each of the stations.

The data, as we find them at page 107 of the second volume of the "Account of the Trigonometrical Survey," are—

Partial Arcs.	Length in Feet.	Astronomical Amplitude.
Dunnose to Greenwich . .	313696	0° 51′ 31″·39
" Blenheim . . .	446498	1 13 19·69
" Arbury . . .	586320	1 36 19·98
" Clifton . . .	1036337	2 50 23·38

Latitude of Dunnose 50° 37′ 7″

If $1-n : 1+n$ be the ratio of the axes of the elliptic meridian, the length of the arc between the latitudes $\lambda + \frac{1}{2}\phi$ and $\lambda - \frac{1}{2}\phi$ is—

$$\sigma = a(1-n)(1-n^2)\left(1+\frac{9}{4}n^2+\cdots\right)\left(\phi - 3n\sin\phi\cos 2\lambda + \cdots\right)$$

and the length, Δ, of a degree in the mean latitude, λ, is therefore—

$$\Delta = a(1-n)(1-n^2)\left(1+\frac{9}{4}n^2+\cdots\right)\left(\frac{\pi}{180} - 3n\sin\frac{\pi}{180}\cos 2\lambda + \cdots\right)$$

Put λ_0 for the latitude of Dunnose, and let the corresponding value of Δ be Δ_c, then, neglecting quantities of the order $a\,n\,\phi^3$, we have—

$$\sigma = \Delta_0 \cdot \frac{180\cdot\phi}{\pi} \cdot \frac{1-3n\cos 2\lambda}{1-3n\cos 2\lambda_0}$$

But supposing the arc ϕ to be measured from Dunnose, $\lambda = \lambda_0 + \frac{1}{2}\phi$ the latitude of the middle point, or rather the mean of the extreme latitudes; hence, supposing ϕ to be expressed in seconds—

$$\begin{aligned} 3600\cdot\sigma &= \Delta_0\phi\left(1+3n\left[\cos 2\lambda_0 - \cos(2\lambda_0+\phi)\right]\right) \\ &= \Delta_0\phi\left(1+6n\sin(2\lambda_0+\tfrac{1}{2}\phi)\sin\tfrac{1}{2}\phi\right) \\ &= \Delta_0\phi\left(1+\phi\sin 3''\sin(2\lambda_0+\tfrac{1}{2}\phi)n\right) \end{aligned}$$

Let 364800 feet be an approximate value of Δ_0, and let the same quantity, increased by the multiplier $1+u$, be the true value, then by substituting this value—

$$\frac{3}{304}\sigma = \phi + \phi^2\sin 3''\sin(2\lambda_0+\tfrac{1}{2}\phi)\cdot n + \phi u$$

In this equation σ is the length in feet of an arc measured from Dunnose northward, and corresponding to the amplitude ϕ. Let us suppose that at each of the five stations named in the table there is an uncertain amount of local attraction, and that in consequence the latitudes have to be increased by small quantities x x_1 x_2 x_3 x_4, x corresponding to Dunnose. Therefore to the four amplitudes given in the above table we have to append the unknown quantities. $x_1 - x$, $x_2 - x$, $x_3 - x$, and $x_4 - x$. Hence, supposing σ_i to be the length and ϕ_i to be the tabular amplitude of the i^{th} partial arc—

$$\frac{3}{304}\sigma_i = \phi_i + x_i - x + \phi_i^2 \sin 3'' \sin\left(2\lambda_0 + \tfrac{1}{2}\phi_i\right) n + \phi_i u$$

$$x_i = \frac{3\,\sigma_i}{304} - \phi_i - \phi_i^2 \sin 3'' \sin\left(2\lambda_0 + \tfrac{1}{2}\phi_i\right) n - \phi_i u + x$$

Substituting in this equation the numerical quantities given in the table of data, we have the following set of four equations, in which, in order to diminish the number of figures, the values of n and u have been multiplied by 1000—

$$\left.\begin{aligned} x_1 &= 4{\cdot}29 - 0{\cdot}1361\,n - 3{\cdot}091\,u + x \\ x_2 &= 6{\cdot}54 - 0{\cdot}2755\,n - 4{\cdot}400\,u + x \\ x_3 &= 6{\cdot}07 - 0{\cdot}4752\,n - 5{\cdot}780\,u + x \\ x_4 &= 3{\cdot}63 - 1{\cdot}4832\,n - 10{\cdot}223\,u + x \end{aligned}\right\} \qquad \text{(A)}$$

These four equations contain all the necessary relations between the radius of curvature of the arc, its rate of variation, and the five effects of local attraction. If we eliminate n and u we shall have two equations of condition between the five quantities $x \cdot \cdot \cdot \cdot$, so that if we assign any arbitrary values to any three of these quantities the other two can be determined. But the most probable values correspond to those values of n and u which make the quantity $x^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2$ a minimum; this is, in fact, determining that particular elliptic curve which will best represent the observations. Making the differential coefficients of the quantity just written with respect to x n and u respectively zero, we have—

$$\begin{aligned} 0 &= x + x_1 + x_2 + x_3 + x_4 \\ 0 &= 0{\cdot}1361\,x_1 + 0{\cdot}2755\,x_2 + 0{\cdot}4752\,x_3 + 1{\cdot}4832\,x_4 \\ 0 &= 3{\cdot}091\,x_1 + 4{\cdot}400\,x_2 + 5{\cdot}780\,x_3 + 10{\cdot}223\,x_4 \end{aligned}$$

and performing the multiplications, we get—

$$\begin{aligned} 0 &= +\ 20{\cdot}53 - 2{\cdot}370\,n - 23{\cdot}494\,u + 5{\cdot}000\,x \\ 0 &= -\ 114{\cdot}23 + 19{\cdot}542\,n + 166{\cdot}832\,u - 23{\cdot}494\,x \\ 0 &= -\ 10{\cdot}65 + 2{\cdot}520\,n + 19{\cdot}542\,u - 2{\cdot}370\,x \end{aligned}$$

Solving these equations, and restoring to n and u their proper values, namely, a thousandth part—

$$\begin{aligned} x &= +\ 0{\cdot}064 \\ u &= +\ 0{\cdot}002088 \\ n &= -\ 0{\cdot}011909 \end{aligned}$$

and the apparent corrections necessary to the observed latitudes—

	″
Dunnose	+ 0·06
Greenwich	− 0·48
Blenheim	+ 0·69
Arbury	− 0·28
Clifton	+ 0·01

These are the smallest corrections that can be applied to the observed latitudes in order that they may agree with the corresponding geodetical arcs in representing any elliptical curve. The ratio of the axes of this curve are—

$$\frac{a}{b} = \frac{1 + n}{1 - n} = \frac{41\cdot48}{42\cdot48}$$

and the length of a degree—

$$368125 + 13152 \cos 2\lambda$$

This is the result most consonant with the observations and measures in this arc, and must be taken as the true length of a degree anywhere between Dunnose and Clifton, provided it be assumed as most probable that the latitudes of any points between these limits determined astronomically would fall in with this curve we have determined; that is, supposing it most probable that there is a large irregularity of the earth's surface between Clifton and Dunnose. But the small number of five stations is quite insufficient to give probability to this hypothesis; and as in general we must assign a greater probability to small irregularities than to great ones, let us ascertain next what are the smallest effects of local attraction that must be attributed as most probable to the different stations, in order that the expression for a degree along the arc may not indicate a large irregularity of the earth's surface. And for this purpose assume an ellipticity of $\frac{1}{300}$, which is very near the mean value for the whole earth; on this supposition n becomes $\frac{1}{600}$, and the equations (A)—

$$\begin{aligned} x_1 &= 4\cdot06 - 3\cdot091\,u + x \\ x_2 &= 6\cdot08 - 4\cdot400\,u + x \\ x_3 &= 5\cdot28 - 5\cdot780\,u + x \\ x_4 &= 1\cdot16 - 10\cdot223\,u + x \end{aligned}$$

$$\therefore \begin{Bmatrix} 0 = + 16\cdot58 + 5\cdot000\,x - 23\cdot494\,u \\ 0 = - 81\cdot68 - 23\cdot494\,x + 166\cdot832\,u \end{Bmatrix}$$

$$x = - 3\cdot00 \qquad u = + \cdot0000668$$

whence the apparent corrections for the effects of local attraction are—

	″
Dunnose	− 3·00
Greenwich	+ 0·85
Blenheim	+ 2·78
Arbury	+ 1·89
Clifton	− 2·52

and the formula for the length of a degree—

$$364469 - 1822 \cos 2\lambda$$

Here the length of the degree exhibits no anomaly, and the probable disturbances at the different stations are not singularly large,—the largest falling to Dunnose in a southern direction, whilst at Arbury the disturbance does not amount to two seconds.

7. We conclude, therefore, that the data of this arc as given in the second volume of the "Account of the Trigonometrical Survey" do not indicate as probable any general irregularity of the earth's surface, nor do they require the supposition of a very large disturbance at any of the stations, as supposed by General Mudge.

If we correct the observed amplitudes accordingly, and thence deduce the lengths of a degree at the different mean latitudes, they will be as follows:—

Partial Arcs.	Lengths in Feet.	Amplitude corrected.	Mean Latitude.	Length of a Degree.	Feet in a Second.
Dunnose—Greenwich . . .	313696	3095″·24	51° 2′ 52″	364852	101·348
Greenwich—Blenheim . .	132802	1310·23	51 39 32	364888	101·358
Blenheim—Arbury . . .	139822	1379·40	52 1 57	364912	101·364
Arbury—Clifton	450017	4438·99	52 50 28	364962	101·378

In obtaining these results we have assumed a value of the ellipticity; they would not, however, have been materially different had we assumed any other value near the truth.

This arc is particularly illsuited to be used in determining a value of the ellipticity, inasmuch as its situation in latitude renders the coefficient of the ellipticity small.

8. Let us next suppose that in the whole arc there is an error of 170 feet, as now appears to be the case. The equations will be altered to—

$$\begin{aligned} x_1 &= 4\cdot06 - 3\cdot091\,u + x \\ x_2 &= 6\cdot08 - 4\cdot400\,u + x \\ x_3 &= 5\cdot28 - 5\cdot780\,u + x \\ x_4 &= 2\cdot84 - 10\cdot223\,u + x \end{aligned}$$

$$\therefore \left\{ \begin{aligned} 0 &= +\,18\cdot26 + 5\cdot000\,x - 23\cdot494\,u \\ 0 &= -\,98\cdot85 - 23\cdot494\,x + 166\cdot832\,u \end{aligned} \right\}$$

$$x = -\,2\cdot565 \qquad u = +\,0\cdot0002313$$

and the apparent corrections for the effects of local attraction become—

Dunnose	− 2″·56
Greenwich	+ 0·78
Blenheim	+ 2·50
Arbury	+ 1·38
Clifton	− 2·09

These quantities are all smaller than before, and the apparent deflection at Dunnose is now reduced by nearly half a second. The length of a degree is also increased to—

$$364529 - 1823 \cos 2\lambda$$

or about 60 feet greater than before.

It appears, then, that the error in the geodetical operations has had a sensible influence in magnifying the discrepancies.

9. As Dunnose is the southern terminus of the arc, it was considered necessary that additional observations should be made at other stations in its neighbourhood, in order to throw light upon the supposed probable existence of disturbance at that point. Three stations were therefore selected by Captain Yolland for this purpose, and the observations for ascertaining their latitudes were made with Airy's Zenith Sector.

The stations Week Down and Boniface Down are connected directly with Dunnose,—see triangles 46, 47, page 472, and the distance Dunnose to Highport Cliff is determined by the calculation of the following triangles:—

Names.	Angles.	Log. Sines.	Log. Distances.
	° ′ ″		
Boniface Down, South East .	175 7 40·96	8·9290573	3·4963342
High Port Cliff	0 48 36·48	8·1504198	2·7176967
Boniface Down	4 3 42·56	8·8502344	3·4175113
	180 0 0·00		
Boniface Down	145 8 13·03	9·7571050	3·9173428
Dunnose	12 31 20·05	9·3360964	3·4963342
High Port Cliff	22 20 26·92	9·5799151	3·7401529
	180 0 0·00		

At Dunnose the observed bearing (page 106) of Boniface Down is 3° 40′ 12″·69, which becomes, by applying the correction + 0″·92 (page 403), 3° 40′ 13″·61. The reading of the referring-object at Dunnose is 339° 12′ 19″·99; but on referring to the results of azimuthal observations, page 177, we find that the true azimuth of the referring-object is 339° 12′ 12″·59. All the readings, therefore, at Dunnose, as given at page 106, have to be corrected by the application of the quantity − 7″·40 to reduce them to the exact meridian. So that we have,

	°	′	″	
At Dunnose, azimuth of Boniface Down	3	40	6·21	S.W.
Angle: Week Down—Dunnose—Boniface Down	45	10	21·33	
∴ At Dunnose, azimuth of Week Down	48	50	27·54	S.W.
Angle: Boniface Down—Dunnose—Highport Cliff . . .	12	31	20·05	
∴ At Dunnose, azimuth of Highport Cliff	8	51	13·84	S.E.

The distances being very small we may neglect the curvature of the surface in determining the meridional distances of Dunnose—Boniface Down and Dunnose—Highport Cliff; we have therefore only to multiply the distance by the cosine of the azimuth in order to get the distance of the parallels. To convert this into seconds of arc, it is almost immaterial what elements are used; the logarithm actually used is derived from the elements of the figure of the earth given by the Astronomer Royal in his treatise in the Encyclopædia Metropolitana.

	Dunnose—Boniface Down.	Dunnose—Week Down.	Dunnose—High Port Cliff.
	° ′ ″	° ′ ″	° ′ ″
Azimuth . . .	3 40 6	48 50 28	8 51 14
Log. cosine . .	9·999109	9·818324	9·994794
Log. distance . .	3·740153	4·053569	3·917343
Tabular log. . .	7·994095	7·994095	7·994095
Log. diff. lat. . .	1·733357	1·865988	1·906232
Diff. lat. .	54″·12	73″·45*	80″·58

To obtain the true difference of latitude of Dunnose and Week Down, a small correction must be added to the quantity 73″·45 just obtained, dependent upon the distance, D, of Week Down from the meridian of Dunnose; this correction is—

$$\frac{1}{2}\left(\frac{D}{R}\right)^2 \frac{\tan \lambda}{\sin 1''}$$

R being the radius of the earth, and λ the latitude. The amount of the correction is + 0″·02.*

The station Highport Cliff is not identical with the point (called Port Valley) at which the astronomical observations were made: the latter is 263 feet from the former, and in a line between it and Boniface Down, S.E. station. The azimuth of this line is 30° 22′, so that the meridian distance is 227 feet, and the difference of latitude 2″·24, which quantity has to be subtracted from the latitude, 50° 35′ 45″·52, of Port Valley, to reduce it to the station at Highport Cliff.

Names.	Latitudes.	Ast. Amp.	Geo. Amp.
	° ′ ′	″	″
Dunnose . . .	50 37 7·15	56·60	54·12
Boniface Down .	50 36 10·55	75·73	73·47
Week Down . .	50 35 51·42	83·87	80·58
High Port Cliff .	50 35 43·28		

Now let x_1 x_2 x_3 x_4 be the disturbances at these four stations due to local attractions, then,

$$56{\cdot}60 + x_1 - x_2 = 54{\cdot}12$$
$$75{\cdot}73 + x_1 - x_3 = 73{\cdot}47$$
$$83{\cdot}87 + x_1 - x_4 = 80{\cdot}58$$
$$x_2 = x_1 + 2{\cdot}48$$
$$x_3 = x_1 + 2{\cdot}26$$
$$x_4 = x_1 + 3{\cdot}29$$
$$x_1^2 + x_2^2 + x_{3_2} + x_4^2 = x_1^2 + (x_1 + 2{\cdot}48)^2 + (x_1 + 2{\cdot}26)^2 + (x_1 + 3{\cdot}29)^2$$

and the value of x which renders this function a minimum is determined by the equation—

$$4x_1 + 8{\cdot}03 = 0$$
$$\therefore x_1 = -2{\cdot}01$$

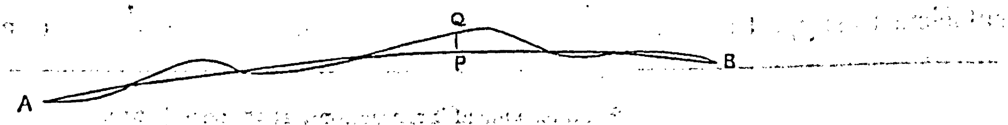
Therefore the disturbances at the several stations, as far as can be determined from these observations alone, are most probably as follows:—

	″
Dunnose	− 2·01
Boniface	+ 0·47
Week Down	+ 0·25
Highport Cliff . . .	+ 1·28

10. The agreement of this result as to the disturbance at Dunnose with that deduced (− 2″·56) from the original data of the arc Dunnose to Clifton, corrected for the error in the geodetical measure, is very satisfactory, and seems to prove that there is a southern deflection at Dunnose to the amount of *about* 2″. It is also worthy of remark in this place, that the correction assigned to the latitude of Dunnose by M. Bessel in his determination of the figure of the earth (Astronomische Nachrichten, No. 438, page 115) is − 1″·816.

In deducing these results for the four stations in the Isle of Wight, the latitudes used are independent of the assumed declinations of the stars observed; they would have been almost exactly the same had we made use of the latitudes deduced from assumed declinations.

11. A fact of the highest importance is also here brought out. We have seen that at Dunnose there is probably a deflection of 2″·01 to the south, and at Highport Cliff a deflection of 1″·28 to the north, and that between the two stations the effect of local attraction alters to the amount of 3″·29, whilst the distance of the parallels of these stations is not much more than a mile and a half. Therefore, if in the original operations the station at Highport Cliff had been used as the southern terminus of the arc instead of Dunnose, the results deduced from that measure would have been very different. And in general it is unsafe to form any conclusion as to the curvature of any entire arc merely from the observed latitudes of its extremities, for it cannot be said with any degree of certainty that there are not *many intervening irregularities*. To illustrate this, suppose A P B to be an elliptic curve passing through the terminal points A B of an arc, at any point P in this curve erect a perpendicular P Q proportional to the deflection (considered positive when the attractive force is to the north, and negative when the attractive force is to the south) at that point. The curve traced by the point Q will intersect the elliptic curve A P B at all points at which there is no local disturbance, and will lie alternately above and below A P Q. By the observed latitudes at A and B we have the directions of the tangents of this irregular curve at those points, and this is all. And if from these data we form a conclusion as to the general curvature of the actual surface between A and B,

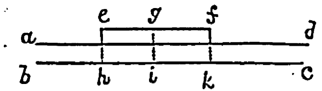


the probability to be attached to such a conclusion will correspond to the probability there is in the assumption that the point Q does not trace any irregularities, but a continuous curve of slowly but evenly varying curvature. The fact that the country between A and B is free from all external causes of disturbance, could not legitimately be brought forward as an argument against the existence of irregularities. Such a conclusion would be unsupported by observation.

By observing the latitude at a great many points between A and B we shall ascertain the directions of the tangents to the curve A Q B at the corresponding points, and thus we shall be able to determine some regular curve A P B which shall most nearly agree on the whole with the curve traced by Q.

12. The irregularities we have been considering, though doubtless in many cases they may be accounted for satisfactorily by the nature of the country around, must still be considered as probably intimately connected with the internal constitution of the earth, of which we know little or nothing with certainty. An important suggestion has been brought forward by the Astronomer Royal* relative to the probable effects of extensive mountain ranges in producing deflection of the plumb-line. It is as follows:—

The general regularity of the physical surface of the earth, the smallness of its elevations and depressions, and the exactness with which it can be represented by a spheroid of revolution, have induced most physicists to suppose either that the interior of the earth is now fluid or that it was fluid when the mountains took the present forms.



Of the probable thickness of the hard crust it would be unsafe to express an opinion, but, for the sake of fixing the ideas, suppose the earth a fluid mass covered with a solid crust 10 miles thick. Now suppose a mass of mountain or table land 100 miles broad in its smaller horizontal dimension and 2 miles high throughout to be placed on the surface; will this mass be supported, or break through the crust and sink partly into the fluid? In the adjoining figure let *a b c d* be part of the earth's crust, *e g f* the table land, and suppose the rocks to be separated by vertical fissures, as indicated by the dotted lines, and conceive these fissures to be opened, as they would be by a sinking of the middle of the mass, the two halves turning upon their lower points of connection with the rest of the crust. Let W be the weight of a square mile of the rock and C the cohesion, or force necessary to separate a square mile. Then the cohesion at $h = 10$ C and at *i* it is 12 C, also the superincumbent weight of each half is $2 \cdot 50$ W; therefore considering one half only, as *e g*, and taking the moments—

$$10^2\,C + 12^2\,C = 2{\cdot}50^2\,W$$

$$\therefore C = \frac{5000}{244}\,W$$

or about twenty times the weight of a cubic mile of rock, so that the cohesion would have to be sufficient to support a hanging column of 20 miles of rock. Had we supposed the thickness of the crust 100 miles we should have had, *cæteris paribus*, $C = \frac{1}{4} W$ nearly. Even in this case the force of cohesion necessary is greater than can be supposed to exist, and therefore the table land will not be supported by the crust.

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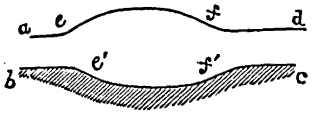
* Philosophical Transactions, 1855, page [101].



It appears, then, probable that such mountain masses must be accompanied with corresponding solid depressions, as $e' f'$, or indentations into the fluid, in order to preserve equilibrium. Now if we suppose a station at a, near the table land $e f$, there will be a disturbance northward owing to the attraction of the superincumbent matter $e f$, but owing to the substitution of the lighter matter $e' f'$ for the denser fluid matter there will be a negative attraction northwards. The diminution of attractive matter below will be sensibly equal to the increase of attractive matter above, and if the point a be not very near to $e f$ there may be no disturbance at all; but if a be close to the table land, and especially if the crust be anything like 100 miles thick, there will be a very sensible disturbance, but less than that which would be due to the attraction of the entire table land.

" The general conclusion, then, is this: In all cases the real disturbance will be less than " that found by computing the effects of the mountains on the law of gravitation. Near to the " elevated country, the part which is to be subtracted from the computed effect is a small " proportion of the whole. At a distance from the elevated country the part which is to be " subtracted is so nearly equal to the whole that the remainder may be neglected as insignificant, " even in cases where the attraction of the elevated country itself would be considerable. But, " in our ignorance of the depth at which the downward immersion of the projecting crust into " the lava takes place, we cannot give greater precision to this statement.

" In all the latter inferences it is supposed that the crust is floating in a state of equi-" librium; but, in our entire ignorance of the *modus operandi* of the forces which have raised " submarine strata to the tops of high mountains, we cannot insist on this as absolutely true. " We know (from the reasoning above) that it will be so to the limits of *breakage* of the table " lands, but within those limits there may be some range of the conditions either way. It is " quite as possible that the immersion of the lower projection in the lava may be too great, as " that the elevation may be too great, and in the former of these cases the attraction on the " distant stations would be negative.

" Again reverting to the condition of *breakage* of the table lands as dominating through " the whole of this reasoning, it will be seen that it does not apply in regard to such com-" putations as that of the attraction of Schehallien and the like. It applies only to the " computation of the attractions of high tracts of very great horizontal extent, such as those to " the north of India."

13. This theory, which has great *à priori* probability, receives strong corroboration from the comparison of the observed effect of the Himalaya Mountains in disturbing the plumb-line at the northern terminus of the Indian arc (Kaliana, in latitude 29° 30′ 48″) with the disturbance that would result from the apparent mass and extent of the mountains. The observed effect of the attraction of the mountains is 5″·236, or more probably 3″·343, as stated by

Colonel Everest at page clxxviii of his account of the Measurement of the Indian Arc; the *exact* amount, being dependent on the assumed elements of the figure of the earth and other circumstances, cannot be positively assigned. But the attraction of the apparent or superincumbent mass of the Himalayas at that point is sufficient to produce a deflection of 27″·853, as calculated by Archdeacon Pratt, of Calcutta.[*]

But the theory does not seem to be borne out in the case of the Alps, where disturbances have been observed of very large amount.

14. The greatest discrepancy ever observed is that brought out by the arc of Beccaria, measured in Piedmont. The terminal stations, Andrate and Mondovi, are situated at the foot of lofty mountain ranges, the intervening country being flat. This measure gave for the length of a degree 57,468 toises,—a quantity much larger than it should be, and corresponding to an error of more than 41″ in the amplitude. In consequence of this anomaly, Laplace and other geometers resolved to make no use of the arc in the determination of the dimensions of the earth, as it was very doubtful whether the observations of Beccaria were not erroneous. In order to throw light on the matter, and to settle the question whether the discrepancy was due to the defective operations of Beccaria, or had a real existence resulting from the attraction of the mountains or the uneven distribution of density in the terrestrial strata, the latitudes of Beccaria's stations at Mondovi and Andrate were re-observed by MM. Plana and Carlini during their operations in that country in 1823. The observations[†] were made with an 18-inch circle by Troughton, and appear satisfactory; the latitudes of the terminal stations were found to be—

	°	′	″
Andrate	45	31	12·36
Mondovi	44	23	45·38
Amplitude	1	7	26·98

This quantity is less than the amplitude assigned by Beccaria by 13″. The geodetical connection of these stations, which had been partly revised by the French Geographical Engineers in their operations in 1809 was now completed, and the length of the arc found to be 126394·6 mètres, 38 mètres greater than obtained by Beccaria. The geodetical latitudes—that is, the latitudes calculated on an assumed figure of the earth, and based on the observed latitude of Paris, are—

	°	′	″
Andrate	45	31	40·45
Mondovi	44	23	25·63
Amplitude	1	8	14·82

which exceeds the astronomical amplitude by 47″·84; showing a deflection 28″·09 to the north at Andrate, and 19″·75 to the south at Mondovi.

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[*] "On the Attraction of the Himalaya Mountains, and of the elevated Regions beyond them, upon the Plumb-line in India." By the Venerable JOHN HENRY PRATT, M.A., Archdeacon of Calcutta.—Phil. Trans. 1855.

[†] "Opérations Géodésiques et Astronomiques pour la Mesure d'un Arc du Parallèle Moyen." Tom. II.

If external causes could suffice to explain these disturbances, they would naturally be sought in the existence of the chain of the Maritime Alps to the south, and of the chain of the Graian Alps to the north. But it is possible, also, that this singular phenomenon may be partly produced by irregularities in the densities of the crust of the earth. There is a difficulty in ascribing the whole amount of the discrepancy to the action of the superincumbent matter of the mountains, inasmuch as the geodetical and astronomical connections of the Observatories of Parma and Milan show a discrepancy of $20''\cdot4$,* although these places are situated in a plain, and so remote from mountains that such a discrepancy could not be explained by their action.

§ II.

On the Deflections produced by given Masses.

15. Although the law of gravity, or of the attraction mutually exercised by all material particles, is in itself very simple, yet, when it is traced to its consequences, whether in the form of motion or mere attraction, difficulties are encountered which, in the present state of analysis, cannot be rigorously overcome. It is only in the most simple cases that exact results can be obtained; in all others, recourse must be had to approximation. Fortunately, in all the important cases that arise in practice, in which it is necessary to trace the effects of attraction, the conditions of the problem are such that the method of approximation can be applied.

When a particle is attracted by a body which subtends, at the particle, a very small angle, then, supposing there is no great disparity between the different dimensions of the attracting body, the attraction will be sensibly the same as if the mass of the body were condensed in its centre of gravity. We may generally get a rough approximation to the true attraction of a body by this rule, but the error will increase rapidly with the increase of the angular magnitude of the body at the attracted particle.

In order to calculate exactly the attraction of a mass—such as a mountain—at any given point, it is necessary to have the equation of the surface, together with the value of the density at any point $x\ y\ z$ in the mass. It is therefore at once evident, that to calculate the attraction exercised by masses of uneven ground upon the plumb-line is theoretically impracticable; but as it is in no case necessary to know the precise amount of attraction, we may have recourse to methods of approximation, which may be carried to any degree of precision that may be necessary.

16. The deflection of the plumb-line caused by a local attraction whose amount is A, is measured by the ratio of A to the attraction of the earth, or the force of gravity.† If λ be

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* Opér. Géod. et Ast. Tom. II., page 348.

† The plumb-line is acted on by gravity G in a vertical direction, and by the disturbing force A acting in a horizontal direction: the resultant of these forces $= \sqrt{A^2 + G^2}$ acts in a direction which makes the angle $\tan^{-1}\frac{A}{G}$ with the direction of gravity. When A is very small in comparison with G, this angle is identical with its tangent.

the latitude of the place of observation, μ the mass of the earth, whose major and minor semi-axes are a and b, ϵ the ellipticity of the surface, and m the ratio of the centrifugal force to gravity at the equator, the attraction of the earth is (see Airy's Mathematical Tracts, pp. 167, 173)—

$$\frac{\mu}{b^2}\left(1 - \epsilon - \frac{3}{2}m + \left(\frac{5}{2}m - \epsilon\right)\sin^2\lambda\right)$$

$$= \frac{\mu}{ab}\left(1 - \frac{3}{2}m + \left(\frac{5}{2}m - \epsilon\right)\sin^2\lambda\right)$$

the values of m and ϵ are $\frac{1}{289}$ and $\frac{1}{300}$ respectively. For the purpose of estimating the amount of disturbance caused by local attraction, it will be quite sufficient to use the principal term only, omitting the terms in m and ϵ. If δ be the mean density of the earth, μ may be taken as the mass of a sphere whose radius $= \sqrt{ab}$ and density $= \delta$, consequently the attraction of the earth—

$$= \frac{4}{3}\pi\delta\frac{(ab)^{\frac{3}{2}}}{ab} = \frac{4}{3}\pi\delta(ab)^{\frac{1}{2}}$$

Therefore if we take 3956 miles as the mean radius of the earth, the deflection ψ caused by an attraction A, will be—

$$\frac{3\,A}{4\pi\delta\cdot 3956}$$

or, expressed in seconds—

$$\psi = \frac{3\cdot 180\cdot 60\cdot 60}{\pi^2\delta\cdot 4\cdot 3956}A = 12''\cdot 447\frac{A}{\delta} \tag{1}$$

17. Let us now investigate the attraction exercised at a given point by an elevated table land whose upper surface is tolerably even, and of which the form is nearly a rectangle. Let the attracted point be taken as the origin, and let the attracting solid be bounded by the planes—

$$x = a \quad y = 0 \quad z = 0$$
$$x = a' \quad y = b \quad z = h$$

so that the dimensions of the parallelopiped are $a' - a$, b, h. We shall also assume h to be small in comparison with a a' and b. If ϱ be the density (assumed uniform) of the solid, and x y z the co-ordinates of an element of the mass whose sides are dx dy dz, the attraction exercised by this infinitely small mass at the origin is—

$$\frac{\varrho\,dx\,dy\,dz}{x^2 + y^2 + z^2}$$

The force in the direction of the axis of x is equal to this quantity multiplied by the cosine of the angle contained by the direction of the element at the origin, and the axis of x; it is, therefore, equal to—

$$\frac{\varrho\,dx\,dy\,dz\cdot x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

Integrating from $y = 0$ to $y = b$, we obtain the attraction in the direction of the axis of x of an elementary rod parallel to the axis of y; this attraction is—

$$\varrho\, dx\, dz\cdot x\int_0^b \frac{dy}{(x^2+y^2+z^2)^{\frac{3}{2}}} = \frac{\varrho\, b\, x\cdot dx\, dz}{(x^2+z^2)\,(x^2+b^2+z^2)^{\frac{1}{2}}}$$

Summing in the direction of x we get the attraction of an elementary slice of the solid parallel to the plane of xy equal to—

$$\varrho\, dz\cdot b\int_a^{a'} \frac{x\, dx}{(x^2+z^2)\,(x^2+b^2+z^2)^{\frac{1}{2}}}$$

The general integral is—

$$\varrho\, dz \log \frac{(x^2+z^2)^{\frac{1}{2}}}{b+(b^2+x^2+z^2)^{\frac{1}{2}}}$$

which taken from a to a' is equal to—

$$\varrho\, dz \log \left\{\left(\frac{a'^2+z^2}{a^2+z^2}\right)^{\frac{1}{2}}\cdot\frac{b+\sqrt{b^2+a^2+z^2}}{b+\sqrt{b^2+a'^2+z^2}}\right\} \qquad (2)$$

Assuming z to be small in comparison with b and a and a', we may neglect quantities of the order z^4; and therefore the quantity within the larger brackets becomes, if we put $b = a\cot\phi = a'\cot\phi'$

$$\left\{\frac{1+\dfrac{z^2}{a'^2}}{1+\dfrac{z^2}{a^2}}\right\}^{\frac{1}{2}} \frac{\cot\phi + \sqrt{\operatorname{cosec}^2\phi + \dfrac{z^2}{a^2}}}{\cot\phi' + \sqrt{\operatorname{cosec}^2\phi' + \dfrac{z^2}{a'^2}}}$$

$$= \frac{1+\frac{1}{2}\dfrac{z^2}{a'^2}}{1+\frac{1}{2}\dfrac{z^2}{a^2}}\;\frac{\cot\phi + \operatorname{cosec}\phi + \frac{1}{2}\dfrac{z^2}{a^2}\sin\phi}{\cot\phi' + \operatorname{cosec}\phi' + \frac{1}{2}\dfrac{z^2}{a'^2}\sin\phi'}$$

Now we have—

$$\cot\phi + \operatorname{cosec}\phi = \frac{1+\cos\phi}{\sin\phi} = \cot\tfrac{1}{2}\phi$$

and therefore the above expression becomes—

$$\frac{\cot\frac{1}{2}\phi}{\cot\frac{1}{2}\phi'}\left(1+\frac{1}{2}\frac{z^2}{a^2}\left(\frac{\sin^2\phi}{1+\cos\phi}-1\right)-\frac{1}{2}\frac{z^2}{a'^2}\left(\frac{\sin^2\phi'}{1+\cos\phi'}-1\right)\right)$$

$$= \frac{\cot\frac{1}{2}\phi}{\cot\frac{1}{2}\phi'}\left(1+\frac{z^2}{2}\left(\frac{\cos\phi'}{a'^2}-\frac{\cos\phi}{a^2}\right)\right)$$

Therefore the attraction of the elementary slice parallel to the plane of $x\,y$ is—

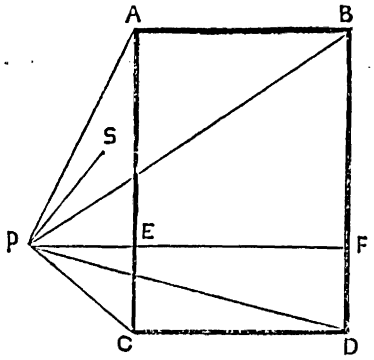
$$\varrho\, dz \log\left\{\frac{\cot\frac{1}{2}\phi}{\cot\frac{1}{2}\phi'}\left(1+\frac{z^2}{2}\left(\frac{\cos\phi'}{a'^2}-\frac{\cos\phi}{a^2}\right)\right)\right\}$$

in which quantities of the order z^4 have been omitted. Since $\log_e(1+u) = u$ when u is a small quantity, the above expression becomes—

$$\varrho\, dz \log\left\{\frac{\cot\frac{1}{2}\phi}{\cot\frac{1}{2}\phi'}\right\} + \tfrac{1}{2}\varrho\, z^2\, dz\left\{\frac{\cos\phi'}{a'^2}-\frac{\cos\phi}{a^2}\right\}$$

Integrating from $z = 0$ to $z = h$, we have the attraction of the whole mass equal to—

$$\varrho\, h \log \left\{\frac{\cot\frac{1}{2}\phi}{\cot\frac{1}{2}\phi'}\right\} + \frac{1}{6}\varrho\, h^3 \left\{\frac{\cos\phi'}{a'^2} - \frac{\cos\phi}{a^2}\right\} \tag{3}$$



18. By the application of this expression we may find the components of the attraction of a table land upon any point. Let ABCD be the rectangular mass of height h, which is small in comparison with PE. Let $\mathrm{PE} = a$, $\mathrm{PF} = a'$, $\mathrm{EC} = b$, $\mathrm{EA} = b'$, so that the dimensions of the rectangle are $a' - a$ and $b' + b$. Let the angles

$$\mathrm{PCE} = \phi \quad \mathrm{PAE} = \theta$$
$$\mathrm{PDF} = \phi' \quad \mathrm{PBF} = \theta'$$

then the attraction in the direction PE is—

$$\varrho\, h \left\{ \log \frac{\cot\frac{1}{2}\theta \cot\frac{1}{2}\phi}{\cot\frac{1}{2}\theta' \cot\frac{1}{2}\phi'} + \frac{h^2}{6}\left(\frac{\cos\theta' + \cos\phi'}{a'^2} - \frac{\cos\theta + \cos\phi}{a^2}\right)\right\}$$

and the attraction in the direction perpendicular to PE is—

$$\varrho\, h \left\{ \log \frac{\cot(45 - \frac{1}{2}\theta)\cot(45 - \frac{1}{2}\phi')}{\cot(45 - \frac{1}{2}\theta')\cot(45 - \frac{1}{2}\phi)} + \frac{h^2}{6}\left(\frac{\sin\theta' - \sin\theta}{b'^2} - \frac{\sin\phi' - \sin\phi}{b}\right)\right\}$$

The corresponding deflections* expressed in seconds are by equation (1)—

$$\begin{aligned} \psi_{,} &= 12''{\cdot}447 \frac{\varrho}{\delta} h \left\{ \log \frac{\cot\frac{1}{2}\theta \cot\frac{1}{2}\phi}{\cot\frac{1}{2}\theta' \cot\frac{1}{2}\phi'} + \frac{h^2}{6}\left(\frac{\cos\theta' + \cos\phi'}{a'^2} - \frac{\cos\theta + \cos\phi}{a^2}\right)\right\} \\ \psi' &= 12''{\cdot}447 \frac{\varrho}{\delta} h \left\{ \log \frac{\cot(45 - \frac{1}{2}\theta)\cot(45 - \frac{1}{2}\phi')}{\cot(45 - \frac{1}{2}\theta')\cot(45 - \frac{1}{2}\phi)} + \frac{h^2}{6}\left(\frac{\sin\theta' - \sin\theta}{b'^2} - \frac{\sin\phi' - \sin\phi}{b^2}\right)\right\} \end{aligned} \tag{4}$$

If the deflection in any particular direction as PS be required, then if $\mathrm{SPE} = \alpha$, the required deflection ψ is—

$$\psi = \psi_{,} \cos\alpha + \psi' \sin\alpha .$$

The absolute amount of deflection is $\sqrt{\psi_{,}^2 + \psi'^2}$, and the direction in which it takes place is determined by the angle whose tangent is the fraction $\psi' : \psi_{,}$.

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* These may be put in the following form: If x and y be the co-ordinates of P measured from the centre of the rectangle parallel to BA, AC respectively, the deflections will be—

$$\psi_{,} = 12''{\cdot}447 \frac{\rho}{\delta} h \log \left\{ \frac{1 + \sqrt{1 + \left(\frac{x - a}{y + b}\right)^2}}{1 + \sqrt{1 + \left(\frac{x + a}{y + b}\right)^2}} \cdot \frac{1 + \sqrt{1 + \left(\frac{x + a}{y - b}\right)^2}}{1 + \sqrt{1 + \left(\frac{x - a}{y - b}\right)^2}} \right\}$$

$$\psi' = 12''{\cdot}447 \frac{\rho}{\delta} h \log \left\{ \frac{1 + \sqrt{1 + \left(\frac{y + b}{x - a}\right)^2}}{1 + \sqrt{1 + \left(\frac{y + b}{x + a}\right)^2}} \cdot \frac{1 + \sqrt{1 + \left(\frac{y - b}{x + a}\right)^2}}{1 + \sqrt{1 + \left(\frac{y - b}{x - a}\right)^2}} \right\}$$

in which $2a$ and $2b$ are the sides of the rectangle AB, BD.

19. Let us now ascertain the actual amount of disturbance that would be caused by a given mass. Suppose a table land whose form is a rectangle of twelve miles by eight miles, having a mean height of 500 feet, and being of half the density of the earth; and let the observer be at a point two miles distant (perpendicularly) from the middle point of the longer side.

In applying the formula the height h must be expressed in miles, as that is the linear unit used in obtaining the equation (1); and if μ be the modulus of the common system of logarithms, we shall have—

$$\psi_{,} = 12''{\cdot}447 \cdot \frac{250}{5280} \cdot \frac{1}{\mu} \cdot \log_{10}\left(\frac{\cot\frac{1}{2}\phi}{\cot\frac{1}{2}\phi'}\right)^2 + \dots$$

$$\psi' = 0$$

The term in h^2 may be neglected, as is easily seen, and the angles ϕ and ϕ', which may be found by construction, are about 18° 30′ and 59° 0′. Hence the numerical value of the deflection due to the mass is—

$$\psi_{,} = 1''{\cdot}472$$

As we approach nearer the mass the deflection increases, being $2''{\cdot}20$ at the distance of one mile. Had the height been a thousand feet instead of five hundred, the term in h^2 might still have been omitted without materially affecting the result.

20. The attraction of a prism of indefinite length, whose section is a trapezoid, upon a point in the plane of one of the parallel faces, admits of a simple expression, from which we may obtain the deflection that would be caused by a rectilinear range of mountains of a somewhat regular section, at a point in the plane of the base.

Let the attracted point be taken as the origin of co-ordinates, and let the prism be contained by the planes—

$$z = 0 \;:\; z = h$$

$$x = a + \alpha z \quad , \quad x = a' + \alpha' z$$

then the attraction of the prism, ϱ being its density, is—

$$2\varrho \iiint \frac{x\,dx\,dy\,dz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

the limits being $y = 0$ to $y = \infty$; $x = a + \alpha z$ to $a' + \alpha' z$; and $z = 0$ to $z = h$—

$$= 2\varrho \iint \frac{x\,dx\,dz}{x^2 + z^2}$$

$$= \varrho \int dz \left\{ \log\left(z^2 + (a' + \alpha' z)^2\right) - \log\left(z^2 + (a + \alpha z)^2\right) \right\}$$

$$= \varrho z \log \frac{z^2 + (a' + \alpha' z)^2}{z^2 + (a + \alpha z)^2} - \varrho \int z \cdot \frac{d}{dz} \log \frac{z^2 + (a' + \alpha' z)^2}{z^2 + (a + \alpha z)^2}\, dz$$

$$= \varrho z \log \frac{z^2 + (a' + \alpha' z)^2}{z^2 + (a + \alpha z)^2} + 2\varrho \int \frac{a'^2 + a'\alpha' z}{z^2 + (a' + \alpha' z)^2} - 2\varrho \int \frac{a^2 + a\alpha z}{z^2 + (a + \alpha z)^2}$$

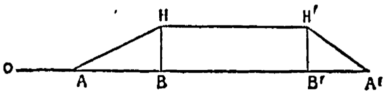
$$\left.\begin{aligned} = \varrho z \log \frac{z^2 + (a' + \alpha' z)^2}{z^2 + (a + \alpha z)^2} &+ \frac{\varrho a' \alpha'}{1 + \alpha'^2} \log\left(z^2 + (a' + \alpha' z)^2\right) + \frac{2\varrho a'}{1 + \alpha'^2} \tan^{-1}\left(z\frac{1 + \alpha'^2}{a'} + \alpha'\right) \\ &- \frac{\varrho a \alpha}{1 + \alpha^2} \log\left(z^2 + (a + \alpha z)^2\right) - \frac{2\varrho a}{1 + \alpha^2} \tan^{-1}\left(z\frac{1 + \alpha^2}{a} + \alpha\right) \end{aligned}\right]$$

Now put $u^2 = z^2 + (a + \alpha z)^2$ and $u'^2 = z^2 + (a' + \alpha' z)^2$; then the definite integral will be—

$$= \varrho \log \left(\frac{u'}{a'}\right)^{\frac{2a'\alpha'}{1+\alpha'^2}} \cdot \left(\frac{a}{u}\right)^{\frac{2a\alpha}{1+\alpha^2}} \cdot \left(\frac{u'}{u}\right)^{2h} + \frac{2\varrho a'}{1+\alpha'^2} \tan^{-1}\left(\frac{h}{a' + \alpha' h}\right) - \frac{2\varrho a}{1+\alpha^2} \tan^{-1}\left(\frac{h}{a + \alpha h}\right) \quad (5)$$

which is the value of the attraction required.

This expression is much simplified by the consideration of the adjoining figure, which is a section of the prism passing through the attracted point O.



If HB, H′B′ are perpendicular to OAA′, we have—

$$\text{HB} = h = \text{H}'\text{B}'$$

$$\text{OA} = a \qquad \text{OA}' = a'$$

$$\text{OB} = a + \alpha h \qquad \text{OB}' = a' + \alpha' h$$

$$\text{OH} = \sqrt{h^2 + (a + \alpha h)^2} \qquad \text{OH}' = \sqrt{h^2 + (a' + \alpha' h)^2}$$

If therefore we suppose OH and OH′ to be joined, and put $<$ HAB $= \theta$, H′A′B′ $= \theta'$, HOB $= \phi$, H′OB′ $= \phi'$

$$\text{A} = \varrho \log_e \left\{ \left(\frac{\text{OA}'}{\text{OH}'}\right)^{a' \sin 2\theta'} \cdot \left(\frac{\text{OA}}{\text{OH}}\right)^{a \sin 2\theta} \cdot \left(\frac{\text{OH}'}{\text{OH}}\right)^{2h} \right\} + 2\varrho \left\{ a' \phi' \sin^2\theta' - a \phi \sin^2\theta \right\} \quad (6)$$

If O coincide with the foot of the range—

$$\text{A} = \varrho \log_e \left(\frac{\text{AH}'}{\text{AH}}\right)^{2h} \cdot \left(\frac{\text{AA}'}{\text{AH}'}\right)^{a' \sin 2\theta'} + 2\varrho a' \phi' \sin^2\theta'$$

If the section of the range be a triangle, so that H and H′ coincide and $\phi = \phi'$, the attraction at any point, O, is—

$$\varrho \log_e \left\{ \left(\frac{\text{OA}'}{\text{OH}}\right)^{a' \sin 2\theta'} \cdot \left(\frac{\text{OA}}{\text{OH}}\right)^{a \sin 2\theta} \cdot \right\} + 2\varrho \phi \left\{ a' \sin^2\theta' - a \sin^2\theta \right\} \quad (7)$$

If the section be an isosceles triangle, the attraction at O is—

$$\varrho \log_e \left\{ \left(\frac{\text{OA}'}{\text{OH}}\right)^{a'} \cdot \left(\frac{\text{OA}}{\text{OH}}\right)^{a} \right\}^{\sin 2\theta} + 2\varrho \phi \sin^2\theta \, (\text{AA}') \quad (8)$$

From these expressions we may ascertain the disturbances existing in the vicinity of a range of mountains. It is true that in no case can the formulæ be supposed to give exact results, but they will serve, as in the following hypothetical cases, to give a general idea of the *amount* of deviation. We shall suppose the matter of which the disturbing mass is composed to have a density equal to half the mean density of the earth; and in the first case suppose a range of indefinite length, whose mean height is a tenth of a mile, or 528 feet, and breadth at base fourteen miles. Suppose, also, that the slope of the range on either side is $\frac{1}{20}$th (or $\tan^{-1} \frac{1}{20}$). The deflection produced by such a mass at a station at the foot of one of the slopes would be 3″·6. At the distance of one mile from this point the deflection is diminished to 2″·5, and at five miles is equal to 1″·3.

If we suppose a range whose mean height is a third of a mile, or 1760 feet, having equal slopes of $\frac{1}{15}$, and a breadth at base of fifteen miles, such a mass would produce a deflection of

7″·9 at a point situated at the foot of one of its slopes. If we remove the station from the foot of the range to a distance of five miles, the deflection at that point would be 3″·5.

As a third instance, suppose a range whose section is triangular, the height of the ridge being eight-tenths of a mile, or 4224 feet, the breadth of the base fifteen miles, and the slopes $\frac{8}{100}$ and $\frac{16}{100}$. The deflection produced by this mass on a point at its base would be 12″·0; at a distance of five miles the effect would be diminished by about one half, at ten miles the deflection would be 4″·2, and at fifteen miles would be diminished to 3″·2.*

21. Hitherto we have neglected the geometrical consideration of the curvature of the earth's surface as affecting deflection. This, indeed, may generally be done with safety, even in the case of a range of mountains when their breadth and the distance of the observer are small compared with the dimensions of the earth. But if it be required to ascertain the disturbing effect of a large tract of country of considerable elevation, the curvature of the surface should not be neglected. Its influence may be determined as follows:

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* The attraction of a mountain whose form is that of a solid of revolution can be obtained only by series. Let the co-ordinates originate at the attracted point, the plane of xy coinciding with the surface of the earth, and let $x = a$, $y = 0$ be the equation of the axis of the solid which is perpendicular to the plane of xy. The attraction of an elementary slice of the solid contained between the horizontal planes z and $z + dz$ respectively is—

$$\rho\, dz \int_{-r}^{r} \left\{ \frac{dy}{(k^2 - 2\,au)^{\frac{1}{2}}} - \frac{dy}{(k^2 + 2\,au)^{\frac{1}{2}}} \right\}$$

where r is the radius of the circular plate $k^2 = a^2 + r^2 + z^2$, $u^2 = r^2 - y^2$. By the binomial theorem this expression is transformed to—

$$\frac{2\,\rho\, dz}{k} \int_{-r}^{r} \Sigma\, \frac{1.3.5 \ldots\ldots (4\,i - 3)}{1.2.3 \ldots\ldots (2\,i - 1)} \cdot \left(\frac{au}{k^2}\right)^{i - 1} \cdot dy$$

where i takes the successive values 1 2 3 4 . . . . ; but we have—

$$\int_{-r}^{r} u^{2i-1} \cdot dy = r^{2i}\, \frac{1.3.5 \ldots\ldots (2\,i - 1)}{1.2.4 \ldots\ldots\ldots (2\,i)}\, \pi$$

Substituting this in the expression for the attraction of the circular plate, and integrating with respect to z, we have the attraction of the mass equal to—

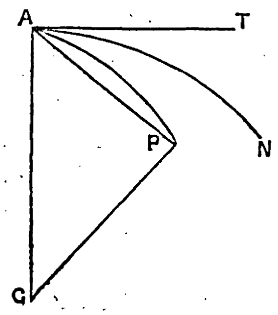
$$\pi\,\rho \int \Sigma\, \frac{r}{i\,k}\, \frac{1.3.5 \ldots\ldots (4\,i - 3)}{2^2.4^2 \ldots\ldots (2\,i - 2)^2} \left(\frac{a\,r}{k^2}\right)^{2i-1} \cdot dz$$

where r is a function of z, and is determined and expressed by the equation of the attracting surface.

If the mass be a paraboloid, the radius of the base $= b$ and height $= h$, the attraction is—

$$= \frac{\text{Mass}}{a^2} \left\{ 1 + \frac{b^2 - h^2}{4\,a^2} + \frac{15\,b^4 - 24\,b^2 h^2 + 16\,h^4}{128\,a^4} + \ldots\ldots \right\}$$

Suppose $h = 2640$ feet, $b =$ a mile and a half; at the distance of a mile and a half from the foot of such a mountain its attraction would produce a deflection of 1″.30; at five miles the deflection would be only a quarter of a second.



In the adjoining diagram let C be the centre of the earth, A the station at which the attraction is required, AN the meridian of A, AP any other great circle passing through A in azimuth α, so that NAP $= \alpha$, and AT a tangent to the great circle AN. Let ACP $= \theta$; then if we consider α and θ as the co-ordinates of P, the element of the surface at P will be the rectangle $r\, d\theta \cdot r \sin\theta\, d\alpha$; and if h be the height of the actual surface of the ground at P, the element of the volume of the superincumbent matter at P is $h\, r^2 \sin\theta\, d\theta\, d\alpha$. Consequently the attraction produced by this prism at A in the direction AP is—

$$\frac{\varrho\, h\, r^2 \sin\theta\, d\theta\, d\alpha}{4\, r^2 \sin^2 \frac{1}{2}\theta}$$

which resolved in the direction AT is—

$$\varrho\frac{h \sin\theta\, d\theta\, d\alpha}{4 \sin^2 \frac{1}{2}\theta} \cos\tfrac{1}{2}\theta \cos\alpha = \frac{\varrho\, h \cos^2 \frac{1}{2}\theta}{2 \sin \frac{1}{2}\theta} \cos\alpha\, d\theta\, d\alpha$$

From this expression we may easily find the attraction of a portion of the surface contained between two vertical planes passing through A in azimuths $\alpha_{,}$ and α' and two small circles having the centre A and radii $\theta_{,}$ and θ', supposing the surface to have an even height and equal to h; the attraction in question is—

$$\frac{\varrho\, h}{2} \int_{\theta_{,}}^{\theta'} \int_{\alpha_{,}}^{\alpha'} \frac{\cos^2 \frac{1}{2}\theta}{\sin\frac{1}{2}\theta} \cos\alpha \,.\, d\theta\, d\alpha$$

$$= \varrho\, h\, (\sin\alpha' - \sin\alpha_{,}) \left(\log \frac{\tan\frac{1}{4}\theta'}{\tan\frac{1}{4}\theta_{,}} + \cos\tfrac{1}{2}\theta' - \cos\tfrac{1}{2}\theta_{,}\right) \qquad (9)$$

When θ amounts to no more than three or four degrees, we may without fear of error omit the higher powers of θ; the corresponding deflection will therefore be, supposing the mean density of the attracting mass to be half that of the earth—

$$6''{\cdot}223\, h\, (\sin\alpha' - \sin\alpha_{,}) \log\frac{\theta'}{\theta_{,}} \qquad (10)$$

22. We shall now apply this formula to the station Tawnaghmore, on the north coast of Mayo in Ireland. If through this station we draw on a map of Ireland a line in a north-east direction making an angle of 75° with the north meridian line, and another line due south, and describe two circles round the station as a centre whose radii are respectively 10′ and 2° 30″; these four lines so drawn will inclose a space which may be taken as equivalent in attraction to the mass of Ireland, supposing the space to be filled up with rock to the height of the mean level of Ireland. Now in Ireland there are 13,243 square miles of country whose height is between 0 and 250 feet, 11,797 square miles whose height is between 250 and 500 feet, 5798 square miles whose height is between 500 and 1000 feet, and 82 square miles whose height is above 2000. From these numbers we may infer that the mean level of Ireland is 400 feet. But the part of Ireland which is more immediately south-west of this station does not attain to the mean level of the country. The mean height of Sligo, Mayo, and Roscommon is 320 feet;

we shall therefore use this quantity instead of 400 feet as the mean height of the disturbing mass. Consequently the deflection caused by this mass at Tawnaghmore is—

$$-6''\cdot223\left(\frac{320}{5280}\right)\sin 75^\circ \log_e 15$$

or 0″·984 south. We have here neglected the local attraction due to the form of the ground within ten or twelve miles of the station, as that requires more exact data, and will be subsequently considered. Similarly it might be shown that at the south end of the Lough Foyle Base the deflection due to the mass of Ireland is nearly two seconds, neglecting the consideration of the purely local attraction.

There is another consideration that must not be overlooked, which is, that the mean density of the sea being less than that of rock, the above effects should be increased; but as the amount depends upon the mean depth of the sea, it cannot in general be very exactly ascertained. The deflection due to this cause will probably be considerably smaller than that due to the land.

23. We now proceed to consider the method by which the local attraction due to the form of the ground in the immediate vicinity of a station is to be calculated. As it is impossible to find an expression for the surface of the ground, the method that naturally suggests itself is the division of the attracting mass into a large number of parts, of each of which the attraction resolved in a certain direction is to be obtained, and thence the sum of all the partial attractions, or the total attraction, or resultant. The case of the computation will materially depend upon the method or law according to which the attracting mass is divided. The method adopted in the calculation of the attraction of the mountain Schehallien by Dr. Hutton is perhaps the most simple and expeditious that could be devised. It is as follows: Having a map showing the nature of the ground, its elevations and depressions, or contours, for some distance round the given station in every direction, draw through the centre of the station a series of radial lines, and describe a series of concentric circles, having the station as their common centre. The ground is thus divided into a number of compartments, whose separate attractions are easily obtained; but before stating the law according to which the circles and radii are to be drawn, we shall investigate an expression for the attraction of the prism of matter standing over any one of these compartments.

Let a horizontal plane be supposed to pass through the given station, and let this point be taken as the origin of co-ordinates $r\ \alpha$ to any point in that plane, and let z be the vertical co-ordinate; let also $\alpha = 0$ correspond to the north meridian line. Let it be required to find the attraction of a mass contained between the horizontal planes $z = 0$, $z = h$, the two vertical planes $\alpha = \alpha_{,}$, $\alpha = \alpha'$, and the two cylindrical surfaces $r = r_{,}$, $r = r'$. The volume of an indefinitely small element of the attracting mass being $r\cdot d\alpha\cdot dr\cdot dz$, if ϱ be its density the corresponding attraction at the origin is—

$$\frac{\varrho\, r\cdot d\alpha\, dr\, dz}{r^2 + z^2}$$

and therefore the attraction of the mass in the direction of the meridian is equal to—

$$\varrho \iiint \frac{r^2 \cos \alpha \, d\alpha \, dr \, dz}{(r^2 + z^2)^{\frac{3}{2}}}$$

the limits of z being 0 and h; of α, $\alpha_{,}$ and α'; of r, $r_{,}$ and r'—

$$= \varrho (\sin \alpha' - \sin \alpha_{,}) \iint \frac{r^2 \, dr \, dz}{(r^2 + z^2)^{\frac{3}{2}}}$$

$$= \varrho (\sin \alpha' - \sin \alpha_{,}) \int \frac{h \, dr}{(r^2 + h^2)^{\frac{1}{2}}}$$

$$= \varrho \, h (\sin \alpha' - \sin \alpha_{,}) \log_e \frac{r' + \sqrt{r'^2 + h^2}}{r_{,} + \sqrt{r_{,}^2 + h^2}}. \qquad (11)$$

If we expand the logarithm, this expression becomes—

$$\varrho \cdot (r' - r_{,}) (\sin \alpha' - \sin \alpha_{,}) \frac{h}{\sqrt{r^2 + h^2}} \left(1 + \frac{(2 r^2 - h^2)(r' - r_{,})^2}{24 \, (r^2 + h^2)^2} + \dots \right)$$

in which $r = \frac{1}{2} (r' + r_{,})$. Therefore taking $r' - r_{,}$ sufficiently small in comparison with $r' + r_{,}$ we have, putting A for the attraction—

$$A = \varrho \, (r' - r_{,}) (\sin \alpha' - \sin \alpha_{,}) \frac{h}{\sqrt{r^2 + h^2}} \qquad (12)$$

This expression suggests immediately a law of division, for if we make the successive radii of the concentric circles drawn round the station to have a common difference, and also draw the radial lines so that the sines of their azimuths shall be in arithmetical progression, then the attraction of any part of the mass is = (constant) (sine elevation), because the last factor of the above expression for A is the sine of the elevation of the middle point of the upper surface of any compartment as seen at the origin or given station, supposing the uneven surface of that compartment to be smoothed down to an even surface or *mean level.* The whole calculation therefore requires merely the sum of the sines of the elevations of the different compartments. Unless the ground be very steep it will be unnecessary to retain h^2, so that in this case we may take—

$$A = \varrho \, (r' - r_{,}) (\sin \alpha' - \sin \alpha_{,}) \frac{h}{r} \qquad (13)$$

24. Let (r) and (s) stand for the common difference of the radii and sines of the azimuths; let r_n be the radius of the n^{th} circle, H_n the sum of the heights of all the compartments on the north side of the station between the n^{th} and $(n + 1)^{\text{th}}$ circles, H'_n being the corresponding quantity for the south side of the station; then since $\frac{1}{2} (r_n + r_{n+1}) = n \, (r) + \frac{1}{2} (r)$, we have—

$$A = \varrho \, (s) \, \Sigma \cdot \frac{H_n - H'_n}{n + \frac{1}{2}}$$

Consequently if ψ be the deflection—

$$\psi = 24''{\cdot}894 \, \frac{\varrho}{\delta} \, (s) \, \Sigma \, \frac{H_n - H'_n}{2 \, n + 1} \qquad (14)$$

where δ is the mean density of the earth: H and H′ are to be expressed in miles.

There is another law of division which may be used when h^2 may be neglected, for in this case we have—

$$A = \varrho \, (\sin \alpha' - \sin \alpha_{,}) \log_e \frac{r'}{r_{,}} \cdot h$$

If therefore the radii of the circles be in geometrical progression and the sines of the azimuths be in arithmetical progression, the total deflection will be—

$$\psi = 12''{\cdot}447 \, \frac{\varrho}{\delta} \, (s) \, (r) \left(\mathrm{H} - \mathrm{H}'\right) \tag{15}$$

where (r) is the Napierian logarithm of the ratio of the geometrical series in which the radii of the concentric circles are drawn, H the sum of the mean heights of all the compartments on the north side, H′ the sum of the mean heights of all the compartments on the south side.

Of the Disturbance produced by a Mass of great Density and compact Form, situated at a short Distance below the Surface.

25. The mathematical surface of the earth being at every point perpendicular to the actual direction of gravity at that point, every disturbance of the direction of gravity must be accompanied with an inclination of the mathematical surface at that point to the *mean* surface. Thus every mountain or series of mountains produces necessarily a *wave* on the mathematical surface of the earth, and the same effect is also produced by irregularities in the density of the matter below the actual surface of the earth. Suppose, for instance, a mass of very dense matter to exist at a certain place at a small distance under the surface of the earth, and, for the sake of illustration, suppose the position of the disturbing mass to be covered by a large lake, then it is clear that the attraction of the dense mass would have the effect of collecting the water towards a point vertically over the centre of the attracting mass where it would accumulate in a permanent wave. The magnitude of the wave will of course be a function of the magnitude and depth of the disturbing mass, and if the latter be given the nature and details of the former can be concluded.

Theoretically therefore, as the matter of the crust of the earth is not of an even density, and the actual surface is everywhere uneven, the mathematical surface of the earth must be a continued series of waves. This is evident *à priori*, but it remains for observation and calculation to ascertain whether these waves are of sufficient magnitude to be detected.

26. The effect of a small and very dense mass below the surface of the earth in producing an irregularity in the mathematical surface in the neighbourhood may be thus investigated. Suppose the earth a sphere of a radius $= a$, and let ah be the depth of the centre of the disturbing mass, and put $1 - k = h$. Let M be the disturbing mass, and P any point of the earth's surface, its distance from the centre of the earth being $= r$, and let the angle subtended by M and P at the centre of the earth be put $= \theta$. We shall suppose the disturbing force to be so small in comparison with the attraction of the earth that its square may be

neglected, consequently in estimating the effect of the disturbing force at P we may neglect $r - a$, or the height of P above the undisturbed surface, and put—

$$\mathrm{PM}^2 = a^2 (1 + k^2 - 2k \cos \theta)$$

Also if ϕ be the angle contained at P between PM the direction of the disturbing mass, and the vertical—

$$\mathrm{PM} \sin \phi = a\, k \sin \theta$$

Hence if we put M for the disturbing mass, its attraction at P resolved horizontally is—

$$\frac{\mathrm{M}\, k \sin \theta}{a^2 (1 + k^2 - 2k \cos \theta)^{\frac{3}{2}}}$$

The ratio of this force to the attraction of the earth will give the angle through which the direction of gravity is displaced by the action of the disturbing force. Now if μ be the ratio of the disturbing mass to the mass of the earth, the attraction of the earth will be expressed by—

$$\frac{\mathrm{M}}{\mu\, a^2}$$

hence the angular displacement of the direction of gravity is—

$$\frac{\mu\, k \sin \theta}{(1 + k^2 - 2k \cos \theta)^{\frac{3}{2}}}$$

The mathematical surface is always perpendicular to the actual direction of gravity; hence r, θ being the polar co-ordinates of the curve formed by the intersection of this surface, with a plane passing through M and the centre of the earth, we have—

$$\frac{dr}{r\, d\theta} = \frac{-\mu\, k \sin \theta}{(1 + k^2 - 2k \cos \theta)^{\frac{3}{2}}}$$

$$\therefore \log r = \int \frac{-\mu\, k \sin \theta\, d\theta}{(1 + k^2 - 2k \cos \theta)^{\frac{3}{2}}}$$

$$\text{or} \cdot \cdot \cdot \cdot \log \frac{r}{a} = \frac{\mu}{(1 + k^2 - 2k \cos \theta)^{\frac{1}{2}}} + \mathrm{C}$$

As the square of μ is neglected, so also may the square of $r - a$; thus the equation of the curve becomes—

$$r = a \left\{ 1 + \frac{\mu}{(1 + k^2 - 2k \cos \theta)^{\frac{1}{2}}} + \mathrm{C} \right\} \tag{16}$$

C being a constant, which is necessarily of the order of the disturbing force: this constant is quite arbitrary, as there are an infinity of surfaces perpendicular to gravity. But if we take for the mathematical surface of the earth the surface of the sea, or still water connected with the sea, then we may determine the constant by the assumption that the volume of the surface generated by the curve (16) is equal to the volume of the sphere whose radius is a. Now the first is equal to—

$$\frac{2}{3} \pi \int_0^{\pi} r^3 \sin \theta\, d\theta = \frac{2}{3} \pi a^3 \int_0^{\pi} \left\{ 1 + 3\mathrm{C} + \frac{3\mu}{(1 + k^2 - 2k \cos \theta)^{\frac{1}{2}}} \right\} \sin \theta\, d\theta$$

$$= \frac{2}{3} \pi a^3 \left\{ 2(1 + 3\mathrm{C}) + 6\mu \right\}$$

and since this is equal to $\frac{4}{3}\pi a^3$ we have $C = -\mu$: hence (16) becomes by putting $y = r - a$,

$$y = a\mu\left\{\frac{1}{(1 + h^2 - 2h\cos\theta)^{\frac{1}{2}}} - 1\right\} \qquad (17)$$

The greatest value of the elevation y of the disturbed surface is that vertically over the disturbing mass, or corresponding to $\theta = 0$, where its value is—

$$\Upsilon = a\mu\,\frac{1 - h}{h}$$

or, as we are considering only small depths h, and Υ is a small quantity—

$$\Upsilon = \frac{a\mu}{h} \qquad (18)$$

The deflection of the plumb-line due to the disturbance of the mass at any distance θ is—

$$\psi = \frac{\mu h \sin\theta}{(1 + h^2 - 2h\cos\theta)^{\frac{3}{2}}} \qquad (19)$$

Or since the effect of the disturbing force is only sensible at a distance which is small compared with a, we may neglect higher powers of θ than the square; thus—

$$\psi = \frac{\mu\theta}{(h^2 + \theta^2)^{\frac{3}{2}}} \qquad (20)$$

When $\theta = 0$ the deflection is zero: as θ increases, the deflection increases up to a certain point, and afterwards diminishes indefinitely. The value of θ, which corresponds to the maximum deflection, is obtained by making—

$$\frac{d\psi}{d\theta} = \mu\,\frac{h^2 - 2\theta^2}{(h^2 + \theta^2)^{\frac{5}{2}}} = 0$$

and therefore the maximum deflection takes place at a point whose distance from the radius of the earth passing through the disturbing mass is to the depth of the disturbing mass as $1 : \sqrt{2}$. The maximum value of the deflection is—

$$\Psi = \frac{2}{3\sqrt{3}}\cdot\frac{\mu}{h^2} \qquad (21)$$

27. We now proceed to show, that although a disturbing mass below the surface may be sufficient to produce a very large deflection, yet the actual *height* of the wave in the mathematical surface produced by its action is quite inconsiderable as regards the figure of the earth. If we suppose Ψ, the maximum deflection, to be expressed in seconds, m the depth of the disturbing mass expressed in miles, x the ratio of the volume of the disturbing mass to that of the earth; then if the excess of the density of the disturbing mass above that of the surrounding matter be equal to half the mean density of the earth—

$$\frac{\pi\Psi}{180\cdot60\cdot60} = \frac{x}{3\sqrt{3}}\left(\frac{4000}{m}\right)^2$$

$$\therefore x = \frac{\sqrt{3}}{2}\,\frac{\pi\Psi m^2}{(12000)^3}$$

The volume of the disturbing mass is therefore to the volume of a sphere a mile in diameter as,

$$\pi\Psi m^2 \quad : \quad \frac{9\sqrt{3}}{4}$$

If we compare Υ, the greatest height of the wave expressed in feet, with Ψ, the greatest deflection expressed in seconds—

$$\Upsilon = \frac{3\sqrt{3}\,\pi\,\Psi\,m}{2\cdot 180\cdot 60\cdot 60} = \frac{11\,\pi\,\Psi\,m}{300\sqrt{3}}$$

Suppose now the disturbing mass to be a sphere of a mile in diameter, the excess of its density above that of the surrounding country being equal to half the density of the earth, and suppose the depth of its centre to be half a mile, then—

$$\Psi = \frac{9\sqrt{3}}{\pi} = 4''\cdot 96$$

$$\Upsilon = \frac{99}{600} = \frac{1}{6} \text{ nearly}$$

so that although the deflection would amount at the maximum to very nearly five seconds, yet the actual vertical magnitude of the wave would be only two inches.

If we had supposed the centre of the mass to be at the depth of three quarters of a mile, the result would have been—

$$\Psi = \frac{4\sqrt{3}}{\pi} = 2''\cdot 20$$

$$\Upsilon = \frac{33}{300} = \frac{1}{9} \text{ nearly}$$

We may from these calculations draw this important conclusion,—that a large observed amount of deflection may arise from, or rather accompany, an irregularity of the surface of the earth whose actual magnitude, as regards height at least, is of an extremely minute order.

28. The above calculations are necessarily only roughly approximative, since the form of the mass is assumed such that its attraction upon any given point is nearly the same as if the whole mass were collected into one point at its centre. The general deduction will, however, remain true.

Let us now examine the effect of the disturbing mass upon the curvature of the surface. The equation of a principal section of the wave we have seen to be—

$$r = a + \frac{\mu\,a}{(h^2 + \theta^2)^{\frac{1}{2}}} + \text{const.}$$

and if R be the radius of curvature of this curve—

$$R = \frac{\left(r^2 + \frac{dr^2}{d\theta^2}\right)^{\frac{3}{2}}}{r^2 + 2\left(\frac{dr}{d\theta}\right)^2 - r\frac{d^2r}{d\theta^2}}$$

but since we may neglect the square of the disturbing force—

$$R = \frac{r}{1 - \frac{1}{r}\,\frac{d^2r}{d\theta^2}}$$

Here we may also neglect $r - a$, and therefore—

$$\text{R} = \frac{a}{1 + \mu \frac{h^2 - 2\theta^2}{(h^2 + \theta^2)^{\frac{5}{2}}}} \tag{22}$$

The radius of curvature will therefore be a maximum or minimum according as $(h^2 - 2\theta^2)(h^2 + \theta^2)^{-\frac{5}{2}}$ is a minimum or a maximum. The differential coefficient of this quantity is—

$$\frac{3\theta(2\theta^2 - 3h^2)}{(h^2 + \theta^2)^{\frac{7}{2}}} = 0.$$

and therefore the maximum and minimum of the radius of curvature correspond to $\theta^2 = \frac{3}{2}h^2$ and $\theta = 0$; hence—

$$\text{Maximum of R} = \frac{a}{1 - \frac{8\sqrt{2}}{25\sqrt{5}} \frac{\mu}{h^3}}$$

$$\text{Minimum of R} = \frac{a}{1 + \frac{\mu}{h^3}}$$

Or expressing μ in terms of Ψ the greatest deflection, the greatest and least values are—

$$\frac{a}{1 - \frac{12\sqrt{6}}{25\sqrt{5}} \cdot \frac{\Psi}{h}} \text{ and } \frac{a}{1 + \frac{3\sqrt{3}}{2} \cdot \frac{\Psi}{h}}$$

The difference of these values is—

$$\frac{3 \cdot 124 \frac{\Psi}{h} a}{1 + 2 \cdot 072 \frac{\Psi}{h} + \cdots}$$

If we suppose Ψ to be expressed in seconds and the depth m in miles, then the difference in miles between the maximum and minimum radii of curvature is—

$$\frac{242 \frac{\Psi}{m}}{1 + 0 \cdot 0402 \frac{\Psi}{m} + \cdots}$$

The inference from this formula is very remarkable, for if we suppose a disturbing mass to be placed at one mile below the surface, then the difference between the greatest and least radii of curvature of the disturbed mathematical surface will be proportional to the maximum deflection, and at the rate of 240 miles per second.

The curvature of the mathematical surface of the earth is therefore affected to a very considerable amount by a disturbing mass of no very great dimensions below the surface; and consequently if the existence of such disturbing masses be admitted, it will follow that if the curvature of the surface could be determined at many *points* of the earth, such curvatures would exhibit very great discrepancies.

29. Let us now inquire whether the disturbing mass which produces a sensible amount of deviation in the plumb-line can also sensibly affect the rate of the seconds pendulum. If L be

the length of the pendulum, g the force of gravity, n the number of vibrations in twenty-four hours—

$$n^2 = \left(\frac{86400}{\pi}\right)^2 \frac{g}{\mathrm{L}}$$

$$\therefore 2\frac{dn}{n} = \frac{dg}{g}$$

Now the vertical component of the disturbing force at the distance $a\theta$ from the point of the surface directly over the mass M is—

$$\frac{\mathrm{M}h}{a^2(h^2+\theta^2)^{\frac{3}{2}}} = dg$$

Also $g = \frac{\mathrm{M}}{\mu a^2}$, therefore if ν be the number of seconds the pendulum is accelerated—

$$2\frac{\nu}{n} = \frac{\mu h}{(h^2+\theta^2)^{\frac{3}{2}}} \text{ or, } \nu = 12 \cdot 60 \cdot 60 \frac{\mu h}{(h^2+\theta^2)^{\frac{3}{2}}} \qquad (23)$$

The disturbance is a maximum for $\theta = 0$, or when the pendulum is just over the disturbing mass, the amount of the greatest acceleration is—

$$\mathrm{N} = 12 \cdot 60 \cdot 60 \frac{\mu}{h^2} \qquad (24)$$

the greatest deflection Ψ in seconds is $\frac{360 \cdot 60 \cdot 60}{3\pi\sqrt{3}} \frac{\mu}{h^2}$ by (21), consequently—

$$\frac{\mathrm{N}}{\Psi} = \frac{\pi\sqrt{3}}{10} \qquad (25)$$

The number of seconds of time which the pendulum is accelerated is therefore, at the maximum, about half the number of seconds of arc in the maximum deviation of the plumb-line.

30. Instead, however, of comparing the *maximum* effects we may compare the *mean* effects in the following manner. Suppose P that point of the surface of the earth which is vertically over M the disturbing mass, and with the centre P and any radius b describe a circle on the surface; draw also a meridian line through P. Let r be the distance of any point of the surface within this circle from P, α its azimuth, and ψ the meridional component of the deflection at that point. In (20) put $\theta = h \tan u$, then ψ expressed in seconds is—

$$\psi = \frac{15\,\mathrm{N}}{\pi h^2} \sin u \cos^2 u \cos \alpha$$

where $\mathrm{N} = 12 \cdot 60 \cdot 60 \cdot \mu$. Now the mean value of ψ within the distance b of P is—

$$\frac{4}{\pi b^2} \int\int \psi\, r\, dr\, d\alpha$$

the limits of α being 0 and $\frac{1}{2}\pi$; of r, 0 and b. But $r = ah \tan u \therefore r\,dr = a^2 h^2 \sin u \sec^3 u \cdot du$; consequently the mean value of ψ is—

$$\frac{60\,\mathrm{N}\,a^2}{\pi^2 b^2} \int_0^u \int_0^{\frac{\pi}{2}} \frac{\sin^2 u}{\cos u} \cos \alpha\, du\, d\alpha$$

$$= \frac{60\,\mathrm{N}\,a^2}{\pi^2 b^2} \int_0^u \left(\frac{du}{\cos u} - \cos u\, du\right)$$

$$= \frac{60\,\mathrm{N}\,a^2}{\pi^2 b^2} \left(\log \cot\left(45 - \frac{u}{2}\right) - \sin u\right) \qquad (26)$$

which is the mean value of the meridional component of the deflection within a distance $b = ah \tan u$ of P.

Similarly we may obtain the mean value of the acceleration of the pendulum within the same limits. By (23) we have—

$$\nu = \frac{N}{h^2} \cos^3 u$$

And the mean value of ν is therefore—

$$\frac{4 N a^2}{\pi b^2} \int_0^u \int_0^{\frac{\pi}{2}} \sin u \, du \, d\alpha$$

$$= \frac{2 N a^2}{b^2} (1 - \cos u)$$

Hence the ratio of the mean value of ψ to the mean value of ν within any given distance of the disturbing mass is—

$$\left(\frac{\psi}{\nu}\right) = \frac{30}{\pi^2} \cdot \frac{\log_e \cot\left(45 - \frac{u}{2}\right) - \sin u}{1 - \cos u} \qquad (27)$$

u	b Depth of M being = 1.	Ratio: $\frac{\text{Mean } \psi}{\text{Mean } \nu}$	u	b Depth of M being = 1.	Ratio: $\frac{\text{Mean } \psi}{\text{Mean } \nu}$	u	b Depth of M being = 1.	Ratio: $\frac{\text{Mean } \psi}{\text{Mean } \nu}$
0°	0·00	0·00	30°	0·58	1·12	60°	1·73	2·74
10	0·18	0·36	40	0·84	1·56	70	2·75	3·68
20	0·36	0·72	50	1·19	2·08	80	5·67	5·34

Of the Disturbance produced by a Mass of great Density and elongated Form, situated at a short Distance below the Surface.

31. In the preceding case we have supposed the disturbing mass to be of such a form that we might, without much altering its effect, replace it by a massive point at a certain depth below the surface. But the results will be different if the disturbing mass be a long prism or cylinder of dense matter; for this reason, that the attraction of a material point acts inversely as the square of the distance of the attracted point, whereas the attraction of a small prism or cylinder of indefinite length upon an external point is inversely as the distance and not the square of the distance. This circumstance is easily explained by considering, that as the point attracted moves away from the cylinder the more distant parts of the cylinder, beginning to act less obliquely, have a greater effect, and therefore the diminution of attraction is not so rapid as in the case of an attracting material point or sphere.

The attraction of a cylindrical shell of indefinite length whose density is ϱ, radius r, and thickness dr, upon a point whose distance from the centre of the cylinder is k, is—

$$\frac{\varrho r}{k} dr \cdot \int_0^{2\pi} \frac{2k(k - r\cos\theta)\,d\theta}{k^2 + r^2 - 2kr\cos\theta}$$

$$= \frac{4\pi\varrho}{k} r \cdot dr.$$

The attraction of the whole cylinder of density ϱ is therefore—

$$\frac{2\pi r^2 \varrho}{k} \tag{28}$$

Let us apply this to the determination of the wave that would result from a long range of very dense matter under and parallel to the surface of the earth. Let, as before, a be the radius of the earth, r the distance of any point P in the curve formed by the intersection with the mathematical surface, of a vertical plane perpendicular to the axis M of the disturbing mass from the centre of the earth. Then θ being the angle subtended by MP at the centre of the earth, and h the ratio of the depth of A to the radius a of the earth, the attraction of the disturbing mass at P is—

$$\frac{2\pi c^2 \varrho}{a(h^2 + \theta^2)^{\frac{1}{2}}}$$

where c is the radius of the section of the disturbing mass and ϱ the excess of its density above that of the surrounding matter. This attraction resolved horizontally is—

$$\frac{2\theta c^2 \varrho \pi}{a(h^2 + \theta^2)}$$

The attraction of the earth being $\frac{4}{3}\pi a \delta$, the deflection is—

$$\psi = -\frac{dr}{r\,d\theta} = \frac{3\varrho}{4\delta} \cdot \frac{c^2}{a^2} \cdot \frac{2\theta}{h^2 + \theta^2} \tag{29}$$

$$\therefore -\log r = \frac{3\varrho c^2}{4\delta a^2} \cdot \log(h^2 + \theta^2) + \mathrm{C}$$

If we suppose that at the distance $\theta = \theta_1$, $r = a$, then—

$$\log\frac{r}{a} = \frac{3\varrho c^2}{4\delta a^2} \log\frac{h^2 + \theta_1^2}{h^2 + \theta^2} \tag{30}$$

which is the equation of a transverse section of the wave.

The maximum value of the deflection takes place when $\frac{\theta}{h^2 + \theta^2}$ is a maximum, that is, when $\theta = h$; the greatest deflection is therefore—

$$\Psi = \frac{3\varrho c^2}{4\delta a^2} \cdot \frac{1}{h}$$

We may express Ψ in seconds, and put m for the actual depth of A in miles, c being also expressed in miles; then—

$$\Psi = \frac{3}{4} \cdot \frac{\varrho}{\delta} \cdot \frac{162}{\pi} \cdot \frac{c^2}{m}$$

$$= 38''\!\cdot\!7 \frac{\varrho \cdot c^2}{\delta \cdot m} \tag{31}$$

If y be the height of any point of the wave surface above the undisturbed surface, g the maximum value of y—

$$\log \frac{a + g}{a + y} = \frac{3\,\varrho\,c^2}{4\,\delta\,a^2} \log \frac{h^2 + \theta^2}{h^2}$$

$$\therefore \frac{g - y}{a} = \frac{3\,\varrho\,c^2}{4\,\delta\,a^2} \log \frac{h^2 + \theta^2}{h^2}$$

$$g - y = ah\,\Psi \log\left(1 + \frac{\theta^2}{h^2}\right) \qquad (32)$$

If θ be large compared with h—

$$g - y = 2\,ah\,\Psi \log \frac{\theta}{h}$$

32. If the excess of the density of the disturbing mass above that of the adjacent matter be equal to half the mean density of the earth, and $g - y$ be expressed in feet—

$$\Psi = 19''{\cdot}3\,\frac{c^2}{m}$$

$$g - y = c^2 \log\left(1 + \frac{\theta^2}{h^2}\right)^{\frac{1}{2}}, \textit{ approximately.}$$

Suppose $c = \frac{1}{2}$, $m = 1$, then the greatest deflection $= 4''{\cdot}8$, and—

At 10 miles $g - y = 0{\cdot}58$ feet.
At 20 miles $g - y = 0{\cdot}75$ feet.
At 30 miles $g - y = 0{\cdot}85$ feet.

In this case therefore, also, a very large deviation of the plumb-line may be due to an irregularity of the surface of which the magnitude is small compared with the dimensions of the earth.

33. Let us now examine the curvature of this wave. Returning to the equation of the curve, we have—

$$\frac{dr}{d\theta} = -\,a\,\frac{3\,\varrho\,c^2}{2\,\delta\,a^2}\,\frac{\theta}{h^2 + \theta^2}$$

$$= -\,2\,ah\,\Psi\,\frac{\theta}{h^2 + \theta^2}$$

$$\frac{d^2r}{d\theta^2} = -\,2\,ah\,\Psi\,\frac{h^2 - \theta^2}{(h^2 + \theta^2)^2}$$

If R be the radius of curvature—

$$\mathrm{R} = \frac{r}{1 - \frac{1}{r}\cdot\frac{d^2r}{d\theta^2}}$$

omitting the square of $\frac{dr}{d\theta}$; we may also omit $r - a$ in this expression; then—

$$\mathrm{R} = \frac{a}{1 + 2\,h\,\Psi\,\frac{h^2 - \theta^2}{(h^2 + \theta^2)^2}} \qquad (33)$$

This will be a maximum or minimum according as—

$$\frac{h^2 - \theta^2}{(h^2 + \theta^2)^2}$$

is a minimum or a maximum: the differential coefficient of this is $- 2\,\theta\,(3\,h^2 - \theta^2)\,(h^2 + \theta^2)^{-3}$, therefore the values required correspond to $\theta^2 = 3\,h^2$ and to $\theta = 0$, so that—

$$\text{Maximum R} = \frac{a}{1 - \dfrac{\Psi}{4\,h}}$$

$$\text{Minimum R} = \frac{a}{1 + \dfrac{2\,\Psi}{h}}$$

The difference of these values is—

$$\frac{\dfrac{9\,\Psi}{4\,h}\,a}{1 + \dfrac{7}{4}\,\dfrac{\Psi}{h} - \dfrac{1}{2}\left(\dfrac{\Psi}{h}\right)^2}$$

or if Ψ be expressed in seconds, and m be the depth in miles of the centre of the disturbing range of matter, the difference of the greatest and least radii of curvature is, in miles—

$$\frac{175\,\dfrac{\Psi}{m}}{1 + \cdot 034 \cdot \dfrac{\Psi}{m} - \cdot 00019\left(\dfrac{\Psi}{m}\right)^2} \tag{34}$$

The disturbance of the curvature is therefore nearly proportional to the area of the section of the disturbing mass, and inversely as the square of its depth below the surface.

34. The maximum effect of the disturbing mass upon the number of vibrations (n) of the pendulum is—

$$2\,\frac{dn}{n} = \frac{3\,\varrho\,c^2}{2\,\delta\,a^2}\;\frac{1}{h} = 2\,\Psi$$

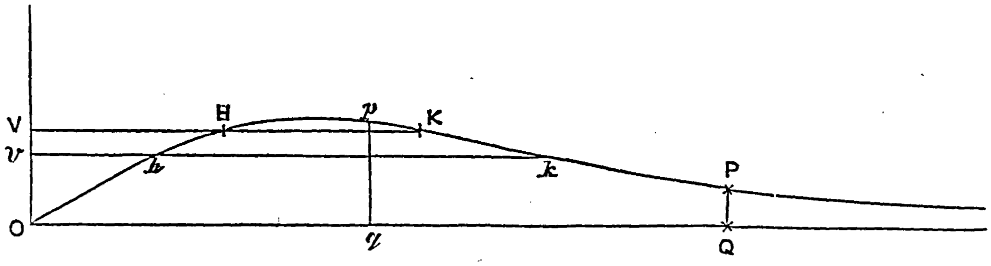
Therefore if Ψ be expressed in seconds—

$$\text{N} = \Psi \cdot \frac{2\,\pi}{15} \tag{35}$$

so that in this case, as in the preceding, the maximum increase of vibrations per day in the seconds pendulum is about half the number of seconds in the maximum deflection.

35. If we wish to have a fair representation of the relative effect of this disturbance upon the plumb-line and upon the pendulum, we should not compare the maximum effects upon either, as there is an infinite number of chances against the maximum being observed in any case.

Let us suppose, then, that a point is taken at random *within a certain distance* from the crest of the wave, and let it be required to ascertain the probable amount of deflection at such a point. The probable deflection will evidently vary with the given distance; if that distance be very small, the probable deflection will be very small; as the distance increases the probable deflection increases; but when the distance is great and increases, then it is clear that the probable deflection will decrease; within a distance that may be determined the deflection to be expected will therefore be a maximum.



Let the curve OHKP be such that at any distance OQ from O the ordinate QP shall represent the deflection at that point, O corresponding to the crest of the wave where the deflection is zero. Draw OV perpendicular to OQ, and through any point *v* draw *vhk* parallel to OQ, cutting the curve in *h* and *k*. If the point taken at random between O and Q (O and Q being the limits) fall within *hk*, the deflection will be greater than O*v*, otherwise it will be less; consequently—

$$\frac{hk}{OQ} \text{ and } 1 - \frac{hk}{OQ}$$

are the probabilities that the deflection that will be encountered will exceed or fall short of O*v*. If we take O*v* such that $hk = \frac{1}{2}$ OQ, which may be done by means of the equation to the curve, then the probabilities of exceeding or falling short of O*v* are equal, and consequently O*v* represents the probable deflection.

If V be determined so that VH = HK, being parallel to OQ, then as P moves up to K, O*v* increases to OV, but if P be taken to the left of K the probable deflection will begin to decrease. The distance VK is therefore that within which, a point being taken at random, the greatest amount of deflection is to be expected.

If O, *q* be the stated limits, then the probable deflection is evidently that which corresponds to the abscissa $\frac{1}{2}$ O*q*, for, for all points in the right half of O*q* the deflection is greater, and for all points in the left half of O*q* the deflection is less, than at the middle point.

36. In the case we have last considered, if ψ be the deflection at any distance x from the crest of the wave, Ψ the greatest value—

$$\psi = \frac{2\,x\,m}{x^2 + m^2}\,\Psi$$

For each value of ψ there are two values of x; these values are determined from the quadratic—

$$x^2 - 2\,m\,\frac{\Psi}{\psi}\,x + m^2 = 0$$

from which we have—

$$x_2 = m\,\frac{\Psi + \sqrt{\Psi^2 - \psi^2}}{\psi}$$

$$x_1 = m\,\frac{\Psi - \sqrt{\Psi^2 - \psi^2}}{\psi}$$

And between these values of x the deflection is greater than without them. These values are equal when $\psi = \Psi$, as is otherwise evident. When $x_2 = 2\,x_1$, which determines the point K in the curve—

$$\Psi + \sqrt{\Psi^2 - \psi^2} = 2\,\Psi - 2\sqrt{\Psi^2 - \psi^2}$$

$$\therefore \psi = \Psi \cdot \frac{2\sqrt{2}}{3}$$

The corresponding value of $x_2 = \mathrm{VK} = m\sqrt{2}$: consequently if the point be selected at random within this distance of the crest of the wave, the probable deflection will be the greatest, and equal to $\frac{94}{100}$ of the maximum deflection.

If r be the given distance ($> m\sqrt{2}$) within which the point is to be taken, the probabilities that the deflection will exceed or fall short of ψ are—

$$2\frac{m}{r}\frac{\sqrt{\Psi^2 - \psi^2}}{\psi} \quad ; \quad 1 - \frac{2\,m}{r}\frac{\sqrt{\Psi^2 - \psi^2}}{\psi}$$

When these are equal—

$$\frac{4m}{r}\sqrt{\frac{\Psi^2}{\psi^2} - 1} = 1$$

$$\frac{\Psi^2}{\psi^2} = 1 + \frac{r^2}{16\,m^2}$$

$$\therefore \psi = \frac{\Psi}{\sqrt{1 + \left(\frac{r}{4\,m}\right)^2}}$$

This, then, is the deflection that may be expected in taking at random a point within the distance r from the crest of the wave, m being the depth of the disturbing mass.

37. Let us now obtain the number of increased vibrations per day that may be expected within the same limits. The ratio of the increased vertical attraction to the attraction of the earth is at any distance x—

$$2\,\frac{dn}{n} = \frac{3\,\varrho\,c^2}{4\,\delta\,a^2} \cdot \frac{2\,a\,m}{m^2 + x^2}$$

Put $dn = \nu$, then—

$$\nu = \frac{n\,m^2}{m^2 + x^2}\,\Psi$$

It is evident that for all positive values of x, $\frac{d\nu}{dx}$ is negative, and ν diminishes continuously as x increases. If therefore we form the curve whose ordinate is ν to the abscissa x, it is clear that all values of ν between $x = 0$ and $x = \frac{1}{2}\,r$ are greater, and all values of ν between $x = \frac{1}{2}\,r$ and $x = r$ are less, than the value of ν corresponding to $x = \frac{1}{2}\,r$, or—

$$\nu = \frac{n\,\Psi}{1 + \frac{r^2}{4\,m^2}}$$

and this, therefore, is the probable increase to the number of vibrations per day to be expected within the distance r of the crest of the wave, taking a point at random.

Comparing this with the probable value of ψ, we have—

$$\frac{\nu}{\psi} = n \frac{(1 + \frac{1}{16} q^2)^{\frac{1}{2}}}{1 + \frac{1}{4} q^2}$$

where $q = \frac{r}{m}$ and is supposed greater than $\sqrt{2}$.

If ψ be expressed in seconds, we must put for ψ—

$$\psi \frac{\pi}{180 \cdot 60 \cdot 60} = \psi \frac{2\pi}{15 n}$$

and therefore—

$$\frac{\nu}{\psi} = \frac{2\pi}{15} \cdot \frac{(q^2 + 16)^{\frac{1}{2}}}{q^2 + 4}$$

§ III.

Observations for determining the Attractions of Mountains.

The first attempt to determine the amount of the attraction of a mountain was made by the French Academicians MM. Bouguer and De la Condamine during their operations in Peru. The great mass of the different mountains in that country invited the experiment, for it appeared that if the attraction of a mountain were at all sensible it could be nowhere apparent in greater amount than among these mountains, though on the other hand, considering the volcanic nature of the country, it was a matter of much doubt whether the probable existence of large internal cavities might not diminish the weight of the result derived from the observations. For this purpose the mountain *Chimboraço** was chosen by M. Bouguer: its height above the level of the sea is 3217 toises, but above the plane of the surrounding country 1700 or 1800 toises; at the base the diameter is about ten or twelve thousand toises, and at half its height the diameter is about 3500 toises. The summit is flat, and three or four hundred toises in diameter.

From these dimensions M. Bouguer estimated the volume of the mountain to be about 20000000000 cubic toises, or the one 7400000000th part of the mass of the earth, whence he inferred that at the distance of 1700 or 1800 toises from the centre of the mass its attraction would amount to the 2000th part of the attraction of the earth, or would produce a deviation of 1′ 43″.

In order to ascertain by observation whether the actual amount of deviation would at all correspond with this rough calculation, a station was chosen on the south side of the mountain as near to the centre of gravity of the mass as convenient, and a second station was fixed upon at about 3700 toises to the east of the first, at which point it was conceived that the meridional component of the attraction of the mass would be very small. The meridional altitudes of several stars were observed at both stations with the quadrant, but unfortunately the observations were few and very discordant, owing to the very inconvenient circumstances in which the observers were placed. The result of the observations, such as they were, gave 7″·5 for the deviation produced by the attraction of the mountain, which is probably erroneous to a considerable extent, and very little weight was attached even by the observers themselves to this experiment.

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* "La Figure de la Terre, déterminée par les Observations de Messieurs Bouguer et de la Condamine, &c. 1749." Page 368.

Schehallien.

In 1772 the Astronomer Royal, Dr. Maskelyne, undertook to repeat the experiment, and after careful search, and the examination of many hills, *Schehallien*, a mountain of 3000 feet in height, in Perthshire, was selected as the most favourable that could be found in the country. The form of *Schehallien* is regular; the summit is an inclined ridge, presenting from some points of view a conical appearance, and rising to the height of about 2000 feet above the general level of the surrounding country. A station was selected on the south side of the hill at about 1200 feet below the vertex, and another station on the north side at about 250 feet below the level of the former. At these stations in 1774 Dr. Maskelyne determined the latitudes with a sector made by Sisson. At the south station 73 observations were made face east and 93 face west; at the north station 68 observations were made face west, and 100 face east. The difference of latitude was found by Dr. Maskelyne to be 54″·60, which agrees with the result given by the Baron de Zach in his work on the "Attraction of Mountains," from his own reduction of all the observations.

The survey was made in the years 1774–75 and 76, by the direction and partly under the inspection of Dr. Maskelyne. Two base-lines were measured with deal rods whose lengths were ascertained in terms of the Royal Society's brass 5-feet scale; the length of the base on the south side of the hill was found to be 3011·4 feet, that on the north side, in Rannoch, was found to be 5895·4. By means of a triangulation founded on these bases, the distance between the parallels passing through the two astronomical stations was found to be 4364·4 feet, which corresponds to a difference of latitude of 42″·94.

This last quantity, determined by geodetical operations and independent of the attraction of the mountain, shows that the effect of that attraction was to increase the astronomical amplitude by $54''\cdot60 - 42''\cdot94 = 11''\cdot66$, which last quantity is the sum of the angles through which the plumb-line was drawn inwards towards the mountain at the two stations.

Mimet.

In 1814 the Baron de Zach published a work on the Attraction of Mountains,* containing a detailed account of observations made by him in the south of France for ascertaining the actual amount of the deviation caused by a disturbing mass. The mountain *Mimet* selected by him for the purpose, is nine or ten miles north of Marseilles, its height is about 2500 feet above the level of the sea, and one of the observatories was situated on the southern face of the mountain, where the ruins of a convent offered some shelter. The second station was in the small island of *Planier*, nine miles south-west of Marseilles, at which distance the attraction due to *Mimet* would be insensible; so that the attraction of the mountain would be represented by the excess of the geodetical over the astronomical difference of latitude of the two stations.

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* "L'Attraction des Montagnes, et ses Effets sur les fils à plomb, &c. Par le Baron de Zach." Avignon, 1814.

The observations for latitude were made with a repeating circle by Reichenbach, of twelve inches diameter. The three stars observed were on the same side of the zenith, and were observed 874 times at the northern station, and 896 times at the island of *Planier*.

The geodetical operations for connecting the two stations were conducted with great care and minuteness. The base-line was measured with a system of three deal rods of ten feet in length, which were guarded against moisture of the atmosphere by coats of oil and paint; the final length of the line was 1182 toises. The angles of the triangles were observed (each being repeated ten times) with an 8-inch repeating theodolite by Reichenbach. The mean error in the sums of the observed angles of the triangles is $3''\cdot 17$. The direction of the meridian was determined both at *Mimet* and *Planier* by numerous observations of the sun.

From these observations the difference of latitude was calculated, using an ellipticity of $\frac{1}{310}$, to be $12'\ 13''\cdot 11$, which exceeded the amplitude derived from the astronomical observations by $1''\cdot 98$, which is the apparent effect of the attraction of the mountain *Mimet*. But it has been shown by M. Arago, in the Additions to the *Connaissance des Tems*, 1819, that the repeating circle used by the Baron de Zach for the purpose, was scarcely adequate to the determination of so small a quantity, and that consequently little weight should be attached to the result of these operations.

ARTHUR'S SEAT, EDINBURGH.

The next experiment upon the attraction of mountains is that by Lieutenant-Colonel James, Royal Engineers, in 1855. The hill is the well-known "Arthur's Seat" in Edinburgh, a compact and abrupt mass rising to the height of 822 feet above the sea. In the choice of this hill there was a second object in view, also in connection with the subject of local disturbances. It has been known for some time that the astronomical latitude of the Calton Hill Observatory, Edinburgh, differs very materially—between five and six seconds—from the latitude inferred by geodetical operations from several other astronomically-determined points not very far distant. This circumstance rendered it very desirable that the latitude of some other points in the immediate vicinity of the Observatory should be determined astronomically, thus to trace the course and magnitude of the disturbance. In the selection of three stations on Arthur's Seat, this object was accomplished, as well as the determination of the attraction of the hill itself: and as a third result, a value of the mean density of the earth was obtained. In form, Arthur's Seat is particularly well adapted for the problem of attraction; the mass, however, is considerably smaller than could be desired, especially for the determination of the density of the earth.

A very convenient place offered itself for the southern station, almost exactly on the meridian passing through the apex of the hill (see plate XX.), and close on the south side of the road called the "Queen's Drive." This station is nearly on the contour '375,' and therefore at about one-third of the effective height of the hill. In selecting stations for the observation of the attraction of a hill, those two points have to be determined, one on the north side and the other on the south, at which the horizontal components of the attraction of the mass resolved in the direction of the meridian are the greatest. These points will not generally be at the base of the hill, but between that and the top, and can only be determined in any particular case by an examination of the ground. In such a mass as Arthur's Seat, the maximum attraction is probably at about a third of the effective altitude, that is, the altitude above the adjacent ground. On the north side of this hill there is more difficulty than on the south in obtaining a favourable position; that chosen for the purpose, and at which it was supposed that the attraction would not be far from the maximum, is in a small hollow to the east of and above the Ruins of St. Anthony's Chapel, and about fifty feet below the horizontal plane passing through the south station. A third station was also chosen on the top of the hill: this point is distant fourteen feet from the Trigonometrical Station, and bearing 18° north-west.

The observations for the determination of the latitudes of these three points were made with Airy's Zenith Sector in the months of August, September, and October, the weather being generally very favourable. The apparent declinations of the stars were obtained principally from the British Association's Catalogue and the constants therein given. The refractions were computed from the tables given in the Appendix to the Greenwich Observations for 1836.

The following table contains an abstract of the results:—

Name of Star.	LATITUDES.						AMPLITUDES.		
	South Station.	No. of Obs.	Summit Station.	No. of Obs.	North Station.	No. of Obs.	South Station and North Station.	SummitStation and South Station.	North Station and SummitStation
	° ′ ″		° ′ ″		° ′ ″		° ′ ″	° ′ ″	° ′ ″
59 Cygni . . f′	55 56 26·63	4	55 56 43·92	8				0 0 17·29	
3416 Groombridge.	25·68	2	42·30	10				16·62	
23 Cephei . . ε	27·36	4	43·93	2				16·57	
3692 Groombridge.	23·09	1	42·98	2				19·89	
3 Lacertæ . . β	28·39	2	46·34	4				17·95	
37 Cassiopeiæ . δ	24·82	2	43·47	6				18·65	
39 Cassiopeiæ . χ	27·27	6	44·17	8				16·90	
362 Groombridge .	25·63	3	42·52	8				16·89	
44 Cassiopeiæ . .	25·26	6	42·85	8				17·59	
1 Persei. . . .	24·98	4	42·07	8				17·09	
9 Cephei . . .			45·52	6	55 57 10·34	4			0 0 24·82
13 Cephei . . μ			42·33	8	8·52	4			26·19
25 Cephei . . .			44·37	6	9·61	10			25·24
4077 Groombridge.			45·07	2	10·50	2			25·43
3224 B. F. . . .			44·96	4	10·15	4			25·19
3 Persei. . . .			42·11	6	6·99	2			24·88
531 Groombridge .			46·69	4	11·73	2			25·04
13 Persei . . θ			43·71	6	8·75	2			25·04
590 Groombridge .			43·93	2	8·05	2			24·12
391 B. F. . . .			46·60	2	10·97	2			24·37
634 Groombridge .			44·47	1	6·64	2			22·17
33 Persei . . α			44·34	4	9·25	2			24·91
45 Draconis . d	27·59	4			10·06	2	0 0 42·47		
46 Draconis . c	24·73	5			7·48	4	42·75		
2677 Groombridge.	22·38	3			4·72	2	42·34		
47 Draconis . o	25·86	6			7·97	2	42·11		
2718 Groombridge.	25·89	2			8·72	2	42·83		

Name of Star.	LATITUDES.						AMPLITUDES.		
	South Station.	No. of Obs.	Summit Station.	No. of Obs.	North Station.	No. of Obs.	South Station and North Station.	SummitStation and South Station.	North Station and SummitStation
	° ′ ″		° ′ ″		° ′ ″		° ′ ″	° ′ ″	° ′ ″
48 Draconis . .	55 56 27·80	3			55 57 10·85	2	0 0 43·05		
49 Draconis . .	23·15	4			8·15	2	45·00		
2777 Groombridge.	28·52	2			10·90	2	42·38		
53 Draconis . .	27·95	2			9·29	6	41·34		
54 Draconis . .	27·64	2			10·05	2	42·41		
2836 Groombridge.	26·16	2			9·33	6	43·17		
10 Cygni . . i^2	26·75	2			8·70	2	41·95		
2865 Groombridge.	27·08	2			10·02	4	42·94		
2894 Groombridge.	24·42	2			7·24	2	42·82		
13 Cygni . . θ	23·67	2			5·90	2	42·23		
16 Cygni (1st) . c^1	28·23	6			10·68	7	42·45		
16 Cygni (2nd) . c^2	26·70	6			9·18	7	42·48		
43 Cygni . . ω^1	23·20	6			7·03	2	43·83		
1137 Oxford, 1843.	26·90	4			9·57	4	42·67		
2 Cephei . . θ	26·78	4			10·83	2	44·05		
20 Andromedæ. ψ	25·31	4			8·27	6	42·96		
7 Cassiopeiæ . ρ	26·64	6			9·17	8	42·53		
9 Cassiopeiæ . .	28·32	2			10·96	6	42·64		
31 Groombridge .	26·63	8			9·32	8	42·69		
8 Cephei . . β	25·77	3			9·38	6	43·61		
3524 Groombridge.	26·48	2			10·83	2	44·35		
80 Cygni . . π^1	25·65	4			7·84	2	42·19		
20 Cygni . . d	28·97	6	55 56 45·46	6	11·71	6	42·74	0 0 16·49	0 0 25·25
2996 Groombridge.	25·98	6	43·28	2	8·01	6	42·03	17·30	24·73
66 Draconis . .	26·35	6	43·26	6	8·42	6	42·07	16·91	25·16
30 Cygni . . o^1	28·12	2	45·90	2	*6·63	4	*38·51	17·78	*20·73
31 Cygni . . o^2	27·90	2	45·59	2	10·33	2	42·43	17·69	24·74
68 Draconis . .	26·45	2	43·47	4	8·60	4	42·15	17·02	25·13
71 Draconis . .	27·34	10	44·13	8	9·75	8	42·41	16·79	25·62
3205 Groombridge.	27·99	3	44·30	4	9·76	4	41·77	16·31	25·46
46 Cygni . . ω^3	27·46	3	43·58	6	9·21	3	41·75	16·12	25·63
50 Cygni . . α	26·72	10	44·10	5	9·66	8	42·94	17·38	25·56
3 Cephei . . η	26·70	6	44·12	8	9·50	4	42·80	17·42	25·38
7268 B. A. C. . .	28·93	6	45·67	4	10·76	6	41·83	16·74	25·09
63 Cygni . . f^2	28·23	8	44·90	10	10·49	8	42·26	16·67	25·59
3415 Groombridge.	26·31	10	42·47	5	8·10	4	41·79	16·16	25·63
5 Cephei . . α	27·04	7	43·80	12	9·25	8	42·21	16·76	25·45
1256 Oxford, 1842.	25·32	2	43·82	4	9·53	2	44·21	18·50	25·71
71 Cygni . . g	25·56	3	42·76	4	8·34	1	42·78	17·20	25·58
10 Cephei . . .	25·63	6	42·70	8	8·70	10	43·07	17·07	26·00
3606 Groombridge.	27·53	7	41·93	1	8·22	8	40·69	14·40	26·29
3652 Groombridge.	27·58	8	43·47	4	10·33	12	42·75	15·89	26·86
7736 B.A.C. (1st star)	27·52	2	44·54	6	10·43	12	42·91	17·02	25·89
7736 B.A.C. (2nd star)	28·18	3	45·18	6	11·15	12	42·97	17·00	25·97
27 Cephei (1st) δ^1	29·35	2	45·81	4	10·58	10	41·23	16·46	24·77
27 Cephei (2nd) δ^2	27·63	2	45·12	6	10·19	10	42·56	17·49	25·07
9 Lacertæ . . .	27·93	4	44·23	2	10·14	6	42·21	16·30	25·91
30 Cephei . . .	25·38	4	44·56	2	10·65	2	45·27	19·18	26·09
3882 Groombridge.	24·25	4	42·26	6	7·12	8	42·87	18·01	24·86
3147 B. F. . . .	26·68	4	43·88	6	9·04	8	42·36	17·20	25·16
1 Cassiopeiæ . .	25·03	4	42·40	2	8·26	10	43·23	17·37	25·86
8 Andromedæ . .	25·71	4	42·48	4	8·57	10	42·86	16·77	26·09

* Not used in mean.

Name of Star.	LATITUDES.						AMPLITUDES.		
	South Station.	No. of Obs.	Summit Station.	No. of Obs.	North Station.	No. of Obs.	South Station and North Station.	SummitStation and South Station.	North Station and SummitStation
4 Cassiopeiæ . .	° ′ ″ 55 56 28·45	4	° ′ ″ 55 56 45·05	4	° ′ ″ 55 57 11·02	8	° ′ ″ 0 0 42·57	° ′ ″ 0 0 16·60	° ′ ″ 0 0 25·97
16 Andromedæ . λ	27·89	4	44·37	2	10·24	10	42·35	16·48	25·87
5 Cassiopeiæ . τ	26·46	4	43·19	4	9·71	4	43·25	16·73	26·52
12 Cassiopeiæ . .	26·96	10	43·09	2	9·53	6	42·57	16·13	26·44
15 Cassiopeiæ . κ	27·59	8	45·10	2	10·02	6	42·43	17·51	24·92
18 Cassiopeiæ . α	27·39	8	44·51	6	10·15	6	42·76	17·12	25·64
24 Cassiopeiæ . η	26·32	10	42·73	8	9·30	4	42·98	16·41	26·57
27 Cassiopeiæ . γ	26·65	10	43·54	10	9·32	4	42·67	16·89	25·78
30 Cassiopeiæ . μ	26·42	10	43·51	8	8·94	4	42·52	17·09	25·43
287 Groombridge .	27·03	4	44·21	8	9·10	2	42·07	17·18	24·89
34 Cassiopeiæ . φ	28·63	10	45·72	8	10·51	2	41·88	17·09	24·79
5 Persei . . h	29·00	2	46·00	6	12·74	2	43·74	17·00	26·74

If we assign to the result given by any one star, a weight equal to the number of observations of that star, we get the following results for the latitudes of the three stations:—

Station.	Latitude.	No. of Obs.
South	° ′ ″ 55 56 26·69	427
Summit . . .	55 56 43·95	425
North	55 57 9·50	411

These results are affected by the errors of the assumed declinations: in order to avoid these the following method is adopted:—Put x for the amplitude or difference of latitude of the first two stations, y for the amplitude of the second and third. Let the stars observed at the first and second stations only, give the values $x = a$, $x = a'$, $x = a''$ · · · · let the stars observed at the second and third stations only, give $y = b$, $y = b'$, $y = b''$ · · · · let the stars observed at the first and third stations only, give $x + y = c$, $x + y = c'$, $x + y = c''$ · · · · and lastly, let the stars observed at all three stations give $x = a_1$, $y = b_1$, $x = a_1'$, $y = b_1'$ · · · · · · Let d, e, and the same letters accented, be taken to represent the number of times the stars of the first set are observed at the first and second stations respectively. Let f, g, and hk, represent the same quantities for stars of the second and third sets; and let n, p, q, and the same letters accented, be taken to represent the number of times the stars of the third set are observed at the three stations respectively.

The probable values of x and y are those which render a minimum the function—

$$\Sigma\left(\frac{de}{d+e}(x-a)^2\right) + \Sigma\left(\frac{fg}{f+g}(y-b)^2\right) + \Sigma\left(\frac{hk}{h+k}(x+y-c)^2\right)$$

$$+ \Sigma\left(\frac{np}{n+p}(x-a_1)^2\right) + \Sigma\left(\frac{pq}{p+q}(y-b_1)^2\right)$$

Making the differential coefficients of this quantity with respect to x and y separately $= 0$, we get—

$$Hx + Ky - L = 0$$
$$Kx + My - N = 0$$

where—

$$H = \Sigma\left(\frac{de}{d+e}\right) + \Sigma\left(\frac{hk}{h+k}\right) + \Sigma\left(\frac{np}{n+p}\right); \; M = \Sigma\left(\frac{fg}{f+g}\right) + \Sigma\left(\frac{hk}{h+k}\right) + \Sigma\left(\frac{pq}{p+q}\right)$$
$$K = \Sigma\left(\frac{hk}{h+k}\right)$$

The quantities L and N are affected by the errors of observation. Let $(d\cdot\varepsilon)$ be taken to represent the sum of the errors of the d observations of a star of the first set at the first station, and let $(e\cdot\varepsilon)$ be the sum of the e errors of observation of the same star at the second station. Then, denoting the other sums of errors in a corresponding manner—

$$L = \Sigma\left(\frac{ed}{e+d}a\right) + \Sigma\left(\frac{pn}{n+p}a_1\right) + \Sigma\left(\frac{hk}{h+k}c\right)$$
$$+ \Sigma\left(\frac{d(e\cdot\varepsilon) - e(d\cdot\varepsilon)}{d+e}\right) + \Sigma\left(\frac{n(p\cdot\varepsilon) - p(n\cdot\varepsilon)}{n+p}\right) + \Sigma\left(\frac{h(k\cdot\varepsilon) - k(h\cdot\varepsilon)}{h+k}\right)$$

$$N = \Sigma\left(\frac{fg}{f+g}b\right) + \Sigma\left(\frac{pq}{p+q}b_1\right) + \Sigma\left(\frac{hk}{h+k}c\right)$$
$$+ \Sigma\left(\frac{f(g\cdot\varepsilon) - g(f\cdot\varepsilon)}{f+g}\right) + \Sigma\left(\frac{p(q\cdot\varepsilon) - q(p\cdot\varepsilon)}{p+q}\right) + \Sigma\left(\frac{h(k\cdot\varepsilon) - k(h\cdot\varepsilon)}{h+k}\right)$$

If we require the probable error of $x + \mu y$, μ being any given multiplier, we have—

$$x + \mu y = \frac{L(M - \mu K) + N(\mu H - K)}{MH - K^2}$$

To find the probable error of $x + \mu y$, we must form the sum of the squares of the coefficients of the errors as they exist in this expression. This sum will be found equal to—

$$\frac{(M - \mu K)^2 H + (\mu H - K)^2 M + 2(M - \mu K)(\mu H - K)(K - P)}{(MH - K^2)^2}$$

where—

$$P = \Sigma\frac{npq}{(n+p)(p+q)}$$

Consequently if E be the probable value of an error of observation (as determined by the discrepancies between the mean and individual results for latitude given by a single star), the probable error of $x + \mu y$ is—

$$\frac{E}{MH - K^2}\left\{M\left(HM - K^2 + 2KP\right) - 2\mu\left(K(MH - K^2) + P(MH + K^2)\right) + \mu^2\left(MH - K^2 + 2KP\right)H\right\}^{\frac{1}{2}}$$

The values of H, M, K, P and L, N are—

$$H = 168{\cdot}93 \quad M = 168{\cdot}52 \quad K = 46{\cdot}06$$
$$L = 362{\cdot}40 \quad N = 182{\cdot}20 \quad P = 49{\cdot}34$$

We have, therefore, for the determination of the astronomical amplitudes x and y,

$$\begin{aligned} 168{\cdot}93\,x + 46{\cdot}06\,y - 362{\cdot}40 &= 0 \\ 46{\cdot}06\,x + 168{\cdot}52\,y - 182{\cdot}20 &= 0 \end{aligned} \qquad (a)$$

by the solution of which, $x = 17''{\cdot}00$; $y = 25''{\cdot}53$; $x + y = 42''{\cdot}53$.

At the South Station the sum of the squares of the apparent errors of observation, 427 in number, is 265·6; at the Summit, the sum of the squares of the apparent errors of observation, 425 in number, is 202·7; and at the North Station the sum of the squares of the apparent errors of observation, 411 in number, is 243·8. From the sum of the whole we find the mean square of error $= 0''{\cdot}56$, and therefore the probable error of an observation $= 0{\cdot}67\sqrt{0{\cdot}56} = 0''{\cdot}50$.

$$\text{Probable error of } x + \mu y = \frac{0''{\cdot}50}{263{\cdot}4}\left\{520{\cdot}5 - 544{\cdot}7\,\mu + 522{\cdot}0\,\mu^2\right\}^{\frac{1}{2}}$$

$$= 0''{\cdot}043\left\{1 - 1{\cdot}05\,\mu + \mu^2\right\}^{\frac{1}{2}}$$

The probable errors, therefore, of the astronomical determinations of the amplitudes x, y, or $x + y$, are each equal to ± 0″·04.

Geodetical Amplitudes.—The stations were very carefully connected by a small network of triangulation based upon the minor triangulation of the Ordnance Survey of the county of Edinburgh. It seems unnecessary to give more than the results of this operation, which are these:—

From	To	Distance in Feet.	Bearing.
Summit (Trigonometrical Station)	South Station .	1426·7	° ′ ″ 179 42 7
	North Station .	2490·0	6 0 17

The bearings are measured from North round by East. The corresponding amplitudes (derived from Airy's Elements of the Figure of the Earth) are 14″·06 and 24″·40. But the Summit Station in the last table is the Trigonometrical Station, which differs 0″·13 in latitude from the Sector Station; therefore the amplitudes corresponding to the astronomical observations are 14″·19 and 24″·27.

Stations.	Astronomical Amplitude.	Geodetical Amplitude.	A—G.
South and Summit . .	″ 17·00	″ 14·19	″ 2·81
Summit and North . .	25·53	24·27	1·26
South and North . . .	42·53	38·46	4·07

It appears, therefore, that the sum of the angles through which the direction of the plumb-line is deflected by the attraction of the hill at the North and South Stations amounts to 4″·07, with a very small probable error.

Mean Density of the Earth.

At the same time that the attraction of Schiehallien was observed astronomically, trigonometrical operations were carried on over its surface, by means of which its mass might be accurately known and its attraction computed and compared with the quantity observed. The altitudes of a great many points on the hill were carefully determined and laid down upon a plan showing also the position of the Observatories. Through each station on the plan a series of 48 radial lines were drawn, the sine of the azimuth of the $(n + 1)^{\text{th}}$ line being equal to the sine of the azimuth of the n^{th} line $\pm \frac{1}{12}$. A series of concentric circles was also drawn round each station as centre, the radius of the $(n + 1)^{\text{th}}$ circle being equal to the radius of the n^{th} circle $+ 666 \cdot 6$ feet. Thus for each station the mass of the hill was divided into 960 prisms. The attraction of any one of these prisms is $\frac{1}{12}\,\varrho \sin \epsilon \times 666 \cdot 6$, where ϱ is the density of the mass and ϵ the elevation of the top of the prism: the deflection for either station is therefore—

$$0'' \cdot 1310 \, \frac{\varrho}{\delta} \, \Sigma (\sin \epsilon)$$

where δ is the mean density of the earth. Dr. Hutton found for the North Station, $\Sigma (\sin \epsilon) = 88 \cdot 644$, and for the South, $\Sigma (\sin \epsilon) = 69 \cdot 967$: this gave—

$$\text{Deflection at North Station} = 11'' \cdot 61 \, \frac{\varrho}{\delta}$$

$$\text{,, South Station} = 9'' \cdot 17 \, \frac{\varrho}{\delta}$$

The sum of these must be equal to the observed quantity $11'' \cdot 66$; this gives $20 \cdot 78 \, \varrho = 11 \cdot 66 \, \delta$ $\therefore \delta = 1 \cdot 78 \, \varrho$. If therefore the density of the hill be 2·75, the mean density of the earth would be 4·9.

It is to be regretted that the inner circle of 666 feet at each station was not subdivided, as it is probable that this might have modified the result sensibly. Had the calculation been extended to a greater distance, the high ground to the south might have diminished the calculated attraction by an appreciable quantity which would have the effect of increasing the resulting value of the mean density of the earth. This calculation was afterwards carefully repeated by Playfair in connection with his lithological survey of the mountain, and from it he obtained 4·713 as his final result.—(*Philosophical Transactions*, 1811, page 376.)

The experiments of Cavendish with the Torsion Balance gave 5·448 for the mean density of the earth (*Memoirs of the Royal Astronomical Society*, Vol. XIV., page 91), with a probable error of ± 0·033. These experiments were repeated in 1836, with some improvements, by Professor Reich, of Freiberg in Saxony, and from his calculations he obtained 5·44, with a probable error of ± 0·023. The volume of the Memoirs of the Royal Astronomical Society mentioned above contains the full account, by Mr. Baily, of his own experiments with the Torsion Balance, conducted at the public expense (a grant of 500*l.* having been applied for and obtained by the Astronomer Royal), and with every imaginable refinement. The result of these valuable experiments was 5·660, with a probable error of ± 0·003.

If we suppose the earth a sphere of radius a, having mean density δ, the force of gravity at its surface is—

$$g = \frac{4}{3}\pi a \delta$$

If we descend below the surface to a depth ah which is small in comparison with a, the force of gravity $g_{,}$ at that depth is—

$$g_{,} = \frac{\frac{4}{3}\pi a^3 \delta - 4\pi a^2 ah \varrho}{a^2 (1 - h)^2}$$

where ϱ is the density of the superficial parts of the globe; consequently—

$$\frac{g_{,}}{g} = 1 + 2h - 3h\frac{\varrho}{\delta}$$

Now if a pendulum make n vibrations per day at the surface, and $n + \sigma$ at the depth ah,

$$1 + 2\frac{\sigma}{n} = \frac{g_{,}}{g} \therefore \frac{\varrho}{\delta} = \frac{2}{3}\left(1 - \frac{\sigma}{nh}\right)$$

So that by observing the rate of a clock at a known depth below the surface and comparing it with another at the surface, the mean density of the earth may be compared with that of the superficial rocks. By this means the Astronomer Royal in 1855 obtained 6·566 ± 0·018 as the mean density of the earth; the acceleration of the seconds' pendulum at the depth of 1256 feet being found equal to $2^{s}{\cdot}24 \pm 0^{s}{\cdot}01$.—(*Philosophical Transactions*, 1856.)

Mean Density derived from the Attraction of Arthur's Seat.

If we add the amplitudes as determined from equations (a) to the observed latitude of the South Station, we get the following values for the latitudes of the three stations:—

	°	′	″
Latitude of South Station	55	56	26·69
" Summit "	55	56	43·69
" North "	55	57	9·22

Now the observed latitude of the Calton Hill Observatory is 55° 57′ 23″·20, and the geodetical amplitude of the Observatory and the Trigonometrical Station on Arthur's Seat is 39·64; consequently the geodetical differs from the astronomical only by 0″·38. It is therefore evident that the disturbing force that acts in a southward direction at the Calton Hill exists at the summit of Arthur's Seat to the same amount very nearly; this may be termed the "general deflection," and exists at each of the stations distinct from the attraction of the hill.

The attraction of the hill was calculated in the same manner as in the case of Schehallien, with only this difference, that the distance of the successive circles is 500 instead of 666·6 feet. The calculation has also been extended to a much greater distance round the stations than was considered necessary for Schehallien. Round the North Station sixteen circles were drawn upon the contoured plan, having their radii in arithmetic progression, the radius of the last being 8000 feet. After these circles, another series of circles was drawn in continuation, having their radii in geometrical progression, the ratio being $\frac{7}{6}$; so that the radius of the $(16 + n)^{\text{th}}$ circle is $(\frac{7}{6})^n \times 8000$ feet. The South Station was treated in a similar manner, as also the Summit Station, with this exception, that at the latter there were only twelve circles in the first (or arithmetic) series, so that the radius of the $(12 + n)^{\text{th}}$ circle is $(\frac{7}{6})^n \times 6000$ feet.

At each of the three stations the inner circle of 500 feet was subdivided by four other circles, the common difference of whose radii was 100 feet: thus each station is surrounded by three series of circles. If we put Σ_n for the sum of the sines of the elevations of the tops of the compartments to the north of a station (up to the end of the second series of circles), Σ_s for the corresponding quantity on the south side: and put H_n for the sum of the altitudes of all the compartments (in the third series of circles) to the north, H_s for the corresponding quantity on the south side, then the deflection to the north is (by pages 584, 585) $+ 0''\cdot01964\ (\Sigma_n - \Sigma_s)\cdot z$ for the first series, $+ 0''\cdot0982\ (\Sigma_n - \Sigma_s)\cdot z$ for the second series, $+ 0''\cdot00003027\ (H_n - H_s)\cdot z$ for the third series; where z is the ratio of the mean densities of the hill and the earth.

By means of the contours on the map the following results were obtained:—

TABLE I.—*First Series of Circles.*

No. of Ring.	South Station.		Summit Station.		North Station.	
	Σ_n	Σ_s	Σ_n	Σ_s	Σ_n	Σ_s
(1)	+1·689	− 1·796	− 7·118	−3·524	−2·002	+4·867
(2)	+5·448	−11·294	−11·281	−9·164	−8·038	+4·347
(3)	+7·705	−11·808	−11·845	−8·603	−8·856	+3·159
(4)	+9·038	−11·360	−12·014	−7·436	−8·424	+2·179
(5)	+8·845	−10·001	−11·719	−6·421	−6·973	+1·932

TABLE II.—*Second Series.*

No. of Ring.	South Station.		Arthur's Seat.		North Station.	
	Σ_n	Σ_s	Σ_n	Σ_s	Σ_n	Σ_s
2	+6·614	−6·299	−10·560	−5·143	−4·644	+2·009
3	+3·187	−3·531	− 7·996	−6·634	−3·094	+1·695
4	+1·088	−2·536	− 5·850	−7·715	−2·859	+0·978
5	+0·025	−2·026	− 4·656	−6·425	−2·361	+1·028
6	−0·095	−1·713	− 4·474	−5·295	−1·954	+1·195
7	−0·088	−1·488	− 4·570	−4·549	−1·623	+0·745
8	−0·418	−1·309	− 4·209	−3·994	−1·422	−0·361
9	−0·926	−1·149	− 3·839	−3·587	−1·258	−0·747
10	−1·004	−0·985	− 3·467	−3·234	−1·176	−0·671
11	−1·037	−0·844	− 3·119	−2·931	−1·117	−0·604
12	−1·004	−0·729	− 2·883	−2·667	−1·048	−0·575
13	−0·908	−0·625			−0·996	−0·558
14	−0·843	−0·539			−0·941	−0·525
15	−0·828	−0·472			−0·935	−0·488
16	−0·806	−0·434			−0·894	−0·449

TABLE III.—*Third Series.*

No. of Ring.	South Station.		Summit Station.		North Station.	
	H_n	$H_{,}$	H_n	$H_{,}$	H_n	$H_{,}$
13			+2643	+4645		
14			+2245	+5025		
15			+1624	+5460		
16			+ 904	+6080		
17	+2385	+ 6060	+ 548	+7460	+520	+4495
18	+1685	+ 6870	+ 427	+8485	+385	+5045
19	+ 935	+ 8295	+ 148	+8845	+277	+5640
20	+ 630	+ 9010	− 92	+8550	− 15	+6475
21	+ 465	+ 8925	− 132	+9595	−200	+8000
22	+ 180	+ 8175			−320	+8175
23	− 10	+10060			−375	+8135

TABLE IV.—*Results.*

Series.	South Station.		Summit Station.		North Station.	
	$\Sigma_n-\Sigma_{,}$	Deflection.	$\Sigma_n-\Sigma_{,}$	Deflection.	$\Sigma_n-\Sigma_{,}$	Deflection.
1	$+78{\cdot}984$	$+1{\cdot}551\,z$	$-18{\cdot}829$	$-0{\cdot}370\,z$	$-50{\cdot}777$	$-0{\cdot}997\,z$
2	$+27{\cdot}636$	$+2{\cdot}714\,z$	$-\ 3{\cdot}441$	$-0{\cdot}338\,z$	$-28{\cdot}994$	$-2{\cdot}847\,z$
	$H_n-H_{,}$		$H_n-H_{,}$		$H_n-H_{,}$	
3	-51665	$-1{\cdot}565\,z$	$-\ 55810$	$-1{\cdot}691\,z$	-45693	$-1{\cdot}393\,z$

The sums of the third, fifth, and seventh columns of this table give—

Deflection at South Station $= +\ 2{\cdot}700\,z$
„ Summit „ $= -\ 2{\cdot}399\,z$
„ North „ $= -\ 5{\cdot}237\,z$

We must now assume that the disturbing force which acts in very nearly the same amount at Calton Hill and the summit of Arthur's Seat acts equally at all the three stations on the hill in a southerly direction. Thus the deflections become $+\ 2\cdot700\,z - v$; $-\ 2\cdot399\,z - v$ and $-\ 5\cdot237\,z - v$. Now let λ be the astronomical latitude of the Summit, so that the latitudes of the three stations are $\lambda - x$, λ, $\lambda + y$, and also if $\lambda_{,}$ be the geodetic latitude of the Summit, the geodetic latitudes will be $\lambda_{,} - 14\cdot19$, $\lambda_{,}$, $\lambda_{,} + 24\cdot27$. Then the deflections north will be (if we put $\lambda_{,} - \lambda = \alpha$) $\alpha + x - 14\cdot19$, α, $\alpha + 24\cdot27 - y$. By equating these with the preceding or calculated values there result—

$$v - 2{\cdot}700\,z + \alpha + x - 14{\cdot}19 = 0$$
$$v + 2{\cdot}399\,z + \alpha = 0$$
$$v + 5{\cdot}237\,z + \alpha - y + 24{\cdot}27 = 0$$

where x and y are known quantities. From this equation by the method of least squares—

$$\left(5{\cdot}099^2 + 7{\cdot}937^2 + 2{\cdot}838^2\right) z = 5{\cdot}099\,(x - 14{\cdot}19) + 7{\cdot}937\,(x + y - 38{\cdot}46) + 2{\cdot}838\,(y - 24{\cdot}27)$$

$$\therefore z = 0{\cdot}13432\,x + 0{\cdot}11102\,y - 4{\cdot}6006$$

Consequently, ϱ being the mean density of Arthur's Seat, δ the mean density of the earth,

$$\delta = \frac{\varrho}{0{\cdot}13432\,x + 0{\cdot}11102\,y - 4{\cdot}6006}$$

By being put in this form, we see more clearly how to estimate the probable error of δ as depending upon the probable errors of the observed quantities x, y, ϱ.

By the examination of a great number of specimens, Lieutenant-Colonel James determined the mean density of the hill to be $= 2{\cdot}75$. Substituting this value, and the previously obtained quantities $x = 17{\cdot}00$, $y = 25{\cdot}53$, we get $\delta = 5{\cdot}316$.

By means of the expression given for the probable error of $x + \mu y$ (page 604), we readily obtain, putting $\mu = {\cdot}83$, the probable error of δ depending on the probable errors of the observed amplitudes, namely $\pm{\cdot}054$. If, therefore, ϵ be the probable error of the observed mean density of Arthur's Seat, we shall have finally*

$$\text{Mean Density of the Earth} = 5{\cdot}316 \pm \sqrt{3{\cdot}725\,\epsilon^2 + {\cdot}0029}$$

§ IV.

On the Influence of the Irregularities of the Surface of the Earth on the Comparison of Geodetical and Astronomical Observations.

1. Let the axis of revolution of the earth be taken as the axis of z, the centre of the earth being the origin, and let—

$$x_1^2 + y_1^2 + (1 + \epsilon)\, z_1^2 - 1 = 0 \tag{1}$$

be the equation of a regular spheroid whose surface coincides very closely with the mathematical surface of the earth. Let the normal to any point $x\,y\,z$ of the latter surface meet the spheroid in $x_1\,y_1\,z_1$, and let v be the part of the normal intercepted by the two surfaces, then—

$$x_1 = x - \lambda v \;:\; y_1 = y - \mu v \;:\; z_1 = z - \nu v$$

where $\lambda\ \mu\ \nu$ are the direction-cosines of the normal. Substituting these values in the equation of the spheroid, we have—

$$(x - \lambda v)^2 + (y - \mu v)^2 + (1 + \epsilon)(z - \nu v)^2 - 1 = 0$$

We may assume v to be so small that terms of the order v^2, ϵv, may be neglected, then the above equation becomes—

$$x^2 + y^2 + (1 + \epsilon)\, z^2 - 2\, v\,(\lambda x + \mu y + \nu z) - 1 = 0$$

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* "On the Deflection of the Plumb-line at Arthur's Seat, and the Mean Specific Gravity of the Earth. Communicated by Lieutenant-Colonel James, R.E., F.R.S., M.R.I.A., &c., Superintendent of the Ordnance Survey." *Philosophical Transactions*, 1856, page 591.

but in the coefficient of v we may neglect ε, so that $x = \lambda, y = \mu, z = \nu$; therefore since $\lambda^2 + \mu^2 + \nu^2 = 1$, we have—

$$u = x^2 + y^2 + (1 + \varepsilon) z^2 - 2 v - 1 = 0 \qquad (2)$$

for the equation of the irregular surface of the earth, where v is the height of the latter at any point, above the surface of the spheroid (1) of reference, and is a function of $x y z$; but as we may substitute for z its value in terms of x and y, v may be considered a function of x and y.

2. The equations of meridians and parallels upon this surface may be expressed as follows: A meridian line upon the irregular surface (2) of the earth is a curve of double curvature, and is the locus of all those points whose longitude is constant. Let ω be this longitude, then the normals for every point in the meridian line are parallel to the plane $y \cos \omega - x \sin \omega = 0$, or perpendicular to the line $y \sin \omega + x \cos \omega = 0$. But the direction-cosines of the normal are proportional to—

$$\frac{du}{dx} : \frac{du}{dy} : \frac{du}{dz} = x - \frac{dv}{dx} : y - \frac{dv}{dy} : (1 + \varepsilon) z$$

Therefore we must have—

$$- \sin \omega \left(x - \frac{dv}{dx}\right) + \cos \omega \left(y - \frac{dv}{dy}\right) = 0$$

and therefore the equation of a meridian line on the surface (2) is—

$$- x \sin \omega + y \cos \omega + \sin \omega \frac{dv}{dx} - \cos \omega \frac{dv}{dy} = 0 \qquad (3)$$

For the equation of a parallel whose latitude is λ, we have—

$$\frac{du^2}{dz^2} = \sin^2 \lambda \left(\frac{du^2}{dx^2} + \frac{du^2}{dy^2} + \frac{du^2}{dz^2}\right)$$

$$\therefore (1 + \varepsilon)^2 z^2 \cot^2 \lambda = x^2 - 2 x \frac{dv}{dx} + y^2 - 2 y \frac{dv}{dy}$$

Or,

$$(1 + \varepsilon) z \left(\frac{1}{1 + \varepsilon} + \cot^2 \lambda\right)^{\frac{1}{2}} - v + x \frac{dv}{dx} + y \frac{dv}{dy} - 1 = 0 \qquad (4)$$

which, together with (2), are the equations of the locus of all points whose latitude is λ.

3. If through any point P on the surface (2) we draw a system of rectangular axes of co-ordinates $\xi \eta \zeta$, the first being directed to the north, the second to the east, and the third to the zenith, then—

$$\begin{aligned} x &= \alpha + l_1 \xi + m_1 \eta + n_1 \zeta \\ y &= \beta + l_2 \xi + m_2 \eta + n_2 \zeta \\ z &= \gamma + l_3 \xi \cdots + n_3 \zeta \end{aligned} \qquad (5)$$

where $\alpha \beta \gamma$ are the co-ordinates of P, and $l_1\ m_1\ n_1\ l_2\ m_2 \cdots\cdots$ the direction-cosines of the new axes. Hence we have—

$$dv = \left(l_1 \frac{dv}{dx} + l_2 \frac{dv}{dy}\right) d\xi + \left(m_1 \frac{dv}{dx} + m_2 \frac{dv}{dy}\right) d\eta + \left(n_1 \frac{dv}{dx} + n_2 \frac{dv}{dy}\right) d\zeta$$

At the point P $d\zeta = 0$; also $l_1 = -\cos\omega\sin\lambda$: $l_2 = -\sin\omega\sin\lambda$: $m_1 = \sin\omega$: $m_2 = -\cos\omega$; consequently at P—

$$-\frac{1}{\sin\lambda}\frac{dv}{d\xi} = \cos\omega\frac{dv}{dx} + \sin\omega\frac{dv}{dy}$$

$$\frac{dv}{d\eta} = \sin\omega\frac{dv}{dx} - \cos\omega\frac{dv}{dy}$$

The quantities $\frac{dv}{d\xi}$ and $\frac{dv}{d\eta}$ are the inclinations of the surface (2) to that of the spheroid of reference (1), measured north and east, or they are the *deflections* north and east, to use a term derived from the expression "deflection of the plumb-line." If we put $i_{\prime}$ and i' for these differential coefficients, the equation $i_{\prime}\,\xi + i'\,\eta + \zeta = 0$ is that of a plane passing through P parallel to the corresponding tangent plane of the spheroid. Let a vertical plane pass through P making an azimuthal angle α with the plane $\xi\eta$, and put $\theta\cos\alpha = \xi$, $\theta\sin\alpha = \eta$, then it is clear that the inclination of the two surfaces measured in the direction determined by the azimuth α is $= i_{\prime}\cos\alpha + i'\sin\alpha$. This may be illustrated mechanically, for if i be the magnitude of a local attraction acting at any point of the earth's surface, $i_{\prime}$ and i' the components of this force in the direction of the meridian and perpendicular to it, then in the direction of the azimuth α the attraction is $i_{\prime}\cos\alpha + i'\sin\alpha$. The maximum, or i, takes place for the azimuth $\tan^{-1}(i' : i_{\prime})$, and is equal to $\sqrt{i_{\prime}^2 + i'^2}$.

By means of the above expressions for $\frac{dv}{d\xi}$ and $\frac{dv}{d\eta}$, the equations of the meridian and parallel become—

$$\left.\begin{aligned} -x\sin\omega + y\cos\omega + \frac{dv}{d\eta} = 0 \\ -z(1+\varepsilon)\left(\frac{1}{1+\varepsilon} + \cot^2\lambda\right)^{\frac{1}{2}} + \cot\lambda\frac{dv}{d\xi} + v + 1 = 0 \end{aligned}\right\} \qquad (6)$$

4. The effect of the irregularities of the surface upon the curvature at any point may be traced in the following manner: Let the system of axes $\xi\eta\zeta$ originate at that point (P) on the surface for which the curvature is required. Let $h\,k\,p$ be the co-ordinates of any point of the surface very near to the origin, then since at that point $\frac{d\zeta}{d\xi} = 0$ and $\frac{d\zeta}{d\eta} = 0$, we have—

$$p = F(h, k) = \tfrac{1}{2}\left(\frac{d^2\zeta}{d\xi^2}h^2 + 2\frac{d^2\zeta}{d\xi\,d\eta} \qquad + \frac{d^2\zeta}{d\eta^2}k^2\right) + \cdots\cdots$$

Put now $h^2 = (h^2 + k^2)\cos^2\alpha$, $k^2 = (h^2 + k^2)\sin^2\alpha$, then—

$$\frac{2p}{h^2 + k^2} = \frac{d^2\zeta}{d\xi^2}\cos^2\alpha + 2\frac{d^2\zeta}{d\xi\,d\eta}\cos\alpha\sin\alpha + \frac{d^2\zeta}{d\eta^2}\sin^2\alpha + \cdots\cdots$$

Now if ϱ be the radius of curvature of the vertical section of the surface by a plane passing through the axis of ζ and making an angle α with the plane of $\zeta\xi$, we have at the limit $h^2 + k^2 = 2p\varrho$. Also it is easy to show that at the point P or the origin—

$$\frac{d^2u}{d\xi^2} + \frac{du}{d\zeta}\frac{d^2\zeta}{d\xi^2} = 0 \qquad \frac{d^2u}{d\xi\,d\eta} + \frac{du}{d\zeta}\frac{d^2\zeta}{d\xi\,d\eta} = 0 \qquad \frac{d^2u}{d\eta^2} + \frac{du}{d\zeta}\frac{d^2\zeta}{d\eta^2} = 0$$

consequently by substitution in the preceding equation, and neglecting the sign of ϱ, we have—

$$\frac{1}{\varrho}\cdot\frac{du}{d\zeta}=\frac{d^2u}{d\xi^2}\cos^2\alpha+2\frac{d^2u}{d\xi\,d\eta}\cos\alpha\sin\alpha+\frac{d^2u}{d\eta^2}\sin^2\alpha \qquad (7)$$

5. By differentiating the equation $u = x^2 + y^2 + (1+\varepsilon)z^2 - 2v - 1$, we obtain through equations (5), and considering v a function of ξ and η only—

$$\tfrac{1}{2}\frac{du}{d\xi}=l_1x+l_2y+(1+\varepsilon)l_3z-\frac{dv}{d\xi} \qquad \tfrac{1}{2}\frac{d^2u}{d\xi^2}=l_1^2+l_2^2+l_3^2(1+\varepsilon)-\frac{d^2v}{d\xi^2}$$

$$\tfrac{1}{2}\frac{du}{d\eta}=m_1x+m_2y\;\cdots\;-\frac{dv}{d\eta} \qquad \tfrac{1}{2}\frac{d^2u}{d\eta^2}=m_1^2+m_2^2\;\cdots\;-\frac{d^2v}{d\eta^2}$$

$$\tfrac{1}{2}\frac{du}{d\zeta}=n_1x+n_2y+(1+\varepsilon)n_3z \qquad \tfrac{1}{2}\frac{d^2u}{d\zeta^2}=n_1^2+n_2^2+n_3^2(1+\varepsilon)$$

$$\tfrac{1}{2}\frac{d^2u}{d\xi\,d\eta}=l_1m_1+l_2m_2-\frac{d^2v}{d\xi\,d\eta}$$

But we have also the following necessary relations—

$$l_1^2+l_2^2+l_3^2=m_1^2+m_2^2=n_1^2+n_2^2+n_3^2=1$$
$$l_1m_1+l_2m_2=0$$

and therefore since $l_3=\cos\lambda$,

$$\tfrac{1}{2}\frac{d^2u}{d\xi^2}=1+\varepsilon\cos^2\lambda-\frac{d^2v}{d\xi^2}$$

$$\tfrac{1}{2}\frac{d^2u}{d\xi\,d\eta}=-\frac{d^2v}{d\xi\,d\eta}\quad:\quad\tfrac{1}{2}\frac{d^2u}{d\eta^2}=1-\frac{d^2v}{d\eta^2}$$

Also if we add the squares of the foregoing values of $\frac{du}{d\xi}\,\frac{du}{d\eta}\,\frac{du}{d\zeta}$ we shall have, remembering that at P the first two of these differential coefficients are zero, and making use of the equation to the surface, (2)

$$\tfrac{1}{4}\frac{du^2}{d\zeta^2}=1+\varepsilon(1+\varepsilon)z^2+2v$$

$$\therefore\ \tfrac{1}{2}\frac{du}{d\zeta}=\frac{1+v}{\left(1-\frac{\varepsilon}{1+\varepsilon}\sin^2\lambda\right)^{\frac{1}{2}}} \qquad (8)$$

6. If $\varrho_1\ \varrho_2$ be the principal radii of curvature, ϕ the azimuth of the plane of the former, and if we put $X\cos^2\alpha+2W\sin\alpha\cos\alpha+Y\sin^2\alpha$ for the reciprocal of ϱ, we shall have—

$$X\cos^2\alpha+2W\sin\alpha\cos\alpha+Y\sin^2\alpha=\frac{1}{\varrho_1}\cos^2(\alpha-\phi)+\frac{1}{\varrho_2}\sin^2(\alpha-\phi)$$

$$\therefore X+Y+(X-Y)\cos2\alpha+2W\sin2\alpha=\frac{1}{\varrho_1}+\frac{1}{\varrho_2}+\cos2\phi\left(\frac{1}{\varrho_1}-\frac{1}{\varrho_2}\right)\cos2\alpha+\sin2\phi\left(\frac{1}{\varrho_1}-\frac{1}{\varrho_2}\right)\sin2\alpha$$

whence we obtain—

$$\frac{1}{\varrho_1}+\frac{1}{\varrho_2}=X+Y$$

$$\cos2\phi\left(\frac{1}{\varrho_1}-\frac{1}{\varrho_2}\right)=X-Y\quad:\quad\sin2\phi\left(\frac{1}{\varrho_1}-\frac{1}{\varrho_2}\right)=2W$$

If we put r_1 r_2 for the principal radii of curvature of the spheroid at a point whose latitude is λ, these equations will become—

$$\begin{aligned}
\frac{1}{\varrho_1} + \frac{1}{\varrho_2} &= \frac{1}{r_1} + \frac{1}{r_2} - \frac{d^2 v}{d\xi^2} - \frac{d^2 v}{d\eta^2} - 2v \\
\cos 2\phi \left(\frac{1}{\varrho_1} - \frac{1}{\varrho_2}\right) &= \frac{1}{r_1} - \frac{1}{r_2} - \frac{d^2 v}{d\xi^2} + \frac{d^2 v}{d\eta^2} \qquad (9) \\
\sin 2\phi \left(\frac{1}{\varrho_1} - \frac{1}{\varrho_2}\right) &= - \frac{2\, d^2 v}{d\xi\, d\eta}
\end{aligned}$$

7. Let us now consider the effect of the irregularities upon the astronomical determination of latitude, longitude, and azimuth. Writing $i_,$ and i' for $\frac{dv}{d\xi}$ and $\frac{dv}{d\eta}$, the equation—

$$i_, \xi + i' \eta + \zeta + v = 0 \qquad (10)$$

is that of the tangent plane to the spheroid (1) at that point (P_1) of its surface which is the projection of P, the origin of the co-ordinates $\xi\, \eta\, \zeta$. Through P draw PQ parallel to the axis of rotation,—this line must be in the plane $\xi\, \zeta$; then, neglecting quantities of the order $v \cdot i$, the meridian plane of P_1 is that which contains PQ and is perpendicular to (10): let $\alpha_, \xi + \alpha' \eta + \zeta = 0$ be this plane, then since this is perpendicular to (10) we must have $\alpha_, i_, + \alpha' i' + 1 = 0$; so that the meridian plane of P_1 is represented by

$$\alpha_, i' \xi - (1 + \alpha_, i_,) \eta + i' \zeta = 0 \qquad (11)$$

The trace of this plane upon the plane $\zeta = 0$ is, omitting quantities of a smaller order, $\eta = \xi \cdot \alpha_, i'$. Also making in (11) $\eta = 0$, since the result is the equation of PQ, we must have $\alpha_, = - \tan \lambda$, λ being the latitude of P. Hence the true meridian line through P makes a horizontal angle $i' \tan \lambda$ with the meridian line traced on the spheroid through the point P_1.

It is also easy to see that the plane (11) makes with the plane $\xi\, \zeta$ an angle equal to the horizontal angle $i' \tan \lambda$ divided by $\sin \lambda$, or $i' \sec \lambda$.

8. It follows, then, that if $\lambda\, \omega$ be the latitude and longitude of P, $\lambda_1\, \omega_1$ the latitude and longitude of P_1, the projection of P upon the spheroid of reference, α the azimuth (measured from north round by east) of any object on the actual surface, α_1 the corresponding azimuth on the spheroid of reference—

$$\begin{aligned}
\lambda_1 &= \lambda + \frac{dv}{d\xi} \\
\omega_1 &= \omega - \frac{dv}{d\eta} \sec \lambda \qquad (12) \\
\alpha_1 &= \alpha + \frac{dv}{d\eta} \tan \lambda
\end{aligned}$$

The first two of these equations are also evident by the consideration of the spherical triangle formed by lines parallel to the normals at P and P_1, and the axis of revolution; the third equation will also be evident by considering the spherical triangle formed by lines parallel to the directions of the meridian at P and P_1, and the axis of revolution.

9. In the operation of spirit levelling, it is clear that the resulting height of any point of the actual surface is the height above the irregular surface (2) of the earth, for the instrumental horizontal line at any point is determined by the direction of gravity at that point. In the measurement of a base-line, also, the horizontality of the apparatus is determined at every place by the normal to the surface, also the height of every part of the line is given with reference to the irregular surface; hence, theoretically at least, the length of the base-line as reduced to the mean level of the sea is the length of the curve traced upon the true mathematical surface of the earth by a plane passing through the extremities of the line, or their projections upon the surface, and containing the normal at either extremity. Let $r = f(\theta)$ be the polar equation of the curve of intersection of this plane with the surface of the spheroid of reference, the origin of r and θ being the intersection with the axis of revolution of that normal which lies in the intersecting plane. Then v the height of the true surface above that of the spheroid of reference at any point in the plane of the base is also a function of θ; let $\varrho = r + v$, then the length of the curve upon the irregular surface is—

$$s = \int \left\{ \varrho^2 \, d\theta^2 + d\varrho^2 \right\}^{\frac{1}{2}}$$

$$= \int \left\{ r^2 \, d\theta^2 + dr^2 \right\}^{\frac{1}{2}} \left\{ 1 + \frac{v}{r} \right\}$$

neglecting terms of the order $e^2 dv$: but if s_1 be the length of the curve of intersection with the surface of the spheroid—

$$s_1 = \int \left\{ r^2 \, d\theta^2 + dr^2 \right\}^{\frac{1}{2}}$$

$$\therefore \; s - s_1 = \int v \, d\theta$$

Consequently if v_0 be the mean value of v between the extremities of the base, the length of the measured line will exceed that of its projection on the speroid of reference by $v_0 s$.

10. Let us now consider the influence of the irregularities of the surface on the measurement of the angles of a triangulation. If we take two points A B on the surface (2), and project them upon the surface (1) of the spheroid of reference, then the azimuth of B_1 (the projection of B) as it would be measured at A_1 (the projection of A) exceeds the azimuth of B at A on account of the altitude of the pole by the quantity $\frac{dv}{d\eta} \tan \lambda$: but if θ be the arc distance of B from A, B is seen at a depression $= \frac{1}{2}\theta$, and is therefore displaced in azimuth by the amount of—

$$\left(\frac{dv}{d\eta} \cos \alpha - \frac{dv}{d\xi} \sin \alpha \right) \tan \tfrac{1}{2}\theta$$

where α is the azimuth of B at A; therefore if α_1 be the azimuth of B_1 at A_1—

$$\alpha_1 = \alpha + \frac{dv}{d\eta} \tan \lambda + \left(\frac{dv}{d\eta} \cos \alpha - \frac{dv}{d\xi} \sin \alpha \right) \tan \frac{\theta}{2}$$

Also if α' be the azimuth of any other point C, and α_1' that of its projection, we have—

$$\alpha_1' = \alpha' + \frac{dv}{d\eta} \tan \lambda + \left(\frac{dv}{d\eta} \cos \alpha' - \frac{dv}{d\xi} \sin \alpha' \right) \tan \frac{\theta'}{2}$$

The effect of the irregularity upon the observed angle is therefore—

$$\frac{dv}{d\eta}\left(\cos\alpha\tan\frac{\theta}{2}-\cos\alpha'\tan\frac{\theta'}{2}\right)-\frac{dv}{d\xi}\left(\sin\alpha\tan\frac{\theta}{2}-\sin\alpha'\tan\frac{\theta'}{2}\right)$$

Hence it follows that the observed angles of a triangulation differ from the angles of the triangles formed by the projection of the trigonometrical points upon the spheroid of reference, only by quantities which are of the second order in comparison with the inclination of the surfaces to each other. At the utmost this influence can never exceed a minute fraction of a second, and is overwhelmed by other sources of error. Hence in calculating the distance between any two points P, Q of a triangulation (the extremities of an arc of meridian, for instance) and using the observed angles, the result will not be sensibly different from the true distance of the projections P_1 Q_1 upon the surface of a spheroid whose surface is parallel to that of the spheroid of reference and at the mean height (v_0) of the base-line from it.

11. In tracing further the influence of the irregularities of the surface of the earth we shall write ξ η instead of the differential coefficients of v with respect to those quantities; thus, at any point ξ denotes the inclination of the actual surface to that of the spheroid of reference measured in the direction of the north; η denotes the inclination of the actual surface to that of the spheroid of reference measured in the direction of the east, the inclination being considered positive when the actual surface is elevated in the stated direction.

12. Now let A B be any two points connected by a triangulation, let λ λ' be their observed or apparent latitudes, ω their observed or apparent difference of longitude, and let the direction of the meridian be observed at each. The points connected by the triangulation are A_1 B_1 the projections of A B, then we have the distance $= s$, and also from the observed bearings of the meridian we obtain from the triangulation the azimuth of B at A $= \alpha$ and the azimuth of A at B $= \alpha'$. We have then,—the quantities λ_1 λ_1' α_1 α_1' ω_1 referring to the points A_1 B_1,

$$\begin{array}{ll} \lambda_1 = \lambda + \xi & \lambda_1' = \lambda' + \xi' \\ \alpha_1 = \alpha + \eta\tan\lambda & \alpha_1' = \alpha' + \eta'\tan\lambda' \\ \multicolumn{2}{c}{\omega_1 = \omega - \eta'\sec\lambda' + \eta\sec\lambda} \end{array} \tag{13}$$

Let (λ') (α') (ω') be the numerical results which we should obtain for the point B by using the observed latitude of A, and the observed direction of the meridian at the same, together with the distance s, then—

$$\begin{aligned} (\lambda') &= f\,(\lambda,\alpha,s) = f\,(\lambda_1-\xi\,,\,\alpha_1-\eta\tan\lambda,\,s\,) \\ (\omega) &= f_1(\lambda,\alpha,s) = f_1(\lambda_1-\xi\,,\,\alpha_1-\eta\tan\lambda,\,s\,) \\ (\alpha') &= f_2(\lambda,\alpha,s) = f_2(\lambda_1-\xi\,,\,\alpha_1-\eta\tan\lambda,\,s\,) \end{aligned}$$

Hence—

$$f\,(\lambda_1,\alpha_1,s) = \lambda_1' = (\lambda') + \xi\cdot\frac{df}{d\lambda_1} + \eta\tan\lambda\,\frac{df}{d\alpha_1}$$

$$f_1(\lambda_1,\alpha_1,s) = \omega_1 = (\omega) + \xi\cdot\frac{df_1}{d\lambda_1} + \eta\tan\lambda\,\frac{df_1}{d\alpha_1}$$

$$f_2(\lambda_1,\alpha_1,s) = \alpha_1' = (\alpha') + \xi\cdot\frac{df_2}{d\lambda_1} + \eta\tan\lambda\,\frac{df_2}{d\alpha_1}$$

and by substitution—

$$\begin{aligned}\xi' &= (\lambda') - \lambda' + \xi \frac{df}{d\lambda_1} + \eta \tan \lambda \frac{df}{d\alpha_1} \\ -\eta' \sec \lambda' &= (\omega) - \omega + \xi \frac{df_1}{d\lambda_1} + \eta \tan \lambda \frac{df_1}{d\alpha_1} - \eta \sec \lambda \\ \eta' \tan \lambda' &= (\alpha') - \alpha' + \xi \frac{df_2}{d\lambda_1} + \eta \tan \lambda \frac{df_2}{d\alpha_1}\end{aligned} \qquad (14)$$

13. The values of the differential coefficients are to be found from the equations at page 249. If we introduce into these equations the condition that azimuths are to be measured from north, by east, and longitudes to be measured from east to west, we get—

$$\begin{aligned}\tan \tfrac{1}{2}(\alpha_1' + \omega_1 - \zeta) &= -\frac{\cos \frac{1}{2}(\frac{1}{2}\pi - \lambda_1 - \theta)}{\cos \frac{1}{2}(\frac{1}{2}\pi - \lambda_1 + \theta)} \cot \tfrac{1}{2}\alpha_1 \\ \tan \tfrac{1}{2}(\alpha_1' - \omega_1 - \zeta) &= -\frac{\sin \frac{1}{2}(\frac{1}{2}\pi - \lambda_1 - \theta)}{\sin \frac{1}{2}(\frac{1}{2}\pi - \lambda_1 + \theta)} \cot \tfrac{1}{2}\alpha_1 \\ \lambda_1' - \lambda_1 &= \frac{s}{\varrho} \frac{\sin \frac{1}{2}(\alpha_1' + \alpha_1 - \zeta)}{\sin \frac{1}{2}(\alpha_1' - \alpha_1 - \zeta)} (1 + Q)\end{aligned} \qquad (15)$$

In differentiating these equations we may neglect the small quantity ζ; and if ϕ be the third side of a spherical triangle of which $90^\circ - \lambda_1$ and θ are two sides including the angle α_1, then by spherical trigonometry we have—

$$\begin{aligned}-\sin(\alpha_1' + \omega_1) \cos^2 \tfrac{1}{2}\phi &= \cos \tfrac{1}{2}(\tfrac{1}{2}\pi - \lambda_1 - \theta) \cos \tfrac{1}{2}(\tfrac{1}{2}\pi - \lambda_1 + \theta) \sin \alpha_1 \\ -\sin(\alpha_1 - \omega_1) \sin^2 \tfrac{1}{2}\phi &= \sin \tfrac{1}{2}(\tfrac{1}{2}\pi - \lambda_1 - \theta) \sin \tfrac{1}{2}(\tfrac{1}{2}\pi - \lambda_1 + \theta) \sin \alpha_1\end{aligned}$$

By means of these equations we get—

$$\frac{d\alpha_1'}{d\alpha_1} + \frac{d\omega_1}{d\alpha_1} = -\frac{\sin(\alpha_1' + \omega_1)}{\sin \alpha_1} \qquad \frac{d\alpha_1'}{d\alpha_1} - \frac{d\omega_1}{d\alpha_1} = -\frac{\sin(\alpha_1' - \omega_1)}{\sin \alpha_1}$$

$$\frac{d\alpha_1'}{d\lambda_1} + \frac{d\omega_1}{d\lambda_1} = \frac{\sin \theta \sin \alpha_1}{2 \cos^2 \frac{1}{2}\phi} \qquad \frac{d\alpha_1'}{d\lambda_1} - \frac{d\omega_1}{d\lambda_1} = \frac{\sin \theta \sin \alpha_1}{2 \sin^2 \frac{1}{2}\phi}$$

$$\frac{d\lambda_1'}{d\alpha_1} = \frac{s}{\varrho}(1 + Q) \frac{\sin \alpha_1' - \text{in } \alpha_1 \frac{d\alpha_1'}{d\alpha_1}}{2 \sin^2 \frac{1}{2}(\alpha_1' - \alpha_1)} \quad : \quad \frac{d\lambda_1'}{d\lambda_1} = 1 - \frac{s}{\varrho}(1 + Q) \frac{\sin \alpha_1 \frac{d\alpha_1'}{d\lambda_1}}{2 \sin^2 \frac{1}{2}(\alpha_1' - \alpha_1)}$$

And therefore since $\sin \theta \sin \alpha_1 + \sin \phi \sin \omega = 0$, and Q may be omitted,

$$\frac{d\alpha_1'}{d\alpha_1} = -\frac{\sin \alpha_1' \cos \omega_1}{\sin \alpha_1} \qquad \frac{d\omega_1}{d\alpha_1} = -\frac{\cos \alpha_1' \sin \omega_1}{\sin \alpha_1}$$

$$\frac{d\alpha_1'}{d\lambda_1} = -\frac{\sin \omega_1}{\sin \phi} \qquad \frac{d\omega_1}{d\lambda_1} = \frac{\cos \phi \sin \omega_1}{\sin \phi}$$

$$\frac{d\lambda_1'}{d\alpha_1} = \frac{v}{\varrho} \sin \alpha_1' \sin \theta \qquad \frac{d\lambda_1'}{d\lambda_1} = 1 - \frac{1}{2}\frac{v}{\varrho} \sin^2 \omega_1$$

where v is the same quantity as at page 249, namely the normal for λ_1. We may here put $v = \varrho$, and $\sin \phi = \cos \lambda'$, and substituting in equations (14) they become—

$$\begin{aligned}\xi' &= (\lambda') - \lambda' + \Big(\cos \omega\Big) \xi + \Big(\sin \lambda \sin \omega\Big) \eta \\ -\sec \lambda' \cdot \eta' &= (\omega) - \omega + \Big(\tan \lambda' \sin \omega\Big) \xi + \left(\frac{\sin \theta \cos \alpha'}{\cot \lambda \cos \lambda'} - \sec \lambda\right) \eta \\ \tan \lambda' \cdot \eta' &= (\alpha') - \alpha' + \Big(-\sec \lambda' \sin \omega\Big) \xi + \left(\frac{\sin \lambda \cos \omega}{\cos \lambda'}\right) \eta\end{aligned} \qquad (16)$$

14. Let us now inquire into the nature of these equations and the information to be derived from them. In obtaining the equations—

$$\lambda_1 = \lambda + \xi$$

$$\omega_1 = \omega - \eta \sec \lambda \quad : \quad \alpha_1 = \alpha + \eta . \tan \lambda$$

it is evident that they do not in any way depend upon the coincidence of the axis of the spheroid of reference with that of rotation of the earth; the only requirement is that the axis of the spheroid should be parallel to the axis of rotation. If therefore, returning to the arc AB or A_1B_1 to which the equations (16) have reference, we suppose the surface of a spheroid whose axes are $2a$ and $2b$, of which the latter is parallel to the axis of rotation of the earth, to pass through the point A or a point A_1 at a very small distance vertically from A, and to have its surface inclined to the true surface at A at a depression $-\xi$ northwards and a depression $-\eta$ eastwards,* we shall be able to determine the quantities ξ' η' with reference to this spheroid for the point B. For if with the observed latitude of A and the observed direction of the meridian at that point we calculate the latitude, longitude, and direction of the meridian at B, and find that the calculated values exceed the observed by quantities ε_1 ε_2 ε_3, then we have from the equations (16)—

$$\begin{aligned} \xi' &= \varepsilon_1 + a_1 \xi + b_1 \eta \\ -\sec \lambda' \cdot \eta' &= \varepsilon_2 + a_2 \xi + b_2 \eta \\ \tan \lambda' \cdot \eta' &= \varepsilon_3 + a_3 \xi + b_3 \eta \end{aligned}$$

Therefore for any assumed values of ξ and η we may obtain the corresponding values of ξ' and η'. If we make the surface of our spheroid of reference parallel to the surface of the earth at A or A_1, then $\xi = 0$, $\eta = 0$, and we have $\xi' = \varepsilon_1$, $\eta' = -\varepsilon_2 \cos \lambda' = \varepsilon_3 \cot \lambda'$.

15. In examining more closely the coefficients a_1 b_1 $\cdots\cdots$ we find that a_1 is very nearly unity, b_1 is a small quantity of the order of θ (the arc distance AB); a_2 is of the order θ; b_2 differs from *sec* λ', the coefficient of η', by a quantity of the order θ; a_3 is of the order θ; b_3 differs from *tan* λ', the coefficient of η', by a quantity of the order θ. Therefore the equations give but little more than—

$$\begin{aligned} \xi' - \xi &= \varepsilon_1 \\ -\sec \lambda (\eta' - \eta) &= \varepsilon_2 \\ \tan \lambda (\eta' - \eta) &= \varepsilon_3 \end{aligned} \qquad (17)$$

From this we see that the observation of difference of longitude gives no further information than is obtained by the observation of the directions of the meridian at both extremities, but is available as a check upon the accuracy of the work.

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* If h k l be the co-ordinates of the centre of the spheroid, its equation is—

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{a^2} + \frac{(z-l)^2}{b^2} = 1$$

It may evidently be made to touch any given place in a given point, for this is equivalent to making it pass through three given points, by which h k l may be determined.

If there be besides B, a number of other points CD······ with respect to which we have by a triangulation their distances from A and also the observed latitude and direction of the meridian corresponding to each, then we have a set of equations—

$$\begin{array}{ll} \xi_1 - \xi = a_1 & \eta_1 - \eta = b_1 \\ \xi_2 - \xi = a_2 & \eta_2 - \eta = b_2 \\ \xi_3 - \xi = a_3 & \eta_3 - \eta = b_3 \\ \vdots & \vdots \\ \xi_n - \xi = a_n & \eta_n - \eta = b_n \end{array}$$

These equations will enable us to determine the position of the given spheroid of reference, that is the quantities ξ and η for the point A, so that it shall upon the whole coincide most nearly with the surface of the earth within the limits of the geodetically-connected points; or rather, we may determine that position corresponding to which the sum of the squares of the quantities $\xi\ \eta$ shall be the least possible; for the sum of the squares is—

$$\begin{array}{l} \xi^2 + (\xi + a_1)^2 + (\xi + a_2)^2 + \cdots\cdots (\xi + a_n)^2 \\ + \eta^2 + (\eta + b_1)^2 + (\eta + b_2)^2 + \cdots\cdots (\eta + b_n)^2 \end{array}$$

$$\therefore \left\{ \begin{array}{l} (n + 1)\,\xi = - \Sigma(a) \\ (n + 1)\,\eta = - \Sigma(b) \end{array} \right\}$$

16. We have supposed the points to be projected on one particular spheroid, and the resulting system of inclinations $\xi\ \eta$ is peculiar to that particular spheroid. Had the calculation been made with reference to another spheroid, another system of values would have resulted, and it is sufficiently obvious that we may assign as many systems of values as we please, each with reference to a certain spheroid. The question, then, is to ascertain which of all the spheroids will represent most nearly the general surface contained within the limits of the triangulation. In this shape the problem can scarcely be solved, but we may ascertain without much difficulty the spheroid which will make the sum of the squares of the apparent irregularities the least possible, or in other words we can determine the spheroid which shall show on the whole the least total amount of discordance with the astronomical observations.

17. If with the latitude of the point A and the distance and bearing of the point B, we calculate the latitude and longitude of B and the back azimuth of A, with the given (numerical) elements of the spheroid, a, b, and would ascertain the same quantities for B with reference to a spheroid whose semiaxes are $a + \delta a$, $b + \delta b$, then we have to add to each of the numerically-determined quantities terms of the form $m \cdot \delta a + n \cdot \delta b$, where the coefficients m and n can be assigned.

By the alteration of a and b the angle θ (which is subtended by the distance $A_1B_1 = s$ at the point of intersection of the normal of A_1 with the axis of revolution) is also altered, for we have—

$$\theta = \frac{s}{a}(1 - e^2 \sin^2 \lambda_1)^{\frac{1}{2}}$$

neglecting smaller terms which are not here required; therefore if u be the result of any calculation made with the angle $\theta = f(a, b)$ and u' the same when $\theta = f(a + \delta a, b + \delta b)$,

$$u' = u + \frac{du}{d\theta}\left(\frac{d\theta}{da}\,\delta a + \frac{d\theta}{db}\,\delta b\right)$$

It is, however, more convenient to use the variation of e^2 than of b, so that if we put $\delta a = \gamma a$, $\delta e^2 = \varepsilon$, and differentiate θ with respect to a and e^2, we get,

$$u' = u - \frac{du}{d\theta}\left(\gamma + \tfrac{1}{2}\,\frac{\sin^2\lambda}{1 - e^2\sin^2\lambda}\cdot\varepsilon\right)\theta$$

The calculated values of λ_1' ω_1 α_1' will therefore become, since ω_1 and α_1' contain θ only, and explicitly—

$$(\lambda') + \delta\lambda_1' \quad , \quad (\omega) + \frac{d\omega_1}{d\theta}\cdot\delta\theta$$

$$(\alpha') + \frac{d\alpha_1'}{d\theta}\cdot\delta\theta$$

By differentiating the first two of equations (15) there results,

$$\frac{d\alpha_1'}{d\theta} = -\tan\lambda'\sin\alpha_1' \quad : \quad \frac{d\omega_1}{d\theta} = \sec\lambda'\sin\alpha_1'$$

and referring to page 248 we find,

$$\lambda_1' - \lambda_1 = \frac{v}{\varrho}(\phi - \lambda_1)$$

$$\therefore \delta\lambda_1' = \frac{v}{\varrho}\,\delta\phi + (\phi - \lambda_1)\,\delta\left(\frac{v}{\varrho}\right)$$

Now by spherical trigonometry we have these equations—

$$\sin\phi = \cos\theta\sin\lambda_1 + \sin\theta\cos\lambda_1\cos\alpha_1$$

$$\cos\alpha_1'\cos\phi = \sin\theta\sin\lambda_1 - \cos\theta\cos\lambda_1\cos\alpha_1$$

Differentiating the first of these equations we get (λ_1 and α_1 being constant) $\cos\phi\,d\phi = -\sin\theta\sin\lambda_1\,d\theta + \cos\theta\cos\lambda_1\cos\alpha_1\,d\theta$, which by the second equation $= -\cos\phi\cos\alpha_1'\,d\theta$; therefore $\delta\phi = -\cos\alpha_1'\,\delta\theta$. We have also—

$$\frac{v}{\varrho} = \frac{\{1 - e^2\sin\frac{1}{2}(\lambda_1' + \lambda_1)}{(1 - e^2)(1 - e^2\sin^2\lambda_1)^{\frac{1}{2}}}$$

from which we obtain, neglecting terms of higher orders—

$$\frac{d}{de^2}\left(\frac{v}{\varrho}\right) = \frac{1}{(1 - e^2)^2}\left\{1 - \tfrac{3}{2}\sin^2\tfrac{1}{2}(\lambda_1' + \lambda_1) + \tfrac{1}{2}\sin^2\lambda_1\right\}$$

$$\therefore \delta\left(\frac{v}{\varrho}\right) = \frac{\cos^2\frac{1}{4}(\lambda + 3\lambda')}{(1 - e^2)^2}\cdot\delta e^2$$

Making these substitutions in the value of $\delta\lambda_1'$ we have—

$$\delta\lambda_1' = -\frac{v}{\varrho}\cos\alpha_1'\cdot\delta\theta + \frac{\varrho}{v}\cdot\frac{\lambda' - \lambda}{(1 - e^2)^2}\cos^2\tfrac{1}{4}(\lambda + 3\lambda')\cdot\delta e^2$$

and from above,

$$\delta\omega_1 = \sec\lambda'\sin\alpha_1'\,\delta\theta$$

$$\delta\alpha_1' = -\tan\lambda'\sin\alpha_1'\,\delta\theta$$

18. The equations (16) thus modified become—

$$\left.\begin{array}{r}
\xi' = (\lambda') - \lambda' + \left(\cos\omega\right)\xi + \left(\sin\lambda\sin\omega\right)\eta - \left(\frac{v}{\varrho}\cos\alpha_1'\right)\delta\theta + Q\cdot\varepsilon \\
-\sec\lambda'\cdot\eta' = (\omega) - \omega + \left(\tan\lambda'\sin\omega\right)\xi + \left(\frac{\sin\theta\cos\alpha'}{\cot\lambda\cos\lambda'} - \sec\lambda\right)\eta + \left(\sec\lambda'\sin\alpha_1'\right)\delta\theta \\
\tan\lambda'\cdot\eta' = (\alpha') - \alpha' - \left(\sec\lambda'\sin\omega\right)\xi + \left(\frac{\sin\lambda\cos\omega}{\cos\lambda'}\right) - \left(\tan\lambda'\sin\alpha'\right)\delta\theta
\end{array}\right\} \quad (18)$$

where—

$$\left\{\begin{array}{l}
Q = \frac{\varrho}{v}\,\frac{\lambda' - \lambda}{(1 - e^2)^2}\cos^2\tfrac{1}{4}(\lambda + 3\lambda') \\
\delta\theta = -\gamma\cdot\theta - \tfrac{1}{2}\cdot\frac{\theta\sin^2\lambda}{1 - e^2\sin^2\lambda}\,\varepsilon
\end{array}\right\}$$

19. Let A and B be the extremities of an arc of the meridian, or rather, lying very nearly in the meridian, at each of which we have the observed latitude and direction of the meridian. Using the observations at A as the basis of the calculation and a spheroid whose semi-axes are a and b, let the latitude of B_1 and the direction of the meridian at that point be calculated, and suppose the quantities so calculated exceed the corresponding observed values by $k_1\, k_2$, then we shall have—

$$0 = k_1 + c_1\gamma + c_1'\varepsilon + \xi - \xi'$$
$$0 = k_2\cot\lambda' + c_2\eta + \eta - \eta'$$

The second of these equations determines nothing further than the relation of the quantities $\eta\,\eta'$. The first gives a relation between γ and ε not free from the influence of ξ and ξ'.

20. If there be a number of meridian arcs there will be a series of equations—

$$\begin{array}{l}
0 = k_1 + c_1\gamma + c_1'\varepsilon + \xi_1 - \xi_1' \\
0 = k_2 + c_2\gamma + c_2'\varepsilon + \xi_2 - \xi_2' \\
0 = k_3 + c_3\gamma + c_3'\varepsilon + \xi_3 - \xi_3' \\
\quad\vdots \\
0 = k_n + c_n\gamma + c_n'\varepsilon + \xi_n - \xi_n'
\end{array}$$

which cannot be all satisfied, but give $n - 2$ relations among the n quantities $\xi - \xi'$.

If there be a number of points ABC······ all on the same meridian, having observed latitudes and being geodetically connected, they will afford a system of equations—

$$\begin{array}{l}
0 = k_1 + c_1\gamma + c_1'\varepsilon + \xi - \xi_1 \\
0 = k_2 + c_2\gamma + c_2'\varepsilon + \xi - \xi_2 \\
\quad\vdots \\
0 = k_n + c_n\gamma + c_n'\varepsilon + \xi - \xi_n
\end{array}$$

From these equations we may determine the values of γ and ε which render the sum of the squares of the quantities ξ a minimum, and these values of γ and ε correspond to and determine the spheroid which shows the least total amount of discordance with the observed latitudes. We have—

$$\Sigma(\xi^2) = \xi^2 + \Sigma(k + c\gamma + c'\varepsilon + \xi)^2$$

The equations for determining the minimum are therefore—

$$\begin{aligned}(n+1)\xi + (c)\gamma + (c')\varepsilon + (k) &= 0\\ (c)\xi + (c^2)\gamma + (cc')\varepsilon + (ck) &= 0\\ (c')\xi + (cc')\gamma + (c'^2)\varepsilon + (c'k) &= 0\end{aligned}$$

and from these we obtain the axes of the required spheroid, and the inclination of the actual surface at each of the stations ABC····

21. If the measured arc be perpendicular to the meridian, the second, or third equation, depending upon the observed difference of longitude or the observed azimuths, will alone give a relation between γ and ε, and this subject to the influence of $\eta - \eta'$, and a small multiple of ξ.

If the line AB lie in any azimuth, we have generally—

$$\begin{aligned}0 &= k_1 + a_1\xi - \xi' + b_1\eta + c_1\gamma + d_1\varepsilon\\ 0 &= k_2 + a_2\xi - \eta' + b_2\eta + c_2\gamma + d_2\varepsilon\end{aligned}$$

but if small multiples of ξ and η are omitted, the first equation will contain only $\xi - \xi'$, and the second only $\eta - \eta'$. If we put $\xi = \xi' = \eta = \eta' = 0$, the spheroid determined from these equations will have its surface parallel to the actual surface of the earth at A and B, and its axis parallel to the axis of rotation of the earth.

22. The application of the equations (18) to the determination of that spheroid which, having its axis parallel to that of the earth, shall most nearly represent the surface of a portion of the earth covered by a network of triangulation, and having the latitudes of several points in it observed, and also several determinations of the direction of the meridian; presents no other difficulty than the assigning of the relative weights of the equations depending upon observed latitudes and the equations depending upon observed azimuths. Let ABC ···· be points in the triangulation, at which the latitude has been observed; these will give a series of equations of the form—

$$\xi_n = k_n + a_n\xi + b_n\eta + c_n\gamma + d_n\varepsilon$$

and if the direction of the meridian has been observed at the points A'B'C' ···· there will be a series of equations of the form—

$$\eta_n' = k_n' + a_n'\xi + b_n'\eta + c_n'\gamma + d_n'\varepsilon$$

In the latitude equations, the absolute term k is subject to two sources of uncertainty—namely, that of the observed latitude and of the transferred latitude. Again, the observed latitude is subject to two distinct sources of error—namely, the irregularity of the earth's surface, and errors of the astronomical determination.

In the equations derived from observed azimuths, the absolute terms are also liable to the errors of observation and of transference, and are affected by the irregularities of the surface. Now the errors (of observation) of the observed azimuths are in general (certainly in the present operations) considerably greater than the errors of the observed latitudes, and the error of transference of azimuth through a long series of triangles is also considerably greater than the error of transference of latitude. Hence the absolute terms of the equations derived from observed azimuths will show discrepancies considerably larger than are shown in the absolute terms of the equations resulting from observed latitudes.

If from the whole of the equations we determine $\xi\ \eta\ \gamma\ \epsilon$ so that $\Sigma\,(\xi^2) + \Sigma\,(\eta^2)$ shall be a minimum, the values thus obtained give the elements of the spheroid most nearly representing, *on the whole*, the observations for latitude and also for azimuth. But the result will be that the azimuths are corrected at the expense of the latitudes, whereas it is more important that the latitude equations should be satisfied than that the azimuths should show small differences. Consequently, the spheroid which makes $\Sigma\,(\xi^2) + \Sigma\,(\eta^2)$ the least possible, does not most nearly represent the surface; but if we could assign a relative weight w to the azimuth equations as compared with the latitude equations, then the spheroid most nearly representing the surface would be that which makes a minimum the quantity—

$$\Sigma\,(\xi^2) + w \cdot \Sigma\,(\eta^2)$$

23. The spheroids which we have been considering in the preceding articles as "most nearly representing" the surface of the earth comprised within the limits of the operations, are evidently not osculating spheroids If geodetical and astronomical observations be made in conjunction at any part of the earth's surface in order to determine strictly the osculating ellipsoid, or the principal radii of curvature and the directions in azimuth in which their planes lie, such operations should be contained within very narrow limits, for the differential coefficients of the quantity v, as there is every reason to believe, may vary very irregularly within a small space upon the earth's surface.

24. Let us suppose that with the object of determining the curvature of the surface of the earth at a certain point, and whether it may for a small extent be represented by a surface of the second order, astronomical observations for latitude and azimuth have been made at three or four stations geodetically connected and at small distances from each other. Let A be one of these stations—say, the central one; at this point let (v) be the value of v, the height of the actual surface above the surface of reference, and let the tangential axes of co-ordinates $\xi\ \eta$ originate at A, the former lying in the direction of the north, the latter to the east.

25. Since we assume v to belong to a surface of the second order, it must be of the form—

$$v = (v) + f\xi + g\eta + h\frac{\xi^2}{2} + i\xi\eta + k\frac{\eta^2}{2}$$

$$\therefore \frac{dv}{d\xi} = f + h\xi + i\eta \quad : \quad \frac{dv}{d\eta} = g + i\xi + k\eta$$

26. Let B be a second point on the surface not far from A, and let a system of rectangular axes of co-ordinates $\xi'\ \eta'\ \zeta'$ originate at B, the first two tangential to the surface and directed to

the north and east respectively, the third ζ' to the zenith of B; then the relations between these quantities and $\xi\ \eta$ are these—

$$\xi = \alpha + \xi' \cos(\xi\ \xi') + \eta' \cos(\xi\ \eta') + \zeta' \cos(\xi\ \zeta')$$
$$\eta = \beta + \xi' \cos(\eta\ \xi') + \eta' \cos(\eta\ \eta') + \zeta' \cos(\eta\ \zeta')$$

and therefore *at the point* B we must have—

$$\frac{dv}{d\xi'} = \frac{dv}{d\xi} \cos(\xi\ \xi') + \frac{dv}{d\eta} \cos(\eta\ \xi')$$
$$\frac{dv}{d\eta'} = \frac{dv}{d\xi} \cos(\xi\ \eta') + \frac{dv}{d\eta} \cos(\eta\ \eta')$$

In order to ascertain the values of these cosines, let $\lambda\omega$, $\lambda'\omega'$ be the latitude and longitude of A and B respectively, then—

$$\begin{aligned}
\cos(\xi\ \xi') &= \cos(\xi\ x)\cos(\xi'\ x) + \cos(\xi\ y)\cos(\xi'\ y) + \cos(\xi\ z)\cos(\xi'\ z) \\
&= \sin\lambda \sin\lambda' (\sin\omega \sin\omega' + \cos\omega \cos\omega') + \cos\lambda \cos\lambda' \\
\cos(\eta\ \xi') &= \cos(\eta\ x)\cos(\xi'\ x) + \cos(\eta\ y)\cos(\xi'\ y) \\
&= \sin\lambda' (-\sin\omega \cos\omega' + \cos\omega \sin\omega') \\
\cos(\xi\ \eta') &= \cos(\xi\ x)\cos(\eta'\ x) + \cos(\xi\ y)\cos(\eta'\ y) \\
&= \sin\lambda (-\cos\omega \sin\omega' + \sin\omega \cos\omega') \\
\cos(\eta\ \eta') &= \cos(\eta\ x)\cos(\eta'\ x) + \cos(\eta\ y)\cos(\eta'\ y) \\
&= \sin\omega \sin\omega' + \cos\omega \cos\omega'
\end{aligned}$$

Therefore if we neglect quantities of the order of the square of the distance of B from A, we have—

$$\cos(\xi\ \xi') = 1 = \cos(\eta\ \eta')$$
$$\cos(\eta\ \xi') = \sin\lambda \sin(\omega' - \omega) = -\cos(\xi\ \eta')$$

But, the longitudes being counted westward, and the azimuth from north round by east and south, it is evident that $\sin\lambda \sin(\omega' - \omega) = \sin(\alpha' - \alpha)$, where α' is the azimuth of A as seen at B: hence at B—

$$\left.\begin{aligned}
\frac{dv}{d\xi'} &= \frac{dv}{d\xi} + \sin(\alpha' - \alpha)\frac{dv}{d\eta} \\
\frac{dv}{d\eta'} &= -\frac{dv}{d\xi}\sin(\alpha' - \alpha) + \frac{dv}{d\eta}
\end{aligned}\right\} \qquad (19)$$

Let $\xi = \theta \cos\alpha$ and $\eta = \theta \sin\alpha$, then—

$$\frac{dv}{d\xi} = f + (h\cos\alpha + i\sin\alpha)\,\theta \quad : \quad \frac{dv}{d\eta} = g + (i\cos\alpha + k\sin\alpha)\,\theta$$

Substituting these equations in the foregoing, and remembering that $\cos(\alpha' - \alpha) + 1 = 0$ very nearly, we get—

$$\frac{dv}{d\xi'} = f + g\sin(\alpha' - \alpha) + h\,\theta\cos\alpha + i\,\theta\sin(\alpha' - 2\alpha) + h\,\theta\sin\alpha\sin(\alpha' - \alpha)$$
$$\frac{dv}{d\eta'} = g - f\sin(\alpha' - \alpha) + k\,\theta\sin\alpha - i\,\theta\cos(\alpha' - 2\alpha) - h\,\theta\cos\alpha\sin(\alpha' - \alpha)$$

If we return to the notation previously used—namely, writing ξ η for the differential coefficients of v, we have—

$$\begin{aligned} \xi' &= \xi + \eta \sin(\alpha' - \alpha) + h\,\theta \cos\alpha + i\,\theta \sin(\alpha' - 2\,\alpha) + k\,\theta \sin\alpha \sin(\alpha' - \alpha) \\ \eta' &= \eta - \xi \sin(\alpha' - \alpha) + k\,\theta \sin\alpha - i\,\theta \cos(\alpha' - 2\,\alpha) - k\,\theta \cos\alpha \sin(\alpha' - \alpha) \end{aligned} \qquad (20)$$

27. These equations express the values of the ξ, η belonging to the point B, in terms of those belonging to A and the three quantities h, i, k, which are to be determined by observation. By equations (16) we have, if we neglect quantities of the order $\theta^2 \xi$, and put δ_1 and δ_2 for the excesses of the observed latitude and azimuth at B above the values as obtained by calculation from A—

$$\begin{aligned} \delta_1 &= \xi - \xi' + \sin\lambda \sin\omega \cdot \eta \\ \cot\lambda' \cdot \delta_2 &= \eta - \eta' - \operatorname{cosec}\lambda \sin\omega \cdot \xi - \cot\lambda \sin(\lambda' - \lambda) \cdot \eta \end{aligned}$$

Substituting the values of ξ' and η' in terms of h, i, k into these equations, we have finally,

$$\begin{aligned} 0 &= h\,\theta \cos\alpha + i\,\theta \sin(\alpha' - 2\,\alpha) + k\,\theta \sin\alpha \sin(\alpha' - \alpha) + \delta_1 \\ 0 &= k\,\theta \sin\alpha - i\,\theta \cos(\alpha' - 2\,\alpha) - h\,\theta \cos\alpha \sin(\alpha' - \alpha) + \delta_2 \cot\lambda' + \theta(-\xi \sin\alpha + \eta \cos\alpha) \cot\lambda' \end{aligned} \qquad (21)$$

28. These two equations are supplied by the point B. A second point C whose latitude is observed, and also the direction of the meridian at it, will, by means of its geodetical connection with A, afford another pair of equations; and since the quantities ξ and η as they enter into these equations have small coefficients, they may be neglected, and thus the two points B and C connected with A, affording four equations between the three quantities h, i, k, will be sufficient to determine these quantities, and also an equation of condition between the calculated values, which equation by its fulfilment or non-fulfilment will determine whether the surface within the small limits of the triangulation can or cannot be represented by a surface of the second order.

Having thus obtained the values of h, i, k, the calculation of the radii of curvature follows from the equations (9), for we have—

$$\frac{d^2 v}{d\xi^2} = h \quad : \quad \frac{d^2 v}{d\xi\, d\eta} = i \quad : \quad \frac{d^2 v}{d\eta^2} = k$$

The appearance of the term $\theta(-\xi \sin\alpha + \eta \cos\alpha) \cot\lambda'$ in the second of equations (21) would make it appear at first sight as if h, i, k were in some way connected with ξ and η, which is not true. The manner in which it has arisen is owing to its having been neglected in the expression for the observed azimuths of B at A, and reciprocally. For if we suppose the points *actually* to observe one another, we shall have to retain those effects of the irregularities of the earth upon the azimuths which depend upon the mutual depressions of these points, namely $\pm \frac{1}{2}\theta(-\xi \sin\alpha + \eta \cos\alpha)$.

SECTION XI.

DETERMINATION OF LOCAL ATTRACTION.

1. The maps of the Ordnance Survey afford the means of calculating with much precision the disturbing effect of the irregularities of the ground in the vicinity of some of the Zenith-sector Stations. Unfortunately, the number of Stations for which this calculation can be made is comparatively small, as the system of contours, which affords the data for this work, has been only of late years carried out. Sufficient data can be obtained for Dunnose, Clifton Beacon, Burleigh-Moor, Calton Hill (observatory), Kellie Law, Monach, and the South end of the Lough Foyle Base in Ireland.

In the six-inch sheets of Ireland the heights of a great number of points are given, which afford data for the determination of the deflection at Forth Mountain, Feaghmaan, and Tawnaghmore, but imperfectly, for each of these Stations being situated on elevated and uncultivated ground, the number of altitude points is smaller than usual in the immediate vicinity of the Station, consequently the calculation for these Stations is imperfect. At Hungry Hill the ground has been specially contoured within a circle of a mile round the Station, at 100 feet intervals. Beyond this circle the altitudes engraved on the map must suffice for the calculation.

2. The method by which the local attraction at each of these Stations has been computed is as follows: Through the Station, as marked on the plan, draw a meridian line, and a system of radial lines making angles—

$$\sin^{-1}\left(\frac{1}{10}\right), \sin^{-1}\left(\frac{2}{10}\right), \sin^{-1}\left(\frac{3}{10}\right), \cdots\cdots \sin^{-1}\left(\frac{10}{10}\right)$$

with the north meridian line on each side; and a similar system with reference to the south meridian line: altogether 40 lines. Describe on the map a series of ten concentric circles round the Station as centre, the radii being successively 500, 1000, 1500,····4500, 5000 feet; and in continuation, another series of concentric circles round the same centre in geometrical progression, such that the radius of the n^{th} circle of this series $= 5000 \times (\frac{7}{6})^n$ feet. So that if we draw altogether 25 circles, as is done in most instances, a circle of 9·55 miles radius will be inclosed, and this area divided into 960 parts, omitting the centre circle, as may be done in most cases. When the ground in the immediate vicinity of the Station is uneven, the inner circle of 500 feet radius is divided by circles having radii of 100, 200, 300, and 400 feet.

3. Then to find the deflection we have the following rule, as proved at page 584: If H_n be the sum of the heights of all the compartments between the n^{th} and $(n+1)^{\text{th}}$ circles of the first series, on the north side of the Station, H'_n the same quantity for the south side, the total

deflection north due to the form of the ground within a circle of 5000 feet radius round the Station is—

$$2''\cdot4894\,\frac{\varrho}{\delta}\,\Sigma\,\frac{H_n - H'_n}{2n+1}$$

where H_n H' are to be expressed in miles: also ϱ is the density of the disturbing mass, and δ the mean density of the earth.

The quantity ϱ varies, indeed, from one compartment or prism to another, but it will be sufficiently accurate to assume a common value for all the compartments, and this must be the mean density of all the rocks within the space over which the calculation extends.

The mean densities of the rocks which exist in the vicinities of the different Stations for which the deflection has been calculated, may be taken as follows:—

Limestone	2·81	Gneiss	2·70
Trap	2·75	Quartz Rock	2·65
Chalk	2·72	Mica Slate	2·65
Mica	2·71	Sandstone	2·50

of which the mean value is 2·69. The mean densities of these rocks differ but little among one another—scarcely more than do different specimens and varieties of the same rock.

For the mean density of the earth we have six independent determinations. Of these, the determinations of Cavendish, Reich, and Baily with the torsion-balance, and the result of the Arthur's Seat experiment, agree very satisfactorily among themselves (see page 605), the mean of the four being 5·46. The value deduced by Playfair from the Schehallien experiment (which must be preferred to Dr. Hutton's result), namely, 4·71, and that obtained by the Astronomer Royal from the pendulum experiments, namely, 6·57, differ so materially from the others that it is clear we are not yet in possession of the truth.

The mean of the whole six values is 5·52, which is not very different from the mean (5·46) of the four which agree among themselves. We may therefore take 5·50 as the mean density of the earth, but affected with considerable probable error, perhaps not less than ± 0·10.

Seeing, then, that in the ratio $\varrho : \delta$ the quantity δ is not assignable with precision, it will be sufficient to assume the relation $\delta = 2\,\varrho$, throughout the calculations.

4. If in the formula for the deflection we express the quantities H_n and H'_n in feet instead of miles, and make $\delta = 2\,\varrho$, the deflection north becomes equal to—

$$0''\cdot00023574\left(\frac{H_1 - H'_1}{3} + \frac{H_2 - H'_2}{5} + \frac{H_3 - H'_3}{7} + \dots\dots + \frac{H_9 - H'_9}{19}\right)$$

Also the deflection due to the conformation of the ground between the tenth and last circles, which are drawn according to a geometric series whose ratio is 7 : 6, is—

$$1''\cdot2447\,\frac{\varrho}{\delta}\,\log_e\frac{7}{6}\left(K - K'\right)$$

K being the sum of all the heights on the north side, K′ the sum of all the heights on the south side, expressed in miles. Expressing these heights in feet, and making the same assumption as previously with respect to the density, we have (since $\log_e 7 = 1\cdot94591$ & $\log_e 6 = 1\cdot79176$) the deflection equal to

$$0''\cdot00001817\,(K - K')$$

If the inner circle of 500 feet be divided by four intermediate circles, the deflection due to the conformation of the ground within this space is

$$0''\cdot62235 \cdot \frac{100}{5280} \cdot (S - S') = 0''\cdot01179\,(S - S')\,.$$

where S is the sum of the sines of the elevations of all the prisms or compartments on the north side, and S′ the sum of the sines of the elevations of all the prisms or compartments on the south side of the Station, and within the 500 feet circle.

The whole deflection at the Station is therefore expressed by the quantity—

$$0''\cdot01179\,(S - S') + 0''\cdot00023574 \left(\frac{H_1 - H'_1}{3} + \frac{H_2 - H'_2}{5} + \cdots + \frac{H_9 - H'_9}{19}\right) + 0''\cdot00001817\,(K - K') \quad \ldots\ldots (1)$$

In a few instances it is necessary to take into account the third power of the apparent elevations or depressions, or, in other words, to discriminate between the sine and tangent. This may be done with sufficient accuracy as follows: Let h be the height of the Station, $h_1\, h_2\, h_3 \ldots h_{20}$ the heights of the different compartments between the n^{th} & $(n + 1)^{\text{th}}$ circles on the north side, then the deflection is proportional to—

$$\Sigma\left(\frac{h_i - h}{r}\right) - \tfrac{1}{2}\,\Sigma\left(\frac{h_i - h}{r}\right)^3$$

Here $\Sigma\,(h_i) = H_n$; and we may for h_i in the second part of the above expression put $\frac{1}{20}H_n$, so that the quantity becomes—

$$\frac{H_n - 20\,h}{r} - \frac{10}{r^3}\left(\frac{H_n}{20} - h\right)^3$$

$$= \frac{1}{r}\left\{H_n - 20\,h - \frac{160}{(2\,n + 1)^2}\left(\frac{H_n}{2000} - \frac{h}{100}\right)^3\right\}$$

because $r = 250 \times (2\,n + 1)$: therefore to H_n, in the formula (1), must be applied the correction—

$$-\frac{160}{(2\,n + 1)^2}\left(\frac{H_n}{2000} - \frac{h}{100}\right)^3 \quad \ldots\ldots\ldots\ldots (a)$$

with a similar correction for H'_n on the south side.

5. If any one of the compartments consist of a depth h of sea, then for this we must take

$$\frac{\rho}{\delta} = \frac{1}{5\cdot5} - \frac{1}{2}$$

supposing the mean density of the earth = 5·5. In this case, therefore, the coefficient given for a disturbing mass of rock must be multiplied by $-\frac{7}{11}$.

In the cases we have to consider, with one exception, the sea occurs only in the compartments formed by the outer series of circles, namely those whose radii form a geometrical series. If, therefore, D be the sums of the depths of the sea in all the compartments covered by it to the north of the Station, D′ the sum of all the depths to the south, the increment to the deflection north is

$$-0''.00001156\,(D - D')$$

And in general, if $r_,$ r' be the radii of the bounding circular arcs of any compartment, $\alpha_,$ α' the azimuths of the bounding straight lines, the deflection due to this compartment is—

$$\left.\begin{array}{ll} \text{For land} \cdots\cdots & +\,0''.002714\,(\sin\alpha' - \sin\alpha_,)\log_{10}\dfrac{r'}{r_,}\cdot(\textit{mean height}) \\ \text{For sea} \cdots\cdots\cdots & -\,0''.001727\,(\sin\alpha' - \sin\alpha_,)\log_{10}\dfrac{r'}{r_,}\cdot(\textit{mean depth}) \end{array}\right\} \qquad (2)$$

6. The mean heights of the different compartments can be obtained with sufficient accuracy and without much trouble when the contoured maps are available for the purpose. Great accuracy in the determination of the mean height is not necessary, as may be seen from the following considerations: In the formula (1) each of the terms H_1 H'_1 &c. consists of 20 estimated mean heights; and if we write E_1 E'_1 E_2 E'_2 &c. for the corresponding sums of the errors, the error of the centre part of the formula is—

$$0''.00024\left\{\frac{E_1 - E'_1}{3} + \frac{E_2 - E'_2}{5} + \cdots\ \frac{E_9 - E'_9}{19}\right\}$$

Each of the quantities K K′ is the sum of 300 estimated mean heights (supposing that there are 15 circles drawn according to the geometric-progression law), and if E and E′ be the sums of the corresponding errors, the error of the last part of the formula is—

$$0''.000018\,(E - E')$$

The error to which the first term of the formula (1) expressing the total deflection is liable, is small, inasmuch as the compartments being small, their mean heights may be estimated pretty accurately. We shall therefore omit that term, and the error of the calculated deflection is—

$$0''.00024\left\{\frac{E_1 - E'_1}{3} + \frac{E_2 - E'_2}{5} + \cdots\ \frac{E_9 - E'_9}{19}\right\} + 0''.000018\,(E - E')$$

Here each of the quantities E_1 E'_1 . . . . are composed of 20 independent errors, and each of the quantities E E′ of 300 independent errors. If we suppose, therefore, that each of the estimated mean heights is liable to an equal probable error e, the square of the probable error of the calculated deflection is—

$$= (0''.00024)^2\left\{\frac{40\,e^2}{3^2} + \frac{40\,e^2}{5^2} + \cdots\ \frac{40\,e^2}{19^2}\right\} + (0''.000018)^2\,600\,e^2$$

$$= e^2\left\{40\,(0.00024)^2\left(\frac{1}{3^2} + \frac{1}{5^2} + \cdots\ \frac{1}{19^2}\right) + 600\,(0.000018)^2\right\}$$

$$= (0.00024\cdot e)^2\left\{40\left(\frac{1}{3^2} + \frac{1}{5^2} + \cdots\ \frac{1}{19^2}\right) + 600\left(\frac{3}{40}\right)^2\right\}$$

so that the probable error is equal to—

$$\pm\,0''.00024\cdot e\left\{40\left(\frac{1}{3^2} + \frac{1}{5^2} + \cdots\ \frac{1}{19^2}\right) + \frac{27}{8}\right\}^{\frac{1}{2}} = \pm\,0''.00082\cdot e$$

So that if all the estimated mean heights of the compartments be liable to a probable error of about 25 feet, the probable error of the computed deflection would only amount to about $\pm 0''\cdot 02$.

7. The probable error of the determination of the mean height of a compartment will increase with the area of the compartment; and therefore, in order that the larger compartments may be estimated as accurately as the smaller, it is necessary to subdivide them. The probable error of the estimated mean height of a compartment does not appear to increase in proportion to the area (not subdivided), but perhaps as the square root of the area. Assuming this, let it be required to find the number of parts into which a compartment whose area is A should be divided, in order that the probable error of its resulting mean height should be equal to that of the estimated mean height of a smaller area a. If x be the number of parts, p the probable error corresponding to a, the probable error of the estimated mean height of one of the subdivisions of A is

$$p\sqrt{\frac{A}{a\,x}}$$

and therefore the probable error of the mean of the mean heights of the x subdivisions is

$$\frac{p}{\sqrt{x}}\sqrt{\frac{A}{a\,x}}=\frac{p}{x}\sqrt{\frac{A}{a}}$$

As this is to be equal to p, we have $x=\sqrt{\frac{A}{a}}$. So that if a compartment be in area equal to eight inches square, it should be divided into eight parts, in order that the mean height should be determined as accurately as a single estimation of a space whose area is one square inch. But in practice this rule cannot always be advantageously employed.

ISLE OF WIGHT SECTOR STATIONS.

Boniface Down.

Plate XXII. shows the contours of the ground in the vicinity of the four Stations, Dunnose, Boniface Down, Week Down, and Highport Cliff, surveyed at intervals of 25 feet vertically. There are two principal features, the one running north from Week Down, about two miles in length and about 700 feet high; the other a curved ridge, having Dunnose Station at its northern extremity, and Boniface Station towards the south-western extremity: the height of this range averages 750 feet. The Station Highport Cliff or Port Valley is close to the sea, and lies S.S.E. from Boniface Down.

The extent of the contours will only allow of 17 circles being drawn round each Station.

The following table contains the heights of the compartments around Boniface Down:—

BONIFACE DOWN. North Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	755	750	740	725	725	725	730	735	740	750	755	755	760	760	765	775	780	780	780	780	15060
2— 3	675	650	645	635	630	630	640	650	650	655	660	670	690	700	715	725	735	750	770	780	13655
3— 4	600	510	515	515	510	510	510	520	530	535	540	555	570	590	610	625	640	655	725	770	11535
4— 5	520	435	430	440	450	465	465	465	465	470	475	485	500	535	550	580	650	720	775	750	10625
5— 6	525	425	385	410	435	450	435	425	450	460	465	480	500	530	610	685	740	750	730	685	10575
6— 7	525	410	350	355	370	385	400	435	475	500	510	530	550	600	710	750	745	715	575	630	10520
7— 8	540	370	330	400	440	465	470	470	470	510	555	625	670	730	760	725	650	575	500	575	10830
8— 9	540	390	325	375	450	485	500	510	525	550	625	720	730	735	725	675	530	465	375	500	10730
9—10	530	390	310	375	450	500	540	565	575	610	675	725	710	605	585	625	520	350	310	385	10335
10—11	430	375	300	360	450	530	660	680	665	700	750	730	630	515	470	520	520	375	260	250	10170
11—12	325	300	275	350	435	540	625	610	560	570	575	560	515	410	350	375	480	370	230		8455
12—13	410	275	240	310	375	450	475	430	390	375	375	375	365	345	300	275	375	380	75		6595
13—14	600	335	250	235	280	345	335	335	270	250	250	275	250	215	210	180	220	125			4960
14—15	625	490	315	210	200	215	225	250	220	215	220	225	230	175	175	145					4135
15—16	480	515	310	225	200	150	180	175	175	135	150	175	135	125	125	110					3365
16—17	325	325	215	160	150	170	135	165	155	165	135	90	90	110	120	50					2560

BONIFACE DOWN. South Side.

Number of Circle.	Number of Sector.																				II
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	730	650	600	580	560	560	570	570	580	600	620	630	650	670	685	710	730	750	755	770	12970
2— 3	705	530	450	460	510	570	620	650	640	610	575	550	525	500	500	510	530	580	600	700	11310
3— 4	700	520	400	420	500	550	520	450	380	320	280	265	260	260	260	265	280	350	435	600	8015
4— 5	700	550	380	350	400	370	310	300	250	200	200	180	180	160	180	180	160	180	300	490	6020
5— 6	680	580	425	315	260	230	220	155	150	135	170	170	160	150	180	190	155	150	175	400	5050
6— 7	640	530	450	325	175	130	110	100	120	115	125	125	115	110	100	115	100	110	120	325	4040
7— 8	530	465	415	270	150	115	100	75	50	50	65	30					20	50	100	260	2745
8— 9	480	450	370	225	125	20													40	170	1880
9—10	430	510	335	185	125															120	1705
10—11	450	520	300	140	35	—40	—60	—60	—70	—70	—70	—70	—70	—70	—70	—70	—70	—60	—60	—40	1445
11—12	500	460	185	75	—20	—60	—70	—70	—70	—80	—80	—80	—80	—80	—80	—80	—70	—70	—70	—70	1220
12—13	560	400	100	—30	—40	—70	—80	—80	—80	—90	—90	—90	—90	—90	—90	—80	—80	—80	—80	—90	1060
13—14	625	350	70	—50	—70	—80	—80	—90	—90	—100	—100	—100	—100	—100	—100	—90	—90	—90	—90	—90	1045
14—15	470	225	25	—60	—70	—90	—110	—120	—110	—150	—150	—160	—150	—130	—120	—110	—100	—100	—90	—90	720
15—16	425	275	20	—60	—90	—110	—140	—170	—190	—190	—200	—200	—200	—190	—180	—120	—110	—100	—90	—90	720
16—17	335	250	—40	—80	—110	—180	—190	—200	—200	—200	—200	—200	—190	—190	—180	—120	—110	—100	—90	—90	585

N.B.—The Sectors are numbered from West to East, and the Circles from the Centre outwards.

The inner circle of 500 feet must here be subdivided, on account of the abruptness of the ground on the southern side, by drawing four concentric circles having radii of 100, 200, 300, and 400 feet. Thus we find for the south side $S' = -10\cdot3$. It is also necessary on the south side to calculate the corrections (*a*), page 627. These quantities are found to be—

45, 65, 185, 225, 195, 185, 190, 180, 150,

and applying them with the positive sign to the quantities H (South) in the table, we have—

$$H_1 - H'_1 = 2045 \qquad H_4 - H'_4 = 4380 \qquad H_7 - H'_7 = 7895$$
$$H_2 - H'_2 = 2280 \qquad H_5 - H'_5 = 5330 \qquad H_8 - H'_8 = 8670$$
$$H_3 - H'_3 = 3335 \qquad H_6 - H'_6 = 6295 \qquad H_9 - H'_9 = 8480$$
$$K = 40240 \qquad K' = 6795 \qquad S = 0 \qquad S' = -10\cdot30$$

and the sum of the depths of sea to the south—

$$D' = 11930.$$

Therefore, as far as the 17th circle the deflection is—

$$0''\cdot01179 \times 10\cdot3 + 0''\cdot00023574 \times 4552 + 0''\cdot00001817 \times 33445 + 0''\cdot00001156 \times 11930$$

amounting to $1''\cdot940$ to the north.

We may extend the calculation to the limits of the Isle of Wight in the following manner: In Plate XXIII. *abcdef* is the 17th circle drawn round Boniface Down; through *a*, *b*, and *c* draw the lines *aa' bb' cc'* passing through the station, and making angles of 70°, 9°, and 33° respectively with the north meridian line, and let these lines be continued in the opposite directions, shown in the plan as *dd' ee' ff'*. Take *aa'* = *dd'* = 9 miles, *cc'* = *ff'* = 6 miles, and, with Boniface as a centre, describe the circular arcs *a'b'*, *c'b,*, and similarly on the south side. That portion of the island which lies south of the line *aa'* may be disregarded, and the two spaces *ab a'b'* and *bc b,c'* may be considered as containing the whole of the effective disturbing force of the island, the opposite and corresponding spaces to the south and south-east being covered with sea. For the land, the mean height is to be taken at 150 feet, and the mean depth of sea estimated from the Admiralty Charts at about 90 feet (by giving greater weight to the nearer parts). This depth of 90 feet of water is equivalent to a *hollow* of $\frac{7}{11} \times 90$, or 57 feet. Thus we may take account of the disturbing effect of the sea by adding, say 60 feet to the mean height of the land in the spaces *ab a'b'*, *bc b,c'*, making that quantity 210. The following table contains the calculation of the deflections due to these spaces:—

Space.	h	α'	$\alpha_{,}$	r'	$r_{,}$	Deflection.
ab a'b'	210	70°	9°	11·8	2·8	+ 0″·278
bc b,c'	210	33	9	8·8	2·8	+ 0·199

Therefore the effect of that part of the island which is external to the 17th circle round Boniface Down is to produce a deflection of 0″·477 to the north.

If we add this quantity to that previously determined, namely, 1″·940, we get for the deflection at Boniface Down 2″·417 to the north.

Dunnose.

The heights of the different compartments around this Station are given, up to the 17th circle, in the following table:—

DUNNOSE. North Side.

Number of Circle.	Number of Sector.																				II
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	720	700	700	685	675	660	650	620	630	630	630	630	640	650	650	660	660	660	650	630	13130
2— 3	650	610	600	575	570	570	560	560	520	510	510	500	500	500	500	510	525	535	520	520	10845
3— 4	630	520	480	455	440	430	430	425	360	410	400	410	415	425	415	410	415	430	420	400	8720
4— 5	630	520	435	400	375	375	380	380	300	335	340	335	350	375	375	350	365	380	375	350	7725
5— 6	620	475	420	380	325	310	325	340	270	285	275	285	310	325	285	300	300	320	350	310	6800
6— 7	565	430	375	345	300	260	255	290	220	230	235	270	285	285	235	235	240	240	280	275	5850
7— 8	515	410	375	320	275	260	225	230	235	210	225	250	250	260	235	225	200	210	220	280	5410
8— 9	425	375	350	330	270	240	225	200	240	180	225	240	230	250	285	215	180	210	200	280	5150
9—10	375	350	315	310	260	235	200	180	130	165	240	235	200	220	235	215	165	200	180	300	4710
10—11	310	300	230	260	240	210	190	165	200	160	230	230	200	165	200	150	160	175	150	250	4175
11—12	225	220	210	230	220	180	180	150	125	140	150	180	175	140	135	140	120	150	110		3180
12—13	210	170	160	180	160	160	160	170	150	120	110	125	125	120	100	125	125	120	40		2630
13—14	290	200	170	135	160	170	150	165	165	150	150	90	85	80	100	120	140	90			2610
14—15	300	160	170	170	130	125	150	150	170	150	130	100	60	70	100	110	100				2345
15—16	280	140	100	120	110	100	130	160	160	170	150	100	60	50	80	80	30				2020
16—17	195	100	75	75	80	100	100	125	150	170	120	90	70	40	50	100					1640

DUNNOSE. South Side.

Number of Circle.	Number of Sector.																				II
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	680	650	650	645	645	650	670	675	700	710	715	725	730	735	735	725	715	700	675	650	13780
2— 3	625	570	565	560	560	550	560	575	615	635	675	700	720	735	735	725	685	650	600	550	12590
3— 4	625	540	520	515	510	510	510	520	550	570	610	620	675	735	755	750	715	650	660	460	12000
4— 5	600	510	500	460	450	475	485	500	500	500	520	545	600	720	755	755	735	675	625	480	11390
5— 6	560	490	485	425	400	430	450	450	460	475	510	530	600	700	755	725	650	625	600	525	10845
6— 7	490	465	450	375	440	455	460	470	490	500	530	550	600	725	755	710	570	500	490	510	10535
7— 8	425	400	400	360	435	460	485	530	575	600	610	600	620	750	755	675	540	425	380	490	10515
8— 9	370	350	340	355	365	455	530	600	680	720	720	700	700	775	690	575	475	370	315	420	10505
9—10	320	300	315	405	415	475	600	690	750	775	775	770	765	780	645	470	400	300	265	345	10560
10—11	290	380	400	455	480	550	670	750	760	770	780	740	700	650	675	680	560	400	210	240	11140
11—12	260	425	525	560	620	700	640	540	570	550	520	450	430	430	430	420	375	250	80		8775
12—13	310	350	500	525	520	600	500	375	385	260	180	180	140	140	130	150	150		—10	—20	5395
13—14	425	390	375	410	460	440	300	170	120	150	100	60	30	—10	—10	—20	—30	— 30	—30	—20	3430
14—15	650	575	525	610	500	275	150	—20	—40	—60	—60	—60	—70	—70	—70	—70	—80	— 90	—80	—20	3285
15—16	500	550	610	410	200	40	—40	—60	—70	—70	—80	—80	—80	—80	—80	—90	—90	— 100	—90	—30	2310
16—17	380	450	430	230	20	—60	—70	—80	—80	—80	—90	—90	—90	—90	—90	—90	—90	— 100	—90	—30	1510

From this table we find—

$H_1 - H'_1 = -\ 650$ $H_4 - H'_4 = -\ 3665$ $H_7 - H'_7 = -\ 5105$

$H_2 - H'_2 = -\ 1745$ $H_5 - H'_5 = -\ 4045$ $H_8 - H'_8 = -\ 5355$

$H_3 - H'_3 = -\ 3280$ $H_6 - H'_6 = -\ 4685$ $H_9 - H'_9 = -\ 5850$

$K = 18600$ $K' = 35845$ $D' = 3230$

The ground in the immediate vicinity of the Station within the 500-feet circle is not sufficiently uneven to require the calculation of S and S′. We have therefore the deflection (as far as the 17th circle) equal to—

$$-\ 0''{\cdot}00023574 \times 3133 - 0''{\cdot}00001817 \times 17245 + 0''{\cdot}00001156 \times 3230$$

or 1″·015 to the south.

It remains to add to this, the effect of that portion of the island which is external to the 17th circle. This quantity will not differ materially from the quantity obtained for Boniface Down, as is evident from the proximity of the two Stations: therefore if we deduct 0″·477 from 1″·015, we obtain 0″·538 as the deflection at Dunnose to the south.

Week Down.

The heights of the several compartments around this Station are given, up to the 17th circle, in the following table:—

WEEK DOWN. North Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	550	555	575	585	610	635	655	670	680	680	680	680	680	675	670	665	650	630	605	655	12785
2— 3	470	510	550	580	610	635	660	675	680	675	665	645	600	575	550	525	515	510	510	575	11715
3— 4	445	500	570	610	650	675	680	680	665	640	600	550	500	475	465	450	430	425	420	500	10930
4— 5	435	480	575	620	660	680	680	675	635	600	540	500	450	425	415	410	390	365	370	470	10375
5— 6	445	500	550	580	635	675	680	660	610	550	505	460	425	380	360	335	350	390	400	475	9965
6— 7	450	500	500	550	615	670	675	660	600	540	490	450	425	390	330	325	385	420	420	430	9825
7— 8	450	465	485	515	580	625	670	670	610	550	460	410	360	335	310	350	410	475	500	430	9660
8— 9	410	450	480	500	525	550	630	700	670	560	450	400	350	300	300	375	450	550	600	465	9715
9—10	350	470	470	455	440	500	600	700	690	560	440	375	325	290	325	400	460	570	615	530	9565
10—11	320	425	385	390	490	550	630	720	630	510	430	345	300	260	325	410	435	450	520	610	9135
11—12	280	315	325	470	530	580	675	690	535	425	370	325	275	250	300	325	365	355	465	610	8465
12—13	230	270	325	400	410	400	500	625	525	410	325	270	230	255	290	320	350	355	430	580	7500
13—14	220	250	270	280	300	325	430	525	500	400	320	240	210	300	375	450	500	455	465	600	7415
14—15	290	160	170	190	210	275	325	325	300	300	260	200	210	320	375	570	665	590	560	500	6795
15—16	300	260	250	200	160	195	225	190	170	175	210	190	200	240	335	350	480	640	650	380	5800
16—17	200	190	180	170	160	150	160	125	125	130	160	170	160	210	210	220	280	350	425	175	3950

WEEK DOWN. South Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	575	600	615	635	640	650	655	655	650	635	620	615	600	600	590	580	570	560	560	610	12215
2— 3	475	520	550	565	575	580	585	580	575	560	535	510	475	460	450	425	415	425	475	600	10335
3— 4	425	435	450	470	480	490	500	495	480	470	450	440	425	385	365	365	400	465	530	625	9145
4— 5	410	400	400	400	410	415	415	400	390	375	370	360	340	325	350	375	400	420	500	620	8075
5— 6	410	380	380	385	385	380	370	345	275	250	225	225	220	220	225	240	270	330	380	555	6450
6— 7	420	380	375	375	375	360	300	215	200	150	140	120	120	115	120	130	130	175	280	490	4970
7— 8	430	375	375	410	400	300	210	175	150	85	60	50	55	60	50	60	75	100	200	450	4070
8— 9	430	360	400	450	410	235	170	115	85	60	50	30		30	20	10	30	80	130	350	3445
9—10	400	350	405	435	300	200	120	70	60	20									95	280	2735
10—11	320	350	390	370	230	135	75	60	— 10	— 20	— 30	— 40	— 40	— 50	— 60	— 60	— 60	— 30		220	2150
11—12	330	400	410	250	135	75	— 10	— 20	— 30	— 30	— 40	— 50	— 60	— 60	— 70	— 70	— 70	— 60	— 40	120	1720
12—13	400	455	330	140	50	— 10	— 30	— 40	— 50	— 60	— 70	— 70	— 80	— 80	— 80	— 80	— 80	— 70	— 50	75	1450
13—14	335	410	250	80	— 30	— 40	— 50	— 60	— 70	— 80	— 90	—100	—110	—110	—110	— 90	— 90	— 80	— 60	60	1135
14—15	300	325	70	— 30	— 40	— 60	— 70	— 80	— 90	—100	—120	—140	—150	—150	—140	—100	— 90	— 80	— 60	— 20	695
15—16	325	300	— 20	— 40	— 60	— 70	— 80	— 90	—110	—120	—150	—180	—190	—190	—200	—180	—180	—120	— 80	— 70	625
16—17	300		— 30	— 50	— 70	— 90	—100	—110	—110	—120	—140	—180	—190	—190	—200	—200	—200	—190	—110	— 70	300

From this table we find the following numbers—

$H_1 - H'_1 = 570$ $H_4 - H'_4 = 2300$ $H_7 - H'_7 = 5590$
$H_2 - H'_2 = 1380$ $H_5 - H'_5 = 3515$ $H_8 - H'_8 = 6270$
$H_3 - H'_3 = 1785$ $H_6 - H'_6 = 4855$ $H_9 - H'_9 = 6830$

$K = 49060$ $K' = 8075$ $D' = 9030$

The quantities S S′ may be neglected, as the ground does not fall abruptly either north or south; the deflection then is—

$$0''{\cdot}00023574 \times 2770 + 0''{\cdot}00001817 \times 40985 + 0''{\cdot}00001156 \times 9030$$

or 1″·502 to the north, up to the 17th circle. To obtain the effect of the remainder of the mass of the island, we may apply the same quantity as obtained for Boniface Down, namely 0″·477; this added to the former gives 1″·979 for the deflection to the north at Week Down.

Port Valley.

This is the name of the point at which the zenith-sector was placed in connection with Highport Cliff. The heights of the various compartments are shown in the following table:—

PORT VALLEY. North Side.

Number of Circle.	Number of Sector.																				II
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	175	190	185	170	170	160	160	165	165	170	160	150	150	150	140	130	125	120	110	80	3025
2— 3	160	200	210	210	210	225	240	245	245	250	250	225	200	170	150	125	120	120	115	90	3760
3— 4	210	280	330	400	450	475	425	450	460	450	430	400	370	345	260	230	195	150	125	40	6475
4— 5	290	500	605	670	670	700	675	650	650	620	550	530	480	450	420	350	260	220	140	30	9460
5— 6	385	620	585	655	750	780	780	780	780	755	700	650	615	560	490	410	350	295	150		11090
6— 7	350	465	625	750	785	780	780	770	765	775	780	770	740	690	600	475	375	315	160		11750
7— 8	365	600	745	760	760	760	750	730	675	720	780	715	615	630	700	600	405	320	150		11780
8— 9	540	710	675	675	670	670	650	620	600	690	760	685	530	450	550	570	400	225	50		10720
9—10	560	660	550	550	550	550	525	530	570	685	750	675	560	425	425	450	345	170			9530
10—11	525	600	460	435	460	465	465	490	565	720	720	600	470	330	300	315	220	75		—30	8215
11—12	530	530	415	350	385	415	490	550	670	750	670	580	450	340	275	180				—30	7580
12—13	490	500	375	350	465	510	530	620	730	625	500	500	515	450	330	140		—25	—25	—40	7630
13—14	440	375	340	325	435	630	670	660	660	550	405	325	325	400	365	60	—25	—30	—30	—40	6965
14—15	570	340	255	310	415	550	500	400	380	355	285	250	210	210			—30	—30	—35	—50	5030
15—16	580	550	285	220	285	340	340	255	240	250	250	175	180	100		—30	—35	—35	—35	—50	4050
16—17	450	600	410	250	175	200	225	180	175	210	150	125	125	70		—30	—35	—35	—40	—70	3345

PORT VALLEY. South Side.

Number of Circle.	Number of Sector.																				II
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	135	100	70	65	45																415
2— 3	155	100	60				—10	—10	—10	—10	—10	—10	—10	—10	—10	—10	—10				315
3— 4	160	80				—10	—10	—15	—15	—15	—15	—15	—15	—15	—10	—10	—10	—10			240
4— 5	110	65			—10	—10	—15	—15	—20	—20	—30	—30	—30	—20	—20	—20	—15	—10	—10		175
5— 6	100			—10	—10	—15	—15	—20	—20	—25	—30	—30	—30	—30	—20	—20	—20	—20	—20	—20	100
6— 7	115		—10	—20	—20	—30	—30	—30	—40	—40	—40	—40	—40	—40	—40	—30	—30	—30	—20	—20	115
7— 8	95		—20	—30	—30	—40	—40	—40	—40	—50	—50	—55	—55	—60	—55	—50	—40	—30	—20	—20	95
8— 9	150	—10	—20	—30	—30	—40	—45	—50	—50	—55	—60	—65	—65	—65	—60	—55	—55	—50	—40	—40	150
9—10	165	—20	—30	—40	—50	—60	—60	—60	—70	—70	—80	—80	—70	—70	—65	—65	—65	—60	—60	—55	165
10—11	200	—40	—70	—70	—80	—80	—80	—80	—90	—90	—90	—90	—90	—90	—90	—90	—90	—80	—80	—90	200
11—12	270	—50	—70	—80	—80	—80	—90	—90	—100	—100	—100	—100	—100	—100	—100	—100	—100	—90	—90	—90	270
12—13	300	—60	—70	—80	—90	—90	—100	—110	—120	—130	—130	—120	—120	—120	—110	—110	—110	—100	—90	—90	300
13—14	275	—60	—80	—90	—100	—120	—150	—180	—180	—180	—190	—190	—180	—180	—150	—120	—110	—100	—90	—90	275
14—15	300	—40	—80	—110	—120	—180	—180	—190	—200	—200	—200	—200	—190	—190	—180	—120	—110	—100	—90	—90	300
15—16	275	—40	—90	—120	—180	—190	—200	—200	—200	—200	—190	—190	—180	—150	—120	—110	—110	—100	—90	—100	275
16—17	320	—60	—100	—150	—190	—200	—200	—190	—190	—180	—150	—120	—110	—110	—100	—100	—100	—100	—90	—100	320

On account of the great apparent elevation of Boniface Down at Highport Cliff, it becomes necessary to apply to the quantities H on the north side the correction (a); these are found to be 10, 10, 55, 130, 150, 130, 100, 55, 30, to be applied with the negative sign.

Also to obtain the quantities H on the south side, the depths of water (up to the 10th circle) are to be multiplied by $\frac{7}{11}$ and treated as hollows, or negative heights. We have then—

$H_1 - H'_1 = 2600 \qquad H_4 - H'_4 = 9330 \qquad H_7 - H'_7 = 12046$

$H_2 - H'_2 = 3505 \qquad H_5 - H'_5 = 11066 \qquad H_8 - H'_8 = 11072$

$H_3 - H'_3 = 6285 \qquad H_6 - H'_6 = 11855 \qquad H_9 - H'_9 = 10054$

$K = 42815 \qquad K' = 1940 \qquad D = 815 \qquad D' = 15830 \qquad S = 19{\cdot}00 \qquad S' = 6{\cdot}48$

the quantities S S′ having been obtained by subdividing the circle of 500 feet by four smaller circles. We have then, up to the 17th circle, the deflection equal to—

$$0''{\cdot}01179 \times 12{\cdot}52 + 0''{\cdot}00023574 \times 7404 + 0''{\cdot}00001817 \times 40875 + 0''{\cdot}00001156 \times 15015$$

or $2''{\cdot}809$ to the north. To this we must add the deflection due to that part of the island which is external to the 17th circle round this Station. This may be taken as equal to the quantity calculated for Boniface Down, namely $0''{\cdot}477$, which added to the preceding, gives $3''{\cdot}286$ for the deflection to the north.

Clifton Beacon.

This Station is in the north of Sheet 290 of Yorkshire. The country to the south is rather higher than to the north; the ground in the immediate vicinity of the Station is tolerably even, and does not require the sines of the depressions or elevations to be computed.

The following table contains the heights of the different compartments up to the 24th circle, the radius of which is 43273 feet, or about $8\frac{1}{4}$ miles.

CLIFTON BEACON. North Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	380	390	400	410	425	430	435	435	435	435	435	435	435	435	430	430	430	430	430	440	8505
2— 3	350	345	355	365	380	405	420	420	420	415	410	415	415	415	415	420	425	425	425	420	8060
3— 4	320	325	325	325	330	335	335	330	340	345	355	375	390	395	400	405	405	410	415	415	7275
4— 5	295	295	290	290	295	295	295	295	295	295	305	320	355	385	400	400	400	400	400	400	6695
5— 6	270	265	270	270	265	265	265	270	270	275	290	320	360	385	395	400	400	400	400	395	6430
6— 7	250	240	245	250	245	240	240	255	265	275	295	320	360	385	395	400	400	400	395	390	6245
7— 8	255	225	215	225	225	220	250	275	275	265	270	300	325	360	385	400	400	400	380	370	6020
8— 9	270	225	200	200	200	210	250	260	265	265	250	265	290	310	350	375	390	380	370	350	5675
9—10	275	225	200	195	185	205	220	215	235	240	240	245	265	285	315	350	375	365	360	345	5340
10—11	270	240	195	175	185	195	185	180	195	210	210	250	260	285	300	315	335	345	350	330	5010
11—12	250	265	240	200	165	170	155	200	250	250	255	270	275	275	275	275	270	310	325	325	5000
12—13	235	275	200	225	195	175	190	175	220	250	270	270	260	250	250	250	255	290	300	305	4900
13—14	225	265	255	240	230	235	230	195	160	200	235	235	245	225	225	245	280	300	290	290	4805
14—15	225	240	245	275	280	275	265	200	120	80	150	155	220	225	220	225	260	280	250	235	4425
15—16	195	155	190	220	220	200	190	100	80	110	95	60	160	205	190	185	205	235	195	210	3400
16—17	125	200	190	145	115	80	65	55	150	245	245	125	140	195	165	130	115	170	150	230	3035
17—18	125	150	120	70	75	50	50	70	230	260	240	210	120	110	150	100	80	85	100	200	2595
18—19	80	70	100	105	115	55	55	110	250	265	225	180	135	85	130	120	85	65	60	100	2390
19—20	140	90	135	130	60	50	105	275	300	265	225	195	170	115	90	65	70	65	50	50	2645
20—21	200	165	120	85	60	110	150	260	290	225	205	180	130	100	70	50	50	25	20	25	2520
21—22	260	160	80	85	100	155	285	310	260	220	155	165	140	75	35	30	50	25	20	25	2635
22—23	250	115	105	115	150	200	305	330	270	180	140	155	145	45	25	25	55	35	45	45	2735
23—24	290	140	110	145	160	200	225	270	270	195	135	90	35	25	25	25	45	45	35	55	2520

CLIFTON BEACON. South Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	380	385	385	390	390	395	400	410	415	420	425	435	440	440	445	450	450	450	450	445	8400
2— 3	345	355	360	365	370	375	375	385	390	400	410	425	425	425	425	425	425	425	435	425	7965
3— 4	330	335	340	350	355	365	375	380	390	400	400	405	410	410	415	415	420	420	420	415	7750
4— 5	300	320	325	335	345	355	365	375	385	395	410	415	410	410	405	400	400	405	400	400	7555
5— 6	290	310	320	330	335	350	355	370	375	395	410	425	415	410	400	400	400	395	390	385	7460
6— 7	275	300	310	325	335	350	370	380	390	400	410	415	410	400	400	400	400	390	380	375	7415
7— 8	295	315	275	330	345	365	380	400	420	430	425	425	420	410	400	400	400	390	375	365	7565
8— 9	310	325	325	345	355	400	440	445	450	445	440	430	415	405	400	400	400	390	370	350	7840
9—10	315	300	310	340	380	450	475	465	455	450	440	430	420	400	395	390	390	380	365	350	7900
10—11	300	285	305	330	375	410	450	450	450	450	440	430	415	395	380	365	375	360	345	335	7645
11—12	270	320	355	365	385	400	440	450	450	450	440	425	425	415	385	360	350	345	330	315	7675
12—13	245	330	370	375	385	400	415	445	450	450	445	440	430	415	385	350	330	330	305	305	7600
13—14	250	320	355	365	360	365	390	440	450	450	445	425	430	405	370	335	325	320	300	300	7400
14—15	310	370	385	375	375	355	335	375	425	445	430	415	390	385	360	340	335	315	285	285	7290
15—16	310	410	400	400	380	340	315	325	345	360	415	410	380	365	370	360	330	290	260	265	7030
16—17	275	405	435	375	350	355	340	310	305	300	330	340	320	350	365	345	300	260	225	245	6530
17—18	240	395	435	450	435	385	360	340	340	400	410	370	295	345	310	290	260	210	180	220	6670
18—19	190	350	470	470	415	375	350	360	380	400	400	375	300	280	305	250	215	160	130	160	6335
19—20	150	275	375	390	390	400	400	380	365	390	350	310	265	225	220	220	195	160	140	110	5710
20—21	135	275	270	250	330	335	375	360	350	380	350	335	315	275	220	160	150	135	110	50	5160
21—22	145	295	190	285	285	355	365	345	350	350	355	360	325	220	210	175	165	115	60	45	4995
22—23	175	185	155	225	305	350	375	350	325	345	385	370	300	215	190	170	160	130	110	100	4920
23—24	270	190	140	210	290	280	310	350	360	360	360	315	230	190	170	170	150	120	95	70	4630

From this table we obtain—

$$\begin{array}{lll} H_1 - H'_1 = 105 & H_4 - H'_4 = -860 & H_7 - H'_7 = -1545 \\ H_2 - H'_2 = 95 & H_5 - H'_5 = -1030 & H_8 - H'_8 = -2165 \\ H_3 - H'_3 = -475 & H_6 - H'_6 = -1170 & H_9 - H'_9 = -2560 \end{array}$$

$$K = 48615 \qquad K' = 89590$$

Therefore by the formula (1) the deflection in seconds is—

$$- 0''\cdot0002357 \times 658 - 0''\cdot00001817 \times 40975$$
$$= - 0''\cdot900$$

The apparent deflection is therefore 0″·90 to the south.

Burleigh Moor.

This Station is situated in the south of Sheet 7 of Yorkshire. The ground falls to the north, at first rapidly, and afterwards more gradually, to the sea, which is distant between two and three miles north. To the south the country gradually rises, until at a distance of nine miles it attains a mean height of about a thousand feet. The sea to the north, though not very deep, adds to the southern deflection: the depths shown with a negative sign in the following table are taken from the Admiralty Charts. At this Station the ground within the interior 500-feet circle, being uneven and falling to the north, was divided by four small circles whose radii were consecutively 100, 200, 300, 400 feet. The sum of the sines of the elevations north was found to be − 22·30, and the corresponding sum south − 12·45.

BURLEIGH MOOR. North Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	375	385	385	390	390	400	400	400	405	410	410	410	415	420	425	430	435	460	500	540	8385
2— 3	325	325	325	325	325	325	325	325	325	325	325	325	325	330	330	345	350	375	410	520	6885
3— 4	275	275	275	275	275	275	275	250	250	250	250	260	265	265	270	290	300	325	400	470	5770
4— 5	250	230	225	225	225	220	220	220	210	210	210	225	225	225	235	250	275	325	375	470	5050
5— 6	225	200	200	200	195	190	190	190	190	190	190	190	195	200	210	225	250	285	350	560	4625
6— 7	225	175	180	180	170	170	170	170	170	170	175	170	170	175	190	195	220	265	345	530	4215
7— 8	225	175	160	170	165	165	160	160	155	155	155	155	145	150	150	170	200	250	300	475	3840
8— 9	225	160	150	150	155	145	145	150	150	150	145	140	130	130	150	165	175	220	290	470	3595
9—10	235	150	125	140	140	130	130	130	130	130	130	125	120	125	140	150	160	180	250	450	3270
10—11	230	150	125	120	125	125	120	120	115	110	105	100	100	125	130	150	160	170	220	365	2965
11—12	235	125	110	100	105	100	95	100	90	85	85	80	85	100	100	125	150	160	190	300	2520
12—13	215	110	95	85	80	80	80	75	75	75	70	70	75	75	75	100	100	150	160	235	2080
13—14	165	80	75	70	70	70	70	60	55	50	55	55	50	60	65	65	100	125	150	210	1700
14—15	125	65	75	55	50	50	50	50	45	40	40	50	50	40	50	55	70	100	150	220	1430
15—16	95	60	50	45	35	35	35	35	35	35	35	30	20	10				20	140	210	925
16—17	85	50	35	35	30	35	30	25	25	20					— 5	— 5	— 5	— 5	40	165	575
17—18	60	35	25	25	20	25	25	20	15			— 20	— 20	— 20	— 30	— 30	— 25	— 25		140	390
18—19	40	25	15	15	15	25	20	10				— 25	— 20	— 20	— 25	— 30	— 35	— 30	— 25	130	295
19—20	35	20	15	15	10					— 20	— 25	— 35	— 55	— 55	— 60	— 60	— 55	— 40	— 35	250	345
20—21					— 5	— 20	— 30	— 35	— 35	— 50	— 60	— 65	— 70	— 80	— 85	— 85	— 65	— 60	— 60	140	140
21—22					— 10	— 20	— 35	— 55	— 60	— 60	— 85	— 90	— 85	— 95	— 90	— 90	— 85	— 85	— 60		
22—23					— 20	— 50	— 60	— 60	— 80	— 85	— 90	— 95	—110	—115	—115	—115	—100	—100	— 85	— 60	
23—24					— 30	— 40	— 65	— 70	— 90	—100	—115	—110	—110	—115	—120	—120	—120	—120	—120	— 70	
24—25					— 25	— 50	— 70	— 90	—110	—110	—120	—120	—120	—130	—130	—130	—130	—130	—120	—110	

BURLEIGH MOOR. South Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	375	380	390	400	410	440	450	475	500	525	540	550	550	550	550	500	545	545	545	545	9815
2— 3	325	330	340	350	360	375	385	400	420	450	460	480	490	500	500	500	510	510	520	520	8725
3— 4	390	300	300	310	320	325	340	350	370	390	400	410	430	490	510	515	500	500	500	510	8160
4— 5	270	280	275	290	300	300	320	325	340	350	370	400	470	500	450	500	480	460	500	535	7715
5— 6	275	280	275	275	275	290	290	300	310	350	370	410	480	500	500	470	440	410	470	535	7505
6— 7	270	280	275	270	260	260	280	290	300	325	380	450	500	500	475	435	400	400	450	540	7340
7— 8	275	290	275	255	250	250	260	275	275	325	400	450	500	460	425	375	350	350	400	510	6950
8— 9	310	300	260	250	260	250	250	250	275	325	375	460	470	425	390	335	300	310	350	400	6545
9—10	300	300	285	275	275	275	275	250	250	300	350	360	360	390	350	325	275	275	295	380	6145
10—11	335	300	280	280	280	280	275	260	260	250	275	300	310	310	300	290	265	230	225	310	5615
11—12	345	310	325	320	320	300	300	290	260	260	275	300	290	290	270	260	240	240	150	250	5595
12—13	335	325	325	340	325	300	300	275	250	270	300	275	275	250	220	240	240	240	200	205	5490
13—14	320	340	350	360	325	310	290	260	250	300	350	325	285	275	265	275	280	290	275	210	5935
14—15	350	375	370	360	325	310	280	250	290	320	385	350	310	300	320	390	420	420	400	240	6765
15—16	395	450	420	475	330	275	275	315	310	335	330	325	330	310	400	600	570	580	480	290	7795
16—17	450	580	510	480	300	300	300	300	300	325	340	360	355	425	555	640	625	500	500	340	8485
17—18	435	530	550	350	310	305	320	340	340	420	460	450	500	590	445	410	410	440	370	275	8250
18—19	415	625	410	330	390	365	375	520	535	725	820	900	800	615	530	580	600	600	450	290	10875
19—20	245	530	310	345	550	600	590	750	940	900	950	870	800	750	750	800	710	660	525	365	12940
20—21	160	435	300	325	465	785	950	900	845	805	910	1010	980	935	900	780	600	570	490	355	13500
21—22	145	330	295	315	475	660	760	765	720	790	750	895	890	880	845	795	620	580	470	285	12265
22—23	105	265	275	300	330	375	680	800	620	720	675	695	765	610	740	845	725	600	535	400	11060
23—24	95	245	265	270	275	310	380	475	610	940	820	830	800	725	635	790	785	735	615	535	11135
24—25	85	240	265	240	250	305	375	460	925	950	1100	1020	650	590	565	525	635	790	615	420	11005

From this table we obtain—

$H_1 - H'_1 = -1430$ $H_4 - H'_4 = -2665$ $H_7 - H'_7 = -3110$
$H_2 - H'_2 = -1840$ $H_5 - H'_5 = -2880$ $H_8 - H'_8 = -2950$
$H_3 - H'_3 = -2390$ $H_6 - H'_6 = -3125$ $H_9 - H'_9 = -2875$

$K = 13365$ $K' = 136710$ $S = -22{\cdot}30$ $S' = -12{\cdot}45$

Also, the sums of the depths of the sea north—

$$D = 7200$$

The deflection, therefore, due to the distribution of matter within a circle of nine miles and a half round the Station is—

$$-0''{\cdot}0118 \times 9{\cdot}85 - 0''{\cdot}0002357 \times 2516 - 0''{\cdot}00001817 \times 123345 - 0''{\cdot}0000116 \times 7200$$
$$= -3''{\cdot}033$$

or $3''{\cdot}033$ to the south.

But it is evident that if the table were extended to a great number of circles, an addition would be made to this quantity, for the high ground to the south continues for eight or ten miles further, whilst on the north there is an increased depth of sea; we may therefore proceed in the following manner: In Plate XXIV., B is the Station on Burleigh Moor, *abmn* the circle within which we have considered the effect of the ground, draw *ag*, *ml* (through B) making each an angle of 25° to the west with the meridian, also *bh* making an angle of 50° to the east, and *nr* perpendicular to the meridian; take $ad = de = 5$ miles, and $ag = 38\cdot5$, and describe the circular arcs *ghrl*, *ef*, *cd*. Now the whole of the space to the west of the lines *ga*, *ml* may be disregarded, as probably producing no meridional component of deflection at B, we have then three spaces left, namely, *lmnr*, *rnbh*, and *abgh*, to consider. Of these the second, namely *nbrh*, may also be disregarded, as the disturbing effect of the strip of land will be counteracted by the greater expanse of water. Within the space *lmnr* the depth of sea varies from 20 to 40 fathoms; the line of 30 fathoms depth runs parallel to the line of the shore, and about 12 miles from it: by giving greater weight to the depths nearer the Station, the effective mean depth may be taken at 30 fathoms, or 180 feet. Within the space *abcd* the mean height of the country, as estimated from the contours, is 1000 feet; within the space *cdef* the mean height, obtained in the same way, is 760 feet; and within the space *efgh* the mean height is 240 feet, estimated also from the contoured maps.

The following table contains the calculation of the deflections due to each of the spaces :—

Space.		α'	$\alpha_{,}$	r'	$r_{,}$	Deflection.
abcd	1000	205°	130°	14·5	9·5	— 0″·592
cdef	760	205	130	19·5	14·5	— 0·315
efgh	240	205	130	48·0	19·5	— 0·303
lmnr	—180	— 25	90	48·0	9·5	— 0·311

The sum of these small quantities amounts to $-1''\cdot521$: this added to the deflection due to the irregular distribution of matter within the circle of $9\cdot5$ miles radius, which was found to be $-3''\cdot033$, gives $-4''\cdot554$ as the total deflection to the south at Burleigh Moor.

Calton Observatory.

This Station is situated in the north-east of the city of Edinburgh: Sheet 2 of Edinburghshire. The ground in the immediate neighbourhood of the Station falls abruptly on all sides: to the north it continues to fall gradually to the sea, which is at the distance of about two miles north; to the south there is a gradual rise, continuing to the borders of the county, where the mean height is about a thousand feet.

The inner space of 1000 feet radius was divided by nine circles, whose radii were 100, 200, · · · 900 feet: the sines of the depressions were then calculated, and were found to be as follows:—

Sum of Sines between	North.	South.
Centre and r_{100}	0·000	0·000
r_{100} — r_{200}	5·380	5·038
r_{200} — r_{300}	5·813	4·250
r_{300} — r_{400}	5·881	4·250
r_{400} — r_{500}	6·295	4·386
r_{500} — r_{600}	5·807	4·366
r_{600} — r_{700}	5·543	3·660
r_{700} — r_{800}	4·868	3·846
r_{800} — r_{900}	4·334	3·965
r_{900} — r_{1000}	3·859	3·757
Total . .	47·780	37·518

$$S = -47{\cdot}780 \quad : \quad S' = -37{\cdot}518$$

The following table contains the mean heights of the compartments, from the 1000 feet circle to nine miles and a half round the Station.

CALTON HILL, EDINBURGH. North Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2																					
2— 3	180	165	145	145	140	140	140	140	135	130	125	120	120	120	115	115	130	150	180	195	2830
3— 4	180	140	120	100	115	110	110	105	110	110	115	115	110	110	110	100	100	115	130	180	2385
4— 5	165	120	110	100	85	80	75	75	75	80	95	95	95	95	95	100	100	100	115	150	2005
5— 6	145	110	85	85	75	75	75	75	75	75	80	80	80	75	75	80	90	95	100	100	1730
6— 7	135	80	75	80	80	80	80	80	75	75	70	75	70	65	70	75	85	90	100	100	1640
7— 8	110	70	55	70	65	75	70	60	65	55	50	50	55	60	60	70	80	90	100	100	1410
8— 9	110	65	35	45	35	30	50	50	50	50	45	45	45	50	50	60	75	95	100	100	1185
9—10	90	70	35	40	35	25	25	35	45	45	45	40	35	45	50	50	70	85	100	95	1060
10—11	85	70	50	45	40	35	25	30	35	40	40	35	30	35	40	45	65	80	90	95	1010
11—12	70	80	90	80	70	65	45	40	40	35	25	25	25	25	25	30	40	70	80	85	1045
12—13	70	80	95	90	80	70	60	60	50	45	30	25	20	20	20	20	25	40	70	80	1050
13—14	85	95	100	95	85	75	70	55	45	30	15	10	5	5	10	10	10	20	55	60	935
14—15	100	100	100	90	75	40	15	10											15	50	595
15—16	150	110	85	45	10															30	430
16—17	170	105	65	5					− 20	− 10	− 10	− 10	− 10	− 10	− 10	− 10	− 10	− 10		5	350
17—18	180	80	20			− 20	− 20	− 25	− 20	− 20	− 15	− 30	− 25	− 25	− 20	− 20	− 20	− 20	− 20		280
18—19	180	15		− 20	− 20	− 20	− 30	− 25	− 25	− 25	− 30	− 30	− 30	− 25	− 25	− 25	− 30	− 25	− 20	− 20	195
19—20	160		− 20	− 25	− 25	− 20	− 20	− 25	− 20	− 20	− 20	− 30	− 25	− 30	− 70	− 40	− 30	− 30	− 25	− 20	160
20—21	90	5	− 25	− 20	− 20	− 20	− 30	− 25	− 30	− 30	− 30	− 25	− 25	− 20	− 20	− 60	− 30	− 30	− 25	− 25	95
21—22			− 20	− 30	− 50	− 40	− 30	− 55	− 90	− 90	− 90	− 40	− 25		− 25	− 35	− 40	− 35	− 30	− 30	
22—23			− 90	− 60	− 90	− 60	− 60	− 90	− 70	− 90	−120	− 90	− 60	− 70	− 40	− 40	− 40	− 35	− 35	− 30	
23—24		− 80	− 70	20	20	30	30	80	50	70	40	− 90	− 95	− 70	− 70	− 60	− 50	− 40	− 40	− 30	340
24—25		30	90	200	180	390	450	530	450	300	130	− 60	− 70	− 70	− 65	− 65	− 65	− 55	− 50	− 30	2750

CALTON HILL, EDINBURGH. South Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2																					
2— 3	235	220	175	165	155	150	150	210	220	220	210	215	220	215	215	215	220	210	205	210	4035
3— 4	230	220	190	180	165	175	180	195	210	200	200	185	180	170	160	155	155	150	155	160	3615
4— 5	225	200	200	200	200	210	205	220	235	200	180	185	160	155	150	140	140	135	135	110	3585
5— 6	210	225	270	290	250	245	210	240	260	240	240	240	190	145	135	125	120	120	115	105	3975
6— 7	220	225	300	250	260	275	270	275	260	260	250	250	200	150	220	350	365	325	210	115	5030
7— 8	200	280	330	300	270	280	275	270	265	260	250	245	235	170	410	500	460	380	280	125	5785
8— 9	190	400	260	280	275	270	260	260	255	250	250	245	240	180	280	490	420	285	220	135	5445
9—10	190	200	230	270	265	250	240	250	240	240	245	245	250	235	220	385	350	210	300	200	5015
10—11	195	225	235	250	250	245	245	245	245	245	245	255	260	260	250	300	330	385	450	320	5435
11—12	175	215	235	245	250	260	260	260	260	255	250	255	255	260	250	275	550	695	500	315	6020
12—13	175	210	240	260	300	300	300	290	290	260	250	235	230	230	230	200	230	445	405	215	5295
13—14	170	200	245	290	300	300	300	285	265	235	215	200	200	200	200	175	160	160	210	160	4470
14—15	175	180	245	290	300	290	275	260	240	225	225	220	195	175	160	150	150	150	150	95	4150
15—16	170	160	225	260	285	260	270	300	390	360	290	245	210	170	150	160	165	150	135	70	4425
16—17	195	160	215	255	310	320	350	380	380	350	300	255	235	205	195	235	235	175	125	70	4945
17—18	250	175	225	370	385	375	450	570	565	500	425	375	350	310	250	225	200	175	150	100	6425
18—19	280	190	235	370	400	400	480	550	575	530	440	425	375	340	305	300	300	260	170	100	7025
19—20	210	200	300	365	425	525	525	550	535	475	425	425	375	400	415	350	275	225	175	75	7250
20—21	175	235	360	450	600	815	900	875	700	475	470	450	450	425	350	280	225	190	170	75	8670
21—22	160	300	450	700	1010	1200	1290	1090	760	575	530	475	420	345	300	260	200	175	100	75	10415
22—23	160	325	525	810	1100	1050	1075	825	625	575	530	465	355	425	330	250	190	180	200	140	10135
23—24	200	400	625	825	1050	1015	1080	750	630	525	535	525	480	415	335	285	360	450	420	370	11275
24—25	250	520	715	925	1250	1350	1040	780	600	735	750	700	620	510	405	450	710	700	510	370	13890

From this table we have—

$$\begin{array}{lll} \dots & H_4 - H'_4 = -1580 & H_7 - H'_7 = -4375 \\ H_2 - H'_2 = -1205 & H_5 - H'_5 = -2245 & H_8 - H'_8 = -4260 \\ H_3 - H'_3 = -1230 & H_6 - H'_6 = -3390 & H_9 - H'_9 = -3955 \end{array}$$

$$S - S' = -10{\cdot}262 \qquad K - K' = -100590$$

$$D = 4970$$

$$-0''{\cdot}0118 \times 10{\cdot}26 - {\cdot}0002357 \times 1808 - {\cdot}00001817 \times 100590 - {\cdot}00001156 \times 4970$$

$$= -2''{\cdot}433$$

which is the deflection due to the distribution of matter within a circle of nine miles and a half round the Calton Hill. But it is not allowable to terminate the calculation here, as will be seen by continuing it as follows: In Plate XXIV., E is the observatory on the Calton Hill, *acegmn* the circle just mentioned; the lines *ab, cd, ef, gh, mk, nl* are each ten miles long, and intersecting in the Station; *ab* and *nl* are inclined (N.W. & S.E.) at 40° to the meridian, *cd, mk* at 14° (N.E. & S.W.), *ef, gh* at 65° (N.E. & S.W.) The effect of the matter within the spaces *abgh* and *efnl* in producing disturbance may be neglected. The mean height of the space *abcd* is 420 feet, obtained from the contoured maps. Of the space *cdef* one-sixteenth part is covered with land, and this is at the further boundary of the space; the remaining fifteen parts are covered with sea to the mean depth of 16 fathoms, or 96 feet: if we suppose the sea to be pressed down or replaced by an equal mass of rock throughout, there would remain a hollow of about 60 feet. This quantity we may add to the mean height of the similar and opposite space *ghmk*, which from the contours is found to be 1060 feet. The mean height of the space *mknl*, which is similar and opposite to *abcd*, is 1100 feet. The following table contains the calculation of these disturbances:—

Space.	h	α'	$\alpha_{,}$	r'	$r_{,}$	Deflection.
ghmk	1060 + 60	245°	194°	19·5	9·5	− 0″·630
mknl	1100—420	194	140	19·5	9·5	− 0·510

making altogether − 1·140. This being added to the quantity − 2·433 previously determined, gives − 3·573 for the total deflection south at the Calton Hill Observatory.

Lough Foyle.

To the south of this Station, which is situated in Sheet 9 of Londonderry, the ground rises continuously, attaining a mean height of about a thousand feet at the southern boundary of the county; to the north the country is flat. In the immediate vicinity of the Station the ground is even, and does not require the calculation of sines.

LOUGH FOYLE.

North Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	100	90	90	80	75	75	75	75	70	70	70	70	70	70	70	70	75	75	85	100	1555
2— 3	100	75	50	60	50	50	50	50	50	50	50	60	60	60	70	70	70	70	75	100	1270
3— 4	90	75	50	50	50	50	50	50	50	50	50	50	60	60	60	60	70	70	75	90	1210
4— 5	70	75	50	50	50	50	50	40	40	50	50	50	50	50	60	60	65	65	65	75	1115
5— 6	50	75	50	50	25	30	30	25	25	30	30	50	40	50	50	50	50	50	75	90	925
6— 7	50	50	25	25	25	25	25	25	25	25	25	25	30	40	50	50	50	50	75	100	795
7— 8	50	40	20	20	20	20	20	20	20	20	25	25	25	30	40	50	50	50	75	100	720
8— 9	50	25	20	20	20	20	20	20	20	20	25	25	25	30	30	50	50	50	75	100	695
9—10	50	25	20	15	15	15	15	15	15	15	20	25	25	25	35	50	50	60	75	75	640
10—11	50	20	15	15	15	15	15	15	15	15	15	25	25	25	25	50	50	60	75	75	615
11—12	30	20	10	10	10	10	10	10	10	10	15	20	25	25	25	25	50	50	50	85	500
12—13	40	10	10	10	10	10	10	10	10	10	10	15	20	25	25	25	40	50	50	85	475
13—14	30						10	10	10	10	10	10	20	25	25	25	40	50	50	85	410
14—15	30							10	10	10	10	20	20	25	25	25	25	40	60	80	390
15—16												25	25	25	25	25	25	50	70	70	340
16—17												25	25	25	25	25	45	40	40	65	315
17—18												25	25	25	25	25	25	35	40	75	300
18—19											25	20	20	25	25	25	35	65	75	75	390
19—20											25	15	20	25	25	25	35	65	95	90	420
20—21												10	25	25	25	60	140	180	150	175	790
21—22												10	25	25	150	620	960	750	370	400	3310
22—23													25	25	240	1000	1050	1010	700	670	5120
23—24													25	25	60	400	940	1000	730	840	4020
24—25													25	25	25	330	850	760	720	700	3435

LOUGH FOYLE.

South Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	100	100	100	100	100	100	100	100	100	100	100	100	100	110	110	120	120	120	120	120	2120
2— 3	100	100	100	100	100	100	100	100	100	100	100	100	100	110	110	110	110	110	100	110	2060
3— 4	100	100	100	100	100	100	120	120	120	100	100	100	100	110	100	100	110	125	100	100	2105
4— 5	100	125	100	100	130	110	140	140	150	120	125	120	120	125	125	120	125	120	100	100	2395
5— 6	75	150	100	120	150	150	150	150	150	130	125	125	125	125	125	120	120	110	100	100	2500
6— 7	100	125	100	130	160	170	160	150	150	150	125	125	125	125	125	120	110	100	100	100	2550
7— 8	120	125	125	150	160	160	150	150	150	150	150	150	125	125	125	120	110	100	100	90	2635
8— 9	120	150	150	150	150	180	190	200	200	200	200	175	150	150	150	120	110	100	100	90	3035
9—10	150	170	150	200	190	180	200	205	225	225	225	200	180	180	160	140	125	100	100	95	3400
10—11	150	160	200	225	225	210	200	225	250	250	250	225	215	210	175	150	145	120	110	100	3795
11—12	180	200	220	240	260	250	250	255	275	280	275	265	250	240	200	175	150	140	125	100	4330
12—13	200	225	280	275	300	300	250	310	320	310	300	300	275	250	210	190	190	175	150	100	4910
13—14	220	300	310	300	330	325	320	330	345	330	325	330	300	270	240	225	225	190	150	125	5490
14—15	250	310	350	350	360	360	350	370	375	375	350	325	300	300	250	250	225	200	185	130	5965
15—16	250	380	420	410	410	415	390	415	430	425	400	400	380	340	295	320	260	250	190	100	6880
16—17	200	470	500	490	480	470	470	490	555	550	550	500	470	425	375	340	315	275	200	90	8215
17—18	180	500	600	580	565	550	600	600	670	700	690	610	550	470	410	400	370	310	220	140	9715
18—19	175	575	680	680	670	750	800	800	800	800	750	680	600	525	460	400	325	300	175	175	11010
19—20	200	560	800	800	900	900	800	750	800	750	690	610	555	500	455	400	300	225	250	180	11425
20—21	310	530	830	1080	1080	800	760	750	800	690	590	470	435	410	400	350	220	340	325	190	9560
21—22	320	550	820	950	1020	850	680	760	850	770	600	460	360	300	245	200	250	325	275	280	10865
22—23	210	520	600	670	710	650	670	725	800	710	580	390	290	230	180	210	240	300	350	560	9595
23—24	200	470	525	510	600	740	840	610	560	550	440	320	275	215	210	295	400	370	550	1050	9730
24—25	270	460	470	575	740	480	460	480	625	640	440	300	250	260	320	560	1130	640	720	790	10610

From this table we obtain—

$$H_1 - H'_1 = -565 \qquad H_4 - H'_4 = -1280 \qquad H_7 - H'_7 = -1915$$
$$H_2 - H'_2 = -790 \qquad H_5 - H'_5 = -1575 \qquad H_8 - H'_8 = -2340$$
$$H_3 - H'_3 = -895 \qquad H_6 - H'_6 = -1755 \qquad H_9 - H'_9 = -2760$$
$$K - K' = 101265$$

Giving a deflection to the amount of—

$$-0''{\cdot}0002357 \times 1305 - {\cdot}00001817 \times 101265$$
$$= -2''{\cdot}148$$

The depth of Lough Foyle is so small that it may safely be neglected; but it is necessary to extend the calculation to a greater distance, which may be done as follows: In Plate XXIV., L is the Station, *adhqnmr* the circle of nine miles and a half radius; *abcuqt* makes an angle of 65° (N.W. & S.E.) with the meridian, *rs* makes an angle of 72° with the south meridian, and *hklv* an angle of 50° with the north meridian; L*s*, L*u*, L*v*, L*t* are each 50 miles. Make $ab = bc = hk = kl = 6$ miles, and describe the circular arcs *cgl*, *bfek*; *de* is drawn in the meridian, and *fg* makes an angle of 15° (N.W.) with the meridian. Draw also *mo* and *np*, each 12 miles long, and making angles 25° (S.W.) and 45° (S.E.) with the south meridian. The effect of the matter within the lines *ua*, *rs*, and *vh*, *qt* in creating disturbance at L may be taken as insensible. The mean height of the space *abde* is 600 feet, of the space *bcfg* 410 feet: the space *edhk* is mostly covered with shallow water, the effect of which may be disregarded; the space *gfkl* is covered with sea to the mean depth of 27 fathoms; the space *ucglv* is covered with sea, the mean depth of which may be taken as 40 fathoms. The mean height of the space *rmopnqts* is 400 feet, and the mean height of the space *mopn* is 900 feet. The calculation of the disturbances due to each of these spaces is shown in the following table:—

Space.	h	α'	$\alpha_{,}$	r'	$r_{,}$	Deflection.
srqt	400	252°	115°	50·0	9·5	−1·454
mnpo	500	205	135	21·5	9·5	− ·544
abde	600	0	295	15·5	9·5	+ ·314
bcfg	410	345	295	21·5	15·5	+ ·102
fgkl	− 160	50	345	21·5	15·5	− ·040
culv	− 240	50	295	50·0	21·5	− ·254

The sum of these partial disturbances is $-1''{\cdot}876$, which being added to the quantity $-2{\cdot}148$ previously determined, gives the total deflection south equal to $-4''{\cdot}024$.

Kellie Law.

This Station is situated in Sheet 19 of Fifeshire. The mean heights of the compartments are given in the following table:—

KELLIE LAW. North Side.

Number of Circle.	Number of Sector. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	II
1— 2	470	480	470	460	450	450	460	460	460	470	480	490	500	500	500	500	500	500	500	500	9600
2— 3	450	450	470	480	480	480	450	440	440	430	420	400	410	400	400	400	420	420	450	450	8740
3— 4	430	480	500	500	500	500	500	500	500	480	480	460	450	430	410	400	380	410	450	400	9160
4— 5	430	480	510	510	510	510	510	510	510	510	510	500	470	460	450	420	400	370	390	350	9310
5— 6	440	480	500	510	510	520	520	520	510	510	510	500	490	480	460	440	420	400	360	350	9430
6— 7	450	480	500	510	520	530	520	520	510	510	500	480	450	440	440	430	430	410	380	330	9340
7— 8	460	480	500	510	530	540	540	530	520	510	490	470	440	420	400	400	400	400	380	320	9240
8— 9	480	480	500	530	540	550	550	550	520	510	500	470	440	420	400	400	400	400	370	320	9330
9—10	500	500	520	550	550	560	560	570	570	550	500	480	460	450	440	430	420	400	360	320	9690
10—11	500	520	540	550	560	580	580	570	550	520	510	480	450	450	450	440	420	400	380	300	9750
11—12	520	550	560	550	550	570	550	540	500	470	450	450	440	450	450	440	420	400	370	325	9555
12—13	550	560	570	560	570	580	550	500	470	450	440	430	420	420	410	420	420	400	360	300	9380
13—14	580	590	580	580	570	580	550	530	520	480	470	450	440	430	400	400	400	390	360	300	9600
14—15	640	600	600	600	560	550	480	480	510	520	510	470	430	400	390	380	370	390	370	275	9525
15—16	600	600	570	550	530	460	460	450	450	420	380	370	370	380	350	360	360	360	360	250	8630
16—17	550	540	530	500	500	490	450	450	420	400	350	320	320	350	350	350	370	340	330	240	8150
17—18	540	580	560	520	500	480	450	420	380	360	320	320	300	300	310	320	320	320	320	220	7840
18—19	600	620	550	520	470	420	370	350	320	300	280	260	280	250	290	300	280	300	300	220	7280
19—20	650	580	500	500	450	420	400	370	350	320	300	330	300	270	250	250	270	260	240	210	7220
20—21	600	550	480	500	470	400	350	300	300	280	300	320	270	240	220	180	200	200	210	170	6540
21—22	550	460	460	550	420	320	200	160	120	100	130	160	150	150	100	110	100	120	130	130	4620
22—23	550	370	400	380	250	200	120	50	20	20	—25	—30	—35	—40		20	20	30	20	70	2520
23—24	450	260	300	350	250	120	50	10		—25	—30	—40	—50	—55	—50	—50	—50	—50	—30		1790
24—25	450	200	90	70	70	20				—30	—40	—50	—50	—55	—55	—55	—55	—60	—70	—60	900

KELLIE LAW. South Side.

Number of Circle.	Number of Sector. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	H
1— 2	470	480	500	500	490	480	460	450	430	430	430	430	430	430	430	430	430	430	440	450	9020
2— 3	450	440	450	450	450	440	440	420	410	400	400	400	400	380	360	360	360	360	350	370	8090
3— 4	430	390	360	360	350	350	350	330	330	320	320	320	320	320	320	320	320	310	300	340	6760
4— 5	410	350	340	310	300	290	290	270	270	260	260	260	260	270	280	290	290	290	280	300	5870
5— 6	380	340	320	300	270	270	250	250	240	240	240	240	240	240	250	250	260	260	250	280	5370
6— 7	390	300	310	290	260	250	250	250	240	240	240	240	240	240	240	240	250	250	240	250	5210
7— 8	400	320	280	260	260	250	240	240	240	240	240	240	230	230	230	240	250	250	230	240	5110
8— 9	400	300	280	250	250	250	230	230	230	230	220	200	210	220	220	230	240	230	220	220	4860
9—10	380	290	270	250	240	230	230	230	230	220	210	200	200	200	210	210	210	210	210	230	4660
10—11	360	280	250	220	230	210	210	200	200	200	200	190	190	180	190	200	200	200	200	220	4330
11—12	380	280	250	210	200	200	190	190	170	150	160	160	160	150	150	170	200	200	180	200	3950
12—13	400	250	220	180	150	160	160	140	130	120	120	110	120	120	120	120	140	160	160	180	3260
13—14	400	260	200	190	170	150	130	100	90	90	90	90	100	100	100	100	120	140	150	180	2950
14—15	400	250	200	180	160	130	110	100	100	100	100	90	90	90	90	90	100	110	130	170	2790
15—16	420	250	180	120	90	70	70	70	90	100	100	90	80	80	80	80	90	90	90	150	2390
16—17	400	200	120	90	70	70	70	70	80	70	60	70	70	70	80	70	70	60	70	150	2010
17—18	400	200	100	90	90	100	90	90	60	60	50	50	40	40	30	40	60	70	60	120	1840
18—19	450	150	80	60	60	60	60	80	60	20								20	20	120	1240
19—20	500	130	80	40	40	60	40	30		—25	—35	—50	—60	—60	—55	—60	—30	—30	—30	70	990
20—21	450	80	20	40	80	30			—35	—50	—100	—100	—110	—95	—100	—85	—90	—85	—60	20	720
21—22	400	20				—30	—60	—70	—90	—110	—115	—120	—140	—145	—155	—150	—140	—120	—90	—50	420
22—23	220	—20	—30	—35	—60	—90	—95	—110	—130	—130	—150	—150	—155	—160	—170	—160	—170	—160	—120	—70	220
23—24	240	—30	—50	—90	—95	—120	—150	—150	—160	—155	—150	—160	—170	—170	—160	—150	—155	—155	—130	—95	240
24—25	300	—60	—90	—110	—130	—140	—150	—150	—170	—150	—180	—160	—170	—175	—150	—155	—150	—150	—120	—110	300

From this table we have—

$H_1 - H'_1 = 580$
$H_2 - H'_2 = 650$
$H_3 - H'_3 = 2400$
$H_4 - H'_4 = 3440$
$H_5 - H'_5 = 4060$
$H_6 - H'_6 = 4130$
$H_7 - H'_7 = 4130$
$H_8 - H'_8 = 4470$
$H_9 - H'_9 = 5030$

$K - K' = 75640$
$D - D' = -9150$

The deflection therefore is—

$$0''{\cdot}0002357 \times 2540 + {\cdot}00001817 \times 75640 + {\cdot}00001156 \times 9150$$
$$= 2''{\cdot}078 \text{ North.}$$

Monach.

This Station is in Sheet 10 of the Island of Lewis. The mean heights of the different compartments are shown in the following table :—

MONACH. North Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	785	765	750	750	735	735	725	725	725	710	700	685	675	675	660	650	650	625	625	615	13965
2— 3	735	700	690	690	690	685	680	675	675	650	625	615	590	575	575	560	550	550	525	525	12560
3— 4	675	600	625	625	625	625	615	600	575	560	550	535	525	525	510	510	510	500	485	475	11250
4— 5	675	525	515	515	515	510	510	500	500	500	500	485	485	475	475	460	460	460	450	450	9965
5— 6	650	500	475	485	485	485	500	500	500	485	485	475	475	450	430	425	425	420	425	425	9500
6— 7	600	485	475	475	475	475	500	490	500	515	515	510	510	475	435	425	400	415	425	425	9525
7— 8	550	485	450	450	460	480	500	510	525	525	535	540	535	510	460	425	400	410	425	410	9585
8— 9	500	475	435	450	500	525	550	570	560	530	525	535	550	510	460	425	400	400	400	400	9700
9—10	475	450	450	500	540	570	575	585	575	525	515	525	525	500	455	435	410	400	380	415	9805
10—11	450	425	485	560	550	525	510	525	525	485	450	450	450	435	435	450	440	400	380	415	9345
11—12	400	415	450	500	500	475	460	460	460	450	415	410	400	400	385	415	440	400	375	385	8595
12—13	375	425	450	460	475	475	450	435	425	415	415	415	400	375	375	375	385	375	365	350	8215
13—14	375	425	415	435	460	470	450	425	410	410	415	400	400	400	400	360	350	375	365	350	8090
14—15	355	385	420	465	450	450	450	435	435	435	430	425	415	400	400	400	375	360	365	335	8185
15—16	360	375	420	410	450	475	475	460	460	450	450	425	410	400	380	400	375	350	300	350	8175
16—17	370	365	375	385	460	475	475	475	475	450	435	425	400	375	375	375	365	350	290	305	7995
17—18	315	280	320	400	420	410	445	400	420	420	410	400	400	375	380	365	340	325	275	250	7350
18—19	275	250	300	350	300	285	315	335	400	400	380	400	400	415	420	400	385	335	280	100	6725
19—20	270	225	200	220	265	275	290	315	320	360	400	385	375	405	415	400	400	370	275		6165
20—21	225	225	160	200	250	245	265	250	260	300	320	395	475	410	385	385	360	365	80		5555
21—22	200	175	140	180	185	210	210	175	200	250	275	320	350	370	335	350	360	210			4495
22—23	140	100	70	100	140	100	120	135	175	225	200	185	240	275	280	290	300	190			3265
23—24	85	10						25	65	135	130	185	215	270	270	80	40				1510
24—25											40	125	160	165	70						560

MONACH. South Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	760	725	715	700	700	685	675	665	665	665	665	600	600	600	600	600	665	670	675	650	13580
2— 3	700	635	615	600	575	560	550	550	550	550	550	550	550	550	550	550	555	560	560	560	11420
3— 4	700	575	525	510	500	500	500	500	490	485	480	475	470	470	470	465	465	460	460	475	9975
4— 5	720	560	490	475	460	475	480	480	475	475	475	475	465	450	450	425	415	400	400	425	9470
5— 6	700	525	450	450	460	460	475	475	470	480	480	475	450	450	450	425	425	400	400	425	9325
6— 7	670	475	435	450	450	450	460	475	460	460	450	450	440	435	430	430	430	430	400	400	9080
7— 8	620	440	435	450	450	440	450	450	450	435	435	435	430	425	425	425	425	425	400	400	8845
8— 9	560	410	450	435	425	425	425	425	425	425	425	425	425	425	425	425	425	425	400	400	8605
9—10	515	420	450	425	415	415	415	425	425	415	415	425	425	425	425	425	415	400	400	400	8475
10—11	470	430	450	415	405	400	400	415	400	400	420	425	425	425	425	425	400	400	400	400	8330
11—12	400	445	390	385	385	385	385	400	400	400	410	415	415	400	400	400	400	400	390	380	7985
12—13	390	390	375	360	360	350	360	375	375	400	400	400	400	400	400	400	400	400	385	375	7695
13—14	360	355	325	310	350	360	375	360	360	375	400	400	400	400	375	385	390	390	365	360	7395
14—15	325	300	285	300	320	330	350	350	360	375	375	375	375	360	360	365	375	360	350	340	6930
15—16	350	325	315	300	285	300	340	335	335	350	350	335	335	350	350	360	365	340	310	325	6655
16—17	375	350	350	350	300	275	280	290	300	315	315	300	315	325	325	320	315	325	300	335	6360
17—18	390	375	375	360	335	300	250	270	230	280	300	300	265	275	300	290	300	325	300	325	6145
18—19	360	375	385	350	330	325	290	260	190	210	250	260	220	295	270	235	260	160	300	270	5595
19—20	325	375	350	335	350	300	300	275	250	135	135	125	105	150	200	75	55	70	120	140	4170
20—21	280	310	345	340	310	260	245	210	200	120	80	35		15	60	45				40	2895
21—22	205	260	315	290	275	255	220	170	135	90	110	50									2375
22—23	150	210	260	260	280	260	225	125	50												1820
23—24	165	300	300	320	315	270	150	90	75	25											2010
24—25	165	355	415	385	320	260	175	25						45	95	70					2310

On account of the abruptness of the ground in the vicinity of the Station, it is necessary to calculate the corrections (*a*), page 627, to the quantities H: they are found to be—

For the North side	30, 40, 50, 60, 50, 35, 25, 20, 15
For the South side	45, 90, 100, 75, 55, 45, 35, 30, 25

These being applied with the positive sign to the quantities H, we get—

$H_1 - H'_1 = 370$ $H_4 - H'_4 = 480$ $H_7 - H'_7 = 730$

$H_2 - H'_2 = 1090$ $H_5 - H'_5 = 170$ $H_8 - H'_8 = 1085$

$H_3 - H'_3 = 1225$ $H_6 - H'_6 = 435$ $H_9 - H'_9 = 1320$

$K - K' = 15555$

So that the deflection is—

$$0''{\cdot}00023574 \times 801 + 0''{\cdot}00001817 \times 15555$$
$$= 0''{\cdot}471 \text{ North.}$$

Hungry Hill.

For the more accurate determination of the disturbance at this Station, the ground enclosed by a circle of a mile radius around it has been contoured at vertical intervals of 100 feet. Beyond the distance of one mile, the mean heights have been obtained from the altitudes found on the 6-inch maps.

HUNGRY HILL. North Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	1950	2020	2060	2120	2120	2150	2160	2160	2160	2170	2180	2190	2200	2200	2200	2210	2210	2220	2220	2170	43070
2— 3	1780	1930	2000	2050	2100	2100	2120	2120	2130	2150	2150	2160	2150	2150	2120	2070	2050	2050	2050	1950	41380
3— 4	1700	1840	1910	1950	1970	1970	1980	2000	2050	2120	2130	2140	2130	2120	2100	1950	1800	1700	1730	1730	39020
4— 5	1580	1690	1790	1850	1850	1850	1840	1850	1890	1950	1990	2060	2050	2000	1900	1750	1610	1500	1450	1500	35950
5— 6	1360	1600	1730	1750	1750	1700	1610	1610	1650	1700	1750	1800	1820	1820	1700	1500	1250	1200	1210	1350	31860
6— 7	1200	1510	1690	1650	1550	1450	1410	1400	1410	1490	1550	1600	1620	1620	1510	1350	1130	1120	1130	1230	28620
7— 8	1200	1500	1490	1420	1350	1280	1200	1150	1200	1250	1310	1400	1520	1520	1450	1400	1320	1150	1150	1150	26410
8— 9	1300	1390	1290	1200	1100	1110	950	900	910	1020	1130	1260	1450	1500	1520	1550	1450	1300	1200	1000	24530
9—10	1180	1220	1100	970	860	760	690	690	740	820	980	1200	1400	1550	1640	1620	1540	1400	1090	800	22250
10—11	1170	1100	900	770	700	530	400	400	500	660	900	1150	1400	1620	1730	1730	1610	1400	950	300	19920
11—12	1300	1170	960	800	650	500	380	380	450	650	950	1200	1600	1650	1750	1750	1630	1350	1100	650	20870
12—13	1350	1350	1130	900	630	700	700	500	400	550	900	1150	1550	1620	1700	1700	1600	1350	1200	700	21680
13—14	1350	1150	900	700	700	700	450	200	150	150	510	860	1050	1100	1150	1500	1500	1200	1100	600	17020
14—15	1400	1100	930	960	960	1000	1000	1000	800	550	350	300	500	800	1200	1550	1400	1300	900	200	18200
15—16	1600	990	1060	1200	1400	1500	1500	1600	1500	1000	600	350	500	600	950	1330	1150	800	800	480	20910
16—17	1370	1050	1530	1850	1900	1000	1000	1600	1900	1000	400	300	70	100	630	1110	900	485	800	650	19645
17—18	1750	950	1600	1920	1250	1000	420	500	500	400	350	300	50	100	620	1190	960	500	700	350	15410
18—19	1500	500	1420	1240	1700	1715	1500	1050	300	250	330	360	25	160	780	1150	1200	800	300	170	16450
19—20	980	500	1180	900	900	1240	1400	1500	880	600	110	95	70	360	775	1200	1600	1090	470	330	16180
20—21	850	490	330	130	160	400	750	650	450	300	100	100	150	930	940	1310	1840	1450	1300	490	13120
21—22	620	120	80	45		20	240	100	60	30	20	90	140	200	400	920	760	1690	1080	930	7545
22—23	280	220	115	75	—70	—60	30		50	100	300	520	580	720	340	1100	1500	1500	1500	1100	10030
23—24	140		—130	—120	—120	—130	—120	—100	—70	80	200	180	300	770	975	1035	1600	1700	1000	800	8780
24—25	40	—150	—140	—120				30	—80		—70	80	330	700	880	1400	1270	1040	670	450	6890

HUNGRY HILL. South Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1— 2	1960	2020	2060	2100	2140	2160	2180	2180	2180	2180	2150	2120	2110	2100	2070	2040	2040	2040	2040	2100	41970
2— 3	1740	1860	1960	2080	2120	2140	2180	2180	2150	2150	2140	2120	2100	1970	1900	1810	1700	1610	1610	1750	39270
3— 4	1550	1790	1920	2050	2120	2160	2160	2180	2180	2150	2160	2100	2100	2050	1980	1910	1730	1550	1300	1400	38540
4— 5	1400	1600	1800	1960	1960	1900	1860	1850	1850	1850	1860	1880	1940	1950	1930	1780	1580	1350	1070	1180	34550
5— 6	1200	1350	1600	1700	1650	1600	1600	1600	1600	1600	1600	1610	1680	1760	1760	1660	1410	1150	1000	1200	30330
6— 7	950	1110	1430	1350	1300	1280	1290	1300	1280	1250	1300	1410	1440	1500	1500	1450	1220	1050	1050	1200	25660
7— 8	800	980	1200	1060	1000	990	1000	1040	1030	980	1000	1080	1190	1280	1330	1330	1200	1010	900	1120	21520
8— 9	650	900	980	870	780	800	820	840	830	830	790	790	900	1100	1160	1180	1150	950	710	900	17930
9—10	520	750	800	680	640	650	670	660	660	650	600	620	720	800	900	1050	1120	920	620	650	14680
10—11	520	570	560	440	420	460	460	480	480	470	460	450	470	580	730	850	990	920	650	320	11280
11—12	750	360	400	350	310	310	330	330	300	300	300	300	310	380	500	620	750	820	720	320	8760
12—13	1050	300	300	260	200	200	210	210	200	200	200	200	200	200	250	310	530	650	720	320	6710
13—14	1050	400	300	180	150	150	150	150	130	120	100	100	120	150	180	180	200	300	550	350	5010
14—15	950	680	320	200	130	120	120	100							100	150	150	150	250	350	3770
15—16	720	700	250	130	75	50	80	100									20	50	110	145	2430
16—17	780	370	150	35								—60	—70	—80	—90	—60				70	1445
17—18	1220	270	110					10	60	70	60		—60	—60	—100	—70		—70		20	1820
18—19	1000	200	25		20	20		45	80	30	—120	—120	—140	—130	—140	—140	—120	—80	—90	30	1450
19—20	540	120		70	400	100	30		—90	—100	—140	—150	—150	—150	—150	—140	—140	—140	—100	40	1300
20—21	480	60	90	400	300	70	80	—80	—150	—160	—160	—160	—160	—150	—150	—140	—130	—140	—120	30	1510
21—22	300		250	650	75	—80	—90	—150	—180	—170	—170	—160	—160	—150	—150	—140	—140	—150	—150	20	1295
22—23	350	30	300	100	—100	—150	—180	—180	—180	—170	—120	—100		40	60	100	50		—140	40	1070
23—24	580	200		—160	—180	—190	—190	—180	—180	—90	150	500	850	750	350	700	650	450	220		5400
24—25	700	250	—100	—180	—180	—180	—190	—190		260	300	200	100	100	40	80	240	200	415		2885

On account of the steepness of the ground on all sides of the Station, it is necessary to calculate the corrections (a), page 627; they are found to be—

For the North side	10,	35,	80,	175,	360,	510,	560,	560,	635
For the South side	55,	140,	105,	270,	510,	840,	1130,	1345,	1510

After correcting the quantities H H′ in the table by these quantities applied with the positive sign, we get—

$H_1 - H'_1 = 1055$ $H_4 - H'_4 = 1305$ $H_7 - H'_7 = 4320$
$H_2 - H'_2 = 2005$ $H_5 - H'_5 = 1380$ $H_8 - H'_8 = 5815$
$H_3 - H'_3 = 455$ $H_6 - H'_6 = 2630$ $H_9 - H'_9 = 6695$
$K = 232650$ $K' = 56135$ $D = 1480$ $D' = 10500$

The deflection resulting from these quantities is—

$$0{\cdot}00023574 \times 2272 + 0{\cdot}00001817 \times 176515 + 0{\cdot}00001156 \times 9020$$

which amounts to $3''{\cdot}847$.

In Plate XXIV., H is the Station on Hungry Hill, *adenmr* is the circle of 9·5 miles radius within which the attraction has just been determined. With a radius of 28 miles describe the circular arc *bcpq*, and with a radius of 50 miles the circular arc *usy*. Draw the following lines: (1), *abg* making an angle of 10° with the meridian, and *mp* in the same straight line with *abg*; (2), *vu* and *xy* making angles of 35° with the north and south meridians; (3), *nqs* making an angle of 60° with the meridian, and *cd* in the same straight line with *nqs*; (4), *hr* making an angle of 60° with the south meridian; (5), *ef* making an angle of 65° with the north meridian. The mean height of the space *abcd* is found to be 600 feet, but as the greater height is in the centre of the space, and the ground is low within a mile and a half of the inner limit *ad*, we shall adopt 450 feet as the effective mean height. The space *bguv*, consisting of land and water, may be considered to produce no sensible effect. The mean height of the space *abgef* is to be taken at 400 feet. The spaces *fenqs*, *nqmp*, and *cdrh* may be neglected as producing no sensible effect. The depth of water in the space *pmrh* may be taken at 40 fathoms, and the depth of water in the space *sqxy* at 65 fathoms. The space to the west of the lines *uvchxy* being covered with an even depth of sea, requires no further consideration.

The calculation of the disturbance up to 50 miles will therefore be as shown in the following table:—

Space.	h	a'	$a_,$	r'	$r_,$	Deflection.
abcd	450	10°	300°	28	9·5	0″·596
hrmp	240	240	190	28	9·5	0·135
aefy	400	65	10	50	9·5	0·574
qsxy	390	215	120	50	28·0	0·244

making altogether $1''{\cdot}549$, which has to be added to the quantity 3·847 previously determined. We have thus, by extending the calculation to a distance of 50 miles, obtained a deflection to the amount of $5''{\cdot}396$ to the north at Hungry Hill.

This determination cannot be considered so satisfactory as those for the Stations previously considered, as the want of a sufficient number of altitudes on the maps throws some uncertainty upon a few of the estimated mean heights. But the error of the determination can scarcely amount to a quarter of a second.

Forth Mountain.

This Station is situated in Sheet 42 of the County of Wexford. In the vicinity of the Station there are very few altitudes marked on the map, and the mean heights cannot be well determined within the 5000 feet circle; there are, however, sufficient altitudes to show that there is little or no disturbance within this circle. The mean heights of the compartments from the 10th to the 24th circle are as follows:—

FORTH. North Side.

Number of Circle.	Number of Sector.																				II
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
10—11	440	295	280	260	250	220	200	220	220	250	290	310	350	400	530	530	510	480	460	515	7010
11—12	370	250	250	240	230	215	205	205	210	220	230	250	330	360	420	500	490	480	450	510	6415
12—13	320	230	210	220	210	210	210	195	200	200	210	210	250	275	350	400	420	440	420	440	5620
13—14	300	250	200	200	190	180	190	210	200	200	210	200	190	190	240	320	390	450	350	320	4980
14—15	330	260	210	180	190	200	210	225	225	180	185	200	160	175	200	225	250	300	275	280	4460
15—16	390	265	175	205	215	200	200	180	140	120	160	160	130	120	150	175	200	210	225	270	3890
16—17	280	185	165	185	180	165	115	95	90	125	120	110	105	85	95	105	120	140	210	260	2935
17—18	200	195	165	165	175	140	145	115	120	120	120	30	30	60	50	40	30	65	235	235	2435
18—19	240	220	180	230	190	200	200	160	140	115	60	30	70	180	180	150	90	60	180	200	3075
19—20	265	225	230	260	210	200	230	165	145	105	30	155	195	195	175	160	50	15	35	150	3195
20—21	170	255	290	305	320	250	210	170	215	75	75	165	215	190	135	135	45		30	30	3280
21—22	135	215	300	320	275	215	160	185	165	90	55	90	170	180	160	125	75	15	35		2965
22—23	180	225	280	290	195	240	190	130	150	160	45	140	190	185	180	155	80	35	15		3065
23—24	170	340	375	265	275	230	180	175	90	80	100	190	195	210	180	195	145	55	10		3465
24—25																					

FORTH. South Side.

Number of Circle.	Number of Sector.																				II
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
10—11	530	620	640	640	640	630	600	540	500	480	470	470	460	450	430	410	390	370	380	450	10100
11—12	440	580	620	590	530	510	500	500	470	420	410	420	420	410	400	400	380	350	360	410	9120
12—13	430	500	540	545	450	430	430	420	410	380	380	400	390	405	400	390	350	325	340	350	8265
13—14	390	415	470	480	430	400	360	360	365	360	340	340	360	370	390	390	350	320	320	325	7535
14—15	315	345	370	430	390	370	340	300	300	290	280	290	300	320	350	320	300	300	310	300	6520
15—16	300	270	280	340	340	300	270	230	230	240	240	240	240	250	270	240	260	270	280	290	5380
16—17	300	265	260	255	255	235	220	215	210	205	200	195	210	230	220	220	215	210	260	250	4630
17—18	250	240	230	240	235	230	190	185	195	170	170	160	180	165	175	185	180	175	160	200	3915
18—19	200	220	205	215	195	195	150	150	160	135	120	120	120	130	135	130	120	120	75	155	3050
19—20	175	150	175	190	185	145	90	95	95	80	95	100	90	95	100	95	70	90	65	70	2250
20—21	170	140	125	135	150	105	70	55	50	60	70	75	80	75	80	70	65	60	65	40	1740
21—22	150	130	120	125	115	85	50	35	45	50	35	50	45	55	60	75	55	45	35		1360
22—23	135	120	95	90	95	50	35	15	15	20	35	50	60	70	70	65	55	40	5		1120
23—24	110	130	100	85	35	15	20	15	10	5	55	55	55	55	55	70	80	65	35	10	1060
24—25																					

From this table we obtain—

$$K = 56790 \qquad K' = 66045$$

giving a deflection of $- 0''\cdot00001817 \times 9255 = - 0''\cdot168$.

In Plate XXIV. *abcd* is the 9·5 mile circle. Draw *aek*, *dhl*, making angles of 60° with the meridian; *bf*, *cg*, making angles of 40° with the meridian; and describe the circular arcs *epf*, *hmg* with a radius of 24 miles. Draw also *pq* and *mn*, making angles of 28° with the meridian, and describe the circular arcs *kq*, *ln* with a radius of 50 miles.

We may neglect the consideration of all matters to the east of the lines *nmgc*, *bfpq*, and of all to the west of *ak*, *dl*. The effective mean height of the space *eabf* is found to be 260 feet. The depth of sea in the corresponding space *dchg* varies from 10 to 35 fathoms; the effective mean depth may be taken at 20 fathoms. The effective mean height of the space *ekpq* is 510 feet, and the effective mean depth of sea in the corresponding space *hlmn* is 37 fathoms. Hence the following calculation, in which to the mean heights of the northern spaces have been added $\frac{7}{11}$ of the mean depths of the corresponding southern spaces:—

Spaces.	h	α'	$\alpha_{,}$	r'	$r_{,}$	Deflection.
cabf, cdhg . .	335	40°	60°	24	9·5	0″·552
ekpq, hlmn . .	650	28	60	50	24·0	0·751

The sum of these quantities is $1''\cdot303$. This being applied to the quantity resulting from the conformation of the ground within the 9·5 mile circle, namely, $- 0''\cdot168$, there results $1''\cdot135$ for the deflection to the north at Forth Mountain. To this should be added the effect within the 5000 feet circle, but this cannot be stated with any precision. It would appear, however, that it is a very small quantity, if at all appreciable.

Feaghmaan.

This Station is in the Island of Valencia, Sheet 78, County of Kerry. As in the preceding case, the data for the calculation are insufficient to determine with accuracy the deflection due to the ground within the 5000 feet circle. But it is obvious that there is a considerable deflection south, and by interpolating between the given altitudes the quantity $- 0''\cdot91$ was obtained. The mean heights of the compartments from the 10th to the 25th circles are as follows:—

FEAGHMAAN. North Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
10—11	—145	—145	—145	—145	— 90	— 90	— 90	— 90	— 90	— 90	— 90	— 90	— 90	— 90	— 90	— 90	— 90		20	110	130
11—12	155	155	155	155	155	155	155	155	155	155	110	110	110	110	110	110	110		15	20	35
12—13	180	180	180	180	150	150	150	150	150	150	120	120	120	120	120	120	120			40	40
13—14	180	180	180	180	180	180	120	120	120	120	130	130	130	130	130	130	130		30	40	70
14—15	190	190	190	190	190	190	190	190	190	190	130	130	130	130	130	130	130		90	20	110
15—16	210	210	210	210	210	210	210	210	150	150	150	150	150	150	90	90	90	— 90	15	10	25
16—17	210	210	210	210	210	210	210	210	210	210	155	155	155	155	+ 120	+ 200	+ 120			5	445
17—18	215	215	215	215	215	215	215	215	180	180	180	180	180	180		450	550	+ 100		5	1105
18—19	240	240	240	240	210	210	210	210	210	210	210	210	150	150	— 150	40	380	150	25	15	610
19—20	240	240	240	240	240	240	240	240	240	240	210	210	210	210	120	— 120	130	155	50	40	375
20—21	270	270	270	270	270	270	270	270	210	210	210	210	210	120	120	120	10	120	250	100	480
21—22	270	270	270	270	270	270	270	270	210	210	210	210	210	120	120	120	200	460	545	500	1705
22—23											210	210	190	180	150	130	50	430	345	255	1080
23—24											210	210	190	150	150	130	— 120	195	640	140	975
24—25											210	210	180	150	150	130	120	540	1205	275	2020

FEAGHMAAN. South Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
10—11	30	170	350	450	350	250	200	200	200	200	180	150	150	130	130	140	190	270	350	450	4540
11—12	30	170	300	350	250	200	150	150	150	140	130	100	90	90	100	130	180	250	350	450	3760
12—13	30	170	300	300	210	200	150	100	100	90	90	90	60	60	75	110	110	110	200	500	3055
13—14	20	130	200	200	210	200	120	100	60	50	50	40	25	15	10	10	15	70	120	350	1995
14—15	20	100	150	200	250	170	120	50	50	10									50	150	1320
15—16		100	140	250	200	110	40	10				10	15							40	915
16—17		100	140	150	100	10	10	10	15	30	90	150	150	60	50	80	60	5	5		1215
17—18	—180	150	120	10	10	15	80	70	100	150	300	350	200	140	140	140	70	30	50	10	2145
18—19	—215	350	100	10	100	160	160	160	180	250	650	600	700	500	600	450	120	70	60	30	5250
19—20	—250	100	100	—150	—120	150	250	250	400	750	950	900	1050	1000	700	650	250	120	80	15	7715
20—21	—270	—240	—180	—180	—150	—120	400	700	500	450	400	350	350	500	250	200	200	350	80	90	4820
21—22	—270	—270	—240	—210	—210	—180	150	300			20	300	600	200	120	120	120	300	150	200	2580
22—23											50	550	600	300	150	50	50	150	90	450	2495
23—24											120	700	200	90	15	5	5	30	100	600	1865
24—25											600	350	—100	—100	— 80	— 70	— 50	30	80	450	1510

From this table K = 9205, K′ = 45180, D = 37360, D′ = 3835, which gives a deflection equal to − 0″·00001817 × 35980 − ·00001156 × 33530, or − 1″·042.

It is unnecessary to extend the calculation further, as the deflection becomes perpendicular to the meridian.

Tawnaghmore.

This Station will be found in Sheet 5 of the county of Mayo, at a distance of about two miles from the sea. The number of levelled heights marked on the map is not sufficient to enable us to calculate with any exactness the deflection close to the Station, but from the heights that are given it would appear that the disturbance due to the distribution of matter in the immediate vicinity of the Station, though decidedly to the north, cannot be large. We shall therefore neglect the consideration of the matter contained between two parallel lines equidistant from the Station, running east and west, and of which that to the north is a tangent to the coast. The table then will be as follows, the depths being taken from the Admiralty Charts :—

TAWNAGHMORE. North Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
15—16							— 90	—110	—120	—120	—120	—120	—100	— 90							870
16—17						—120	120	150	150	150	150	150	150	120	—120	—120					1500
17—18				—120	—120	130	150	150	180	180	180	180	170	160	150	130	—120	—120			2240
18—19			—120	130	130	150	160	160	185	185	185	185	170	160	160	145	130	125			2480
19—20		—120	130	145	150	160	175	180	200	200	200	200	180	180	180	150	145	130	—120		2945
20—21		130	145	155	160	175	180	180	210	210	210	210	190	180	180	160	150	140	125	—120	3210
21—22		150	155	170	175	180	180	190	220	220	220	230	210	190	190	180	155	145	130	140	3430
22—23		180	170	180	190	210	210	210	230	230	230	235	220	205	190	180	170	150	140	145	3675
23—24		190	180	210	210	230	210	230	245	240	240	240	230	210	190	180	170	155	145	145	3850
24—25		210	210	240	240	245	250	260	260	260	260	240	240	210	190	180	170	160	150	145	4120

TAWNAGHMORE. South Side.

Number of Circle.	Number of Sector.																				H
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
15—16							100	220	200	200	200	200	180	150							1450
16—17					180	250	180	180	180	160	180	200	200	200	180	160					2250
17—18			30	100	160	200	160	140	180	140	150	150	200	350	490	600	480	150			3680
18—19			100	70	150	170	130	130	150	150	250	280	300	600	800	1000	600	700			5580
19—20		70	90	55	70	100	120	140	180	250	400	500	400	500	500	700	700	900	560		6235
20—21		100	80	50	100	110	160	160	300	500	650	580	470	450	500	700	700	700	850		7160
21—22		60	60	110	150	400	400	400	400	600	650	600	500	450	450	400	460	550	850	200	7690
22—23		75	150	300	400	900	900	800	500	600	520	500	500	400	380	380	550	650	700	250	9455
23—24	50	90	140	230	700	800	1000	950	750	500	400	450	360	350	350	370	650	600	600	350	9690
24—25	280	250	70	80	270	350	500	640	560	380	320	330	330	330	370	440	480	650	430	250	7310

$$K = 0 \quad K' = 60500 \quad D = 28320$$

We have, therefore, the deflection due to the matter with a circle of nine miles and a half, equal to—

$$-\cdot 00001817 \times 60500 - \cdot 00001156 \times 28320$$
$$= -1''\cdot 426$$

At the distance of about seven miles from land, proceeding northwards, the depth of the sea is pretty evenly 40 fathoms; it increases gradually and evenly, and at 60 miles it is between 60 and 70 fathoms: the effective mean depth may be taken at 50 fathoms. On the north side of the Station, extend the series of circles already drawn, by eleven more, and produce the radii: then, neglecting all matter to the left of the meridian of 10° West Longitude (which passes through the north-west corner of Mayo), we find that 160 compartments are covered with a mean depth of 50 fathoms of sea: thus the effect of the sea to the north is to produce a deflection of $-0''\cdot 00001156 \times 160 \times 300 = 0''\cdot 555$. On the south side (Plate XXIV.) draw with a radius of 27 miles the circular arc *defg*, and with a radius of 50 miles the circular arc *hk*: *cgk* is inclined at 80° to the meridian; *bf* at 32°; *eh* at 27°; and *ad* at 36°. The effective mean height of the space *abdf* is 380 feet, of the space *bcfg* 130 feet, and of the space *ehgk* the effective mean height is 270 feet.

Spaces.	h	a'	$a_{,}$	r'	$r_{,}$	Deflection.
abdf	380	216°	148°	27	9·5	0″·193
bcfg	130	148	100	27	9·5	0·027
ehgk	270	207	100	50	27·0	0·104

The sum of these quantities is 0″·324, which added to 0″·555 and 1″·426, give 2″·305 for the deflection, to the south, at Tawnaghmore. To this should be added the disturbance due to the configuration of the ground within two miles of the Station. This would diminish the above quantity, but by how much cannot be precisely stated.

Ben Hutig.

This Station is within a mile and a half or two miles of the sea on the north coast of Sutherlandshire. In a south-by-west direction from the hill, runs a ridge of high ground called the Moin. To the west this ground slopes down gradually to Loch Hope, a long and narrow freshwater lake lying parallel to the ridge, and on the east the ground falls gradually to the Kyle of Tongue, an inlet of the sea, which also lies parallel to the same direction. This ridge extends southwards about ten miles, where its regularity of form is interrupted by the abrupt masses of Ben Hope and Ben Loyal, of which the former exceeds 3000 feet in height. The width of the Moin is about five miles and three quarters; the breadth or base of the eastern slope may be taken at three miles, that of the western at two miles, leaving along the summit a horizontal plane of about three quarters of a mile in width, the height of which is very uniform, and 750 feet above the sea. The regular form of this piece of ground enables us easily to estimate its effect upon the plumb-line at Ben Hutig; but it is impossible at present to extend the calculation any further on the south side of the Station. In the immediate vicinity of the Station there is not much irregularity, and within the 5000 feet circle the deflection must be very small: from an examination of the ground, unaided by instruments, it is difficult to say whether it would be to the north or to the south. If a straight line be drawn east and west, at the distance of a mile and a half south from the Station, it may be assumed that the deflection due to the configuration of the ground to the north of this line is very nearly zero. By sketching in upon a map the contours of the ridge of ground described above, and drawing a series of circles such that $5\,r_{n+1} = 6\,r_n$,—the radial lines being as in all the preceding cases,—we find the following mean heights:—

Number of Circle.	Number of Sector.																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1— 2						360	500	710	700	700	670	650	620	500	270					
2— 3				300	750	750	750	710	700	670	650	610	580	550	530	500	220			
3— 4			420	720	750	750	750	720	700	650	620	580	550	500	450	400	350	150		
4— 5		150	510	680	750	750	750	720	700	650	600	550	490	450	380	320	250	200	100	
5— 6		150	380	600	720	750	750	720	690	600	550	500	450	350	280	220	150			
6— 7		50	200	450	630	730	750	720	690	600	520	450	350	210	150	100				
7— 8			50	250	500	650	750	730	690	600	500	400	210	100						
8— 9					260	510	700	750	710	600	460	300	120							
9—10						350	660	750	720	600	420	200								
10—11						300	720	800	740	620	410	150								

The sum of the heights in this table is 63750, and the corresponding deflection is therefore

$$0''\cdot0002714 \times 63750 \times \log_{10}\frac{6}{5} = 1''\cdot370$$

to the south.

We must next consider the effect of the sea to the north of the Station. The depth of the sea is, at the distance of three quarters of a mile from shore, about 20 fathoms; the line of 30 fathoms depth runs between three and four miles distant from shore. At ten miles north from the Station the depth ranges between 30 and 40 fathoms; and at the same distance north-east the depth is about 45 fathoms. Twenty miles north is the Whiten-head Bank, where the depth is in some parts as small as 25 fathoms.

In Plate XXIV., U is the Station, *abcd* a rectangle having its sides parallel and perpendicular to the meridian: the side *ab* is 50 miles distant from U, and *cd* is 5 miles; the lengths of these sides are 36 miles, U being opposite to the middle point of *cd*. The mean depth of water covered by this rectangle is 35 fathoms.

By the formula (3), page 578, we have the attraction of this mass, or rather its defect of attraction—

$$= 2 \times (1 - 2\cdot75) \times 210 \times \log_e\left(\frac{\cot\frac{1}{2}\phi}{\cot\frac{1}{2}\phi'}\right)$$

and the corresponding deflection by (1), page 576—

$$= -2 \times \frac{12\cdot447 \times 1\cdot75 \times 21}{5\cdot5 \times 528 \times \cdot4343}\log_{10}\left(\frac{\cot\frac{1}{2}\phi}{\cot\frac{1}{2}\phi'}\right)$$

$$= -0''\cdot725\log_{10}\left(\frac{\cot\frac{1}{2}\phi}{\cot\frac{1}{2}\phi'}\right)$$

Now the angles ϕ and ϕ' are 15° 30′ and 70°, which makes this quantity $= -0''\cdot516$.

A second and smaller rectangle may also be taken into account, covering a mean depth of about 22 fathoms. The outer side of this rectangle coincides with *cd*; the inner side is parallel to it, and two and a half miles distant from the Station U; thus the meridional dimension of this rectangle is 2·5 miles, and that perpendicular to the meridian is 16 miles. Taking the depth as 130 feet the deflection will be—

$$= -0''\cdot449 \log_{10}\left(\frac{\cot\frac{1}{2}\phi}{\cot\frac{1}{2}\phi'}\right)$$

The angles ϕ and ϕ' are 17° 30′ and 32°, which makes this quantity equal to $-0''\cdot121$.

Adding together the three quantities we have obtained, they amount to $2''\cdot007$; to this should be added the deflection due to the mass of the county of Sutherland generally; but at present there are not sufficient data. We know, however, that the deflection south must certainly exceed $2''\cdot007$.

Cowhythe.

It has been shown in the volume of the Ordnance Survey containing the "Observations for Latitude made with Airy's Zenith-Sector" that the deflection observed at Cowhythe is very large, and probably greater than can be accounted for satisfactorily. At present there are no sufficient data for calculating exactly the disturbance, but we may endeavour to ascertain a limit which it cannot be supposed to exceed.

The Station is on a small flat-topped hill, or hillock, the height of which is only 256 feet (by recent levelling) above the sea. To the north, the ground slopes down gradually for 200 yards, and then falls, somewhat abruptly, to the sea, which is within a quarter of a mile from the Station. To the south, east, or west, the ground is flat for about 200 yards round the Station. At about three quarters of a mile south, the ground reaches the level of about 100 feet above the plane of the sea; the country then rises gradually in a southerly direction, and attains at the visible horizon—about seven miles distant—a height of about 500 feet. In a south-south-west direction the ground does not rise regularly to the visible horizon. At a distance of three miles is a piece of rising ground, in horizontal extent about 15° or 20°, and exceeding Cowhythe in height by something under 100 feet. In the same direction, and about seven miles distant, the height of the furthest visible ground is about 700 feet; a little to the west of this is Knock Hill, eight miles distant. Although this hill exceeds 1400 feet in height, yet its horizontal extent is so small that it can produce no material effect on the latitude of Cowhythe. In the south-west direction, and between one and two miles from Cowhythe, the ground is from 100 to 150 feet above the sea; thence it rises gradually, and at the distance of Knock, and immediately to the right of it, attains a height of 650 feet.

In the south-east direction the ground falls for about three quarters of a mile, then rises, and attains at the distance of a mile and a half a height of 350 or 400 feet; at two and a half or three miles it falls to 150 feet. Alva Hill, about 650 feet high, is six miles distant in this direction. Manor Lee, 770 feet high, is seven miles distant in a south-south-east direction from Cowhythe.

To the north, the line of 20 fathoms depth is about two miles distant from shore, the line of 30 fathoms between three and four miles. Eight miles north or north-east the depth is between 40 and 50 fathoms; this depth continues to the distance of about twenty miles. At a greater distance north or north-west the depths diminish, and are between 20 and 30 fathoms. The effect of the sea to the north of this Station will therefore not be much different from the effect of the sea at Ben Hutig; we may say that probably it will not exceed $0''\cdot 7$. The disturbance produced by the configuration of the ground within a circle of one mile round the Station will be to the south, but cannot exceed $0''\cdot 5$.

If we suppose the height of the ground at the distance of one mile from the Station to be 100 feet, and that it rises regularly in every direction (not to the north) until it attains at the distance of seven miles an altitude of 700 feet, the deflection calculated on this assumption would exceed the truth, but would not be very far from it. The height h of the country at any distance r would be $h = \frac{100}{5280}\,r$, and the deflection would be—

$$6''\cdot 223 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{1}^{7} \frac{100}{5280} \cos\theta \, d\theta \, dr$$

$$= \frac{62''\cdot 23}{528} \times 2 \times 6 = 1''\cdot 4$$

This quantity probably exceeds the true deflection, that is, what would be obtained from the actual contours of the country between the distances of one and seven miles from the Station.

Proceeding in a direct line south from Cowhythe, at the distance of ten miles the height of the river Deveron is 250 feet above the sea. At Huntly, 17 miles from Cowhythe and a little to the west of south, the height of the same river is 400 feet, and six miles south of Huntly its height is 500 feet. Thirty miles south, in a direct line from Cowhythe, we come upon the river Don, of which the height is, at that point, 450 feet: following hence the course of the river westward, at five miles its height is 550 feet, at ten miles 750, at fifteen miles its height is 1000 feet. The source of the Don is about forty miles south-west of Cowhythe, and is at the altitude of 1600 feet. The altitude of the Dee, forty miles south of Cowhythe, is about 300 feet; at Castletown of Braemar, fifty miles south-west of Cowhythe, its altitude is over 1000 feet above the level of the sea. In the neighbourhood of Braemar are the highest points of the Grampians, many of them exceeding 4000 feet.

In general, then, the country rises as we proceed south, but still more in a south-west direction; in the south-east direction there is nothing remarkable in the height of the country: it may not exceed in the mean 400 feet.

In order to make sure of not underrating the probable deflection, let us suppose that between the distances of seven and twenty-five miles from Cowhythe, and between the directions of west and south, the height of the country is 800 feet, and that between the same directions, and between the distances of 25 and 50 miles, the mean height is 2000 feet; the deflection would be—formula (2), page 628—

$$2''\cdot 17 \log \frac{25}{7} + 5''\cdot 42 \log \frac{50}{25} = 2''\cdot 8$$

Between the directions of south and east, and between the distances of seven and forty-two miles, suppose the mean height as much as 500 feet, then we shall have to add to the preceding quantity

$$1\cdot35 \log 6 = 1''\cdot0$$

making altogether 3″·8. We have also estimated,—for the sea 0″·7; for the immediate vicinity up to one mile distance, 0″·5; from one to seven miles, 1″·4; the whole estimate is therefore 6″·4, and this quantity is certainly greater than would be obtained if we had sufficient data; probably it would be found not to exceed 5″·0.

Summary.

We may sum up the results of this Section in the following table:—

Stations.	Deflection.	Distance* to which Calculation is carried.	Remarks.
Dunnose	− 0·54	Twelve miles . .	Determinations satisfactory. But by extending the calculations, each of these deflections would receive a small + correction due to the mass of England to the north and the English Channel to the south.
Boniface	+ 2·42	" " . .	
Week Down	+ 1·98	" " . .	
Port Valley	+ 3·29	" " . .	
Clifton	− 0·90	Eight miles . . .	Determination satisfactory. There are not sufficient data for extending the calculations.
Burleigh Moor . . .	− 4·55	Forty-eight miles .	Determination satisfactory.
Calton Hill	− 3·57	Nineteen and a half miles.	Determination satisfactory.
Lough Foyle	− 4·02	Fifty miles . . .	Determination satisfactory.
Kellie Law	+ 2·08	Nine and a half miles	Determination satisfactory.
Monach	+ 0·47	......	Determination satisfactory.
Ben Hutig	− 2·01	Fifty miles north and ten miles south.	Determination tolerably satisfactory: by extending the calculation the amount would be increased.
Hungry Hill . . .	+ 5·40	Fifty miles . . .	Determination satisfactory.
Forth	+ 1·13	" " . . .	Determination tolerably satisfactory.
Feaghmaan	− 1·95	Nine and a half miles	Determination not quite satisfactory.
Tawnaghmore . . .	− 2·30	Fifty miles . . .	Determination not quite satisfactory.
Cowhythe	...	......	Not sufficient data for this calculation, but the deflection may be about − 5″·0; certainly not amounting to − 6″·0.

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* It is difficult to know where to stop in these calculations. I have assumed fifty miles as a distance probably sufficient, but in some cases the result would be modified by extending the calculation to a still greater distance. There is, however, some uncertainty as to the propriety of going beyond this, especially when there is sea on the one side and land on the other. The final results in the following Section may, perhaps, throw some light on this question.—A.R.C.

SECTION XII.

DETERMINATION OF THE SPHEROID MOST NEARLY REPRESENTING THE SURFACE OF GREAT BRITAIN AND IRELAND.

§ I.

When a number of astronomical observations for latitude have been made at each of a series of Stations, we may deduce the final results by one or the other of two methods: we may either determine the latitude of each Station by means of the observations made at itself, and assumed mean declinations of the stars, without reference to the observations made at the other Stations; or we may obtain differences of latitude which shall be independent of any assumed mean places of the stars,—and using these differences, make the absolute latitudes such as will agree best, upon the whole, with the results derived from the former process. Thus, suppose by the former process that we obtain the independent results $\lambda_1, \lambda_2, \lambda_3, \cdots \lambda_n$ for the latitudes of the n Stations, and suppose that by the latter we obtain the successive differences of latitude (or amplitudes), $\alpha_1, \alpha_2, \alpha_3, \cdots \alpha_{n-1}$, which are independent of the errors of the assumed mean declinations, and let $x, x+\alpha_1, x+\alpha_2, \cdots x+\alpha_{n-1}$ be the latitudes, then x may be determined by making $(x-\lambda_1)^2+(x+\alpha_1-\lambda_2)^2+(x+\alpha_2-\lambda_3)^2+\cdots (x+\alpha_{n-1}-\lambda_n)^2$ a minimum. Thus the absolute latitudes will rest upon the whole of the assumed mean declinations, but the differences of latitude will be independent of them altogether.

In the first method, if $\epsilon_1, \epsilon_2, \epsilon_3, \cdots \epsilon_i$ be the errors of the assumed declinations of the stars used at a Station, $n_1, n_2, n_3, \cdots n_i$ the number of times they are observed, respectively; the error in the finally obtained latitude will be—

$$\frac{n_1\epsilon_1+n_2\epsilon_2+n_3\epsilon_3+\cdots n_i\epsilon_i}{n_1+n_2+n_3+\cdots n_i}$$

or if ϵ be taken as the probable error of an assumed mean declination, the probable error of the latitude will be—

$$\frac{\sqrt{n_1^2+n_2^2+n_3^2+\cdots n_i^2}}{n_1+n_2+n_3+\cdots n_i}\cdot\epsilon$$

If we suppose the stars to be observed nearly the same number of times each, then i being the number of stars observed at the Station, the probable error of the latitude will be $\epsilon \div \sqrt{i}$.

In examining the results of the observations for latitude made with Airy's Zenith Sector, it was found that stars north of the zenith generally gave the greatest, and stars south of the zenith the smallest, resulting latitudes. The cause of this discrepancy appears to have been the pressure of the upper pivot screw upon the revolving frame of the instrument, which, by compression, disturbed the true ratio of the radius to the divided arc. Since the discovery of this source of error, the upper pivot screw has been used with less pressure; and although the same degree of precision is not found in the Zenith Points, yet the difference between the results given by north and south stars has almost entirely vanished.

All zenith distances, (not including those at Arthur's Seat) resulting from observations with this instrument have therefore received a correction of the form $+ \alpha z$, where α is a quantity constant for each Station, and determined by a comparison of the latitudes given by well known north and south stars. The determinations of these corrections, which vary at the different Stations from 0″·038 to 0″·308 for each degree of zenith distance, are liable to considerable probable error; but it is to be remarked, that as stars are generally observed equally on both sides of the zenith, the effect of any uncertainty in these quantities will be neutralized in the mean resulting latitudes obtained as in the first process mentioned.

§ II.

Observations made with Ramsden's Zenith Sector.

It has been found necessary to revise the calculation of the results of the observations made with Ramsden's Zenith Sector, in consequence of the corrections obtained from a micrometric examination of the divisions of the limb having been applied in the printed volume with a wrong sign. Of the existence of this error, which had been long suspected, no proof sufficiently satisfactory to justify the alterations now effected could be obtained, until it appeared by a communication from Professor Peters, of Altona,* to the Superintendent of the Ordnance Survey, that the corrections determined by the late Professor Schumacher were opposite in sign to those printed in the volume of 1842.

By changing the signs of the corrections, the observations have been in general improved, and discrepancies diminished.

The following table of corrections for collimation errors is to be substituted for that given at page 132 of the "*Astronomical Observations made with Ramsden's Zenith Sector.*"

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* "According to Professor Schumacher's observations, the signs of corrections given in the table page 2 of the "work 'Astronomical Observations made with Ramsden's Zenith Sector.—London, 1842,' are to be inverted. At the "right hand of the limb, for instance, is, according to Schumacher's measurements, the interval between 4° and 4° 5′, "= $5^{r}\,5^{d}\cdot55$, and between 4° 5′ and 4° 10′, = $5^{r}\,2^{d}\cdot72$. The mean value of an interval is $5^{r}\,4^{d}\cdot50$. Accordingly, the "correction for 4° 5′ is nearly 1″·05 greater than for 4° 0′, and for 4° 10′, nearly 1″·77 less than for 4° 5′. But in the "table above mentioned the corrections are given as follow :—

°	′		″
4	0,	correction	− 0·05
4	5	„	− 1·17
4	10	„	+ 0·36

"contrary to those found by Professor Schumacher."—*Extract from the letter referred to above.*

Greenwich, 1802.

Date.	Sector Face.	c.
April . 12	W	+4″·34*
13	W	+4·34*
15	W	+4·34
16	W	+3·67
19	W	+4·44
21	E	−4·34
,,	W	+3·87
22	E	−4·91
23	E	−3·89
24	E	−3·87
,,	W	+4·83
25	W	+3·77
26	W	+4·92

Dunnose, 1802.

Date.	Sector Face.	c.
May . 9	E	−3″·98*
10	E	−3·98
11	E	−3·19
,,	W	+3·93
12	E	−3·56*
,,	W	+3·19
13	E	−3·46
14	E	−3·79
,,	W	+3·17
15	E	−4·18
,,	W	+3·46
16	E	−2·70
,,	W	+3·85
17	E	−2·68
20	E	−3·16*
June . 5	W	+3·64
8	E	−3·56
,,	W	+4·09
11	E	−4·09
,,	W	+3·69
12	E	−4·27
13	E	−3·40
,,	W	+3·84
14	E	−3·80
,,	W	+3·32
15	E	−2·72
,,	W	+2·81*
16	E	−2·91
,,	W	+2·31
17	E	−2·27
,,	W	+3·11
18	E	−3·38
,,	W	+2·58
20	E	−2·32
,,	W	+2·94
21	E	−2·69
,,	W	+2·29
22	W	+2·72

* The quantities marked with an asterisk are inferred or interpolated.

Clifton Beacon, 1802.

Date.	Sector Face.	c.
July . 20	W	+3″·25
21	E	−3·36
22	W	+3·55
23	E	−2·92
,,	W	+3·66*
26	E	−4·40
,,	W	+3·13
28	E	−3·81
,,	W	+3·06
29	E	−3·40
,,	W	+3·90
30	W	+3·94
31	E	−3·81
,,	W	+4·31
August 1	E	−4·31
,,	W	+2·70
3	E	−1·44
,,	W	+3·64
4	W	+3·10
5	E	−3·47
,,	W	+3·02
7	W	+3·52
8	E	−3·06
,,	W	−2·77*
9	E	−2·77*
,,	W	+2·49
10	W	+1·32
12	E	−3·85
,,	W	+3·59
13	E	−2·45
,,	W	+3·85
17	E	−3·64
18	E	−2·54
,,	W	+3·67
19	E	−3·91
,,	W	+2·54

Arbury Hill, 1802.

Date.	Sector Face.	c.
Sept. . 7	E	−3″·07
8	E	−3·07*
,,	W	+3·07
10	E	−3·85
,,	W	+1·91
11	E	−1·91
12	E	−2·24
14	W	+2·47
15	W	+3·85
16	W	+3·35
18	E	−2·87
,,	W	+2·34
19	E	−2·34
,,	W	+2·67
20	E	−2·88
21	W	+3·13
22	E	−2·63
23	W	+2·42
24	E	−2·42
25	E	−1·92
,,	W	+2·33
26	E	−2·66
,,	W	+ ·06
27	W	+3·71
28	E	−2·81
,,	W	+2·80
29	E	−2·35
,,	W	+2·55
30	E	−2·87*
,,	W	+3·19
October 1	E	−3·14
2	E	−3·02
3	E	−2·80*
,,	W	+2·59

Delamere Forest, 1806.

Date.	Sector Face.	c.
May . 30	W	+1″·52
31	E	−2·55
June . 1	E	−2·79*
,,	W	+3·03
2	E	−2·10
3	E	−3·24
,,	W	+3·81
4	E	−1·39
,,	W	+1·97
5	E	−1·60
,,	W	+2·02*
6	E	−2·45
,,	W	+1·99
7	E	−3·10
,,	W	+1·50
9	E	−1·55
,,	W	+1·28

Burleigh Moor, 1806.

Date.	Sector Face.	c.
July . 4	W	+2″·11*
6	W	+2·11
7	E	−2·02
9	E	−2·42
10	E	−1·68
11	W	+1·70
12	E	−3·55
14	W	+2·58*
15	W	+1·61
16	W	+2·52
17	E	−3·39
18	W	+3·34
19	E	−2·69
,,	W	+3·39

Kellie Law, 1813.

Date.	Sector Face.	c.
June . 23	E	−2″·65
25	W	+2·58
26	E	−2·28
27	W	+2·22
28	W	+2·24
July . 1	W	+3·25
2	E	−2·32
3	E	−2·92
,,	W	+2·59
4	E	−2·61
7	W	+3·20
8	E	−3·24
,,	W	+2·85
9	W	+2·57
11	E	−2·66
,,	W	+3·23
12	E	−3·18
,,	W	+3·29
13	E	−2·61
,,	W	+3·25
17	E	−2·62
18	E	−3·60
20	E	−3·80*
22	W	+4·01
24	E	−3·60
,,	W	+3·35
25	E	−3·86
26	E	−3·41
,,	W	+3·62
27	E	−3·74
,,	W	+4·13
28	W	+3·46*
29	W	+2·79
30	E	−2·79

Cowhythe, 1813.

Date.	Sector Face.	c.
Sept. . 3	W	+2″·83*
4	W	+2·83
5	E	−2·89
,,	W	+2·84
6	E	−2·84
,,	W	+3·00
7	E	−3·01
,,	W	+2·95
8	E	−2·92
,,	W	+2·77
9	E	−2·78
,,	W	+3·00
10	E	−3·06
11	W	+3·13
14	E	−3·85
15	W	+3·35
16	E	−3·08*
,,	W	+2·82
18	E	−2·73
19	W	+0·67
20	E	−2·87
,,	W	+3·21
21	E	−3·21
,,	W	+2·98
22	E	−3·72
24	W	+2·75
26	W	+2·83
27	E	−3·59
,,	W	+3·25*
28	E	−2·91
29	E	−2·73
,,	W	+3·31
30	W	−2·45
Oct. . 2	E	−3·63
,,	W	+2·92*
3	E	−2·22
,,	W	+3·93
7	W	+3·63*
8	E	−3·63*
12	E	−3·63*
,,	W	+3·64

Balta, 1817.

Date.	Sector Face.	c.
August 13	W	+1″·41
14	E	−1·89
15	E	−2·24
16	E	−2·31
17	E	−1·75
,,	W	+2·59
18	W	+1·99
19	W	+1·76
20	E	−1·99
,,	W	+2·93
21	E	−3·04
,,	W	+2·81
22	E	−2·57
,,	W	+2·29
23	E	−2·74
,,	W	+2·52
24	E	−2·55
,,	W	+2·89
25	E	−2·83
26	E	−2·91
27	E	−2·99
,,	W	+3·18
28	W	+2·96
29	E	−3·83
,,	W	+2·99

Dunkirk, 1818.

Date.	Sector Face.	c.
October 19	W	+2″·89
20	E	−2·94
,,	W	+3·23
21	E	−3·19
22	E	−3·13
,,	W	+2·94
23	E	−3·11
,,	W	+3·01
24	E	−2·93
,,	W	+3·61
25	E	−3·13
,,	W	+2·89
26	E	−3·02*
,,	W	+3·16
27	E	−2·79
,,	W	+2·80*
28	E	−2·80*
,,	W	+2·82
29	W	+3·36

Greenwich, 1836.

Date.	Sector Face.	c.
May . 31	W	−1″·84
June . 1	W	−1·84
3	W	−1·84
8	E	+1·84
9	E	+1·84
13	E	+1·84
15	W	−1·84
16	W	−1·84
20	W	+2·10
24	E	−2·10
25	W	+0·22
27	E	−0·22
28	W	+2·35
29	E	−2·35
July . 4	E	−2·35
6	W	−3·51*
7	W	−3·51
8	E	+3·51
9	E	+3·51
,,	W	−3·51*
11	W	−0·75
12	E	+0·75
14	W	−0·75
15	W	−0·75
18	E	+0·75
20	E	+0·75
21	E	+0·75
23	W	−0·75
27	W	−2·52
August 1	E	+2·52
2	E	+2·52
4	W	−2·52
8	W	−2·52
9	W	−2·52
10	E	+2·52
16	E	+2·52
17	E	−1·84
18	W	+1·84
19	W	+1·84
22	E	−1·84
24	E	+2·11
26	W	−2·11
30	W	−2·11
31	W	−2·11

RESULTS FOR LATITUDE WITH RAMSDEN'S ZENITH SECTOR.

No. in Cat.	Name of Star.	Greenwich, 1802.		Dunnose, 1802.		Clifton Beacon, 1802.		Arbury Hill, 1802.		Delamere, 1806.		Burleigh Moor, 1806.		Kellie Law, 1813.		Cowhythe, 1813.		Balta, 1817.		Dunkirk, 1818.	
		Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.
		° ′ 51 28		° ′ 50 37		° ′ 53 27		° ′ 52 13		° ′ 53 13		° ′ 54 34		° ′ 56 14		° ′ 57 41		° ′ 60 44		° ′ 51 1	
		″		″		″		″		″		″		″		″		″		″	
1	12 Cassiopeiæ.........																	61·47	7		
2	*																			59·07	7
3	15 Cassiopeiæ, κ													50·56	4	8·66	9	61·77	8		
4	18 Cassiopeiæ, α			5·24	2									51·21	6	10·48	13	63·24	9	58·40	5
5	24 Cassiopeiæ, η											17·73	2	51·52	6	9·64	13	62·82	7	57·98	7
6	Cassiopeiæ.........																	61·36	5		
7	27 Cassiopeiæ, γ											15·04	3	51·03	3	9·94	12	61·66	5		
8	30 Cassiopeiæ, μ																			59·86	7
9	33 Cassiopeiæ, θ													51·89	1	11·55	11	63·77	6		
10	*																	62·15	4		
11	34 Cassiopeiæ, φ																	61·11	5	58·67	7
12	37 Cassiopeiæ, δ													50·92	6	9·99	11	62·26	7		
13	39 Cassiopeiæ, χ																	62·66	6	57·72	7
14	*																	62·37	3		
15	44 Cassiopeiæ.........																	62·84	6		
16	54 Andromedæ																			58·39	6
17	1 Persei																			59·09	6
18	45 Cassiopeiæ, ε													50·72	5	9·76	11	62·04	8		
19	53 Cassiopeiæ.........																	63·12	4		
20	4 Persei																			59·82	5
21	5 Persei, h																			57·77	5
22	55 Cassiopeiæ.........																	61·43	5		
23	*																	62·99	2	56·86	3
24	*																	63·81	3		
26	Cassiopeiæ, i																	62·81	5		
27	13 Persei, θ																			57·91	5
28	15 Persei, η																	62·71	5	58·33	5
29	23 Persei, γ																			59·29	5
30	27 Persei, κ																			58·39	4
31	Camelopardi......																	60·94	3		
32	33 Persei, α			8·91	2	28·87	5	26·23	9			19·51	3	52·64	5					57·45	4
33	39 Persei, δ																			57·70	5
34	Camelopardi																	61·18	3		
35	13 Aurigæ, α..........	38·96	3	8·70	9	31·97	5	28·55	12	19·50	1									59·81	3
36	18 Ursæ Majoris, e .	35·80	3																		
37	25 Ursæ Majoris, θ .	41·07	1																		
38	30 Ursæ Majoris, φ .	38·30	3																		
39	48 Ursæ Majoris, β .									19·92	1			49·37	3	9·56	4	64·09	1	58·02	8
40	50 Ursæ Majoris, α .																	60·83	1		
41	52 Ursæ Majoris, ψ .	41·69	6	8·16	3																
42	63 Ursæ Majoris, χ.			7·99	4																
43	64 Ursæ Majoris, γ .	39·25	4	7·22	14	27·30	1	27·57	4	18·06	4	20·04	1	49·57	2	10·42	4	61·45	3	57·91	9
44	69 Ursæ Majoris, δ .													50·45	2	10·12	3			58·13	6
45	77 Ursæ Majoris, ε .			7·27	1									49·96	3	10·37	4	61·08	3	57·10	9
46	79 Ursæ Majoris, ζ .	38·36	1	6·64	11	31·16	5	27·52	4	18·85	8	22·21	1	50·09	3	8·67	1	63·01	3	57·10	9
47	85 Ursæ Majoris, η .	38·39	2	6·08	17	30·75	5	26·97	9	18·72	8	22·26	1	51·06	4	10·23	5			57·62	10
50	11 Draconis, α																	60·64	2		
51	17 Boötis, κ²			7·07	10									48·69	2						
53	23 Boötis, θ			7·61	11	32·05	1														
62	12 Draconis, ι									18·23	7	19·08	2	49·69	5	9·30	2	61·08	2		
67	6 Herculis, υ			7·01	8	30·63	3			18·78	7										
68	13 Draconis, θ													50·98	6	8·65	2	62·70	1		
69	22 Herculis, τ			7·92	14	31·20	6	29·35	2	19·69	3	19·61	2								
72	14 Draconis, η													49·51	4	8·61	4	60·98	4		
73	16 Draconis			6·16	1									50·37	2						
74	17 Draconis			6·96	8	31·65	3			18·61	3	19·67	8	51·83	4						
77	52 Herculis			7·15	12	31·45	4														
78	19 Draconis, h¹																	63·03	1		
80	21 Draconis, μ	36·66	1	6·44	12	30·23	6			18·70	5	19·37	4	48·64	8						
81	22 Draconis, ζ																	59·74	2		
84	23 Draconis, β	39·28	4	7·51	14	30·76	15	26·87	14	19·35	6	20·68	8	51·65	11	10·24	7			58·57	4
85	24 Draconis, ν¹.......																	61·77	3		
86	25 Draconis, ν².......																	61·29	3		
87	26 Draconis													48·55	10	6·48	5	61·40	1		
88	85 Herculis, ι	37·11	3	7·84	5	31·04	8			21·47	3									58·39	1
90	32 Draconis, ξ																	61·85	7	58·41	2

RESULTS FOR LATITUDE WITH RAMSDEN'S ZENITH SECTOR—*continued.*

No. in Cat.	Name of Star.	Greenwich, 1802.		Dunnose, 1802.		Clifton Beacon, 1802.		Arbury Hill, 1802.		Delamere, 1806.		Burleigh Moor, 1806.		Kellie Law, 1813.		Cowhythe, 1813.		Balta, 1817.		Dunkirk, 1818.	
		Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.	Lat.	No. Obs.
		51° 28′		50° 37′		53° 27′		52° 13′		53° 13′		54° 34′		56° 14′		57° 41′		60° 44′		51° 1′	
91	33 Draconis, γ	38″·73	5	7″·11	13	30″·14	15	26″·49	15	18″·79	6	20″·19	7	50″·66	12	9″·52	9	″		57″·53	4
95	36 Draconis																	61·05	4		
96	39 Draconis, *b*													48·65	11	7·80	8	60·57	4		
97	42 Draconis																	59·27	5		
98	45 Draconis, *d*	36·73	3	5·07	6	29·41	9	25·34	16	16·42	4	18·20	4	49·11	10	7·74	8			55·77	4
99	46 Draconis, *c*	38·19	1	7·07	5	30·35	12	27·43	18	17·44	4	20·25	5	50·65	9	9·39	9	62·03	6	58·23	5
101	47 Draconis, *o*													49·22	12	8·10	8	61·21	8		
103	48 Draconis													49·76	15	8·34	9	61·18	6	56·36	4
104	49 Draconis																	61·70	3	57·52	1
106	51 Draconis	38·26	2	6·76	5	29·54	9	26·82	17	17·87	2	18·56	5	50·27	13	9·46	10	60·72	7	56·91	4
108	53 Draconis, *n*	35·12	1	7·19	5					17·02	4	19·31	5	50·17	13	8·78	10	61·45	8	58·13	6
109	54 Draconis																			57·25	3
111	57 Draconis, δ																	60·70	6		
112	1 Cygni, κ	37·23	2	7·57	5	29·87	14	26·93	17	17·81	4	18·77	4	50·86	14	9·79	9			56·39	5
113	58 Draconis, π																	60·70	7		
114	10 Cygni, ι²	37·21	2	6·72	5	30·18	12	26·78	15	18·59	4	19·30	5	50·65	14	9·44	9			58·19	6
115	13 Cygni, θ					30·66	2													58·08	5
116	16 Cygni, *c*																			57·69	2
117	*																			53·73	1
119	20 Cygni, *d*			7·78	2	30·83	3							51·44	14	10·06	11			58·76	7
120	Cephei													48·79	15	6·88	11	60·60	8	56·60	6
121	66 Draconis																	60·79	6		
122	*																	60·12	6		
123	31 Cygni, *o*²			6·93	3																
124	68 Draconis																	60·23	6		
125	71 Draconis																	60·56	7		
126	43 Cygni, ω¹			6·09	3	30·06	2													57·51	5
127	46 Cygni, ω³			6·34	2	31·40	1														
128	2 Cephei, θ																	59·84	9		
129	50 Cygni, α			6·62	2															57·97	6
130	Cephei, χ			7·40	4	29·70	2													56·68	6
131	3 Cephei, η																	61·93	8		
132	59 Cygni, *f*¹			7·26	4	31·21	3														
133	5 Cephei, α													49·22	10	8·11	18	61·80	8		
134	9 Cephei																	61·20	6		
135	80 Cygni, π¹			5·79	4	28·59	3														
136	81 Cygni, π²			6·68	4	29·98	3														
137	10 Cephei, ν																	61·29	5		
138	*																	63·58	4		
139	19 Cephei																	59·49	6		
140	23 Cephei, ε																	62·06	5		
141	25 Cephei																	61·59	5		
142	3 Lacertæ, β			7·12	1															58·21	4
143	27 Cephei, δ																	62·27	8		
144	7 Lacertæ, α			5·28	1															57·38	4
145	9 Lacertæ............																			57·21	4
146	30 Cephei																	61·32	6		
147	32 Cephei, ι																	61·43	8		
148	1 Cassiopeiæ.........																	60·63	7	56·53	7
149	2 Cassiopeiæ.........																	59·92	1		
150	34 Cephei, *o*																	60·48	7		
151	4 Cassiopeiæ.........																	61·31	8		
152	Cassiopeiæ.........																	61·58	7	57·36	1
153	18 Andromedæ																			56·89	5
154	5 Cassiopeiæ, τ ...																	62·34	4		
155	6 Cassiopeiæ.........																	61·49	4		
156	*																			57·32	4
157	9 Cassiopeiæ.........																	61·94	7		
158	11 Cassiopeiæ, β ...													52·09	6	10·91	8	62·49	9	58·20	7
	Resulting Latitudes	38″·51	47	7″·07	232	30″·35	157	26″·90	152	18″·55	84	19·39	70	50″·22	273	9″·30	258	61″·55	382	57·86	270

The results for the latitude of Greenwich, from the observations made by Lieutenant Denison, R.E., in 1836, not included in the foregoing table, are as follows:—

GREENWICH, 1836.

No. in Cat.	Name of Star.	Lat.	No. Obs.	No. in Cat.	Name of Star.	Lat.	No. Obs.	No. in Cat.	Name of Star.	Lat.	No. Obs.
		51° 28′				51° 28′				51° 28′	
45	77 Ursæ Majoris, ε	37″·63	1	66	*	37″·86	14	94	*	38″·71	21
46	79 Ursæ Majoris, ζ	41·57	2	68	13 Draconis, θ	37·60	16	96	39 Draconis, *b*	39·19	25
47	85 Ursæ Majoris, η	37·64	7	69	22 Herculis, τ	39·14	17	98	45 Draconis, *d*	39·29	22
48	86 Ursæ Majoris	44·98	1	70	*	40·17	8	99	46 Draconis, *c*	38·60	26
49	*	39·37	2	71	*	40·37	1	100	*	38·28	20
52	19 Boötis, λ	40·21	5	73	16 Draconis	38·42	10	102	13 Lyræ	38·00	21
53	23 Boötis, θ	34·43	4	75	42 Herculis	37·79	6	105	*	37·60	2
54	*	38·12	2	76	*	39·48	13	106	51 Draconis	38·74	17
55	38 Boötis, *h*	34·34	3	77	52 Herculis	39·46	17	107	*	36·61	1
56	*	35·94	4	79	*	40·05	20	110	*	35·59	1
57	44 Boötis, *i*	39·66	8	82	74 Herculis	38·25	21	112	1 Cygni, κ	37·73	14
58	*	40·33	1	83	77 Herculis, *x*	38·55	17	114	10 Cygni, ι^2	40·10	18
59	*	40·13	1	84	23 Draconis, β	38·74	23	115	13 Cygni, θ	39·42	18
60	*	40·09	1	88	85 Herculis, ι	39·13	23	118	18 Cygni, δ	38·09	19
61	*	35·70	3	89	30 Draconis	39·39	24				
63	*	35·06	2	90	32 Draconis, ξ	39·09	23				
64	*	40·50	7	92	*	40·23	18		*Resulting Latitude..*	38″·81	586
65	*	38·11	10	93	*	38·71	26				

If we put the amplitudes—

$$
\begin{array}{lll}
\textit{Dunnose and Greenwich} & = & 0^{\circ}\ 51'\ 30'' + x_1 \\
\quad\text{,,}\qquad \textit{Arbury} & = & 1\ 36\ 20 + x_2 \\
\quad\text{,,}\qquad \textit{Delamere} & = & 2\ 36\ 10 + x_3 \\
\quad\text{,,}\qquad \textit{Clifton} & = & 2\ 50\ 20 + x_4 \\
\quad\text{,,}\qquad \textit{Burleigh Moor} & = & 3\ 57\ 10 + x_5
\end{array}
$$

we shall have 87 equations for the determination of these five quantities: these equations being treated according to the method of least squares, give—

$$
\begin{array}{lcl}
0 = -\ 34{\cdot}75 + 27{\cdot}54\, x_1 & \qquad \therefore & x_1 = +\ 1''{\cdot}26 \\
0 = +\ 1{\cdot}97 + 54{\cdot}18\, x_2 & & x_2 = -\ 0{\cdot}04 \\
0 = -\ 80{\cdot}56 + 50{\cdot}45\, x_3 - 1{\cdot}56\, x_5 & & x_3 = +\ 1{\cdot}68 \\
0 = -\ 251{\cdot}54 + 73{\cdot}74\, x_4 & & x_4 = +\ 3{\cdot}41 \\
0 = -\ 102{\cdot}73 - 1{\cdot}56\, x_3 + 38{\cdot}56\, x_5 & & x_5 = +\ 2{\cdot}71
\end{array}
$$

So that the required amplitudes are—

		°	′	″
Dunnose and Greenwich	=	0	51	31·26
„ *Arbury*	=	1	36	19·96
„ *Delamere*	=	2	36	11·68
„ *Clifton*	=	2	50	23·41
„ *Burleigh Moor*	=	3	57	12·71

The quantity which has to be applied to these amplitudes, in order to give the latitudes which shall most nearly agree with the independent results in page 669, is easily found to be 50° 37′ 6″·96, whence the following latitudes:—

	°	′	″
Dunnose	50	37	6·96
Greenwich	51	28	38·22
Arbury	52	13	26·92
Delamere	53	13	18·64
Clifton	53	27	30·37
Burleigh Moor	54	34	19·67

Combining next the observations made from 1813 to 1818 at Kellie Law, Cowhythe, Balta, and Dunkirk, we get 85 equations for determining the three amplitudes. These equations treated according to the method of least squares, resolve themselves into the following:—

$$\begin{aligned} 0 &= -178{\cdot}22 + 132{\cdot}52\,x_1 - 40{\cdot}38\,x_2 - 31{\cdot}75\,x_3 & \therefore\ x_1 &= +2''{\cdot}90 \\ 0 &= -\ \ 72{\cdot}63 - \ \ 40{\cdot}38\,x_1 + 99{\cdot}59\,x_2 & x_2 &= +1{\cdot}90 \\ 0 &= -264{\cdot}94 - \ \ 31{\cdot}75\,x_1 + 87{\cdot}84\,x_3 & x_3 &= +4{\cdot}06 \end{aligned}$$

Hence the amplitudes—

	°	′	″			°	′	″
Dunkirk and Kellie Law	5	12	50	$+ x_1 =$		5	12	52·90
„ *Cowhythe*	6	39	10	$+ x_2 =$		6	39	11·90
„ *Balta*	9	43	0	$+ x_3 =$		9	43	4·06

The quantity, which has to be applied to these amplitudes, to give those latitudes which shall most nearly agree with the independent results at page 669, is easily found to be 51° 1′ 57″·52, whence the following latitudes:—

	°	′	″
Dunkirk	51	1	57·52
Kellie Law	56	14	50·42
Cowhythe	57	41	9·42
Balta	60	45	1·58

The amplitudes just deduced are dependent upon and affected with the error of the assumption that the points 0° and 7° (Right Limb) are relatively correct with respect to the radius of the divided arc. This assumption is, of course, erroneous, but the error is very small, as a comparison of the results for latitude given by north and south stars does not consistently indicate any error in the angular measures. We must, therefore, allow the assumption to remain.

The following table contains the corrected results of the observations with Ramsden's Zenith Sector, together with results given in the volume of 1842 :—

Stations.	Latitudes.			Stations.	Latitudes.		
	1st Method.	2d Method.	Vol.: 1842.		1st Method.	2d Method.	Vol.: 1842.
	° ′ ″	″	″		° ′ ″	″	″
Dunnose . . .	50 37 7·07	6·96	7·03	Clifton	53 27 30·35	30·37	30·45
Dunkirk . . .	51 1 57·86	57·52	57·93	Burleigh Moor .	54 34 19·39	19·67	19·48
Greenwich . . .	51 28 38·51	38·22	38·30	Kellie Law . .	56 14 50·22	50·42	50·51
Arbury	52 13 26·90	26·92	27·14	Cowhythe . . .	57 41 9·30	9·42	9·74
Delamere . . .	53 13 18·55	18·64	18·77	Balta	60 45 1·55	1·58	2·31

The results now obtained agree very closely with those given at page 198, which are, however, affected with the wrongly applied corrections for the divisions of the limb.

There is some difficulty in deciding whether the results of the first method of reduction contained in the second column of this table, or those of the second method, contained in the third column, are to be preferred as probably nearest the truth. The results in the second column are affected by errors of assumed mean declinations, those in the third column are affected with the error involved in the assumption that the length of the arc from 0° to 7° (Right Limb) of Ramsden's Zenith Sector was correct.

In all the subsequent calculations in this work the results in the third column have been used, and similarly with respect to the observations made with Airy's Zenith Sector, the quantities used being those in the 6th column of the table, pages 198, 199. But the final results of this Section will be put in such a form as to admit of easily determining the effect of any changes of latitude that may be considered necessary or advisable. For the latitude of Greenwich, the quantity 51° 28′ 38″·30 has been used in all calculations. In the following table the latitudes have, when necessary, received the corrections to reduce them to the Trigonometrical Stations. For Dunnose, Cowhythe, and Balta, the means between the results furnished by the two Zenith Sectors have been used.

Stations.	Latitudes.	Stations.	Latitudes.
	° ′ ″		° ′ ″
Greenwich	51 28 38·30	Forth	52 18 57·91
St. Agnes	49 53 33·93	Tawnaghmore . . .	54 17 41·34
Goonhilly	50 2 50·07	Lough Foyle	55 2 38·74
Hensbarrow	50 23 1·84	South Berule	54 8 56·40
Blackdown	50 41 8·89	Ben Lomond	56 11 26·27
Dunnose	50 37 7·08	Kellie Law	56 14 51·52
Boniface Down . . .	50 36 10·55	Ben Heynish	56 27 16·88
Week Down	50 35 51·42	Great Stirling . . .	57 27 49·12
High Port Cliff . . .	50 35 43·28	Cowhythe	57 41 8·96
Southampton	50 54 46·97	Monach	58 21 20·84
Precelly	51 56 45·18	Ben Hutig	58 33 6·47
Arbury	52 13 26·59	North Rona	59 7 15·19
Delamere	53 13 18·61	Balta	60 45 1·66
Clifton	53 27 30·40	Gerth of Scaw . . .	60 48 56·43
Burleigh Moor . . .	54 34 19·67	Saxavord	60 49 38·58
Hungry Hill	51 41 10·26		
Feaghmaan	51 55 22·85	*Dunkirk*	51 2 8·41

At page 1 of "*Astronomical Observations made with Ramsden's Zenith Sector*," it is stated that the point at which the observations were made was "1438 feet S. and 1104 feet W. of the Flagstaff on the great Church Tower." On looking back, however, to the original documents, we find that in order to connect the Sector Station with the Tower a small base-line was measured 1008·6 feet in length, and at each extremity of this base the Sector Station and the Flagstaff on the Church Tower were observed. By means of these angles the distance of the Sector Station from the Tower is found to be 1813·1 feet. From observations taken* on the 14th of October, we find that the Flagstaff reads 44° 46′ 6″, while the Eastern Elongation of the pole star reads† 174° 55′ 12″. On the following day the Western Elongation of the pole star reads 169° 38′ 48″, and the Flagstaff on the Tower 44° 45′ 58″. Hence we infer that the Tower was 52° 29′ 2″ north-east of the Sector Station; from this bearing and the distance 1813·1 feet, it appears that the Zenith Sector was placed 1104 feet south and 1438 feet west of the Flagstaff on the Tower, the numbers being erroneously transposed in the printed volume of Observations. The distance 1104 corresponds to 10″·89, so that the latitude of Dunkirk Tower is 51° 2′ 8″·41.

The latitude of Dunkirk here given differs slightly from Delambre's determination, which we find at page 295, tom. II., "*Base du Système Mètrique*" to be 51° 2′ 9″·7. On examining the observations upon which this result depends, we find that they were taken generally under unfavourable circumstances, and not to the satisfaction of the observer himself (tom. II. pp. 259—262). Two stars only (α and β Ursæ Minoris) were observed, and the results furnished by these stars differ by 1″. By rejecting the observations of β Ursæ Minoris, which Delambre thinks not improper (tom. III., page 297), the latitude would be slightly increased. At page 548, tom. III., the latitude finally adopted as most probable is 51° 2′ 8″·50, but expressly subject to a doubt of 0″·5. The agreement of this with the result with Ramsden's Zenith Sector is very remarkable. If we may judge of the accuracy of the determination resulting from the observations in 1818 by the accordance of those observations among themselves, or by the accordance of the results furnished by the 53 different stars observed, we find by comparing these different results with their mean, that the mean square of error is 1·091, and the probable error ± 0″·095.

At the same time that observations were being taken by Major-General Mudge and Major Colby, with Ramsden's Zenith Sector, for the latitude of Dunkirk, the French Astronomers MM. Biot and Arago were engaged in the same determination with the Repeating Circle. In the volume entitled "*Recueil d'Observations Géodésiques, &c.—Paris*, 1821," it is stated by M. Biot that the result of their observations agreed perfectly, when reduced to the tower of Dunkirk, with the result furnished by Ramsden's Zenith Sector, and also with Delambre's determination.* In 1809 M. Biot had made a great number of observations with his Repeating Circle for the latitude of Dunkirk, but the result he obtained was 4″ less than Delambre's latitude. This discrepancy he attributes to the constant error of his Repeating Circle. A similar discrepancy is seen in his determination of the latitude of Unst, in the Shetland Isles, when engaged in conjunction with Colonel Mudge and Captain Colby. The point at which his Repeating Circle was placed, was found trigonometrically to be 26″·94 to the north of Balta Station: now the latitude of Balta is 60° 45′ 1″·66, hence the latitude of M. Biot's Station must have been 60° 45′ 28″·60, which exceeds his determination by 3″·60 (see page 560 "*Recueil d' Observations Géodésiques, &c.*").

---

* With Ramsden's large theodolite, placed over the centre of the *Zenith Sector Station*.

† The circle is divided into two semicircles, each reading from 0° to 180°: otherwise the reading of the Flagstaff would be, on the first day 224° 46′ 6″, and on the second day 224° 45′ 58″.

§ III.

Calculation of Latitudes, Longitudes, &c.

In the treatise on the Figure of the Earth in the Encyclopædia Metropolitana, by the Astronomer Royal, he has assigned the following elements for the elliptic meridian:—

$$a = 20923713$$
$$b = 20853810$$
$$\therefore e^2 = 0{\cdot}00667054$$

These quantities very nearly represent the surface of Great Britain, but require,—the former at least,—to be slightly increased. In the present Section we shall determine small corrections, δa, δb, $\delta(e^2)$, to these elements, such as shall correspond to the surface sought after.

The first step in this operation is the calculation of the latitudes and longitudes, and directions of the meridian, at the various points at which Astronomical Observations have been made, using the above values of a and b. Let ϱ and ϱ' be the principal radii of curvature of the surface of revolution corresponding to this ellipse, and let $n = a - b : a + b$; then

$$\log(\varrho \sin 1'')^{-1} = \log\left\{\frac{648000}{\pi a^2 b^2}\left(\frac{a+b}{2}\right)^3\right\} + 3\,\mathrm{M}\left\{n\cos 2\lambda - \frac{n^2}{2}\cos 4\lambda + \frac{n^3}{3}\cos 6\lambda + \cdot\cdot\right\}$$

$$\log(\varrho' \sin 1'')^{-1} = \log\left\{\frac{648000}{\pi a^2}\left(\frac{a+b}{2}\right)\right\} + \mathrm{M}\left\{n\cos 2\lambda - \frac{n^2}{2}\cos 4\lambda + \frac{n^3}{3}\cos 6\lambda + \cdot\cdot\right\}$$

Now $\log a = 7{\cdot}3206387544$, $\log b = 7{\cdot}3191854123$, $\log(a+b) = 7{\cdot}6209426870$, and the values of the above series are consequently—

$$\log(\varrho \sin 1'')^{-1} = 7{\cdot}9945148738 + {\cdot}0021800110\cos 2\lambda - {\cdot}0000018237\cos 4\lambda + {\cdot}0000000020\cos 6\lambda$$

$$\log(\varrho' \sin 1'')^{-1} = 7{\cdot}9930603157 + {\cdot}0007266703\cos 2\lambda - {\cdot}0000006079\cos 4\lambda + {\cdot}0000000007\cos 6\lambda$$

ϱ being the radius of curvature of the meridian, and ϱ' that perpendicular to the meridian at latitude $= \lambda$.

---

* Page 516, ".... alors notre latitude obtenue par la moyenne des distances zénithales, étant réduite à la tour de "Dunkerque, s'est trouvée parfaitement d'accord avec celle que les savans anglais ont obtenue en même tems par le "grand Secteur de Ramsden ; et ce qui était sur-tout un résultat désirable, cette latitude s'est trouvée exactement la "même que M. Delambre avait autrefois obtenue."

The following table contains the above quantities for every ten minutes, from $\lambda = 49^\circ$ to $\lambda = 61^\circ$:—

λ	In the Meridian.		Perp. to Meridian.		λ	In the Meridian.		Perp. to Meridian.	
	log $\frac{1}{\rho \sin 1''}$	Diff.	log $\frac{1}{\rho' \sin 1''}$	Diff.		log $\frac{1}{\rho \sin 1''}$	Diff.	log $\frac{1}{\rho' \sin 1''}$	Diff.
° ′					° ′				
49 0	7·9942132289	125604	7·9929597676	41868	55 10	7·9937587461	118933	7·9928082733	39644
10	2006685	125496	9555808	41832	20	7468528	118677	8043089	39559
20	1881189	125388	9513976	41796	30	7349851	118415	8003530	39472
30	1755801	125275	9472180	41758	40	7231436	118147	7964058	39382
40	1630526	125155	9430422	41719	50	7113289	117878	7924676	39293
50	1505371	125035	9388703	41678	56 0	6995411	117603	7885383	39201
50 0	1380336	124910	9347025	41637	10	6877808	117322	7846182	39107
10	1255426	124778	9305388	41592	20	6760486	117042	7807075	39014
20	1130648	124646	9263796	41549	30	6643444	116754	7768061	38918
30	1006002	124508	9222247	41503	40	6526690	116462	7729143	38820
40	0881494	124363	9180744	41454	50	6410228	116169	7690323	38724
50	0757131	124218	9139290	41406	57 0	6294059	115872	7651599	38624
51 0	0632913	124067	9097884	41356	10	6178187	115568	7612975	38523
10	0508846	123910	9056528	41303	20	6062619	115264	7574452	38421
20	0384936	123752	9015225	41251	30	5947355	114953	7536031	38317
30	0261184	123587	8973974	41196	40	5832402	114636	7497714	38212
40	0137597	123418	8932778	41139	50	5717766	114320	7459502	38107
50	0014179	123247	8891639	41082	58 0	5603446	113998	7421395	37999
52 0	7·9939890932	123072	8850557	41024	10	5489448	113670	7383396	37890
10	9767860	122890	8809533	40963	20	5375778	113342	7345506	37781
20	9644970	122707	8768570	40903	30	5262436	113009	7307725	37670
30	9522263	122517	8727667	40839	40	5149427	112670	7270055	37556
40	9399746	122323	8686828	40774	50	5036757	112331	7232499	37444
50	9277423	122127	8646054	40709	59 0	4924426	111986	7195055	37329
53 0	9155296	121926	8605345	40642	10	4812440	111635	7157726	37211
10	9033370	121719	8564703	40573	20	4700805	111284	7120515	37095
20	8911651	121512	8524130	40504	30	4589521	110928	7083420	36976
30	8790139	121298	8483626	40433	40	4478593	110566	7046444	36855
40	8668841	121078	8443193	40359	50	4368027	110204	7009589	36735
50	8547763	120858	8402834	40286	60 0	4257823	109836	6972854	36612
54 0	8426905	120632	8362548	40211	10	4147987	109463	6936242	36488
10	8306273	120399	8322337	40133	20	4038524	109090	6899754	36363
20	8185874	120168	8282204	40056	30	3929434	108711	6863391	36237
30	8065706	119930	8242148	39977	40	3820723	108327	6827154	36109
40	7945776	119685	8202171	39894	50	3712396	107942	6791045	35981
50	7826091	119441	8162277	39814	61 0	3604454		6755064	
55 0	7706650	119189	8122463	39730					

The Trigonometrical Station at Chingford was ascertained by careful measurement to be 0·454 feet to the west of the meridian plane passing through the centre of the meridian mark at Chingford. This distance subtends at Greenwich an angle of 1″·62. In the Greenwich Observations for 1842, page xiii, it will be found that the azimuth at Greenwich of the (centre of the) meridian mark is 0″·02 west of north (it is added that "this result may be considered particularly good"); hence the azimuth of Chingford Station at Greenwich is 0° 0′ 1″·64 west. The angle between Chingford and Leith Hill at Greenwich is

142° 19′ 11″·423 (see first triangle, page 498); therefore the azimuth of Leith Hill at Greenwich is 142° 19′ 13″·063. Now let

α = azimuth of Leith Hill at Greenwich.
α' = azimuth of Greenwich at Leith Hill.
$\varkappa$ = colatitude of Greenwich = 90 − λ.
λ' = latitude of Leith Hill.
ω = longitude of Leith Hill.
s = distance from Greenwich to Leith Hill.

Then—

$$\tan \tfrac{1}{2}(\alpha' + \omega + \zeta) = \frac{\cos \frac{1}{2}(\varkappa - \theta)}{\cos \frac{1}{2}(\varkappa + \theta)} \cot \tfrac{1}{2}\alpha$$

$$\tan \tfrac{1}{2}(\alpha' - \omega + \zeta) = \frac{\sin \frac{1}{2}(\varkappa - \theta)}{\sin \frac{1}{2}(\varkappa + \theta)} \cot \tfrac{1}{2}\alpha$$

$$\lambda' - \lambda = \frac{s}{\varrho \sin 1''} \; \frac{\sin \frac{1}{2}(\alpha' - \alpha + \zeta)}{\sin \frac{1}{2}(\alpha' + \alpha + \zeta)} \left(1 + \frac{\theta^2}{12}\cos^2 \tfrac{1}{2}(\alpha' - \alpha)\right)$$

$$\theta = \frac{s}{\varrho' \sin 1''} \left(1 + \frac{1}{6} \cdot \frac{e^2\,\theta^2}{1 - e^2} \cos^2 \lambda \cos^2 \alpha\right)$$

$$\zeta = \frac{1}{\sin 4''} \; \frac{e^2\,\theta^2}{1 - e^2} \cos^2 \lambda \sin 2\,\alpha$$

It would be practically sufficient to take this small correction ζ from the table at page 250; it has, however, been calculated for each required case.

By these formulæ we obtain the latitude and longitude of Leith Hill and the bearing of Greenwich. This last angle is found to be 37° 23′ 27″·755; and by page 498 the angle at Leith Hill between Greenwich and Beachy Head is 100° 16′ 40″·258, hence the azimuth of Beachy Head at Leith Hill is 137° 40′ 8″·013. Now in the abstract at page 128, Beachy Hill (− 180°) reads 137° 40′ 1″·56, to which add the correction (page 387) + 0″·574, making 137° 40′ 2″·134 for the abstract reading of Beachy Head at Leith Hill. Therefore it is evident that the *reading of the north meridian*, if inserted amongst the objects observed at Leith Hill, would be 180° − (8″·013 − 2″·134) = 179° 59′ 54″·121.

Knowing now the latitude and longitude of Leith Hill, with the reading of the meridian in the abstract, we may proceed to determine the same quantities for any other point of which we know the distance and bearing from Leith Hill.

The following table contains an abstract of the calculations of latitude, longitude, &c., for those points which are more immediately required for the results in the present Section. The first column contains the name of the Station for which the latitude, &c., are given in the fifth, sixth, and seventh columns; the second column contains the name of the point from which the calculation starts; the third column contains the azimuth of the point in the first column, from the corresponding point in the second, the logarithmic distance, and the value of θ; the fourth column contains log tan $\frac{1}{2}(\alpha' + \omega + \zeta)$, log tan $\frac{1}{2}(\alpha' - \omega + \zeta)$, and the log (distance of parallels). The seventh column contains the calculated *reading of the north meridian*, found by adding the angle α' to the corresponding corrected "abstract reading," found between pages 72 and 166, or subtracting it, as the case may require.

Stations.	Obtained from	Azimuth Log (Dist.) θ	Logs $\tan\frac{1}{2}(\alpha'+\omega+\zeta)$ $\tan\frac{1}{2}(\alpha'-\omega+\zeta)$ Merid. Dist.	Latitude.	Longitude.	N. Meridian.
		° ′ ″		° ′ ″	° ′ ″	° ′ ″
LEITH TOWER.	*Greenwich* . .	142 19 13·064 5·14229329 0 22 45·184	9·53401869 9·52478814 5·04155604	51 10 32·887	0 22 11·049	179 59 54·121
BUTSER . .	*Leith Tower* .	117 7 7·639 5·19773428 0 25 51·100	9·78745911 9·77704015 4·86003794	50 58 38·233	0 58 43·780	180 0 25·036
BEACON HILL.	*Butser* . . .	65 56 50·381 5·27048029 0 30 33·963	0·18928804 0·17702159 4·87576069	51 10 59·233	1 43 15·506	180 0 3·996
DUNNOSE . .	*Butser* . . .	158 51 33·254 5·14802983 0 23 3·376	9·27196683 9·26271435 5·11801869	50 37 3·748	1 11 50·136	180 0 11·025
SOUTHAMPTON.	*Dunnose* . .	23 38 38·759 5·07098936 0 19 18·534	0·68007770 0·67238850 5·03264761	50 54 47·226	1 24 7·054	179 59 12·217
BLACK DOWN.	*Dunnose* . .	84 55 4·584 5·49740321 0 51 32·591	0·04090233 0·02037416 4·39779596	50 41 10·287	2 32 52·432	180 0 5·854
BARROW HILL.	*Black Down* .	121 39 24·140 5·49842221 0 51 39·844	9·74914607 9·72853971 5·22384314	50 13 38·527	3 41 37·149	180 0 11·463
HENSBARROW .	*Barrow Hill* .	77 22 22·286 5·42951547 0 44 5·115	0·09851101 0·08109797 4·75429230	50 22 58·810	4 49 5·137	180 0 4·224
GOONHILLY .	*Hensbarrow* .	145 29 10·961 5·17326056 0 24 26·175	9·49336734 9·48368445 5·08990294	50 2 45·352	5 10 38·794	180 0 35·725
PERTINNEY .	*Hensbarrow* .	117 16 6·356 5·33879090 0 35 46·427	9·78666084 9·77248490 5·00445727	50 6 22·081	5 38 39·961	179 59 58·702
SAINT AGNES LIGHTHOUSE	*Pertinney* . .	115 7 32·795 5·26078600 0 29 53·569	9·80450541 9·79272877 4·89308822	49 53 30·776	6 20 40·633	Non-observing Station.
LUNDY ISLAND.	*Hensbarrow* .	6 41 58·912 5·45940875 0 47 13·570	1·23475116 1·21603605 5·45638249	51 10 0·196	4 40 17·983	179 59 44·569
PRECELLY . .	*Lundy Island* .	4 37 3·563 5·45527895 0 46 46·626	1·39657885 1·37772839 5·45384262	51 56 44·755	4 46 24·516	179 59 44·144
HIGH WILHAYS	*Black Down* .	89 31 45·297 5·53015693 0 55 34·836	0·00607646 9·98390745 2·74915518	50 41 4·750	4 0 36·100	179 59 59·427
PRECELLY . .	*High Wilhays* .	20 28 43·569 5·69221059 1 20 43·153	0·74683344 0·71463244 5·66301646	51 56 44·755	4 46 24·515	179 59 44·148

Stations.	Obtained from	Azimuth Log (Dist.) θ	Logs $\tan\frac{1}{2}(\alpha'+\omega+\zeta)$ $\tan\frac{1}{2}(\alpha'-\omega+\zeta)$ Merid. Dist.	Latitude.	Longitude.	N. Meridian.
		° ′ ″		° ′ ″	° ′ ″	° ′ ″
Whitehorse Hill	*Leith Tower* .	61 27 58·754 5·49083617 0 50 46·065	0·22807935 0·20761612 5·16341190	51 34 29·856	1 33 57·038	179 59 45·908
Cradle . .	*Whitehorse Hill*	68 4 38·121 5·57766991 1 2 0·193	0·17311088 0·14789821 5·13814190	51 57 5·508	3 7 16·885	180 0 4·971
Precelly .	*Cradle* . . .	89 40 22·788 5·57129508 1 1 5·906	0·00514013 9·98008733 3·32308415	51 56 44·756	4 46 24·517	179 59 44·155
Arbury Hill.	*Whitehorse Hill*	18 36 3·875 5·39840001 0 41 2·043	0·78758297 0·77089923 5·37474244	52 13 27·287	1 12 35·080	179 59 21·592
Arbury Hill.	*Greenwich* . .	44 36 41·861 5·58674535 1 3 18·773	0·38974809 0·36405782 5·43558864	52 13 27·287	1 12 35·080	179 59 21·593
Bardon Hill.	*Arbury Hill* .	7 43 12·294 5·25640924 0 29 35·379	1·17215195 1·15994672 5·25241267	52 42 50·754	1 19 8·751	180 0 6·038
Mowcopt .	*Bardon Hill* .	53 3 26·684 5·38966198 0 40 12·858	0·30343303 0·28665886 5·16494917	53 6 52·441	2 12 41·654	179 59 58·329
Delamere .	*Mowcopt* . .	69 11 24·407 5·04460003 0 18 10·084	0·16208853 0·15444049 4·59136983	53 13 17·274	2 41 3·562	180 0 19·365
Snowdon .	*Mowcopt* . .	91 37 19·817 5·61266064 1 7 12·001	9·99053371 9·96223755 4·22870175	53 4 5·481	4 4 30·075	180 0 4·208
Precelly .	*Snowdon* . .	158 55 50·351 5·64174764 1 11 51·283	9·27246278 9·24223801 5·61250768	51 56 44·755	4 46 24·516	179 59 44·161
South Berule	*Snowdon* . .	17 49 51·425 5·61827517 1 8 4·454	0·80732492 0·77869133 5·59631465	54 8 57·597	4 40 5·328	180 0 20·787
Forth . .	*Precelly* . .	70 42 34·976 5·62711621 1 9 28·719	0·15208818 0·12360094 5·13054873	52 18 56·829	6 33 41·461	180 0 2·065
Knockanaffrin	*Forth* . . .	92 4 54·610 5·35876256 0 37 27·190	9·98583112 9·97034988 3·99590095	52 17 19·135	7 34 52·941	179 59 57·809
Baurtregaum	*Knockanaffrin* .	92 30 37·789 5·70283550 1 22 42·648	9·98453331 9·95035443 4·47589836	52 12 24·103	9 49 44·622	180 0 1·366
Feaghmaan .	*Baurtregaum* .	131 33 9·489 5·19277728 0 25 33·406	9·65423540 9·64369798 5·01623307	51 55 20·297	10 20 45·313	180 0 0·808

Stations.	Obtained from	Azimuth Log (Dist.) θ	Logs $\tan \frac{1}{2}(\alpha' + \omega + \zeta)$ $\tan \frac{1}{2}(\alpha' - \omega + \zeta)$ Merid. Dist.	Latitude.	Longitude.	N. Meridian.
		° ′ ″		° ′ ″	° ′ ″	° ′ ″
Hungry Hill	*Baurtregaum*	177 27 46·290 5·27887967 0 31 9·647	8·34661062 8·33376233 5·27845865	51 41 11·474	9 47 31·122	179 59 59·796
Kippure	*Knockanaffrin*	39 58 51·918 5·63020198 1 9 58·363	0·44217618 0·41326456 5·51140877	53 10 40·564	6 19 52·120	179 59 59·253
Kippure	*Precelly*	36 59 39·974 5·75418108 1 33 5·570	0·47960618 0·44142800 5·65302506	53 10 40·563	6 19 52·118	179 59 59·250
South Berule	*Kippure*	44 48 32·211 5·70396709 1 22 55·346	0·38831434 0·35334099 5·54983279	54 8 57·598	4 40 5·331	180 0 20·777
Keeper	*Knockanaffrin*	41 35 31·023 5·35545620 0 37 10·150	0·44206698 0·40671257 5·22747917	52 45 4·195	8 15 38·740	179 59 58·505
Cuilcagh	*Keeper*	10 18 44·638 5·73100149 1 28 15·026	1·04837577 1·01151916 5·72368006	54 12 2·877	7 48 38·560	179 59 59·068
Cuilcagh	*Kippure*	40 3 15·754 5·69215523 1 20 41·851	0·44168693 0·40765273 5·57225559	54 12 2·877	7 48 38·559	179 59 59·070
Nephin	*Keeper*	27 14 3·065 5·71622432 1 25 17·890	0·61938366 0·58376118 5·66346414	54 0 47·289	9 22 4·159	180 0 4·708
Tawnaghmore	*Nephin*	25 31 39·927 5·05643344 0 18 40·132	0·64563733 0·63760970 5·01148426	54 17 39·605	9 35 51·255	180 0 7·617
Slieve Snaght	*Cuilcagh*	15 19 23·856 5·57672864 1 1 51·591	0·87372521 0·84699842 5·56060450	55 11 47·115	7 20 0·253	180 0 0·749
Lough Foyle	*Slieve Snaght*	129 15 50·944 4·94653490 0 14 29·646	9·67648876 9·67007252 4·74910780	55 2 33·931	7 0 25·105	180 0 1·872
Clifton Beacon	*Arbury Hill*	0 15 3·786 5·65343405 1 13 49·082	2·66257966 2·63212247 5·65342977	53 27 27·022	1 13 7·668	180 0 15·343
Clifton Beacon	*Bardon Hill*	4 36 14·352 5·43504841 0 44 38·650	1·39768758 1·37906519 5·43362037	53 27 27·022	1 13 7·668	180 0 15·341
Easington	*Clifton Beacon*	11 14 15·324 5·61537072 1 7 37·142	1·00990458 0·98120227 5·60673306	54 33 53·367	0 50 24·132	180 0 0·913
York Minster	*Clifton Beacon*	9 10 53·921 5·27097897 0 30 35·813	1·09657755 1·08359316 5·26531037	53 57 43·266	1 4 49·778	180 0 2·458

Stations.	Obtained from	Azimuth Log (Dist.) θ	Logs $\tan\frac{1}{2}(\alpha'+\omega+\zeta)$ $\tan\frac{1}{2}(\alpha'-\omega+\zeta)$ Merid. Dist.	Latitude.	Longitude.	N. Meridian.
		° ′ ″		° ′ ″	° ′ ″	° ′ ″
Wordeslow	*Easington*	49 43 3·868 5·20752595 0 26 26·166	0·33519312 0·32367200 5·01599566	54 50 56·160	1 25 25·842	179 59 54·696
Cheviot	*Easington*	38 49 3·612 5·63494110 1 10 43·887	0·45590655 0·42507180 5·52329532	55 28 42·372	2 8 38·127	179 59 50·391
Sca Fell	*Cheviot*	148 38 43·843 5·63951494 1 11 28·603	9·45098448 9·41910561 5·57297050	54 27 14·774	3 12 36·725	180 0 7·217
Sca Fell	*South Berule*	69 44 26·592 5·51936549 0 54 12·356	0·15908159 0·13569226 5·04645578	54 27 14·774	3 12 36·726	180 0 7·219
Hart Fell	*Cheviot*	95 6 45·483 5·41838451 0 42 57·421	9·96288159 9·94372722 4·41008644	55 24 28·961	3 24 0·212	180 0 13·769
Goat Fell	*Hart Fell*	77 9 55·436 5·57919191 1 2 12·454	0·10055575 0·07286296 4·90054796	55 37 32·899	5 11 26·512	180 0 8·555
South Berule	*Goat Fell*	168 15 16·732 5·74060133 1 30 12·510	9·01576312 8·97536941 5·73174915	54 8 57·597	4 40 5·329	180 0 20·799
Divis	*South Berule*	59 3 51·727 5·52296532 0 54 39·427	0·24902258 0·22543849 5·22700631	54 36 40·327	6 1 2·656	180 0 1·284
Divis	*Slieve Snaght*	127 7 8·170 5·54382112 0 57 20·557	9·69887703 9·67348694 5·32984340	54 36 40·326	6 1 2·651	180 0 1·273
Sayrs Law	*Cheviot*	38 37 10·275 5·23766058 0 28 20·049	0·45656512 0·44393200 5·12916926	55 50 49·439	2 40 8·139	180 0 0·304
Jura	*Slieve Snaght*	46 20 22·842 5·57669069 1 1 51·063	0·37102799 0·34364060 5·41140732	55 54 8·900	6 0 11·227	180 0 4·625
Ben Nevis	*Jura*	31 25 56·881 5·58509189 1 3 3·404	0·55312957 0·52470152 5·51424025	56 47 49·339	5 0 8·432	180 0 14·782
Ben Wyvis	*Ben Nevis*	14 26 26·343 5·52252507 0 54 35·631	0·89931464 0·87411937 5·50823491	57 40 45·100	4 34 40·727	179 59 53·164
Ben Heynish	*Jura*	42 27 32·651 5·44006639 0 45 9·301	0·41239458 0·39204021 5·30532732	56 27 19·638	6 55 20·981	180 0 1·367
Ben More, S. U.	*Ben Heynish*	14 3 42·661 5·48115201 0 49 38·036	0·91081238 0·88811354 5·46764395	57 15 32·205	7 17 38·743	180 0 7·077

Stations.	Obtained from	Azimuth Log (Dist.) θ	Logs $\tan\frac{1}{2}(\alpha'+\omega+\xi)$ $\tan\frac{1}{2}(\alpha'-\omega+\xi)$ Merid. Dist.	Latitude.	Longitude.	N. Meridian.
		° ′ ″		° ′ ″	° ′ ″	° ′ ″
SCOURNALAPICH	*Ben More, S.U.*	83 50 8·089 5·64708655 1 12 43·608	0·04951495 0·01552482 4·60641681	57 22 10·334	5 3 31·679	180 0 12·240
BEN LOMOND .	*Goat Fell* . .	28 51 0·919 5·37281477 0 38 40·663	0·59120005 0·57388944 5·31429474	56 11 25·260	4 37 54·183	179 59 58·802
EAST LOMOND .	*Ben Lomond* .	85 37 51·056 5·45939307 0 47 12·547	0·03496231 0·01352225 4·27875000	56 14 32·515	3 13 11·223	179 59 53·951
CALTON HILL .	*E.L omond* .	175 47 35·218 5·02218968 0 17 15·077	8·56566434 8·55782019 5·02102669	55 57 18·042	3 10 55·592	179 59 55·472
KELLIE LAW .	*E. Lomond* .	88 26 39·915 4·95283913 0 14 42·312	0·01235621 0·00566980 3·33211581	56 14 53·689	2 46 43·752	179 59 47·025
BEN CLEUGH .	*Ben Lomond* .	90 11 17·763 5·24547749 0 28 50·858	9·99968054 9·98658077 3·22550101	56 11 8·695	3 46 3·873	179 59 59·890
BEN MACDUI .	*Ben Cleugh* .	3 31 26·584 5·51034299 0 53 5·130	1·51401200 1·48990500 5·50950153	57 4 14·043	3 40 3·749	179 59 51·212
SCOURNALAPICH	*Ben Macdui* .	67 48 14·203 5·47207618 0 48 36·350	0·17443125 0·15183542 5·03831537	57 22 10·334	5 3 31·683	180 0 12·277
BEN MACDUI .	*Hart Fell* . .	5 0 42·769 5·78512472 1 39 56·908	1·36280698 1·31829316 5·78338734	57 4 14·043	3 40 3·749	179 59 51·213
BEN MACDUI .	*Sayrs Law* .	23 54 23·513 5·69059228 1 20 23·747	0·67739068 0·64119055 5·65023965	57 4 14·043	3 40 3·749	179 59 51·216
DUDWICK .	*Ben Macdui* .	67 9 45·141 5·54283857 0 57 12·433	0·18001544 0·15341903 5·11927532	57 25 50·886	2 2 6·952	179 59 49·199
MOUNT BATTOCK	*Ben Macdui* .	103 4 48·011 5·27887092 0 31 9·108	9·90114584 9·88666566 4·64608625	56 56 57·814	2 44 25·376	179 59 45·302
CLEISHAM .	*Ben More, S.U.*	19 57 49·067 5·43844003 0 44 58·977	0·75615735 0·73513944 5·41096550	57 57 50·561	6 48 41·648	179 59 58·856
MONACH . .	*Cleisham* . .	33 52 2·538 5·23842308 0 28 22·809	0·51749114 0·50397229 5·15658744	58 21 23·543	6 18 32·931	179 59 55·306
BEN CLIBRIG .	*Monach* . .	96 5 32·526 5·56589633 1 0 19·348	9·95589298 9·92683237 4·64564665	58 14 7·844	4 24 35·534	179 59 53·071

Stations.	Obtained from	Azimuth Log (Dist.) θ	Logs $\tan\frac{1}{2}(\alpha'+\omega+\xi)$ $\tan\frac{1}{2}(\alpha'-\omega+\xi)$ Merid. Dist.	Latitude.	Longitude.	N. Meridian.
		° ' "		° ' "	° ' "	° ' "
Ben Macdui .	*Ben Clibrig* .	160 49 47·206 5·65297290 1 13 42·927	9·23020235 9·19480537 5·62902369	57 4 14·043	3 40 3·744	179 59 51·130
Corryhabbie .	*Ben Nevis* .	60 8 58·903 5·61442657 1 7 27·589	0·23979234 0·20865545 5·30129905	57 20 41·415	3 11 41·826	179 59 58·971
Mount Battock	*Corryhabbie* .	147 48 38·209 5·23133844 0 27 55·312	9·46123158 9·44815616 5·15976935	56 56 57·813	2 44 25·371	179 59 45·252
Mormonth .	*Corryhabbie* .	67 8 18·786 5·39385588 0 40 35·640	0·17957909 0·16056812 4·97434578	57 36 10·258	2 1 52·860	179 59 41·208
Cowhythe .	*Corryhabbie* .	40 11 25·557 5·21084748 0 26 38·103	0·43767866 0·42520591 5·09243744	57 41 0·492	2 39 32·726	179 59 55·574
Scarabin .	*Corryhabbie* .	13 22 50·273 5·51742292 0 53 57·279	0·93269266 0·90742187 5·50516967	58 13 14·567	3 35 24·240	179 59 57·292
Ben Hutig .	*Ben Wyvis* .	2 16 20·291 5·50373052 0 52 16·751	1·70454330 1·67983218 5·50338045	58 33 5·124	4 30 42·372	179 59 54·943
North Rona .	*Ben Hutig* .	49 15 30·675 5·50903727 0 52 55·169	0·34058686 0·31495285 5·31879034	59 7 17·674	5 48 49·941	No Observations.
North Rona .	*Cleisham* . .	23 49 58·075 5·66644395 1 16 2·333	0·67840956 0·64217299 5·62631191	59 7 17·673	5 48 49·942	No Observations.
Fitty Hill .	*Ben Hutig* .	46 5 2·207 5·59340827 1 4 16·008	0·37355004 0·34241572 5·42940309	59 17 13·020	3 0 3·669	179 59 55·970
Fitty Hill .	*Ben Clibrig* .	34 14 35·080 5·67036490 1 16 43·644	0·51413438 0·47728964 5·58457749	59 17 13·020	3 0 3·669	179 59 55·972
Foula . .	*Fitty Hill* . .	27 49 44·258 5·54914673 0 58 2·249	0·60801187 0·57929165 5·49420109	60 8 26·609	2 5 38·668	179 59 55·435
Yell . .	*Foula* . . .	50 25 16·360 5·37153330 0 38 33·276	0·32847110 0·30890232 5·17173825	60 32 49·262	1 5 12·594	179 59 56·139

The following table contains, in continuation, those calculations which, on account of short distances, have been made by the shorter method, page 253. The fourth column contains the azimuth, and under it the quantity log ($\lambda_1 - \lambda$) ; the fifth column contains log ω, and under it log ν.

Stations.	Obtained from	α log $(\lambda_{,} - \lambda)$	log ω log ν	Latitude.	Longitude.	N. Meridian.
		° ′ ″		° ′ ″	° ′ ″	° ′ ″
WROTHAM	*Leith*	70 54 21·317 2·7096411	3·3733176 3·2653261	51 18 58·708	−0 17 11·156	180 0 5·566
FAIRLIGHT	*Wrotham*	154 17 36·955 3·1987924	3·0801327 2·9712307	50 52 36·494	−0 37 13·787	179 59 53·495
GADS HILL	*Wrotham*	48 51 58·617 2·5463514	2·8090361 2·7017654	51 24 50·061	−0 27 55·379	179 59 57·669
SOUTHAMPTON	*Dunnose*	23 38 38·759 3·0269921	2·8674192 2·7564774	50 54 47·226	1 24 7·054	179 59 12·217
WEEK DOWN	*Dunnose*	228 50 23·916 1·8659956	2·1205966 2·0086731	50 35 50·277	1 14 2·143	180 1 45·759
BONIFACE	*Dunnose*	183 40 2·5865 1·7333557	0·7364515 0·6245440	50 36 9·628	1 11 55·587	180 0 15·237
H. P. CLIFF	*Dunnose*	171 8 42·536 1·9062282	1·2949586 1·1830290	50 35 43·168	1 11 30·414	Nil.
DUNKERY	*High Wilhays*	29 9 24·059 3·2360686	3·1840991 3·0741358	51 9 44·119	3 35 8·095	180 0 0·502
DANBURY	*Gads Hill*	12 46 55·418 3·0366381	2·5991922 2·4931295	51 42 57·897	−0 34 32·746	179 59 35·378
THAXTED	*Danbury*	31 6 29·961 2·9327734	2·9224987 2·8180515	51 57 13·663	−0 20 36·183	179 59 55·604
BALSHAM	*Thaxted*	4 36 20·221 2·8090972	1·9260432 1·8228320	52 7 57·968	−0 19 11·841	179 59 53·391
ELY	*Balsham*	7 18 5·535 2·9805123	2·3016187 2·1997171	52 23 54·041	−0 15 51·570	180 0 14·310
WALPOLE, ST. PETER'S	*Ely*	4 12 10·398 3·0747678	2·1577078 2·0575441	52 43 41·884	−0 13 27·787	179 59 49·457
WALTON	*Danbury*	71 15 24·207 2·7324202	3·4100293 3·3053156	51 51 50·130	−1 17 23·315	179 59 57·616
ORFORD CASTLE	*Walton*	33 1 0·660 2·9181246	2·9413812 2·8377890	52 5 37·411	−1 31 57·053	180 1 59·342
NAUGHTON	*Danbury*	31 3 48·749 3·1426472	3·1330371 3·0290306	52 6 4·547	−0 57 11·175	180 0 2·584
SOUTH LOPHAM	*Naughton*	5 20 15·618 3·0248051	2·2087172 2·1067134	52 23 43·295	−0 59 52·878	179 59 59·587
CAMBRIDGE	*Balsham*	300 46 5·775 2·4677743	2·9046345 2·8021908	52 12 50·821	−0 5 48·991	Nil.
BURLEIGH MOOR	*Easington*	273 9 29·525 1·3610169	2·8550821 2·7661350	54 34 15·742	1 2 20·411	179 59 50·197
BRANDON	*Wordeslow*	237 20 45·471 2·5283780	2·9594180 2·8717277	54 45 17·629	1 40 36·632	179 59 56·006
DURHAM OBSERVATORY	*Brandon*	76 1 37·042 1·6794005	2·5214408 2·4335340	54 46 5·300	1 35 4·400	Nil.

Stations.	Obtained from	α $\log(\lambda_{,} - \lambda)$	$\log \omega$ $\log \nu$	Latitude.	Longitude.	N. Meridian.
		° ′ ″		° ′ ″	° ′ ″	° ′ ″
Howth . .	*Kippure* . .	38 48 6·692 2·8473306	2·9758969 2·8798123	53 22 23·130	6 4 6·105	180 0 1·084
Kingstown Observatory*	*Howth* . . .	204 56 13·586 2·4631639	2·3530473 2·2572850	53 17 32·559	6 7 51·553	Nil.
Dunrich .	*Hart Fell* . .	36 42 45·700 2·7720841	2·8913927 2·8073356	55 34 19·950	3 11 1·472	179 59 56·249
Little Stirling .	*Dudwick* . .	76 42 49·587 2·0131167	2·9084092 2·8341728	57 27 33·231	1 48 37·094	179 59 49·333
Great Stirling .	*Little Stirling* .	68 44 38·874 1·2284260	1·9069917 1·8328352	57 27 50·143	1 47 16·371	179 59 11·992
Ronas . .	*Yell* . . .	266 3 4·746 1·6405025	3·1089032 3·0487738	60 32 3·843	1 26 37·594	179 59 54·228
Balta . .	*Yell* . . .	35 46 37·300 2·8687152	3·0367695 2·9771064	60 45 7·156	0 47 4·242	179 59 57·859
Saxavord .	*Yell* . . .	23 14 54·114 3·0058461	2·9502972 2·8907972	60 49 41·992	0 50 20·733	179 59 57·673
Gerth of Scaw	*Saxavord* . .	108 5 41·450 1·6032376	2·4002641 2·3413365	60 49 1·818	0 46 9·391	179 57 37·295
Gerth of Scaw	*Balta* . . .	6 30 44·798 2·3704483	1·7392048 1·6801149	60 49 1·818	0 46 9·389	179 57 37·293
Docking .	*Walpole, St. Peter's* .	54 24 27·781 2·7948356	3·1585636 3·0598500	52 54 2·956	—0 37 28·456	179 59 52·426
Mordington .	*Sayrs Law* .	96 22 25·573 2·1326731	3·3339717 3·2516624	55 48 28·453	2 4 10·535	180 0 7·596
Burnswark .	*Hart Fell* . .	167 18 14·733 3·0517190	2·6459455 2·5606406	55 5 42·270	3 16 37·679	180 0 2·176

Let us now compare the latitude of Saxavord arrived at by these calculations with the distance of the parallels of Dunnose and Saxavord calculated at page 508. For this purpose it will suffice to calculate the length of the meridian arc between the latitude 50° 37′ 3″·748 (the latitude of Dunnose, see page 677,) and the latitude 60° 49′ 41″·992.

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* Kingstown Observatory is fixed from Howth and Poolbeg Lighthouse. All the angles of the triangle Kingstown-Howth-Poolbeg are observed, and Poolbeg is observed from three points,—Lyons Hill, Dublin Observatory, and Howth. This connection gives two equations of condition, whence we obtain the following corrections—

Lyons to Poolbeg	+ 0″·4314	Poolbeg to Howth	— 0″·6694
Dublin to Poolbeg	— 0·0215	Poolbeg to Kingstown	+ 0·5454
Howth to Kingstown	— 5·2898	Kingstown to Poolbeg	— 3·0158
Howth to Poolbeg	+ 0·2668	Kingstown to Howth	+ 0·2803

to be applied to the observed bearings.

From these corrections it is easily found that the angle at Howth between Dublin and Kingstown is 70° 8′ 37″·048; and that the log. distance from Howth to Kingstown Observatory is 4·5117759.

By page 267 we have this distance S—

$$S = \frac{8\,a^2\,b^2}{(a+b)^3}\left(m\,\phi - m_1 \sin\phi \cos 2\,\lambda + m_2 \sin 2\,\phi \cos 4\,\lambda - m_3 \sin 3\,\phi \cos 6\,\lambda\right)$$

$$m = 1 + \frac{9}{4}\,n^2 \qquad m_1 = 3n + \frac{45}{8}\,n^3$$

$$m_2 = \frac{15}{8}\,n^2 \qquad m_3 = \frac{35}{24}\,n^3$$

In this calculation we must use the same values of a and b as in the calculations of latitudes and longitudes; namely, $a = 20923713$, $b = 20853810$. These give $m = 1\cdot0000062993$; $m_1 = 00501969$; $m_2 = \cdot0000052494$; $m_3 = \cdot0000000068$; also $\phi = 10^\circ\ 12'\ 38''\cdot244$; $2\,\lambda = 111^\circ\ 26'\ 45''\cdot74$; $\log 8\,a^2\,b^2\,(a+b)^{-3} = 7\cdot31991025$.

First Term +		*Second Term* +		*Third Term* −		*Fourth Term* −	
	7·31991026		7·3199103		7·31991		7·3199
m ·	·00000274	m_1 ·	$\bar{3}\cdot7006768$	m_2 ·	$\bar{6}\cdot72011$	m_3 ·	$\bar{9}\cdot8325$
ϕ'' ·	4·56535475	sin ϕ ·	$\bar{1}\cdot2486285$	sin 2 ϕ ·	$\bar{1}\cdot54272$	sin 3 ϕ ·	$\bar{1}\cdot7072$
sin 1″ ·	$\bar{6}\cdot68557487$	cos 2 λ ·	$\bar{1}\cdot5630357$	cos 4 λ ·	$\bar{1}\cdot86489$	cos 6 λ ·	$\bar{1}\cdot9549$
	6·57084262		3·8322513		1·44763		$\bar{2}\cdot8145$

First	3722567·82
Second	+ 6795·97
Third	− 28·03
Fourth	− ·07
Distance of Parallels	3729335·69

which exceeds the quantity calculated at page 508, namely 3729334·07, by 1·62 feet, or less than 20 inches. This difference must be admitted to be immaterial, when we consider the multitude of figures through which the present distance is obtained.

We may apply the same test to the calculated latitudes of Hensbarrow and Ben Hutig and their meridional distance calculated at page 506. The latitudes are $50^\circ\ 22'\ 58''\cdot810$ and $58^\circ\ 33'\ 5''\cdot124$, hence $\phi = 8^\circ\ 10'\ 6''\cdot314$ and $2\,\lambda = 108^\circ\ 56'\ 3''\cdot934$: the calculation is as follows:—

First Term +		*Second Term* +		*Third Term* −		*Fourth Term* −	
	7·31991026		7·3199103		7·31991		7·3199
m ·	0·00000274	m_1 ·	$\bar{3}\cdot7006768$	m_2 ·	$\bar{6}\cdot72011$	m_3 ·	$\bar{9}\cdot8325$
ϕ'' ·	4·46844059	sin ϕ ·	$\bar{1}\cdot1525433$	sin 2 ϕ ·	$\bar{1}\cdot44914$	sin 3 ϕ ·	$\bar{1}\cdot6178$
sin 1″ ·	$\bar{6}\ 68557487$	cos 2 λ ·	$\bar{1}\cdot5111957$	cos 4 λ ·	$\bar{1}\cdot89731$	cos 6 λ ·	$\bar{1}\cdot9226$
	6·47392846		3·6843261		1·38647		$\bar{2}\cdot6928$

First	2978025·78
Second	+ 4834·22
Third	− 24·35
Fourth	− ·05
Distance of Parallels	2982835·60

which differs only ·09 of a foot from the quantity given at page 506.

§ IV.

Equations of Condition.

Latitudes.—Besides the thirty-two points whose latitudes have been determined with Ramsden's or Airy's Zenith Sector in the course of the Ordnance Survey, we may also take account in our equations of condition, without increasing them to an unmanageable number, of the observed latitudes of the Observatories at Cambridge, Durham, and Edinburgh.

The latitude of Cambridge Observatory, kindly supplied by Professor Challis to Captain Yolland in 1851, is 52° 12′ 51″·63. A first determination was made by the present Astronomer Royal in 1833, an account of which is given in the volume of the Cambridge Observations of that year, and in the Transactions of the Cambridge Philosophical Society, vol. v. An account of a subsequent determination by Professor Challis, is given in the volume of the Cambridge Observations for 1838, page lviii. The (weighted) mean of the two determinations is the latitude above stated.

The latitude of the Durham Observatory, kindly supplied by Professor Temple Chevallier, is 54° 46′ 6″·20.

The latitude of the Royal Observatory, Edinburgh, is 55° 57′ 23″·20; kindly supplied by Professor Piazzi Smyth. Hence the latitude of Calton Hill Trigonometrical Station (which is 6·08 feet more to the south than the Centre of the Dome or of the Altitude and Azimuth Circle) is 55° 57′ 23″·14.

The following table contains the comparison of the astronomical latitudes with those calculated with Airy's elements:—

Stations.	Observed Latitude.			Calculated Latitude.			Diff.
	°	′	″	°	′	″	″
Greenwich	51	28	38·30		...		...
Saint Agnes	49	53	33·93	49	53	30·776	− 3·154
Goonhilly	50	2	50·07	50	2	45·352	− 4·718
Hensbarrow	50	23	1·84	50	22	58·810	− 3·030
Black Down	50	41	8·89	50	41	10·287	+ 1·397
Dunnose	50	37	7·08	50	37	3·748	− 3·332
Boniface Down	50	36	10·55	50	36	9·628	− 0·922
Week Down	50	35	51·42	50	35	50·277	− 1·143
High Port Cliff	50	35	43·28	50	35	43·168	− 0·112
Southampton	50	54	46·97	50	54	47·226	+ 0·256
Precelly	51	56	45·18	51	56	44·755	− 0·425
Arbury Hill	52	13	26·59	52	13	27·287	+ 0·697
Delamere	53	13	18·61	53	13	17·274	− 1·336
Clifton Beacon	53	27	30·40	53	27	27·022	− 3·378
Burleigh Moor	54	34	19·67	54	34	15·742	− 3·928
Hungry Hill	51	41	10·26	51	41	11·474	+ 1·214
Feaghmaan	51	55	22·85	51	55	20·297	− 2·553
Forth	52	18	57·91	52	18	56·829	− 1·081
Tawnaghmore	54	17	41·34	54	17	39·605	− 1·735
Lough Foyle	55	2	38·74	55	2	33·931	− 4·809
South Berule	54	8	56·40	54	8	57·597	+ 1·197
Ben Lomond	56	11	26·27	56	11	25·260	− 1·010

Stations.	Observed Latitude.			Calculated Latitude.			Diff.
	°	′	″	°	′	″	″
Kellie Law	56	14	51·52	56	14	53·689	+ 2·169
Ben Heynish	56	27	16·88	56	27	19·638	+ 2·758
Great Sterling . . .	57	27	49·12	57	27	50·143	+ 1·023
Cowhythe	57	41	8·96	57	41	0·491	− 8·469
Monach	58	21	20·84	58	21	23·543	+ 2·703
Ben Hutig	58	33	6·47	58	33	5·124	− 1·346
North Rona	59	7	15·19	59	7	17·673	+ 2·483
Balta	60	45	1·66	60	45	7·156	+ 5·496
Gerth of Scaw . . .	60	48	56·43	60	49	1·818	+ 5·388
Saxavord	60	49	38·58	60	49	41·992	+ 3·412
Calton Hill	55	57	23·14	55	57	18·042	− 5·098
Durham	54	46	6·20	54	46	5·300	− 0·900
Cambridge	52	12	51·63	52	12	50·821	− 0·809

A careful examination of the differences in the last column of this table will make it clear that the elements used will require modification, as, with some exceptions, we have negative signs in the southern, and positive signs in the northern Station. The required modification will increase the discrepancy at Cowhythe by more than one second; we shall therefore omit this Station in the Equations of Condition.

Longitudes.—The Astronomical Observations taken at different Stations of the Triangulation during the progress of the Ordnance Survey, do not include any determination of longitude. The longitude of Feaghmaan has, however, been determined by the Astronomer Royal, assisted by Captain Gosset, R.E., and a party of Sappers from the Ordnance Survey. Of these operations an account is given in the volume of the Greenwich Observations for 1845. The arc of longitude from Greenwich to Valentia was subdivided at Kingstown near Dublin, where a temporary observatory was erected. The transit instruments used at Kingstown and Valentia were the property of the late Mr. Sheepshanks, who took an active part in the undertaking; the length of the former was 42 inches, that of the latter 30 inches. For comparison of the transit clocks at the different Stations, thirty pocket chronometers were used, carefully packed in two cases. From Greenwich to Liverpool the chronometers were transmitted by railway, thence by steamer to Kingstown, thence by railway to Dublin, and thence by mail coach to Limerick. From Limerick to Tralee the chronometers were transmitted by mail coach, under charge of a Sapper, and thence under another Sapper to Cahirsaveen Ferry. The chronometers left Greenwich the first time on the morning of June 27, 1844, and reached Kingstown the last time on the morning of July 27, having made nine journeys the one way and eight the other: they then left Kingstown for Liverpool Observatory on the evening of July 27, and were returned to Kingstown the last time on the morning of August 4th, having made four journeys each way: they then left Kingstown for Feaghmaan on the evening of August 5, and were returned to Kingstown the last time on the morning of September 14, having made ten journeys each way.

For all the details respecting the reduction of the observations, determinations of clock errors, corrections for personal equations, &c., the reader is referred to the Greenwich Observations for 1845, where every point is very fully treated. The final results, page ccxxiii, are:—

	m	s
Liverpool Observatory, West of Greenwich	12	0·05
Kingstown „ „ „	24	31·20
Feaghmaan „ „ „	41	23·23

The longitude of Cambridge Observatory, as finally determined by Professor Challis, from Observations of Galvanic Signals, is 22^{s}·689 East of Greenwich. This determination will be found in the Monthly Notices of the Royal Astronomical Society, vol. xiv, May 1854. The previously adopted longitude was 23^{s}·54, determined by transits of chronometers between Greenwich and Cambridge in 1828, of which an account is given in vol. iii, of the Transactions of the Cambridge Philosophical Society. The dome being 45·4 feet east of the transit instrument, its eastern longitude is greater by $0''$·729 or 0^{s}·049. The longitude of the dome is therefore 22^{s}·738.

The longitude of the Durham Observatory is 6^{m} 19^{s}·75. The dome being 12·5 feet west of the transit instrument, its longitude is greater by $0''$·213 or 0^{s}·014; hence the longitude of the dome is 6^{m} 19^{s}·764.

The longitude of the Royal Observatory, Edinburgh, is 12^{m} 43^{s}·00; that of the Trigonometrical Station is greater by $0''$·056 or 0^{s}·004.

Stations.	Obsd. Longitude.	Calc. Longitude.	Diff.	D × cos λ.
	° ′ ″	° ′ ″		
Calton Hill	3 10 45·06	3 10 55·592	− 10·532	− 5·896
Durham	1 34 56·46	1 35 4·400	− 7·940	− 4·580
Cambridge	− 0 5 41·06	− 0 5 48·991	+ 7·931	+ 4·859
Kingstown	6 7 48·00	6 7 51·553	− 3·553	− 2·123
Feaghmaan	10 20 48·45	10 20 45·313	+ 3·137	+ 1·935

Azimuths.—Pages 167 to 196 contain observations for the determination of absolute azimuth at thirty Stations: page 197 contains an abstract of the results of similar observations at thirty-one additional Stations, which being less accurate or less important from their position in the triangulation, it was not considered necessary to print in full. The calculated probable error is given, which will show the amount of dependence that may be placed on each. Observations for azimuth have been also taken at a few other Stations, but have been cancelled, having been taken either under very unfavorable circumstances, or by men under instruction, or merely for practice. The sixty-one results given in this volume are more than sufficient for the determination of the absolute azimuth of the triangulation as a whole.

It would swell the number of equations of condition to an unnecessary amount were all these 61 results included; it has been considered sufficient to take the first thirty (given in full) together with the azimuths at Docking, Orford Castle, Walpole St. Peters, Sayrs Law, and Scournalapich.

For the comparison of the observed and calculated azimuths, let P be a Station at which these quantities have to be compared.

A = Abstract Reading of Q at P (pages 72—166)
R = „ „ Referring Object at P
R' = Observed azimuth of „ „ „ (pages 166—197)
C = Calculated correction to A (pages 372—413)
$180^\circ - E$ = Calculated reading of North Meridian at P

$$\text{At P}\begin{cases}\text{Calculated azimuth of Q} = A + C + E\\ \text{Observed azimuth of Q} = A + C + R' - R\end{cases}$$

$$\therefore \text{(Calc. — obsd.) azimuth} = E + R - R'$$

In the following table the calculated and observed azimuths at thirty-five Stations are contrasted. The sixth column contains the required difference.

Stations.	Abstract Reading of Referring Object.			Observed Azimuth of Referring Object.			$R - R'$	E	$R - R' + E = S$	$S \times \cot\lambda$.
	°	′	″	°	′	″	″	″	″	″
Blackdown	316	31	58·96	316	32	1·55	− 2·59	− 5·854	− 8·444	− 6·915
Dunnose	339	12	19·99	339	12	12·59	+ 7·40	− 11·025	− 3·625	− 2·976
Precelly	211	58	30·09	211	58	46·38	− 16·29	+ 15·851	− 0·439	− 0·344
Cowhythe	66	27	31·99	66	27	36·91	− 4·92	+ 4·426	− 0·494	− 0·312
Wrotham	274	56	45·11	274	56	37·50	+ 7·61	− 5·566	+ 2·044	+ 1·637
Gads Hill	307	11	50·07	307	11	48·95	+ 1·12	+ 2·331	+ 3·451	+ 2·754
Fairlight	276	5	8·93	276	5	11·57	− 2·64	+ 6·505	+ 3·865	+ 3·144
Orford Castle	68	51	29·77	68	49	23·65	+126·12	−119·342	+ 6·778	+ 5·278
Balsham	150	59	14·94	150	59	12·77	+ 2·17	+ 6·609	+ 8·779	+ 6·826
Walpole, St. Peter's	194	47	29·96	194	47	31·91	− 1·95	+ 10·543	+ 8·593	+ 6·539
Docking	40	47	20·10	40	47	21·26	− 1·16	+ 7·574	+ 6·414	+ 4·851
South Lopham	190	11	6·82	190	11	2·68	+ 4·14	+ 0·413	+ 4·553	+ 3·507
Leith Hill	221	32	45·05	221	32	53·17	− 8·12	+ 5·879	− 2·241	− 1·803
Butser	108	11	14·96	108	10	54·92	+ 20·04	− 25·036	− 4·996	− 4·049
Beacon Hill	212	52	57·12	212	52	55·08	+ 2·04	− 3·996	− 1·956	− 1·574
Barrow Hill	352	32	29·98	352	32	27·79	+ 2·19	− 11·463	− 9·273	− 7·718
Lundy Island	194	25	50·99	194	26	4·99	− 14·00	+ 15·431	+ 1·431	+ 1·152
Dunkery	84	41	9·98	84	41	14·46	− 4·48	− 0·502	− 4·982	− 4·011
Mowcopt	280	37	6·87	280	37	2·55	+ 4·32	+ 1·671	+ 5·991	+ 4·496
York Minster	260	26	25·03	260	26	26·63	− 1·60	− 2·458	− 4·058	− 2·952
Easington	128	38	55·03	128	38	60·49	− 5·46	− 0·913	− 6·373	− 4·535
Wordeslow	57	51	4·98	57	51	18·05	− 13·07	+ 5·304	− 7·766	− 5·468
Hart Fell	350	20	29·89	350	20	11·11	+ 18·78	− 13·769	+ 5·011	+ 3·456
Cheviot	246	9	15·05	246	9	30·63	− 15·58	+ 9·609	− 5·971	− 4·107
Dunrich	242	28	42·12	242	28	56·85	− 14·73	+ 3·751	−10·979	− 7·525
Goat Fell	233	2	2·31	233	1	57·98	+ 4·33	− 8·555	− 4·225	− 2·890
Sayrs Law	113	50	20·00	113	50	21·61	− 1·61	− 0·304	− 1·914	− 1·298
Mordington	148	27	44·92	148	27	40·51	+ 4·41	− 7·596	− 3·186	− 2·165
Burnswark	204	52	12·04	204	52	7·20	+ 4·84	− 2·176	+ 2·664	+ 1·859
Ben More, S. Uist	305	41	44·45	305	41	41·70	+ 2·75	− 7·077	− 4.327	− 2·782
Ben Macdui	161	48	21·98	161	48	29·96	− 7·98	+ 8·786	+ 0·806	+ 0·522
Ben Nevis	81	52	46·07	81	52	22·51	+ 23·56	− 14·782	+ 8·778	+ 5·745
Scournalapich	351	58	39·99	351	58	35·14	+ 4·85	− 12·240	− 7·390	− 4·732
Mount Battock	238	28	21·50	238	28	44·26	− 22·76	+ 14·698	− 8·062	− 5·246
Mormonth	84	55	17·02	84	55	47·19	− 30·17	+ 18·792	−11·378	− 7·220

Equations of Condition.—If the sides a, b, and the included angle C of a spherical triangle receive increments da, db, and dC, the corresponding variations of the other parts are given by the following equations:—

$$\begin{aligned}
\text{Sin}\, c\, dA &= \sin B\, da - \cos c \sin A\, db - \sin a \cos B\, dC \\
\text{Sin}\, c\, dB &= \sin A\, db - \cos c \sin B\, da - \sin b \cos A\, dC \\
dc &= \cos B\, da + \cos A\, db + \sin b \sin A\, dC
\end{aligned}$$

These are without much trouble transformed into the equations (18), page 620. If in the latter we put—

$$100\,\frac{v}{\varrho} = \mu\ ,\ \mu' = \frac{\varrho}{v}\,\frac{100}{(1-e^2)^2}\cos^2\tfrac{1}{4}(\lambda + 3\lambda')$$

and also for γ and ε, write u(arc 100″) and v(arc 100″) respectively, then—

$$\begin{aligned}
Q\,\varepsilon &= \mu'(\lambda' - \lambda)\sin 1''\cdot v \\
-\delta\theta &= 100\,\theta \sin 1''\cdot u + 100\,g\,\theta \sin 1''\cdot v
\end{aligned}$$

Where g is a constant quantity, namely the value of $\frac{\frac{1}{2}\sin 2\lambda}{1 - e^2 \sin^2\lambda}$ for the latitude of Greenwich; $= 0\cdot307299 = \log^{-1}\bar{1}\cdot4875618$. Divide each of the three equations by $\sin 1''$, and the second and third by $-\sec\lambda'$ and $\tan\lambda'$ respectively; they will then assume the form—

$$\begin{aligned}
\xi^1 &= k_1 + a_1\,\xi + b_1\,\eta + c_1\,u + e_1\,v \\
\eta^1 &= k_2 + a_2\,\xi + b_2\,\eta + c_2\,u + e_2\,v \\
\eta^1 &= k_3 + a_3\,\xi + b_3\,\eta + c_3\,u + e_3\,v
\end{aligned}$$

The values of the absolute terms are—

$$\begin{aligned}
k_1 &= (\text{calculated} - \text{observed})\ \text{latitude} \\
k_2 &= (\text{observed} - \text{calculated})\ \text{longitude} \times \cos\lambda' \\
k_3 &= (\text{calculated} - \text{observed})\ \text{azimuth} \times \cot\lambda'
\end{aligned}$$

The values of the coefficients are—

$$\begin{array}{ll}
a_1 = \cos\omega & b_1 = \sin\lambda \sin\omega \\
c_1 = \mu\,\theta\cos\alpha' & e_1 = \mu\,g\,\theta\cos\alpha' + \mu'(\lambda' - \lambda) \\
a_2 = -\sin\lambda'\sin\omega & b_2 = \dfrac{\cos\lambda'}{\cos\lambda} - \dfrac{\sin\theta\cos\alpha'}{\cot\lambda} \\
c_2 = 100\,\theta\sin\alpha' & e_2 = 100\,g\,\theta\sin\alpha' \\
a_3 = -\dfrac{\sin\omega}{\sin\lambda'} & b_3 = \dfrac{\sin\lambda\cos\omega}{\sin\lambda'} \\
c_3 = 100\,\theta\sin\alpha' & e_3 = 100\,g\,\theta\sin\alpha'
\end{array}$$

The following table contains the logarithms of μ sin 1″ and μ' sin 1″ :—

λ'	Log μ sin 1″	Log μ' sin 1″	Δ	λ'	Log μ sin 1″	Log μ' sin 1″	Δ
° ′				° ′			
49 0	6·68680	6·31313	—	55 0	6·68657	6·22633	—
20	78	6·30867	446	20	56	6·22108	525
40	77	6·30417	450	40	55	6·21578	530
50 0	6·68676	6·29962	455	56 0	6·68654	6·21043	535
20	75	6·29504	458	20	53	6·20503	540
40	73	6·29041	463	40	51	6·19958	545
51 0	6·68672	6·28575	466	57 0	6·68650	6·19408	550
20	71	6·28104	471	20	49	6·18853	555
40	70	6·27629	475	40	48	6·18292	561
52 0	6·68669	6·27149	480	58 0	6·68647	6·17726	566
20	67	6·26665	484	20	45	6·17155	571
40	66	6·26177	488	40	44	6·16578	577
53 0	6·68665	6·25685	492	59 0	6·68643	6·15996	582
20	64	6·25188	497	20	42	6·15408	588
40	62	6·24686	502	40	41	6·14814	594
54 0	6·68661	6·24180	506	60 0	6·68639	6·14215	599
20	60	6·23669	511	20	38	6·13610	605
40	59	6·23153	526	40	37	6·12999	611
55 0	6·68657	6·22633	520	61 0	6·68636	6·12381	618

In the application of the equations (18), page 620, to the different points in the triangulation at which astronomical observations have been taken, it is to be observed that θ corresponds to the distance of any such point from Greenwich, and α' is the azimuth of Greenwich from that point. All azimuths are measured from north by east, and longitudes westward. It is immaterial that the latitude, longitude, and direction of the meridian at any point have been arrived at, not by means of the direct distance from Greenwich in one step, but by several steps. The quantities θ and α' are required for every point entering into the equations of condition; their values, computed from the latitudes and longitudes, are as follows:—

Stations.	θ	α'	Stations.	θ	α'
	° ′ ″	° ′ ″		° ′ ″	° ′ ″
Arbury	1 3 19	134 26 13	Cowhythe	6 22 51	164 55 42
Balsham	0 40 59	196 57 44	Cheviot	4 11 26	161 24 23
Barrow Hill	2 38 37	60 26 50	Corryhabbie	6 8 31	161 4 5
Bardon	1 28 34	146 10 38	Cambridge	0 44 15	184 41 49
Butser	0 47 24	50 29 16	Cradle	1 59 27	102 30 55
Ben Lomond	5 26 14	147 56 34	Dunnose	1 8 27	40 49 2
Ben Hutig	7 30 55	158 0 0	Delamere	2 23 26	135 38 12
Ben Heynish . . .	6 25 5	137 48 20	Dunkery	2 15 45	80 37 9
Beacon Hill	1 6 53	74 3 55	Dunrich	4 30 6	153 51 1
Ben More, South Uist .	7 9 25	140 36 14	Durham	3 25 5	163 12 32
Ben Macdui	5 58 33	157 29 52	Ditchling	0 34 42	6 33 28
Ben Nevis	6 3 36	149 2 22	Dunstable	0 30 30	139 4 22
Black Down	1 47 4	62 46 19	Docking	1 28 16	195 20 21
Boniface Down . . .	1 9 9	40 22 52	Easington	3 7 17	170 20 48
Burleigh Moor . . .	3 8 48	168 7 46	Edinburgh Observatory (Calton Hill) . . .	4 50 49	155 50 51
Burnswark	4 6 19	150 10 50	Feaghmaan	6 25 20	89 53 8
Balta	9 16 3	176 57 42	Forth Mountain . . .	4 7 59	99 4 30
Beachy Head . . .	0 45 11	347 51 27	Fairlight	0 42 51	327 14 30
Bunwell	1 13 40	215 0 29	Frittenfield	0 35 1	297 47 26
Clifton	2 6 37	158 54 51			

Stations.	θ	α′	Stations.	θ	α′
	° ′ ″	° ′ ″		° ′ ″	° ′ ″
Goonhilly	3 34 18	64 27 25	Orford Castle . . .	1 7 46	237 38 21
Gerth of Scaw . . .	9 19 57	177 2 29	H. P. Cliff	1 9 20	39 59 13
Gad's Hill	0 17 49	282 28 50	Precelly	2 59 38	97 5 54
Goat Fell.	5 9 31	141 11 40	Pertinny	3 49 11	66 51 30
Hensbarrow	3 13 34	68 22 11	Paracombe	2 25 48	81 22 28
Hungry Hill	6 5 1	88 7 11	Ryder's Hill	2 37 59	66 54 0
Hart Fell	4 24 46	151 18 33	Scournalapich . . .	6 34 15	151 19 7
Inkpen	0 55 16	81 37 51	Sayrs Law	4 38 14	158 58 40
Kingstown	4 9 20	113 22 4	Saint Agnes	4 19 4	66 5 36
Kellie Law	5 2 1	159 52 17	South Berule . . .	3 52 49	131 29 32
Great Stirling . . .	6 3 48	169 23 47	Saxavord.	9 20 46	176 46 49
Leith Hill	0 22 45	37 23 28	Slieve Donard . . .	4 28 36	124 37 18
Lough Foyle. . . .	5 29 35	127 28 7	Southampton. . . .	1 2 36	56 48 59
Lundy Island . . .	2 56 8	82 7 23	South Lopham . . .	1 6 11	214 17 36
Lumsden	4 36 29	162 38 59	Tharfield Tower. . .	0 32 19	177 47 50
Laxfield Tower . . .	1 10 42	226 18 2	Tofts Tower	1 24 29	224 9 12
Mormonth	6 13 28	168 15 5	Tawnaghmore . . .	6 25 49	111 59 54
Mount Battock . . .	5 41 24	162 31 18	Week Down	1 10 18	40 59 55
Merrick	4 31 23	142 2 9	Wrotham.	0 14 25	312 3 9
Monach	7 45 23	149 31 36	Wordeslow	3 28 22	165 1 5
Mowcopt	2 7 13	139 29 15	Wingreen	1 24 27	68 45 13
Mordington	4 29 27	163 18 28	Walpole, St. Peter's .	1 15 19	186 23 30
Mendip	1 36 33	79 45 46	York Minster . . .	2 33 49	164 46 33
North Rona	8 18 34	154 6 55			

EQUATIONS OF CONDITION

DERIVED FROM OBSERVED LATITUDES.

Greenwich	$\xi_0 = + 0{\cdot}000 + 1{\cdot}0000\,\xi + 0{\cdot}0000\,\eta + 0{\cdot}0000\,u + 0{\cdot}0000\,v$
Saint Agnes	$\xi_1 = - 3{\cdot}154 + 0{\cdot}9939\,\xi + 0{\cdot}0865\,\eta + 3{\cdot}0623\,u - 0{\cdot}2007\,v$
Goonhilly	$\xi_2 = - 4{\cdot}718 + 0{\cdot}9959\,\xi + 0{\cdot}0706\,\eta + 2{\cdot}6952\,u - 0{\cdot}1975\,v$
Hensbarrow	$\xi_3 = - 3{\cdot}030 + 0{\cdot}9965\,\xi + 0{\cdot}0657\,\eta + 2{\cdot}0811\,u - 0{\cdot}1364\,v$
Black Down	$\xi_4 = + 1{\cdot}397 + 0{\cdot}9990\,\xi + 0{\cdot}0348\,\eta + 1{\cdot}4287\,u - 0{\cdot}1118\,v$
Dunnose	$\xi_5 = - 3{\cdot}332 + 0{\cdot}9998\,\xi + 0{\cdot}0163\,\eta + 1{\cdot}5109\,u - 0{\cdot}1406\,v$
Boniface Down	$\xi_6 = - 0{\cdot}922 + 0{\cdot}9998\,\xi + 0{\cdot}0164\,\eta + 1{\cdot}5361\,u - 0{\cdot}1438\,v$
Week Down	$\xi_7 = - 1{\cdot}143 + 0{\cdot}9998\,\xi + 0{\cdot}0168\,\eta + 1{\cdot}5475\,u - 0{\cdot}1442\,v$
H. P. Cliff	$\xi_8 = - 0{\cdot}112 + 0{\cdot}9998\,\xi + 0{\cdot}0163\,\eta + 1{\cdot}5494\,u - 0{\cdot}1445\,v$
Southampton	$\xi_9 = + 0{\cdot}256 + 0{\cdot}9997\,\xi + 0{\cdot}0191\,\eta + 0{\cdot}9993\,u - 0{\cdot}0862\,v$
Precelly	$\xi_{10} = - 0{\cdot}425 + 0{\cdot}9965\,\xi + 0{\cdot}0651\,\eta - 0{\cdot}6473\,u + 0{\cdot}1168\,v$
Arbury	$\xi_{11} = + 0{\cdot}697 + 0{\cdot}9998\,\xi + 0{\cdot}0165\,\eta - 1{\cdot}2927\,u + 0{\cdot}1015\,v$
Delamere	$\xi_{12} = - 1{\cdot}336 + 0{\cdot}9989\,\xi + 0{\cdot}0366\,\eta - 2{\cdot}9903\,u + 0{\cdot}2072\,v$
Clifton	$\xi_{13} = - 3{\cdot}378 + 0{\cdot}9998\,\xi + 0{\cdot}0166\,\eta - 3{\cdot}4447\,u + 0{\cdot}2091\,v$
Burleigh Moor	$\xi_{14} = - 3{\cdot}928 + 0{\cdot}9998\,\xi + 0{\cdot}0142\,\eta - 5{\cdot}3873\,u + 0{\cdot}2492\,v$
Hungry Hill	$\xi_{15} = + 1{\cdot}214 + 0{\cdot}9854\,\xi + 0{\cdot}1331\,\eta + 0{\cdot}3492\,u + 0{\cdot}2495\,v$
Feaghmaan	$\xi_{16} = - 2{\cdot}553 + 0{\cdot}9837\,\xi + 0{\cdot}1405\,\eta + 0{\cdot}0224\,u + 0{\cdot}3070\,v$
Forth	$\xi_{17} = - 1{\cdot}081 + 0{\cdot}9935\,\xi + 0{\cdot}0894\,\eta - 1{\cdot}1406\,u + 0{\cdot}2076\,v$
Tawnaghmore	$\xi_{18} = - 1{\cdot}735 + 0{\cdot}9860\,\xi + 0{\cdot}1304\,\eta - 4{\cdot}2138\,u + 0{\cdot}4563\,v$
Lough Foyle	$\xi_{19} = - 4{\cdot}809 + 0{\cdot}9925\,\xi + 0{\cdot}0955\,\eta - 5{\cdot}8460\,u + 0{\cdot}3616\,v$
South Berule	$\xi_{20} = + 1{\cdot}197 + 0{\cdot}9967\,\xi + 0{\cdot}0637\,\eta - 4{\cdot}4975\,u + 0{\cdot}2877\,v$
Ben Lomond	$\xi_{21} = - 1{\cdot}010 + 0{\cdot}9967\,\xi + 0{\cdot}0632\,\eta - 8{\cdot}0605\,u + 0{\cdot}2580\,v$
Kellie Law	$\xi_{22} = + 2{\cdot}169 + 0{\cdot}9988\,\xi + 0{\cdot}0379\,\eta - 8{\cdot}2669\,u + 0{\cdot}2221\,v$
Ben Heynish	$\xi_{23} = + 2{\cdot}758 + 0{\cdot}9927\,\xi + 0{\cdot}0943\,\eta - 8{\cdot}3184\,u + 0{\cdot}3041\,v$
Great Stirling	$\xi_{24} = + 1{\cdot}023 + 0{\cdot}9995\,\xi + 0{\cdot}0244\,\eta - 10{\cdot}4233\,u + 0{\cdot}1089\,v$
Monach	$\xi_{25} = + 2{\cdot}703 + 0{\cdot}9939\,\xi + 0{\cdot}0860\,\eta - 11{\cdot}6910\,u + 0{\cdot}0801\,v$
Ben Hutig	$\xi_{26} = - 1{\cdot}346 + 0{\cdot}9969\,\xi + 0{\cdot}0615\,\eta - 12{\cdot}1860\,u + 0{\cdot}0030\,v$
North Rona	$\xi_{27} = + 2{\cdot}483 + 0{\cdot}9949\,\xi + 0{\cdot}0793\,\eta - 13{\cdot}0730\,u - 0{\cdot}0595\,v$
Balta	$\xi_{28} = + 5{\cdot}496 + 0{\cdot}9999\,\xi + 0{\cdot}0107\,\eta - 16{\cdot}1815\,u - 0{\cdot}4851\,v$
Gerth of Scaw	$\xi_{29} = + 5{\cdot}388 + 0{\cdot}9999\,\xi + 0{\cdot}0105\,\eta - 16{\cdot}2960\,u - 0{\cdot}5012\,v$
Saxavord	$\xi_{30} = + 3{\cdot}412 + 0{\cdot}9999\,\xi + 0{\cdot}0115\,\eta - 16{\cdot}3154\,u - 0{\cdot}5040\,v$
Edinburgh	$\xi_{31} = - 5{\cdot}098 + 0{\cdot}9985\,\xi + 0{\cdot}0434\,\eta - 7{\cdot}7362\,u + 0{\cdot}2440\,v$
Durham	$\xi_{32} = - 0{\cdot}900 + 0{\cdot}9996\,\xi + 0{\cdot}0216\,\eta - 5{\cdot}7248\,u + 0{\cdot}2610\,v$
Cambridge	$\xi_{33} = - 0{\cdot}809 + 1{\cdot}0000\,\xi + 0{\cdot}0013\,\eta - 1{\cdot}2860\,u - 0{\cdot}1486\,v$
Cowhythe	$(\xi) = - 8{\cdot}469 + 0{\cdot}9989\,\xi + 0{\cdot}0363\,\eta - 10{\cdot}7758\,u + 0{\cdot}0909\,v$

EQUATIONS OF CONDITION

DERIVED FROM OBSERVED LONGITUDES AND AZIMUTHS.

Greenwich	$\eta_{0} = +0.000 + 0.0000\,\xi + 1.0000\,\eta + 0.0000\,u + 0.0000\,v$
Edinburgh	$\eta_{1} = -5.896 - 0.0460\,\xi + 0.9957\,\eta + 3.4614\,u + 1.0588\,v$
Durham	$\eta_{2} = -4.580 - 0.0226\,\xi + 0.9979\,\eta + 1.7234\,u + 0.5296\,v$
Cambridge	$\eta_{3} = +4.859 + 0.0013\,\xi + 0.9999\,\eta - 0.1054\,u - 0.0324\,v$
Kingstown	$\eta_{4} = -2.123 - 0.0856\,\xi + 0.9958\,\eta + 6.6575\,u + 2.0459\,v$
Feaghmaan	$\eta_{5} = +1.935 - 0.1413\,\xi + 0.9899\,\eta + 11.2087\,u + 3.4445\,v$
Blackdown	$\eta_{6} = -6.915 - 0.0575\,\xi + 1.0102\,\eta + 2.7685\,u + 0.8507\,v$
Dunnose	$\eta_{7} = -2.976 - 0.0270\,\xi + 1.0120\,\eta + 1.3014\,u + 0.3999\,v$
Precelly	$\eta_{8} = -0.344 - 0.1057\,\xi + 0.9901\,\eta + 5.1853\,u + 1.5934\,v$
Cowhythe	$\eta_{9} = -0.312 - 0.0549\,\xi + 0.9248\,\eta + 2.8958\,u + 0.8899\,v$
Wrotham	$\eta_{10} = +1.637 + 0.0064\,\xi + 1.0023\,\eta - 0.3114\,u - 0.0957\,v$
Gad's Hill	$\eta_{11} = +2.754 + 0.0104\,\xi + 1.0008\,\eta - 0.5059\,u - 0.1555\,v$
Fairlight	$\eta_{12} = +3.144 + 0.0140\,\xi + 1.0084\,\eta - 0.6744\,u - 0.2073\,v$
Orford Castle	$\eta_{13} = +5.278 + 0.0339\,\xi + 0.9912\,\eta - 1.6653\,u - 0.5117\,v$
Balsham	$\eta_{14} = +6.826 + 0.0071\,\xi + 0.9910\,\eta - 0.3478\,u - 0.1069\,v$
Walpole, St. Peter's	$\eta_{15} = +6.539 + 0.0049\,\xi + 0.9832\,\eta - 0.2439\,u - 0.0750\,v$
Docking	$\eta_{16} = +4.851 + 0.0137\,\xi + 0.9809\,\eta - 0.6793\,u - 0.2087\,v$
South Lopham	$\eta_{17} = +3.507 + 0.0220\,\xi + 0.9874\,\eta - 1.0847\,u - 0.3333\,v$
Leith Hill	$\eta_{18} = -1.803 - 0.0083\,\xi + 1.0042\,\eta + 0.4019\,u + 0.1235\,v$
Butser Hill	$\eta_{19} = -4.049 - 0.0220\,\xi + 1.0069\,\eta + 1.0639\,u + 0.3269\,v$
Beacon Hill	$\eta_{20} = -1.574 - 0.0385\,\xi + 1.0037\,\eta + 1.8707\,u + 0.5748\,v$
Barrow Hill	$\eta_{21} = -7.718 - 0.0838\,\xi + 1.0158\,\eta + 4.0137\,u + 1.2334\,v$
Lundy Island	$\eta_{22} = +1.152 - 0.1046\,\xi + 1.0010\,\eta + 5.0747\,u + 1.5595\,v$
Dunkery	$\eta_{23} = -4.011 - 0.0803\,\xi + 1.0024\,\eta + 3.8960\,u + 1.1972\,v$
Mowcopt	$\eta_{24} = +4.496 - 0.0483\,\xi + 0.9774\,\eta + 2.4040\,u + 0.7387\,v$
York Minster	$\eta_{25} = -2.952 - 0.0233\,\xi + 0.9673\,\eta + 1.1749\,u + 0.3611\,v$
Easington	$\eta_{26} = -4.535 - 0.0180\,\xi + 0.9601\,\eta + 0.9135\,u + 0.2807\,v$
Wordeslow	$\eta_{27} = -5.468 - 0.0308\,\xi + 0.9565\,\eta + 1.5670\,u + 0.4815\,v$
Hart Fell	$\eta_{28} = +3.456 - 0.0720\,\xi + 0.9487\,\eta + 3.6975\,u + 1.1362\,v$
Cheviot	$\eta_{29} = -4.107 - 0.0454\,\xi + 0.9489\,\eta + 2.3321\,u + 0.7166\,v$
Dunrich	$\eta_{30} = -7.525 - 0.0673\,\xi + 0.9470\,\eta + 3.4619\,u + 1.0641\,v$
Goat Fell	$\eta_{31} = -2.890 - 0.1096\,\xi + 0.9440\,\eta + 5.6426\,u + 1.7340\,v$
Sayrs Law	$\eta_{32} = -1.298 - 0.0263\,\xi + 0.9444\,\eta + 2.9034\,u + 0.8926\,v$
Mordington	$\eta_{33} = -2.165 - 0.0437\,\xi + 0.9453\,\eta + 2.2513\,u + 0.6918\,v$
Burnswark	$\eta_{34} = +1.859 - 0.0697\,\xi + 0.9524\,\eta + 3.5630\,u + 1.0949\,v$
B. More, S. Uist	$\eta_{35} = -2.782 - 0.1509\,\xi + 0.9226\,\eta + 7.9278\,u + 2.4362\,v$
B. Macdui	$\eta_{36} = +0.522 - 0.0762\,\xi + 0.9302\,\eta + 3.9916\,u + 1.2266\,v$
B. Nevis	$\eta_{37} = +5.745 - 0.1042\,\xi + 0.9314\,\eta + 5.4411\,u + 1.6720\,v$
Scournalapich	$\eta_{38} = -4.732 - 0.1047\,\xi + 0.9254\,\eta + 5.5040\,u + 1.6914\,v$
Mount Battock	$\eta_{39} = -5.246 - 0.0570\,\xi + 0.9323\,\eta + 2.9826\,u + 0.9166\,v$
Mormonth	$\eta_{40} = -7.220 - 0.0420\,\xi + 0.9260\,\eta + 2.2120\,u + 0.6798\,v$

§ V.

Solution of the Equations.

We have in the preceding equations all the requisite data for determining the elements of that spheroid which most nearly coincides with the surface of Great Britain and Ireland. These elements are—

$$\text{Semiaxis major} = 20923713 + 10144 \cdot u$$
$$\text{Square of excent.} = {\cdot}00667054 + {\cdot}0004848\ v$$

The quantities u and v, together with the quantities ξ and η for Greenwich, have to be determined so that the ξ's and η's for all the stations shall be in the aggregate the least possible. The equations for latitude must be considered much more important than those for observed longitudes or azimuths, inasmuch as the probable error of an observed latitude is much smaller than the probable error either of a determination of longitude or of the direction of the meridian. Were all three classes of observations equally to be depended on, we might determine $\xi\ \eta\ u\ v$ by making $\Sigma\,(\xi^2) + \Sigma\,(\eta^2)$ a minimum. This will, indeed, give us that surface which best represents the whole of the observations, but we must allow an advantage to the latitude equations, and a disadvantage to the others, in order to obtain that surface which should be considered as the best representation of the whole; that is, we must make $\Sigma\,(\xi^2) + w\,\Sigma\,(\eta^2)$ a minimum where w is a quantity less than unity.

Let $\xi_p = m + a\xi + b\eta + cu + ev$ be the type of a latitude equation: $\eta_q = n + a'\xi + b'\eta + c'u + e'v$, the type of an equation derived from an observed longitude or azimuth, then $\xi\ \eta\ u\ v$ are to be determined so as to make a minimum the quantity—

$$\Sigma\,(m + a\xi + b\eta + cu + ev)^2 + w\,\Sigma\,(n + a'\xi + b'\eta + c'u + e'v)^2$$

This gives the four equations—

$$0 = (am) + w\,(a'n) + [a^2]\,\xi + [ab]\,\eta + [ac]\,u + [ae]\,v$$
$$0 = (bm) + w\,(b'n) + [ab]\,\xi + [b^2]\,\eta + [bc]\,u + [be]\,v$$
$$0 = (cm) + w\,(c'n) + [ac]\,\xi + [bc]\,\eta + [c^2]\,u + [ce]\,v$$
$$0 = (em) + w\,(e'n) + [ae]\,\xi + [be]\,\eta + [ce]\,u + [e^2]\,v$$

where $[a^2] = (a^2) + w\,(a'^2)$: $[ab] = (ab) + w\,(a'b')$, &c.

1.

In the first instance, we shall determine those values of $\xi\ \eta\ u\ v$ which make $\Sigma\,(\xi^2) + \Sigma\,(\eta^2)$ a minimum, the value of w being unity.

From actual multiplications the following values are found—

$(a^2) = 33{\cdot}7768$ $\quad (ab) = 1{\cdot}6788$ $\quad (ac) = -\ 147{\cdot}8547$

$(ae) = 1{\cdot}2083$ $\quad (b^2) = 0{\cdot}1356$ $\quad (bc) = -\ 6{\cdot}6372$

$(be) = 0{\cdot}2341$ $\quad (c^2) = 1814{\cdot}0613$ $\quad (ce) = 2{\cdot}5100$

$(e^2) = 2{\cdot}1146$ $\quad (a'^2) = 0{\cdot}1676$ $\quad (a'b') = -\ 1{\cdot}7844$

$(a'c') = -\ 9{\cdot}4035$ $\quad (a'e') = -\ 2{\cdot}8895$ $\quad (b'^2) = 38{\cdot}9743$

$(b'c') = 100{\cdot}1245$ $\quad (b'e') = 30{\cdot}7643$ $\quad (c'^2) = 548{\cdot}8328$

$(c'e') = 168{\cdot}6411$ $\quad (e'^2) = 51{\cdot}8189$

If we put

$$\begin{aligned}
A &= ma + m_1\,a_1 + m_2\,a_2 + \cdots\cdot\; na' + n_1a_1' + \cdots\cdot \\
B &= mb + m_1\,b_1 + m_2\,b_2 + \cdots\cdot\; nb' + n_1b_1' + \cdots\cdot \\
C &= mc + m_1\,c_1 + m_2\,c_2 + \cdots\cdot\; nc' + n_1c_1' + \cdots\cdot \\
D &= me + m_1\,e_1 + m_2\,e_2 + \cdots\cdot\; ne' + n_1e_1' + \cdots\cdot
\end{aligned}$$

We get

$$\begin{aligned}
0 &= A + 33{\cdot}9444\,\xi - 0{\cdot}1056\,\eta - 151{\cdot}2582\,u - 1{\cdot}6812\,v \\
0 &= B - 0{\cdot}1056\,\xi + 39{\cdot}1108\,\eta + 93{\cdot}4873\,u + 30{\cdot}9984\,v \\
0 &= C - 157{\cdot}2582\,\xi + 93{\cdot}4873\,\eta + 2362{\cdot}8941\,u + 171{\cdot}1511\,v \\
0 &= D - 1{\cdot}6812\,\xi + 30{\cdot}9984\,\eta + 171{\cdot}1511\,u + 52{\cdot}9335\,v
\end{aligned}$$

From these we find the values of ξ η u and v in terms of A B C D, as follows:—

$$\begin{aligned}
\xi &= -\,{\cdot}0471729\,A + {\cdot}00100431\,B - {\cdot}00393545\,C + {\cdot}01044100\,D \\
\eta &= +\,{\cdot}0010043\,A - {\cdot}04701100\,B - {\cdot}00004228\,C + {\cdot}02718515\,D \\
u &= -\,{\cdot}0039354\,A - {\cdot}00004228\,B - {\cdot}00087818\,C + {\cdot}00268843\,D \\
v &= +\,{\cdot}0104410\,A + {\cdot}02718515\,B + {\cdot}00268843\,C - {\cdot}04237210\,D
\end{aligned}$$

By substituting the values of A B C and D, we have finally ξ η u v in terms of the observed quantities $m\,m_{,}\cdots n\,n_{,}\cdots$ with numerical coefficients.

Suppose $\xi = \alpha A + \beta B + \gamma C + \delta D$, then by substitution

$$\xi = (\alpha a + \beta b + \gamma c + \delta e)\,m + (\alpha a_{,} + \beta b_{,} + \gamma c_{,} + \delta e_{,})\,m_{,} + \cdots\cdot$$

Let also

$$\begin{aligned}
\alpha a + \beta b + \gamma c + \delta e &= \lambda \\
\alpha a_1 + \beta b_1 + \gamma c_1 + \delta e_1 &= \lambda_1 \\
\alpha a_2 + \beta b_2 + \gamma c_2 + \delta e_2 &= \lambda_2
\end{aligned}$$

and so on. Multiply these equations by $\lambda\;\lambda_1\cdots\cdot\cdot$ and add, then we have $\lambda^2 + \lambda_1^2 + \cdots\cdot$ equal to

$$\alpha\,\Sigma(\lambda a) + \beta\,\Sigma(\lambda b) + \gamma\,\Sigma(\lambda c) + \delta\,\Sigma(\lambda e)$$

Now multiply the original equations—

$$\left.\begin{aligned}
\xi &= m + a\,\xi + b\,\eta + c\,u + e\,v \\
\xi_1 &= m_1 + a_1\,\xi + b_1\,\eta + c_1\,u + e_1\,v \\
\xi_2 &= m_2 + a_2\,\xi + b_2\,\eta + c_2\,u + e_2\,v
\end{aligned}\right\}\ \&c.$$

by λ, λ_1, $\lambda_2\cdots\cdot$ and take their sum, then of necessity $\lambda\xi + \lambda_1\,\xi_1 + \lambda_2\,\xi_2 + \cdots = 0$, and since $\xi = \lambda m + \lambda_1\,m_1 + \lambda_2\,m_2 + \cdots$ we have identically $\Sigma(\lambda a) = -1$; $\Sigma(\lambda b) = 0$; $\Sigma(\lambda c) = 0$; $\Sigma(\lambda e) = 0$; consequently the sum of the squares of the coefficients of the observed quantities in the expression for ξ is—

$$\lambda^2 + \lambda_1^2 + \lambda_2^2 + \cdots\cdot = -\alpha$$

Similarly, the sum of the squares of the coefficients of the observed quantities in the expression for η is the coefficient of B taken with an opposite sign (that is, ·0470110), and similarly for u and v.

By substitution of A B C D in the linear expressions for $\xi\ \eta\ u\ v$, we get, after the necessary multiplications, the following values :—

$$\begin{aligned}
\xi = & - {\cdot}047172\, m_0 \quad - {\cdot}060944\, m_1 \quad - {\cdot}059576\, m_2 \quad - {\cdot}056555\, m_3 \quad - {\cdot}053878\, m_4 \quad - {\cdot}054560\, m_5 \quad - {\cdot}054693\, m_6 \\
& - {\cdot}054741\, m_7 \quad - {\cdot}054752\, m_8 \quad - {\cdot}051971\, m_9 \quad - {\cdot}043175\, m_{10} \quad - {\cdot}040999\, m_{11} \quad - {\cdot}033152\, m_{12} \quad - {\cdot}031406\, m_{13} \\
& - {\cdot}023345\, m_{14} \quad - {\cdot}045118\, m_{15} \quad - {\cdot}043144\, m_{16} \quad - {\cdot}040119\, m_{17} \quad - {\cdot}025033\, m_{18} \quad - {\cdot}019940\, m_{19} \quad - {\cdot}026249\, m_{20} \\
& - {\cdot}012537\, m_{21} \quad - {\cdot}012224\, m_{22} \quad - {\cdot}010821\, m_{23} \quad - {\cdot}004966\, m_{24} \quad + {\cdot}000048\, m_{25} \quad + {\cdot}001025\, m_{26} \quad + {\cdot}003975\, m_{27} \\
& + {\cdot}011460\, m_{28} \quad + {\cdot}011742\, m_{29} \quad + {\cdot}011790\, m_{30} \quad - {\cdot}014064\, m_{31} \quad - {\cdot}021876\, m_{32} \quad - {\cdot}043661\, m_{33} \quad + {\cdot}001004\, n_0 \\
& + {\cdot}000603\, n_1 \quad + {\cdot}000815\, n_2 \quad + {\cdot}001019\, n_3 \quad + {\cdot}000199\, n_4 \quad - {\cdot}000488\, n_5 \quad + {\cdot}001714\, n_6 \quad + {\cdot}001344\, n_7 \\
& + {\cdot}002211\, n_8 \quad + {\cdot}001414\, n_9 \quad + {\cdot}000931\, n_{10} \quad + {\cdot}000882\, n_{11} \quad + {\cdot}000842\, n_{12} \quad + {\cdot}000607\, n_{13} \quad + {\cdot}000913\, n_{14} \\
& + {\cdot}000933\, n_{15} \quad + {\cdot}000833\, n_{16} \quad + {\cdot}000743\, n_{17} \quad + {\cdot}001108\, n_{18} \quad + {\cdot}001275\, n_{19} \quad + {\cdot}001464\, n_{20} \quad + {\cdot}002056\, n_{21} \\
& + {\cdot}002251\, n_{22} \quad + {\cdot}001962\, n_{23} \quad + {\cdot}001512\, n_{24} \quad + {\cdot}001217\, n_{25} \quad + {\cdot}001149\, n_{26} \quad + {\cdot}001274\, n_{27} \quad + {\cdot}001661\, n_{28} \\
& + {\cdot}001399\, n_{29} \quad + {\cdot}001612\, n_{30} \quad + {\cdot}002017\, n_{31} \quad + {\cdot}000083\, n_{32} \quad + {\cdot}001374\, n_{33} \quad + {\cdot}001654\, n_{34} \quad + {\cdot}002282\, n_{35} \\
& + {\cdot}001627\, n_{36} \quad + {\cdot}001895\, n_{37} \quad + {\cdot}001867\, n_{38} \quad + {\cdot}001457\, n_{39} \quad + {\cdot}001304\, n_{40}
\end{aligned}$$

$$\begin{aligned}
\eta = & + {\cdot}001004\, m_0 \quad - {\cdot}008654\, m_1 \quad - {\cdot}007802\, m_2 \quad - {\cdot}005884\, m_3 \quad - {\cdot}003732\, m_4 \quad - {\cdot}003648\, m_5 \quad - {\cdot}003741\, m_6 \\
& - {\cdot}003771\, m_7 \quad - {\cdot}003756\, m_8 \quad - {\cdot}002279\, m_9 \quad + {\cdot}001143\, m_{10} \quad + {\cdot}003042\, m_{11} \quad + {\cdot}005042\, m_{12} \quad + {\cdot}006054\, m_{13} \\
& + {\cdot}007339\, m_{14} \quad + {\cdot}001500\, m_{15} \quad + {\cdot}002728\, m_{16} \quad + {\cdot}002487\, m_{17} \quad + {\cdot}007443\, m_{18} \quad + {\cdot}006585\, m_{19} \quad + {\cdot}006018\, m_{20} \\
& + {\cdot}005384\, m_{21} \quad + {\cdot}005609\, m_{22} \quad + {\cdot}005183\, m_{23} \quad + {\cdot}003258\, m_{24} \quad - {\cdot}000373\, m_{25} \quad - {\cdot}001293\, m_{26} \quad - {\cdot}003794\, m_{27} \\
& - {\cdot}012002\, m_{28} \quad - {\cdot}012426\, m_{29} \quad - {\cdot}012548\, m_{30} \quad + {\cdot}005923\, m_{31} \quad + {\cdot}007326\, m_{32} \quad - {\cdot}003042\, m_{33} \quad - {\cdot}047011\, n_0 \\
& - {\cdot}018218\, n_1 \quad - {\cdot}032611\, n_2 \quad - {\cdot}047881\, n_3 \quad + {\cdot}008437\, n_4 \quad + {\cdot}046487\, n_5 \quad - {\cdot}024539\, n_6 \quad - {\cdot}036786\, n_7 \\
& - {\cdot}003554\, n_8 \quad - {\cdot}019461\, n_9 \quad - {\cdot}049701\, n_{10} \quad - {\cdot}051244\, n_{11} \quad - {\cdot}052999\, n_{12} \quad - {\cdot}060404\, n_{13} \quad - {\cdot}049472\, n_{14} \\
& - {\cdot}048245\, n_{15} \quad - {\cdot}051744\, n_{16} \quad - {\cdot}055412\, n_{17} \quad - {\cdot}043876\, n_{18} \quad - {\cdot}038516\, n_{19} \quad - {\cdot}031677\, n_{20} \quad - {\cdot}014477\, n_{21} \\
& - {\cdot}004982\, n_{22} \quad - {\cdot}014823\, n_{23} \quad - {\cdot}026017\, n_{24} \quad - {\cdot}035730\, n_{25} \quad - {\cdot}037561\, n_{26} \quad - {\cdot}031974\, n_{27} \quad - {\cdot}013940\, n_{28} \\
& - {\cdot}025272\, n_{29} \quad - {\cdot}015806\, n_{30} \quad + {\cdot}002412\, n_{31} \quad - {\cdot}020281\, n_{32} \quad - {\cdot}025772\, n_{33} \quad - {\cdot}015229\, n_{34} \quad + {\cdot}022369\, n_{35} \\
& - {\cdot}010630\, n_{36} \quad + {\cdot}001333\, n_{37} \quad + {\cdot}002139\, n_{38} \quad - {\cdot}019094\, n_{39} \quad - {\cdot}025187\, n_{40}
\end{aligned}$$

$$\begin{aligned}
u = & - {\cdot}003935\, m_0 \quad - {\cdot}007144\, m_1 \quad - {\cdot}006820\, m_2 \quad - {\cdot}006119\, m_3 \quad - {\cdot}005488\, m_4 \quad - {\cdot}005640\, m_5 \quad - {\cdot}005671\, m_6 \\
& - {\cdot}005682\, m_7 \quad - {\cdot}005685\, m_8 \quad - {\cdot}005044\, m_9 \quad - {\cdot}003042\, m_{10} \quad - {\cdot}002527\, m_{11} \quad - {\cdot}000750\, m_{12} \quad - {\cdot}000348\, m_{13} \\
& + {\cdot}001466\, m_{14} \quad - {\cdot}003520\, m_{15} \quad - {\cdot}003072\, m_{16} \quad - {\cdot}002354\, m_{17} \quad + {\cdot}001041\, m_{18} \quad + {\cdot}002196\, m_{19} \quad + {\cdot}000798\, m_{20} \\
& + {\cdot}003847\, m_{21} \quad + {\cdot}003925\, m_{22} \quad + {\cdot}004212\, m_{23} \quad + {\cdot}005512\, m_{24} \quad + {\cdot}006567\, m_{25} \quad + {\cdot}006784\, m_{26} \quad + {\cdot}007402\, m_{27} \\
& + {\cdot}008971\, m_{28} \quad + {\cdot}009028\, m_{29} \quad + {\cdot}009037\, m_{30} \quad + {\cdot}003518\, m_{31} \quad + {\cdot}001794\, m_{32} \quad - {\cdot}003206\, m_{33} \quad - {\cdot}000042\, n_0 \\
& - {\cdot}000054\, n_1 \quad - {\cdot}000043\, n_2 \quad - {\cdot}000042\, n_3 \quad - {\cdot}000051\, n_4 \quad - {\cdot}000069\, n_5 \quad + {\cdot}000039\, n_6 \quad - {\cdot}000004\, n_7 \\
& + {\cdot}000104\, n_8 \quad + {\cdot}000026\, n_9 \quad - {\cdot}000051\, n_{10} \quad - {\cdot}000057\, n_{11} \quad - {\cdot}000063\, n_{12} \quad - {\cdot}000089\, n_{13} \quad - {\cdot}000052\, n_{14} \\
& - {\cdot}000048\, n_{15} \quad - {\cdot}000060\, n_{16} \quad - {\cdot}000072\, n_{17} \quad - {\cdot}000031\, n_{18} \quad - {\cdot}000011\, n_{19} \quad + {\cdot}000011\, n_{20} \quad + {\cdot}000078\, n_{21} \\
& + {\cdot}000105\, n_{22} \quad + {\cdot}000071\, n_{23} \quad + {\cdot}000024\, n_{24} \quad - {\cdot}000010\, n_{25} \quad - {\cdot}000017\, n_{26} \quad - {\cdot}000001\, n_{27} \quad + {\cdot}000051\, n_{28} \\
& + {\cdot}000017\, n_{29} \quad + {\cdot}000046\, n_{30} \quad + {\cdot}000098\, n_{31} \quad - {\cdot}000086\, n_{32} \quad + {\cdot}000015\, n_{33} \quad + {\cdot}000049\, n_{34} \quad + {\cdot}000142\, n_{35} \\
& + {\cdot}000053\, n_{36} \quad + {\cdot}000088\, n_{37} \quad + {\cdot}000086\, n_{38} \quad + {\cdot}000030\, n_{39} \quad + {\cdot}000011\, n_{40}
\end{aligned}$$

$$\begin{aligned}
v = & + {\cdot}010441\, m_0 \quad + {\cdot}029465\, m_1 \quad + {\cdot}027932\, m_2 \quad + {\cdot}023565\, m_3 \quad + {\cdot}019953\, m_4 \quad + {\cdot}020901\, m_5 \quad + {\cdot}021107\, m_6 \\
& + {\cdot}021166\, m_7 \quad + {\cdot}021170\, m_8 \quad + {\cdot}017296\, m_9 \quad + {\cdot}005485\, m_{10} \quad + {\cdot}003111\, m_{11} \quad - {\cdot}005394\, m_{12} \quad - {\cdot}007231\, m_{13} \\
& - {\cdot}014218\, m_{14} \quad + {\cdot}004274\, m_{15} \quad + {\cdot}001142\, m_{16} \quad + {\cdot}000941\, m_{17} \quad - {\cdot}016823\, m_{18} \quad - {\cdot}018079\, m_{19} \quad - {\cdot}012143\, m_{20} \\
& - {\cdot}020477\, m_{21} \quad - {\cdot}020177\, m_{22} \quad - {\cdot}022320\, m_{23} \quad - {\cdot}021537\, m_{24} \quad - {\cdot}022109\, m_{25} \quad - {\cdot}020808\, m_{26} \quad - {\cdot}020081\, m_{27} \\
& - {\cdot}012217\, m_{28} \quad - {\cdot}011848\, m_{29} \quad - {\cdot}011755\, m_{30} \quad - {\cdot}019532\, m_{31} \quad - {\cdot}015426\, m_{32} \quad + {\cdot}013316\, m_{33} \quad + {\cdot}027185\, n_0 \\
& - {\cdot}008970\, n_1 \quad + {\cdot}009085\, n_2 \quad + {\cdot}028285\, n_3 \quad - {\cdot}042613\, n_4 \quad - {\cdot}090382\, n_5 \quad - {\cdot}001741\, n_6 \quad + {\cdot}013784\, n_7 \\
& - {\cdot}027763\, n_8 \quad - {\cdot}005354\, n_9 \quad + {\cdot}030533\, n_{10} \quad + {\cdot}032544\, n_{11} \quad + {\cdot}034530\, n_{12} \quad + {\cdot}044505\, n_{13} \quad + {\cdot}030609\, n_{14} \\
& + {\cdot}029302\, n_{15} \quad + {\cdot}033826\, n_{16} \quad + {\cdot}038279\, n_{17} \quad + {\cdot}023059\, n_{18} \quad + {\cdot}016152\, n_{19} \quad + {\cdot}007557\, n_{20} \quad - {\cdot}014731\, n_{21} \\
& - {\cdot}026316\, n_{22} \quad - {\cdot}013842\, n_{23} \quad + {\cdot}001230\, n_{24} \quad + {\cdot}013911\, n_{25} \quad + {\cdot}016474\, n_{26} \quad + {\cdot}009492\, n_{27} \quad - {\cdot}013164\, n_{28} \\
& + {\cdot}001228\, n_{29} \quad - {\cdot}010739\, n_{30} \quad - {\cdot}033785\, n_{31} \quad - {\cdot}004617\, n_{32} \quad + {\cdot}001981\, n_{33} \quad - {\cdot}011651\, n_{34} \quad - {\cdot}058408\, n_{35} \\
& - {\cdot}016751\, n_{36} \quad - {\cdot}031986\, n_{37} \quad - {\cdot}032807\, n_{38} \quad - {\cdot}006070\, n_{39} \quad + {\cdot}001877\, n_{40}
\end{aligned}$$

By putting the final values of $\xi\ \eta\ u\ v$ in this form, we can at sight estimate the effect that would be produced in them by an alteration of any of the observed latitudes. When the numerical values of the observed quantities $m_1\ m_2$, &c., are supplied in these expressions, there results—

$$\begin{aligned} \xi &= +\ 1''{\cdot}479 \\ \eta &= -\ 0{\cdot}404 \\ u &= +\ 0{\cdot}2433 \\ v &= +\ 0{\cdot}8918 \end{aligned}$$

By substituting these in the Equations of Condition we get the following values :—

Name.	ξ	Name.	ξ	Name.	η	Name.	η
Greenwich . .	+1″·479	Tawnaghmore .	−0″·948	Greenwich . .	−0″·404	Barrow Hill .	−6″·176
St. Agnes . .	−1·153	Lough Foyle .	−4·480	Edinburgh . .	−4·580	Lundy Island .	+3·218
Goonhilly . .	−2·794	South Berule .	+1·808	Durham . . .	−4·125	Dunkerry . .	−2·519
Hensbarrow .	−1·198	Ben Lomond .	−1·292	Cambridge . .	+4·402	Mowcopt . .	+5·273
Black Down .	+3·108	Kellie Law . .	+1·818	Kingstown . .	+0·792	York Minster .	−2·769
Dunnose . .	−1·618	Ben Heynish .	+2·435	Feaghmaan . .	+7·125	Easington . .	−4·477
Boniface . .	+0·796	Great Stirling .	+0·052	Black Down . .	−5·976	Wordeslow . .	−5·089
Week Down .	+0·577	Monach . . .	+1·365	Dunnose . . .	−2·751	Hart Fell . .	+4·879
Port Valley . .	+1·608	Ben Hutig . .	−2·858	Precelly . . .	+1·782	Cheviot . . .	−3·351
Southampton .	+1·893	North Rona .	+0·689	Cowhythe . .	+0·731	Dunrich . .	−6·216
Precelly. . .	+0·969	Balta . . .	+2·601	Wrotham . .	+1·080	Goat Fell . .	−0·514
Arbury . . .	+1·945	Gerth of Scaw .	+2·451	Gad's Hill . .	+2·103	Sayrs Law . .	−0·216
Delamere . .	−0·416	Saxavord . .	+0·467	Fairlight . . .	+2·408	Mordington .	−1·447
Clifton . . .	−2·558	Edinburgh . .	−5·303	Orford Castle .	+4·066	Burnswark . .	+3·214
Burleigh Moor .	−3·543	Durham . . .	−0·590	Balsham . . .	+6·256	Ben More . .	+0·723
Hungry Hill .	+2·925	Cambridge . .	+0·443	Walpole, St. Peter's	+6·023	Ben Macdui .	+2·098
Feaghmaan . .	−0·876			Docking . . .	+4·124	Ben Nevis . .	+8·029
Forth . . .	+0·260			South Lopham .	+2·579	Scournalapich .	−2·413
				Leith Hill . .	−2·013	Mount Battock .	−4·164
				Butser . . .	−3·938	Mormonth . .	−6·512
Cowhythe (ξ) = − 9″·548				Beacon Hill . .	−1·069		

The system of quantities $\xi\ \eta$ contained in this table have this property, that the sum of their squares is less than that of any other system that can be obtained in reference to any spheroidal surface whatever. They are therefore to be understood as the *actual inclinations* of the surface (north or east) to that of the spheroid we have above determined. They are, however, affected with errors of observation, more especially the quantities η; for it is clear that their mean value should not exceed the mean value of the quantities ξ, whereas it is nearly double.

Since this spheroid so nearly agrees with the observed quantities, it might be expected, with some probability, that the corresponding inclinations would agree in some measure with those derived from calculations based upon a knowledge of the magnitude and position of disturbing masses in the vicinity; namely, the quantities given at page 664. The comparison of these is given in the following table:—

Names.	From the Ground.	By Geodetical Inference.	Difference.	Names.	From the Ground.	By Geodetical Inference.	Difference.
	″	″	″		″	″	″
Dunnose	− 0·54	− 1·62	− 1·08	Forth	+ 1·13	+ 0·26	− 0·87
Boniface	+ 2·42	+ 0·80	− 1·62	Tawnaghmore . . .	− 2·30	− 0·95	+ 1·35
Week Down . . .	+ 1·98	+ 0·58	− 1·40	Lough Foyle . . .	− 4·02	− 4·48	− 0·46
Port Valley. . . .	+ 3·29	+ 1·61	− 1·68	Kellie Law	+ 2·08	+ 1·82	− 0·26
Clifton	− 0·90	− 2·56	− 1·66	Monach	+ 0·47	+ 1·36	+ 0·89
Burleigh Moor . . .	− 4·55	− 3·54	+ 1·01	Ben Hutig	− 2·01	− 2·86	− 0·85
Hungry Hill . . .	+ 5·40	+ 2·92	− 2·48	Calton Hill, Edinburgh	− 3·57	− 5·30	− 1·73
Feaghmaan	− 1·95	− 0·88	+ 1·07	(*Cowhythe*)	(−5″)	− 9·55	− 4·55

If we disregard signs in this table, the average amount (omitting Cowhythe) of the deflections accounted for by the ground is 2″·44, while the average of the corresponding deflections, as inferred from the connection of geodetical and astronomical observations, is 2″·10. The average amount of the differences in the third column is 1″·23.

The agreement between the quantities in the second and third columns is not, perhaps, so close as might have been anticipated, and shows that the deflections cannot be entirely accounted for by the inequalities of the surface. In the Isle of Wight, for instance, we have a second and a half to the south which cannot be accounted for; and here it must be remembered that the attraction of the nearer parts of the mass of England have not been taken into account in assigning the deflections at page 664: this attraction, if included, would increase the discrepancy to at least 2″.

At Hungry Hill also there is apparently 2″·5 unaccounted for, as though the attractive power of the mountains had been overrated; while, on the other hand, at the Calton Hill, Edinburgh, there are nearly two seconds to the south unaccounted for, as if the attractive power of the mountains had been underrated.

At Cowhythe the ground will only account for half the disturbance observed, and leaves four or five seconds unaccounted for. At Lough Foyle and Kellie Law the ground seems to account satisfactorily for the observed deflections.

The hypothesis of the Astronomer Royal respecting the influence of masses of high land upon the plumb-line (pages 572-3) makes it very doubtful to what distance from any given point their apparent attraction should be taken into account. We shall therefore divide our *ground deflections* into two systems: the first, which for convenience we shall call A, results from the consideration of the irregularities of the ground more immediately in the neighbourhood of the Station; that is, excluding all masses at a distance greater than nine or ten miles (in the Isle of Wight Stations, three miles). In the second system, which we shall designate B, the action of more distant masses is generally included. This system is the one we have just been considering.

Let us next compare the system of deflections A with those resulting from the surface which renders $\Sigma\,(\xi^2) + \Sigma\,(\eta^2)$ a minimum:—

Names.	From the Ground.	By Geodetical Inference.	Difference.	Names.	From the Ground.	By Geodetical Inference.	Difference.
	″	″	″		″	″	″
Dunnose	− 1·02	− 1·62	− 0·60	Forth	− 0·17	+ 0·26	+ 0·43
Boniface	+ 1·94	+ 0·80	− 1·14	Tawnaghmore . . .	− 1·43	− 0·95	+ 0·48
Week Down . . .	+ 1·50	+ 0·58	− 0·92	Lough Foyle . . .	− 2·15	− 4·48	− 2·33
Port Valley	+ 2·81	+ 1·61	− 1·20	Kellie Law	+ 2·08	+ 1·82	...
Clifton	− 0·90	− 2·56	...	Monach	+ 0·47	+ 1·36	...
Burleigh Moor . . .	− 3·03	− 3·54	− 0·51	Ben Hutig	− 1·63	− 2·86	− 1·23
Hungry Hill . . .	+ 3·85	+ 2·92	− 0·93	Calton Hill, Edinburgh	− 2·43	− 5·30	− 2·87
Feaghmaan	− 1·95	− 0·88	...	(*Cowhythe*)	(− 2″)	− 9·55	− 7·55

It appears from this, that by omitting the deflection apparently due to *distant* masses, the two systems are brought to more close agreement in eight cases, and in four cases are rendered more discordant. In the remaining four cases, it is uncertain what would be the effect of extending the calculation over a greater extent of country. The average amount of the quantities in the second column is now 1″·82.

The equatorial radius of the spheroid we are considering is—

$$a = 20923713 + 10144\,u = 20926181 \text{ feet.}$$

It will be convenient to use a symbol 2ϵ for the square of the eccentricity: and for the *compression*, let

$$\frac{a-b}{a} = \frac{1}{c}$$

Then we shall have—

$$\frac{1}{c} = 1 - \sqrt{1 - 2\epsilon} = \epsilon + \tfrac{1}{2}\epsilon^2 + \cdots\cdot$$

Now, $\epsilon = 0{\cdot}00333527 + v\,.\,\text{arc}\ 50''$, therefore in the present instance $\epsilon = {\cdot}00355145$, and

$$\frac{1}{c} = {\cdot}00355775 \qquad \therefore\ c = 281{\cdot}08$$

If u and v be liable to errors δu and δv, then a will be liable to the error 10144 δu, and to find the error of the quantity c resulting from δv we have—

$$-\frac{1}{c^2}\,\frac{dc}{dv} = (1+\epsilon)\,\frac{d\epsilon}{dv} = (1+\epsilon)\ \text{arc}\ 50''$$

Hence, with sufficient precision—

$$\delta c = -\,c^2\,.\,\text{arc}\ 50''\,.\,\delta v$$

In the expression for u, page 697, the sum of the squares of the coefficients of the m's is 0·000878, and the sum of the squares of the coefficients of the n's is 0·0000001 .... In the expression for v these quantities are 0·0102 and 0·0322 respectively. The mean square of the quantities ξ is 4·49, and the mean square of the quantities η is 16·13; hence—

$$\text{Probable error of } u = \pm {\cdot}674 \sqrt{\left({\cdot}000878 \times 4{\cdot}49 + \cdots\cdots \quad \right)} = \pm {\cdot}042 \cdots$$

$$\text{Probable error of } v = \pm {\cdot}674 \sqrt{\left({\cdot}0102 \times 4{\cdot}49 + {\cdot}0322 \times 16{\cdot}13\right)} = \pm {\cdot}507 \cdots$$

$$\begin{aligned} \therefore \quad a &= 20926181 \pm 430 \\ c &= 281{\cdot}08 \pm 9{\cdot}75 \end{aligned}$$

It is to be observed that these probable errors are not to be regarded as results of errors of observation: they are the probable errors of these quantities as representatives of an irregular surface. The probable error of a depending upon the errors of observed latitudes would not perhaps exceed ± 50 feet.

2.

There is no general principle by which we can assign to the equations derived from longitudes and azimuths their value relative to the equations derived from observed latitudes. The probable error of a determination of either longitude or azimuth is considerably greater than that of a latitude, but their ratio cannot be generally assigned. In consequence of this difficulty, the equations themselves have been made to contribute to the determination of a relative weight; for the quantities η are much greater than the quantities ξ in the table, page 698, whereas they should not be, but for errors of observation. Let x^2 and y^2 be the mean squares of these systems of quantities, and put $x^2 = i^2 + \alpha^2 + \alpha'^2$, $y^2 = i^2 + \beta^2 + \beta'^2$, where i is the mean value of a *deflection*, α the error due to the astronomical determination of the latitude, α' the error due to the corresponding geodetical determinations; and let β and β' be the errors of the astronomical and geodetical determinations of azimuth or longitude. The quantity y^2 (16·13) is nearly four times x^2 (4·49), but $\beta^2 + \beta'^2$ is considerably more than four times $\alpha^2 + \alpha'^2$.

After due consideration, the quantity $\frac{1}{6}$ was chosen as the representative of the weight that should attach to the equations derived from longitudes and azimuths in obtaining any final conclusions.

We proceed to determine that spheroid which makes $\Sigma\,(\xi^2) + \frac{1}{6}\,\Sigma\,(\eta)^2$ a minimum.

If in the equations at page 695 we make $w = \frac{1}{6}$ and substitute the numerical values of $(a^2)\,(ab)\cdots\cdot$ they become—

$$\begin{aligned} 0 &= \text{A} + 33{\cdot}8047\,\xi + 1{\cdot}3814\,\eta - 149{\cdot}4219\,u + 0{\cdot}7267\,v \\ 0 &= \text{B} + 1{\cdot}3814\,\xi + 6{\cdot}6322\,\eta + 10{\cdot}0502\,u + 5{\cdot}3615\,v \\ 0 &= \text{C} - 149{\cdot}4219\,\xi + 10{\cdot}0502\,\eta + 1905{\cdot}5334\,u + 30{\cdot}6168\,v \\ 0 &= \text{D} + 0{\cdot}7267\,\xi + 5{\cdot}3615\,\eta + 30{\cdot}6168\,u + 10{\cdot}7511\,v \end{aligned}$$

Where the values of A B C D are—

$$
\begin{aligned}
\mathrm{A} &= ma + m_1 a_1 + m_2 a_2 + \cdots + \tfrac{1}{6}(na' + n_1 a_1' + n_2 a_2' + \cdots) \\
\mathrm{B} &= mb + m_1 b_1 + m_2 b_2 + \cdots + \tfrac{1}{6}(nb' + n_1 b_1' + n_2 b_2' + \cdots) \\
\mathrm{C} &= mc + m_1 c_1 + m_2 c_2 + \cdots + \tfrac{1}{6}(nc' + n_1 c_1' + n_2 c_2' + \cdots) \\
\mathrm{D} &= me + m_1 e_1 + m_2 e_2 + \cdots + \tfrac{1}{6}(ne' + n_1 e_1' + n_2 e_2' + \cdots)
\end{aligned}
$$

By elimination we obtain—

$$
\begin{aligned}
\xi &= - {\cdot}04751080\,\mathrm{A} + {\cdot}00703928\,\mathrm{B} - {\cdot}00393806\,\mathrm{C} + {\cdot}01091570\,\mathrm{D} \\
\eta &= + {\cdot}00703928\,\mathrm{A} - {\cdot}25463240\,\mathrm{B} - {\cdot}00014427\,\mathrm{C} + {\cdot}12691850\,\mathrm{D} \\
u &= - {\cdot}00393806\,\mathrm{A} - {\cdot}00014427\,\mathrm{B} - {\cdot}00087846\,\mathrm{C} + {\cdot}00283978\,\mathrm{D} \\
v &= + {\cdot}01091570\,\mathrm{A} + {\cdot}12691850\,\mathrm{B} + {\cdot}00283978\,\mathrm{C} - {\cdot}16513200\,\mathrm{D}
\end{aligned}
$$

Finally, by substituting the values of A B C and D, we get the values of $\xi\ \eta\ u\ v$ in terms of the observed quantities $m, m_1, m_2 \cdots n, n_1, n_2 \cdots$. The values are as follows:—

$$
\begin{aligned}
\xi = & -{\cdot}047511\, m_0 - {\cdot}060862\, m_1 - {\cdot}059589\, m_2 - {\cdot}056566\, m_3 - {\cdot}054063\, m_4 - {\cdot}054871\, m_5 - {\cdot}055005\, m_6 \\
& -{\cdot}055051\, m_7 - {\cdot}055065\, m_8 - {\cdot}052238\, m_9 - {\cdot}043062\, m_{10} - {\cdot}041186\, m_{11} - {\cdot}033163\, m_{12} - {\cdot}031536\, m_{13} \\
& -{\cdot}023466\, m_{14} - {\cdot}044532\, m_{15} - {\cdot}042484\, m_{16} - {\cdot}039815\, m_{17} - {\cdot}024353\, m_{18} - {\cdot}019513\, m_{19} - {\cdot}026054\, m_{20} \\
& -{\cdot}012350\, m_{21} - {\cdot}012207\, m_{22} - {\cdot}010422\, m_{23} - {\cdot}005079\, m_{24} + {\cdot}000298\, m_{25} + {\cdot}001091\, m_{26} + {\cdot}004122\, m_{27} \\
& +{\cdot}010998\, m_{28} + {\cdot}011271\, m_{29} + {\cdot}011324\, m_{30} - {\cdot}014005\, m_{31} - {\cdot}021946\, m_{32} - {\cdot}044060\, m_{33} + {\cdot}001173\, n_0 \\
& +{\cdot}001187\, n_1 + {\cdot}001182\, n_2 + {\cdot}001173\, n_3 + {\cdot}001198\, n_4 + {\cdot}001190\, n_5 + {\cdot}001371\, n_6 + {\cdot}001274\, n_7 \\
& +{\cdot}001494\, n_8 + {\cdot}001238\, n_9 + {\cdot}001155\, n_{10} + {\cdot}001141\, n_{11} + {\cdot}001138\, n_{12} + {\cdot}001057\, n_{13} + {\cdot}001140\, n_{14} \\
& +{\cdot}001138\, n_{15} + {\cdot}001108\, n_{16} + {\cdot}001090\, n_{17} + {\cdot}001205\, n_{18} + {\cdot}001252\, n_{19} + {\cdot}001300\, n_{20} + {\cdot}001465\, n_{21} \\
& +{\cdot}001509\, n_{22} + {\cdot}001433\, n_{23} + {\cdot}001295\, n_{24} + {\cdot}001205\, n_{25} + {\cdot}001180\, n_{26} + {\cdot}001214\, n_{27} + {\cdot}001323\, n_{28} \\
& +{\cdot}001246\, n_{29} + {\cdot}001308\, n_{30} + {\cdot}001427\, n_{31} + {\cdot}001034\, n_{32} + {\cdot}001236\, n_{33} + {\cdot}001323\, n_{34} + {\cdot}001506\, n_{35} \\
& +{\cdot}001306\, n_{36} + {\cdot}001388\, n_{37} + {\cdot}001379\, n_{38} + {\cdot}001255\, n_{39} + {\cdot}001204\, n_{40}
\end{aligned}
$$

$$
\begin{aligned}
\eta = & +{\cdot}007039\, m_0 - {\cdot}040944\, m_1 - {\cdot}036422\, m_2 - {\cdot}027327\, m_3 - {\cdot}016224\, m_4 - {\cdot}015175\, m_5 - {\cdot}015610\, m_6 \\
& -{\cdot}015765\, m_7 - {\cdot}015676\, m_8 - {\cdot}008911\, m_9 + {\cdot}005356\, m_{10} + {\cdot}015905\, m_{11} + {\cdot}024441\, m_{12} + {\cdot}029847\, m_{13} \\
& +{\cdot}035827\, m_{14} + {\cdot}004661\, m_{15} + {\cdot}010110\, m_{16} + {\cdot}010742\, m_{17} + {\cdot}032257\, m_{18} + {\cdot}029407\, m_{19} + {\cdot}027959\, m_{20} \\
& +{\cdot}024831\, m_{21} + {\cdot}026761\, m_{22} + {\cdot}022772\, m_{23} + {\cdot}016148\, m_{24} - {\cdot}003049\, m_{25} - {\cdot}006504\, m_{26} - {\cdot}018855\, m_{27} \\
& -{\cdot}054920\, m_{28} - {\cdot}056896\, m_{29} - {\cdot}057503\, m_{30} + {\cdot}028062\, m_{31} + {\cdot}035488\, m_{32} - {\cdot}011966\, m_{33} - {\cdot}042439\, n_0 \\
& -{\cdot}019996\, n_1 - {\cdot}031215\, n_2 - {\cdot}043116\, n_3 + {\cdot}000756\, n_4 + {\cdot}030416\, n_5 - {\cdot}025011\, n_6 - {\cdot}034552\, n_7 \\
& -{\cdot}008562\, n_8 - {\cdot}020557\, n_9 - {\cdot}044546\, n_{10} - {\cdot}045738\, n_{11} - {\cdot}047148\, n_{12} - {\cdot}052809\, n_{13} - {\cdot}044301\, n_{14} \\
& -{\cdot}043301\, n_{15} - {\cdot}046010\, n_{16} - {\cdot}048902\, n_{17} - {\cdot}040024\, n_{18} - {\cdot}035868\, n_{19} - {\cdot}030527\, n_{20} - {\cdot}017214\, n_{21} \\
& -{\cdot}009738\, n_{22} - {\cdot}017404\, n_{23} - {\cdot}025968\, n_{24} - {\cdot}033468\, n_{25} - {\cdot}034851\, n_{26} - {\cdot}030481\, n_{27} - {\cdot}016401\, n_{28} \\
& -{\cdot}025221\, n_{29} - {\cdot}017843\, n_{30} - {\cdot}003647\, n_{31} - {\cdot}021299\, n_{32} - {\cdot}025589\, n_{33} - {\cdot}017426\, n_{34} + {\cdot}012011\, n_{35} \\
& -{\cdot}013715\, n_{36} - {\cdot}004413\, n_{37} - {\cdot}003750\, n_{38} - {\cdot}020315\, n_{39} - {\cdot}025021\, n_{40}
\end{aligned}
$$

$$
\begin{aligned}
u = & -{\cdot}003938\, m_0 - {\cdot}007186\, m_1 - {\cdot}006860\, m_2 - {\cdot}006149\, m_3 - {\cdot}005511\, m_4 - {\cdot}005666\, m_5 - {\cdot}005697\, m_6 \\
& -{\cdot}005708\, m_7 - {\cdot}005711\, m_8 - {\cdot}005062\, m_9 - {\cdot}003033\, m_{10} - {\cdot}002516\, m_{11} - {\cdot}000724\, m_{12} - {\cdot}000320\, m_{13} \\
& +{\cdot}001500\, m_{14} - {\cdot}003498\, m_{15} - {\cdot}003042\, m_{16} - {\cdot}002334\, m_{17} + {\cdot}001096\, m_{18} + {\cdot}002240\, m_{19} + {\cdot}000834\, m_{20} \\
& +{\cdot}003879\, m_{21} + {\cdot}003954\, m_{22} + {\cdot}004248\, m_{23} + {\cdot}005526\, m_{24} + {\cdot}006571\, m_{25} + {\cdot}006779\, m_{26} + {\cdot}007386\, m_{27} \\
& +{\cdot}008898\, m_{28} + {\cdot}008953\, m_{29} + {\cdot}008962\, m_{30} + {\cdot}003550\, m_{31} + {\cdot}001831\, m_{32} - {\cdot}003231\, m_{33} - {\cdot}000024\, n_0 \\
& +{\cdot}000001\, n_1 - {\cdot}000011\, n_2 - {\cdot}000025\, n_3 + {\cdot}000026\, n_4 + {\cdot}000058\, n_5 + {\cdot}000011\, n_6 - {\cdot}000008\, n_7 \\
& +{\cdot}000041\, n_8 + {\cdot}000011\, n_9 - {\cdot}000028\, n_{10} - {\cdot}000030\, n_{11} - {\cdot}000033\, n_{12} - {\cdot}000044\, n_{13} - {\cdot}000028\, n_{14} \\
& -{\cdot}000027\, n_{15} - {\cdot}000032\, n_{16} - {\cdot}000037\, n_{17} - {\cdot}000019\, n_{18} - {\cdot}000011\, n_{19} - {\cdot}000001\, n_{20} + {\cdot}000027\, n_{21} \\
& +{\cdot}000040\, n_{22} + {\cdot}000025\, n_{23} + {\cdot}000006\, n_{24} - {\cdot}000009\, n_{25} - {\cdot}000012\, n_{26} - {\cdot}000004\, n_{27} + {\cdot}000021\, n_{28} \\
& +{\cdot}000005\, n_{29} + {\cdot}000018\, n_{30} + {\cdot}000044\, n_{31} - {\cdot}000008\, n_{32} + {\cdot}000004\, n_{33} + {\cdot}000019\, n_{34} + {\cdot}000069\, n_{35} \\
& +{\cdot}000024\, n_{36} + {\cdot}000041\, n_{37} + {\cdot}000041\, n_{38} + {\cdot}000012\, n_{39} + {\cdot}000003\, n_{40}
\end{aligned}
$$

$$
\begin{aligned}
v = & +{\cdot}010916\, m_0 + {\cdot}063666\, m_1 + {\cdot}060099\, m_2 + {\cdot}047650\, m_3 + {\cdot}037839\, m_4 + {\cdot}040490\, m_5 + {\cdot}041103\, m_6 \\
& +{\cdot}041252\, m_7 + {\cdot}041244\, m_8 + {\cdot}030409\, m_9 - {\cdot}001986\, m_{10} - {\cdot}007424\, m_{11} - {\cdot}027158\, m_{12} - {\cdot}031291\, m_{13} \\
& -{\cdot}043734\, m_{14} - {\cdot}012560\, m_{15} - {\cdot}022062\, m_{16} - {\cdot}015329\, m_{17} - {\cdot}060003\, m_{18} - {\cdot}053359\, m_{19} - {\cdot}041316\, m_{20} \\
& -{\cdot}046593\, m_{21} - {\cdot}044440\, m_{22} - {\cdot}051035\, m_{23} - {\cdot}033576\, m_{24} - {\cdot}024663\, m_{25} - {\cdot}016414\, m_{26} - {\cdot}006375\, m_{27} \\
& +{\cdot}046426\, m_{28} + {\cdot}048734\, m_{29} + {\cdot}049269\, m_{30} - {\cdot}045854\, m_{31} - {\cdot}045704\, m_{32} + {\cdot}031967\, m_{33} + {\cdot}021153\, n_0 \\
& -{\cdot}006524\, n_1 + {\cdot}007308\, n_2 + {\cdot}021995\, n_3 - {\cdot}032248\, n_4 - {\cdot}068812\, n_5 - {\cdot}000838\, n_6 + {\cdot}010967\, n_7 \\
& -{\cdot}020648\, n_8 - {\cdot}003659\, n_9 + {\cdot}023700\, n_{10} + {\cdot}025229\, n_{11} + {\cdot}026742\, n_{12} + {\cdot}034323\, n_{13} + {\cdot}023753\, n_{14} \\
& +{\cdot}022755\, n_{15} + {\cdot}026196\, n_{16} + {\cdot}029586\, n_{17} + {\cdot}018018\, n_{18} + {\cdot}012766\, n_{19} + {\cdot}006227\, n_{20} - {\cdot}010711\, n_{21} \\
& -{\cdot}019535\, n_{22} - {\cdot}010048\, n_{23} + {\cdot}001394\, n_{24} + {\cdot}011037\, n_{25} + {\cdot}012983\, n_{26} + {\cdot}007667\, n_{27} - {\cdot}009583\, n_{28} \\
& +{\cdot}001371\, n_{29} - {\cdot}007738\, n_{30} - {\cdot}025283\, n_{31} - {\cdot}003263\, n_{32} + {\cdot}001942\, n_{33} - {\cdot}008428\, n_{34} - {\cdot}044056\, n_{35} \\
& -{\cdot}012331\, n_{36} - {\cdot}023929\, n_{37} - {\cdot}024561\, n_{38} - {\cdot}004198\, n_{39} + {\cdot}001849\, n_{40}
\end{aligned}
$$

Substituting the numerical values of the *m*'s and *n*'s from the Equations of Condition at pages 693, 694, the values of ξ η u and v are found to be—

$$\begin{aligned} \xi &= +1''{\cdot}491 \\ \eta &= -0{\cdot}888 \\ u &= +0{\cdot}2479 \\ v &= +1{\cdot}5397 \end{aligned}$$

These quantities do not differ materially, with the exception of the value of v, from those which render the sum of the squares of all a minimum.

The system of corrections resulting from these values are as in the following table:—

Names.	ξ	Names.	ξ	Names.	η	Names.	η
Greenwich . .	+1″·491	Forth . . .	+0″·358	Greenwich . .	−0″·888	Barrow Hill . .	−5″·851
St. Agnes . .	−1·299	Tawnaghmore .	−0·723	Edinburgh . .	−4·361	Lundy Island .	+3·766
Goonhilly . .	−2·932	Lough Foyle .	−4·306	Durham . . .	−4·257	Dunkery . . .	−2·212
Hensbarrow .	−1·297	South Berule .	+1·955	Cambridge . .	+3·897	Mowcopt . . .	+5·290
Black Down .	+3·038	Ben Lomond .	−1·181	Kingstown . .	+1·666	York Minster .	−2·998
Dunnose . .	−1·698	Kellie Law . .	+1·917	Feaghmaan . .	+8·928	Easington . .	−4·756
Boniface Down.	+0·714	Ben Heynish .	+2·560	Black Down .	−5·902	Wordeslow . .	−5·233
Week Down .	+0·494	Great Stirling .	+0·075	Dunnose . .	−2·977	Hart Fell . .	+5·172
H. P. Cliff . .	+1·526	Monach . . .	+1·334	Precelly . . .	+2·358	Cheviot . . .	−3·336
Southampton .	+1·845	Ben Hutig . .	−2·931	Cowhythe . .	+0·873	Dunrich . . .	−5·970
Precelly . . .	+1·022	North Rona. .	+0·564	Wrotham . .	+0·532	Goat Fell. . .	+0·177
Arbury . . .	+2·009	Balta . . .	+2·219	Gad's Hill . .	+1·516	Sayrs Law . .	−0·082
Delamere . .	−0·301	Gerth of Scaw .	+2·058	Fairlight Down	+1·783	Mordington . .	−1·446
Clifton . . .	−2·434	Saxavord . .	+0·072	Orford Castle .	+3·248	Burnswark . .	+3·479
Burleigh Moor .	−3·402	Edinburgh . .	−5·190	Balsham . .	+5·706	Ben More, S. U..	+1·890
Hungry Hill .	+3·036	Durham . .	−0·446	Walpole, S. P. .	+5·497	Ben Macdui . .	+2·461
Feaghmaan . .	−0·733	Cambridge . .	+0·511	Docking . .	+3·511	Ben Nevis . .	+8·686
				South Lopham .	+1·881	Scournalapich .	−1·741
				Leith Hill . .	−2·417	Mount Battock .	−4·008
Cowhythe (ξ) = − 9″·543				Butser Hill . .	−4·209	Mormonth . .	−6·510
				Beacon Hill .	−1·174		

The comparison of these quantities with the deflections A and B, is as follows:—

Names.	By Geod. Inference.	Defl. A	Defl. B	Diff. A	Diff. B
Dunnose	− 1″·70	− 1″·02	− 0″·54	− 0″·68	− 1″·16
Boniface	+ 0·71	+ 1·94	+ 2·42	− 1·23	− 1·71
Week Down	+ 0·49	+ 1·50	+ 1·98	− 1·01	− 1·49
Port Valley	+ 1·53	+ 2·81	+ 3·29	− 1·28	− 1·76
Clifton	− 2·43	− 0·90	...	− 1·53	...
Burleigh Moor . . .	− 3·40	− 3·03	− 4·55	− 0·37	+ 1·15
Hungry Hill	+ 3·04	+ 3·85	+ 5·40	− 0·81	− 2·36
Feaghmaan	− 0·73	− 1·95	...	+ 1·22	...
Forth	+ 0·36	− 0·17	+ 1·13	+ 0·53	− 0·77
Tawnaghmore . . .	− 0·72	− 1·43	− 2·30	+ 0·71	+ 1·58
Lough Foyle	− 4·31	− 2·15	− 4·02	− 2·16	− 0·29
Kellie Law	+ 1·92	+ 2·08	...	− 0·16	...
Monach	+ 1·33	+ 0·47	...	+ 0·86	...
Ben Hutig	− 2·93	− 1·63	− 2·01	− 1·30	− 0·92
Calton Hill, Edinburgh .	− 5·19	− 2·43	− 3·57	− 2·76	− 1·62
(*Cowhythe*)	− 9·54	(− 2″)	(− 5″)	− 7·54	− 4·54

It appears, therefore, that, as in the case of the spheroid that renders $\Sigma(\xi^2) + \Sigma(\eta^2)$ a minimum, the deflections A agree with the quantities in the first column more closely in eight cases, and less closely in four cases, than the deflections B. If we neglect Cowhythe, the average amount (independent of sign) of the differences A is $1''\cdot 11$, while the average of the differences B is $1''\cdot 35$. If we include Cowhythe, the average of the differences A will still be less than the average of the differences B.

The elements of this spheroid are—

$$\begin{aligned} a &= 20923713 + 10144 \,.\, u = 20926228 \\ \epsilon &= \cdot 00333527 + \text{arc } 50'' \,.\, v = \cdot 00370850 \\ c &= 269\cdot 15 \end{aligned}$$

The sum of the squares of the 34 quantities ξ corresponding to this spheroid is 146·86, and the sum of the squares of the 41 quantities η is 680·41.

The sum of the squares of the coefficients of the *m*'s in the expression for *u*, page 703, is ·000878, and the sum of the squares of the coefficients of the *n*'s may be called zero. The corresponding quantities with reference to *v* are ·0528 and ·0187. Therefore, for the probable errors of *u* and *v* we have—

$$\text{Probable error of } u = \cdot 674 \sqrt{\left(\cdot 000878 \times 4\cdot 32 + \cdots\cdot \qquad\right)} = \pm \cdot 042 \cdots$$

$$\text{Probable error of } v = \cdot 674 \sqrt{\left(\cdot 0528 \times 4\cdot 32 + \cdot 0187 \times 16\cdot 60\right)} = \pm \cdot 495 \cdots$$

$$\begin{aligned} \therefore \quad a &= 20926228 \pm 422 \\ c &= 269\cdot 15 \pm 8\cdot 69 \end{aligned}$$

3.

We have now determined two spheroids which represent the actual surface of Great Britain and Ireland with some precision. Of these, the former represents the *whole* of the Astronomical Observations with the greatest exactness, without distinguishing between determinations of latitude and longitude or azimuth. In the latter spheroid allowance is made—but necessarily in a somewhat arbitrary manner—for the greater exactness of the latitude determinations. The major axes of these spheroids are very nearly identical, but the minor axis of the latter is considerably less than that of the former.

In Section XI. we have determined for several Stations the apparent effects of the irregularities of the distribution of matter on the surface of the adjacent country, and by making use of these quantities we may obtain the latitudes that would belong to those points, supposing the hills and valleys to be cut down on every side to one level plane. If it were proved by observation that latitudes observed in perfectly flat countries presented no discrepancies when geodetically compared, it might be assumed as probable that latitudes corrected for the effect of external irregularities would coincide with the mean figure of the earth. But inasmuch as latitudes observed in flat countries have not been found free from discrepancies when geodetically compared, it must be assumed that every latitude is affected by two distinct sources of disturbance; namely, superincumbent irregularly-disposed masses, and irregularities in the distribution of matter below the surface. Of the amount of error attributable to the latter source we must be in every case totally ignorant, but it is always possible to make an estimate of the effect due to the former. By thus removing *one* of these sources of error we shall in all probability arrive at a result more near the mean latitude than is the immediately observed value.

If we take any three of the equations at page 693, and put $\eta = 0$, a spheroid will be determined which will have its surface parallel to the actual surface at those points (that is, in a northerly direction). This spheroid will, however, be restricted to having its surface at Greenwich parallel to the actual surface in an east and west direction: this, however, is no material restriction, for we have seen already that η is a very small quantity, and, moreover, it has little effect in the latitude equations.

1. Let us first examine the points Dunnose, Clifton, and Burleigh Moor. Leaving the absolute term symbolical, the equations are—

$$\begin{aligned} 0 &= m_1 + 0{\cdot}9998\,\xi + 1{\cdot}5109\,u - 0{\cdot}1406\,v \\ 0 &= m_2 + 0{\cdot}9998\,\xi - 3{\cdot}4447\,u + 0{\cdot}2091\,v \\ 0 &= m_3 + 0{\cdot}9998\,\xi - 5{\cdot}3873\,u + 0{\cdot}2492\,v \end{aligned}$$

Whence—

$$\begin{aligned} \xi &= -\ 0{\cdot}5579\,m_1 - 0{\cdot}7928\,m_2 + 0{\cdot}3504\,m_3 \\ u &= +\ 0{\cdot}0834\,m_1 - 0{\cdot}8110\,m_2 + 0{\cdot}7276\,m_3 \\ v &= +\ 4{\cdot}0419\,m_1 - 14{\cdot}3527\,m_2 + 10{\cdot}3109\,m_3 \end{aligned}$$

The system of deflections A makes—

$$m_1 = -\ 2{\cdot}312 \qquad m_2 = -\ 2{\cdot}478 \qquad m_3 = -\ 0{\cdot}898$$

which give—

$$\xi = +\ 2''{\cdot}94 \qquad u = +\ 1{\cdot}1635 \qquad v = +\ 16{\cdot}96$$

The system of deflections B makes—

$$m_1 = -\ 2{\cdot}792 \qquad m_2 = -\ 2{\cdot}478 \qquad m_3 = +\ 0{\cdot}622$$

which give—

$$\xi = +\ 3''{\cdot}74 \qquad u = +\ 2{\cdot}2294 \qquad v = +\ 30{\cdot}69$$

2. Take now the points Boniface Down, Kellie Law, Monach. If we put m_1, m_2, m_3 for the absolute terms, we find—

$$\begin{aligned} \xi &= -\ 0{\cdot}7306\,m_1 - 0{\cdot}5885\,m_2 + 0{\cdot}3201\,m_3 \\ u &= -\ 0{\cdot}0532\,m_1 - 0{\cdot}0842\,m_2 + 0{\cdot}1381\,m_3 \\ v &= +\ 1{\cdot}3069\,m_1 - 4{\cdot}9910\,m_2 + 3{\cdot}7010\,m_3 \end{aligned}$$

By using the system A of deflections, we get—

$$\xi = +\ 2''{\cdot}75 \qquad u = +\ 0{\cdot}4530 \qquad v = +\ 4{\cdot}08$$

The system of deflections B gives—

$$\xi = +\,3''\!\cdot\!10 \qquad u = +\,0\cdot4785 \qquad v = +\,3\cdot45$$

3. The three points Week Down, Burleigh Moor, Calton Hill, treated in like manner, give—

$$\begin{aligned} \xi &= -\,0\cdot6387\,m_1 - 0\cdot7682\,m_2 + 0\cdot4072\,m_3 \\ u &= -\,0\cdot0051\,m_1 - 0\cdot4038\,m_2 + 0\cdot4095\,m_3 \\ v &= +\,2\cdot4518\,m_1 - 9\cdot6607\,m_2 + 7\cdot2179\,m_3 \end{aligned}$$

Using the system A of deflections, we have—

$$\xi = +\,1''\!\cdot\!29 \qquad u = -\,0\cdot7163 \qquad v = -\,17\cdot06$$

The system of deflections B gives—

$$\xi = +\,0''\!\cdot\!89 \qquad u = -\,\cdot8609 \qquad v = -\,24\cdot69$$

4. The points High Port Cliff, Lough Foyle, and Monach, treated in like manner, give—

$$\begin{aligned} \xi &= -\,0\cdot7475\,m_1 - 0\cdot3112\,m_2 + 0\cdot0566\,m_3 \\ u &= -\,0\cdot0556\,m_1 - 0\cdot0445\,m_2 + 0\cdot1004\,m_3 \\ v &= +\,1\cdot1519\,m_1 - 2\cdot6303\,m_2 + 1\cdot4679\,m_3 \end{aligned}$$

By using the system A of deflections, we have—

$$\xi = +\,3''\!\cdot\!14 \qquad u = +\,0\cdot5051 \qquad v = +\,6\cdot91$$

The system B of deflections gives—

$$\xi = +\,2''\!\cdot\!91 \qquad u = +\,0\cdot4486 \qquad v = +\,1\cdot43$$

The elements of these eight spheroids are shown concisely in the following table:—

Spheroids.	Deflections A			Deflections B		
	ξ	u	v	ξ	u	v
1	+ 2″·94	+ 1·16	+16·96	+ 3″·74	+ 2·23	+30·69
2	+ 2·75	+ 0·45	+ 4·08	+ 3·10	+ 0·48	+ 3·45
3	+ 1·29	− 0·72	−17·06	+ 0·89	− 0·86	−24·69
4	+ 3·14	+ 0·50	+ 6·91	+ 2·91	+ 0·45	+ 1·43

In examining and comparing these results, it must be borne in mind that each quantity is liable to considerable probable error, resulting from the probable errors of the astronomical determinations of latitude, on account of the magnitude of the coefficients of the quantities m, especially in 1 and 3. Giving due weight to this consideration, it is a fair conclusion that the points we have considered do not lay upon the surface of any one spheroid, whether we use the deflections A or the deflections B. The results produced by the former system are, however, *less discordant* than those produced by the latter.

5. Let us next determine the spheroid which best represents the latitudes of the firs thirteen points in the table at page 664, when corrected for calculated deflection. If σ^2 be the sum of the squares of the thirteen quantities ξ for these points, then we must make $\sigma^2 + \xi^2 + \eta^2$ a minimum, otherwise η will come out a large quantity altogether beyond the truth, and ξ will be similarly affected, though only slightly.

If we correct the thirteen latitudes for the system of deflections A, the thirteen absolute terms become—

$$\begin{array}{lll}
m_5 = -\,2{\cdot}312 & m_{13} = -\,2{\cdot}478 & m_{19} = -\,2{\cdot}659 \\
m_6 = -\,2{\cdot}862 & m_{14} = -\,0{\cdot}898 & m_{22} = +\,0{\cdot}089 \\
m_7 = -\,2{\cdot}643 & m_{15} = -\,2{\cdot}636 & m_{25} = +\,2{\cdot}233 \\
m_8 = -\,2{\cdot}922 & m_{17} = -\,0{\cdot}911 & m_{26} = +\,0{\cdot}284 \\
 & m_{31} = -\,2{\cdot}668 &
\end{array}$$

The equations are found to be—

$$\begin{aligned}
0 &= -\,20{\cdot}3264 + 13{\cdot}9170\,\xi + 0{\cdot}6393\,\eta - 49{\cdot}0302\,u + 1{\cdot}2441\,v \\
0 &= -\,0{\cdot}8196 + 0{\cdot}6393\,\xi + 1{\cdot}0509\,\eta - 3{\cdot}0501\,u + 0{\cdot}1100\,v \\
0 &= +\,2{\cdot}8677 - 49{\cdot}0302\,\xi - 3{\cdot}0501\,\eta + 499{\cdot}2936\,u - 9{\cdot}9034\,v \\
0 &= -\,1{\cdot}4616 + 1{\cdot}2441\,\xi + 0{\cdot}1100\,\eta - 9{\cdot}9034\,u + 0{\cdot}5395\,v
\end{aligned}$$

From which by elimination—

$$\begin{aligned}
\xi &= +\,2{\cdot}132 \\
\eta &= -\,0{\cdot}040 \\
u &= +\,0{\cdot}2512 \\
v &= +\,2{\cdot}4124
\end{aligned}$$

Substituting these in the equations from which they were derived, we have the following system—

	″		″
Greenwich	+ 2·132	*Hungry Hill*	+ 0·149
Dunnose	− 0·141	*Forth*	+ 1·418
Boniface Down	− 0·692	*Lough Foyle*	− 1·143
Week Down	− 0·471	*Kellie Law*	+ 0·675
H. P. Cliff	− 0·751	*Monach*	+ 1·605
Clifton	− 0·707	*Ben Hutig*	− 0·647
Burleigh Moor	+ 0·481	*Edinburgh*	− 1·896

The sum of the squares of these quantities, together with that of η, is 16·94 ···· whence the mean error—

$$= \sqrt{\frac{16{\cdot}94}{15-4}} = \pm\,1''{\cdot}241$$

If we apply these values of $\xi\ \eta\ u\ v$ to the northern portions of the arc, we get for Saxavord + 0·226, which is not much out of the way. But when we apply them to the longitude of Feaghmaan, we get for that point $\eta = +\,12{\cdot}719$.

6. If we had used the system B of deflections, we should have obtained a different system of quantities. In this case—

$$\begin{array}{lll} m_5 = -\ 2\cdot792 & m_{13} = -\ 2\cdot478 & m_{19} = -\ 0\cdot789 \\ m_6 = -\ 3\cdot342 & m_{14} = +\ 0\cdot622 & m_{22} = +\ 0\cdot089 \\ m_7 = -\ 3\cdot123 & m_{15} = -\ 4\cdot186 & m_{25} = +\ 2\cdot233 \\ m_8 = -\ 3\cdot402 & m_{17} = -\ 2\cdot211 & m_{26} = +\ 0\cdot664 \\ & m_{31} = -\ 1\cdot528 & \end{array}$$

And the equations are—

$$\begin{aligned} 0 &= -\ 20\cdot1721 + 13\cdot9170\,\xi + 0\cdot6393\,\eta - 49\cdot0302\,u + 1\cdot2441\,v \\ 0 &= -\ 0\cdot9010 + 0\cdot6393\,\xi + 1\cdot0509\,\eta - 3\cdot0501\,u + 0\cdot1100\,v \\ 0 &= -\ 31\cdot7104 - 49\cdot0302\,\xi - 3\cdot0501\,\eta + 499\cdot2936\,u - 9\cdot9034\,v \\ 0 &= -\ 0\cdot5088 + 1\cdot2441\,\xi + 0\cdot1100\,\eta - 9\cdot9034\,u + 0\cdot5395\,v \end{aligned}$$

From these by elimination—

$$\begin{aligned} \xi &= +\ 2\cdot513 \\ \eta &= +\ 0\cdot171 \\ u &= +\ 0\cdot3372 \\ v &= +\ 1\cdot3021 \end{aligned}$$

Substituting these in the equations from which they are derived, we have for

	″		″
Greenwich	+ 2·513	*Hungry Hill*	− 1·244
Dunnose	+ 0·050	*Forth*	+ 0·187
Boniface Down . . .	− 0·496	*Lough Foyle*	+ 0·221
Week Down	− 0·274	*Kellie Law*	+ 0·107
H. P. Cliff	− 0·552	*Monach*	+ 0·908
Clifton	− 0·852	*Ben Hutig*	− 0·925
Burleigh Moor . . .	+ 1·645	*Edinburgh*	− 1·302

The sum of the squares of these quantities, together with that of η, is $15\cdot42$ .... whence the mean error—

$$\sqrt{\frac{15\cdot42}{15-4}} = \pm\ 1\cdot184$$

which is rather smaller than the corresponding quantity from the system A of deflections.

If we apply the present values of $\xi\ \eta\ u\ v$ to the extremities of the Triangulation, namely, Feaghmaan and Saxavord, we obtain for the latter $\xi = -\ 0''\cdot230$, and for the former $+\ 11''\cdot175$. These are nearer the probable truth than the corresponding quantities derived in the last article by using the deflections A.

7. It appears, then, that whether we use the system A or B of deflections, the corrected latitudes will still present discrepancies greater than the probable errors of observation, and of which the mean amount is $\pm\ 1''\cdot2$. A comparison of the results in the last two articles is in favour of the second system of deflections.

4.

The values of $\xi\ \eta\ u\ v$ which make $\Sigma\,(\xi^2) + \frac{1}{6}\,\Sigma\,(\eta^2)$ a minimum, as given at page 703, being in terms of the observations explicitly, we can easily find the modifications the quantities already obtained will undergo from the correction of the observed latitudes by calculated deflections; and in the first place we shall use the deflections A.

If A_n be the coefficient of m_n in the expression for ξ, then the value of ξ at page 704, namely, 1·491, will receive the increment—

$$\begin{aligned} &1{\cdot}02\,A_5 - 1{\cdot}94\,A_6 - 1{\cdot}50\,A_7 - 2{\cdot}81\,A_8 + 0{\cdot}90\,A_{13} + 3{\cdot}03\,A_{14} - 3{\cdot}85\,A_{15} \\ &+ 0{\cdot}17\,A_{17} + 2{\cdot}15\,A_{19} - 2{\cdot}08\,A_{22} - 0{\cdot}47\,A_{25} + 1{\cdot}63\,A_{26} + 2{\cdot}43\,A_{31} \end{aligned}$$

By actual multiplication the value of this quantity is found to be $+\,0\cdot304$. In the same manner the quantities

$$+\,0{\cdot}268 \qquad +\,0{\cdot}0604 \qquad -\,0{\cdot}4800$$

are obtained for the increments to the values of $\eta\ u\ v$ at page 704. Thus we have—

$$\begin{array}{cccc} +\,1{\cdot}491 & -\,0{\cdot}888 & +\,0{\cdot}2479 & +\,1{\cdot}5397 \\ \underline{+\,0{\cdot}304} & \underline{+\,0{\cdot}268} & \underline{+\,0{\cdot}0604} & \underline{-\,0{\cdot}4800} \\ \therefore \xi = +\,1{\cdot}795 & \therefore \eta = -\,0{\cdot}620 & \therefore u = +\,0{\cdot}3083 & \therefore v = +\,1{\cdot}0597 \end{array}$$

The substitution of these quantities in the Equations of Condition—the absolute terms being corrected for the deflections A—furnishes the results shown in the following table:—

Stations.	ξ	Stations.	ξ	Stations.	η	Stations.	η
Greenwich	+1″·795	Forth	+0″·685	Greenwich	−0″·620	Barrow Hill	−5″·953
St. Agnes	−0·692	Tawnaghmore	−0·861	Edinburgh	−4·406	Lundy Island	+3·561
Goonhilly	−2·353	Lough Foyle	−2·356	Durham	−4·147	Dunkery	−2·306
Hensbarrow	−0·785	South Berule	+1·865	Cambridge	+4·175	Mowcopt	+5·327
Blackdown	+3·490	Ben Lomond	−1·472	Kingstown	+1·327	York Minster	−2·849
Dunnose	−0·211	Kellie Law	−0·455	Feaghmaan	+8·175	Easington	−4·583
Boniface Down	−0·756	Ben Heynish	+2·239	Black Down	−5·889	Wordeslow	−5·123
Week Down	−0·535	Great Stirling	−0·296	Dunnose	−2·827	Hart Fell	+5·083
H. P. Cliff	−0·813	Monach	+0·444	Precelly	+2·140	Cheviot	−3·298
Southampton	+2·255	Ben Hutig	−1·718	Cowhythe	+0·852	Dunrich	−6·038
Precelly	+1·248	North Rona	+0·126	Wrotham	+0·830	Goat Fell	−0·094
Arbury	+2·191	Balta	+1·781	Gad's Hill	+1·831	Sayrs Law	−0·089
Delamere	−0·268	Gerth of Scaw	+1·621	Fairlight	+2·116	Mordington	−1·402
Clifton	−1·534	Saxavord	−0·364	Orford Castle	+3·668	Burnswark	+3·402
Burleigh Moor	−0·509	Edinburgh	−3·029	Balsham	+6·004	Ben More, S. U.	+1·402
Hungry Hill	−0·578	Durham	−0·607	Walpole, S. P.	+5·783	Ben Macdui	+2·339
Feaghmaan	−0·542	Cambridge	+0·691	Docking	+3·837	Ben Nevis	+8·430
				South Lopham	+2·247	Scournalapich	−2·004
				Leith Hill	−2·186	Mount Battock	−4·035
				Butser	−4·038	Mormonth	−6·467
				Beacon Hill	−1·079		

Cowhythe (ξ) = − 9″·925

The elements of the spheroid under consideration are—

$$a = 20923713 + 10144.1\,u = 20926840$$
$$\varepsilon = \cdot 00333527 + \text{arc } 50''.\,v = \cdot 003359215$$
$$c = 277\cdot 88$$

The sums of the squares of the quantities ξ, η, are $74\cdot 81 \cdots$ and $669\cdot 00 \cdots$ respectively: hence—

$$\text{Probable error of } u = \cdot 674 \sqrt{\left(\cdot 000878 \times 2\cdot 20 + \cdots\cdots \right)} = \pm \cdot 030 \cdots$$

$$\text{Probable error of } v = \cdot 674 \sqrt{\left(\cdot 0528 \times 2\cdot 20 + \cdot 0187 \times 16\cdot 32\right)} = \pm \cdot 437 \cdots$$

$$\therefore\; a = 20926840 \pm 300$$
$$c = 277\cdot 88 \pm 8\cdot 20$$

5.

If A_n be the coefficient of m_n in the expression for ξ at page 703, then if we correct the latitudes observed, for the calculated deflections B, we shall have—

$$\xi = +\,1\cdot 491 \quad +\,0\cdot 54\,A_5 - 2\cdot 42\,A_6 - 1\cdot 98\,A_7 - 3\cdot 29\,A_8 + 0\cdot 90\,A_{13} + 4\cdot 55\,A_{14}$$
$$-\,5\cdot 40\,A_{15} - 1\cdot 13\,A_{17} + 4\cdot 02\,A_{19} - 2\cdot 08\,A_{22} - 0\cdot 47\,A_{25} + 2\cdot 01\,A_{26} + 3\cdot 57\,A_{31}$$

Performing these multiplications, and similar for η u and v, we have finally—

$$\begin{array}{cccc} +\,1\cdot 491 & -\,0\cdot 888 & +\,0\cdot 2479 & +\,1\cdot 5397 \\ \underline{+\,0\cdot 443} & \underline{+\,0\cdot 416} & \underline{+\,0\cdot 0929} & \underline{-\,0\cdot 7441} \end{array}$$

$$\xi = +\,1\cdot 934 \;:\; \eta = -\,0\cdot 472 \;:\; u = +\,0\cdot 3408 \;:\; v = +\,0\cdot 7956$$

The substitution of these quantities in the Equations of Condition—the absolute terms being corrected for the deflections B—furnishes the results shown in the following table:—

Stations.	ξ	Stations.	ξ	Stations.	η	Stations.	η
Greenwich . .	+1″·934	Forth . . .	−0″·555	Greenwich . .	−0″·472	Barrow Hill . .	−6″·010
St. Agnes . .	−0·389	Tawnaghmore .	−0·963	Edinburgh . .	−4·433	Lundy Island .	+3·447
Goonhilly . .	−2·064	Lough Foyle .	−0·619	Durham. . .	−4·086	Dunkery . . .	−2·359
Hensbarrow .	−0·533	South Berule .	+1·791	Cambridge . .	+4·328	Mowcopt . . .	+5·348
Black Down .	+3·711	Ben Lomond .	−1·654	Kingstown . .	+1·138	York Minster .	−2·766
Dunnose . .	−0·463	Kellie Law . .	−0·638	Feaghmaan . .	+7·755	Easington . .	−4·488
Boniface Down .	−1·007	Ben Heynish .	+2·040	Black Down .	−5·883	Wordeslow . .	−5·062
Week Down .	−0·785	Great Stirling .	−0·521	Dunnose . .	−2·744	Hart Fell . .	+5·033
H. P. Cliff . .	−1·063	Monach . . .	+0·194	Precelly. . .	+2·019	Cheviot . . .	−3·278
Southampton .	+2·452	Ben Hutig . .	−1·588	Cowhythe . .	+0·840	Dunrich . . .	−6·076
Precelly. . .	+1·344	North Rona .	−0·133	Wrotham . .	+0·994	Goat Fell . .	−0·245
Arbury . . .	+2·263	Balta	+1·524	Gad's Hill . .	+2·006	Sayrs Law . .	−0·095
Delamere . .	−0·276	Gerth of Scaw .	+1·364	Fairlight Down	+2·300	Mordington . .	−1·378
Clifton . . .	−1·560	Saxavord . .	−0·621	Orford Castle .	+3·901	Burnswark . .	+3·360
Burleigh Moor .	+0·911	Edinburgh . .	−2·060	Balsham . .	+6·169	Ben More, S. U. .	+1·131
Hungry Hill .	−2·026	Durham . .	−0·720	Walpole, S. P. .	+5·942	Ben Macdui . .	+2·272
Feaghmaan . .	−0·465	Cambridge . .	+0·763	Docking . .	+4·017	Ben Nevis . .	+8·288
				South Lopham .	+2·449	Scournalapich .	−2·150
				Leith Hill . .	−2·058	Mount Battock .	−4·051
				Butser Hill . .	−3·944	Mormonth . .	−6·444
				Beacon Hill. .	−1·027		

Cowhythe (ξ) = − 10″·155

The elements of this spheroid are—

$$a = 20923713 + 10144 . 1 u = 20927170$$
$$\epsilon = \cdot 00333527 + \text{arc } 50'' . v = \cdot 00352813$$
$$c = 282 \cdot 94$$

The sums of the squares of the quantities ξ, η, are $70 \cdot 32 \cdots$ and $665 \cdot 01 \cdots$ respectively: hence—

$$\text{Probable error of } u = \cdot 674 \sqrt{\left(\cdot 000878 \times 2 \cdot 07 + \cdots\cdots \qquad \right)} = \pm \cdot 029 \cdots$$

$$\text{Probable error of } v = \cdot 674 \sqrt{\left(\cdot 0528 \times 2 \cdot 07 + \cdot 0187 \times 16 \cdot 22\right)} = \pm \cdot 433 \cdots$$

$$\therefore \; a = 20927170 \pm 291$$
$$c = 282 \cdot 94 \pm 8 \cdot 41$$

6.

Either of the last two spheroids may be taken as a very close representative of the actual surface of Great Britain between the latitudes of 49° and 61°, and for 12° of longitude. We shall adopt the mean of the two sets of values, namely—

$$\xi = + 1 \cdot 864$$
$$\eta = - 0 \cdot 546$$
$$u = + 0 \cdot 3245$$
$$v = + 0 \cdot 9276$$

$$\therefore \; a = 20927005 \pm 295$$
$$c = 280 \cdot 4 \pm 8 \cdot 3$$

These values of $\xi \; \eta \; u \; v$ being substituted in the original Equations of Condition, page 693, we obtain the following results for the probable deflections at the different Stations :—

Stations.	ξ	Unexplained.		Stations.	ξ	Unexplained.	
		Defl. A	Defl. B			Defl. A	Defl. B
	″	″	″		″	″	″
Greenwich	+ 1·864			Forth	+ 0·545	+ 0·715	− 0·585
St. Agnes	− 0·541			Tawnaghmore . . .	− 0·912		
Goonhilly	− 2·209			Lough Foyle . . .	− 4·573	− 2·423	− 0·553
Hensbarrow . . .	− 0·660			South Berule . . .	+ 1·828		
Blackdown	+ 3·600			Ben Lomond . . .	− 1·563		
Dunnose	− 1·117	− 0·097	− 0·577	Kellie Law	+ 1·533	− 0·547	
Boniface Down . .	+ 1·298	− 0·642	− 1·122	Ben Heynish . . .	+ 2·140		
Week Down . . .	+ 1·080	− 0·420	− 0·900	Great Stirling . . .	− 0·409		
H. P. Cliff	+ 2·111	− 0·699	− 1·179	Monach	+ 0·789	+ 0·319	
Southampton . . .	+ 2·353			Ben Hutig	− 3·473	− 1·843	− 1·463
Precelly	+ 1·295			North Rona. . . .	− 0·003		
Arbury	+ 2·226			Balta	+ 1·653		
Delamere	− 0·272			Gerth of Scaw . . .	+ 1·493		
Clifton	− 2·447	− 1·547		Saxavord	− 0·492		
Burleigh Moor . . .	− 3·589	− 0·559	+ 0·961	Edinburgh	− 5·545	− 3·115	− 1·975
Hungry Hill	+ 3·323	− 0·527	− 2·077	Durham	− 0·664		
Feaghmaan	− 0·504			Cambridge	+ 0·727		

In this table, the figures in the third column are the apparent errors of latitude unaccounted for by the external irregularities of ground, not considering the effect of disturbing masses above nine miles distant. The figures in the fourth column are apparent errors of latitude unaccounted for after including the attractions of more distant masses.

It appears, therefore, that in the Isle of Wight there is (as compared with any spheroidal surface that nearly represents the actual surface of Great Britain) a general *southward deflection*, whereas from the mass of England to the north we might have anticipated a general northward deflection at the four Zenith Sector Stations.

At Burleigh Moor it would appear from the fourth column that the calculation of local attraction has been extended too far, and the same at Hungry Hill. At Feaghmaan the computed deflection, $-1''\cdot95$, is apparently too great, but the determination of this quantity was not very satisfactory.

At Lough Foyle the ground within ten miles will only account for half the probable deflection: the same observation applies to Ben Hutig and Edinburgh. At these three Stations it seems probable that the plumb-line is drawn southward by dense subterranean masses.

The largest remaining deflection is $3''\cdot60$, at Blackdown. This is the highest point of a range of low hills in Dorsetshire: to the north of the Station the country is considerably higher than to the south, but from the very undulating nature of the surface it is impossible, without contours, to estimate the amount of the deflection. It may amount to $2''$, but it seems improbable that the ground would account for a larger quantity than this.

At Southampton there appears to be a deflection of $2''\cdot35$ to the north. From the Sector Station the country rises as we proceed northwards, but falls to the south; the sea being distant from half a mile to a mile and a half. This inequality of ground would account for $1''$ or perhaps $1''\cdot5$ to the north, but not for the whole amount of $2''\cdot35$.

At Greenwich we have seen that *all* our calculations have assigned a positive value to ξ, and yet the ground will not account for any such positive value, but rather the opposite

At Balta and Gerth of Scaw all our calculations have indicated a northerly deflection, which the ground will scarcely account for. These Stations, together with Saxavord, were visited by the Astronomer Royal in 1849, who has expressed his opinion that all three are fairly good as far as the form of the country is concerned; that Balta is the best, but that Gerth of Scaw is also a good Station, and that it was impossible to say on which side, north or south, there is any preponderance of attraction: however, nearly three miles north from Balta, and two miles south from the Gerth of Scaw, lies Nive Hill, rising to the height of about 516 feet, so that if any local deflection of the plumb-line exists at these Stations, it might be expected to be in opposite directions. Saxavord he considered to be more exceptionable than the others, on account of the steepness of the declivity on the north and west sides, as compared with that on the south and east.

At Cowhythe there is a deflection amounting to nearly, if not quite, $10''$ to the south, the greater part of which is certainly due to dense masses below the surface.

7.

It is important to remember, with respect to the probable errors of the adopted values of a and c, that they do not belong to the *figure of the earth*, but merely to a very small portion of its surface. The precision of the surface obtained will be better comprehended if viewed in another light. At Durham Observatory, for instance (being nearly a central point in respect of latitude), we have $\xi = -0''\cdot664$, $\eta = -4''\cdot117$: these quantities give the actual position, or direction rather, of the tangent plane to the surface. In this tangent plane, through the point of contact, suppose the meridian and the perpendicular to it, traced; take these lines as axes of co-ordinates x, y, then the surface will be defined by a third co-ordinate z measured vertically downwards from the point whose co-ordinates are x, y; the probable error of z will give the best idea of the precision of the adopted surface.

The equation of a spheroid whose semi-axes are a and b when the co-ordinates originate at the centre, is—

$$\frac{x'^2 + y'^2}{a^2} + \frac{z'^2}{b^2} = 1$$

Transfer the origin of co-ordinates to a point on the surface in the plane of $x\,z$ whose latitude is λ, and let the axes of x and y be tangents to the surface, the former pointing northwards, the latter eastwards; let z be measured vertically downwards. If $h\ k$ be the distances of this new origin on the surface from the axis of revolution and from the plane of the equator respectively, and if we put $p = z\cos\lambda + x\sin\lambda$, $q = z\sin\lambda - x\cos\lambda$, then—

$$h = x' + p$$
$$k = z' + q$$

Also $y = y'$, whence by substitution—

$$(1 - e^2)\,y^2 + (1 - e^2)\,(h - p)^2 + (k - q)^2 - b^2 = 0$$

Now if n be the normal, $h = n\cos\lambda$, $k = n\,(1 - e^2)\sin\lambda$, consequently

$$(1 - e^2)\,h^2 + k^2 - b^2 = 0$$

Also $(1 - e^2)\,hp + kq = n\,(1 - e^2)\,(p\cos\lambda + q\sin\lambda) = nz\,(1 - e^2)$; so that the equation is reduced to

$$(1 - e^2)\,y^2 + (1 - e^2)\,p^2 + q^2 - 2\,nz\,(1 - e^2) = 0$$

Now $(1 - e^2)\,p^2 + q^2 = x^2\,(1 - e^2\sin^2\lambda) + z^2\,(1 - e^2\cos^2\lambda) - e^2\,xz\sin 2\lambda$; consequently the equation of the surface is finally

$$2\,\phi = (1 - e^2)\,y^2 + x^2\,(1 - e^2\sin^2\lambda) + z^2\,(1 - e^2\cos^2\lambda) - e^2\,xz\sin 2\lambda - \frac{2\,a\,(1 - e^2)\,z}{(1 - e^2\sin^2\lambda)^{\frac{1}{2}}} = 0$$

In this equation, suppose x, y, and λ constant, and a, e, z variable, then—

$$\frac{d\phi}{da}\,\delta a + \frac{d\phi}{de}\,\delta e + \frac{d\phi}{dz}\,\delta z = 0$$

By differentiating we get—

$$\frac{d\phi}{da} = \frac{-z(1-e^2)}{(1-e^2\sin^2\lambda)^{\frac{1}{2}}}$$

$$\frac{1}{e}\,\frac{d\phi}{de} = -x^2\sin^2\lambda - y^2 - z^2\cos^2\lambda - xz\sin 2\lambda + 2\,az\,\frac{1-\frac{1}{2}(1+e^2)\sin^2\lambda}{(1-e^2\sin^2\lambda)^{\frac{3}{2}}}$$

$$\frac{d\phi}{dz} = z(1-e^2\cos\lambda) - e^2\,x\sin\lambda\cos\lambda - \frac{a(1-e^2)}{(1-e^2\sin^2\lambda)}$$

In forming the value of

$$\delta z = -\frac{\frac{d\phi}{da}}{\frac{d\phi}{dz}}\,\delta a - \frac{\frac{d\phi}{de}}{\frac{d\phi}{dz}}\,\delta e$$

it is clear that, δa and δe being very small, it will suffice to retain the principal terms of the three differential coefficients, and reject those that are comparatively small. Now from the equation of the surface we have—

$$x^2 + y^2 = 2\,az - z^2 + \text{terms in } e^2\,x^2,\ e^2\,y^2 \dots$$

Let $x^2 = r^2\cos^2\alpha$, $y^2 = r^2\sin^2\alpha$, then—

$$\frac{d\phi}{da} = -\frac{r^2}{a}\ ;\ \frac{d\phi}{dz} = -a$$

$$\frac{d\phi}{de} = e\,r^2\left(\cos^2\alpha\cos^2\lambda - \tfrac{1}{2}\sin^2\lambda - \frac{x}{2\,a}\sin 2\lambda\right)$$

$$\delta z = -\frac{r^2}{a^2}\,\delta a + \frac{r^2}{a^2}\left(\cos^2\alpha\cos^2\lambda - \tfrac{1}{2}\sin^2\lambda - \frac{r}{a}\cos\alpha\cos\lambda\sin\lambda\right) ae\,.\,\delta e$$

Suppose $r = 400$ miles, the greatest distance to which our surface refers, then the probable error of z, corresponding to the estimated probable error of a (viz. ± 295 feet), will amount to only ± 3·0 feet. Again, taking $\lambda = 55°$ and $\alpha = 0$, we have—

$$\cos^2\lambda - \tfrac{1}{2}\sin^2\lambda = -0{\cdot}0065$$

$$-\tfrac{1}{2}\sin 2\lambda = -0{\cdot}4698$$

$$ae\,\delta e = a\delta\epsilon = 5072\,\delta v$$

So that the error δv will produce an error

$$\delta z = -({\cdot}000065 + {\cdot}00047)\,5072\,\delta v = -2{\cdot}7\,\delta v$$

Therefore the probable error of z, as depending upon that of v or c, will barely exceed one foot. It will, however, be greater in the direction perpendicular to the meridian: and for the same value of r, equal to

$$\delta z = -\frac{{\cdot}35}{100}\,ae\,\delta e = -17{\cdot}7\,\delta v$$

which, taking $\delta v = \pm\,{\cdot}43$, will amount to ± 7·7 feet. If, however, we take our origin at a point centrally situated with respect to longitude, we shall not require to assume such a large value of r as the maximum, but 200 miles will be sufficient, and thus, instead of $\pm 7 \cdot 7$ feet, we should have $\pm 1 \cdot 9$ feet.

Equation of the Surface.

Taking Durham Observatory as the origin; the tangent plane to the surface (determined by $\xi = - 0'' \cdot 664$, $\eta = - 4'' \cdot 117$), as the plane of x and y, the former measured northwards, and z measured vertically downwards, the equation to the surface is—

$$\cdot 99524953 \, x^2 + \cdot 99288005 \, y^2 + \cdot 99763052 \, z^2 - \cdot 00671003 \, xz - 41655070 \, z = 0$$

SECTION XIII.

LATITUDES AND LONGITUDES OF THE STATIONS, LENGTH OF THE DEGREE, &c.

§ I.

Having determined in the preceding Section the axes of that spheroid which corresponds most nearly with the surface of Great Britain, we proceed to develop the expression for the radius of curvature in and perpendicular to the meridian, the length of a degree in and perpendicular to the meridian, &c. By page 267 it appears that the radius of curvature of the meridian is—

$$\varrho = a' \left(n_0 - 2\, n_1 \cos 2\,\lambda + 2\, n_2 \cos 4\,\lambda + \cdots \right) \qquad (1)$$

In which expression—

$$a' = a\,(1 - n)\,(1 - n^2)$$

$$n = \frac{a - b}{a + b}$$

$$n_0 = 1 + \frac{9}{4}\, n^2 + \cdots\cdot$$

$$n_1 = \frac{3}{2}\, n + \frac{3 \cdot 5 \cdot 3}{2 \cdot 4 \cdot 2}\, n^3 + \cdots\cdot$$

$$n_2 = \frac{3 \cdot 5}{2 \cdot 4}\, n^2 + \cdots\cdot$$

1. If s be the length of a meridian arc between the latitudes of λ_1 and λ_2, and if we put

$$\lambda_2 - \lambda_1 = \varphi \quad ; \quad \lambda_2 + \lambda_1 = 2\,\lambda$$

we shall have—

$$s = \int_{\lambda - \frac{1}{2}\phi}^{\lambda + \frac{1}{2}\phi} \varrho\, d\lambda$$

$$s = a' \left(n_0\, \varphi - 2\, n_1 \sin \varphi \cos 2\,\lambda + n_2 \sin 2\,\varphi \cos 4\,\lambda + \cdots\cdot \right) \qquad (2)$$

The whole perimeter is therefore $2\,\pi\, a'\, n_0$, and hence,

$$\text{Mean degree} = \frac{\pi\, a'\, n_0}{180}$$

2. If x and y be the coordinates (originating at the centre, and measured parallel to the directions of a and b respectively) of the point whose latitude is λ, then since $ds = \varrho d\lambda$,

$$\begin{aligned} dx &= - ds \sin \lambda = - \varrho \sin \lambda \, d\lambda \\ dy &= ds \cos \lambda = \varrho \cos \lambda \, d\lambda \end{aligned}$$

Also—

$$\begin{aligned} 2 \cos 2 \lambda \sin \lambda &= \sin 3 \lambda - \sin \lambda \\ 2 \cos 4 \lambda \sin \lambda &= \sin 5 \lambda - \sin 3\lambda \\ 2 \cos 2 \lambda \cos \lambda &= \cos 3 \lambda + \cos \lambda \\ 2 \cos 4 \lambda \cos \lambda &= \cos 5 \lambda + \cos 3\lambda \end{aligned}$$

Multiplying the value of ϱ by $\sin \lambda$ and $\cos \lambda$, we have—

$$\begin{aligned} x &= - a' \int \Big((n_0 + n_1) \sin \lambda - (n_1 + n_2) \sin 3 \lambda + n_2 \sin 5 \lambda \Big) \, d\lambda \\ y &= a' \int \Big((n_0 - n_1) \cos \lambda - (n_1 - n_2) \cos 3 \lambda + n_2 \cos 5 \lambda \Big) \, d\lambda \end{aligned}$$

$$x = a' (n_0 + n_1) \cos \lambda - \frac{a'}{3} (n_1 + n_2) \cos 3 \lambda + \frac{a'}{5} n_2 \cos 5 \lambda \qquad (3)$$

$$y = a' (n_0 - n_1) \sin \lambda - \frac{a'}{3} (n_1 - n_2) \sin 3 \lambda + \frac{a'}{5} n_2 \sin 5 \lambda \qquad (4)$$

3. Let ϱ' be the radius of curvature perpendicular to the meridian in the latitude λ, then—

$$\varrho' \cos \lambda = x$$

Assume $\varrho' = a_0 + 2 a_1 \cos 2 \lambda + 2 a_2 \cos 4 \lambda$, then—

$$\varrho' \cos \lambda = (a_0 + a_1) \cos \lambda + (a_1 + a_2) \cos 3 \lambda + a_2 \cos 5 \lambda$$

Comparing this with the value of x in equation (3)—

$$\left.\begin{aligned} a_0 + a_1 &= a' (n_0 + n_1) \\ - 3 (a_1 + a_2) &= a' (n_1 + n_2) \\ 5 a_2 &= a' n_2 \end{aligned}\right\} \therefore \left\{\begin{aligned} a_0 &= a' (n_0 + n_1) + \frac{a'}{3} (n_1 + n_2) + \frac{a'}{5} n_2 \\ a_1 &= \qquad\qquad - \frac{a'}{3} (n_1 + n_2) - \frac{a'}{5} n_2 \\ a_2 &= \qquad\qquad\qquad\qquad\qquad \frac{a'}{5} n_2 \end{aligned}\right.$$

Therefore the value of the radius of curvature ϱ' is—

$$\varrho' = a' (n_0 + n_1) + \frac{a'}{3} (n_1 + n_2) + \frac{a'}{5} n_2 - \left(\frac{2 a'}{3} (n_1 + n_2) + \frac{2 a'}{5} n_2 \right) \cos 2 \lambda + \frac{2 a}{5} n_2 \cos 4 \lambda \qquad (5)$$

4. For finding the spherical excess of a triangle we require the logarithm of the quantity $2 \varrho \varrho' \sin 1''$. Referring to page 229 we have—

$$\frac{1}{\varrho \varrho'} = \frac{\Delta}{a} \cdot \frac{\Delta^3}{a (1 - e^2)} = \frac{\Delta^4}{a^2 (1 - e^2)}$$

$$\begin{aligned} \therefore \log \frac{1}{2 \varrho \varrho' \sin 1''} &= - \log 2 b^2 \sin 1'' + 4 \log \Delta \\ &= - \log \Big(2 a^2 (1 - n^2)^2 \sin 1'' \Big) + 4 \mathrm{M} \Big(n \cos 2 \lambda - \frac{n^2}{2} \cos 4 \lambda \Big) \qquad (6) \end{aligned}$$

where M is the quantity 0·43429448 . . . .

5. Let us now determine the area contained between any given meridians and parallels. Let dA be the area of an indefinitely small rectangle on the surface, of which two sides are formed by the parallels of the latitudes λ and $\lambda + d\lambda$, and the other two sides by meridians whose difference of longitude is $d\omega$; the length of either of the two first sides is $\varrho d\lambda$, and that of either of the latter is $\varrho' \cos \lambda \, d\omega$.

$$\therefore \; dA = \varrho \, \varrho' \cos \lambda \, d\lambda \, d\omega$$

$$\therefore \quad A = a^2 (1 - e^2) \int\!\!\int \Delta^{-4} \cos \lambda \, d\lambda \, d\omega$$

$$= ab\omega \int \frac{\frac{1-n}{1+n} \, d \, . \sin \lambda}{\left(1 - \frac{4n}{(1+n)^2} \sin^2 \lambda\right)^2}$$

Now we have—

$$\int \frac{dx}{(1 - x^2)^2} = \frac{1}{2} \cdot \frac{x}{1 - x^2} + \frac{1}{2} \int \frac{dx}{1 - x^2}$$

which applied to the preceding value of A gives—

$$A = \frac{ab\omega}{2} (1 - n^2) \left(\frac{\sin \lambda}{1 + 2n \cos 2\lambda + n^2} + \int \frac{\cos \lambda}{1 + 2n \cos 2\lambda + n^2} \, d\lambda \right)$$

But—

$$\frac{1 - n^2}{1 + 2n \cos 2\lambda + n^2} = 1 - 2n \cos 2\lambda + 2n^2 \cos 4\lambda - 2n^3 \cos 6\lambda + \cdots$$

This quantity multiplied by $\sin \lambda$ and $\cos \lambda$, becomes the following—

$$(1 + n)(\sin \lambda - n \sin 3\lambda + n^2 \sin 5\lambda + \cdots)$$
$$(1 - n)(\cos \lambda - n \cos 3\lambda + n^2 \cos 5\lambda + \cdots)$$

If we substitute these in the expression for A, the general expression for the i^{th} term is—

$$\frac{ab\omega}{2} \left((1 + n) \sin (2i - 1)\lambda + \frac{1 - n}{2i - 1} \sin (2i - 1)\lambda \right) \left(- n \right)^{i-1}$$

$$= ab\omega \, \frac{i + (i - 1)n}{2i - 1} \sin (2i - 1)\lambda \, . \left(- n \right)^{i-1}$$

Hence we have, making $i = 1, 2, 3,$ &c.—

$$A = ab\omega \left(\sin \lambda - \frac{n}{3}(2 + n) \sin 3\lambda + \frac{n^2}{5}(3 + 2n) \sin 5\lambda \cdots \right) \tag{7}$$

the integral being taken from the equator to the latitude λ. The area between the limits $\lambda - \frac{1}{2}\phi$ and $\lambda + \frac{1}{2}\phi$ is—

$$A = ab\omega \, \phi \left(\phi_1 \cos \lambda - n(2 + n) \phi_3 \cos 3\lambda + n^2 (3 + 2n) \phi_5 \cos 5\lambda - \cdots \right) \tag{8}$$

$$\phi_i = \frac{\sin \frac{i}{2} \phi}{\frac{i}{2} \phi}$$

Numerical Values.

At page 712 it will be seen that the elliptic elements which most nearly represent the meridian curve of Great Britain are—

$$a = 20927005$$

$$\frac{b}{a} = \frac{279\cdot4}{280\cdot4}$$

$$b = 20852372$$

$$n = \frac{1}{559\cdot8}$$

From these quantities we get the following—

$$\begin{array}{ll} a' = 20889555 & a'n_0 = 20889705 \\ n_0 = 1\cdot00000718 & a'n_1 = 55974 \\ n_1 = 0\cdot00267954 & a'n_2 = 125 \\ n_2 = 0\cdot00000598 & \end{array}$$

These values being substituted in the values of ϱ ϱ', &c. given in the preceding paragraphs, we get—

1.—*Radius of Curvature of the Meridian,*

$$= 20889705 - 111949 \cos 2\lambda + 250 \cos 4\lambda$$

2.—*Radius of Curvature perpendicular to the Meridian,*

$$= 20964404 - 37450 \cos 2\lambda + 50 \cos 4\lambda$$

3.—*Radius of Parallel,*

$$= 20945679 \cos \lambda - 18700 \cos 3\lambda + 25 \cos 5\lambda$$

4.—*Length of a Meridian Arc whose amplitude is ϕ, and mean latitude λ,*

$$= 20889705\,\phi - 111949 \sin\phi \cos 2\lambda + 125 \sin 2\phi \cos 4\lambda$$

5.—*Length of a Degree of the Meridian,*

$$= 364594\cdot1 - 1953\cdot8 \cos 2\lambda + 4\cdot4 \cos 4\lambda$$

6.—*Length of a Degree of Longitude,*

$$= 365571\cdot0 \cos \lambda - 326\cdot4 \cos 3\lambda + 0\cdot4 \cos 5\lambda$$

For converting distances in the meridian and perpendicular to it into seconds of arc, we have—

$$\text{Log}\,\frac{1}{\varrho \sin 1''} = 7{\cdot}9944959375 + {\cdot}0023274088 \cos 2\lambda - {\cdot}0000020788 \cos 4\lambda + {\cdot}0000000024 \cos 6\lambda$$

$$\text{Log}\,\frac{1}{\varrho' \sin 1''} = 7{\cdot}9929429442 + {\cdot}0007758029 \cos 2\lambda - {\cdot}0000006929 \cos 4\lambda + {\cdot}0000000008 \cos 6\lambda$$

And for computing the spherical excesses of triangles—

$$\text{Log}\,\frac{1}{2\varrho\varrho' \sin 1''} = 0{\cdot}3719838 + {\cdot}0031032 \cos 2\lambda - {\cdot}0000028 \cos 4\lambda$$

Length of a Degree, Minute, and Second in Latitude and Longitude.

Lat.	Degree of Latitude.		Minute of Latitude.	Second of Latitude.	Degree of Longitude.		Minute of Longitude.	Second of Longitude.	Lat.
	Feet.	Miles.	Miles.	Feet.	Feet.	Miles.	Miles.	Feet.	
49°	364861·8	69·103	1·1517	101·3505	240109·7	45·475	0·7579	66·6972	49°
50	364929·2	69·115	1·1519	101·3693	235267·0	44·558	0·7426	65·3520	50
51	364996·3	69·128	1·1521	101·3879	230352·0	43·627	0·7271	63·9867	51
52	365062·9	69·141	1·1523	101·4064	225366·1	42·683	0·7114	62·6017	52
53	365128·9	69·153	1·1526	101·4247	220310·8	41·726	0·6954	61·1975	53
54	365194·3	69·166	1·1528	101·4429	215187·7	40·755	0·6793	59·7744	54
55	365259·0	69·178	1·1530	101·4608	209998·2	39·772	0·6629	58·3328	55
56	365322·8	69·190	1·1532	101·4786	204744·0	38·777	0·6463	56·8734	56
57	365385·8	69·202	1·1534	101·4961	199426·7	37·770	0·6295	55·3963	57
58	365447·9	69·214	1·1536	101·5133	194047·9	36·751	0·6125	53·9022	58
59	365508·9	69·225	1·1538	101·5303	188609·1	35·721	0·5954	52·3914	59
60	365568·8	69·237	1·1539	101·5469	183112·1	34·680	0·5780	50·8645	60
61	365627·5	69·248	1·1541	101·5632	177558·5	33·629	0·5605	49·3218	61

The following table contains the logarithms of

$$\frac{1}{\varrho \sin 1''},\quad \frac{1}{\varrho' \sin 1''},\quad \frac{1}{2\varrho\varrho' \sin 1'}$$

VALUES OF

$\text{Log}\left(\frac{1}{\varrho \sin 1''}\right)$, $\text{Log}\left(\frac{1}{\varrho' \sin 1''}\right)$, $\text{Log}\left(\frac{1}{2\varrho\varrho' \sin 1''}\right)$

FOR THE SURFACE OF GREAT BRITAIN.

Lat.	$\text{Log}\left(\frac{1}{\rho \sin 1''}\right)$	Diff.	$\text{Log}\left(\frac{1}{\rho' \sin 1''}\right)$	Diff.	$\text{Log}\left(\frac{1}{2\rho\rho' \sin 1''}\right)$	Diff.	Lat.	$\text{Log}\left(\frac{1}{\rho \sin 1''}\right)$	Diff.	$\text{Log}\left(\frac{1}{\rho' \sin 1''}\right)$	Diff.	$\text{Log}\left(\frac{1}{2\rho\rho' \sin 1''}\right)$	Diff.
49°	7·99417402	—	7·99283564	—	0·371555	—	55°	7·99370151	—	7·99267814	—	0·370925	—
10′	416061	1341	83117	447	537	18	10′	368879	1272	67389	425	908	17
20	414722	1339	82670	447	519	18	20	367609	1270	66966	423	891	17
30	413383	1339	82224	446	501	18	30	366342	1267	66544	422	874	17
40	412045	1338	81778	446	483	18	40	365078	1264	66122	422	857	17
50	410709	1336	81333	445	465	18	50	363816	1262	65702	420	840	17
50°	7·99409374	1335	7·99280888	445	0·371447	18	56°	7·99362557	1259	7·99265282	420	0·370823	17
10′	408041	1333	80443	445	430	17	10′	361302	1255	64864	418	807	16
20	406708	1333	79999	444	412	18	20	360049	1253	64446	418	790	17
30	405378	1330	79556	443	394	18	30	358799	1250	64030	416	773	17
40	404048	1330	79113	443	376	18	40	357553	1246	63614	416	757	16
50	402720	1328	78670	443	359	17	50	356309	1244	63200	414	740	17
51°	7·99401394	1326	7·99278228	442	0·371341	18	57°	7·99355069	1240	7·99262786	414	0·370723	17
10′	400070	1324	77786	442	323	18	10′	353832	1237	62374	412	707	16
20	398747	1323	77345	441	306	17	20	352598	1234	61962	412	690	17
30	397425	1322	76905	440	288	18	30	351367	1231	61552	410	674	16
40	396106	1319	76465	440	271	17	40	350140	1227	61143	409	658	16
50	394788	1318	76026	439	253	18	50	348916	1224	60735	408	641	17
52°	7·99393472	1316	7·99275587	439	0·371235	18	58°	7·99347695	1221	7·99260328	407	0·370625	16
10′	392158	1314	75149	438	218	17	10′	346478	1217	59922	406	609	16
20	390846	1312	74712	437	200	18	20	345264	1214	59518	404	593	16
30	389536	1310	74275	437	183	17	30	344054	1210	59115	403	577	16
40	388228	1308	73839	436	166	17	40	342847	1207	58712	403	560	17
50	386922	1306	73404	435	148	18	50	341644	1203	58311	401	544	16
53°	7·99385618	1304	7·99272969	435	0·371131	17	59°	7·99340445	1199	7·99257912	399	0·370528	16
10′	384316	1302	72535	434	113	18	10′	339249	1196	57513	399	512	16
20	383017	1299	72102	433	096	17	20	338057	1192	57116	397	497	15
30	381719	1298	71670	432	079	17	30	336869	1188	56720	396	481	16
40	380424	1295	71238	432	061	18	40	335685	1184	56325	395	465	16
50	379132	1292	70807	431	044	17	50	334504	1181	55931	394	449	16
54°	7·99377841	1291	7·99270377	430	0·371027	17	60°	7·99333327	1177	7·99255539	392	0·370434	15
10′	376553	1288	69948	429	010	17	10′	332155	1172	55148	391	418	16
20	375268	1285	69519	429	0·370993	17	20	330986	1169	54758	390	402	16
30	373985	1283	69091	428	976	17	30	329821	1165	54370	388	387	15
40	372704	1281	68665	426	959	17	40	328660	1161	53983	387	371	16
50	371426	1278	68239	426	942	17	50	327504	1156	53598	385	356	15
55°	7·99370151	1275	7·99267814	425	0·370925	17	61°	7·99326351	1153	7·99253214	384	0·370341	15

§ II.

Latitudes and Longitudes.

The following table contains the latitudes and longitudes of the principal points of the triangulation, calculated from the approximate values, $\xi = 1''\cdot 40$, $\eta = 0$, $u = 0\cdot 2500$, and $v = 0$:—

Names of Stations.	Latitude.	Longitude W.	North Meridian.
	° ′ ″	° ′ ″	° ′ ″
Acklam Wold	54 3 1·98	0 47 0·32	180 0 17·52
Arbury Hill	52 13 28·36	1 12 34·58	179 59 21·22
Arrenig	52 55 1·32	3 44 39·42	...
Axedge	53 13 56·78	1 56 50·25	180 0 11·82
Back Tor	53 24 54·90	1 42 8·70	...
Baconsthorpe Tower	52 53 12·98	− 1 9 43·56	180 0 1·35
Ballycreen	52 55 6·16	6 21 53·68	179 59 50·29
Balsham Tower	52 7 59·08	− 0 19 11·71	179 59 53·49
Balta	60 45 4·51	0 47 3·84	179 59 57·52
Banstead	51 19 1·31	0 12 46·58	180 0 4·94
Bardon Hill	52 42 51·62	1 19 8·20	180 0 5·62
Barrow Hill	50 13 40·49	3 41 35·69	180 0 10·40
Baurtregaum	52 12 25·34	9 49 40·59	179 59 58·34
Beachy Head	50 44 23·66	− 0 15 15·43	180 0 2·97
Beacon Hill	51 11 0·77	1 43 14·81	180 0 3·48
Beacon Hill, Trescow	49 57 38·70	6 20 22·86	180 6 52·73
Ben Cheilt	58 19 15·44	3 22 37·72	179 59 55·40
Ben Cleugh	56 11 8·07	3 46 2·17	179 59 58·53
Ben Clibrig	58 14 6·33	4 24 33·43	179 59 51·34
Ben Corr	53 30 23·43	9 47 25·03	179 59 53·41
Ben Heynish	56 27 18·95	6 55 17·83	179 59 58·84
Ben Hutig	58 33 3·47	4 30 40·20	179 59 53·12
Ben Lawers	56 32 41·42	4 13 8·61	180 0 30·65
Ben Lomond	56 11 24·64	4 37 52·09	179 59 57·12
Ben Macdui	57 4 13·03	3 40 2·05	179 59 49·84
Ben More, Mull	56 25 29·20	6 0 44·00	179 59 57·77
Ben More, S. Uist	57 15 31·18	7 17 35·35	180 0 4·32
Ben Nevis	56 47 48·46	5 0 6·13	180 0 12·93
Ben Tartevil	55 43 32·38	6 26 32·55	180 0 1·77
Ben Wyvis	57 40 43·83	4 34 38·57	179 59 51·40
Berkhampstead	51 45 23·58	0 7 25·05	179 59 56·47
Black Comb	54 15 27·52	3 19 37·06	180 0 12·96
Blackdown	50 41 12·04	2 32 51·41	180 0 5·10
Blackheddon	55 38 18·52	1 55 38·39	180 0 24·50
Blue Hill	57 5 40·05	2 7 30·40	179 59 44·42
Boniface Down	50 36 11·41	1 11 55·11	180 0 20·63
Boniface, S.E.	50 36 7·21	1 11 50·44	180 0 46·45
Boston Church Tower	52 58 42·13	0 1 26·57	179 59 30·29
Botton Head	54 24 22·41	1 5 3·07	179 59 55·60
Brandon, Suffolk	52 24 20·82	− 0 37 20·78	180 0 16·59
Brandon Down, Durham	54 45 17·61	1 40 35·89	179 59 55·42
Brandon, Kerry	52 14 6·43	10 15 10·05	179 59 59·99
Brassa	60 7 47·79	1 5 40·61	179 59 57·71
Brimmond	57 10 20·15	2 14 11·85	179 59 48·96
Broadway Tower	52 1 26·80	1 50 2·81	179 59 53·40

Names of Stations.	Latitude.	Longitude W.	North Meridian.
	° ′ ″	° ′ ″	° ′ ″
Brown Willy	50 35 25·95	4 36 4·11	179 59 50·04
Buckminster Spire	52 47 52·68	0 41 45·43	...
Bunwell Church Tower	52 29 28·46	— 1 7 51·16	180 2 52·79
Burleigh Moor	54 34 15·80	1 2 19·96	179 59 49·84
Burnswark	55 5 42·11	3 16 36·24	180 0 1·04
Butser Hill	50 58 39·85	0 58 43·39	180 0 24·75
Cader Idris	52 41 58·31	3 54 25·70	...
Caherbarnagh	52 1 51·67	9 10 36·51	179 59 54·41
Calton Hill	55 57 17·51	3 10 54·16	179 59 54·33
Carrigfadda	51 38 7·88	9 5 36·11	179 59 59·10
Cheviot	55 28 42·03	2 8 37·17	179 59 49·64
Chingford	51 38 10·19	0 0 0·01	179 59 55·80
Cleisham	57 57 49·21	6 48 38·42	179 59 56·21
Clifton Beacon	53 27 27·56	1 13 7·15	180 0 14·95
Cnoc Ghiubhais	58 35 3·86	4 58 55·73	179 59 52·43
Collier Law	54 46 15·56	1 58 28·35	179 59 42·62
Coringdon	50 37 49·03	1 59 16·78	180 0 2·60
Corryhabbie	57 20 40·27	3 11 40·34	179 59 57·76
Cowhythe	57 40 59·20	2 39 31·47	179 59 54·55
Cradle	51 57 6·72	3 7 15·61	180 0 4·01
Criffel	54 56 26·38	3 37 37·16	180 0 13·65
Croghan	53 20 46·37	7 16 36·51	179 59 57·83
Cross Fell	54 42 10·76	2 29 6·73	180 0 17·58
Crowborough	51 3 19·37	— 0 9 21·09	179 59 56·64
Crowle Beacon	53 36 37·45	0 49 29·08	180 0 29·13
Cuilcagh	54 12 3·19	7 48 35·20	179 59 56·46
Cundtham	55 11 2·71	7 7 27·40	179 59 57·73
Cyrn-y-Brain	53 2 17·61	3 10 20·97	179 59 49·17
Danbury Church Spire	51 42 57·19	— 0 34 32·52	179 59 35·55
Deadman	50 13 17·22	4 48 0·21	180 0 1·90
Dean Hill	51 1 49·71	1 39 5·59	180 0 28·40
Deerness	58 57 5·22	2 44 56·26	179 59 55·30
Delamere	53 13 17·93	2 41 2·43	180 0 18·50
Ditchling	50 54 5·69	0 6 21·80	180 0 6·02
Divis	54 36 40·42	6 1 0·04	179 59 59·24
Docking Church Tower	52 54 3·74	— 0 37 28·20	179 59 52·63
Doolieve	51 47 22·92	8 27 24·65	179 59 56·18
Drung Point	55 9 11·36	7 8 53·34	179 59 58·53
Dublin Observatory Dome	53 23 14·21	6 20 13·04	179 59 58·05
Dudwick	57 25 49·69	2 2 6·00	179 59 48·39
Dunkery Beacon	51 9 45·68	3 35 6·65	179 59 59·42
Dunnet Head	58 40 8·25	3 22 11·43	179 59 55·92
Dunnose	50 37 5·53	1 11 49·66	180 0 10·67
Dunrich	55 34 19·58	3 11 0·05	179 59 55·12
Dunstable	51 51 50·79	0 32 5·09	180 0 2·48
Easington	54 33 53·42	0 50 23·77	180 0 0·63
East Lomond	56 14 31·85	3 13 9·76	179 59 52·78
Easton Church Tower	52 37 50·65	0 30 21·96	179 59 29·15
Ely Minster	52 23 55·04	— 0 15 51·46	180 0 14·39
Epping Cupola	51 42 15·89	0 7 28·09	180 0 3·56
Fair Isle	59 32 45·04	1 37 50·23	179 59 54·73
Fairlight	50 52 38·16	—0 37 13·54	179 59 53·68
Fashven	58 33 42·74	4 53 55·48	179 59 54·70
Feaghmaan	51 55 21·68	10 20 41·09	179 59 57·64
Fetlar	60 37 12·71	0 51 43·82	179 59 56·96
Fitty Hill	59 17 11·03	3 0 2·19	179 59 54·72
Forth Mountain	52 18 57·93	6 33 38·76	180 0 0·02
Foula	60 8 24·24	2 5 37·61	179 59 54·56

Names of Stations.	Latitude.	Longitude W.	North Meridian.
	° ′ ″	° ′ ″	° ′ ″
Four Mile Stone	51 7 7·48	1 51 17·55	180 0 12·00
Frittenfield	51 12 19·60	— 0 50 10·62	179 59 32·47
Gad's Hill	51 24 51·49	— 0 27 55·19	179 59 57·81
Galtymore	52 21 57·41	8 10 38·93	...
Garforth Cliff	53 46 55·54	1 22 7·40	179 59 46·46
Garreg	53 17 38·55	3 17 58·19	180 0 22·81
Glashmeal	56 52 22·76	3 21 59·23	179 59 55·29
Gerth of Scaw	60 48 59·15	0 46 8·99	179 57 36·96
Goat Fell	55 37 32·54	5 11 24·20	180 0 6·72
Goonhilly	50 2 47·42	5 10 36·76	180 0 34·24
Gorleston Church Tower	52 34 43·13	— 1 43 33·39	179 59 36·90
Great Sterling	57 27 48·94	1 47 15·53	179 59 11·29
Great Whernside	54 9 39·04	1 59 48·04	179 59 56·85
Gringley	53 24 30·47	0 53 3·66	179 59 57·52
Gwaunysgaer	53 19 18·60	3 23 40·65	179 59 40·24
Hampton Poorhouse	51 25 34·86	0 21 50·83	...
Hanger Hill Tower	51 31 23·36	0 17 50·98	179 59 48·96
Hanslope Spire	52 6 46·08	0 49 33·63	...
Happisburgh Church Tower	52 49 30·42	— 1 31 56·18	180 0 20·70
Hart Fell	55 24 28·67	3 23 58·70	180 0 12·57
Hensbarrow	50 23 0·73	4 49 3·23	180 0 2·83
High Port Cliff	50 35 44·95	1 11 29·94	180 0 49·38
High Willhays	50 41 6·52	4 0 34·41	179 59 58·25
Hingham Church Tower	52 34 45·15	— 0 59 1·54	180 1 17·21
Holme Moss	53 32 19·14	1 52 54·55	180 0 10·85
Horton's Gazebo	50 51 36·37	1 57 23·66	...
Howth Hill	53 22 23·76	6 4 3·54	179 59 59·13
Hungry Hill	51 41 12·94	9 47 27·14	179 59 56·82
Ingleborough	54 9 58·73	2 23 45·87	...
Inkpen Beacon	51 21 8·54	1 27 48·57	179 59 58·23
Jura North Pap	55 54 8·43	6 0 8·53	180 0 2·48
Karnbonellis	50 10 56·56	5 13 38·56	179 59 58·76
Karn Galver	50 9 54·67	5 36 41·91	180 0 7·96
Karnminnis	50 11 41·00	5 31 56·38	179 59 57·84
Keeper	52 45 5·15	8 15 35·30	179 59 55·89
Kellie Law	56 14 53·02	2 46 42·49	179 59 46·02
Keysoe Spire	52 14 59·81	0 25 36·74	180 0 2·20
King's Arbour	51 28 46·89	0 26 55·27	179 59 48·69
Kippure	53 10 41·29	6 19 49·46	179 59 57·21
Knock	57 35 2·60	2 46 25·36	179 59 51·93
Knockalongy	54 11 39·02	8 45 30·95	179 59 56·17
Knockanaffrin	52 17 20·28	7 34 49·82	179 59 55·45
Knocklayd	55 9 43·43	6 14 57·42	179 59 57·54
Knockmealdown	52 13 40·23	7 54 51·10	...
Knocknadober	51 59 35·12	10 10 24·84	179 59 59·35
Knocknagante	51 53 37·21	9 56 3·00	180 0 41·31
Knocknaskeagh	52 6 25·57	8 25 55·25	179 59 53·54
Lawshall Church Tower	52 9 17·94	— 0 43 31·25	180 0 29·96
Laxfield Church Tower	52 18 6·65	— 1 22 3·91	179 59 50·23
Layton Hill	57 16 49·06	2 1 29·05	179 59 47·04
Leith Hill Tower	51 10 34·42	0 22 10·90	179 59 54·01
Lincoln Minster	53 14 3·00	0 32 4·46	179 59 35·60
Little Sterling	57 27 32·02	1 48 36·24	179 59 48·64
Littletown Down	50 35 59·21	1 12 4·41	179 59 13·97
Llanelian	53 15 20·81	3 43 46·13	179 59 45·91
Longmount Pole	52 32 40·39	2 51 43·77	179 59 50·45
Lough Foyle Base, North End	55 9 5·96	6 56 50·58	179 59 58·73
Lough Foyle Base, South End	55 2 33·86	7 0 22·03	179 59 59·45

Names of Stations.	Latitude.	Longitude W.	North Meridian.
Lumsden	55° 54′ 26″·64	2° 12′ 16″·22	179° 59′ 53″·09
Lundy Island	51 10 1·77	4 40 16·10	179 59 43·17
Lynn Old Tower	52 45 4·48	0 24 2·29	179 59 42·15
Lyons Hill	53 17 25·30	6 32 2·42	179 59 54·75
Maker Church Tower	50 20 49·90	4 11 1·19	180 0 4·50
Malvern	52 6 16·95	2 20 14·25	180 0 6·72
Mamsuil	57 16 47·65	5 7 8·98	180 0 13·67
Mendip	51 13 6·39	2 32 35·52	179 59 45·15
Merrick	55 8 21·05	4 27 59·91	179 59 54·79
Merrington Church	54 40 38·75	1 35 35·23	179 31 22·48
Mickfield Church Tower	52 12 43·97	− 1 7 29·37	179 59 47·06
Milk Hill	51 22 34·33	1 50 57·75	179 59 54·88
Misterton Carr Base, North End	53 31 45·60	0 54 14·64	180 1 4·31
Misterton Carr Base, South End	53 27 29·02	0 55 22·17	180 1 57·69
Moelfre Issa	53 14 48·91	3 34 17·10	179 59 44·26
Monach	58 21 22·01	6 18 29·91	179 59 52·81
Mordington	55 48 27·97	2 4 9·61	180 0 6·87
Mormonth	57 36 8·99	2 1 51·91	179 59 40·43
Mount Battock	56 56 56·84	2 44 24·11	179 59 44·28
Mount Leinster	52 37 5·22	6 46 43·36	...
Mount Sandy	55 10 54·80	6 55 51·67	179 59 57·93
Mowcopt	53 6 53·13	2 12 40·72	179 59 57·62
Naseby Church Tower	52 23 48·42	0 59 14·92	180 0 1·69
Naughton Church Tower	52 6 5·68	− 0 57 10·78	180 0 2·88
Nephin	54 0 47·73	9 22 0·15	180 0 1·60
Nive Hill	60 47 33·36	0 46 56·66	179 59 47·02
Nodes Beacon	50 40 0·07	1 32 24·09	180 0 0·75
North Rona	59 7 15·80	5 48 47·10	...
North Ronaldshay Lighthouse	59 23 3·15	2 22 9·05	...
Norwich Spire	52 37 54·44	− 1 18 10·37	180 2 1·04
Norwood	51 24 50·63	− 0 50 6·11	179 59 47·11
Old Lodge	51 8 3·71	1 38 45·87	180 0 8·79
Old Sarum Castle	51 5 35·31	1 48 13·49	179 59 28·90
Old Sarum Gun	51 5 43·70	1 47 48·36	180 0 11·38
Orme's Head	53 19 58·76	3 51 6·89	...
Orford Castle	52 5 38·54	− 1 31 56·42	180 1 59·82
Otley Church Tower	52 8 54·90	− 1 13 17·46	179 59 48·74
Over Hill	57 15 18·91	2 6 20·15	179 59 49·90
Paddlesworth	51 6 48·95	− 1 8 23·38	179 59 25·99
Paracombe	51 10 32·17	3 51 33·99	179 59 50·76
Pendle Hill	53 52 6·43	2 17 48·68	180 0 12·53
Peninnis Windmill	49 54 28·96	6 18 12·84	179 59 58·14
Pertinny	50 6 24·13	5 38 37·74	179 59 57·08
Peterhead Old Windmill	57 30 43·78	1 47 31·94	179 59 2·66
Pillesdon	50 48 24·37	2 49 55·49	180 0 27·45
Plynlimmon	52 28 0·74	3 46 53·05	...
Precelly	51 56 45·99	4 46 22·56	179 59 42·69
Reform Monument	57 29 30·87	1 47 49·07	...
Rhuddlan Base, East End	53 17 11·83	3 28 13·65	180 37 48·50
Rhuddlan Base, West End	53 17 41·05	3 34 54·11	180 43 12·70
Ronas	60 32 1·30	1 26 36·86	179 59 55·61
Ru Rea	57 50 8·07	5 45 53·21	180 0 11·21
Ryder's Hill	50 30 21·27	3 53 23·17	180 0 13·80
St. Agnes Beacon	50 18 24·28	5 12 56·41	179 59 57·89
St. Agnes Lighthouse	49 53 32·93	6 20 38·14	...
St. Ann's Hill	51 23 50·98	0 31 22·64	179 43 27·91
St. Martin's Head	49 57 59·53	6 15 53·10	179 59 56·04
St. Paul's Cathedral	51 30 49·14	0 5 48·32	179 59 51·14

Names of Stations.	Latitude.	Longitude W.	North Meridian.
	° ′ ″	° ′ ″	° ′ ″
St. Peter's Church Tower	51 21 54·55	— 1 25 12·22	179 59 49·60
Sawel	54 49 10·82	7 2 15·83	180 0 1·58
Saxavord	60 49 39·31	0 50 20·30	179 59 57·30
Sayrs Law	55 50 48·94	2 40 6·94	179 59 59·35
Sea Fell	54 27 14·89	3 12 35·34	180 0 6·14
Scarabin	58 13 13·05	3 35 22·53	179 59 55·88
Scournalapich	57 22 9·21	5 3 29·32	180 0 10·36
Severndroog	51 27 59·49	— 0 3 41·59	180 0 7·32
Shanklin Down	50 37 5·59	1 12 21·34	180 0 29·73
Slieve Donard	54 10 48·56	5 55 9·51	179 59 53·67
Slieve League	54 39 4·50	8 42 19·12	180 0 0·14
Slieve More	54 0 35·00	10 3 26·32	179 59 50·33
Slieve Snaght	55 11 46·98	7 19 57·02	179 59 58·20
Snowdon	53 4 6·21	4 4 28·37	180 0 2·91
Southampton	50 54 48·88	1 24 6·49	179 59 11·80
South Berule	54 8 57·87	4 40 3·32	180 0 19·23
South Lopham Church Tower	52 23 44·30	— 0 59 52·46	179 59 59·90
South Ronaldshay	58 46 54·84	2 56 30·11	179 59 55·23
Southwold Church Tower	52 19 41·05	— 1 40 48·00	179 59 55·00
Start Lighthouse	59 16 39·60	2 22 28·86	...
Stoke Church Tower	51 59 21·30	— 0 53 33·35	180 0 18·15
Stoke Hill	51 16 18·73	2 3 40·71	179 59 57·22
Storr	57 30 25·38	6 10 52·48	180 0 12·35
Stronsay	59 5 37·60	2 32 34·16	179 59 55·51
Swaffham Church Spire	52 38 52·84	— 0 41 29·07	180 0 19·42
Swyre Barrow	50 36 20·63	2 5 34·35	180 0 0·04
Tara	52 41 56·41	6 12 58·01	179 59 53·48
Tarbathy	57 12 42·09	2 4 6·34	179 59 49·90
Taur	52 14 17·69	9 7 27·61	179 59 53·72
Tawnaghmore	54 17 39·93	9 35 47·12	180 0 4·40
Telegraph Tower	49 55 43·62	6 18 7·53	179 59 58·36
Tharfield	52 1 2·43	0 1 59·66	179 59 51·03
Thaxted Church Spire	51 57 14·86	— 0 20 36·04	179 59 55·71
Tilton	52 38 43·19	0 51 56·09	179 59 46·67
Tofts Church Tower	52 30 3·96	— 1 34 29·24	179 59 38·67
Trevose Head	50 32 54·39	5 1 52·73	180 0 7·32
Trostan	55 2 44·41	6 9 13·04	179 59 58·33
Upcot Down	51 28 44·19	1 48 20·93	180 0 12·68
Vicars Carn	54 17 53·71	6 35 42·38	179 59 55·42
Walpole, St. Peter's Church Tower	52 43 42·74	— 0 13 27·69	179 59 49·53
Walton Church Tower	51 51 51·37	— 1 17 22·79	179 59 58·01
Wart Hill Hoy	58 54 1·98	3 20 18·73	179 59 54·15
Water Crag	54 26 12·47	2 6 32·98	179 59 57·85
Week Down	50 35 52·06	1 14 1·66	180 1 45·40
Westbury Down	51 15 36·50	2 8 30·74	179 59 38·34
Whitehorse Hill	51 34 31·22	1 33 56·40	179 59 45·44
Whittle	53 39 59·66	2 15 52·68	180 0 1·27
Wingreen	50 59 6·36	2 6 23·89	179 59 54·13
Wisp	55 17 3·50	2 57 56·78	179 59 56·24
Wolf Rock	49 56 43·38	5 48 27·40	...
Wordeslow	54 50 56·09	1 25 25·21	179 59 54·20
Wrotham	51 19 0·18	— 0 17 11·04	180 0 5·65
Wroxall	50 36 7·53	1 12 23·78	180 0 39·92
Yell	60 32 46·71	1 5 12·04	179 59 55·67
York Minster	53 57 43·58	1 4 49·32	180 0 2·11

Positions of Observatories.

Observatory.	Latitude.	Longitude W.
	° ′ ″	° ′ ″
Armagh Observatory	54 21 10·56	6 38 53·71
Brisbane "	55 49 1·82	4 51 33·15
Cambridge "	52 12 51·90	− 0 5 48·95
Edinburgh "	55 57 17·57	3 10 54·10
Durham "	54 46 5·27	1 35 3·71
Dublin "	53 23 14·21	6 20 13·04
Liverpool "	53 24 47·06	3 0 3·88
Markree "	54 10 30·51	8 27 21·30
Oxford "	51 45 38·56	1 15 43·69

§ III.

Comparison of observed Azimuths.

If we suppose a spheroid whose semiaxes are $a = 20926249$ and $b = 20856337$ feet, so placed with its minor axis parallel to that of the earth's revolution, that there shall be at Greenwich an apparent deflection of $1''\cdot 40$ to the north, and no deflection in the direction perpendicular to the meridian; and if we suppose all the points of the triangulation to be projected upon this regular surface, the latitudes and longitudes of these projected points, together with the directions of the meridians, will be as given in the preceding table. The surface just named, as also its position, were obtained from the approximate solution of the equations at pages 693, 694; if the final values of $\xi\ \eta\ u$ and v at page 712 had been used, we should have obtained quantities differing slightly, but immaterially, from those calculated with the first approximation to the nearest spheroid.

The comparison of the different observed azimuths with the transferred azimuth of the meridian mark at Greenwich, is shown in the following table:—

Stations.	Abstract Reading of Referring Object.	Observed Azimuth of Referring Object.	Diff.	180° − N. Merid.	Error.	(Err.)²
	° ′ ″	° ′ ″	″	″	″	″
Arbury Hill	185 7 13·94	185 7 50·69	− 36·75	+ 38·78	+ 2·03	4·12
Balsham	150 59 14·94	150 59 12·77	+ 2·17	+ 6·51	+ 8·68	75·34
Bardon Hill	213 47 12·42	213 47 10·13	+ 2·29	− 5·62	− 3·33	11·09
Barrow Hill	352 32 29·98	352 32 27·79	+ 2·19	− 10·39	− 8·20	67·24
Beacon Hill	212 52 57·12	212 52 55·08	+ 2·04	− 3·48	− 1·44	2·07
Ben More, South Uist .	305 41 44·45	305 41 41·70	+ 2·75	− 4·32	− 1·57	2·46
Ben Macdui	161 48 21·98	161 48 29·96	− 7·98	+ 10·16	+ 2·18	4·75
Ben Nevis	81 52 46·07	81 52 22·51	+ 23·56	− 12·93	+ 10·63	113·00
Blackdown	316 31 58·96	316 32 1·55	− 2·59	− 5·11	− 7·70	59·29
Brandon (Suffolk) . . .	126 15 30·04	126 15 11·35	+ 18·69	− 16·59	+ 2·10	4·41
Bunwell Tower	183 23 30·00	183 20 28·23	+ 181·77	− 172·79	+ 8·98	80·64
Burnswark	204 52 12·04	204 52 7·20	+ 4·84	− 1·04	+ 3·80	14·44
Butser	108 11 14·96	108 10 54·92	+ 20·04	− 24·75	− 4·71	22·18

Stations.	Abstract Reading of Referring Object.	Observed Azimuth of Referring Object.	Diff.	180° − N. Merid.	Error.	(Err.)²
	° ′ ″	° ′ ″	″	″	″	″
Clifton Beacon	21 34 35·89	21 34 22·58	+ 13·31	− 14·95	− 1·64	2·69
Cheviot	246 9 15·05	246 9 30·63	− 15·58	+ 10·36	− 5·22	27·25
Corryhabbie	204 40 10·96	204 40 1·59	+ 9·37	+ 2·24	+ 11·61	134·79
Cowhythe	66 27 31·99	66 27 36·91	− 4·92	+ 5·45	+ 0·53	0·28
Cradle	130 50 37·98	130 50 37·59	+ 0·39	− 4·01	− 3·62	13·10
Ditchling	274 23 17·16	274 23 10·05	+ 7·11	− 6·02	+ 1·09	1·19
Docking	40 47 20·10	40 47 21·26	− 1·16	+ 7·37	+ 6·21	38·56
Dunkery	84 41 9·98	84 41 14·46	− 4·48	+ 0·58	− 3·90	15·21
Dunnose	339 12 19·99	339 12 12·59	+ 7·40	− 10·67	− 3·27	10·69
Dunrich	242 28 42·12	242 28 56·85	− 14·73	+ 4·88	− 9·85	97·02
Easington	128 38 55·03	128 38 60·49	− 5·46	− 0·63	− 6·09	37·09
Epping Poorhouse	342 30 4·97	342 29 56·23	+ 8·74	− 3·56	+ 5·18	26·83
Fairlight	276 5 8·93	276 5 11·57	− 2·64	+ 6·32	+ 3·68	13·54
Frittenfield	231 8 10·11	231 8 35·44	− 25·33	+ 27·53	+ 2·20	4·84
Gad's Hill	307 11 50·07	307 11 48·95	+ 1·12	+ 2·19	+ 3·31	10·96
Goonhilly	342 58 11·99	342 57 35·14	+ 36·85	− 34·24	+ 2·61	6·81
Goat Fell	233 2 2·31	233 1 57·98	+ 4·33	− 6·72	− 2·39	5·71
Happisburgh	145 27 59·61	145 27 39·70	+ 19·91	− 20·70	− 0·79	0·62
Hart Fell	350 20 29·89	350 20 11·11	+ 18·78	− 12·57	+ 6·21	38·56
Hensbarrow	78 25 55·01	78 25 49·82	+ 5·19	− 2·83	+ 2·36	5·57
Inkpen	319 9 39·94	319 9 45·78	− 5·84	+ 1·77	− 4·07	16·56
Laxfield Tower	145 54 30·03	145 54 34·09	− 4·06	+ 9·77	+ 5·71	32·60
Leith Hill Tower	221 32 45·05	221 32 53·17	− 8·12	+ 5·99	− 2·13	4·54
Lumsden	144 53 47·86	144 53 59·95	− 12·09	+ 6·91	− 5·18	26·83
Lundy Island	194 25 50·99	194 26 4·99	− 14·00	+ 16·83	+ 2·83	8·01
Mendip	114 54 10·06	114 54 24·91	− 14·85	+ 14·85	0·00	0·00
Merrick	71 27 0·36	71 27 6·48	− 6·12	+ 5·21	− 0·91	0·83
Mordington	148 27 44·92	148 27 40·51	+ 4·41	− 6·87	− 2·46	6·05
Mormonth	84 55 17·02	84 55 47·19	− 30·17	+ 19·57	− 10·60	112·36
Mount Battock	238 28 21·50	238 28 44·26	− 22·76	+ 15·72	− 7·04	49·56
Mowcopt	280 37 6·87	280 37 2·55	+ 4·32	+ 2·38	+ 6·70	44·89
Orford Castle	68 51 29·77	68 49 23·65	+ 126·12	− 119·82	+ 6·30	39·69
Paracombe	87 45 7·06	87 45 11·24	− 4·18	+ 9·24	+ 5·06	25·60
Pertinny	31 21 40·02	31 21 43·70	− 3·68	+ 2·92	− 0·76	0·58
Precelly	211 58 30·09	211 58 46·38	− 16·29	+ 17·31	+ 1·02	1·04
Ryder's Hill	310 0 44·97	310 0 36·12	+ 8·85	− 13·80	− 4·95	24·50
Sayrs Law	113 50 20·00	113 50 21·61	− 1·61	+ 0·65	− 0·96	0·92
Scournalapich	351 58 39·99	351 58 35·14	+ 4·85	− 10·36	− 5·51	30·36
Slieve Donard	130 5 13·06	130 5 23·07	− 10·01	+ 6·33	− 3·68	13·54
South Lopham	190 11 6·82	190 11 2·68	+ 4·14	+ 0·10	+ 4·24	17·98
Stoke Tower	272 34 49·95	272 34 28·67	+ 21·28	− 18·15	+ 3·13	9·80
Tharfield	111 38 29·96	111 38 39·58	− 9·62	+ 8·97	− 0·65	0·42
Tofts Tower	273 34 59·98	273 35 14·96	− 14·98	+ 21·33	+ 6·35	40·32
Walpole, St. Peter's	194 47 29·96	194 47 31·91	− 1·95	+ 10·47	+ 8·52	72·59
Wingreen	85 58 34·88	85 58 44·84	− 9·96	+ 5·87	− 4·09	16·73
Wordeslow	57 51 4·98	57 51 18·05	− 13·07	+ 5·80	− 7·27	52·85
Wrotham	274 56 45·11	274 56 37·50	+ 7·61	− 5·65	+ 1·96	3·84
York Minster	260 26 25·03	260 26 26·63	− 1·60	− 2·11	− 3·71	13·76
						1619·09

The sum of the positive errors is + 135·41, and the sum of the negative is − 127·69, which shows that the observed azimuth of the Chingford mark at Greenwich is very exact. The sum of the squares of the 61 errors is 1619·09, whence the mean square of error is 26·54, and the probable error ± 3″·47. In the comparison of azimuths observed at two different Stations we have seven independent sources of error: at the one Station, (1), the error ϵ_1 of the astronomical determination of the angle between the meridian and the terrestrial object observed in connection; (2), the error ϵ_2 in the bearing of this point relatively to all the Stations around; (3), the error ϵ_3, due to local attraction; (4), (5), (6), or $\epsilon_1' \, \epsilon_2' \, \epsilon_3'$, are the corresponding errors at the other Station; and (7), the error ϵ_0 of transference due to errors in the intermediate triangulation. But as we have not the means of separating these different sources of error, we shall suppose the error of comparison due to: (1), errors of observation; (2) and (3), the local deflections at the two Stations whose azimuthal determinations are to be compared. In comparing the different observed azimuths with that at Greenwich, we may suppose the influence of local attraction at Greenwich upon the azimuth as zero, so that the comparison will be affected by: (1), errors of observation (every kind combined); (2), the effect of local attraction at the Station under consideration. Now it will be seen in the Section on the Figure of the Earth that the probable value of the disturbance of the plumb-line due to accidental local attraction amounts to $\pm 1''\cdot75$; and therefore the probable error of an observed azimuth due to local attraction is $\pm 1''\cdot75 \tan\lambda$. Let τ be the mean value of $\tan^2\lambda$ for Great Britain, then $3\cdot062\,\tau$ will be the square of the probable error at any Station. From λ_1 to λ_2 the value of τ is,

$$\tau = \frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} \tan^2\lambda \, d\lambda$$

$$\therefore \; \tau = \frac{\tan\lambda_2 - \tan\lambda_1}{\lambda_2 - \lambda_1} - 1$$

Taking $\lambda_1 = 50°$, $\lambda_2 = 60°$, we have $\tau = 2\cdot096$, $\therefore \; 3\cdot062\,\tau = 6\cdot418$; hence the probable error of a comparison of observed azimuths, separated from the effect of local attraction, is

$$\pm \sqrt{3\cdot47^2 - 6\cdot418} = \pm 2''\cdot37$$

Of this the greater part is doubtless due to the astronomical determinations.

The probable error of η (page 712) is $\pm 0''\cdot55$, consequently the probable error of the absolute azimuth of the whole triangulation is $\pm 0''\cdot69$.

§ IV.

Distances of Parallels.

The logarithmic values of the distances of parallels will be found from page 677 to page 682. They are, however, necessarily dependent, within small limits, upon the assumed figure of the earth, and must receive corrections corresponding to the quantities $\xi \; \eta \; u \; v$, page 712. Let s be the distance of any Station from Greenwich, α its azimuth, and θ the angle subtended by s at the point where the normal at Greenwich intersects the axis of revolution: let λ be the latitude of Greenwich, and $90° - \phi$ the third side of a spherical triangle of which $90 - \lambda$, θ, and α are two sides and the included angle; then by spherical trigonometry—

$$\sin\phi = \sin\lambda \cos\theta + \cos\lambda \sin\theta \cos\alpha$$

$$\cos\omega \cos\phi = \cos\lambda \cos\theta - \sin\lambda \sin\theta \cos\alpha$$

$$\cos\alpha' \cos\phi = \sin\lambda \sin\theta - \cos\lambda \cos\theta \cos\alpha$$

Differentiate the first of these equations, and we have—

$$\begin{aligned}\cos\varphi\, d\varphi &= \cos\lambda\cos\theta\, d\lambda - \sin\lambda\sin\theta\cos\alpha\, d\lambda\\ &\quad - \sin\lambda\sin\theta\, d\theta + \cos\lambda\cos\theta\cos\alpha\, d\theta - \cos\lambda\sin\theta\sin\alpha\, d\alpha\\ &= \cos\omega\cos\varphi\, d\lambda - \cos\alpha'\cos\varphi\, d\theta - \cos\lambda\sin\theta\sin\alpha\, d\alpha\end{aligned}$$

by reason of the second and third equations;

$$\therefore\ d\varphi = \cos\omega\, d\lambda - \cos\alpha'\, d\theta - \sin\theta\sin\alpha'\, d\alpha$$

which might have been deduced directly from the expression for dc, page 690. If S be the distance of parallels, we have by page 248—

$$S = s\,\frac{\varphi - \lambda}{\theta}$$

$$dS = s\,\frac{\theta\, d\varphi - \theta\, d\lambda - (\varphi - \lambda)\, d\theta}{\theta^2}$$

$$= -\frac{s}{\theta}\left((1 - \cos\omega)\, d\lambda + \left(\frac{\varphi - \lambda}{\theta} + \cos\alpha'\right) d\theta + \sin\theta\sin\alpha'\, d\alpha\right)$$

Also—

$$\frac{\varphi - \lambda}{\theta} = \frac{\sin\frac{1}{2}(\alpha' - \alpha)}{\sin\frac{1}{2}(\alpha' + \alpha)}$$

$$\therefore\ \cos\alpha' + \frac{\varphi - \lambda}{\theta} = \cot\tfrac{1}{2}(\alpha' + \alpha)\sin\alpha'$$

$$= \tfrac{1}{2}\,\omega\sin\tfrac{1}{2}(\varphi + \lambda)\sin\alpha' \cdots \text{nearly.}$$

which may be taken as the coefficient of $d\theta$. Again, $2\,\epsilon$ being put for the square of the excentricity—

$$\theta = \frac{s}{a}\sqrt{1 - 2\,\epsilon\sin^2\lambda}$$

Taking the logarithmic differential—

$$\frac{d\theta}{\theta} = -\,\frac{\sin^2\lambda}{1 - 2\,\epsilon\sin^2\lambda}\, d\epsilon - \frac{da}{a}$$

the values of $\xi\ \eta\ u\ v$ are—

$$\begin{aligned}\xi &= d\lambda & \xi &= +\ 1{\cdot}864\\ \tan\lambda\,.\,\eta &= d\alpha & \eta &= -\ 0{\cdot}546\\ \text{arc } 100''\,.\,u &= \frac{da}{a} & u &= +\ 0{\cdot}3245\\ \text{arc } 100''\,.\,v &= 2\, d\epsilon & v &= +\ 0{\cdot}9276\end{aligned}$$

Remembering that azimuths are to be counted from the north round by east, south, and west, and longitudes measured westwards, we have for dS, by putting $s = a\theta$ in the common factor, and substituting for $\theta\sin\alpha'\sin\frac{1}{2}(\phi + \lambda)$ its approximate value $\frac{1}{2}\,\omega\sin 2\,\lambda$,

$$dS = a\sin 1''\left(-\,(1 - \cos\omega)\,\xi + (1 - \cos\omega)(u + gv)\,50\sin 2\,\lambda + \eta\sin\lambda\sin\omega\right)$$

where, as in page 690, $g = {\cdot}3073$. Substituting the numerical values of $\xi\ \eta\ u\ v$, there results finally—

$$dS = 2824\,(1 - \cos\omega) - 43{\cdot}3\sin\omega$$

The following table contains the distances of the parallels as derived from pages 677—682, with the values corrected to the true spheroidal figure.

Distances of Parallels.

Stations.	On Approximate Spheroid.	Corrections.	Final Results.
Saint Agnes	0·0	+ 0·0	0·0
Goonhilly	56210·9	− 4·9	56206·0
Hensbarrow	179210·2	− 6·1	179204·1
High Port Cliff	256691·2	− 12·8	256678·4
Week Down	257411·9	− 12·7	257399·2
Boniface Down	259373·5	− 12·7	259360·8
Dunnose	264859·7	− 12·8	264846·9
Black Down	289851·4	− 11·6	289839·8
Southampton	372666·8	− 12·6	372654·2
Greenwich	578576·6	− 12·4	578564·2
Hungry Hill	654938·3	+ 21·0	654959·3
Feaghmaan	741000·7	+ 25·3	741026·0
Precelly	749564·1	− 6·3	749557·8
Cambridge	847518·4	− 12·5	847505·9
Arbury Hill	851216·0	− 12·8	851203·2
Forth	884631·0	+ 1·0	884632·0
Delamere	1215262·6	− 11·4	1215251·2
Clifton Beacon	1301441·2	− 12·7	1301428·5
South Berule	1554046·6	− 6·6	1554040·0
Tawnaghmore	1606994·7	+ 19·6	1607014·3
Burleigh Moor	1708038·1	− 12·8	1708025·3
Durham	1780015·0	− 12·6	1780002·4
Lough Foyle Base	1880304·8	+ 3·3	1880308·1
Calton Hill	2203487·7	− 10·5	2203477·2
Ben Lomond	2299448·6	− 6·7	2299441·9
Kellie Law	2320596·8	− 11·2	2320585·6
Ben Heynish	2396286·2	+ 2·8	2396289·0
Great Stirling	2764699·1	− 12·4	2764686·7
Cowhythe	2844909·0	− 11·4	2844897·6
Monach	3090834·7	− 0·2	3090834·5
Ben Hutig	3162045·8	− 7·1	3162038·7
North Rona	3370394·2	− 2·4	3370391·8
Balta	3966289·3	− 12·8	3966276·5
Gerth of Scaw	3990116·3	− 12·8	3990103·5
Saxavord	3994195·5	− 12·8	3994182·7

SECTION XIV.

FIGURE OF THE EARTH.

It is proved in the mechanical theory of the Figure of the Earth that, neglecting small quantities of the second order, the generating meridian is an ellipse. When the investigation is pushed to include quantities of the order of the square of the ellipticity,* a second parameter or constant is introduced, the value of which can only be obtained from observation. The determination of the Figure of the Earth from geodetical operations is virtually the determination of that curve, generally assumed elliptic, which will bring the different measured arcs into the best accordance with their observed terminal latitudes: this curve may be said to represent the real Figure of the Earth with the greatest measure of probability.

§ I.

Previous Determinations.

The attention of many eminent writers has been turned to this problem, and it may be instructive to compare some of their results. The results arrived at by Laplace in the *Mécanique Céleste* were very unsatisfactory, owing entirely to the very imperfect state of geodetical measures at that time. In Bowditch's notes† to that work, Vol. II., page 453, we find the following expression for the length of an arc of the meridian between latitudes 0 and ψ, in feet—

$$s = 101 \cdot 259564\, \psi'' - 50209 \cdot 2 \sin 2\psi - 60 \cdot 0 \sin 4\psi$$

where ψ'' is the latitude expressed in seconds. This result is deduced from the arcs in Peru, India, France, England, and Sweden. The curve is not restricted to the elliptic form, but differs very slightly from it; the axes are—

$$a = 20919768 \qquad \frac{b}{a} = \frac{311 \cdot 1}{312 \cdot 2}$$
$$b = 20852822$$

The curve is depressed below an ellipse of the same axes, but the maximum depression is only 58·8 feet, which takes place at the latitude of 45°.

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* Phil. Trans. 1826, page 548; and Phil. Trans. 1841, page 75.

† Mécanique Céleste, by the Marquis de Laplace, translated, with a Commentary, by Nathaniel Bowditch, LL.D.—Boston, 1832.

The most celebrated determinations of the Figure of the Earth are those by Professors Airy and Bessel. The former is given in the Encyclopædia Metropolitana (Art. *Figure of the Earth*), being based upon the discussion of fourteen meridian arcs and four arcs of parallel: the resulting quantities are—

$$a = 20923713 \qquad \frac{b}{a} = \frac{298{\cdot}33}{299{\cdot}33}$$
$$b = 20853810$$

Bessel's determination will be found in Nos. 333 and 438 of the *Astronomische Nachrichten.* The results contained in No. 333 were subsequently found to be vitiated by an error in the geodetic distance of the parallels of Mountjoy and Mola. The corrected values were found to be $a = 3272077{\cdot}14$ toises; $b = 3261139{\cdot}33$ toises; or,

$$a = 20923600 \qquad \frac{b}{a} = \frac{298{\cdot}15}{299{\cdot}15}$$
$$b = 20853656$$

The probable error of the determination of the mean degree of the meridian is ± 1·916 toises, or ± 12·25 feet; and the probable error of the quantity 299·15 is ± 3·15.

The agreement of these two sets of results with one another is very remarkable, but must perhaps be considered rather in the light of a singular coincidence than as a proof of their accuracy.

In Colonel Everest's "Account of the Measurement of Two Sections of the Meridional Arc of India between the Parallels of 18° 3′ and 29° 30′," he has, by the consideration of twelve meridian arcs, deduced the following values—

$$a = 20920902 \qquad \frac{b}{a} = \frac{310{\cdot}04}{311{\cdot}04}$$
$$b = 20853642$$

in which the value of a is considerably less than in the two preceding determinations.

In the Philosophical Transactions for 1856 there is a paper containing a new determination of the Figure of the Earth founded upon the same arcs as used by Bessel, but including the extensions of the Indian and English arcs. The sum of all the total arcs amounts to 63° 21′, the whole distance, measured geodetically, being about twenty-three millions of feet, or a tenth part greater than the radius of the earth: the number of Astronomical Latitudes employed is 38. The results of this calculation are—

$$a = 20924933 \qquad \frac{b}{a} = \frac{297{\cdot}07}{298{\cdot}07}$$
$$b = 20854731$$

The probable errors assigned to a and b are $\pm$ 539 and $\pm$ 408 feet respectively, and that of the denominator of the fraction representing the compression (298·07) is $\pm$ 1·82. Of the accuracy of these estimates there may be a question; they were obtained as follows: Using the notation of that paper, the quantities $p\ q$ are such as render a minimum the quantity $(m_0 + x)^2 + (m_1 + a_1 p + b_1 q + x)^2 + (m_2 + a_2 p + b_2 q + x)^2 + \cdots + (m_0' + x')^2 + (m_1' + a_1' p + b_1' q + x')^2 + (m_2' + a_2' p + b_2' q + x')^2 + \cdots$, where $x\ x'\ x'' \cdots$ are the corrections to the initial points of the several arcs. Suppose p finally expressed thus, $\lambda_0 m_0 + \lambda_1 m_1 + \cdots + \lambda_0' m_0' + \lambda_1' m_1' + \cdots$, then the weight of p was taken as equal to the reciprocal of $\lambda_1^2 + \lambda_2^2 + \cdots + \lambda_1'^2 + \lambda_2'^2 + \cdots$: for the quantities $m_0\ m_0'\ m_0'' \cdots$ are zero. By including the sum of the squares of $\lambda_0\ \lambda_0' \cdots$ the probable errors of a and b would be ± 626 and ± 503, and that of the denominator of the fraction representing the compression would be ± 2·37.

By omitting the point *Evaux* in the French arc from the data, on account of the extraordinary deflection at that point, we should have obtained—

$$a = 20925174 \qquad \frac{b}{a} = \frac{296{\cdot}72}{297{\cdot}72}$$
$$b = 20854914$$

and for the mean value of the corrections to the latitudes $\pm\ 2''{\cdot}05$.

If with the data employed in the paper just noticed, omitting *Evaux* in the French arc, we determine the meridian curve without restricting it to the elliptic form, we get the following expression for the length of an arc of the meridian from 0 to ψ,

$$s = 101{\cdot}285925\ \psi'' - 54160 \sin 2\psi + 765 \sin 4\psi$$

and the values of the axes—

$$a = 20927623 \qquad \frac{b}{a} = \frac{287{\cdot}80}{288{\cdot}80}$$
$$b = 20855410$$

This curve is more protuberant in middle latitudes than the ellipse of the same axes: the maximum elevation above the elliptic curve, which takes place at the latitude of 45°, amounts to 377 feet. The mean degree of the meridian is 364,629 feet. The mean value of the corrections to the latitudes necessary to bring them into accordance with this curve is $\pm\ 1''{\cdot}97$.

Although this curve does not differ in form but by a very minute quantity from an ellipse, yet the comparison of the axes with those of the ellipse best representing the same observations, shows a difference of about 2450 feet in the equatorial radius. Such a difference is very remarkable when we remember that the sum of the measured arcs employed amounts to more than the sixth part of the circumference of the earth.

For the surface of Great Britain we have found $a = 20927005$, and the denominator of the fraction representing the compression $c = 280{\cdot}4 \pm 8{\cdot}3$. Comparing these with the two last-mentioned determinations of the Figure of the Earth, we infer that the surface of Great Britain does not present any anomaly with respect to the general figure, as from its insular position might perhaps have been expected.

Since the publication of the last-mentioned determination of the Figure of the Earth, the data of the problem have received a most important increase by the completion of the Russian Arc of Meridian, which extends from Ismail, in latitude 45° 20′, to Fuglenæs in the Island of Qual-oe, in latitude 70° 40′,—an arc of 25° 20′.

§ II.

Method of Calculation.

Let ϱ be the radius of curvature of the meridian at the point of which the latitude is λ, and put

$$\varrho = A + 2B\cos 2\lambda + 2C\cos 4\lambda \qquad (1)$$

The Figure of the Earth will be known by the determination of the quantities A, B, C.

The distance s, measured along the meridian, of two points whose latitudes are $\lambda - \frac{1}{2}\phi$ and $\lambda + \frac{1}{2}\phi$, is

$$s = \mathrm{A}\,\phi + 2\,\mathrm{B}\sin\phi\cos 2\lambda + \mathrm{C}\sin 2\phi\cos 4\lambda$$

Suppose now x_1 x_2 are the requisite corrections to the two observed terminal latitudes, and y the correction to the length as actually measured, then—

$$s + y = \mathrm{A}\,(\phi + x_2 - x_1) + 2\,\mathrm{B}\sin(\phi + x_2 - x_1)\cos 2\lambda + \mathrm{C}\sin 2\,(\phi + x_2 - x_1)\cos 4\lambda$$

where s, $\lambda - \frac{1}{2}\phi$, $\lambda + \frac{1}{2}\phi$, are the actual results of measurement and observation. The effects of x_1 and x_2 upon the mean latitude of the arc may be safely neglected; we may also neglect x_1 and x_2 in the term in C, and thus by transformation, neglecting the squares and products of the very small quantities x, we have—

$$0 = \mathrm{A}\,\phi - s - y + (\mathrm{A} + 2\,\mathrm{B}\cos 2\lambda\cos\phi)\,(x_2 - x_1) + 2\,\mathrm{B}\sin\phi\cos 2\lambda + \mathrm{C}\sin 2\phi\cos 4\lambda$$

or, putting $\mathrm{A} + 2\,\mathrm{B}\cos 2\lambda\cos\phi = \mathrm{A}\,\mu^{-1}$,

$$\mathrm{A}\,(x_2 - x_1) - \mu\,y = (s - \mathrm{A}\,\phi)\,\mu - 2\,\mathrm{B}\,\mu\sin\phi\cos 2\lambda - \mathrm{C}\,\mu\sin 2\phi\cos 4\lambda$$

But for the errors x_1 x_2 y, each pair of corresponding quantities s ϕ would give an accurate relation between A, B, C, and three arcs would therefore determine the Figure of the Earth. But the relation is affected by various sources of error: the latitudes observed are affected by errors of observation and accidental local attraction, and the measured length is affected by errors in the geodetical operations and the error of the assumed relation of the measuring unit of length to the standard or reference unit. Suppose that the probable amount of local attraction is $\pm\,1''\cdot75$, which is not far from the truth, and that the probable error of the astronomical determination of latitude is $\pm\,0''\cdot25$; then the probable error of an observed amplitude is $\pm\sqrt{2\,(1\cdot75^2 + 0\cdot25^2)} = \pm\,2''\cdot50$; this quantity multiplied by A, or the radius of the earth, is equivalent to about ± 253 feet. The probable error of the actual length of the arc depends entirely upon the accuracy of the different operations, and must be much larger in the earlier arcs than in the modern. The probable error of the Russian arc of 25° 20′ is less than ± 40 feet; the probable error of the partial arc of about 11° from the southern point to Jacobstadt is ± 24·9 feet. The probable error of the English arc cannot at most exceed this quantity; but in the southern portion of the great Indian arc, the first Indian arc, and the Peruvian arc, the uncertainty is doubtless much greater. Still, even in these earlier measures, there can be no doubt that the errors in the equations will be due principally to the latitudes. From this it follows that the correct method of calculation is to make the sum of the squares of the corrections to the observed latitudes a minimum, remembering that the corrections so obtained will be affected with the errors of the different terrestrial measures.

Omitting, then, the quantity y, we have—

$$x_2 - x_1 = \left(\frac{s}{A} - \phi\right)\mu - 2\frac{B}{A}\mu \sin\phi\cos 2\lambda - \frac{C}{A}\mu\sin 2\phi\cos 4\lambda$$

Assume three unknown quantities $u\ v\ z$ such that

$$\frac{1}{A} = \frac{1 + \frac{u}{10000}}{20890000}$$

$$-\frac{2B}{A} = \frac{1}{200} + \frac{v}{10000} \qquad (2)$$

$$-\frac{C}{A} = \frac{z}{10000}$$

Put 20890000 sin 1″, which is the approximate *mean second* of latitude $= \sigma$, then, expressing x_1 x_2 and ϕ in seconds, we have—

$$\frac{1}{\mu}(x_2 - x_1) = \frac{s}{\sigma} + \frac{su}{10000 \cdot \sigma} - \phi + \frac{\sin\phi\cos 2\lambda}{200\sin 1''} + \frac{\sin\phi\cos 2\lambda}{10000\sin 1''}v + \frac{\sin 2\phi\cos 4\lambda}{10000\sin 1''}z$$

which may be put in the form—

$$x_2 - x_1 = m + au + bv + cz \qquad (3)$$

where the values of m, a, b, c are as follows :—

$$m = \left(\frac{s}{\sigma} - \phi + \frac{\sin\phi\cos 2\lambda}{200\sin 1''}\right)\mu$$

$$a = \frac{s}{10000 \cdot \sigma}\mu$$

$$b = \frac{\sin\phi\cos 2\lambda}{10000\sin 1''}\mu$$

$$c = \frac{\sin 2\phi\cos 4\lambda}{10000\sin 1''}\mu$$

$$\mu = 1 + \frac{1}{200}\cos\phi\cos 2\lambda$$

If a measured arc have only the latitudes of the two extremities observed, there will result from this arc only one equation between the quantities $u\ v\ z$ and the two corrections x_1 and x_2; and in general if there be i observed latitudes, or $i - 2$ intermediate Stations, there will be $i - 1$ equations of the form

$$x' = m + x + au + bv + cz$$

$$x'' = m' + x + a'u + 'bv + c'z$$

If $2\,U_1$ be the sum of the squares of the corrections of the observed latitudes in one meridian arc, $2\,U_2$ the same quantity with reference to another arc; and so on; $x_1\ x_2\ x_3$ being the corrections to the initial points of the different total arcs, then—

$$2\,U_1 = x_1^2 + \left(m_1 + x_1 + a_1u + b_1v + c_1z\right)^2 + \left(m_1' + x_1 + a_1'u + b_1'v + c_1'z\right)^2 + \left(m_1'' + x_1 + \cdots\right)^2 + \cdots$$
$$2\,U_2 = x_2^2 + \left(m_2 + x_2 + a_2u + b_2v + c_2z\right)^2 + \left(m_2' + x_2 + a_2'u + b_2'v + c_2'z\right)^2 + \left(m_2'' + x_2 + \cdots\right)^2 + \cdots$$
$$2\,U_3 = x_3^2 + \left(m_3 + x_3 + a_3u + b_3v + c_3z\right)^2 + \left(m_3' + x_3 + a_3'u + b_3'v + c_3'z\right)^2 + \left(m_3'' + x_3 + \cdots\right)^2 + \cdots$$

The values of $u\ v\ z,\ x_1\ x_2\ x_3\ \cdots\cdot$ which render the quantity

$$U = U_1 + U_2 + U_3 + \cdots$$

a minimum, are the most probable values of those quantities. They are obtained as follows:—

$$\frac{dU}{du} = \frac{dU_1}{du} + \frac{dU_2}{du} + \frac{dU_3}{du} + \cdots = \Sigma\,(am) + \Sigma\,(a^2)\,u + \Sigma\,(ab)\,v + \Sigma\,(ac)\,z$$
$$+ \quad (a_1)\,x_1 + (a_2)\,x_2 + (a_3)\,x_3 + \cdots = 0$$
$$\frac{dU}{dv} = \frac{dU_1}{dv} + \frac{dU_2}{dv} + \frac{dU_3}{dv} + \cdots = \Sigma\,(bm) + \Sigma\,(ab)\,u + \Sigma\,(b^2)\,v + \Sigma\,(bc)\,z$$
$$+ \quad (b_1)\,x_1 + (b_2)\,x_2 + (b_3)\,x_3 + \cdots = 0$$
$$\frac{dU}{dz} = \frac{dU_1}{dz} + \frac{dU_2}{dz} + \frac{dU_3}{dz} + \cdots = \Sigma\,(cm) + \Sigma\,(ac)\,u + \Sigma\,(bc)\,v + \Sigma\,(c^2)\,z$$
$$+ \quad (c_1)\,x_1 + (c_2)\,x_2 + (c_3)\,x_3 + \cdots = 0$$
$$\frac{dU}{dx_1} = \frac{dU_1}{dx_1} + \frac{dU_2}{dx_1} + \frac{dU_3}{dx_1} + \cdots = (m_1) + i_1\,x_1 + (a_1)\,u + (b_1)\,v + (c_1)\,z = 0$$
$$\frac{dU}{dx_2} = \frac{dU_1}{dx_2} + \frac{dU_2}{dx_2} + \frac{dU_3}{dx_2} + \cdots = (m_2) + i_2\,x_2 + (a_2)\,u + (b_2)\,v + (c_2)\,z = 0$$

and so on, in which for abbreviation—

$$\Sigma\,(a\,b) = (a_1\,b_1) + (a_2\,b_2) + (a_3\,b_3) + \cdots$$

From the last equations we have—

$$-\,x_1 = \frac{(m_1)}{i_1} + \frac{(a_1)}{i_1}\,u + \frac{(b_1)}{i_1}\,v + \frac{(c_1)}{i_1}\,z$$
$$-\,x_2 = \frac{(m_2)}{i_2} + \frac{(a_2)}{i_2}\,u + \frac{(b_2)}{i_2}\,v + \frac{(c_2)}{i_2}\,z$$
$$-\,x_3 = \frac{(m_3)}{i_3} + \frac{(a_3)}{i_3}\,u + \frac{(b_3)}{i_3}\,v + \frac{(c_3)}{i_3}\,z$$

and so on. Substituting these values in the first equations, we have for the determination of $u\ v$ and z the following equations: (4)

$$0 = \Sigma\left((am) - \frac{(a)\,(m)}{i}\right) + \Sigma\left((a^2) - \frac{(a)^2}{i}\right)u + \Sigma\left((ab) - \frac{(a)\,(b)}{i}\right)v + \Sigma\left((ac) - \frac{(a)\,(c)}{i}\right)z$$
$$0 = \Sigma\left((bm) - \frac{(b)\,(m)}{i}\right) + \Sigma\left((ab) - \frac{(a)\,(b)}{i}\right)u + \Sigma\left((b^2) - \frac{(b)^2}{i}\right)v + \Sigma\left((bc) - \frac{(b)\,(c)}{i}\right)z$$
$$0 = \Sigma\left((cm) - \frac{(c)\,(m)}{i}\right) + \Sigma\left((ac) - \frac{(a)\,(c)}{i}\right)u + \Sigma\left((bc) - \frac{(b)\,(c)}{i}\right)v + \Sigma\left((c^2) - \frac{(c)^2}{i}\right)z$$

Probable Errors.—Having thus obtained $u\ v\ z$, the next step is to determine their probable errors, or that of the quantity $\alpha u + \beta v + \gamma z$. For this purpose suppose p unknown quantities $u\ v\ z\ y\ x\ \cdots$ to be contained in $q\ (> p)$ equations—

$$\begin{array}{l} 0 = n_1 + a_1 u + b_1 v + c_1 z + e_1 y + \cdots \\ 0 = n_2 + a_2 u + b_2 v + c_2 z + e_2 y + \cdots . \\ \vdots \\ 0 = n_q + a_q u + b_q v + c_q z + e_q y + \cdots \end{array}$$

Then for the determination of $u\ v\ z\ y\ \cdots$ we have—

$$\begin{array}{l} 0 = (an) + (a^2)\, u + (ab)\, v + (ac)\, z + \cdots \\ 0 = (bn) + (ab)\, u + (b^2)\, v + (bc)\, z + \cdots \\ 0 = (cn) + (ac)\, u + (bc)\, v + (c^2)\, z + \cdots \\ \vdots \end{array} \qquad (a)$$

and so on. From these we have, by eliminating $u\ v\ z$,

$$\begin{array}{l} 0 = u + \lambda_{a^2}(an) + \lambda_{ab}(bn) + \lambda_{ac}(cn) + \cdots \\ 0 = v + \lambda_{ab}(an) + \lambda_{b^2}(bn) + \lambda_{bc}(cn) + \cdots \\ 0 = z + \lambda_{ac}(an) + \lambda_{bc}(bn) + \lambda_{c^2}(cn) + \cdots \\ 0 = y + \lambda_{ae}(an) + \lambda_{be}(bn) + \cdots\cdots \end{array} \qquad (b)$$

Multiply these equations by $\alpha\ \beta\ \gamma\ \cdots$, and put

$$\begin{array}{l} \mathrm{A} = \alpha\lambda_{a^2} + \beta\lambda_{ab} + \gamma\lambda_{ac} + \cdots \\ \mathrm{B} = \alpha\lambda_{ab} + \beta\lambda_{b^2} + \gamma\lambda_{bc} + \cdots \\ \mathrm{C} = \alpha\lambda_{ac} + \beta\lambda_{bc} + \gamma\lambda_{c^2} + \cdots \end{array} \qquad (c)$$

and so on; then

$$\begin{array}{rl} 0 = \alpha u + \beta v + \gamma z + \cdots & + \Big(\mathrm{A}a_1 + \mathrm{B}b_1 + \mathrm{C}c_1 + \cdots\Big)\, n_1 \\ & + \Big(\mathrm{A}a_2 + \mathrm{B}b_2 + \mathrm{C}c_2 + \cdots\Big)\, n_2 \\ & + \Big(\mathrm{A}a_3 + \mathrm{B}b_3 + \mathrm{C}c_3 + \cdots\Big)\, n_3 \\ & + \cdots\cdots\cdots\cdots \end{array}$$

The sum of the squares of the coefficients of n is

$$\mathrm{A}\Big(\mathrm{A}\,(a^2) + \mathrm{B}\,(ab) + \mathrm{C}\,(ac) + \cdots\Big) + \mathrm{B}\Big(\mathrm{A}\,(ab) + \mathrm{B}\,(b^2) + \mathrm{C}\,(bc) + \cdots\Big) + \cdots \qquad (d)$$

If in equations (a) we substitute the equations (b), the results must be identically zero, or the different coefficients of (an) (bn) (cn) $\cdots\cdot$ must vanish; hence we have—

$$
\begin{aligned}
1 &= (a^2)\,\lambda_{a^2} + (ab)\,\lambda_{ab} + (ac)\,\lambda_{ac} + \cdots\\
1 &= (ab)\,\lambda_{ab} + (b^2)\,\lambda_{b^2} + (bc)\,\lambda_{bc} + \cdot\cdot\\
1 &= (ac)\,\lambda_{ac} + (bc)\,\lambda_{bc} + (c^2)\,\lambda_{c^2} + \cdots\\
0 &= (a^2)\,\lambda_{ab} + (ab)\,\lambda_{b^2} + (ac)\,\lambda_{bc} + \cdots\\
0 &= (a^2)\,\lambda_{ac} + (ab)\,\lambda_{bc} + (ac)\,\lambda_{c^2} + \cdots
\end{aligned}
$$

and so on. By help of these relations, and the equations (c),

$$
\begin{aligned}
\alpha &= \mathrm{A}(a^2) + \mathrm{B}(ab) + \mathrm{C}(ac) + \cdots\\
\beta &= \mathrm{A}(ab) + \mathrm{B}(b^2) + \mathrm{C}(bc) + \cdots\\
\gamma &= \mathrm{A}(ac) + \mathrm{B}(cb) + \mathrm{C}(c^2) + \cdots
\end{aligned}
$$

Therefore the sum of the squares of the coefficients of the quantities n in the expression for $\alpha u + \beta v + \gamma z + \cdots\cdot$ is $\mathrm{A}\alpha + \mathrm{B}\beta + \mathrm{C}\gamma + \cdots\cdot$, and consequently if W be the weight of the determination of $\alpha u + \beta v + \gamma z + \cdots\cdot$, we have by equations ($c$),

$$\frac{1}{\mathrm{W}} = \mathrm{A}\alpha + \mathrm{B}\beta + \mathrm{C}\gamma + \cdots$$

$$\therefore\ \mathrm{W} = \left(\alpha^2\,\lambda_{a^2} + \beta^2\,\lambda_{b^2} + \gamma^2\,\lambda_{c^2} + 2\,\alpha\beta\,\lambda_{ab} + 2\,\alpha\gamma\,\lambda_{ac} + 2\,\beta\gamma\,\lambda_{bc} + \cdots\right)^{-1}$$

By means of this theorem it may be proved that if the resolution of the equations (4) give

$$
\begin{aligned}
0 &= u + f\mathrm{A} + g\mathrm{B} + h\mathrm{C}\\
0 &= v + g\mathrm{A} + i\mathrm{B} + j\mathrm{C}\\
0 &= z + h\mathrm{A} + j\mathrm{B} + k\mathrm{C}
\end{aligned}
\qquad (5)
$$

where A, B, C represent the absolute terms of the three equations (4), then the weight of the resulting value of any quantity $\alpha u + \beta v + \gamma z$ is the reciprocal of

$$f\alpha^2 + i\beta^2 + k\gamma^2 + 2\,g\alpha\beta + 2\,h\alpha\gamma + 2\,j\beta\gamma \qquad (6)$$

Axes of the Curve, &c.—After obtaining u v z, we get finally through equations (2) the value of ϱ given in equation (1). The axes of the curve are determined as follows: If x y be the co-ordinates (measured from the centre of the curve) of the point whose latitude is λ,

$$
\begin{aligned}
x &= -\int \varrho \sin\lambda\, d\lambda = (\mathrm{A} - \mathrm{B})\cos\lambda + \frac{1}{3}(\mathrm{B} - \mathrm{C})\cos 3\lambda + \frac{1}{5}\mathrm{C}\cos 5\lambda\\
y &= \int \varrho \cos\lambda\, d\lambda = (\mathrm{A} + \mathrm{B})\sin\lambda + \frac{1}{3}(\mathrm{B} + \mathrm{C})\sin 3\lambda + \frac{1}{5}\mathrm{C}\sin 5\lambda
\end{aligned}
$$

By taking the integral from $\lambda = 0$ to $\lambda = \frac{1}{2}\pi$ we have, a and b being the semi-axes—

$$\begin{aligned} a &= A - B + \frac{1}{3}(B - C) + \frac{1}{5}C = A - \frac{2}{3}B - \frac{2}{15}C \\ b &= A + B - \frac{1}{3}(B + C) + \frac{1}{5}C = A + \frac{2}{3}B - \frac{2}{15}C \end{aligned} \qquad (7)$$

Taking the sums and differences—

$$a - b = -\frac{4}{3}B$$

$$a + b = 2A - \frac{4}{15}C$$

$$A = \frac{a + b}{2} + \frac{2}{15}C$$

$$B = -3\frac{a - b}{4}$$

Whence by substitution—

$$\varrho = \left(\frac{a + b}{2} + \frac{2}{15}C\right) - \frac{3}{2}\left(a - b\right)\cos 2\lambda + 2C\cos 4\lambda$$

$$x = \left(\frac{a + b}{2} + \frac{3}{4}(a - b) + \frac{2}{15}C\right)\cos\lambda - \left(\frac{a - b}{4} + \frac{C}{3}\right)\cos 3\lambda + \frac{C}{5}\cos 5\lambda$$

$$y = \left(\frac{a + b}{2} - \frac{3}{4}(a - b) + \frac{2}{15}C\right)\sin\lambda - \left(\frac{a - b}{4} - \frac{C}{3}\right)\sin 3\lambda + \frac{C}{5}\sin 5\lambda$$

The curve will be an ellipse if

$$C = \frac{15}{8}\left(\frac{a - b}{a + b}\right)^2 a$$

Consequently if $x'\,y'$ be the co-ordinates of a point whose latitude is λ in an ellipse whose axes are a and b,

$$x - x' = \left\{C - \frac{15}{8}\left(\frac{a - b}{a + b}\right)^2 a\right\}\left(\frac{2}{15}\cos\lambda - \frac{1}{3}\cos 3\lambda + \frac{1}{5}\cos 5\lambda\right)$$

$$y - y' = \left\{C - \frac{15}{8}\left(\frac{a - b}{a + b}\right)^2 a\right\}\left(\frac{2}{15}\sin\lambda + \frac{1}{3}\sin 3\lambda + \frac{1}{5}\sin 5\lambda\right)$$

Suppose the curve (1) to be actually described, and also on the same axes an ellipse: Let P be the point whose latitude is λ on the ellipse, and Q the point whose latitude is λ on the curve (1). The points P and Q would coincide if C were equal to $\frac{15}{8}n^2 a$ (putting $n = a - b : a + b$), otherwise they depart from each other in proportion as C increases or diminishes. Measure PS along the curve of the ellipse and SQ perpendicular to it, put $PS = \delta s$ and $SQ = \delta r$, then—

$$\begin{aligned} \delta s &= -(x - x')\sin\lambda + (y - y')\cos\lambda \\ \delta r &= (x - x')\cos\lambda + (y - y)\sin\lambda \end{aligned}$$

Substituting the values of $x - x'$ and $y - y'$ in these expressions, and reducing, we get—

$$\delta s = \left\{\frac{8}{15}C - \left(\frac{a-b}{a+b}\right)^2 a\right\}\sin 4\lambda$$

$$\delta r = \tfrac{1}{2}\left\{\frac{8}{15}C - \left(\frac{a-b}{a+b}\right)^2 a\right\}\sin^2 2\lambda$$

which again are equivalent to the following—

$$\begin{aligned}\delta s &= \frac{4A}{45}\left\{6\frac{C}{A} - 5\frac{B^2}{A^2}\right\}\sin 4\lambda \\ \delta r &= \frac{2A}{45}\left\{6\frac{C}{A} - 5\frac{B^2}{A^2}\right\}\sin^2 2\lambda\end{aligned} \qquad (8)$$

This last quantity measures the protuberance of the curve (1) above the ellipse on the same axes in any given latitude.

The semiaxes a and b in terms of u v and z are obtained by substituting (2) in (7); thus we find—

$$\begin{aligned}a &= 20924817 - 2092{\cdot}5\,u + 696{\cdot}3\,v + 278{\cdot}5\,z \\ b &= 20855183 - 2085{\cdot}5\,u - 696{\cdot}3\,v + 278{\cdot}5\,z\end{aligned} \qquad (9)$$

$$\tfrac{1}{2}\frac{a+b}{a-b} = \left(300 + \frac{z}{250}\right)\left(1 + \frac{v}{50}\right)^{-1}$$

$$\delta r = -\,29{\cdot}01 - 1{\cdot}16\,v - 557{\cdot}07\,z$$

§ III.

Data of the Calculation, and Corrections in terms of u v z, &c.

1.

The English triangulation differs essentially from the other meridian measures, first, in the extent of the triangulation across the meridian; and secondly, in the great number of astronomically determined latitudes. In general it is objectionable that the triangulation for determining the meridional distance of two points should be oblique to the meridian, on account of the influence of uncertain errors in the observed azimuths upon the calculated result; but in the present instance the number of observed azimuths at different points, and their sufficiently satisfactory agreement, preclude any doubt as to the true azimuth of the whole network. The number of points having astronomical latitudes is so great that we may dispense with some of them without fear of lessening the accuracy of the final results. We shall therefore omit Cowhythe, Calton Hill (Royal Observatory, Edinburgh), and all the Irish points. With respect to the observed latitudes, we shall make use of the quantities given at page 664,—the influence of superincumbent masses upon the observed latitudes at certain points. There can be little or no doubt that the latitudes so corrected will be more probably true, or be more near the mean latitudes, than the direct astronomical results. But as this point may be questioned, the results will be given in such a form that the influence of the calculated corrections upon the Figure of the Earth may be ascertained by a very short computation.

English Arc.

Stations.	Latitudes.	Amplitudes.	Distances
	° ′ ″	° ′ ″	Feet.
St. Agnes	49 53 33·93	....	.........0·0
Goonhilly	50 2 50·07	0 9 16·14	56206·0
Hensbarrow	50 23 1·84	0 29 27·91	179204·1
High Port Cliff	50 35 46·57	0 42 12·64	256678·4
Week Down	50 35 53·40	0 42 19·47	257399·2
Boniface Down	50 36 12·97	0 42 39·04	259360·8
Dunnose	50 37 6·54	0 43 32·61	264846·9
Black Down	50 41 8·89	0 47 34·96	289839·8
Southampton	50 54 46·97	1 1 13·04	372654·2
Greenwich	51 28 38·30	1 35 4·37	578564·2
Precelly	51 56 45·18	2 3 11·25	749557·8
Cambridge	52 12 51·63	2 19 17·70	847505·9
Arbury	52 13 26·59	2 19 52·66	851203·2
Delamere	53 13 18·61	3 19 44·68	1215251·2
Clifton	53 27 29·50	3 33 55·57	1301428·5
South Berule	54 8 56·40	4 15 22·47	1554040·0
Burleigh Moor	54 34 15·12	4 40 41·19	1708025·3
Durham	54 46 6·20	4 52 32·27	1780002·4
Ben Lomond	56 11 26·27	6 17 52·34	2299441·9
Kellie Law	56 14 53·60	6 21 19·67	2320585·6
Ben Heynish	56 27 16·88	6 33 42·95	2396289·0
Great Stirling	57 27 49·12	7 34 15·19	2764686·7
Monach	58 21 21·31	8 27 47·38	3090834·5
Ben Hutig	58 33 4·46	8 39 30·53	3162038·7
North Rona	59 7 15·19	9 13 41·26	3370391·8
Balta	60 45 1·66	10 51 27·73	3966276·5
Gerth of Scaw	60 48 56·43	10 55 22·50	3990103·5
Saxavord	60 49 38·58	10 56 4·65	3994182·7

If x_1 be the correction to the latitude of St. Agnes, the corrections to the other points will be as follows:—

Goonhilly	$-1{\cdot}649+0{\cdot}0554\,u-0{\cdot}0096\,v-0{\cdot}1045\,z+x_1$
Hensbarrow	$-0{\cdot}052+0{\cdot}1768\,u-0{\cdot}0315\,v-0{\cdot}3308\,z+x_1$
High Port Cliff	$-0{\cdot}537+0{\cdot}2532\,u-0{\cdot}0460\,v-0{\cdot}4719\,z+x_1$
Week Down	$-0{\cdot}258+0{\cdot}2539\,u-0{\cdot}0462\,v-0{\cdot}4734\,z+x_1$
Boniface Down	$-0{\cdot}478+0{\cdot}2559\,u-0{\cdot}0465\,v-0{\cdot}4769\,z+x_1$
Dunnose	$+0{\cdot}066+0{\cdot}2613\,u-0{\cdot}0476\,v-0{\cdot}4873\,z+x_1$
Black Down	$+4{\cdot}251+0{\cdot}2859\,u-0{\cdot}0524\,v-0{\cdot}5319\,z+x_1$
Southampton	$+3{\cdot}047+0{\cdot}3676\,u-0{\cdot}0688\,v-0{\cdot}6822\,z+x_1$
Greenwich	$+2{\cdot}663+0{\cdot}5707\,u-0{\cdot}1123\,v-1{\cdot}0506\,z+x_1$
Precelly	$+2{\cdot}192+0{\cdot}7393\,u-0{\cdot}1514\,v-1{\cdot}3513\,z+x_1$
Cambridge	$+1{\cdot}685+0{\cdot}8360\,u-0{\cdot}1751\,v-1{\cdot}5212\,z+x_1$
Arbury	$+3{\cdot}187+0{\cdot}8396\,u-0{\cdot}1759\,v-1{\cdot}5273\,z+x_1$

Delamere	$+ 0{\cdot}943 + 1{\cdot}1986\,u - 0{\cdot}2715\,v - 2{\cdot}1429\,z + x_1$
Clifton	$- 0{\cdot}266 + 1{\cdot}2835\,u - 0{\cdot}2959\,v - 2{\cdot}2849\,z + x_1$
South Berule	$+ 3{\cdot}319 + 1{\cdot}5325\,u - 0{\cdot}3710\,v - 2{\cdot}6905\,z + x_1$
Burleigh Moor	$+ 2{\cdot}583 + 1{\cdot}6845\,u - 0{\cdot}4198\,v - 2{\cdot}9320\,z + x_1$
Durham	$+ 1{\cdot}026 + 1{\cdot}7553\,u - 0{\cdot}4432\,v - 3{\cdot}0421\,z + x_1$
Ben Lomond	$+ 0{\cdot}670 + 2{\cdot}2673\,u - 0{\cdot}6260\,v - 3{\cdot}8023\,z + x_1$
Kellie Law	$+ 1{\cdot}717 + 2{\cdot}2882\,u - 0{\cdot}6338\,v - 3{\cdot}8314\,z + x_1$
Ben Heynish	$+ 4{\cdot}479 + 2{\cdot}3628\,u - 0{\cdot}6625\,v - 3{\cdot}9358\,z + x_1$
Great Stirling	$+ 2{\cdot}384 + 2{\cdot}7258\,u - 0{\cdot}8095\,v - 4{\cdot}4223\,z + x_1$
Monach	$+ 3{\cdot}534 + 3{\cdot}0471\,u - 0{\cdot}9491\,v - 4{\cdot}8200\,z + x_1$
Ben Hutig	$+ 1{\cdot}850 + 3{\cdot}1173\,u - 0{\cdot}9810\,v - 4{\cdot}9031\,z + x_1$
North Rona	$+ 3{\cdot}607 + 3{\cdot}3225\,u - 1{\cdot}0759\,v - 5{\cdot}1355\,z + x_1$
Balta	$+ 6{\cdot}203 + 3{\cdot}9095\,u - 1{\cdot}3674\,v - 5{\cdot}7247\,z + x_1$
Gerth of Scaw	$+ 6{\cdot}083 + 3{\cdot}9330\,u - 1{\cdot}3797\,v - 5{\cdot}7459\,z + x_1$
Saxavord	$+ 4{\cdot}104 + 3{\cdot}9370\,u - 1{\cdot}3819\,v - 5{\cdot}7496\,z + x_1$

2.

The latitudes and distances of parallels of the points in the French arc will be found at pages 548, 549 of the *Base du Système Métrique Décimal.* We shall, however, omit the point Evaux, for although there is no reason to doubt the accuracy of the astronomical determination of the latitude, yet there is certainly a very unusual amount of local disturbance at that point, amounting probably to — 8″ (*Phil. Trans.* 1856, Part II. page 625). There appears, however, no reason for excluding the point Barcelona from the data; the determination of the latitude was considered by Delambre as in itself unexceptionable, but some doubt was thrown upon it from its presenting a discrepancy of 3″ with the transferred latitude of Mountjoy, the two Stations being only 1094 toises apart. The distance of the parallels of Mountjoy and Formentera given in the *Base du Syst. Met.* is erroneous; in Number 438 of the *Astronomische Nachrichten* will be found an elaborate recalculation of the distance of parallels by Bessel.

French Arc.

Stations.	Latitudes.	Amplitudes.	Distances.
	° ′ ″	° ′ ″	Toises.
Formentera	38 39 56·11	...	...
Montjouy	41 21 44·96	2 41 48·85	153673·61
Barcelona	41 22 47·90	2 42 51·79	154616·74
Carcassonne	43 12 54·30	4 32 58·19	259172·61
Pantheon	48 50 49·37	10 10 53·26	580312·41
Dunkirk	51 2 8·41	12 22 12·30	705257·21

The data for Barcelona are given in the second volume of the *Base du Syst. Met.*, pages 565 and 615.

The correction to the observed latitude of Formentera being called x_2, the corrections to the latitudes of the other points will be as follows:—

Montjouy	$+ 2{\cdot}321 + 0{\cdot}9711\,u + 0{\cdot}1682\,v - 1{\cdot}8242\,z + x_2$
Barcelona	$- 1{\cdot}034 + 0{\cdot}9771\,u + 0{\cdot}1690\,v - 1{\cdot}8364\,z + x_2$
Carcassonne	$- 2{\cdot}793 + 1{\cdot}6375\,u + 0{\cdot}2312\,v - 3{\cdot}1340\,z + x_2$
Pantheon	$+ 5{\cdot}125 + 3{\cdot}6648\,u + 0{\cdot}1583\,v - 7{\cdot}1518\,z + x_2$
Dunkirk	$- 2{\cdot}061 + 4{\cdot}4530\,u + 0{\cdot}0230\,v - 8{\cdot}6320\,z + x_2$

In obtaining these equations, the toise has been taken as equal to 6·39454378 feet of O_1. This quantity was obtained in the following manner: A series of 54 comparisons was made by the late Lieutenant Murphy, R.E., in the summer of 1834, between the Ordnance Standard O_2 and two lengths of the Royal Astronomical Society's 5-feet Brass Scale; these comparisons are given in detail at pages [29] and [30] of the *Account of the Measurement of the Lough Foyle Base.* The result was—

$O_2 = 2\,A - {\cdot}001950$ inch, "Lough Foyle Base," p. 85;
but $O_2 = O_1 - {\cdot}000034$ inch,
and $A = 59{\cdot}999712$ mean inches of *centre yard* of A,
(Mem. R. A. Soc., vol. 9, page 100),
whence $O_1 = 119{\cdot}997508$ mean inches of *centre yard* of A.

In the months of May, June, and December of the following year, Mr. Baily compared the mètre *à traits*,—which had been previously compared in 1818 by Captain Kater with Sir George Shuckburgh's Scale,—with the Royal Astronomical Society's Scale. The mean of 102 comparisons gave this result—

Standard mètre $= 39{\cdot}369678$ mean inches of *centre yard* of A.

Combining these two results we have—

Mètre $= 3{\cdot}28087463$ feet of O_1 log $= 0{\cdot}5159896351$
Toise $= 6{\cdot}39454378$ feet of O_1 log $= 0{\cdot}8058095651$

It appears from later observations on the Royal Astronomical Society's Scale that it has undergone an alteration in length. This makes it necessary to remark that the interval between the two comparisons just quoted was only from one year to a year and a half; during the whole of this period the scale was in Mr. Baily's hands and under examination, and had any change in length been going on at that time it could not have escaped his observation.

3.

If we apply the corrections given at page 624 of the *Philosophical Transactions* for 1856 to the latitudes of Dunkirk and Greenwich, and thence deduce the distance of the parallels of those points, it will be found to be about 161,142 feet, being less than Delambre's determination by about 260 feet. From this it is evidently necessary that the geodetical connection of Greenwich and Dunkirk should be taken into consideration in the Figure of the Earth, forming an entire arc of 22° 10′ from Formentera to Saxavord. The connection of Nôtre Dame, Calais, and Blancnez with the English triangulation is unexceptionable, and Colonel Blondel (le Directeur du Dépôt de la Guerre) has kindly supplied the new triangulation connecting these points and Dunkirk. The data supplied by him is contained in the first three columns of the following tables; the fourth column contains the *inferred spherical angles;* the fifth contains the seconds of the sexagesimal angles corresponding to the angles in the second column. The second part of the table contains the calculation of the direct distance of Dunkirk and Blancnez, the computed angles being marked with an asterisk.

Noms des Stations.	Angles Moyens.	Côtés en Mètres.	Inferred Spherical Angles.	Seconds of red. Ang.	Log. Sines.	Log. Distances.
	G. ″	M.	° ′ ″	″		
Watten, tour signalée . .	82 7422·4	27458·60	74 28 5·26	4·86	9·9838432	4·4386784
Cassel, clocher	70 4628·7	25486·52	63 24 60·10	59·70	9·9514754	4·4063106
Dunkerque, tour du pavillon	46 7948·9	19112·42	42 6 55·83	55·44	9·8264804	4·2813156
			ε = 1·19			
Gravelines, tour du beffroi .	96 0492·1	25486·52	86 26 39·73	39·44	9·9991633	4·4063106
Watten	51 1219·0	18371·83	46 0 35·25	34·96	9·8570051	4·2641524
Dunkerque	52 8288·9	18840·72	47 32 45·89	45·60	9·8679502	4·2750975
			ε = 0·87			
Hondrecoutre, signal . .	61 6295·1	18840·72	55 27 59·87	59·61	9·9158194	4·2750975
Watten	86 1610·1	22332·37	77 32 41·93	41·67	9·9896568	4·3489349
Gravelines	52 2094·8	16723·40	46 59 18·98	18·72	9·8640464	4·2233245
			ε = 0·78			
Calais, grande flèche . .	78 2112·6	22332·37	70 23 24·78	24·48	9·9740508	4·3489349
Hondrecoutre	61 1640·5	19431·27	55 2 51·82	51·52	9·9136172	4·2885013
Gravelines	60 6246·9	19315·51	54 33 44·30	44·00	9·9110221	4·2859062
			ε = 0·90			
Fiennes, signal	62 1836·9	22332·37	55 57 55·37	55·16	9·9183968	4·3489349
Hondrecoutre	110 5938·8	26576·57	99 32 4·37	4·17	9·9939588	4·4244969
Gravelines	27 2224·3	11175·58	24 30 0·88	0·67	9·6177301	4·0482682
			ε = 0·62			
Fiennes	49 9197·3	19431·27	44 55 40·14	39·92	9·8489367	4·2885013
Gravelines	33 4013·3	13782·78	30 3 40·53	40·31	9·6997724	4·1393370
Calais	116 6789·4	26575·77	105 0 39·99	39·77	9·9849213	4·4244859
			ε = 0·66			
Fiennes	112 1042·7	19315·51	100 53 37·96	37·84	9·9921022	4·2859062
Hondrecoutre	49 4272·7	13783·09	44 29 4·48	4·35	9·8455425	4·1393465
Calais	38 4684·6	11175·58	34 37 17·94	17·81	9·7544662	4·0482702
			ε = 0·38			
Blancnez	60 4347·4	19315·51	54 23 28·73	28·56	9·9100969	4·2859062
Hondrecoutre	29 5408·3	10632·93	26 35 12·46	12·29	9·6508437	4·0266530
Calais	110 0244·3	23464·04	99 1 19·32	19·15	9·9945935	4·3704028
			ε = 0·51			
Blancnez	77 0905·3	13782·94	69 22 53·43	53·32	9·9712507	4·1393418
Fiennes	51 3539·3	10632·09	46 13 6·84	6·73	9·8585275	4·0266186
Calais	71 5555·4	13280·58	64 23 60·06	59·95	9·9551259	4·1232170
			ε = 0·33			

Calculation of the Distance Dunkirk—Blancnez.

Noms des Stations.	Angles Moyens.	Côtés en Mètres.	Inferred Spherical Angles.	Seconds of red. Ang.	Log. Sines.	Log. Distances.
		M.	° ′ ″	″		
Hondrecoutre . . . . *	. . . . .	25486·52	34 34 50·35	50·05	9·7540152	4·4063106
Dunkerque . . . . . *	. . . . .	16723·40	21 51 53·37	53·07	9·5710291	4·2233245
Watten	. . . . .	. . .	123 33 17·18	16·88	9·9208320	4·5731274
			ε = 0·90			
Hondrecoutre . . . . *	. . . . .	18371·83	20 53 9·52	9·27	9·5520695	4·2641524
Dunkerque . . . . . *	. . . . .	22332·37	25 40 52·52	52·27	9·6368520	4·3489349
Gravelines	. . . . .	. . .	133 25 58·71	58·46	9·8610443	4·5731272
			ε = 0·75			
Blancnez . . . . . . *	. . . . .	. . .	49 9 41·90	41·18	9·8788406	4·5731273
Dunkerque . . . . . *	. . . . .	23464·04	28 19 6·46	5·74	9·6761161	4·3704028
Hondrecoutre . . . . *	. . . . .	. . .	102 31 13·80	13·08	9·9895474	4·6838341
			ε = 2·16			
Calais . . . . . . *	. . . . .	. . .	74 16 32·39	31·80	9·9834350	4·5731273
Dunkerque . . . . . *	. . . . .	19315·51	29 47 28·04	27·45	9·6962139	4·2859062
Hondrecoutre . . . . *	. . . . .	. . .	75 56 1·34	0·75	9·9867782	4·5764705
			ε = 1·77			

The following table contains the triangulation from Greenwich to Blancnez, Calais, and Montlambert; the second part contains the calculation of the direct distance of Blancnez from Greenwich:—

Stations.	Angles.	$-\frac{\text{Error}}{3}$	Seconds of red. Ang.	Log. Sines.	Log. Sides.	Sides in Feet.
	° ′ ″	″	″			
Wrotham.	50 5 40·59	− 0·93	39·66	9·8848531	5·1390655	137741·70
Fairlight	31 56 4·44		3·51	9·7234118	4·9776242	94978·26
Stede Hill	97 58 17·77		16·83	9·9957833	5·2499957	177826·17
	2·80					
	ε = 3·06					
Wrotham.	93 16 48·50	− 0·74	47·77	9·9992880	5·1516375	141787·35
Crowborough	41 58 20·52		19·78	9·8252764	4·9776259	94978·63
Stede Hill	44 44 53·19		52·45	9·8475657	4·9999152	99980·47
	2·21					
	ε = 2·24					
Fairlight	65 2 36·06	− 1·15	34·91	9·9574276	5·1516362	141786·93
Crowborough	61 44 2·81		1·66	9·9448560	5·1390646	137741·37
Stede Hill	53 13 24·58		23·43	9·9036182	5·0978268	125264·14
	3·45					
	ε = 3·70					
Fairlight	110 30 30·76	− 0·39	30·37	9·9715637	5·3286260	213120·90
Crowborough	36 5 23·01		22·62	9·7701521	5·1272144	134033·83
Tolsford	33 24 7·40		7·01	9·7407645	5·0978268	125264·14
	1·17					
	ε = 3·70					

Stations.	Angles.	$\frac{\varepsilon}{3}$	Seconds of red. Ang.	Log. Sines.	Log. Sides.	Sides in Feet.
	° ′ ″	″	″			
Fairlight	45 27 54·70	− 1·67	53·03	9·8529792	5·0215083	105077·15
Stede Hill	65 24 11·67		10·00	9·9586853	5·1272154	134034·13
Tolsford	69 7 58·65		56·97	9·9705359	5·1390650	137741·54
	5·02					
	ε = 3·11					
Crowborough	25 38 39·80	− 2·10	37·70	9·6362623	5·0215109	105077·78
Stede Hill	118 37 36·25		34·15	9·9433779	5·3286265	213121·13
Tolsford	35 43 50·25		48·15	9·7663882	5·1516368	141787·14
	6·30					
	ε = 3·09					
Fairlight	6 50 15·35	+ 0·55	15·90	9·0757592	4·4306515	26955·76
Tolsford	136 51 46·63		47·17	9·8348934	5·1897857	154805·25
Folkstone	36 17 56·38		56·93	9·7723226	5·1272149	134034·00
	− 1·64					
	ε = 0·58					
Fairlight	51 18 58·89	− 0·78	58·11	9·8924322	5·2849000	192705·53
Tolsford	95 48 1·68		0·91	9·9977708	5·3902378	245605·34
Montlambert	32 53 1·76		0·98	9·7347471	5·1272149	134034·00
	2·33					
	ε = 6·07					
Fairlight	44 28 43·54	− 0·90	42·64	9·8454982	5·2387842	173294·26
Folkstone	96 46 26·44		25·54	9·9969578	5·3902438	245608·75
Montlambert	38 44 52·71		51·82	9·7964997	5·1897857	154805·25
	2·69					
	ε = 6·29					
Fairlight	33 39 33·72	− 1·07	32·65	9·7437057	5·2027355	159490·74
Tolsford	118 34 49·06		47·98	9·9435687	5·4025995	252696·67
Blancnez	27 45 40·44		39·37	9·6681841	5·1272149	134034·00
	3·22					
	ε = 4·44					
Fairlight	26 49 18·37	− 0·77	17·60	9·6543819	5·1276314	134162·60
Folkstone	121 48 12·92		12·15	9·9293482	5·4025977	252695·61
Blancnez	31 22 31·02		30·25	9·7165362	5·1897857	154805·25
	2·31					
	ε = 4·17					
Tolsford	22 46 47·38	− 0·44	46·94	9·5879230	4·8853604	76799·86
Montlambert	53 31 9·36		8·92	9·9052861	5·2027235	159486·34
Blancnez	103 42 4·59		4·14	9·9874626	5·2849000	192705·53
	1·33					
	ε = 2·81					
Fairlight	17 39 25·17	− 0·15	25·02	9·4818970	4·8853473	76797·54
Montlambert	86 24 11·12		10·98	9·9991436	5·4025939	252693·40
Blancnez	75 56 24·15		24·00	9·9867905	5·3902408	245607·05
	0·44					
	ε = 4·45					
Folkstone	25 1 46·48	− 0·02	46·46	9·6264288	4·8853546	76798·82
Montlambert	47 39 18·41		18·39	9·8687052	5·1276310	134162·47
Blancnez	107 18 55·17		55·15	9·9798584	5·2387842	173294·26
	0·06					
	ε = 2·32					

Stations.	Angles.	$\frac{\varepsilon}{3}$	Seconds of red. Ang.	Log. Sines.	Log. Sides.	Sides in Feet.
Tolsford	41° 3′ 44″·95	+ 0″·43	45″·38	9·8174882	5·2387783	173291·91
Montlambert	5 51 50·95		51·38	9·0093330	4·4306231	26953·99
Folkstone	133 4 22·82		23·24	9·8636099	5·2849000	192705·53
	— 1·28					
	ε = 0·80					
Tolsford	18 16 57·57	+ 0·85	58·42	9·4965270	5·1276151	134157·55
Blancnez	3 36 50·58		51·43	8·7996118	4·4306999	26958·76
Folkstone	158 6 9·30		10·15	9·5716416	5·2027295	159488·54
	— 2·55					
	ε = 0·32					
Tolsford	17 30 15·54	— 0·91	14·63	9·4782396	4·7791956	60144·46
Montlambert	87 58 48·88		47·97	9·9997300	5·3006860	199841·65
Fiennes *	74 30 58·30		57·40	9·9839440	5·2849000	192705·53
	ε = 2·72					
Tolsford	5 16 31·84	— 0·24	31·60	8·9635224	4·6392529	43576·56
Blancnez	155 3 36·66		36·43	9·6249697	5·3007002	199848·16
Fiennes *	19 39 52·21		51·97	9·5269990	5·2027295	159488·54
	ε = 0·71					
Montlambert	34 27 39·52	— 0·21	39·31	9·7526967	4·6392077	43572·02
Blancnez	51 21 32·07		31·86	9·8926912	4·7792022	60145·37
Fiennes *	94 10 49·03		48·83	9·9988431	4·8853541	76798·74
	ε = 0·62					
Tolsford	7 32 41·70	— 0·30	41·40	9·1182553	4·5425591	34878·60
Blancnez	135 33 29·47	— 0·31	29·16	9·8452132	5·2695170	186001·75
Calais *	36 53 49·75		49·44	9·7784257	5·2027295	159488·54
	ε = 0·92					
Folkstone	9 21 21·83	— 0·27	21·56	9·2110353	4·5426184	34883·37
Blancnez	131 56 38·89		38·42	9·8714548	5·2030379	159601·80
Calais *	38 41 60.10		59·82	9·7960481	5·1276312	134162·53
	ε = 0·82					
Blancnez (1)	69 22 53·43	— 0·11	53·32	9·9712507	4·6553325	45220·20
Fiennes	46 13 6·84		6·73	9·8585275	4·5426093	34882·64
Calais	64 23 60·06		59·95	9·9551259	4·6392077	43572·02
	ε = 0·33					

(1) The observed value of this angle by Captain Kater is 69° 22′ 53″·87.

Calculation of the Distance Greenwich—Blancnez.

Stations.	Angles.	$\frac{\varepsilon}{3}$	Seconds of red. Ang.	Log. Sines.	Log. Sides.	Sides in Feet.
Severndroog	149° 30′ 35″·740	−0″·044	35″·696	9·7053413	4·9440771	87917·85
Wrotham . . . . . *	4 50 14·812		14·768	8·9259765	4·1647123	14612·09
Greenwich . . . . . *	25 39 9·579		9·536	9·6364018	4·8751376	75013·19
	$\varepsilon = 0·131$					
Epping	33 35 54·811	−0·555	54·256	9·7430143	4·9440770	87917·83
Wrotham . . . . . *	33 24 11·841		11·286	9·7407781	4·9418408	87466·32
Greenwich . . . . . *	112 59 55·013		54·458	9·9640310	5·1650937	146249·27
	$\varepsilon = 1·665$					
Wrotham	157 45 32·185	−0·464	31·721	9·5780731	5·4171897	261330·24
Fairlight . . . . . *	7 18 57·464		57·000	9·1049604	4·9440770	87917·83
Greenwich . . . . . *	14 55 31·743		31·279	9·4108791	5·2499957	177826·17
	$\varepsilon = 1·392$					
Beachy Head . . . . *	71 22 16·870	−2·015	14·855	9·9766277	5·4171897	261330·24
Greenwich . . . . . *	20 54 18·980		16·966	9·5524429	4·9930049	98402·22
Fairlight . . . . . *	87 43 30·194		28·179	9·9996574	5·4402194	275562·04
	$\varepsilon = 6·044$					
Fairlight . . . . . . *	118 22 30·32	−4·56	25·76	9·9444164	5·6449201	441489·19
Greenwich . . . . . *	30 14 25·09		20·54	9·7020933	5·4025970	252695·32
Blancnez . . . . . . *	31 23 18·26		13·70	9·7166860	5·4171897	261330·24
	$\varepsilon = 13·67$					

At Greenwich the observed azimuth of Chingford is 359° 59′ 58″·360. This has to be corrected by the quantity $\eta \tan \lambda = -0''·686$, which gives the azimuth of Chingford finally 359° 59′ 57″·674. From the preceding table,

The angle Chingford—Greenwich—Severndroog . . . . = 106° 10′ 35″·365
∴ Azimuth of Severndroog . . . . . . = 106 10 33·039
The angle Severndroog—Greenwich—Wrotham . . . . = 25 39 9·579
∴ Azimuth of Wrotham . . . . = 131 49 42·618
The angle Wrotham—Greenwich—Fairlight . . . . . . = 14 55 31·743
∴ Azimuth of Fairlight . . . . = 146 45 14·361
The angle Fairlight—Greenwich—Blancnez . . . . . . = 30 14 25·09
∴ Azimuth of Blancnez . . . . = 116 30 49·27

Taking the latitude of Greenwich 51° 28′ 38″·30 + 1″·864; by means of the distance and bearing of Blancnez we find for the distance of parallels of

Greenwich—Blancnez = 201693·1 feet.

Also, the back azimuth of Greenwich at Blancnez is 297° 50′ 53″·888. If from this we subtract the angle Greenwich—Blancnez—Fairlight, or 31° 23′ 18″·26, we have 266° 27′ 35″·628 for the azimuth of Fairlight at Blancnez. This agrees satisfactorily with the result of Captain Kater's observations of the Pole Star at Blancnez: he obtained 266° 27′ 33″·40 (*Philosophical Transactions*, 1828, page 233).

To find the azimuth of Dunkirk at Blancnez we have—

	°	′	″
The angle Calais—Blancnez—Fairlight . . . . =	163	19	9·91
„ Calais—Blancnez—Dunkirk . . . . =	5	13	46·83
Azimuth of Fairlight at Blancnez . . . . =	266	27	35·63
„ Dunkirk at Blancnez . . . . =	75	0	32·37

The log. distance of Dunkirk—Blancnez is 4·6838341; if to this we add the logarithm of a mètre expressed in feet, namely 0·5159896, we find 5·1998237 as the log. distance of Dunkirk and Blancnez in feet. This distance, together with the above bearing, gives the distance of parallels of

Blancnez—Dunkirk = 40289·7 feet.

This quantity subtracted from the meridional distance of Greenwich and Blancnez gives the

Distance of Parallels of Greenwich and Dunkirk = 161403·4 feet.

The quantity used by Delambre was (*Base du Syst. Met.*, III. 192) 25241·9 toises, which is equivalent to 161410·5 feet of the Ordnance Standard O_1: so that the present determination differs seven feet from that of Delambre. By Delambre's observations for azimuth at Watten, he found, after transferring the azimuth geodetically to Dunkirk, that the azimuth of Watten at Dunkirk was 205° 19′ 42″·14 (III. 14); if from this we deduct the angle Watten—Dunkirk—Cassel, viz. 42° 6′ 9″·73 (II. 800), we get 163° 13′ 32″·41 as the azimuth of Cassel at Dunkirk. From the preceding table of triangles we get—

	°	′	″
Blancnez—Dunkirk—Hondrecoutre . . . . =	28	19	6·46
Hondrecoutre—Dunkirk—Watten . . . . =	21	51	53·37
Watten—Dunkirk—Cassel . . . . =	42	6	55·83

Now the back azimuth of Blancnez from Dunkirk, as obtained in the same calculation which gave the distance of parallels above (40289·7), is found 255° 31′ 31″·98. By subtracting from this the sum of the three angles specified above, namely 92° 17′ 55″·66, we have 163° 13′ 36″·32 as the azimuth of Cassel at Dunkirk. The difference between this and Delambre's determination is only 3″·91.

We are now able to assign an equation between the corrections to the latitudes of Formentera and St. Agnes, and thus to eliminate the former quantity x_2: the equation is—

$$x_1 - x_2 = -2{\cdot}788 + 4{\cdot}0415\,u + 0{\cdot}1010\,v - 7{\cdot}8692\,z.$$

Thus the corrections to the different points of the French arc become—

Formentera . . . $+2{\cdot}788 - 4{\cdot}0415\,u - 0{\cdot}1010\,v + 7{\cdot}8692\,z + x_1$
Montjouy . . . $+5{\cdot}109 - 3{\cdot}0704\,u + 0{\cdot}0671\,v + 6{\cdot}0450\,z + x_1$
Barcelona . . . $+1{\cdot}754 - 3{\cdot}0644\,u + 0{\cdot}0679\,v + 6{\cdot}0328\,z + x_1$
Carcassonne . . $-0{\cdot}005 - 2{\cdot}4040\,u + 0{\cdot}1301\,v + 4{\cdot}7352\,z + x_1$
Pantheon . . . $-2{\cdot}337 - 0{\cdot}3767\,u + 0{\cdot}0572\,v + 0{\cdot}7174\,z + x_1$
Dunkirk . . . . $+0{\cdot}727 + 0{\cdot}4115\,u - 0{\cdot}0781\,v - 0{\cdot}7628\,z + x_1$

where x_1 is the correction to the latitude of St. Agnes. These equations are now to be treated in one group with those of the English Triangulation.

4.

Professor Struve of Pulkowa has kindly furnished this department with the first two sections of the account of the Russian "Meridian Arc of 25° 20′," containing the whole of the geodetical operations, but only part of the astronomical. The observations and calculations of this magnificent work have been conducted in such a systematic and masterly manner, that we find in connexion with every measured base-line or calculated quantity its probable error, and these probable errors are carried through to the final results,—the distances of the parallels, an advantage which no other arc possesses. The possibility of assigning the probable errors of the final distances arises from the nature of the triangulation, which is very simple and direct, and presents very few equations of condition other than those resulting from the sum of the three angles of the different triangles. The sum of the lengths of the ten measured bases is 29,863 toises, or 190,960 feet, so that the average length of a base-line is 19,096 feet: in the British Triangulation the total length of measured base-lines is 183,001 feet, and the average length of a base 30,500 feet. The length of the total arc is 1753 miles, with a probable error of ± 39·8 feet.

The following table contains the data of this arc. The latitudes are not final, but the probable errors of the quantities stated are *less than half a second.* That we may have a correct idea of the influence of this uncertainty, suppose the probable error due to local attraction to be $\pm 1''\cdot 75$, then the uncertainty of the astronomical determination, assumed as $\pm 0\cdot 50$, will raise the probable error of the adopted latitudes in the proportion $1\cdot 75 : \sqrt{1\cdot 75^2 + 0\cdot 50^2} = 100 : 104$.

Russian Arc.

Stations.	Latitudes.	Amplitudes.	Distances.
	° ′ ″	° ′ ″	Toises.
Staro-Nekrassowka .	45 20 2·8	....	...............0 ± 0
Wodolui	47 1 25·2	1 41 22·4	96415·136 ± 0·651
Ssuprunkowzi . .	48 45 3·1	3 25 0·3	194973·124 ± 1·646
Kremenetz . . .	50 5 50·0	4 45 47·2	271724·510 ± 2·039
Belin	52 2 42·2	6 42 39·4	382943·521 ± 2·611
Nemesch	54 39 5·9	9 19 3·1	531753·042 ± 3·453
Jacobstadt . . .	56 30 4·8	11 10 2·0	637483·921 ± 3·893
Dorpat	58 22 47·6	13 2 44·8	744764·484 ± 4·177
Hogland	60 5 10·1	14 45 7·3	842303·102 ± 4·372
Kilpi-Maki . . .	62 38 5·0	17 18 2·2	988016·669 ± 4·502
Tornea	65 49 44·7	20 29 41·9	1170810·973 ± 4·957
Stuor-Oivi . . .	68 40 58·4	23 20 55·6	1334032·877 ± 5·539
Fuglenœs . . .	70 40 11·3	25 20 8·5	1447786·783 ± 6·226

If x_2 be the correction to the latitude of Staro-Nekrassowka, the corrections to the other points will be as follows:—

Wodolui	$+\ 3{\cdot}883 + 0{\cdot}6086\,u - 0{\cdot}0250\,v - 1{\cdot}2114\,z + x_2$
Ssuprunkowzi	$+\ 5{\cdot}686 + 1{\cdot}2306\,u - 0{\cdot}0875\,v - 2{\cdot}4285\,z + x_2$
Kremenetz	$+\ 1{\cdot}051 + 1{\cdot}7148\,u - 0{\cdot}1620\,v - 3{\cdot}3509\,z + x_2$
Belin	$+\ 3{\cdot}710 + 2{\cdot}4163\,u - 0{\cdot}3094\,v - 4{\cdot}6269\,z + x_2$
Nemesch	$+\ 2{\cdot}188 + 3{\cdot}3546\,u - 0{\cdot}5786\,v - 6{\cdot}1893\,z + x_2$
Jacobstadt	$+\ 6{\cdot}989 + 4{\cdot}0209\,u - 0{\cdot}8185\,v - 7{\cdot}1716\,z + x_2$
Dorpat	$+\ 3{\cdot}534 + 4{\cdot}6969\,u - 1{\cdot}1025\,v - 8{\cdot}0426\,z + x_2$
Hogland	$+\ 4{\cdot}866 + 5{\cdot}3114\,u - 1{\cdot}3948\,v - 8{\cdot}7106\,z + x_2$
Kilpi Maki	$+\ 5{\cdot}353 + 6{\cdot}2290\,u - 1{\cdot}8895\,v - 9{\cdot}4695\,z + x_2$
Tornea	$+\ 11{\cdot}306 + 7{\cdot}3799\,u - 2{\cdot}6029\,v - 9{\cdot}9857\,z + x_2$
Stuor Oivi	$+\ 7{\cdot}247 + 8{\cdot}4072\,u - 3{\cdot}3210\,v - 10{\cdot}0188\,z + x_2$
Fuglenœs	$+\ 9{\cdot}497 + 9{\cdot}1231\,u - 3{\cdot}8622\,v - 9{\cdot}8018\,z + x_2$

5.

There is some little uncertainty relative to the unit of measure in the earlier geodetical operations in India. In Volume VIII. of *Asiatic Researches*, page 138, it is stated by Colonel Lambton that his Standard Chain, made by Ramsden, was measured off "by the Standard in London when the temperature was 50° by Fahrenheit's thermometer." It is not stated whether *the Standard* was Ramsden's Bar or his Brass Scale: the former supposition has in itself the greater probability, as the Bar was constructed from the Brass Scale expressly for the laying off of the 100-feet chains used in the English Base Lines. In the *Philosophical Transactions*, 1823, page 28, Colonel Lambton states, in reference to this same Standard Chain, that it "was laid off from Ramsden's Bar at the temperature of 52°." This temperature (52°) is probably a mistake, as Colonel Everest, at page 51 of his first volume,* states that the chain was "set off at the temperature of 50° Fahrenheit from his (Mr. Ramsden's) Bar." These three statements all differ, but upon the whole it would seem probable that the chain was measured from Ramsden's Bar at the temperature of 50°, which is in fact Colonel Everest's statement. But on examining the "Reduction to the Temperature of 62°" in Colonel Lambton's different bases, we find that he has supposed the chain to have been laid off from the Brass Standard at 50°: for if τ be the mean temperature in which the base was measured, he has applied the correction

$$\frac{B}{100}\left((\tau^\circ - 50^\circ)\frac{\cdot 0074}{12} - (62^\circ - 50^\circ)\frac{\cdot 01237}{12}\right)$$

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* "An Account of the Measurement of an Arc of the Meridian between the Parallels of 18° 3′ and 24° 7′, &c. By Captain George Everest."—London, 1830.

where B is the length of the base, and ·0074 and ·01237 are the expansions (in inches) of 100 feet of iron and 100 feet of brass for each degree of Fahrenheit. This correction is erroneous if, as seems most probable, the chain was laid off from Ramsden's Bar; but it would scarcely be prudent to alter Colonel Lambton's reductions for temperature at this distance of time; we shall therefore consider his first bases, as he himself has done, to be expressed in terms of Ramsden's Brass Scale at 62° Fahrenheit. The northern part of his arc is in terms of his own Standard.

Now from Captain Kater's comparison of this Standard with the others mentioned, page 61 of the *Philosophical Transactions* for 1821, it appears that Ramsden's Brass Scale exceeded Colonel Lambton's in the proportion of

$$36{\cdot}003147 : 36{\cdot}000000 = 1{\cdot}0000874$$

Therefore to reduce his latter distances to the same standard as the earlier, namely Ramsden's Brass Scale, they must be divided by 1·0000874. Again, the factor for reducing measures in terms of Ramsden's Brass Scale at 62° to the Ordnance Standard O_1 (see page 212) is (1·0000328) (1·0000383) = 1·0000711.

The distances in this arc are as follows:—

	Col. Lambton.	Factor.	Corr.
Punnœ to Putchapolliam	1029100·5	+ ·0000711	+ 73·2
Putchapolliam to Dodagoontah	727334·6	+ ·0000711	+ 51·7
Dodagoontah to Yerracondah	332662·3	+ ·0000711	+ 23·7
Yerracondah to Namthabad	429134·3	− ·0000161	− 6·9
Namthabad to Daumergida	1073428·2	− ·0000161	− 17·3

Hence in terms of O_1 the distances from Punnœ are—

Punnoe to Putchapolliam	1029173·7
„ Dodagoontah	1756560·0
„ Namthabad	2518373·4
„ Daumergida	3591784·3

A small portion of the northern part of this arc was revised by Colonel Everest, who found a sensible difference between the lengths of sides as determined by Colonel Lambton and himself. Unfortunately, but few of Colonel Lambton's Stations could be refound, and from those that were identified it appears that Colonel Lambton's distances were short in proportion of about 0·99993 : 1·00000. If we suppose this error to have accumulated gradually during the work, it would indicate an error of − 126 feet in the arc between Punnœ and Daumergida. It seems more probable that the discrepancy is due to his base-line measures.

The Station Takal Khera is omitted in Colonel Everest's second volume; its meridional position in the new work may be found as follows:—

Let MN be the points in which the meridian of Takal Khera cuts the sides Ashti-Ner and Ashti-Badali; then, by Colonel Everest's first work, putting A for Ashti, which is the same as the South End of the Takal Khera base, and T for Takal Khera—

$$AM = 4879{\cdot}4 \quad , \quad AN = 5972{\cdot}2$$
$$TM = 17397{\cdot}8 \quad , \quad TN = 22888{\cdot}0$$

The points in which the meridian of Kalianpur cuts the same two sides being designated by M′N′, we have by Colonel Everest's second volume*—

$$AM' = 10188{\cdot}03 \qquad AN' = 12469{\cdot}62$$
$$AM'N' = 70^\circ\ 2'\ 48'' \qquad AN'M' = 50^\circ\ 10'\ 23''$$

From these quantities we have $MM' = 5308 \cdot 6$ feet, $NN' = 6497 \cdot 4$, and the difference of parallels of

$$\text{Takal Khera and } M' = 15586{\cdot}0$$
$$\text{Takal Khera and } N' = 27049{\cdot}2$$

Now by page 53 of Colonel Everest's work the distance Kalianpur to M′ is 1112955·7, to which must be added the proportional part of the correction 11·87 at the foot of the page; thus we get—

$$\text{Kalianpur} - M' = 1112961{\cdot}7$$
$$\text{Kalianpur} - N' = 1124424{\cdot}8$$

The first of these, with the meridional distance of Takal Khera and M′, gives the distance of Kalianpur and Takal Khera = 1097375·7; the second, with the meridional distance of Takal Khera and N′, gives 1097375·6.

The measurements given by Colonel Everest in his last volume are expressed partly in terms of his 10-feet Standard A and his 6-inch Brass Scale A, twenty parts of any result being in terms of the former and one part in terms of the latter. He has also given at the end of the same volume the results of comparisons made between his 10-feet Standards A and B, and also between the 6-inch Brass Scales A and B; and in Lieutenant-Colonel Yolland's *Account of the Measurement of the Lough Foyle Base* we have the comparisons of the two Indian Standards B with the Ordnance Standards. Let

x = a foot of the Ordnance Standard O_1 ten feet.
y = " " $O_{6\text{in.}}$ six inch.
x' = a foot of the Indian Standard A ten feet.
x'' = " " B ten feet.
y' = " " A six inch.
y'' = " " B six inch.

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* "An Account of the Measurement of Two Sections of the Meridional Arc of India, &c. &c. By Lieut.-Colonel Everest, F.R.S., &c."—London, 1847.

At page [40] Appendix VI. of the *Measurement of the Lough Foyle Base* we find the result of 60 comparisons between the Ordnance 6-inch Scale and the Indian 6-inch Scale B, giving

$$\tfrac{1}{2}y - \tfrac{1}{2}y'' = + {\cdot}000151 \text{ inch.}$$

And at page 101 of the same volume—

$$\begin{aligned} 10\,x'' - 10\,x &= - {\cdot}0000721 \text{ foot.} \\ \tfrac{1}{2}y - \tfrac{1}{2}x &= - {\cdot}0000222 \quad \text{,,} \end{aligned}$$

At pages 436 and 437 of the *Account of the Indian Arc*—

$$\begin{aligned} 10\,x'' - 10\,x' &= + {\cdot}0000459 \text{ inch.} \\ \tfrac{1}{2}y'' - \tfrac{1}{2}y' &= - {\cdot}0001008 \quad \text{,,} \end{aligned}$$

From these equations we have the following :—

$$\begin{aligned} y - y'' &= + {\cdot}0000252 \text{ foot.} \\ y - x &= - {\cdot}0000444 \quad \text{,,} \\ y'' - y' &= - {\cdot}0000168 \quad \text{,,} \\ x'' - x &= - {\cdot}00000721 \quad \text{,,} \\ x'' - x' &= + {\cdot}00000038 \quad \text{,,} \\ \therefore \quad x' &= x - {\cdot}00000759 \\ y' &= x - {\cdot}00005280 \\ \therefore \quad \frac{20\,x' + y'}{21} &= x - {\cdot}00000974 \end{aligned}$$

Consequently, in order to reduce the Indian measures to feet of the Ordnance Standard, they have to be multiplied by $1 - 0{\cdot}00000974$. The actual corrections will be as follows :—

	Col. Everest.	Corr.	Reduced to O_1
Daumergida to Takal Khera	1105550·6	− 10·8	1105539·8
,, Kalianpur	2202926·2	− 21·5	2202904·7
,, Kaliana	4164083·3	− 40·6	4164042·7

In Numbers 334, 335 of the *Astronomische Nachrichten* will be found a discussion of the earlier zenith sector observations of this arc by Bessel. The latitudes he obtains, dependent upon assumed declinations, are—

	°	′	″
Punnœ	8	9	31·132
Putchapolliam	10	59	42·276
Dodagoontah	12	59	52·165
Namthabad	15	5	53·562
Daumergida	18	3	16·245
Takal Khera	21	5	51·532
Kalianpur	24	7	11·860

By eliminating the declinations of the stars, he obtains the following amplitudes :—

	°	′	″
Punnœ, Putchapolliam	2	50	10·985
Putchapolliam, Dodagoontah . .	2	0	10·530
Dodagoontah, Namthabad . .	2	6	0·788
Namthabad, Daumergida . . .	2	57	22·532
Daumergida, Takal Khera . .	3	2	35·950
Takal Khera, Kalianpur	3	1	20·392

These amplitudes do not agree precisely with the differences of the previously determined latitudes. Of the two sets of results, however, the former have the greater *weight*, and are therefore entitled to the preference.

The latitudes of Daumergida and Kalianpur have been redetermined by Colonel Everest in his later operations in India, together with that of the present northern terminus, Kaliana. At pages lxx and lxxi we find the amplitudes of Daumergida and Kalianpur = 6° 3′ 55″·97, and of Kalianpur and Kaliana = 5° 23′ 37″·06. At page clxxiii the latitude of Kalianpur is given as 24° 7′ 11″·262.

In consequence of the small uncertainty in the absolute distances south of Daumergida, we shall arrange the equations in such a manner that the two portions can be separated if necessary. This will be effected by making Daumergida the initial point, and measuring southwards and northwards from that point.

Indian Arc.

Stations.	Latitudes.	Amplitudes.	Distances.
	° ′ ″	° ′ ″	Feet.
Punnœ	8 9 31·132	− 9 53 44·160	− 3591784·3
Putchapolliam . .	10 59 42·276	− 7 3 33·016	− 2562610·6
Dodagoontah . .	12 59 52·165	− 5 3 23·127	− 1835224·3
Namthabad. . .	15 5 53·562	− 2 57 21·730	− 1073410·9
Daumergida . .	18 3 15·292	....	.... 0·0
Takal Khera . .	21 5 51·532	3 2 36·240	1105539·8
Kalianpur . . .	24 7 11·262	6 3 55·970	2202904·7
Kaliana . . .	29 30 48·322	11 27 33·030	4164042·7

If x_3 be the correction to the latitude of Daumergida, the corrections to the other latitudes will be as follows :—

Punnœ	$+ 0{\cdot}402 - 3{\cdot}5622\,u - 3{\cdot}1942\,v - 4{\cdot}2775\,z + x_3$
Putchapolliam . . .	$- 0{\cdot}629 - 2{\cdot}5413\,u - 2{\cdot}2256\,v - 2{\cdot}6704\,z + x_3$
Dodagoontah . . .	$+ 4{\cdot}538 - 1{\cdot}8198\,u - 1{\cdot}5641\,v - 1{\cdot}7017\,z + x_3$
Namthabad	$- 1{\cdot}506 - 1{\cdot}0643\,u - 0{\cdot}8943\,v - 0{\cdot}8574\,z + x_3$
Takal Khera	$+ 2{\cdot}168 + 1{\cdot}0958\,u + 0{\cdot}8525\,v + 0{\cdot}4451\,z + x_3$
Kalianpur	$- 4{\cdot}063 + 2{\cdot}1831\,u + 1{\cdot}6212\,v + 0{\cdot}4285\,z + x_3$
Kaliana	$+ 0{\cdot}365 + 4{\cdot}1251\,u + 2{\cdot}7741\,v - 0{\cdot}7213\,z + x_3$

Of Colonel Lambton's first arc in Southern India the particulars will be found in the eighth volume of the *Asiatic Researches*. The data are as follows :—

Indian Arc.

tations.	Latitudes.	Amplitude.	Distance.
	° ′ ″	° ′ ″	Feet.
Trivandeporum . .	11 44 52·590	...	...
Paudree	13 19 49·018	1 34 56·428	574327·9

If x_4 be the correction to the latitude of Trivandeporum, that to Paudree will be—

Paudree . . . . . . $+ 0{\cdot}197 + 0{\cdot}5697\,u + 0{\cdot}5182\,v + 0{\cdot}7329\,z + x_4$

6.

The account of the Prussian Arc by Bessel will be found in the work entitled *Gradmessung in Ostpreussen, &c.—Berlin*, 1838. The latitudes of the three Astronomical Stations are finally deduced at page 419, and the distances of the parallels at page 448.

Prussian Arc.

Stations.	Latitudes.	Amplitudes.	Distances.
	° ′ ″	° ′ ″	Toises.
Trunz	54 13 11·466	...	...
Königsberg . . .	54 42 50·500	0 29 39·034	28211·629
Memel	55 43 40·446	1 30 28·980	86176·975

If x_5 be the correction to the observed latitude of Trunz, the corrections for the other points will be—

Königsberg . . . . . $- 0{\cdot}671 + 0{\cdot}1778\,u - 0{\cdot}0576\,v - 0{\cdot}2804\,z + x_5$
Memel . . . . . . $+ 2{\cdot}865 + 0{\cdot}5432\,u - 0{\cdot}1849\,v - 0{\cdot}8312\,z + x_5$

The astronomical observations for the determination of the latitudes of the extreme points of the Peruvian arc have been recomputed by Delambre (*Base du Syst. Met.*, III. pages 112—133), and also by the Baron Von Zach (*Mon Corresp.*, XXVI. page 52). The amplitudes assigned by these differ by 0″·67; the mean value is 3° 7′ 3″·455. The latitudes found by Delambre are—

	°	′	″	
Tarqui	3	4	31·90	S.
Cotchesqui	0	2	31·22	N.

The difference of these differs by 0″·335 from the mean amplitude just stated: by adding 0″·168 to the latitude of Tarqui and 0″·167 to that of Cotchesqui, we get for the latitude of Tarqui 3° 4′ 32″·068, and for that of Cotchesqui 0° 2′ 31″·387: these are the values used by Bessel. The mean of the two values of the distance of parallels assigned by Delambre and Von Zach is 176875·5 toises. The determinations in this arc cannot be considered as very accurate, which is much to be regretted, as the position of the measurement on the surface of the earth otherwise renders it most valuable.

The data for the Hanoverian and Danish arcs are taken from the *Astronomische Nachrichten*, Number 333.

Peruvian Arc.

Stations.	Latitudes.	Amplitude.	Distance.
	° ′ ″	° ′ ″	Toises.
Tarqui	−3 4 32·068	...	...
Cotchesqui . . .	0 2 31·387	3 7 3·455	176875·5

If x_6 be the correction to the latitude of Tarqui, that of Cotchesqui will be—

Cotchesqui . . . . . $+ 0{\cdot}262 + 1{\cdot}1223\,u + 1{\cdot}1258\,v + 2{\cdot}2388\,z + x_6$

Hanoverian Arc.

Stations.	Latitudes.	Amplitude.	Distance.
	° ′ ″	° ′ ″	Toises.
Göttingen	51 31 47·85	...	...
Altona	53 32 45·27	2 0 57·42	115163·725

If x_7 be the correction to the latitude of Göttingen, that of Altona will be—

Altona . . . . . $+ 4{\cdot}436 + 0{\cdot}7262\,u - 0{\cdot}1885\,v - 1{\cdot}2524\,z + x_7$

Danish Arc.

Stations.	Latitudes.	Amplitude.	Distance.
	° ′ ″	° ′ ″	Toises.
Lauenburg . . .	53 22 17·046	...	...
Lysabbell	54 54 10·352	1 31 53·306	87436·538

If x_8 be the correction to the latitude of Lauenburg, that of Lysabbell will be—

Lysabbell . . . . . . $- 1{\cdot}310 + 0{\cdot}5512\,u - 0{\cdot}1726\,v - 0{\cdot}8840\,z + x_8$

§ IV.

Determination of $u\ v\ z, x_1 \cdots\cdots x_8$, &c.

By making the sums of the squares of the 66 corrections a minimum, the following equations are arrived at—

$$\begin{array}{rrrrrr}
0 = & +\ 64{\cdot}389 & +34{\cdot}0000\,x_1 & +\ 30{\cdot}7150\,u & -\ 12{\cdot}4884\,v & -\ 45{\cdot}5355\,z \\
0 = & +\ 65{\cdot}310 & +13{\cdot}0000\,x_2 & +\ 54{\cdot}4933\,u & -\ 16{\cdot}1539\,v & -\ 81{\cdot}0076\,z \\
0 = & +\ 1{\cdot}275 & +\ 8{\cdot}0000\,x_3 & -\ 1{\cdot}5836\,u & -\ 2{\cdot}6304\,v & -\ 9{\cdot}3547\,z \\
0 = & +\ 0{\cdot}197 & +\ 2{\cdot}0000\,x_4 & +\ 0{\cdot}5697\,u & +\ 0{\cdot}5182\,v & +\ 0{\cdot}7329\,z \\
0 = & +\ 2{\cdot}194 & +\ 3{\cdot}0000\,x_5 & +\ 0{\cdot}7210\,u & -\ 0{\cdot}2425\,v & -\ 1{\cdot}1116\,z \\
0 = & +\ 0{\cdot}262 & +\ 2{\cdot}0000\,x_6 & +\ 1{\cdot}1223\,u & +\ 1{\cdot}1258\,v & +\ 2{\cdot}2388\,z \\
0 = & +\ 4{\cdot}436 & +\ 2{\cdot}0000\,x_7 & +\ 0{\cdot}7262\,u & -\ 0{\cdot}1885\,v & -\ 1{\cdot}2524\,z \\
0 = & -\ 1{\cdot}310 & +\ 2{\cdot}0000\,x_8 & +\ 0{\cdot}5512\,u & -\ 0{\cdot}1726\,v & -\ 0{\cdot}8840\,z \\
0 = & +460{\cdot}6524 & +F_1(x_1\,x_2\cdots) & +539{\cdot}9160\,u & -111{\cdot}2099\,v & -\ 694{\cdot}3329\,z \\
0 = & -172{\cdot}7428 & +F_2(x_1\,x_2\cdots) & -111{\cdot}2099\,u & +\ 83{\cdot}1016\,v & +\ 229{\cdot}4756\,z \\
0 = & -668{\cdot}9672 & +F_3(x_1\,x_2\cdots) & -694{\cdot}3329\,u & +229{\cdot}4756\,v & +1134{\cdot}0821\,z
\end{array}$$

In which the values of $F_1\ F_2\ F_3$ are—

$$\begin{aligned}
F_1 = & +\ 30{\cdot}7150\,x_1 + 54{\cdot}4933\,x_2 - 1{\cdot}5836\,x_3 + 0{\cdot}5697\,x_4 \\
& +\ \ 0{\cdot}7210\,x_5 + \ \ 1{\cdot}1223\,x_6 + 0{\cdot}7262\,x_7 + 0{\cdot}5512\,x_8 \\
F_2 = & -\ 12{\cdot}4884\,x_1 - 16{\cdot}1539\,x_2 - 2{\cdot}6304\,x_3 + 0{\cdot}5182\,x_4 \\
& -\ \ 0{\cdot}2425\,x_5 + \ \ 1{\cdot}1258\,x_6 - 0{\cdot}1885\,x_7 - 0{\cdot}1726\,x_8 \\
F_3 = & -\ 45{\cdot}5355\,x_1 - 81{\cdot}0076\,x_2 - 9{\cdot}3547\,x_3 + 0{\cdot}7329\,x_4 \\
& -\ \ 1{\cdot}1116\,x_5 + \ \ 2{\cdot}2388\,x_6 - 1{\cdot}2524\,x_7 - 0{\cdot}8840\,x_8
\end{aligned}$$

From the first eight equations obtain the eight quantities x in terms of $u\ v\ z$; substitute these values in $F_1\ F_2\ F_3$, and we have finally—

$$\begin{array}{rrrrr}
0 = & +126{\cdot}9908 & +\ 282{\cdot}0495\,u & -\ 33{\cdot}3400\,v & -\ 315{\cdot}9814\,z \\
0 = & -\ 67{\cdot}2347 & -\ \ 33{\cdot}3400\,u & +\ 56{\cdot}7563\,v & +\ 107{\cdot}2795\,z \\
0 = & -171{\cdot}6256 & -\ 315{\cdot}9814\,u & +107{\cdot}2795\,v & +\ 553{\cdot}0100\,z
\end{array} \qquad (\omega)$$

which is the final equation for the determination of $u\ v$ and z. For convenience of future reference the details of the multiplications for the different arcs are contained in the following table:—

Arcs.	(ma)	(mb)	(mc)	(ab)	(ac)	(bc)	(a^2)	(b^2)	(c^2)
Anglo-Gallic . .	+105·4240	− 42·8052	−152·6414	− 36·0739	−259·2221	+ 56·2108	+155·0671	+11·4230	+439·3151
Russian . . .	+362·3634	−119·8721	−502·9558	−113·1013	−460·6233	+147·6981	+335·5318	+40·5868	+656·1373
Indian, II. . .	− 11·4775	− 9·3614	− 7·5102	+ 36·7493	+ 24·4806	+ 22·1080	+ 46·5750	+29·4531	+ 29·9609
Indian, I. . .	+ 0·1122	+ 0·1021	+ 0·1444	+ 0·2952	+ 0·4175	+ 0·3798	0·3246	0·2685	0·5371
Prussian . .	+ 1·4370	− 0·4911	− 2·1932	− 0·1107	− 0·5014	+ 0·1698	0·3267	0·0375	0·7695
Peruvian . .	+ 0·2940	+ 0·2950	+ 0·5866	+ 1·2635	+ 2·5126	+ 2·5204	1·2596	1·2674	5·0122
Hanoverian .	+ 3·2214	− 0·8362	− 5·5556	− 0·1369	− 0·9095	+ 0·2361	0·5274	0·0355	1·5685
Danish . . .	− 0·7221	+ 0·2261	+ 1·1580	− 0·0951	− 0·4873	+ 0·1526	0·3038	0·0298	0·7815
	+460·6524	−172·7428	−668·9672	−111·2099	−694·3329	+229·4756	539·9160	83·1016	1134·0821

Arcs.	$\frac{(m)(a)}{i}$	$\frac{(m)(b)}{i}$	$\frac{(m)(c)}{i}$	$\frac{(a)(b)}{i}$	$\frac{(a)(c)}{i}$	$\frac{(b)(c)}{i}$	$\frac{(a)^2}{i}$	$\frac{(b)^2}{i}$	$\frac{(c)^2}{i}$
Anglo-Gallic . .	+ 58·1679	− 23·6504	− 86·2348	−11·2818	− 41·1360	+ 16·7255	+ 27·7474	+ 4·5871	+ 60·9847
Russian . . .	+273·7660	− 81·1547	−406·9697	−67·7138	−339·5670	+100·6606	+228·4246	+20·0730	+504·7870
Indian, II. . .	− 0·2524	− 0·4192	− 1·4909	+ 0·5207	+ 1·8518	+ 3·0758	+ 0·3135	+ 0·8649	+ 10·9388
Indian, I. . .	+ 0·0561	+ 0·0510	+ 0·0722	+ 0·1476	+ 0·2088	+ 0·1899	0·1623	0·1343	0·2686
Prussian . .	+ 0·5273	− 0·1773	− 0·8129	− 0·0583	− 0·2671	+ 0·0898	0·1733	0·0196	0·4119
Peruvian . .	+ 0·1470	+ 0·1475	+ 0·2933	+ 0·6317	+ 1·2563	+ 1·2602	0·6298	0·6337	2·5061
Hanoverian .	+ 1·6107	− 0·4181	− 2·7778	− 0·0684	− 0·4547	+ 0·1180	0·2637	0·0178	0·7843
Danish . . .	− 0·3610	+ 0·1131	+ 0·5790	− 0·0476	− 0·2436	+ 0·0763	0·1519	0·0149	0·3907
	+333·6616	−105·5081	−497·3416	−77·8699	−378·3515	+122·1961	257·8665	26·3453	581·0721

Arcs.	MA	MB	MC	AB	AC	BC	A^2	B^2	C^2
Anglo-Gallic . .	+ 47·2561	− 19·1548	− 66·4066	−24·7921	−218·0861	+ 39·4853	+127·3197	+ 6·8359	+378·3304
Russian . .	+ 88·5974	− 38·7174	− 95·9861	−45·3875	−121·0563	+ 47·0375	+107·1072	+20·5138	+151·3503
Indian, II. . .	− 11·2251	− 8·9422	− 6·0193	+36·2286	+ 22·6288	+ 19·0322	+ 46·2615	+28·5882	+ 19·0221
Indian, I. . .	+ 0·0561	+ 0·0511	+ 0·0722	+ 0·1476	+ 0·2087	+ 0·1899	0·1623	0·1342	0·2685
Prussian . .	+ 0·9097	− 0·3138	− 1·3803	− 0·0524	− 0·2343	+ 0·0800	0·1534	0·0179	0·3576
Peruvian . .	+ 0·1470	+ 0·1475	+ 0·2933	+ 0·6318	+ 1·2563	+ 1·2602	0·6298	0·6337	2·5061
Hanoverian .	+ 1·6107	− 0·4181	− 2·7778	− 0·0685	− 0·4548	+ 0·1181	0·2637	0·0177	0·7842
Danish . . .	− 0·3611	+ 0·1130	+ 0·5790	− 0·0475	− 0·2437	+ 0·0763	0·1519	0·0149	0·3908
	+126·9908	− 67·2347	−171·6256	−33·3400	−315·9814	+107·2795	282·0495	56·7563	553·0100

If we write P Q R for the three absolute terms of the equations (ω), and then express $u\ v\ z$ in terms of P Q R, the result is—

$$\begin{aligned}
0 &= u + \cdot0125381\,\mathrm{P} - \cdot0097521\,\mathrm{Q} + \cdot0090559\,\mathrm{R} \\
0 &= v - \cdot0097521\,\mathrm{P} + \cdot0354054\,\mathrm{Q} - \cdot0124405\,\mathrm{R} \\
0 &= z + \cdot0090559\,\mathrm{P} - \cdot0124405\,\mathrm{Q} + \cdot0093960\,\mathrm{R}
\end{aligned} \qquad (\omega')$$

The weight of $\alpha u + \beta v + \gamma z$ is the reciprocal of

$$\begin{aligned}
&+ \cdot012538\,\alpha^2 + \cdot035405\,\beta^2 + \cdot009396\,\gamma^2 \\
&- \cdot019504\,\alpha\beta + \cdot018112\,\alpha\gamma - \cdot024881\,\beta\gamma
\end{aligned}$$

The numerical values of $u\ v\ z$ are—

$$\begin{aligned}
u &= - 0\cdot6937 \\
v &= + 1\cdot4838 \\
z &= + 0\cdot3739
\end{aligned}$$

Suppose that the correction to the latitude of the r^{th} Station in the n^{th} arc is represented by

$$[n\cdot r] + a_{n.r}\,u + b_{n.r}\,v + c_{n.r}\,z + x_n$$

When r designates the point taken as initial, this all vanishes except x_n: in general, for the initial point $r = 1$, but in the Anglo-Gallic arc $r = 7$ (St. Agnes): in the Indian $r = 5$ (Daumergida). So that in general $[n\cdot1] = 0$, but in the first and third arcs $[1\cdot7] = 0$ and $[3\cdot5] = 0$.

The following expression is the value of $\alpha u + \beta v + \gamma z$ in terms of the observed latitudes:—

$$\begin{array}{lllll}
\textit{Anglo-Gallic}\ \ldots\ldots\ldots\ldots\ * & +\ (-\ \cdot018796\,\alpha & +\ \cdot056910\,\beta & -\ \cdot038431\,\gamma) & [1\cdot1] \\
 & +\ (-\ \cdot012812\,\alpha & +\ \cdot037731\,\beta & -\ \cdot027992\,\gamma) & [1\cdot2] \\
 & +\ (-\ \cdot012769\,\alpha & +\ \cdot037609\,\beta & -\ \cdot027922\,\gamma) & [1\cdot3] \\
 & +\ (-\ \cdot008691\,\alpha & +\ \cdot025705\,\beta & -\ \cdot020936\,\gamma) & [1\cdot4] \\
 & +\ (+\ \cdot001564\,\alpha & -\ \cdot001927\,\beta & -\ \cdot002451\,\gamma) & [1\cdot5] \\
 & +\ (+\ \cdot003767\,\alpha & -\ \cdot007865\,\beta & +\ \cdot002636\,\gamma) & [1\cdot6] \\
 & +\ (+\ \cdot002780\,\alpha & -\ \cdot005153\,\beta & +\ \cdot000167\,\gamma) & [1\cdot7] \\
 & +\ (+\ \cdot002938\,\alpha & -\ \cdot005573\,\beta & +\ \cdot000527\,\gamma) & [1\cdot8] \\
 & +\ (+\ \cdot003252\,\alpha & -\ \cdot006429\,\beta & +\ \cdot001282\,\gamma) & [1\cdot9] \\
 & +\ (+\ \cdot003430\,\alpha & -\ \cdot006926\,\beta & +\ \cdot001735\,\gamma) & [1\cdot10] \\
 & +\ (+\ \cdot003433\,\alpha & -\ \cdot006931\,\beta & +\ \cdot001741\,\gamma) & [1\cdot11] \\
 & +\ (+\ \cdot003437\,\alpha & -\ \cdot006944\,\beta & +\ \cdot001752\,\gamma) & [1\cdot12] \\
 & +\ (+\ \cdot003453\,\alpha & -\ \cdot006982\,\beta & +\ \cdot001787\,\gamma) & [1\cdot13] \\
 & +\ (+\ \cdot003501\,\alpha & -\ \cdot007127\,\beta & +\ \cdot001923\,\gamma) & [1\cdot14] \\
 & +\ (+\ \cdot003678\,\alpha & -\ \cdot007619\,\beta & +\ \cdot002392\,\gamma) & [1\cdot15] \\
 & +\ (+\ \cdot004044\,\alpha & -\ \cdot008682\,\beta & +\ \cdot003473\,\gamma) & [1\cdot16] \\
 & +\ (+\ \cdot004272\,\alpha & -\ \cdot009394\,\beta & +\ \cdot004285\,\gamma) & [1\cdot17] \\
 & +\ (+\ \cdot004367\,\alpha & -\ \cdot009725\,\beta & +\ \cdot004711\,\gamma) & [1\cdot18]
\end{array}$$

Anglo-Gallic—(continued) *	$+ (+ {\cdot}004369\,\alpha - {\cdot}009738\,\beta + {\cdot}004725\,\gamma)$	[1·19]
	$+ (+ {\cdot}004510\,\alpha - {\cdot}010510\,\beta + {\cdot}006069\,\gamma)$	[1·20]
	$+ (+ {\cdot}004494\,\alpha - {\cdot}010585\,\beta + {\cdot}006331\,\gamma)$	[1·21]
	$+ (+ {\cdot}004313\,\alpha - {\cdot}010544\,\beta + {\cdot}006953\,\gamma)$	[1·22]
	$+ (+ {\cdot}004118\,\alpha - {\cdot}010338\,\beta + {\cdot}007238\,\gamma)$	[1·23]
	$+ (+ {\cdot}003999\,\alpha - {\cdot}010189\,\beta + {\cdot}007341\,\gamma)$	[1·24]
	$+ (+ {\cdot}002681\,\alpha - {\cdot}008181\,\beta + {\cdot}007573\,\gamma)$	[1·25]
	$+ (+ {\cdot}002607\,\alpha - {\cdot}008063\,\beta + {\cdot}007560\,\gamma)$	[1·26]
	$+ (+ {\cdot}002337\,\alpha - {\cdot}007618\,\beta + {\cdot}007508\,\gamma)$	[1·27]
	$+ (+ {\cdot}000758\,\alpha - {\cdot}004926\,\beta + {\cdot}006963\,\gamma)$	[1·28]
	$+ (- {\cdot}001031\,\alpha - {\cdot}001798\,\beta + {\cdot}006054\,\gamma)$	[1·29]
	$+ (- {\cdot}001469\,\alpha - {\cdot}001017\,\beta + {\cdot}005802\,\gamma)$	[1·30]
	$+ (- {\cdot}002863\,\alpha + {\cdot}001453\,\beta + {\cdot}004947\,\gamma)$	[1·31]
	$+ (- {\cdot}007730\,\alpha + {\cdot}010168\,\beta + {\cdot}001541\,\gamma)$	[1·32]
	$+ (- {\cdot}007952\,\alpha + {\cdot}010569\,\beta + {\cdot}001374\,\gamma)$	[1·33]
	$+ (- {\cdot}007990\,\alpha + {\cdot}010640\,\beta + {\cdot}001345\,\gamma)$	[1·34]
Russian .... *	$+ (+ {\cdot}008244\,\alpha - {\cdot}007353\,\beta - {\cdot}005131\,\gamma)$	[2·1]
	$+ (+ {\cdot}011340\,\alpha - {\cdot}015603\,\beta + {\cdot}000429\,\gamma)$	[2·2]
	$+ (+ {\cdot}013954\,\alpha - {\cdot}022466\,\beta + {\cdot}005455\,\gamma)$	[2·3]
	$+ (+ {\cdot}015510\,\alpha - {\cdot}026581\,\beta + {\cdot}008810\,\gamma)$	[2·4]
	$+ (+ {\cdot}016832\,\alpha - {\cdot}030395\,\beta + {\cdot}012613\,\gamma)$	[2·5]
	$+ (+ {\cdot}016591\,\alpha - {\cdot}031151\,\beta + {\cdot}015447\,\gamma)$	[2·6]
	$+ (+ {\cdot}014793\,\alpha - {\cdot}028379\,\beta + {\cdot}015658\,\gamma)$	[2·7]
	$+ (+ {\cdot}011436\,\alpha - {\cdot}022568\,\beta + {\cdot}014187\,\gamma)$	[2·8]
	$+ (+ {\cdot}006930\,\alpha - {\cdot}014536\,\beta + {\cdot}011263\,\gamma)$	[2·9]
	$+ (- {\cdot}002527\,\alpha + {\cdot}002486\,\beta + {\cdot}003929\,\gamma)$	[2·10]
	$+ (- {\cdot}019239\,\alpha + {\cdot}032546\,\beta - {\cdot}010518\,\gamma)$	[2·11]
	$+ (- {\cdot}038823\,\alpha + {\cdot}067578\,\beta - {\cdot}028444\,\gamma)$	[2·12]
	$+ (- {\cdot}055042\,\alpha + {\cdot}096420\,\beta - {\cdot}043699\,\gamma)$	[2·13]
Indian, II. .... *	$+ (+ {\cdot}042385\,\alpha + {\cdot}029975\,\beta + {\cdot}024024\,\gamma)$	[3·1]
	$+ (+ {\cdot}024477\,\alpha + {\cdot}025630\,\beta + {\cdot}011728\,\gamma)$	[3·2]
	$+ (+ {\cdot}013109\,\alpha + {\cdot}021297\,\beta + {\cdot}004322\,\gamma)$	[3·3]
	$+ (+ {\cdot}002523\,\alpha + {\cdot}015454\,\beta - {\cdot}002120\,\gamma)$	[3·4]
	$+ (- {\cdot}009865\,\alpha + {\cdot}004836\,\beta - {\cdot}008689\,\gamma)$	[3·5]
	$+ (- {\cdot}019321\,\alpha - {\cdot}009123\,\beta - {\cdot}012189\,\gamma)$	[3·6]
	$+ (- {\cdot}025307\,\alpha - {\cdot}025942\,\beta - {\cdot}012317\,\gamma)$	[3·7]
	$+ (- {\cdot}028000\,\alpha - {\cdot}062127\,\beta - {\cdot}004757\,\gamma)$	[3·8]
Indian, I................... *	$+ (+ {\cdot}004363\,\alpha + {\cdot}001837\,\beta + {\cdot}002799\,\gamma)$	[4·1]
	$+ (- {\cdot}004363\,\alpha - {\cdot}001837\,\beta - {\cdot}002799\,\gamma)$	[4·2]
Prussian.................... *	$+ (+ {\cdot}000446\,\alpha - {\cdot}000596\,\beta - {\cdot}000299\,\gamma)$	[5·1]
	$+ (+ {\cdot}000194\,\alpha - {\cdot}000311\,\beta + {\cdot}000008\,\gamma)$	[5·2]
	$+ (- {\cdot}000640\,\alpha + {\cdot}000907\,\beta + {\cdot}000291\,\gamma)$	[5·3]
Peruvian..................... *	$+ (+ {\cdot}011683\,\alpha + {\cdot}000531\,\beta + {\cdot}008597\,\gamma)$	[6·1]
	$+ (- {\cdot}011683\,\alpha - {\cdot}000531\,\beta - {\cdot}008597\,\gamma)$	[6·2]
Hanoverian.............. *	$+ (- {\cdot}000199\,\alpha + {\cdot}000912\,\beta - {\cdot}001423\,\gamma)$	[7·1]
	$+ (+ {\cdot}000199\,\alpha - {\cdot}000912\,\beta + {\cdot}001423\,\gamma)$	[7·2]
Danish...................... *	$+ (+ {\cdot}000294\,\alpha - {\cdot}000244\,\beta - {\cdot}000584\,\gamma)$	[8·1]
	$+ (- {\cdot}000294\,\alpha + {\cdot}000244\,\beta + {\cdot}000584\,\gamma)$	[8·2]

By inspection of this formula we see at a glance the influence of each observed latitude upon the final result. The points which have the greatest influence are the extremities of the Indian arc, the south point of the Anglo-Gallic, and the north point of the Russian arc. An error of a second at Formentera would affect the major semiaxis by 69 feet, and the minor by only 11; an error at Saxavord of the same amount would affect the axes by smaller quantities, viz. 25 and 9 feet; at Fuglenœs an error of one second affects the semiaxis major by 170 feet, and the minor by 35; at Punnœ an error of a second affects the major semiaxis by 60 feet, and the minor by 102; at Kaliana the same amount of error would affect the axes by 14 and 100 feet respectively. The small arcs have very little influence at all.

Corrections to the Latitudes.

By substituting the values of u v and z which we have found in the expressions for the corrections to the different latitudes, we get the results given in the following table:—

Stations.	Correction.	Stations.	Correction.
Formentera	+ 1·276″	Staro-Nekrassowka	− 2·602″
Montjouy	+ 3·855	Wodolui	+ 1·274
Barcelona	+ 0·502	Ssuprunkowzi	+ 3·008
Carcassonne	− 1·138	Kremenetz	− 1·728
Pantheon	− 3·482	Belin	+ 0·703
Dunkirk	− 0·612	Nemesch	− 1·286
		Jacobstadt	+ 3·065
St. Agnes	− 1·223	Dorpat	− 0·955
Goonhilly	− 2·886	Hogland	− 0·233
Hensbarrow	− 1·321	Kilpi-Maki	− 0·833
High Port Cliff	− 1·827	Tornea	+ 3·456
Week Down	− 1·549	Stuor-Oivi	− 2·369
Boniface Down	− 1·769	Fuglenœs	− 1·500
Dunnose	− 1·227		
Black Down	+ 2·951	Punnœ	− 0·513
Southampton	+ 1·722	Putchapolliam	− 1·416
Greenwich	+ 1·270	Dodagoontah	+ 3·870
Precelly	+ 0·737	Namthabad	− 2·020
Cambridge	+ 0·191	Daumergida	− 0·246
Arbury	+ 1·692	Takal Khera	+ 2·260
Delamere	− 0·713	Kalianpur	− 3·578
Clifton	− 1·964	Kaliana	+ 1·643
South Berule	+ 1·488		
Burleigh Moor	+ 0·665	Trivandeporum	− 0·148
Durham	− 0·935	Paudree	+ 0·148
Ben Lomond	− 1·633		
Kellie Law	− 0·601	Trunz	− 0·583
Ben Heynish	+ 2·106	Königsberg	− 1·358
Great Stirling	− 0·277	Memel	+ 1·941
Monach	+ 0·591		
Ben Hutig	− 1·158	Tarqui	− 0·158
North Rona	+ 0·403	Cotchesqui	+ 0·158
Balta	+ 2·380		
Gerth of Scaw	+ 2·233	Göttingen	− 2·060
Saxavord	+ 0·249	Altona	+ 2·060
		Lauenburg	+ 0·809
		Lysabbel	− 0·809

The sum of the squares of these 66 corrections is 208·486; consequently the probable error of a latitude is

$$\pm\,\cdot6745\sqrt{\frac{208\cdot486}{66-11}}=\pm\,1''\cdot313$$

This, however, is not to be taken as the probable error of an observed latitude, as some of the points in the English arc have received corrections for the apparent effects of the irregular distribution of attractive matter in their vicinity. The corrections to the actually observed latitudes at these points are as follows:—

High Port Cliff	+ 1″·463	*Dunnose*	− 1″·767	*Kellie Law*	+ 1″·479
Week Down	+ 0·431	*Clifton*	− 2·864	*Monach*	+ 1·061
Boniface Down	+ 0·651	*Burleigh*	− 3·885	*Ben Hutig*	− 3·168

The sum of the squares of these nine quantities is 42·5173, and the sum of the squares of the corresponding quantities in the table is 16·7232. Hence the sum of the squares of the corrections to the 66 observed latitudes is 208·486 + 42·517 − 16·723, and the probable error of an observed latitude, in consequence,

$$=\pm\,\cdot6745\sqrt{\frac{234\cdot28}{55}}=\pm\,1''\cdot392$$

Values of the Axes, &c.

Substituting the values of *u v z* in the expressions for *a* and *b*, we have—

$$a=20927197$$
$$b=20855493$$
$$a:b=291\cdot86:290\cdot86$$

With respect to the weights of the determinations—

$$a \text{ involves} \cdots -2092\cdot5\,u+696\cdot3\,v+278\cdot5\,z$$
$$b \text{ involves} \cdots -2085\cdot5\,u-696\cdot3\,v+278\cdot5\,z$$
$$a:b \text{ involves} \cdots -6\,v+\frac{3}{25}\,v^2+\frac{1}{250}\,z$$

From these we easily find—

$$\text{Probable error of } a=\pm\,2092\cdot5\left(\cdot01960\right)^{\frac{1}{2}}1\cdot313=\pm\,385 \text{ feet.}$$

$$\text{,, ,, } b=\pm\,2085\cdot5\left(\cdot00883\right)^{\frac{1}{2}}1\cdot313=\pm\,257 \text{ feet.}$$

In estimating the probable error of the denominator (291·86) of the fraction representing the compression, we may neglect the small uncertainty due to the quantity *z*, then—

$$\text{Probable error of } 291\cdot86=\pm\left(6-\frac{6}{25}\,v\right)\left(\cdot03540\right)^{\frac{1}{2}}1\cdot313=\pm\,1\cdot39$$

The quantity δr by which the curve is more protuberant than an ellipse of the same axes is—

$$\delta r = \left(177{\cdot}5 \pm 70{\cdot}9\right) \sin^2 2\lambda$$

This last quantity is remarkably well determined, and shows decisively that the figure of the earth does not differ sensibly from the *exact* elliptic form.

For the radius of curvature of the meridian, and the length of an arc of amplitude ϕ, we have—

$$\rho = 20891449 - 107557 \cos 2\lambda + 1562 \cos 4\lambda$$
$$s = 20891449\,\phi - 107557 \sin\phi \cos 2\lambda + 781 \sin 2\phi \cos 4\lambda$$

The meridional quadrant $= (364624{\cdot}58 \pm 5{\cdot}36) \times 90 = 32816212 \pm 482$ feet; consequently the *mètre* $= 39{\cdot}3794544 \pm {\cdot}0005784$ inches.

§ V.

Determination of the Ellipse most nearly representing the Observations.

The curve $\rho = A + 2B \cos 2\lambda + 2C \cos 4\lambda$ becomes an ellipse if $6AC - 5B^2 = 0$; that is, if neglecting the square of v,

$$z = -\frac{5}{96} - \frac{v}{480}$$

If we substitute this value of z in the expressions for the corrections to the latitudes of the different points, they become, on the elliptic hypothesis, as follows:—

Formentera	$+ 2{\cdot}378$	$- 4{\cdot}0415\,u$	$- 0{\cdot}1175\,v$	$+ x_1$
Montjouy	$+ 4{\cdot}794$	$- 3{\cdot}0704\,u$	$+ 0{\cdot}0545\,v$	$+ x_1$
Barcelona	$+ 1{\cdot}440$	$- 3{\cdot}0644\,u$	$+ 0{\cdot}0553\,v$	$+ x_1$
Carcassonne	$- 0{\cdot}252$	$- 2{\cdot}4040\,u$	$+ 0{\cdot}1202\,v$	$+ x_1$
Pantheon	$- 2{\cdot}374$	$- 0{\cdot}3767\,u$	$+ 0{\cdot}0557\,v$	$+ x_1$
Dunkirk	$+ 0{\cdot}767$	$+ 0{\cdot}4115\,u$	$- 0{\cdot}0765\,v$	$+ x_1$
St. Agnes	$0{\cdot}$	$0{\cdot}$	$0{\cdot}$	$+ x_1$
Goonhilly	$- 1{\cdot}644$	$+ 0{\cdot}0554\,u$	$- 0{\cdot}0094\,v$	$+ x_1$
Hensbarrow	$- 0{\cdot}035$	$+ 0{\cdot}1768\,u$	$- 0{\cdot}0308\,v$	$+ x_1$
High Port Cliff	$- 0{\cdot}512$	$+ 0{\cdot}2532\,u$	$- 0{\cdot}0450\,v$	$+ x_1$
Week Down	$- 0{\cdot}233$	$+ 0{\cdot}2539\,u$	$- 0{\cdot}0452\,v$	$+ x_1$
Boniface Down	$- 0{\cdot}453$	$+ 0{\cdot}2559\,u$	$- 0{\cdot}0455\,v$	$+ x_1$
Dunnose	$+ 0{\cdot}091$	$+ 0{\cdot}2613\,u$	$- 0{\cdot}0466\,v$	$+ x_1$
Blackdown	$+ 4{\cdot}279$	$+ 0{\cdot}2859\,u$	$- 0{\cdot}0513\,v$	$+ x_1$
Southampton	$+ 3{\cdot}083$	$+ 0{\cdot}3676\,u$	$- 0{\cdot}0674\,v$	$+ x_1$
Greenwich	$+ 2{\cdot}718$	$+ 0{\cdot}5707\,u$	$- 0{\cdot}1101\,v$	$+ x_1$
Precelly	$+ 2{\cdot}262$	$+ 0{\cdot}7393\,u$	$- 0{\cdot}1486\,v$	$+ x_1$
Cambridge	$+ 1{\cdot}764$	$+ 0{\cdot}8360\,u$	$- 0{\cdot}1719\,v$	$+ x_1$

Arbury	$+3{\cdot}267$	$+0{\cdot}8396\,u$	$-0{\cdot}1727\,v$	$+x_1$
Delamere	$+1{\cdot}055$	$+1{\cdot}1986\,u$	$-0{\cdot}2670\,v$	$+x_1$
Clifton	$-0{\cdot}147$	$+1{\cdot}2835\,u$	$-0{\cdot}2911\,v$	$+x_1$
South Berule	$+3{\cdot}459$	$+1{\cdot}5325\,u$	$-0{\cdot}3654\,v$	$+x_1$
Burleigh Moor	$+2{\cdot}736$	$+1{\cdot}6845\,u$	$-0{\cdot}4137\,v$	$+x_1$
Durham	$+1{\cdot}184$	$+1{\cdot}7553\,u$	$-0{\cdot}4369\,v$	$+x_1$
Ben Lomond	$+0{\cdot}868$	$+2{\cdot}2673\,u$	$-0{\cdot}6181\,v$	$+x_1$
Kellie Law	$+1{\cdot}917$	$+2{\cdot}2882\,u$	$-0{\cdot}6258\,v$	$+x_1$
Ben Heynish	$+4{\cdot}684$	$+2{\cdot}3628\,u$	$-0{\cdot}6543\,v$	$+x_1$
Great Stirling	$+2{\cdot}614$	$+2{\cdot}7258\,u$	$-0{\cdot}8003\,v$	$+x_1$
Monach	$+3{\cdot}785$	$+3{\cdot}0471\,u$	$-0{\cdot}9391\,v$	$+x_1$
Ben Hutig	$+2{\cdot}105$	$+3{\cdot}1173\,u$	$-0{\cdot}9708\,v$	$+x_1$
North Rona	$+3{\cdot}874$	$+3{\cdot}3225\,u$	$-1{\cdot}0652\,v$	$+x_1$
Balta	$+6{\cdot}501$	$+3{\cdot}9095\,u$	$-1{\cdot}3555\,v$	$+x_1$
Gerth of Scaw	$+6{\cdot}382$	$+3{\cdot}9330\,u$	$-1{\cdot}3677\,v$	$+x_1$
Saxavord	$+4{\cdot}403$	$+3{\cdot}9370\,u$	$-1{\cdot}3699\,v$	$+x_1$
Staro-Nekrassowka	$0{\cdot}$	$0{\cdot}$	$0{\cdot}$	$+x_2$
Wodolui	$+3{\cdot}946$	$+0{\cdot}6086\,u$	$-0{\cdot}0225\,v$	$+x_2$
Ssuprunkowzi	$+5{\cdot}813$	$+1{\cdot}2306\,u$	$-0{\cdot}0824\,v$	$+x_2$
Kremenetz	$+1{\cdot}226$	$+1{\cdot}7148\,u$	$-0{\cdot}1550\,v$	$+x_2$
Belin	$+3{\cdot}951$	$+2{\cdot}4163\,u$	$-0{\cdot}2998\,v$	$+x_2$
Nemesch	$+2{\cdot}510$	$+3{\cdot}3546\,u$	$-0{\cdot}5657\,v$	$+x_2$
Jacobstadt	$+7{\cdot}363$	$+4{\cdot}0209\,u$	$-0{\cdot}8036\,v$	$+x_2$
Dorpat	$+3{\cdot}953$	$+4{\cdot}6969\,u$	$-1{\cdot}0857\,v$	$+x_2$
Hogland	$+5{\cdot}320$	$+5{\cdot}3114\,u$	$-1{\cdot}3767\,v$	$+x_2$
Kilpi Maki	$+5{\cdot}846$	$+6{\cdot}2290\,u$	$-1{\cdot}8698\,v$	$+x_2$
Tornea	$+11{\cdot}826$	$+7{\cdot}3799\,u$	$-2{\cdot}5821\,v$	$+x_2$
Stuor Oivi	$+7{\cdot}769$	$+8{\cdot}4072\,u$	$-3{\cdot}3001\,v$	$+x_2$
Fuglenœs	$+10{\cdot}008$	$+9{\cdot}1231\,u$	$-3{\cdot}8418\,v$	$+x_2$
Punnœ	$+0{\cdot}625$	$-3{\cdot}5622\,u$	$-3{\cdot}1853\,v$	$+x_3$
Putchapolliam	$-0{\cdot}490$	$-2{\cdot}5413\,u$	$-2{\cdot}2200\,v$	$+x_3$
Dodagoontah	$+4{\cdot}627$	$-1{\cdot}8198\,u$	$-1{\cdot}5606\,v$	$+x_3$
Namthabad	$-1{\cdot}461$	$-1{\cdot}0643\,u$	$-0{\cdot}8925\,v$	$+x_3$
Daumergida	$0{\cdot}$	$0{\cdot}$	$0{\cdot}$	x_3
Takal Khera	$+2{\cdot}145$	$+1{\cdot}0958\,u$	$+0{\cdot}8516\,v$	$+x_3$
Kalianpur	$-4{\cdot}085$	$+2{\cdot}1831\,u$	$+1{\cdot}6203\,v$	$+x_3$
Kaliana	$+0{\cdot}403$	$+4{\cdot}1251\,u$	$+2{\cdot}7756\,v$	$+x_3$
Trivandeporum	$0{\cdot}$	$0{\cdot}$	$0{\cdot}$	x_4
Paudree	$+0{\cdot}159$	$+0{\cdot}5697\,u$	$+0{\cdot}5167\,v$	$+x_4$
Trunz	$0{\cdot}$	$0{\cdot}$	$0{\cdot}$	x_5
Königsberg	$-0{\cdot}656$	$+0{\cdot}1778\,u$	$-0{\cdot}0570\,v$	$+x_5$
Memel	$+2{\cdot}908$	$+0{\cdot}5432\,u$	$-0{\cdot}1832\,v$	$+x_5$
Tarqui	$0{\cdot}$	$0{\cdot}$	$0{\cdot}$	$+x_6$
Cotchesqui	$+0{\cdot}145$	$+1{\cdot}1223\,u$	$+1{\cdot}1211\,v$	$+x_6$
Göttingen	$0{\cdot}$	$0{\cdot}$	$0{\cdot}$	$+x_7$
Altona	$+4{\cdot}501$	$+0{\cdot}7262\,u$	$-0{\cdot}1859\,v$	$+x_7$
Lauenburg	$0{\cdot}$	$0{\cdot}$	$0{\cdot}$	$+x_8$
Lysabbel	$-1{\cdot}264$	$+0{\cdot}5512\,u$	$-0{\cdot}1708\,v$	$+x_8$

The following table contains the results of the multiplications for the different arcs in order to render the sum of the squares of these corrections a minimum :—

Arcs.	(ma)	(mb)	(a^2)	(ab)	(b^2)
Anglo-Gallic	+118·9207	− 45·3653	+155·0671	− 35·5339	+ 11·1906
Russian	+386·3623	−126·4488	335·5318	−112·1421	39·9745
Indian, II.	− 12·7516	− 10·4936	46·5750	+ 36·6984	29·3612
Indian, I.	+ 0·0906	+ 0·0822	0·3246	+ 0·2944	0·2670
Prussian	+ 1·4630	− 0·4953	0·3267	− 0·1096	0·0368
Peruvian	+ 0·1627	+ 0·1626	1·2596	+ 1·2582	1·2569
Hanoverian	+ 3·2686	− 0·8367	0·5274	− 0·1350	0·0346
Danish	− 0·6967	+ 0·2159	0·3038	− 0·0941	0·0292
	+496·8196	−183·1790	+539·9160	−109·7637	+ 82·1508

Arcs.	$-\frac{(m)(a)}{i}$	$-\frac{(m)(b)}{i}$	$-\frac{(a)^2}{i}$	$-\frac{(a)(b)}{i}$	$-\frac{(b)^2}{i}$
Anglo-Gallic	− 60·3098	+ 24·3352	− 27·7474	+ 11·1962	− 4·5177
Russian	−291·4511	+ 85·4950	−228·4246	+ 67·0066	− 19·6559
Indian, II.	+ 0·3492	+ 0·5757	− 0·3135	− 0·5168	− 0·8521
Indian, I.	− 0·0453	− 0·0411	− 0·1623	− 0·1472	− 0·1335
Prussian	− 0·5412	+ 0·1803	− 0·1733	+ 0·0577	− 0·0192
Peruvian	− 0·0814	− 0·0813	− 0·6298	− 0·6291	− 0·6284
Hanoverian	− 1·6343	+ 0·4183	− 0·2637	+ 0·0675	− 0·0173
Danish	+ 0·3483	− 0·1079	− 0·1519	+ 0·0471	− 0·0146
	−353·3656	+110·7742	−257·8665	+ 77·0820	− 25·8387

Arcs.	MA	MB	A^2	AB	B^2
Anglo-Gallic	+ 58·6109	− 21·0301	+127·3197	− 24·3377	+ 6·6729
Russian	+ 94·9112	− 40·9538	107·1072	− 45·1355	20·3186
Indian, II.	− 12·4024	− 9·9179	46·2615	+ 36·1816	28·5091
Indian, I.	+ 0·0453	+ 0·0411	0·1623	+ 0·1472	0·1335
Prussian	+ 0·9218	− 0·3150	0·1534	− 0·0519	0·0176
Peruvian	+ 0·0813	+ 0·0813	0·6298	+ 0·6291	0·6285
Hanoverian	+ 1·6343	− 0·4184	0·2637	− 0·0675	0·0173
Danish	− 0·3484	+ 0·1080	0·1519	− 0·0470	0·0146
	+143·4540	− 72·4048	+282·0495	− 32·6817	+ 56·3121

From this it appears that the final equation for the determination of u and v is the following :—

$$0 = +\ 143{\cdot}4540 + 282{\cdot}0495\,u - 32{\cdot}6817\,v$$
$$0 = -\ \ 72{\cdot}4048 - \ 32{\cdot}6817\,u + 56{\cdot}3121\,v$$

If we write P Q for the absolute terms of this equation, it becomes by transformation—

$$0 = u + {\cdot}0038011\,P + {\cdot}0022060\,Q$$
$$0 = v + {\cdot}0022060\,P + {\cdot}0190385\,Q$$

Whence it follows that the weight of the determination of $\alpha u + \beta v$ is the reciprocal of

$$\cdot003801\,\alpha^2 + \cdot004412\,\alpha\beta + \cdot019038\,\beta^2$$

The actual values of u and v are—

$$u = -\,0\cdot3856$$
$$v = +\,1\cdot0620$$

We may, as in the preceding Section, obtain an expression for the value of $\alpha u + \beta v$ in terms of the observations. Adopting the same notation, the value will be as follows:—

*Anglo-Gallic**	$+ (+ \cdot018251\,\alpha + \cdot006206\,\beta)$	[1·1]
	$+ (+ \cdot014180\,\alpha + \cdot000789\,\beta)$	[1·2]
	$+ (+ \cdot014156\,\alpha + \cdot000760\,\beta)$	[1·3]
	$+ (+ \cdot011502\,\alpha - \cdot001932\,\beta)$	[1·4]
	$+ (+ \cdot003939\,\alpha - \cdot005176\,\beta)$	[1·5]
	$+ (+ \cdot001234\,\alpha - \cdot004398\,\beta)$	[1·6]
	$+ (+ \cdot002630\,\alpha - \cdot004947\,\beta)$	[1·7]
	$+ (+ \cdot002440\,\alpha - \cdot004890\,\beta)$	[1·8]
	$+ (+ \cdot002026\,\alpha - \cdot004751\,\beta)$	[1·9]
	$+ (+ \cdot001767\,\alpha - \cdot004649\,\beta)$	[1·10]
	$+ (+ \cdot001764\,\alpha - \cdot004647\,\beta)$	[1·11]
	$+ (+ \cdot001757\,\alpha - \cdot004645\,\beta)$	[1·12]
	$+ (+ \cdot001739\,\alpha - \cdot004636\,\beta)$	[1·13]
	$+ (+ \cdot001656\,\alpha - \cdot004601\,\beta)$	[1·14]
	$+ (+ \cdot001381\,\alpha - \cdot004475\,\beta)$	[1·15]
	$+ (+ \cdot000703\,\alpha - \cdot004110\,\beta)$	[1·16]
	$+ (+ \cdot000147\,\alpha - \cdot003749\,\beta)$	[1·17]
	$+ (- \cdot000169\,\alpha - \cdot003519\,\beta)$	[1·18]
	$+ (- \cdot000181\,\alpha - \cdot003511\,\beta)$	[1·19]
	$+ (- \cdot001337\,\alpha - \cdot002508\,\beta)$	[1·20]
	$+ (- \cdot001607\,\alpha - \cdot002236\,\beta)$	[1·21]
	$+ (- \cdot002389\,\alpha - \cdot001371\,\beta)$	[1·22]
	$+ (- \cdot002861\,\alpha - \cdot000787\,\beta)$	[1·23]
	$+ (- \cdot003079\,\alpha - \cdot000501\,\beta)$	[1·24]
	$+ (- \cdot004625\,\alpha + \cdot001819\,\beta)$	[1·25]
	$+ (- \cdot004687\,\alpha + \cdot001920\,\beta)$	[1·26]
	$+ (- \cdot004908\,\alpha + \cdot002298\,\beta)$	[1·27]
	$+ (- \cdot005966\,\alpha + \cdot004276\,\beta)$	[1·28]
	$+ (- \cdot006881\,\alpha + \cdot006210\,\beta)$	[1·29]
	$+ (- \cdot007078\,\alpha + \cdot006659\,\beta)$	[1·30]
	$+ (- \cdot007650\,\alpha + \cdot008003\,\beta)$	[1·31]
	$+ (- \cdot009240\,\alpha + \cdot012235\,\beta)$	[1·32]
	$+ (- \cdot009303\,\alpha + \cdot012416\,\beta)$	[1·33]
	$+ (- \cdot009313\,\alpha + \cdot012449\,\beta)$	[1·34]
*Russian ... **	$+ (+ \cdot013221\,\alpha - \cdot014163\,\beta)$	[2·1]
	$+ (+ \cdot010957\,\alpha - \cdot015077\,\beta)$	[2·2]
	$+ (+ \cdot008725\,\alpha - \cdot015309\,\beta)$	[2·3]
	$+ (+ \cdot007045\,\alpha - \cdot014995\,\beta)$	[2·4]
	$+ (+ \cdot004698\,\alpha - \cdot013786\,\beta)$	[2·5]
	$+ (+ \cdot001718\,\alpha - \cdot010793\,\beta)$	[2·6]
	$+ (- \cdot000290\,\alpha - \cdot007734\,\beta)$	[2·7]
	$+ (- \cdot002237\,\alpha - \cdot003854\,\beta)$	[2·8]
	$+ (- \cdot003931\,\alpha + \cdot000330\,\beta)$	[2·9]
	$+ (- \cdot006331\,\alpha + \cdot007694\,\beta)$	[2·10]
	$+ (- \cdot009135\,\alpha + \cdot018716\,\beta)$	[2·11]
	$+ (- \cdot011456\,\alpha + \cdot030119\,\beta)$	[2·12]
	$+ (- \cdot012982\,\alpha + \cdot038853\,\beta)$	[2·13]
*Indian, II. **	$+ (+ \cdot019095\,\alpha + \cdot061851\,\beta)$	[3·1]
	$+ (+ \cdot013085\,\alpha + \cdot041221\,\beta)$	[3·2]
	$+ (+ \cdot008888\,\alpha + \cdot027076\,\beta)$	[3·3]
	$+ (+ \cdot004542\,\alpha + \cdot012690\,\beta)$	[3·4]
	$+ (- \cdot001472\,\alpha - \cdot006650\,\beta)$	[3·5]
	$+ (- \cdot007516\,\alpha - \cdot025281\,\beta)$	[3·6]
	$+ (- \cdot013345\,\alpha - \cdot042314\,\beta)$	[3·7]
	$+ (- \cdot023275\,\alpha - \cdot068593\,\beta)$	[3·8]
*Indian, I...**	$+ (+ \cdot001653\,\alpha + \cdot005547\,\beta)$	[4·1]
	$+ (- \cdot001653\,\alpha - \cdot005547\,\beta)$	[4·2]
*Prussian... **	$+ (+ \cdot000737\,\alpha - \cdot000994\,\beta)$	[5·1]
	$+ (+ \cdot000187\,\alpha - \cdot000301\,\beta)$	[5·2]
	$+ (- \cdot000924\,\alpha + \cdot001295\,\beta)$	[5·3]
*Peruvian...**	$+ (+ \cdot003370\,\alpha + \cdot011910\,\beta)$	[6·1]
	$+ (- \cdot003370\,\alpha - \cdot011910\,\beta)$	[6·2]
*Hanoverian**	$+ (+ \cdot001175\,\alpha - \cdot000969\,\beta)$	[7·1]
	$+ (- \cdot001175\,\alpha + \cdot000969\,\beta)$	[7·2]
*Danish ... **	$+ (+ \cdot000859\,\alpha - \cdot001018\,\beta)$	[8·1]
	$+ (- \cdot000859\,\alpha + \cdot001018\,\beta)$	[8·2]

Corrections to the Latitudes.

If in the expressions for the corrections to the latitudes we substitute the numerical values of *u* and *v* just obtained, we get the following system of corrections:—

Stations.	Corrections.	Stations.	Corrections.
Formentera	+ 2″·584	Staro-Nekrassowka	− 2″·426
Montjouy	+ 4·808	Wodolui	+ 1·261
Barcelona	+ 1·452	Ssuprunkowzi	+ 2·825
Carcassonne	− 0·425	Kremenetz	− 2·026
Pantheon	− 3·398	Belin	+ 0·275
Dunkirk	− 0·701	Nemesch	− 1·811
		Jacobstadt	+ 2·533
St. Agnes	− 1·228	Dorpat	− 1·437
Goonhilly	− 2·903	Hogland	− 0·616
Hensbarrow	− 1·364	Kilpi Maki	− 0·968
High Port Cliff	− 1·885	Tornea	+ 3·812
Week Down	− 1·607	Stuor Oivi	− 1·404
Boniface Down	− 1·828	Fuglenœs	− 0·016
Dunnose	− 1·287		
Blackdown	+ 2·886	Punnœ	− 1·334
Southampton	+ 1·642	Putchapolliam	− 1·818
Greenwich	+ 1·153	Dodagoontah	+ 3·721
Precelly	+ 0·591	Namthabad	− 1·949
Cambridge	+ 0·031	Daumergida	+ 0·050
Arbury	+ 1·532	Takal Khera	+ 2·677
Delamere	− 0·919	Kalianpur	− 3·156
Clifton	− 2·179	Kaliana	+ 1·810
South Berule	+ 1·252		
Burleigh Moor	+ 0·419	Trivandeporum	− 0·244
Durham	− 1·185	Paudree	+ 0·244
Ben Lomond	− 1·891		
Kellie Law	− 0·858	Trunz	− 0·573
Ben Heynish	+ 1·850	Königsberg	− 1·358
Great Stirling	− 0·515	Memel	+ 1·931
Monach	+ 0·385		
Ben Hutig	− 1·356	Tarqui	− 0·451
North Rona	+ 0·234	Cotchesqui	+ 0·451
Balta	+ 2·326		
Gerth of Scaw	+ 2·185	Göttingen	− 2·012
Saxavord	+ 0·202	Altona	+ 2·012
		Lauenburg	+ 0·829
		Lysabbel	− 0·829

The sum of the squares of these 66 corrections is 219·360, and hence the probable error of a latitude

$$= \pm \cdot 6745 \sqrt{\frac{219 \cdot 360}{66 - 10}} = \pm 1'' \cdot 335$$

This is not, however, the probable error of an observed latitude,—for this reason, that some of the points in the English arc have already received a correction for local attraction. The corrections to the *observed* latitudes of these points will be—

High Port Cliff	+ 1″·405	*Dunnose*	− 1″·827	*Kellie Law*	+ 1″·222
Week Down	+ 0·373	*Clifton*	− 3·079	*Monach*	+ 0·855
Boniface Down	+ 0·592	*Burleigh Moor*	− 4·131	*Ben Hutig*	− 3·366

The sum of the squares of these corrections is 45·901; that of the corresponding quantities in the preceding table is 18·780, so that the sum of the squares of the corrections to the observed latitudes is 219·360 + 45·901 − 18·780 = 246·481, and the probable error of an observed latitude becomes—

$$\pm \cdot 6745 \sqrt{\frac{246 \cdot 481}{66 - 10}} = \pm 1'' \cdot 415$$

This quantity is somewhat larger than that obtained in the non-elliptic figure, but the difference is immaterial.

If we would obtain the probable error of the observed latitude of any point taken at random, it must be remembered that in the preceding calculation two Stations have been omitted on account of the large amount of deflection at those points; namely, Cowhythe and Evaux. The expressions for the corrections of the latitudes of these two points are—

Cowhythe $- 6 \cdot 915 + 2 \cdot 8048\, u - 0 \cdot 8340\, v + x_1$
Evaux . $- 7 \cdot 045 - 1 \cdot 3378\, u + 0 \cdot 1360\, v + x_1$

Substituting in these expressions the values of u v and x_1, they become—

Cowhythe $- 10'' \cdot 110$
Evaux . $- 7 \cdot 613$

of which the sum of the squares is 160·170. We may therefore take as the *probable error of an observed latitude*—

$$\pm \cdot 6745 \sqrt{\frac{246 \cdot 481 + 160 \cdot 170}{56 + 2}} = \pm 1'' \cdot 786$$

This quantity is affected to a small extent with errors in the different geodetical operations: the influence of these, however, is certainly very small; but their effect is obviously to render this quantity too large as the representative of local attraction. We shall not probably be far from the truth in taking ± 1″·75 as the probable effect of local attraction upon an observed latitude.

Determination of the Axes, &c.

By means of the values of u and v, and the relation $6\,A\,C - 5\,B^2 = 0$, we get for the radius of curvature of the meridian the following expression—

$$\varrho = 20890805 - 106673 \cos 2\lambda + 227 \cos 4\lambda$$

And for the length of a meridian arc whose amplitude is ϕ and mean latitude $= \lambda$,

$$s = 20890805\, \phi - 106673 \cos 2\lambda \sin \phi + 113 \cdot 5 \cos 4\lambda \sin 2\phi$$

For the axes a and b we find—

$$a = 20926348$$
$$b = 20855233$$
$$a : b = 294 \cdot 26 : 293 \cdot 26$$

To find the probable errors of these determinations, we remark that—

$$a \text{ involves } -\ 2093{\cdot}1\ u + 696{\cdot}3\ v$$

$$b \text{ involves } -\ 2086{\cdot}1\ u - 696{\cdot}3\ v$$

$$\tfrac{1}{2}\,\frac{a+b}{a-b} \text{ involves } -6\ v + \frac{3}{25}\,v^2$$

The probable error of $\alpha u + \beta v$ we have seen to be—

$$1{\cdot}335\left({\cdot}003801\ \alpha^2 + {\cdot}004412\ \alpha\beta + {\cdot}019038\ \beta^2\right)^{\frac{1}{2}}$$

which, by making the proper substitutions, gives—

$$a = 20926348 \pm 186$$

$$b = 20855233 \pm 239$$

$$\tfrac{1}{2}\,\frac{a+b}{a-b} = 293{\cdot}76 \pm 1{\cdot}06$$

The mean degree of the meridian is 20890805 into the arc of one degree: it involves 36·46 u; hence,

Mean degree = 364613·33 ± 3·00 feet.

from which it would follow that the length of the imaginary mètre = 39·378240 ± ·000324 inches.

§ VI.

Examination of the preceding Results.

We have now obtained results for the Figure of the Earth on two different hypotheses: in the first, the curve has been left altogether indeterminate as to the constants A, B, C; in the second, a relation has been assumed between the three constants such that the curve must be an ellipse. In the first hypothesis, the mean value of a correction to an observed latitude is ± 2″·064; in the second, the mean value is ± 2″·098. On examination of the corrections in the two tables, it appears that the English arc and the Indian arc are equally well represented on either hypothesis; the Russian arc is rather better represented by the elliptic corrections; but the French arc, especially towards its southern extremity, is not so well satisfied by the elliptic as by the non-elliptic curve. On the first hypothesis, the sum of the squares of the corrections to the six latitudes in the French arc is 30·513; on the second, it is 44·1207. This difference is due almost entirely to the station Montjouy, at which there is apparently a northerly deflection exceeding in amount the mean quantity. Upon the whole, the number of Stations in the southern part of this arc is too small to bear much weight against the elliptic hypothesis.

The following table shows the differences of the two systems of values, and from it will be seen how small is the difference of the two figures, as far as represented by observed latitudes:—

Stations.	$c—c'$	Stations.	$c—c'$
Formentera	+ 1″·308	Staro-Nekrassowka	+ 0″·176
Montjouy	+ 0·953	Wodolui	− 0·013
Barcelona	+ 0·950	Ssuprunkowzi	− 0·183
Carcassonne	+ 0·713	Kremenetz	− 0·298
Pantheon	+ 0·084	Belin	− 0·428
Dunkirk	− 0·089	Nemesch	− 0·525
		Jacobstadt	− 0·532
St. Agnes	− 0·005	Dorpat	− 0·482
Goonhilly	− 0·017	Hogland	− 0·383
Hensbarrow	− 0·043	Kilpi Maki	− 0·135
High Port Cliff	− 0·058	Tornea	+ 0·356
Week Down	− 0·058	Stuor-Oivi	+ 0·965
Boniface Down	− 0·059	Fuglenœs	+ 1·484
Dunnose	− 0·060		
Blackdown	− 0·065	Punnœ	− 0·821
Southampton	− 0·080	Putchapolliam	− 0·402
Greenwich	− 0·117	Dodagoontah	− 0·149
Precelly	− 0·146	Namthabad	+ 0·071
Cambridge	− 0·160	Daumergida	+ 0·296
Arbury	− 0·160	Takal Khera	+ 0·417
Delamere	− 0·206	Kalianpur	+ 0·422
Clifton	− 0·215	Kaliana	+ 0·167
South Berule	− 0·236		
Burleigh Moor	− 0·246	Trivandeporum	− 0·096
Durham	− 0·250	Paudree	+ 0·096
Ben Lomond	− 0·258		
Kellie Law	− 0·257	Trunz	+ 0·010
Ben Heynish	− 0·256	Königsberg	0·000
Great Stirling	− 0·238	Memel	− 0·010
Monach	− 0·206		
Ben Hutig	− 0·198	Tarqui	− 0·293
North Rona	− 0·169	Cotchesqui	+ 0·293
Balta	− 0·054		
Gerth of Scaw	− 0·048	Göttingen	+ 0·048
Saxavord	− 0·047	Altona	− 0·048
		Lauenburg	+ 0·020
		Lysabbel	− 0·020

In the non-elliptic curve the axes, mean degree, and compression are larger than the corresponding quantities in the elliptic, as may be seen by the following table:—

Quantities.	Elliptic.	Non-Elliptic.	Diff.
a	20926348 ± 186	20927197 ± 385	849
b	20855233 ± 239	20855493 ± 257	260
$\frac{1}{2}\frac{a+b}{a-b}$	293·76 ± 1·06	291·36 ± 1·39	2·40
Degree	364613·33 ± 3·00	364624·57 ± 5·36	11·24

The quantities in the elliptic curve are all much more accurately determined, for it is more easy to determine a curve of two parameters than one of three. By combining these axes as if they were independent results, we should obtain—

$$a = 20926509$$
$$b = 20855354$$
$$\frac{1}{2}\frac{a+b}{a-b} = 293\cdot60$$

All these linear results are in terms of O_1, and depending upon the assumed value of the toise, and also assuming the toise to be identical in the different Continental measures. These assumptions must be to a certain extent erroneous; and therefore let us suppose that to reduce them all to the Ordnance Standard, the distances in the different arcs require the multipliers,

French . . . . $1 + \frac{\theta_1}{100000}$

Russian . . . . $1 + \frac{\theta_2}{100000}$

Indian, South Portion . . . . . . $1 + \frac{\theta_3}{100000}$

Indian, First Arc . . . . $1 + \frac{\theta_4}{100000}$

Prussian . . . . $1 + \frac{\theta_5}{100000}$

Peruvian . . . . $1 + \frac{\theta_6}{100000}$

Hanoverian . . . . $1 + \frac{\theta_7}{100000}$

Danish . . . . $1 + \frac{\theta_8}{100000}$

The consequent increments to the different absolute terms of the expressions for the corrections to the latitudes will be as follows, omitting the multiplier μ, page 737, which differs from unity by a very small quantity:—

French.	Russian.	Indian, II.	Indian, I.	Prussian.	Peruvian.	Hanoverian.	Danish.
$-\cdot404\,\theta_1$	$+\cdot061\,\theta_2$	$-\cdot355\,\theta_3$	$+\cdot057\,\theta_4$	$+\cdot018\,\theta_5$	$+\cdot112\,\theta_6$	$+\cdot073\,\theta_7$	$+\cdot055\,\theta_8$
$-\cdot307\,\theta_1$	$+\cdot123\,\theta_2$	$-\cdot253\,\theta_3$		$+\cdot054\,\theta_5$			
$-\cdot306\,\theta_1$	$+\cdot172\,\theta_2$	$-\cdot181\,\theta_3$					
$-\cdot240\,\theta_1$	$+\cdot242\,\theta_2$	$-\cdot106\,\theta_3$					
$-\cdot038\,\theta_1$	$+\cdot336\,\theta_2$						
$+\cdot041\,\theta_1$	$+\cdot402\,\theta_2$						
	$+\cdot470\,\theta_2$						
	$+\cdot532\,\theta_2$						
	$+\cdot624\,\theta_2$						
	$+\cdot739\,\theta_2$						
	$+\cdot842\,\theta_2$						
	$+\cdot914\,\theta_2$						

The consequent increment of $\alpha u + \beta v$ is easily determined by means of the expression for that quantity previously given; it is found to be—

$$\begin{aligned}&+\left(-\cdot 018917\,\alpha - \cdot 002501\,\beta\right)\theta_1 + \left(-\cdot 030806\,\alpha + \cdot 062416\,\beta\right)\theta_2\\&+\left(-\cdot 012180\,\alpha - \cdot 038632\,\beta\right)\theta_3 + \left(-\cdot 000094\,\alpha - \cdot 000316\,\beta\right)\theta_4\\&+\left(-\cdot 000047\,\alpha + \cdot 000065\,\beta\right)\theta_5 + \left(-\cdot 000377\,\alpha - \cdot 001334\,\beta\right)\theta_6\\&+\left(-\cdot 000086\,\alpha + \cdot 000071\,\beta\right)\theta_7 + \left(-\cdot 000047\,\alpha + \cdot 000056\,\beta\right)\theta_8\end{aligned}$$

If in this expression we make $\alpha = -2093$, $\beta = +696$, we shall obtain the increment to a; and similarly, putting $\alpha = -2086$, $\beta = -696$, the increment to b will result: their values are—

$$\begin{aligned}da = &+ 37\cdot 85\,\theta_1 + 107\cdot 92\,\theta_2 - 1\cdot 40\,\theta_3 - 0\cdot 02\,\theta_4\\&+ 0\cdot 14\,\theta_5 - 0\cdot 14\,\theta_6 + 0\cdot 23\,\theta_7 + 0\cdot 14\,\theta_8\\db = &+ 41\cdot 20\,\theta_1 + 20\cdot 82\,\theta_2 + 52\cdot 30\,\theta_3 + 0\cdot 42\,\theta_4\\&+ 0\cdot 05\,\theta_5 + 1\cdot 71\,\theta_6 + 0\cdot 13\,\theta_7 + 0\cdot 06\,\theta_8\end{aligned}$$

We may get rid of O_1 as the unit of reference altogether, in the following manner: Let the distances we have adopted in the different arcs require the following multipliers to reduce them to one standard unit:—

English	$1 + \dfrac{\theta_0'}{100000}$
French	$1 + \dfrac{\theta_1'}{100000}$
Russian	$1 + \dfrac{\theta_2'}{100000}$
&c. &c.	&c.
Danish	$1 + \dfrac{\theta_8'}{100000}$

Comparing these multipliers with the former system—

$$\left(1 + \frac{\theta_n}{10000}\right)\left(1 + \frac{\theta_0'}{100000}\right) = 1 + \frac{\theta_n'}{100000}$$

$$\therefore\quad \theta_n = \theta_n' - \theta_0'$$

The value of a and b will receive the increments $209\cdot 26\,\theta_0'$ and $208\cdot 55\,\theta_0'$; wherefore—

$$\begin{aligned}a = 20926348 &+ 64\cdot 54\,\theta_0' + 37\cdot 85\,\theta_1' + 107\cdot 92\,\theta_2' - 1\cdot 40\,\theta_3' - 0\cdot 02\,\theta_4'\\&+ 0\cdot 14\,\theta_5' - 0\cdot 14\,\theta_6' + 0\cdot 23\,\theta_7' + 0\cdot 14\,\theta_8'\\b = 20855233 &+ 91\cdot 86\,\theta_0' + 41\cdot 20\,\theta_1' + 20\cdot 82\,\theta_2' + 52\cdot 30\,\theta_3' + 0\cdot 42\,\theta_4'\\&+ 0\cdot 05\,\theta_5' + 1\cdot 71\,\theta_6' + 0\cdot 13\,\theta_7' + 0\cdot 06\,\theta_8'\\\tfrac{1}{2}\frac{a+b}{a-b} = 293\cdot 76 &+ \cdot 1139\,\theta_0' + \cdot 0144\,\theta_1' - \cdot 3589\,\theta_2' + \cdot 2222\,\theta_3' + \cdot 0018\,\theta_4'\\&- \cdot 0004\,\theta_5' + \cdot 0076\,\theta_6' - \cdot 0004\,\theta_7' - \cdot 0003\,\theta_8'\end{aligned}$$

From these expressions we see at a glance the relative influence of the different units of measure, and also how small is the influence of the smaller arcs in the final results. Colonel Lambton's arc from Punnœ to Daumergida, about which there is some doubt as to the unit of measure, has unfortunately considerable influence in the determination of the ellipticity, and also of the minor axis.

If we combine Colonel Everest's arc from Daumergida to Kaliana with the English arc from St. Agnes to Saxavord, we get for u and v this equation—

$$0 = +\,51{\cdot}0759 + 56{\cdot}2645\,u - 9{\cdot}8335\,v$$
$$0 = -\,20{\cdot}1055 - 9{\cdot}8335\,u + 9{\cdot}7506\,v$$
$$\therefore\quad u = -\,0{\cdot}6645 \quad : \quad v = +\,1{\cdot}3918$$
$$\text{Weight of } \alpha u + \beta v = \left({\cdot}0216\,\alpha^2 + {\cdot}0435\,\alpha\beta + {\cdot}1245\,\beta^2\right)^{-1}$$

From which we obtain the value of the mean degree = 364623·50 ± 7·15 and $a : b =$ 292·39 : 291·39; probable error ± 2·67.

By combining the same Indian arc with the Russian arc, the relations for determining u and v are—

$$0 = +\,92{\cdot}8511 + 116{\cdot}3957\,u - 38{\cdot}9285\,v$$
$$0 = -\,42{\cdot}6110 - 38{\cdot}9285\,u + 24{\cdot}4891\,v$$
$$\therefore\quad u = -\,0{\cdot}4607 \quad : \quad v = +\,1{\cdot}0076$$
$$\text{Weight of } \alpha u + \beta v = \left({\cdot}0183\,\alpha^2 + {\cdot}0583\,\alpha\beta + {\cdot}0872\,\beta^2\right)^{-1}$$

From which we obtain the value of the mean degree = 364616·07 ± 6·59 and $a : b =$ 294·58 : 293·58; probable error ± 2·27.

The three arcs combined give the following equation:—

$$0 = +\,145{\cdot}9871 + 163{\cdot}3818\,u - 54{\cdot}9690\,v$$
$$0 = -\,61{\cdot}0593 - 54{\cdot}9690\,u + 30{\cdot}0692\,v$$
$$\therefore\quad u = -\,0{\cdot}5464 \quad : \quad v = +\,1{\cdot}0317$$
$$\text{Weight of } \alpha u + \beta v = \left({\cdot}0159\,\alpha^2 + {\cdot}0581\,\alpha\beta + {\cdot}0864\,\beta^2\right)^{-1}$$

From which we obtain the value of the mean degree = 364619·19 ± 6·14 and $a : b =$ 294·44 : 293·44; probable error ± 2·26.

Until the exact latitudes of the thirteen Stations in the Russian arc are known, we cannot state the precise elements best representing all geodetical operations. They cannot, however, be far from the following:—

Mean degree of meridian ······	364616 feet of Ord. Standard.
Ratio of semiaxes ············	293 : 294.

With respect to the Indian arc, an examination of the corrections to the observed latitudes, either those at page 764 or those at page 770, will show that there is no indication whatever of any disturbance from the Himalayan masses. If there had been a larger number of astronomical points in this arc—an observed latitude at each degree, for instance—they would have added very greatly to the weight of the determination of the earth's figure, and would, besides, have thrown much light upon the question of Himalayan influence. As it is, the points are so few and far between that the hypothesis of Archdeacon Pratt (at page 100 *Phil. Trans.* 1855) as to the non-appearance of any disturbing effect of the Himalayan masses has little weight. It may, however, be improved in the following manner: Making use of the points Daumergida, Kalianpur, and Kaliana only, we may determine for any assigned system of latitudes an ellipse which will make the latitudes accord exactly with the measured distances. Let λ_1 λ_2 λ_3 be the latitudes of these points as corrected to the mean ellipse, so that the observed latitudes are—

$$\lambda_1 - 0''{\cdot}05 \quad , \quad \lambda_2 + 3''{\cdot}16 \quad , \quad \lambda_3 - 1''{\cdot}81$$

Now by means of the two equations in u and v connecting the corrections to these three latitudes (page 767), it is easy to show by transformation that

$$\lambda_1 + \varepsilon_1 \quad , \quad \lambda_2 + \varepsilon_2 \quad , \quad \lambda_3 + \varepsilon_3$$

will give these values—

$$\begin{aligned} u &= -\,0{\cdot}3856 + 1{\cdot}8500\,\varepsilon_1 - 4{\cdot}4446\,\varepsilon_2 + 2{\cdot}5946\,\varepsilon_3 \\ v &= +\,1{\cdot}0620 - 3{\cdot}1098\,\varepsilon_1 + 6{\cdot}6056\,\varepsilon_2 - 3{\cdot}4958\,\varepsilon_3 \end{aligned}$$

From this it appears that (making $\varepsilon_1 = -\,0''{\cdot}05$; $\varepsilon_2 = +\,3''{\cdot}16$; $\varepsilon_3 = -\,1''{\cdot}81$) extremely erroneous values will result from the use of the *observed latitudes;* more erroneous, indeed, than if (making $\varepsilon_1 = +\,6''{\cdot}9$; $\varepsilon_2 = 12''{\cdot}0$; $\varepsilon_3 = 27''{\cdot}8$) we adopt the corrections for attraction assigned by Archdeacon Pratt. Let us therefore ascertain that ellipse whose deviation from the known figure of the earth ($u = -\,0{\cdot}3856$, $v = +\,1{\cdot}0620$) shall be a minimum, while it exhibits errors in the observed latitudes the sum of the squares of the differences of which from the computed deflections shall be also a minimum; that is, determine ε_1 ε_2 ε_3 such as to render the quantity

$$\begin{aligned} &\left(\varepsilon_1 - 6{\cdot}9\right)^2 + \left(\varepsilon_2 - 12{\cdot}0\right)^2 + \left(\varepsilon_3 - 27{\cdot}8\right)^2 \\ &+ \alpha\left(\quad 1{\cdot}8500\,\varepsilon_1 - 4{\cdot}4446\,\varepsilon_2 + 2{\cdot}5946\,\varepsilon_3\right)^2 \\ &+ \alpha\left(-\,3{\cdot}1098\,\varepsilon_1 + 6{\cdot}6056\,\varepsilon_2 - 3{\cdot}4958\,\varepsilon_3\right)^2 \end{aligned}$$

a minimum. The quantity α is arbitrary; by increasing it our ellipse will approach the mean ellipse, but the quantities ε_1 ε_2 ε_3 will increase their differences from the computed deflections, and *vice versâ*.

By making $\alpha = 1$ we get—

$$\varepsilon_1 = 5{\cdot}80 \quad ; \quad \varepsilon_2 = 16{\cdot}11 \quad ; \quad \varepsilon_3 = 24{\cdot}79$$

$$\begin{aligned} u &= -\,0{\cdot}3856 + 3{\cdot}448 = 3{\cdot}062 \\ v &= +\,1{\cdot}0620 + 1{\cdot}718 = 2{\cdot}780 \end{aligned}$$

If we make $\alpha = 5$ we get, $\epsilon_1 = 8{\cdot}04$; $\epsilon_2 = 16{\cdot}01$; $\epsilon_3 = 22{\cdot}65$; and,

$$u = -\,0{\cdot}3856 + 2{\cdot}483 = 2{\cdot}097$$
$$v = +\,1{\cdot}0620 + 1{\cdot}573 = 2{\cdot}635$$

Neither of these values of u and v differ very egregiously from the mean values. The corresponding corrections to the *observed* latitudes are as follows:—

$\alpha = 1$ $a = 20920328$ $a : b = 284{\cdot}7 : 283{\cdot}7$	″	$\alpha = 5$ $a = 20922247$ $a : b = 285{\cdot}5 : 284{\cdot}5$	″
Daumergida	+ 5·85	Daumergida	+ 8·09
Kalianpur	+ 12·95	Kalianpur	+ 12·85
Kaliana	+ 26·60	Kaliana	+ 24·46

The first of these sets of corrections is remarkably near to the computed results of attraction, while the curve differs very little from the mean figure of the earth; in fact, the value of a differs only 574 feet from the value adopted by Colonel Everest for the basis of the Indian Atlas. It would make the arc more convex, raising the middle parts by fifty feet. There are, however, no grounds for the supposition of an irregularity of the earth extending over such a surface; and if we take into consideration the station Takal Khera, it becomes the more improbable, as the latitude of that point will differ altogether from what it should become on the supposition of Himalayan influence.

The hypothesis of an irregularity in the form of the Indian arc is merely the supposition of the existence of another disturbing force whose action is opposed to that of the mountains. Having in itself no *à priori* probability, and being unsupported by evidence, we must have recourse to the hypothesis of the Astronomer Royal for the most probable explanation of this phenomenon. It must be admitted, however, that the value of this measure is considerably diminished since the investigations of Archdeacon Pratt, contained in the *Philosophical Transactions* for 1855.

The account of the Arc at the Cape of Good Hope is not yet sufficiently advanced to make it available in the determination of the Figure of the Earth. If the approximate data given at page 42 of the Professional Papers of the Corps of Royal Engineers (Vol. I.) be at all near the truth, the disturbances in this arc are extremely large: so much so that the measure would appear to be of little use, unless the number of observed latitudes has been considerably increased.

GEOGRAPHICAL POSITIONS

OF VARIOUS

LIGHTHOUSES

ON THE

COAST OF GREAT BRITAIN AND IRELAND.

5 F 2

Lighthouse.	Latitude.	Longitude W.
	° ′ ″	° ′ ″
Aberdeen	57 8 32·9	2 4 5·6
Aberystwith	52 24 48·9	4 5 24·2
Agnes (Saint) . . .	49 53 30·8	6 20 40·6
Air (mouth of Dee) . .	53 21 24·1	3 19 15·3
Annan River . . .	54 57 58·5	3 15 53·5
Arran (Galway Bay) .	53 7 38·3	9 42 6·3
Avon	51 30 2·4	2 42 15·1
Ayr (Firth of Clyde) .	55 28 8·7	4 38 11·0
Ayre Point (Isle of Man)	54 24 56·3	4 22 1·1
Bailey, S. E. Point of Howth Peninsula .	53 21 40·8	6 3 5·3
Bardsey	52 44 58·7	4 47 55·0
Barnaby Moor . . .	54 33 24·3	1 7 14·6
Barra Head	56 47 8·2	7 39 9·5
Beachy Head . . .	50 44 15·0	E. 0 12 57·8
Beeves	52 38 59·7	9 1 18·4
Bell Rock	56 26 4·2	2 23 6·9
Berwick	55 45 53·3	1 58 57·4
Bidstone	53 24 2·4	3 4 22·8
Black Rock (Liverpool)	53 26 38·7	3 2 27·6
Black Rock (Sligo Bay)	54 18 26·8	8 37 1·0
Bran Sands	54 37 41·3	1 8 17·5
Braunton Sands, Lower	51 4 16·9	4 12 19·1
Braunton Sands *or* Bideford	51 4 13·8	4 12 5·3
Buchanness	57 28 14·5	1 46 22·0
Budden Ness, Higher .	56 28 7·9	2 44 53·6
Budden Ness, Lower .	56 28 1·5	2 44 37·0
Burnham *or* Bridgewater	51 14 54·3	2 59 52·1
Caldy	51 37 52·1	4 40 59·5
Calf of Man, Upper . .	54 3 13·9	4 49 37·0
Cantyre	55 18 37·6	5 48 8·0
Cape Clear	51 26 2·3	9 29 2·7
Cape Wrath	58 37 33·1	4 59 52·3
Cardiff, East . . .	51 27 46·0	3 9 45·4
Cardiff, West . . .	51 27 46·0	3 9 47·4
Carlingford	54 1 11·2	6 4 40·7
Claro Island	53 49 38·1	9 58 58·3
Copeland	54 41 45·0	5 31 20·7
Coquet	55 20 1·8	1 32 17·4
Corsewall	55 0 25·4	5 9 29·8
Cromer	52 55 27·4	E. 1 19 5·6
Crosby	53 30, 49·9	3 3 45·2
Cumbrae, New . . .	55 43 16·2	4 57 56·8
Cumbrae, Old . . .	55 43 15·4	4 57 24·3
Dungeness	50 54 46·6	E. 0 58 18·2
Dunnet Head . . .	58 40 18·6	3 22 29·5
Eagle Island* . . .	54 16 59·9	10 5 32·3
Eddystone	50 10 49·4	4 15 53·4
Fannet	55 16 34·3	7 37 52·4
Farn Island, N. W. . .	55 36 59·5	1 39 20·7
Farn Island, S. W. . .	55 36 55·2	1 39 15·0
Flamborough, New . .	54 6 58·1	0 4 51·4
Flamborough, Old, *or* Telegraph Tower .	54 7 4·0	0 5 16·2

Lighthouse.	Latitude.	Longitude W.
	° ′ ″	° ′ ″
Flatholm	51 22 30·6	3 7 2·8
Fleetwood, New . . .	53 55 34·7	3 0 22·3
Fleetwood, Old . . .	53 55 42·1	3 0 27·9
Formby	53 32, 19·3	3 3 55·6
Happisburgh, Lower .	52 49 5·2	E. 1 32 56·0
Happisburgh, Upper .	52 49 12·2	E. 1 32 19·6
Hartlepool	54 41 47·0	1 10 26·7
Harwich	51 56 38·3	E. 1 17 25·1
Holyhead	53 18 50·3	4 37 7·6
Hook Tower	52 7 24·3	6 55 42·9
Howth	53 23 35·2	6 3 56·8
Hoylake	53 23 30·6	3 10 52·8
Hunstanton	52 56 57·2	E. 0 29 43·2
Hurst, East	50 42 26·4	1 32 56·7
Hurst, West	50 42 20·6	1 33 3·9
Ila Rhins	55 40 23·3	6 30 44·0
Inchkeith	56 2 0·8	3 8 4·9
Inisgort	53 49 35·4	9 40 12·5
Inistrahul	55 25 56·6	7 13 37·4
Isle of May	56 11 8·5	2 33 21·5
Isle of Glass	57 51 25·9	6 38 27·4
Kilcradan	52 34 46·8	9 42 34·1
Killingholm, High . .	53 38 48·8	0 13 1·7
Killingholm, Low . .	53 38 46·5	0 12 52·2
Kingstown (East Pier) .	53 18 7·1	6 7 30·8
Kinsale, Old Head . .	51 36 44·9	8 31 59·6
Kirkcudbright . . .	54 45 56·4	4 5 1·2
Leasowe	53 24 46·4	3 7 28·1
Lee Scar	54 51 46·0	3 24 43·2
Lismore	56 27 20·7	5 36 22·6
Lizard, East	49 57 34·5	5 12 4·0
Lizard, West . . .	49 57 34·1	5 12 7·3
Llanddwyn Point . .	53 8 5·7	4 24 51·8
Longstone	55 38 37·5	1 36 33·1
Loophead	52 33 38·2	9 55 54·7
Lowestoft	52 29 11·6	E. 1 45 27·6
Lundy	51 10 0·1	4 40 20·2
Maiden Rock, N. . .	54 55 47·7	5 44 17·4
Maiden Rock, S. . .	54 55 41·8	5 43 50·2
Maryport	54 43 0·3	3 30 21·2
Metal Man	54 18 13·3	8 34 30·3
Mull of Galloway . .	54 38 5·5	4 51 21·9
Mumbles	51 33 58·9	3 58 11·7
Mutton Island . . .	53 15 14·0	9 3 10·7
Nash Point, East . .	51 24 1·3	3 33 3·3
Nash Point, West . .	51 24 2·4	3 33 19·0
Needles	50 39 40·1	1 34 31·6
North Foreland . . .	51 22 27·6	E. 1 26 48·4
North Wall (Dublin) .	53 20 47·3	6 13 33·5
Orfordness, N. . . .	52 5 35·6	E. 1 35 12·4
Orfordness, S. . . .	52 5 0·2	E. 1 34 33·7
Paull	53 43 7·2	0 13 57·7
Peel	54 12 45·5	4 42 32·9
Pentland Skerries, Lower	58 41 25·1	2 55 23·1
Pentland Skerries, Upper	58 41 26·0	2 55 23·4

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* In the "List of Lighthouses" published by the Admiralty, the latitude of this point is erroneously printed 54° 7′ 0″.

Lighthouse.	Latitude.	Longitude W.	Lighthouse.	Latitude.	Longitude W.
	° ′ ″	° ′ ″		° ′ ″	° ′ ″
Pladda	55 25 30·5	5 7 1·7	St. Catherine, Old	50 35 29·7	1 18 5·9
Plymouth Breakwater	50 20 2·2	4 9 27·3	St. John's Point	54 13 34·2	5 39 30·2
Poolbeg	53 20 30·7	6 9 1·4	Stack (South)	53 18 23·4	4 41 54·5
Portland, North Bill	50 31 17·9	2 27 17·9	Staples, East	55 38 2·8	1 37 26·0
Rhins of Ila	55 40 23·3	6 30 44·0	Staples, West	55 37 1·3	1 39 14·9
Roche's Point (Queenstown)	51 47 33·4	8 15 14·6	Start Point (Devon)	50 13 18·4	3 38 28·2
Ronaldshay, North	59 23 5·2	2 22 10·2	Start Point (Orkney)	59 16 41·6	2 22 30·0
Seaham	54 50 19·8	1 19 33·5	Stornoway	58 11 29·9	6 22 9·7
Seaton	54 40 7·9	1 12 14·1	Sumburgh	59 51 16·8	1 16 22·7
Shields, North	55 0 31·0	1 26 10·3	Sunderland, No. 1	54 55 5·4	1 21 36·7
Skelligs, Great	51 46 5·8	10 32 28·8	Sunderland, No. 2	54 55 7·5	1 21 30·1
Skerries	53 25 15·4	4 36 25·8	Sunderland, No. 3	54 55 2·5	1 21 31·2
Skerryvore	56 19 23·9	7 6 45·2	Swansea	51 36 54·9	3 55 44·6
Skinburness	54 52 46·6	3 22 46·1	Tarbert	52 35 30·1	9 21 47·1
Slyne Head, N.	53 23 59·4	10 14 1·4	Tarbetness	57 51 55·2	3 46 30·7
Slyne Head, S.	53 23 55·5	10 13 58·9	Toward	55 51 43·7	4 58 42·8
Smalls	51 43 14·0	5 40 8·6	Trevose Head	50 32 55·2	5 2 2·9
South Foreland	51 8 23·3	E. 1 22 22·1	Troon	55 34 37·5	4 41 38·6
South Rock	54 23 56·5	5 25 4·1	Tuscar	52 12 9·3	6 12 21·9
Southampton Pier	50 53 41·7	1 24 22·7	Tynemouth	55 1 5·0	1 24 52·3
Southerness	54 52 21·9	3 35 37·4	Usk	51 32 24·5	2 59 33·3
Southsea Castle	50 46 38·9	1 5 14·5	Valentia	51 56 0·1	10 19 15·1
Spurn Head, High	53 34 41·4	.0 7 10·8	Walney Island	54 2 54·5	3 10 33·3
Spurn Head, Low	53 34 39·8	.0 7 16·7	Whitehaven, Pier Head	54 33 9·7	3 35 50·2
St. Agnes	49 53 30·8	6 20 40·6	Wicklow, Lower	52 57 54·5	5 59 58·6
St. Ann's, Lower	51 40 50·8	5 10 21·5	Wicklow, Upper	52 57 54·0	6 0 5·4
St. Ann's, Upper	51 40 55·1	5 10 28·2	Winterton	52 42 45·2	E. 1 41 49·9
St. Bees	54 30 48·3	3 38 8·1	Wyre River	53 57 12·5	3 1 46·3
St. Catherine, New	50 34 30·3	1 17 47·1	Ynys Gadarn	53 23 13·4	4 15 11·6

The positions of these points were calculated with the following approximate data—

$$a = 20923713\ ,\ a : b = 299{\cdot}33 : 298{\cdot}33$$

Latitude of Greenwich...... 51° 28′ 38″·30

In order to correct these results, if necessary: Let λ ω be the latitude and longitude calculated as above, and λ' ω' the values that would have resulted from the following data (see page 712)—

$$a = 20927005\ ,\ a : b = 280{\cdot}4 : 279{\cdot}4$$

Latitude of Greenwich...... 51° 28′ 40″·16

The corrections $\lambda' - \lambda$, and $\omega' - \omega$, are given in the following tables:—

VALUES OF $\lambda' - \lambda$.

Lat.	Longitude.											Lat.
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	
49°	+2″·80	+2″·80	+2″·82	+2″·84	+2″·86	+2″·90	+2″·95	+3″·00	+3″·06	+3″·13	+3″·21	49°
50	+2·45	+2·46	+2·47	+2·49	+2·52	+2·56	+2·60	+2·66	+2·72	+2·79	+2·87	50
51	+2·06	+2·07	+2·08	+2·10	+2·13	+2·17	+2·22	+2·27	+2·34	+2·41	+2·49	51
52	+1·63	+1·64	+1·65	+1·67	+1·70	+1·74	+1·79	+1·84	+1·91	+1·98	+2·06	52
53	+1·16	+1·17	+1·18	+1·20	+1·23	+1·27	+1·32	+1·37	+1·44	+1·51	+1·59	53
54	+0·65	+0·65	+0·67	+0·69	+0·72	+0·76	+0·81	+0·86	+0·93	+1·00	+1·08	54
55	+0·10	+0·10	+0·11	+0·13	+0·17	+0·21	+0·25	+0·31	+0·38	+0·45	+0·54	55
56	−0·50	−0·49	−0·48	−0·46	−0·43	−0·39	−0·34	−0·28	−0·21	−0·14	−0·05	56
57	−1·13	−1·12	−1·11	−1·09	−1·06	−1·02	−0·97	−0·91	−0·84	−0·77	−0·68	57
58	−1·80	−1·80	−1·78	−1·76	−1·73	−1·69	−1·64	−1·58	−1·51	−1·43	−1·35	58
59	−2·51	−2·50	−2·49	−2·47	−2·44	−2·39	−2·34	−2·29	−2·22	−2·14	−2·05	59
60	−3·26	−3·25	−3·24	−3·21	−3·18	−3·14	−3·09	−3·03	−2·96	−2·88	−2·80	60
61	−4·04	−4·03	−4·02	−4·00	−3·97	−3·92	−3·87	−3·81	−3·74	−3·66	−3·58	61

VALUES OF $\omega' - \omega$.

Lat.	Longitude.											Lat.
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	
49°	0″·00	−0″·97	−1″·95	−2″·92	−3″·89	−4″·87	−5″·83	−6″·80	− 7″·77	− 8″·73	− 9″·70	49°
50	0·00	−0·99	−1·98	−1·98	−3·97	−4·96	−5·95	−6·93	− 7·92	− 8·91	− 9·89	50
51	0·00	−1·01	−2·02	−3·04	−4·05	−5·06	−6·07	−7·08	− 8·08	− 9·09	−10·10	51
52	0·00	−1·04	−2·07	−3·10	−4·14	−5·17	−6·20	−7·23	− 8·26	− 9·29	−10·31	52
53	0·00	−1·06	−2·12	−3·17	−4·23	−5·28	−6·34	−7·39	− 8·45	− 9·50	−10·55	53
54	0·00	−1·08	−2·17	−3·25	−4·33	−5·41	−6·49	−8·57	− 8·65	− 9·72	−10·79	54
55	0·00	−1·11	−2·22	−3·33	−4·44	−5·54	−6·65	−7·75	− 8·86	− 9·96	−11·06	55
56	0·00	−1·14	−2·28	−3·41	−4·55	−5·68	−6·82	−7·95	− 9·08	−10·21	−11·34	56
57	0·00	−1·17	−2·34	−3·50	−4·67	−5·84	−7·00	−8·16	− 9·33	−10·49	−11·64	57
58	0·00	−1·20	−2·40	−3·60	−4·80	−6·00	−7·20	−8·39	− 9·59	−10·78	−11·97	58
59	0·00	−1·24	−2·47	−3·71	−4·94	−6·17	−7·41	−8·64	− 9·87	−11·09	−12·32	59
60	0·00	−1·27	−2·55	−3·82	−5·09	−6·36	−7·63	−8·90	−10·17	−11·43	−12·69	60
61	0·00	−1·31	−2·63	−3·94	−5·25	−6·56	−7·87	−9·18	−10·49	−11·79	−13·10	61

FINIS.

ERRATA AND CORRECTIONS.

Page 42, line 7 from bottom,	*for*	"Plate I."	*read*	"the Frontispiece.
60 „ 9 „ top,	„	"Ben Hulig"	„	"Ben Hutig."
90 „ 2 „ bottom, insert reading of R. O.				183° 23′ 30″·00
117, Ref. Obj. at Happisburg,	*for*	145 28 59·61	*read*	145 27 59·61
165 at Stede Hill,	„	104 12 1·93	„	284 12 1·93
198, line 3 from bottom,	„	1836	„	1846
199 „ 4 „ „	„	− 2″·31	„	− 2″·24
229 „ 6 „ top,	„	$\Delta^{\frac{3}{2}}$	„	Δ^2
247 „ 2 „ „	„	$\left(\frac{s}{v}\right)$	„	$\left(\frac{s}{v}\right)^3$
247 „ 2 „ bottom,	„	$\frac{\sin\frac{1}{2}(\alpha + \zeta - \alpha)}{\sin\frac{1}{2}(\alpha' + \zeta + \alpha)}$	„	$\frac{\sin\frac{1}{2}(\alpha' + \zeta - \alpha)}{\sin\frac{1}{2}(\alpha' + \zeta + \alpha)}$
248 „ 4 „ „	„	$\frac{\sin\frac{1}{2}(\alpha' - \alpha)}{\cos\frac{1}{2}(\alpha' + \alpha)}$	„	$\frac{\sin\frac{1}{2}(\alpha' - \alpha)}{\sin\frac{1}{2}(\alpha' + \alpha)}$
249 „ 5 „ „	„	$\frac{\cos\frac{1}{2}(\kappa - \theta)}{\cos\frac{1}{2}(\kappa - \theta)}$	„	$\frac{\cos\frac{1}{2}(\kappa - \theta)}{\cos\frac{1}{2}(\kappa + \theta)}$
251 „ 12 „ „	„	ζ	„	ξ
264 „ 2 „ „	„	(33)	„	(35)
266 „ 12 „ „	„	$u_1, \ v_1,$	„	$u, \ v,$
268 „ 12 „ „	„	$\frac{d\,\alpha_1}{d\,\alpha_1}$	„	$\frac{d\,\alpha_1'}{d\,\alpha_1}$
270 „ 4 „ „	„	289	„	259
396 „ 11 „ top,	„	GCB′	„	GC′B′
432, in Δ 16....	„	"Knockaskagh"	„	"Knocknaskagh."
435, in Δ 18....	„	+ 1·85	„	−1·85
440, in Δ 40....	„	422782·84	„	42278·28
445, in Δ 9....	„	158109·76	„	158109·67
488, in Δ 1....	„	5·00062260	„	5·00062460
558, Height of Boniface Down,	„	483·9	„	783·9
559 „ Knock,	„	1149·2	„	1416·9
619, line 9 from bottom,	„	$\{1 - e^2 \sin\frac{1}{2}(\lambda_1' + \lambda_1)\}$	„	$\{1 - e^2 \sin^2\frac{1}{2}(\lambda_1' + \lambda_1)\}^{\frac{3}{2}}$
624 „ 4 „ top,	„	$- h\,\theta \cos\alpha \sin(\alpha' - \alpha)$	„	$- h\,\theta \cos\alpha \sin(\alpha' - \alpha)$
693 „ 2 „ bottom,	„	$- 0{\cdot}1486\, v$	„	$+ 0{\cdot}0969\, v$
715 „ 12 „ top,	„	$-\frac{r^2}{a}$	„	$-\frac{r^2}{2a}$
„ „ 14 „ „	„	$-\frac{r^2}{a^2}\delta a + \ldots.$	„	$-\frac{r^2}{2a^2}\delta a + \ldots.$
„ „ 17 „ „	„	± 3·0	„	± 1·5

Page 198. The latitudes given at page 198 as resulting from observations made with Ramsden's Zenith Sector are affected with small errors: the correct results will be found at page 672.

Page 270. The last figures, *i.e.*, *tenth* decimals, in the small table page 270, are not exact.

Page 507. In the calculation of the distances of parallels between Clifton and Easington at the bottom of page 507, the errors of the *observed* azimuths, which were used, have produced an error of about − 2 feet: and in the next calculation, page 508, of the distance of the parallels of Easington and Burleigh Moor, a similar error, amounting to about + 1 foot, is produced; hence the distance of parallels of Dunnose and Burleigh Moor, as given at page 508, is about a foot too small in consequence of the errors with which the azimuths employed are affected. See page 685.

Page 558. The (recently) levelled height of Cowhythe is 254·4 feet.

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